1 MARK

1. If $x \neq 0$, $\frac{x}{2} = y^2$ and $\frac{x}{4} = 4y$, then x equals -

(1) 8 (2) 16 (3) 32 (4) 128

2. When x^5, $x + \frac{1}{x}$ and $1 + \frac{2}{x} + \frac{3}{x^3}$ are multiplied, the product is a polynomial of degree

(1) 2 (2) 3 (3) 6 (4) 7

3. Let $f(x) = \frac{x+1}{x-1}$. Then for $x^2 \neq 1$, $f(-x)$ is

(1) $\frac{1}{f(x)}$ (2) $-f(x)$ (3) $\frac{1}{f(-x)}$ (4) $-f(-x)$

4. If xy = a, xz = b and yz = c and none of these quantities is 0, then $x^2 + y^2 + z^2$ equals

(1) $\frac{ab+ac+bc}{abc}$ (2) $\frac{a^2+b^2+c^2}{abc}$

(3) $\frac{(a+b+c)^2}{abc}$ (4) $\frac{(ab)^2+(ac)^2+(bc)^2}{abc}$

5. If $60^a = 3$ and $60^b = 5$, then $12^{[(1-a-b)/2(1-b)]}$ is

(1) $\sqrt{3}$ (2) 2 (3) $\sqrt{5}$ (4) $\sqrt{12}$

6. If x, y and $y - \frac{1}{x}$ are not 0, then $\frac{x-\frac{1}{y}}{y-\frac{1}{x}}$ equals

(1) 1 (2) x/y (3) y/x (4) $\frac{x}{y} - \frac{y}{x}$

7. The product of all real roots of the equation $x^{\log_{10} x} = 10$ is

(1) 1 (2) –1 (3) 10 (4) 10^{-1}

8. If 2x + 1 = 8 , then 4x + 1 =

(1) 15 (2) 16 (3) 17 (4) 18

9. Let a, a', b, b' be real numbers with a and a' nonzero. The solution to ax + b = 0 is less than the solution to a'x + b' = 0 if and only if

(1) a'b < ab' (2) ab' < a'b

(3) ab < a'b' (4) $\frac{b'}{a'} < \frac{b}{a}$

10. [x – (y – z)] – [(x–y) – z] =

(1) 2y (2) 2z (3) –2y (4) –2z

11. Suppose hops, skips and jumps are specific units of length. If b hops equals c skips, d jumps equals e hops, and f jumps equals g meters, then one meter equals how many skips ?

(1) $\frac{bdg}{cef}$ (2) $\frac{cdf}{beg}$ (3) $\frac{cdg}{bef}$ (4) $\frac{cef}{bdg}$

12. Let $p(x) = x^2 + bx + c$, where b and c are integers. If p(x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, what is p(l) ?

(1) 0 (2) 1

(3) 2 (4) 4

13. $(1+x^2)(1-x^3)$ equals

(1) $1-x^5$ (2) $1-x^6$

(3) $1+x^2-x^3$ (4) $1+x^2-x^3-x^5$

14. If a – 1 = b + 2 = c – 3 = d + 4, which of the four quantities a,b,c,d is the largest?

(1) a (2) b (3) c (4) d

15. If (x,y) is a solution to the system xy = 6 and $x^2y + xy^2 + x + y = 63$, find $x^2 + y^2$

(1) 13 (2) $\frac{1173}{32}$ (3) 55 (4) 69

16. It takes A algebra books (all the same thickness) and H geometry books (all the same thickness, which is greater than that of an algebra book) to completely fill a certain shelf. Also, S of the algebra books and M of the geometry books would fill the same shelf. Finally, E of the algebra books alone would fill this shelf. Given that A, H, S, M, E are distinct positive integers, it follows that E is

(1) $\frac{AM+SH}{M+H}$ (2) $\frac{AM^2+SH^2}{M^2+H^2}$

(3) $\frac{AH-SM}{M-H}$ (4) $\frac{AM-SH}{M-H}$

17. Evaluate $\log_{10}(\tan 1°) + \log_{10}(\tan 2°) + \log_{10}(\tan 3°) + ... + \log_{10}(\tan 88°) + \log_{10}(\tan 89°)$

(1) 0 (2) $\frac{1}{2}\log_{10}\left(\frac{1}{2}\sqrt{3}\right)$

(3) $\frac{1}{2}\log_{10} 2$ (4) 1

18. If p is a prime and both roots of $x^2 + px - 444p = 0$ are integers, then

(1) $1 < p \le 11$ (2) $11 < p \le 21$

(3) $21 < p \le 31$ (4) $31 < p \le 21$

19. If b and c are constants and $(x+2)(x+b)=x^2 + cx + 6$, then c is

(1) –5 (2) –3

(3) –1 (4) 5

20. If a and b are integers such that $x^2 - x - 1$ is a factor of $ax^3 + bx^2 + 1$, then b is

(1) –2 (2) –1 (3) 0 (4) 1

21. Suppose that p and q are positive numbers for which $\log_9(p) = \log_{12}(q) = \log_{16}(p+q)$. What is the value of q/p?

(1) 4/3 (2) $\frac{1}{2}(1+\sqrt{3})$

(3) 8/5 (4) $\frac{1}{2}(1+\sqrt{5})$

22. If $a + b = 1$ and $a^2 + b^2 = 2$ then $a^3 + b^3$ equals-

(1) 4 (2) $3\frac{1}{2}$ (3) 3 (4) $2\frac{1}{2}$

23. Which of the following is a factor of $ab(c^2 + d^2) + cd(a^2 + b^2)$?

(1) ab (2) $c^2 + d^2$

(3) ab + cd (4) ac + bd

24. If n is a real number, then the simultaneous system to the right has no solution if and only if *n* is equal to -

$nx + y = 1,\ ny + z = 1,\ x + nz = 1$

(1) –1 (2) 0

(3) 1 (4) 0 or 1

25. If $x \ne 0$, or 4 and $y \ne 0$ or 6, then $\frac{2}{x}+\frac{3}{y}=\frac{1}{2}$ is equivalent to -

(1) $4x + 3y = xy$ (2) $y=\frac{4x}{6-y}$

(3) $\frac{x}{2}+\frac{y}{3}=3$ (4) $\frac{4y}{y-6}=x$

26. What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

(1) 0 (2) 1

(3) 49 (4) 50

27. For positive real numbers x and y define $x * y=\frac{x-y}{x+y}$; then -

(1) "*" is commutative but not associative

(2) "*" is associative but not commutative

(3) "*" is neither commutative nor associative

(4) "*" is commutative and associative

28. What is the smallest integral value of k such that $2x(kx - 4) - x^2 + 6 = 0$ has no real roots ?

(1) –1 (2) 2 (3) 3 (4) 4

29. If $g(x) = 1 - x^2$ and $f(g(x)) = \frac{1-x^2}{x^2}$ when $x \ne 0$ then $f(1/2)$ equals -

(1) 3/4 (2) 1

(3) 3 (4) $\sqrt{2}/2$

30. Which statement is correct ?

(1) If $x < 0$, then $x^2 > x$.

(2) If $x^2 > 0$, then $x > 0$

(3) If $x^2 > x$, then $x > 0$.

(4) If $x^2 > x$, then $x < 0$.

31. If $x < -2$ then $|1 - |1 + x||$ equals -

(1) 2 + x (2) –2–x (3) x (4) –x

32. If $f(x) = 3x + 2$ for all real x, then the statement "$|f(x) + 4| < a$ whenever $|x + 2| < b$ and $a > 0$ and $b > 0$" is true when

(1) $b \le a/3$ (2) $b > a/3$

(3) $a \le b/3$ (4) $a > b/3$

33. Which of the following inequalities are satisfied for all numbers a,b,c,x,y,z which satisfy the conditions $x < a < y < b$, and $z < c$?

I. $xy + yz + zx < ab + bc + ca$

II. $x^2 + y^2 + z^2 < a^2 + b^2 + c^2$

III. $xyz < abc$

(1) None are satisfied (2) I only

(3) II only (4) III only

34. For which non-zero real numbers x is $\frac{|x-|x||}{x}$ a positive integer ?

(1) for negative x only

(2) for positive x only

(3) only for x and even integer

(4) for no non-zero real numbers x

35. If $a \neq b$, $a^3 - b^3 = 19x^3$ and $a - b = x$, which of the following conclusions is correct ?

(1) $a = 3x$ (2) $a = 3x$ or $a = -2x$
(3) $a = -3x$ or $a = 2x$ (4) $a = 3x$ or $a = 2x$

36. The equation $x^6 - 3x^5 - 6x^3 - x + 8 = 0$ has -

(1) no real roots
(2) exactly two distinct negative roots
(3) exactly one negative root
(4) no negative roots, but at least one positive root

37. Which positive numbers x satisfy the equation $(\log_3 x)(\log_3 5) = \log_3 5$?

(1) 3 only
(2) 3, 5 and 15 only
(3) only numbers of the form $5^n.3^m$. where n and m are positive integers
(4) all positive x = 1

38. Suppose $f(x)$ is defined for all real numbers x; $f(x) > 0$ for all x; and $f(a)\,f(b) = f(a + b)$ for all a and b. Which of the following statements are true ?

