CHAPTER
8

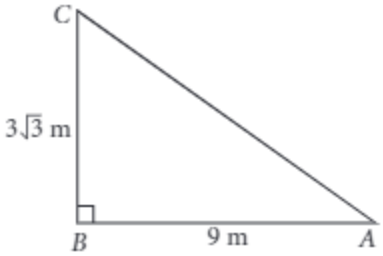
Introduction to Trigonometry

CASE STUDY / PASSAGE BASED QUESTIONS

1

Hide and Seek

Three friends – Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal have to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right angled triangle such that $AB = 9$ m, $BC = 3\sqrt{3}$ m and $\angle B = 90°$, then answer the following questions.



(i) The measure of $\angle A$ is

(a) 30° (b) 45° (c) 60° (d) None of these

(ii) The measure of $\angle C$ is

(a) 30° (b) 45° (c) 60° (d) None of these

(iii) The length of AC is

(a) $2\sqrt{3}$ m (b) $\sqrt{3}$ m (c) $4\sqrt{3}$ m (d) $6\sqrt{3}$ m

(iv) $\cos 2A =$

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

(v) $\sin\left(\frac{C}{2}\right) =$

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

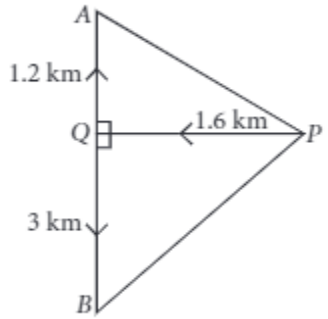
Syllabus

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined). Values of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios. Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given.

2

Two Flights

Two aeroplanes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes is 400 km/hr and 500 km/hr respectively. Considering PQ as runway and A and B are any two points in the path followed by two planes, then answer the following questions.



(i) Find $\tan\theta$; if $\angle APQ = \theta$.

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{4}$

(ii) Find $\cot B$.

(a) $\frac{3}{4}$ (b) $\frac{15}{4}$ (c) $\frac{3}{8}$ (d) $\frac{15}{8}$

(iii) Find $\tan A$.

(a) 2 (b) $\sqrt{2}$ (c) $\frac{4}{3}$ (d) $\frac{2}{\sqrt{3}}$

(iv) Find $\sec A$.

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

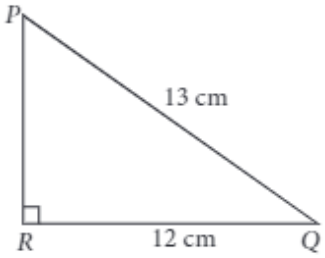
(v) Find $\text{cosec} B$.

(a) $\frac{17}{8}$ (b) $\frac{12}{5}$ (c) $\frac{5}{12}$ (d) $\frac{8}{17}$

3

Bird House

Anita, a student of class 10^{th}, has to made a project on 'Introduction to Trigonometry'. She decides to make a bird house which is triangular in shape. She uses cardboard to make the bird house as shown in the figure. Considering the front side of bird house as right angled triangle PQR, right angled at R, answer the following questions.



(i) If $\angle PQR = \theta$, then $\cos\theta =$

(a) $\frac{12}{5}$ (b) $\frac{5}{12}$ (c) $\frac{12}{13}$ (d) $\frac{13}{12}$

(ii) The value of $\sec\theta =$

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{13}{12}$ (d) $\frac{12}{13}$

(iii) The value of $\frac{\tan\theta}{1+\tan^2\theta} =$

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{60}{169}$ (d) $\frac{169}{60}$

(iv) The value of $\cot^2\theta - \text{cosec}^2\theta =$

(a) -1 (b) 0 (c) 1 (d) 2

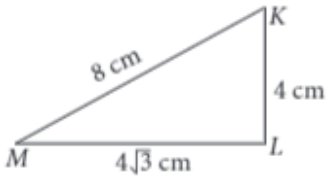
(v) The value of $\sin^2\theta + \cos^2\theta =$

(a) 0 (b) 1 (c) -1 (d) 2

4

Sandwich Making

Ritu's daughter is feeling so hungry and so thought to eat something. She looked into the fridge and found some bread pieces. She decided to make a sandwich. She cut the piece of bread diagonally and found that it forms a right angled triangle, with sides 4 cm, $4\sqrt{3}$ cm and 8 cm.



On the basis of above information, answer the following questions.

(i) The value of $\angle M =$

(a) 30° (b) 60° (c) 45° (d) None of these

(ii) The value of $\angle K =$

(a) 45° (b) 30° (c) 60° (d) None of these

(iii) Find the value of $\tan M$.

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) None of these

(iv) $\sec^2 M - 1 =$

(a) $\tan M$ (b) $\tan 2M$ (c) $\tan^2 M$ (d) None of these

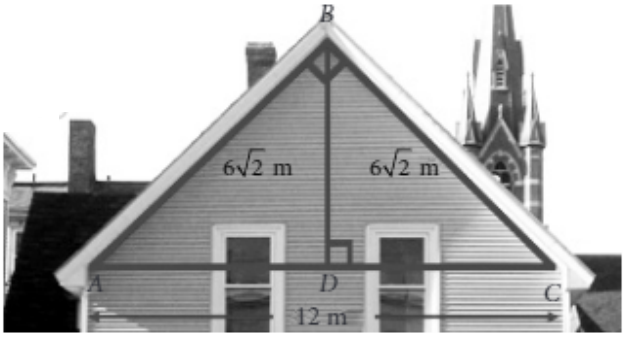
(v) The value of $\frac{\tan^2 45° - 1}{\tan^2 45° + 1}$ is

(a) 0 (b) 1 (c) 2 (d) -1

5

Roof Top of House

Aanya and her father go to meet her friend Juhi for a party. When they reached to Juhi's place, Aanya saw the roof of the house, which is triangular in shape. If she imagined the dimensions of the roof as given in the figure, then answer the following questions.