I. $f(0) = 1$ II. $f(-a) = 1/f(a)$ for all x
III. $f(a) = \sqrt[3]{f(3a)}$ for all a
IV. $f(b) > f(a)$ if $b > a$

(1) III and IV only (2) I,III and IV only
(3) I, II and IV only (4) I, II and III only

39. If p and q are prime and $x^2 - px + q = 0$ has distinct positive integral roots, then which of the following statements are true ?

I. The difference of the roots is odd.
II. At least one root is prime.
III. $p^2 - q$ is prime

(1) I only (2) II only
(3) II and III only (4)All are true

40. If one minus the reciprocal of $(1-x)$ equals the reciprocal of $(1-x)$ then x equals-

(1) –2 (2) –1 (3) 1/2 (4) 1

41. If c is a real number and the negative of one of the solutions of $x^2 - 3x + c = 0$ is a solution of $x^2 + 3x - c = 0$ then solutions of $x^2 - 3x + c = 0$ are-

(1) 1,2 (2) –1,–2
(3) 0,3 (4) 0,–3

42. If x is a real number, then the quantity $(1 - |x|)(1+x)$ is positive if and only if -

(1) $|x| < 1$ (2) $x < 1$
(3) $|x| > 1$ (4) $x < -1$ or $-1 < x < 1$

43. If x cows give x + 1 cans of milk in x + 2 days how many days will it take x + 3 cows to give x + 5 cans of milk ?

(1) $\frac{x(x+2)(x+5)}{(x+1)(x+3)}$

(2) $\frac{x(x+1)(x+5)}{(x+2)(x+3)}$

(3) $\frac{(x+1)(x+3)(x+5)}{x(x+2)}$

(4) $\frac{(x+1)(x+3)}{x(x+2)(x+5)}$

44. A polynomial p(x) has reminder three when divided by x -1 and remainder five when divided by x-3. The remainder when p(x) is divided by (x-1)(x-3) is-

(1) x – 2 (2) x + 2 (3) 2 (4) 8

45. Let a,b and x be positive real numbers distinct from one. Then

$4(\log_a x)^2 + 3(\log_b x)^2 = 8(\log_a x)(\log_b x)$

(1) for all values of a,b and x
(2) if and only if $a = b^2$
(3) if and only if $b = a^2$
(4) none of these

46. If y = 2x and z = 2y then x + y + z equals -

(1) x (2) 3x (3) 5x (4) 7x

47. If x, y and $2x + \frac{y}{2}$ are not zero, then

$\left(2x + \frac{y}{2}\right)^{-1}\left[(2x)^{-1} + \left(\frac{y}{2}\right)^{-1}\right]$ equals

(1) 1 (2) xy^{-1}
(3) $x^{-1}y$ (4) none of these

48. For every triple (a, b, c) of non-zero real numbers, form the number

$\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}$.

The set of all numbers formed is

(1) {0} (2) {–4, 0, 4}
(3) {–4,–2,0,1,4} (4) (–4, –2, 2, 4}

49. If $y = (\log_2 3)(\log_3 4)$ ...... $(\log_n[n + 1])$ .... $(\log_{31} 32)$ then

(1) $4 < y < 5$ (2) $y = 5$

(3) $5 < y < 6$ (4) $y = 6$

50. For how many values of the coefficient a do the equations $x^2 + ax + 1 = 0$, $x^2 - x - a = 0$ have a common real solution ?

(1) 0 (2) 1 (3) 2 (4) 3

51. If $f(a)$ is a real valued function of the real variable x and $f(x)$ is not identically zero, and for all a and b $f(a + b) + f(a - b) = 2f(a) + 2f(b)$, then for all x and y.

(1) $f(0) = 1$ (2) $f(-x) = -f(x)$

(3) $f(-x) = f(x)$ (4) $f(x + y) = f(x) + f(y)$

52. If the solutions of the equation $x^2 + px + q = 0$ are the cubes of the solutions of the equation $x' + mx + n = 0$, then

(1) $p = m^3 + 3mn$ (2) $p = m^3 - 3mn$

(3) $p + q = m^3$ (4) $\left(\frac{m}{n}\right)^3 = \frac{p}{q}$

53. Find the smallest integer n such that $(x^2 + y^2 + z^2)^2 \le n(x^4 + y^4 + z^4)$ for all real numbers x,y and z.

(1) 2 (2) 3 (3) 4 (4) 6

54. If $1 - \frac{4}{x} + \frac{4}{x^2} = 0$, then $\frac{2}{x}$ equals -

(1) –1 (2) 1 (3) 2 (4) –1 or 2

55. For all non-zero numbers x and y such that $x = 1/y$. $\left(x - \frac{1}{y}\right)\left(y + \frac{1}{y}\right)$ equals-

(1) $2x^2$ (2) $2y^2$

(3) $x^2 + y^2$ (4) $x^2 - y^2$

56. If a = 1, b = 10, c = 100 and d = 1000, then $(a + b + c - d) + (a + b - c + d) + (a - b + c + d) + (-a + b + c + d)$ is equal to

(1) 1111 (2) 2222

(3) 3333 (4) 1212

57. If $x < 0$, then $\left|x - \sqrt{(x-1)^2}\right|$ equals-

(1) 1 (2) $1 - 2x$

(3) $-2x - 1$ (4) $1 + 2x$

58. If a,b,c and d are non-zero numbers such that c and d are the solutions of $x^2 + ax + b = 0$ and a and b are the solutions of $x^2 + cx + d = 0$, then a + b + c + d equals

(1) 0 (2) –2 (3) 2 (4) 4

59. If an integer n, greater than 8, is a solution of the equation $x^2 - ax + b = 0$ and the representation of a in the base n numeration system is 18, then the base n representation of b is-

(1) 18 (2) 28

(3) 80 (4) 81

60. If k is a positive number and f is a function such that, for every positive number x. $\left[f(x^2+1)\right]^{\sqrt{x}} = k$; then, for every positive number y, $\left[f\left(\frac{9+y^2}{y^2}\right)\right]^{\sqrt{\frac{12}{y}}}$ is equal to -

(1) $\sqrt{k}$ (2) 2k

(3) $k\sqrt{k}$ (4) k^2

61. If a,b,c are non-zero real numbers such that $\frac{a+b-c}{c} = \frac{a-b+c}{b} = \frac{-a+b+c}{a}$ and $x = \frac{(a+b)(b+c)(c+a)}{abc}$, then x equals -

(1) 8 (2) 6 (3) 4 (4) 2

62. For all positive numbers x distinct from 1. $\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x}$ equals -

(1) $\frac{1}{\log_{60} x}$

(2) $\frac{1}{\log_x 60}$

(3) $\frac{1}{(\log_3 x)(\log_4 x)(\log_5 x)}$

(4) $\frac{12}{(\log x) + (\log x) + (\log x)}$

63. To the nearest thousandth. $\log_{10}2$ is .301 and $\log_{10}3$ is 0.477. Which of the following is the best approximation of $\log_5 10$?

(1) 8/7 (2) 9/7

(3) 10/7 (4) 11/7

64. Find the sum of the squares of all real numbers satisfying the equation : $x^{256} - 256^{32} = 0$.

(1) 8 (2) 128

(3) 512 (4) 65.536

65. A positive number x satisfies the inequality $\sqrt{x} < 2x$ if and only if :-

(1) $x > \frac{1}{4}$ (2) $x > 2$ (3) $x > 4$ (4) $x < \frac{1}{4}$

66. The sum $\sqrt[3]{5+2\sqrt{13}}+\sqrt[3]{5-2\sqrt{13}}$ equals :-

(1) $\frac{3}{2}$ (2) $\frac{\sqrt[3]{65}}{4}$

(3) $\frac{1+\sqrt[6]{13}}{2}$ (4) none of these

67. For $x \neq 0$, $\frac{1}{x}+\frac{1}{2x}+\frac{1}{3x}$ equals

(1) $\frac{1}{2x}$ (2) $\frac{1}{6x}$ (3) $\frac{5}{6x}$ (4) $\frac{11}{6x}$

68. If $\frac{x}{x-1}=\frac{y^2+2y-1}{y^2+2y-2}$, then x equals

(1) $y^2 + 2y - 1$ (2) $y^2 + 2y - 2$

(3) $y^2 + 2y + 2$ (4) $y^2 + 2y + 1$

69. For all positive numbers x, y, z the product

$$\left(\frac{1}{x+y+z}\right)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\left(\frac{1}{xy+yz+zx}\right)\left(\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}\right)$$

equals

(1) $x^{-2}y^{-2}z^{-2}$ (2) $x^{-2} + y^{-2} + z^{-2}$

(3) $(x + y + z)^{-2}$ (4) $\frac{1}{xyz}$

70. If a, b, c, d are the solutions of the equation $x^4 - bx - 3 = 0$. then an equation whose solutions are $\frac{a+b+c}{d^2}, \frac{a+b+d}{c^2}, \frac{a+c+d}{b^2}, \frac{b+c+d}{a^2}$

(1) $3x^4 + bx - 1 = 0$ (2) $3x^4 - bx + 1 = 0$

(3) $3x^4 + bx^3 - 1 = 0$ (4) $3x^4 - bx^3 - 1 = 0$

71. When the polynomial x^3-2 is divided by the polynomial $x^2 - 2$, the remainder is :-

(1) 2 (2) –2 (3) –2x–2 (4) 2x + 2

72. Two positive numbers x and y are in the ratio a : b. where $0 < a < b$. If $x + y = c$. then the smaller of x and y is :-

(1) $\frac{ac}{b}$ (2) $\frac{bc-ac}{b}$ (3) $\frac{ac}{a+b}$ (4) $\frac{bc}{a+b}$

73. If the operation $x* y$ is defined by $x* y = (x + 1) (y + 1)- 1$, then which one of the following is false ?

(1) $x*y = y* x$ for all real x and y.

(2) $x* (y + z) - (x * y) + (x * z)$ for all real x,y, and z

(3) $(x - 1)*(x + 1) = (x * x) -1$ for all real x.

(4) $x * 0 = z$ for all real x.

74. Let $f(x) = ax^7 + bx^3 + cx - 5$, where a. b and c are constants. If $f(-7) = 7$, then f(7) equals

(1) –17 (2) –7

(3) 14 (4) 21

75. If $a > 1$, $b > 1$ and $p=\frac{\log_b(\log_b a)}{\log_b a}$ then a^p equals :-

(1) 1 (2) b

(3) $\log_a b$ (4) $\log_b a$

76. Let |Z| denote the greatest integer not exceeding x Let x and y satisfy the simultaneous equations : $y = 2|x| +3$, $y = 3|x-2| + 5$.

If x is not an integer, then x + y is

(1) an integer (2) between 4 and 5

(3) between –4 and 4 (4) between 15 and 16

77. How many real numbers x satisfy the equation $3^{2x+2} - 3^{x+3} - 3^x + 3 = 0$?

(1) 0 (2) 1 (3) 2 (4) 3

78. Let $f(x) = |x - 2| + |x - 4| - |2x - 6|$. for $2 \leq x \leq 8$. The sum of the largest and smallest value of f(x) is :-

(1) 1 (2) 2 (3) 4 (4) 6

79. If $x^2 + x + 1 = 0$, then the value of $x^{2002} + x^{2003}$ is :-

(1) – 1 (2) 1 (3) 2 (4) –2

80. On simplifying $\frac{999813\times 999815+1}{(999814)^2}$, we get :-

(1) 1 (2) 2 (3) 4 (4) 4

81. The value of y which will satisfy the equations $2x^2 + 6x + 5y + 1 = 0$ and $2x + y + 3 = 0$ may be found by solving :-

(1) $y^2 + 14y - 7 = 0$ (2) $y^2 + 8y + 1 = 0$

(3) $y^2 + 10y - 7 = 0$ (4) $y^2 + y - 12 = 0$

82. a, b, c, d are real numbers.
If $a^2 + b^2 + c^2 + d^2 = ab + bc + cd + da$ then

(1) $a = b = c = d$ (2) $a = b$ but $c \neq d$

(3) $a = c$ but $b \neq d$ (4) $a = d$ but $b \neq c$

83. If $(am + bn + m)^2 - (am + bn + n)^2 = (m - n)^2$ with $m \neq n$, m, $n \neq 0$, then :-

(1) $a = 0, b = -1$ (2) $a = 0, b = 1$

(3) $a = 1, b = 0$ (4) $a = -1, b = 0$

84. The sum of the squares of the roots of the equation $x^3 - x^2 - 2x + 2 = 0$ is :-

(1) 1 (2) 3 (3) 5 (4) 7

85. What is the smallest positive integer n such that $\sqrt{n} - \sqrt{n-1} < .01$?

(1) 2499 (2) 2500

(3) 2501 (4) 10,000

2 MARKS

1. If $\log_7 (\log_3(\log_2 x) = 0$, then $x^{-1/2}$ equals

(1) $\frac{1}{3}$ (2) $\frac{1}{2\sqrt{3}}$

(3) $\frac{1}{3\sqrt{3}}$ (4) none of these

2. Let f be a polynomial function such that, for all real x, $f(x^2 + 1) = x^4 + 5x^2 + 3$. For all real x, $f(x^2 - 1)$ is

(1) $x^4 + 5x^2 + 1$ (2) $x^4 + x^2 - 3$

(3) $x^4 - 5x^2 + 1$ (4) $x^4 + x^2 + 3$

3. If $\tan \alpha$ and $\tan \beta$ are the roots of $x^2 - px + q = 0$, and $\cot \alpha$ and $\cot \beta$ are the roots of $x^2 - rx + s = 0$, then rs is necessarily

(1) pq (2) 1/pq (3) p/q^2 (4) q/p^2

4. Consider the two functions $f(x) = x^2 + 2bx + 1$ and $g(x) = 2a(x + b)$. Where the variable x and the constants a and b are real numbers. Each pair of constants a and b may be considered as a point (a, b) in an ab-plane. Let S be the set of such points (a,b) for which the graphs of $y = f(x)$ and $y = g(x)$ do not intersect (in the xy-plane). The area of S is

(1) 1 (2) π (3) 4 (4) 4π

5. The function f(x) satisfies $f(2+x) = f(2-x)$ for all real numbers x. If the equation $f(x) = 0$ has exactly four distinct real roots, then the sum of these roots is

(1) 0 (2) 2 (3) 4 (4) 8

6. The number of triples (a,b,c) of positive integers which satisfy the simultaneous equations $ab + bc = 44$, $ac + bc = 23$, is

(1) 0 (2) 1 (3) 2 (4) 3

7. If a and b are positive real numbers and each of the equations $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ has real roots, then the smallest possible value of a + b is

(1) 2 (2) 3 (3) 4 (4) 6

8. How many integers x satisfy the equation $(x^2 - x - 1)^{x+2} = 1$?

(1) 2 (2) 3 (3) 4 (4) 5

9. Let [x] be the greatest integer less than or equal to x. Then the number of real solutions to $4x^2 - 40[x] + 51 = 0$ is

(1) 0 (2) 1 (3) 2 (4) 4

10. The sum of the greatest integer less than or equal to x and the least integer greater than or equal to x is 5. The solution set for x is

(1) 5/2 (2) $2 < x \leq 3$

(3) $2 \leq x \leq 3$ (4) $2 \leq x < 3$

11. A student attempted to compute the average A, of x, y and z by computing the average of x and y, and then computing the average of the result and z. Whenever $x < y < z$, the student's final result is

(1) Correct

(2) Always less than A

(3) Always greater than A

(4) Sometimes less than A and sometimes equal to A

12. How many polynomial functions f of degree ≥ 1 satisfy $f(x^2) = [f(x)]^2 = f(f(x))$?

(1) 0

(2) 1

(3) 2

(4) finitely man but more than 2

13. If $|x| + x + y = 10$ and $x + |y| - y = 12$, find x + y.

(1) -2 (2) 2

(3) 18/5 (4) 22/3

14. Simplify

$$\frac{bx(a^2x^2 + 2a^2y^2 + b^2y^2) + ay(a^2x^2 + 2b^2x^2 + b^2y^2)}{bx + ay}$$

(1) $a^2x^2 + b^2y^2$ (2) $(ax+by)^2$

(3) $(ax+by)(bx+ay)$ (4) $2(a^2x^2+b^2y^2)$

15. For how many integers x does a triangle with side length 10, 24 and x have all its angles acute ?

(1) 4 (2) 5 (3) 6 (4) 7

16. Let $f(x) = 4x - x^2$. Given x_0, consider the sequence defined by $x_n = f(x_{n-1})$ for all $n \geq 1$. For how many real numbers x_0 will the sequence x_0, x_1, x_2 .... take on only a finite number of different values ?

(1) 0 (2) 1 or 2

(3) 3,4,5 or 6 (4) infinitely many

17. If $a \geq 1$ then the sum of the real solutions of :- $\sqrt{a-\sqrt{a+x}} = x$ is equal to

(1) $\sqrt{a}-1$ (2) $\frac{\sqrt{a}-1}{2}$

(3) $\sqrt{a-1}$ (4) $\frac{\sqrt{4a-3}-1}{2}$

18. The discriminant of a quadratic equation with integer coefficients cannot be -

(1) 23 (2) 24

(3) 25 (4) 28

19. For all real numbers x and integers $n \geq 1$, define $f_1(x) = |1 - x|$ and $f_{n+1}(x) = |1 - f_n(x)|$. The equation $f_{1981}(x) = f_{1982}(x)$ has how many real solutions ?

(1) 1981 (2) 1982

(3) 3962 (4) 3963

20. If $\log_8 3 = p$ and $\log_3 5 = q$, then in terms of p and q, log5 equals -

(1) pq (2) $\frac{3p+q}{5}$

(3) $\frac{1+3pq}{p+q}$ (4) $\frac{3pq}{1+3pq}$

21. If p,q and r are distinct roots of $x^3 - x^2 + x - 2 = 0$, then $p^3 + q^3 + r^3$ equals -

(1) –1 (2) 10

(3) 3 (4) none of these

22. How many distinct ordered triples (x,y,z) satisfy the equations

$$x + 2y + 4z = 12$$
$$xy + 4xz + 2x = 22$$
$$xyz = 6$$

(1) none (2) 1

(3) 2 (4) 6

23. Let $g(x) = x^5 + x^4 + x^3 + x^2 + x + 1$. What is the remainder when the polynomial $g(x^{12})$ is divided by the polynomial g(x) ?