(i) If D is the mid point of AC, then BD =

(a) 2 m (b) 3 m (c) 4 m (d) 6 m

(ii) Measure of $\angle A$ =

(a) 30° (b) 60° (c) 45° (d) None of these

(iii) Measure of $\angle C$ =

(a) 30° (b) 60° (c) 45° (d) None of these

(iv) Find the value of $\sin A + \cos C$.

(a) 0 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

(v) Find the value of $\tan^2 C + \tan^2 A$.

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

HINTS & EXPLANATIONS

1. (i) (a): We have, $AB = 9$ m, $BC = 3\sqrt{3}$ m

In ΔABC, we have

$$\tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \tan A = \tan 30° \Rightarrow \angle A = 30°$ ...(1)

(ii) (c): Similarly, $\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$

$\Rightarrow \tan C = \tan 60° \Rightarrow \angle C = 60°$ ...(2)

(iii) (d): Since, $\sin A = \frac{BC}{AC} \Rightarrow \sin 30° = \frac{BC}{AC}$ [Using (1)]

$\Rightarrow \frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow AC = 6\sqrt{3}$ m

(iv) (b): $\because \angle A = 30°$ [From (1)]

$\therefore \cos 2A = \cos(2 \times 30°) = \cos 60° = \frac{1}{2}$

(v) (b): $\because \angle C = 60°$ [Using (2)]

$\therefore \sin\left(\frac{C}{2}\right) = \sin\left(\frac{60°}{2}\right) = \sin 30° = \frac{1}{2}$

2. (i) (d): In ΔAPQ, $\tan\theta = \frac{AQ}{PQ} = \frac{1.2}{1.6} = \frac{3}{4}$

(ii) (d): In ΔPBQ, $\cot B = \frac{QB}{PQ} = \frac{3}{1.6} = \frac{15}{8}$ ...(1)

(iii) (c): In ΔAPQ, $\tan A = \frac{PQ}{AQ} = \frac{1.6}{1.2} = \frac{4}{3}$ ...(2)

(iv) (d): We have, $\tan^2 A + 1 = \sec^2 A$

$\Rightarrow \sec A = \sqrt{\left(\frac{4}{3}\right)^2 + 1}$ [Using (2)]

$= \sqrt{\frac{16}{9} + 1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$

(v) (a): Since, $\text{cosec}B = \sqrt{\cot^2 B + 1}$

$= \sqrt{\left(\frac{15}{8}\right)^2 + 1}$ [Using (1)]

$= \frac{17}{8}$

3. $\because$ ΔPQR is a right angled triangle.

$\therefore$ $PR^2 + RQ^2 = PQ^2$

$\Rightarrow$ $PR^2 = (13)^2 - (12)^2 = 25 \Rightarrow PR = 5$ cm

(i) (c): $\cos\theta = \frac{QR}{PQ} = \frac{12}{13}$

(ii) (c): $\sec\theta = \frac{1}{\cos\theta} = \frac{13}{12}$

(iii) (c): $\tan\theta = \frac{PR}{RQ} = \frac{5}{12}$ ...(1)

$\therefore$ $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\frac{5}{12}}{1+\frac{25}{144}} = \frac{\frac{5}{12}}{\frac{169}{144}} = \frac{60}{169}$

(iv) (a): $\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$ [Using (1)]

$\text{cosec}\theta = \frac{PQ}{PR} = \frac{13}{5}$

$\therefore$ $\cot^2\theta - \text{cosec}^2\theta = \frac{144}{25} - \frac{169}{25} = -1$

(v) (b): $\sin^2\theta + \cos^2\theta = 1$ (Using identity)

4. We have, $KL = 4$ cm, $ML = 4\sqrt{3}$ cm, $KM = 8$ cm

(i) (a): $\tan M = \frac{KL}{LM} = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan M = \tan 30^\circ \Rightarrow \angle M = 30^\circ$

(ii) (c): $\tan K = \frac{ML}{KL} = \frac{4\sqrt{3}}{4} = \sqrt{3} = \tan 60^\circ$

$\Rightarrow$ $\angle K = 60^\circ$

(iii) (b) (iv) (c)

(v) (a): $\frac{\tan^2 45^\circ - 1}{\tan^2 45^\circ + 1} = \frac{(1)^2 - 1}{1^2 + 1} = \frac{0}{2} = 0$

5. We have, $AB = BC = 6\sqrt{2}$ m
and $AC = 12$ m.

(i) (d): $\because$ D is mid point of AC.

$\therefore$ $AD = DC = 6$ m

Now, $AB^2 = BD^2 + AD^2$ ($\because$ ΔABD is a right triangle)

$\Rightarrow$ $BD^2 = (6\sqrt{2})^2 - 6^2 = 72 - 36 = 36$

$\Rightarrow$ $BD = 6$ m ...(1)

(ii) (c): In ΔABD, $\sin A = \frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$ [Using (1)]

$\Rightarrow$ $\sin A = \sin 45^\circ \Rightarrow \angle A = 45^\circ$

(iii) (c): In ΔBDC, $\tan C = \frac{BD}{DC} = \frac{6}{6}$ [Using (1)]

$\Rightarrow$ $\tan C = 1 = \tan 45^\circ \Rightarrow \angle C = 45^\circ$

(iv) (d): $\sin A = \frac{1}{\sqrt{2}}$, $\cos C = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$\therefore$ $\sin A + \cos C = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

(v) (c): $\tan C = 1$, $\tan A = \tan 45^\circ = 1$

$\Rightarrow$ $\tan^2 C + \tan^2 A = 1 + 1 = 2$