(1) 6 (2) $5 - x$

(3) $4 - x + x^2$ (4) $3 - x + x^2 - x^3$

24. The inequality $y - x < \sqrt{x^2}$ satisfied if and only if

(1) $y < 0$ or $y < 2x$ (or both inequalities hold)

(2) $y > 0$ or $y < 2x$ (or both inequalities hold)

(3) $y^2 < 2xy$

(4) $y < 0$

25. If q_1 (x) and r_1 are the quotient and remainder, respectively when the polynomial x^8 is divided by $x + \frac{1}{2}$ and if $q_2(x)$ and if $q_2(x)$, and r_2 are the quotient and reminder, respectively, when $q_1(x)$ is divided by $x + \frac{1}{2}$. then r_2 equals

(1) $\frac{1}{256}$ (2) $-\frac{1}{16}$ (3) 1 (4) –16

26. The function f satisfies the functional equation $f(x)+f(y)=f(x+y)-xy-1$

for every pair x, y of real numbers. If f(l) = 1. then the number of integer $n \neq 1$ for which f(n) = n is :-

(1) 0 (2) 1 (3) 2 (4) 3

27. For each positive number x, let

$$f(x) = \frac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$$

The minimum value of f(x) is :-

(1) 1 (2) 2 (3) 3 (4)6

28. If the function f defined by

$f(x) = \frac{cx}{2x+3}$, $x \neq -\frac{3}{2}$. c a constant, satisfies f(f(x)) = x for all real numbers x except $-\frac{1}{2}$. then c is

(1) –3 (2) –3/2 (3) 3/2 (4) 3

29. For some real number r, the polynomial $8x^3 - 4x^2 + 45$ is divisible by $(x-r)^2$. Which of the following numbers is closest to r ?

(1) 1.22 (2) 1.32 (3) 1.42 (4) 1.52

30. The polynomial $x^{2n} + 1 + (x + 1)^{2n}$ is not divisible by $x^2 + x + 1$ if n equals

(1) 17 (2) 20 (3) 21 (4) 64

31. How many ordered triples (x, y, z) of integers satisfy the system of equations below ?

$x^2 - 3xy + 2y^2 - z^2 = 31.$ $-x^2 + 6yz + 2z^2 = 44.$ $x^2 + xy + 8z^2 = 100.$

(1) 0

(2) 1

(3) 2

(4) a finite number greater than two

32. If h > 1, x > 0 and $(2x)^{\log_h^2} - (3x)^{\log_h^3} = 0$. then x is :-

(1) $\frac{1}{216}$ (2) $\frac{1}{6}$ (3) 1 (4) 6

33. The function f is not defined for x = 0. but. for all non-zero real numbers x, $f(x) + 2f\left(\frac{1}{x}\right) = 3x$.

The equation f(x) = f(–x) is satisfied by :-

(1) exactly one real number.

(2) exactly two real numbers

(3) no real numbers

(4) infinitely many, but not all. non-zero real numbers

34. The number of real solutions to the equation $\frac{x}{100} = \sin x$ is :-

(1) 61 (2) 62

(3) 63 (4) 64

35. If a and b are positive numbers such that $a^b = b^a$ and b = 9a, then the value of a is

(1) 9 (2) 1/9

(3) $\sqrt[9]{9}$ (4) $\sqrt[4]{3}$

1 MARK

1. $\frac{x}{2} = y^2 \Rightarrow x = 2y^2$ .... (1)

$\frac{x}{4} = 4y \Rightarrow x = 16y$ ... (2)

from eq. (1) & (2)

$2y^2 = 16y \Rightarrow y = 8$ $\because x \neq 0$

$\Rightarrow y \neq 0$

$\Rightarrow x = 2y^2 = 2(8)^2 = 128$

2. $x^5\left(x+\frac{1}{x}\right)\left(1+\frac{2}{x}+\frac{3}{x^3}\right)$

$= x^5 \frac{(x^2+1)}{x}\left(\frac{x^2+2x^2+3}{x^3}\right)$

$= x(x^2 + 1)(x^2 + 2x^2 + 3)$

$\Rightarrow$ degree of polynomial = 6

3. $f(-x) = \frac{-x+1}{-x-1} = \frac{x+1}{x+1}$

$f(x) = \frac{1}{\left(\frac{x+1}{x-1}\right)} = \frac{1}{f(x)}$

4. $(xy)(xz)(yz) = abc \Rightarrow x^2y^2z^2 = abc$

$\Rightarrow (xy)^2z^2 = abc \Rightarrow a^2z^2 = abc$

$\Rightarrow z^2 = \frac{bc}{a}$

Now $x^2(yz)^2 = abc \Rightarrow x^2(c)^2 = abc$

$\Rightarrow x^2 = \frac{ab}{c}$

$(xz)^2y^2 = abc \Rightarrow y^2 = \frac{ac}{b}$

$\Rightarrow x^2 + y^2 + z^2 = \frac{ab}{c}+\frac{ac}{b}+\frac{bc}{a}$

$x^2 + y^2 + z^2 = \frac{(ab)^2+(ac)^2+(bc)^2}{abc}$

5. $60^a = 3$ & $60^b = 5$

$\Rightarrow \log_{60}60^a = \log_{60}{}^3$

$\Rightarrow a = \log_{60}{}^3$

$12^{\frac{1-a-b}{2(1-b)}} = 12^{\frac{\log_{60}60-\log_{60}3-\log_{60}5}{2(\log_{60}60-\log_{60}5)}}$

$=(12)^{\left(\frac{\log_{60}\frac{60}{3\times5}}{2\log_{60}\left(\frac{60}{5}\right)}\right)} = \frac{\log_{60}4}{12^{2\log_{60}12}}$

$= 12^{\frac{2\log_{60}2}{2\log_{60}12}} = (12)^{\log_{12}2} = 2$

6. $\frac{x-\frac{1}{y}}{y-\frac{1}{x}} = \frac{\left(\frac{xy-1}{y}\right)}{\frac{(xy-1)}{x}} = \frac{x}{y}$

7. $x^{\log_{10}x} = 10$

taking $\log_{10}$ on both sides

$\log_{10}x^{\log_{10}x} = \log_{10}10$

$\Rightarrow (\log_{10}x)(\log_{10}x) = 1$

$\Rightarrow \log_{10}x_1 + \log_{10}x_2 = 0$

$\Rightarrow x_1x_2 = 10^\circ = 1$

$x_1x_2 = 1$

8. $2x + 1 = 8 \Rightarrow 2x = 7$ ... (1)

$\Rightarrow 4x + 1 = 2(2x) + 1 = 2(7) + 1$

$4x + 1 = 15$

9. $ax = -b \quad \Rightarrow x = \frac{-b}{a} \quad \ldots (1)$

$a'x = -b' \quad \Rightarrow x = \frac{-b'}{a'} \quad \ldots (2)$

$\because$ solution of eq. (1) is less than solution of eq. (2)

$\Rightarrow -\frac{b}{a} < \frac{-b'}{a'}$

$\Rightarrow \frac{b}{a} > \frac{b'}{a'}$

10. $x - (y - z) - [(x - y) - z]$

$= x - y + z - x + y + z = 2z$

11. Hopes = H units, skips = S units,

Jumps = J units

according to question

$bH = cS$, $dJ = eH$, $fJ = g$ meter

$\Rightarrow 1 \text{ meter} = \frac{f(J)}{g} = \frac{f}{g}\left(\frac{eH}{d}\right)$

$= \frac{f}{g}\frac{e}{d}(H) \quad = \frac{f}{g}\cdot\frac{e}{d}\cdot\frac{cS}{b}$

$= \left(\frac{fec}{gdb}\right)$ skips

12. $x^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \quad \ldots (1)$

$\Rightarrow \alpha^4 + 6\alpha^2 + 25 = 0$

$\Rightarrow \alpha^4 = -6\alpha^2 - 25 \quad \ldots (2)$

& $3\alpha^4 + 4\alpha^2 + 28\alpha + 5 = 0$

$3(-6\alpha^2 - 25) + 4\alpha^2 + 28\alpha + 5 = 0$

from eq. (2)

$-14\alpha^2 + 28\alpha - 70 = 0$

$\alpha^2 - 2\alpha + 5 = 0 \quad \ldots (3)$

eq. (1) & (3) are same

$\Rightarrow b = -2,\ c = 5$

$\Rightarrow p(x) = x^2 + bx + c = x^2 - 2x + 5$

$\Rightarrow p(1) = 1 - 2 + 5 = 4$

13. $(1 + x^2)(1 - x^3) = 1 - x^3 + x^2 - x^5$

$= 1 + x^2 - x^3 - x^5$

14. $a - 1 = b + 2 \Rightarrow b = a - 3$

$a - 1 = c - 3 \Rightarrow c = a + 2$

$a - 1 = d + 4 \Rightarrow d = a - 5$

from above $a + 2 > a > a - 3 > a - 5$

$c > a > b > d$

$\Rightarrow$ c is largest.

15. $x^2y + xy^2 + x + y = 63 \quad \& \ xy = 6$

$xy(x + y) + (x + y) = 63$

$(x + y)(xy + 1) = 63$

$(x + y)(6 + 1) = 63 \Rightarrow (x + y) = 9$

$\Rightarrow x^2 + y^2 = (x + y)^2 - 2xy = 81 - 12 = 69$

$x^2 + y^2 = 69$

16. Let x is thickness of one book algebra & y is thickness of one book of geometry.

$\Rightarrow Ax + Hy = Sx + My = Ex$

Now $\quad Ax + Hy = Ex$

$\Rightarrow \quad (A - E)x + Hy = 0 \quad \ldots. (1)$

$Sx + My = Ex$

$\Rightarrow \quad (S - E)x + My = 0 \quad \ldots (2)$

(1) $\quad M - (2)\ H$

$\Rightarrow \quad AM - ME - HS + HE = 0$

$AM - HS = E(M - H)$

$E = \frac{AM - HS}{M - H}$

17. $\log_{10}(\tan 1°.\tan 2°......\ \tan 88°\ \tan 89°)$

$= \log_{10}(\tan 1°.\tan 2°\tan 3°......\ \tan 45\cot 44°\ ....\ \cot 1°)$

$= \log_{10}(\tan 45°) = \log_{10}1 = 0$

18. $x^2 + px - 444p = 0 \begin{array}{l} \alpha \\ \beta \end{array}$

$\alpha + \beta = -p,\quad \alpha\beta = -444p$

$\Rightarrow$ both roots are multiple of p

$$\Rightarrow \left(\frac{x}{p}\right)^2 + \left(\frac{x}{p}\right) = \frac{444}{p}$$

$$\left(\frac{x}{p}\right)\left(\frac{x}{p}+1\right) = \frac{444}{p} = \frac{2\times2\times3\times37}{p}$$

$\Rightarrow$ p may be 2, 3 or 37 $\quad$ ($\because$ p is prime)

Now when $p = 2, \Rightarrow x^2 + 2x - 888 = 0$

roots are not integer

when $p = 3 \Rightarrow x^2 + 3x - 1332 = 0$

roots are not ineter

when $p = 37, \Rightarrow x^2 + 37x - 16428 = 0$

$x^2 + 148x - 111x - 16428=0$

$x = -148,\ x = 111$

$\Rightarrow p = 37$ $\quad$ integer roots

19. $(x + 2)(x + b) = x^2 + cx + 6$

$x^2 + (2 + b)x + 2b = x^2 + cx + 6$

comparing coefficiant of x & constant term

$2b = 6$

$\Rightarrow \quad b = 3$

$2 + b = c$

$c = 2 + 3 = 5$

20. $x^2 - x - 1 = 0 \begin{array}{l} \alpha \\ \beta \end{array}$

$\Rightarrow \alpha^2 - \alpha - 1 = 0 \quad ...\ (1)$

$\alpha^2 = \alpha + 1$

$ax^3 + bx^2 + 1 = 0 \begin{array}{l} \alpha \\ \beta \\ \gamma \end{array}$

$\Rightarrow \quad a\alpha^3 + b\alpha^2 + 1 = 0$

$a\alpha\cdot\alpha^2 + b\alpha^2 + 1 = 0$

$a\alpha(\alpha + 1) + b(\alpha + 1) + 1 = 0$

from eq. (1)

$a\alpha^2 + (a + b)\alpha + b + 1 = 0 \quad ...\ (2)$

(1) & (2) are same

$$\Rightarrow \quad \frac{a}{1} = \frac{a+b}{-1} = \frac{b+1}{-1}$$

$\Rightarrow \quad a + b = b + 1 \Rightarrow a = 1$

$$\Rightarrow \quad \frac{a}{1} = \frac{a+b}{-1} \Rightarrow -a = a + b$$

$\Rightarrow \quad b = -2a = -2\times 1$

$b = -2$

21. $\log_9 p = \log_{12} q = \log_{16}(p + q) = k$ (say)

$p = 9^k,\ q = 12^k,\ p + q = 16^k$

$$\frac{q}{p} = \frac{12^k}{9^k} = \left(\frac{4}{3}\right)^k$$

$\because p + q = 16^k$

$9^k + 12^k = 16^k$ dividing by 9^k

$$1 + \left(\frac{4}{3}\right)^k = \left(\left(\frac{4}{3}\right)^k\right)^2$$

$\Rightarrow \alpha^2 - \alpha - 1 = 0$ where $\alpha = \dfrac{q}{p} = \left(\dfrac{4}{3}\right)^k$

$$\alpha = \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2} \quad \text{reject}$$

$$\Rightarrow \frac{q}{p} = \frac{1+\sqrt{5}}{2}$$

22. $a + b = 1 \Rightarrow a^2 + b^2 + 2ab = 1$

$2 + 2ab = 1 \qquad \because a^2 + b^2 = 2$

$\Rightarrow \quad ab = -\frac{1}{2}$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$= 1\left(2+\frac{1}{2}\right)=\frac{5}{2}$

23. $ab(c^2 + d^2) + cd(a^2 + b^2)$

$= abc^2 + \underbrace{abd^2 + cda^2} + cdb^2$

$= abc^2 + cdb^2 + abd^2 + cda^2$

$= bc(ac + bd) + ad(bd + ac)$

$= (ac + bd)(bc + ad)$

24. $\Delta = 0$

$$\begin{vmatrix} n & 1 & 0 \\ 0 & n & 1 \\ 1 & 0 & n \end{vmatrix} = 0$$

$n(n^2 - 0) - 1(0 - 1) = 0$

$n^3 + 1 = 0$

$\Rightarrow n = -1$

25. $\frac{2}{x}+\frac{3}{y}=\frac{1}{2}$

$\frac{2y+3x}{xy}=\frac{1}{2} \Rightarrow 4y + 6x = xy$

$\Rightarrow 4y = xy - 6x$ or $6x = y(x - y)$

$x=\frac{4y}{y-6} \qquad y=\frac{6x}{x-4}$

26. Let $p(x) = x^{51} + 51$ then remainder when $p(x)$ is divided by $(x + 1) = p(-1)$

[by remainder theorem]

remainder $= p(-1) = (-1)^{51} + 51 = 50$

27. $x*y = \frac{x-y}{x+y} \qquad \ldots (1)$

$y*x = \frac{y-x}{y+x} \qquad \ldots (2)$

from eq. (1) & (2) $x*y \neq y*x$

$\Rightarrow$ * is not commutative.

For associative

$$(x*y)*z = \left(\frac{x-y}{x+y}\right)*z=\frac{\frac{x-y}{x+y}-z}{\frac{x-y}{x+y}+z}$$

$$(x*y)*z = \frac{x-y-zx-zy}{x-y+zx+zy} \qquad \ldots (3)$$

$$x*(y*z) = x*\left(\frac{y-z}{y+z}\right)=\frac{x-\frac{(y-z)}{y+z}}{x+\left(\frac{y-z}{y+z}\right)}$$

$$x*(y*z) = \frac{xy+xz-y+z}{xy+xz+y-z} \qquad \ldots (4)$$

from eq. (3) & (4)

$(x*y)*z \neq x*(y*z)$

$\Rightarrow$ * is not associative.

28. $(2k - 1)x^2 - 8x + 6 = 0$

no real roots $\Rightarrow D < 0$

$64 - 4(2k - 1)6 < 0$ Dividing by 8

$8 - 3(2k - 1) < 0$

$8 - 6k + 3 < 0$

$k>\frac{11}{6}$

$\Rightarrow k_{\text{min integer}} = 2$

29. $f(g(x)) = \frac{1-x^2}{x^2}$

$f(g(x)) = \frac{1-x^2}{1-(1-x^2)} = \frac{g(x)}{1-g(x)}$

put $g(x) = \frac{1}{2}$

$\Rightarrow f\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{1-\frac{1}{2}}=1 \Rightarrow f\left(\frac{1}{2}\right) = 1$

30. $x^2 > x \Rightarrow x(x-1) > 0$

$+ \quad - \quad +$
$0 \qquad 1$

$x \in (-\infty, 0) \cup (1, +\infty)$

$\Rightarrow$ if $x < 0 \Rightarrow x^2 > x$

31. $x < -2 \Rightarrow x + 2 < 0 \Rightarrow |x+2| = -x-2$

$\Rightarrow x + 1 < 0$

$\Rightarrow |x+1| = -(x+1)$

Now $|1 - |1+x|| = |1 + (1+x)|$

$= |2+x| = -2-x$

$$\because |x| = \begin{cases} -x & \text{if } x<0 \\ x & \text{if } x\geq 0 \end{cases}$$

32. $|f(x) + 4| < a$

$\Rightarrow |3x+6| < a \qquad \because f(x) = 3x+2$

$3|x+2| < a \qquad$ Let $T = x + 2$

$3|T| < a$ whereever $|T| < b$

$T\in\left(-\frac{a}{3}, \frac{a}{3}\right)$ lies in $(-b, b)$

$\Rightarrow \frac{a}{3} < b$

33. (I) None are satisfied

Take counter example for

(I) $xy + yz + zx < ab + bc + ca$ false

for $x = -1, y = -2, z = -3, a = 1$

$b = 0$

$c = 0$

(II) $x^2 + y^2 + z^2 < a^2 + b^2 + c^2$

false for above values

(III) $xyz < abc$ false for $x = -1, a = 0$

$y = -2, b = 1$

$z = 3, \quad c = 5$

34. Case I if $x < 0 \qquad |x| = -x$

$$\Rightarrow \frac{|x-|x||}{x} = \frac{|x-(-x)|}{x}$$

$$= \frac{|2x|}{x} = \frac{-2x}{x} = -2$$

Case II if $x > 0 \qquad |x| = x$

$$\Rightarrow \frac{|x-|x||}{x} = \frac{|x-x|}{x} = 0$$

$\Rightarrow$ (4) is Ans.

35. $b = a - x$

$a^3 - b^3 = 19x^3$

$\Rightarrow a^3 - (a-x)^3 = 19x^3$

$a^2 - ax - 6x^2 = 0$

$\Rightarrow a = 3x$ or $a = -2x$

36. $f(x) = x^6 - 3x^5 - 6x^3 - x + 8$

$f(\text{-ve}) > 0 \Rightarrow$ no negative roots

$f(1) = 1 - 3 - 6 - 1 + 8 = -3 < 0$

$f(0) = 8$

$f(1)\, f(0) < 0 \Rightarrow$ one root lies between 0 & 1

$\Rightarrow$ option (4) Ans.

37. $(\log_3 x)(\log_3 5) = \log_3 5$

$\Rightarrow \log_3 x = 1$

$\Rightarrow x = 3$

38. Let $f(x) = \lambda p^x$; $\lambda, p, > 0$

$\Rightarrow f(a) \cdot f(b) = f(a+b)$

$\lambda p^a \cdot \lambda p^b = \lambda p^{a+b}$

$\Rightarrow \lambda = 1$

$\Rightarrow f(x) = p^x$

$\Rightarrow f(0) = 1, \; f(a) = p^a, \; f(-a) = p^{-a}$

$$\Rightarrow f(-a) = \frac{1}{p^a} = \frac{1}{f(a)}$$

$$(f(3a))^{\frac{1}{3}} = (p^{3a})^{\frac{1}{3}} = p^a = f(a)$$

if $\quad 0 < p < 1$; $f(b) > f(a)$

$\Rightarrow \quad p^b > p^a \Rightarrow b < a$

if $\quad p > 1$; $f(b) > f(a)$

$\Rightarrow \quad p^b > p^a \Rightarrow b > a$

$\Rightarrow$ (I) (II) & (III) are true

Method-2

$f(a)\, f(b) = f(a+b) \qquad \ldots (1)$

Put $a = b = 0$

$(f(0))^2 = f(0) \Rightarrow f(0) = 1 \quad \because\ f(x) > 0\ \forall x$

put $\quad b = -a$ in eq. (1)

$f(a)\ f(-a) = f(0) \Rightarrow f(-a) = \dfrac{1}{f(a)}$

put $\quad b = 2a$ in (1)

$f(a)\ f(2a) = f(3a) \Rightarrow f(a)\cdot(f(a))^2 = f(3a)$

$\Rightarrow f(a) = \sqrt[3]{f(3a)}$

39. $x^2 - px + q = 0 <^{\alpha}_{\beta}$

$\alpha\cdot\beta = q, \qquad \alpha + \beta = p$

but p & q are prime & α, β are natural numbers

$\Rightarrow \alpha = 1,\ \beta = \underbrace{q \quad 1 + \beta}\ = p$

$1 + q = p$

$\Rightarrow p - q = 1$ difference of two prime is one

$\Rightarrow p = 3\ \&\ q = 2$

Now given eq. become $x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0 \Rightarrow x = 2, 1$

40. $1 - \dfrac{1}{1-x} = \dfrac{1}{1-x}$

$\Rightarrow 1 = \dfrac{2}{1-x} \quad \Rightarrow 1 - x = 2$

$x = -1$

41. $x^2 - 3x + c = 0 <^{\alpha}_{\beta}, \quad x^2 - 3x - c = 0 <^{-\alpha}_{\gamma}$

$\alpha^2 - 3\alpha + c = 0 \quad \dots (1)$

$\alpha^2 - 3\alpha - c = 0 \quad \dots (2)$

$(1) - (2) \Rightarrow c = 0$ put in $x^2 - 3x + c = 0$

$\Rightarrow x^2 - 3x = 0$

$x = 0, 3$

42. $(1 - |x|)(1 + x) > 0 \quad \dots (1)$

Case I $\quad x < 0 \quad \dots (A)$

$\Rightarrow$ eq. (1) become $(1 + x)(1 + x) > 0$

$\Rightarrow (x + 1)^2 > 0 \quad \dots (B)$

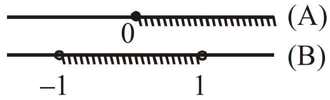
$(A) \cap (B)\ x \in (-\infty, -1) \cup (-1, 0) \quad \dots (\alpha)$

Case II $\quad x \geq 0 \quad \dots (A)'$

$\Rightarrow$ eq. (1) become

$(1 - x)(1 + x) > 0$

$(x - 1)(x + 1) < (B)'$



$(A)' \cap (B)'$

$x \in [0, 1) \quad \dots (\beta)$

final answer will be $(\alpha) \cup (\beta)$

$\Rightarrow x < -1$ or $-1 < x < 1$

43. One cow in one day give milk $= \dfrac{(x+1)}{x(x+2)}$ cans

Let $(x + 3)$ cow take α days to give $(x + 5)$ cans of milk.

$\Rightarrow (x + 3)\left(\dfrac{x+1}{x(x+2)}\right)\alpha = (x + 5)$

$\alpha = \dfrac{x(x+2)(x+5)}{(x+1)(x+3)}$

44. $p(1) = 3, \quad p(3) = 5$

$p(x) = (x - 1)(x - 3)Q(x) + (ax + b) \quad \dots (1)$

[$\because$ Here p(x) is divided by $(x - 1)(x - 3)$ is quadratic $\Rightarrow$ take remainder linear (i.e. one degree less than divisor)]

put $x = 1$ in (1)

$p(1) = 0 + a + b$

$\Rightarrow a + b = 3 \quad \dots (2)$

put $x = 3$ in (1)

$p(3) = 3a + b$

$\Rightarrow 3a + b = 5 \quad \dots (3)$

solve eq. (2) & (3) $\quad a = 1, b = 2$

$\Rightarrow$ remainder $= ax + b = x + 2$

45. $4(\log_a x)^2 + 3(\log_b x)^2 = 8(\log_a x)(\log_b x)$

Dividing by $(\log_b x)^2$

$\Rightarrow 4(\log_a b)^2 + 3 \ = at\ (\log_a b)$

$4p^2 - 8p + 3 = 0$ where $p = \log_a b$

$4p^2 - 6p - 2p + 3 = 0$

$2p(2p-3) - 1(2p-3) = 0$

$(2p-3)(2p-1) = 0$

$\Rightarrow p = \frac{3}{2}$ or $p = \frac{1}{2}$

$\Rightarrow \log_a b = \frac{3}{2}$ $\quad \log_a b = \frac{1}{2}$

$b = a^{3/2}$ $\quad b = a^{\frac{1}{2}}$

$\Rightarrow b^2 = a^3$ or $a = b^2$

$\Rightarrow b^2 = a^3$ or $a = b^2$

46. $x + y + z = x + (2x) + (2y) \quad \because y = 2x$

$= x + 2x + 2(2x)$

$x + y + z = 7x$

47. $\left(2x + \frac{y}{2}\right)^{-1}\left[(2x)^{-1} + \left(\frac{y}{2}\right)^{-1}\right]$

$= \frac{2}{(4x+y)}\left[\frac{1}{2x} + \frac{2}{y}\right]$

$= \frac{2}{(4x+y)}\left[\frac{y+4x}{2xy}\right] = \frac{1}{xy}$

48. $\frac{a}{|a|} = \pm 1, \frac{b}{|b|} = \pm 1, \frac{c}{|c|} = \pm 1, \frac{abc}{|abc|} = \pm 1$

Case I All +ve is $a > 0, b > 0, c > 0$

$\Rightarrow \frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|} = 1 + 1 + 1 + 1 = 4$

Case II One -ve and two positive is

$a < 0, b > 0, c > 0$

$\Rightarrow \frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|} = -1 + 1 + 1 - 1 = 0$

Case III one +ve and two -ve is

$a > 0, b < 0 < c < 0$

$\Rightarrow \frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|} = 1 - 1 - 1 + 1 = 0$

Case IV all are -ve

$\Rightarrow \frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|} = -1 -1 -1 -1 = -4$

Ans. Union of all cases is $\{4, 0, -4\}$

49. $y = (\log_2 3)(\log_3 4) \ldots\ldots (\log_n(n+1)) \ldots\ldots (\log_{31} 32)$

$y = \frac{\log 32}{\log 2} = \frac{5\log 2}{\log 2} = 5$

50. $x^2 + ax + 1 = 0$

$x^2 - x - a = 0$

$$\begin{vmatrix} 1 & 1 \\ 1 & -a \end{vmatrix}^2 = \begin{vmatrix} 1 & a \\ 1 & -1 \end{vmatrix}\begin{vmatrix} a & 1 \\ -1 & -a \end{vmatrix}$$

$(a+1)^2 = (-1-a)(-a^2+1)$

$(a+1)^2 = (a+1)(a^2-1)$

$(a+1)(a+1)(a-1) - (a+1)^2 = 0$

$(a+1)^2 [a - 1 - 1] = 0$

$a = -1, a = 2$

51. $f(a+b) + f(a-b) = 2f(a) + 2f(b) \ldots. (1)$

put $a = 0, b = 0$ in (1)

$\Rightarrow f(0) + f(0) = 4(0) \quad \Rightarrow f(0) = 0$

Now put $a = 0$ in eq. (1)

$f(b) + f(-b) = 2f(0) + 2f(b)$

$\Rightarrow f(-b) = f(b)$

$\Rightarrow f(-x) = f(x)$

52. $x^2 + px + q = 0 \begin{cases} \alpha^3 \\ \beta^3 \end{cases}$

$x^2 + mx + n = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$-p = (-m)^3 - 3n(-m)$

$p = m^3 - 3mn$

53. $(x^2 + y^2 + z^2)^2 \le n(x^4 + y^4 + z^4)$

$x^4 + y^4 + z^4 + 2(x^2y^2 + y^2z^2 + z^2x^2)$

$\le n(x^4 + y^4 + z^4)$

$\Rightarrow (n-1)x^4 + (n-1)y^4 + (n-1)z^4$

$- 2(x^2y^2 + y^2z^2 + z^2x^2) \ge 0$

$2(x^2)^2 + 2(y^2)^2 + 2(z^2)^2$

$-2(x^2y^2 + y^2z^2 + z^2x^2) \ge 0$

if $n - 1 = 2 \quad \Rightarrow n = 3$

$(x^2 - y^2)^2 + (y^2 - z^2)^2 + (z^2 - x^2)^2 \ge 0$

which is always true

$\Rightarrow n = 3$ Ans.

54. $1-\frac{4}{x}+\left(\frac{2}{x}\right)^2=0$

$\Rightarrow \left(1-\frac{2}{x}\right)^2=0$

$\Rightarrow \frac{2}{x}=1$

55. If a + b + c = 0

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \frac{(p-q)^3+(q-r)^3+(r-p)^3}{(p-q)(q-r)(r-p)}$

$= \frac{3(p-q)(q-r)(r-p)}{(p-q)(q-r)(r-p)} = 3$

56. (a + b + c – d) + (a + b – c + d)

+ (a – b + c + d) + (–a + b + c + d)

= 2 (a + b + c + d)

= 2 (1 + 10 + 100 + 1000)

= 2 × 1111 = 2222

57. $\left|x-\sqrt{(x-1)^2}\right| = \left|x-|x-1|\right|$

$= |x + (x - 1)|$ $\left[\begin{array}{c} \because x<0 \\ \Rightarrow x-1<0 \\ \Rightarrow |x-1|=-(x-1) \end{array}\right.$

$= |2x - 1|$

$= 1 - 2x$

58. $x^2+ax+b=0 \begin{array}{l} \nearrow c \\ \searrow d \end{array}$

c + d = – a ... (1)

cd = b ... (2)

$x^2+cx+d=0 \begin{array}{l} \nearrow a \\ \searrow b \end{array}$

a + b = – c ... (3)

ab = d ... (4)

from eq. (1) & (3)

add $\begin{array}{c} c+d=-a \\ -c=a+b \\ \hline d=b \end{array}$

put in (2) & (4)

c = 1, a = 1

put in (1)

1 + d = – 1 $\Rightarrow$ d = – 2 = b

a + b + c + d = – 2

59. $x^3 + 8y^3 + x^3y^3 = 6x^2y^2$

$x^3 + (2y)^3 + (xy)^3 - 3.(x)(2y)(xy) = 0$

$\Rightarrow x + 2y + xy = 0$ or $x = 2y = xy$

but in ques. $x \neq 2y$

$\Rightarrow x + 2y = - xy$

Dividing by xy

$\frac{1}{y}+\frac{2}{x}=-1$

60. $\left[f\left(x^2+1\right)\right]^{\sqrt{x}}=k$

put $x^2 + 1 = \frac{9+y^2}{y^2}$

$\Rightarrow x^2 = \frac{9}{y^2} \Rightarrow \sqrt{x} = \sqrt{\frac{3}{y}}$: x, y > 0

$\Rightarrow \left[f\left(\frac{9+y^2}{y^2}\right)\right]^{\sqrt{\frac{12}{y}}} = \left(\left[f\left(\frac{9+y^2}{y^2}\right)\right]^{\sqrt{\frac{3}{y}}}\right)^2$

$= \left(\left[f\left(x^2+1\right)\right]^{\sqrt{x}}\right)^2 = k^2$

61. $\frac{a+b-c}{c} = \frac{a-b+c}{b} = \frac{-a+b+c}{a} = \lambda$ (let)

$a + b - c = c\lambda$ ... (1)

$a - b + c = b\lambda$ .... (2)

$- a + b + c = a\lambda$ .... (3)

eq. (1) + (2) + (3)

$(a + b + c) = \lambda (a + b + c)$

$\Rightarrow \lambda = 1$

put in (1), (2) & (3)

a + b = 2c, a + c = 2b, b + c = 2a

$$\Rightarrow\ x = \frac{(a+b)(b+c)(c+a)}{abc} = \frac{(2c)(2a)(2b)}{abc}$$

$x = 8$

62. $\dfrac{1}{\log_3 x}+\dfrac{1}{\log_4 x}+\dfrac{1}{\log_5 x}$

$= \log_x 3 + \log_x 4 + \log_x 5$

$= \log_x(3.4.5) = \log_x 60 = \dfrac{1}{\log_{60} x}$

63. $\log_5 10 = \dfrac{\log_{10} 10}{\lg_{10} 5} = \dfrac{1}{\log_{10}\left(\dfrac{10}{2}\right)}$

$= \dfrac{1}{\log_{10} 10 - \log_{10} 2}$

$= \dfrac{1}{1-0.301} \approx \dfrac{1}{0.7} = \dfrac{10}{7}$

64. $x^{256} - 256^{32} = 0$

$x^{256} - (2^8)^{32} = 0$

$x^{256} - 2^{256} = 0$

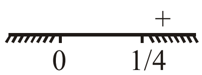
$x^{256} = (\pm 2)^{256}$

$\Rightarrow x = +2, -2$

$\Rightarrow x_1^2 + x_2^2 = (2)^2 + (-2)^2 = 8$

65. $\sqrt{x} < 2x$

$\Rightarrow x < 4x^2$



$\because x > 0$

$4x^2 - x > 0$

$x(4x - 1) > 0$

$\Rightarrow\ x > \dfrac{1}{4}$

66. $x = \sqrt[3]{5+2\sqrt{13}} + \sqrt[3]{5-2\sqrt{13}}$

$x^3 = 5 + 2\sqrt{13} + 5$

$- 2\sqrt{13} + 3\left[\left(5+2\sqrt{13}\right)\left(5-2\sqrt{13}\right)\right]^{1/3} x$

$x^3 = 10 + 3(25-52)^{\frac{1}{3}} x$

$x^3 = 10 + 3(-27)^{\frac{1}{3}} x$

$x^3 + 9x - 10 = 0$

$\Rightarrow x = 1$

[$\because$ $x^2(x-1) + x(x-1) + 10(x-1) = 0$

$(x-1)(x^2 + x + 10) = 0$]

67. $\dfrac{1}{x}+\dfrac{1}{2x}+\dfrac{1}{3x} = \dfrac{1}{x}\left(1+\dfrac{1}{2}+\dfrac{1}{3}\right) = \dfrac{11}{6x}$

68. $\dfrac{x}{x-1} = \dfrac{y^2+2y-1}{y^2+2y-2}$

Using componendo & dividendo

$\dfrac{x+x-1}{x-(x-1)} = \dfrac{y^2+2y-1+y^2+2y-2}{y^2+2y-1-(y^2+2y-2)}$

$\dfrac{2x-1}{1} = \dfrac{2y^2+4y-3}{1}$

$2x = 2y^2 + 4y - 2$

$x = y^2 + 2y - 1$

69. $\left(\dfrac{1}{x+y+z}\right)\left(\dfrac{yz+xz+xy}{xyz}\right).$

$\dfrac{1}{(xy+yz+zx)}\ \left(\dfrac{z+x+y}{xyz}\right)$

$= \dfrac{1}{x^2y^2z^2}$

70. $x^4 - bx - 3 = 0$ with roots a, b, c, d

$a + b + c + d = 0 \Rightarrow a + b + c = -d$

$\Rightarrow \dfrac{a+b+c}{d^2}, \dfrac{a+b+d}{c^2}, \dfrac{a+c+d}{b^2}, \dfrac{b+c+d}{a^2}$

$= \dfrac{-d}{d^2}, \dfrac{-c}{c^2}, \dfrac{-b}{b^2}, \dfrac{-a}{a^2}$

$= \dfrac{-1}{d}, \dfrac{-1}{c}, \dfrac{-1}{b}, \dfrac{-1}{a}$

replace $x \to -\dfrac{1}{x}$ in $x^4 - bx - 3 = 0$

$\Rightarrow \dfrac{1}{x^4} + \dfrac{b}{x} - 3 = 0$

$\Rightarrow 3x^4 - bx^3 - 1 = 0$

71. $x^2-2\overline{)\,x^3-2\,}(x$

$$\begin{array}{l} x^3-2 \\ x^3-2x \\ \underline{-\quad +} \\ 2x-2 \end{array}$$

$\Rightarrow$ remainder = 2x − 2

72. $\frac{x}{y}=\frac{a}{b}$

$\Rightarrow \frac{x}{y}+1 = \frac{a}{b}+1 \Rightarrow \frac{x+y}{y} = \frac{a+b}{b}$

$\Rightarrow \frac{c}{y} = \frac{a+b}{b} \Rightarrow y=\frac{bc}{a+b}$

$\because \frac{x}{y}=\frac{a}{b} \Rightarrow x = \frac{a}{b}y$

$x=\frac{a}{b}\left(\frac{bc}{a+b}\right)=\frac{ac}{a+b}$

$x - y = \frac{(a-b)c}{a+b}<0$

$\Rightarrow x < y$

$\Rightarrow x=\frac{ac}{a+b}$

73. (1) x*y = (x + 1) (y + 1) − 1

y*x = (y + 1) (x + 1) − 1

$\Rightarrow$ x*y = y*x is true

(2) x*(y + z) = (x + 1) (y + z + 1) − 1

= (x + 1) [(y + 1) + (z + 1) − 1]

= (x + 1) (y + 1)+(x + 1)(z + 1) − x − 1 −1

= {(x+1) (y+1)−1} + {(x+1) (2+1)−1} − x

= x*y + x*z − x ≠ x*y + x*z

(3) (x−1)*(x+1) = (x−1+1) (x+1+1) −1

= x^2 + 2x − 1

= [(x + 1) (x + 1) −1] − 1

= x*x − 1

(4) x*0 = (x + 1) (0 + 1) − 1

= x

$\Rightarrow$ option (2) is Ans.

74. $f(x) = ax^7 + bx^3 + cx - 5$ ... (1)

$f(-x) = -ax^7 - bx^3 - cx - 5$ ... (2)

(1) + (2)

f(x) + f(−x) = − 10

put x = 7

$\Rightarrow$ f(7) + f(−7) = − 10

f(7) + 7 = − 10

f(7) = − 17

75. $p=\frac{\log_b(\log_b a)}{\log_b a} = \frac{\frac{\log(\log_b a)}{\log b}}{\left(\frac{\log a}{\log b}\right)}$

$= \frac{\log(\log_b a)}{\log a} = \log_a(\log_b a)$

(Using base canging theoram)

$\Rightarrow a^P = a^{\log_a(\log_b a)} = \log_b a$

$\boxed{a^P=\log_b a}$ $\quad [\because a^{\log_a x}=x]$

76. y = 2|x| + 3 ...(1)

y = 3|x − 2| + 5 ...(2)

(1) = (2)

Here |x| = [x]

2[x] + 3 = 3 [x − 2] + 5

2[x] + 3 = 3 [x] − 6 + 5

[x] = 4

$\Rightarrow x\in[4, 5)$ but x is not an integer

$\Rightarrow x\in (4, 5)$

$\Rightarrow$ y = 2[x] + 3 = 2 × 4 + 3 = 11

$\Rightarrow \because$ 4 < x < 5 & y = 11

$\Rightarrow$ 15 < x + y < 16

77. $3^{2x}\cdot 3^2 - 3^x \cdot 3^3 - 3^x + 3 = 0$

$9(3^x)^2 - 28.3^x + 3 = 0$

$9(3^x)^2\ 27.3^x - 3^x + 3 = 0$

$9.3^x\ (3^x - 3) - 1\ (3^x - 3) = 0$

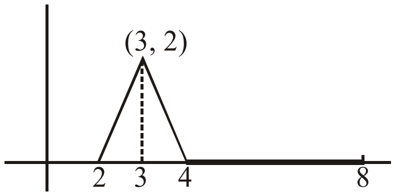
$(3^x - 3)\ (9.3^x - 1) = 0 \Rightarrow 3^x = 3$ or $3^x = 3^{-2}$

$\Rightarrow$ x = 1, x = −2

$\Rightarrow \boxed{x=-2,1}$

number of values of x = 2

78. $f(2) = 0,\ f(3) = 2,\ f(4) = 0,\ f(8) = 0$



$f(x)_{min} = 0 \quad 2 \le x \le 8$

$f(x)_{max} = 2 \quad$ at $x = 3$

$\boxed{f_{min} + f_{max} = 2}$

79. $x^2 + x + 1 = 0$

$(x - 1)(x^2 + x + 1) = 0 \Rightarrow x^3 = 1$

Now $x^{2002} + x^{2003} = x^{2001}1\ (x + x^2)$

$(x^3)^{667}\ (1 + x + x^2 - 1)$

$= 1\ (0 - 1) = -1$

80. $\dfrac{999813 \times 999815 + 1}{(999814)^2}$

$\dfrac{(999814 - 1)(999814 + 1)}{(999814)^2}$

$\dfrac{(999814)^2 - 1 + 1}{(999814)^2} = 1$

81. $2x^2 + 6x + 5x + 1 = 0 \qquad .....(1)$

& $2x + y + 3 = 0$

$\Rightarrow x = -\dfrac{(y+3)}{2}$ put in (1)

$2\left(\dfrac{y+3}{2}\right)^2 - 6\left(\dfrac{y+3}{2}\right) + 5y + 1 = 0$

$\dfrac{(y+3)^2}{2} - 6\left(\dfrac{y+3}{2}\right) + 5y + 1 = 0$

$y^2 + 9 + 6y - 6y - 18 + 10y + 2 = 0$

$\boxed{y^2 + 10y - 7 = 0}$

82. $a^2 + b^2 + c^2 + d^2 = ab + bc + cd + da$

$\Rightarrow 2a^2+2b^2+2c^2+2d^2-2ab-2bc-2cd-2da=0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (c - d)^2 + (d - a)^2 = 0$

Possible only when

$a - b = 0$ & $b - c = 0$ & $c - d = 0$ & $d - a = 0$

$\Rightarrow \boxed{a = b = c = d}$

83. $(am + bn + m)^2 - (am + bn + n)^2 = (m - n)^2$
$(am + bn + m - am - bn - n)\ (am + bn + m + am + bn + n) = (m-n)^2$

$(m-n)\ (2am + 2bn + m + n) = (m - n)^2$

$\Rightarrow 2am + 2bn + m + n = m-n$

$2am + 2(b+1)n = 0$

$\Rightarrow \boxed{a = 0,\ b = -1}$

84. $x^3 - x^2 - 2x + 2 = 0$

$x^2(x - 1) - 2(x - 1) = 0$

$(x - 1)\ (x^2 - 2) = 0$

$x = 1,\ \sqrt{2}, -\sqrt{2}$

$\Rightarrow \boxed{\text{sum of squares of roots} = 1+2+2=5}$

or $x^3 - x^2 - 2x + 2 = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2\ (\alpha\beta + \beta\gamma + \gamma\alpha)$

$= 1^2 - 2(-2) = 5$

85. $\sqrt{n} - \sqrt{n-1} < 0.01$

$\dfrac{\left(\sqrt{n} - \sqrt{n-1}\right)\left(\sqrt{n} + \sqrt{n-1}\right)}{\sqrt{n} + \sqrt{n-1}} < \dfrac{1}{100}$

$\dfrac{n - (n-1)}{\sqrt{n} + \sqrt{n-1}} < \dfrac{1}{100}$

$\dfrac{1}{\sqrt{n} + \sqrt{n-1}} < \dfrac{1}{100}$

$\sqrt{n} + \sqrt{n-1} > 100$

$\left[\because \dfrac{1}{101} < \dfrac{1}{100}\right]$

$\Rightarrow \sqrt{n} + \sqrt{n-1} > 100$

$\Rightarrow$ least positive integer at $\sqrt{n-1} = 50$

$\boxed{n = 2501}$

2 MARK

1. $\log_7(\log_3(\log_2 x)) = 0$

$\Rightarrow \log_3(\log_2 x) = 7^\circ = 1$

$\log_2 x = 3^1$

$x = 2^3 = 8$

$\Rightarrow \boxed{(x)^{-1/2} = (8)^{-1/2} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}}$

2. $f(x^2 + 1) = x^4 + 5x^2 + 3$

$f(x^2 + 1) = (x^2)^2 + 5x^2 + 3$

replace $x^2 \to x^2 - 2$

$f(x^2 - 2 + 1) = (x^2 - 2)^2 + 5(x^2 - 2) + 3$

$\boxed{f(x^2 - 1) = x^4 + x^2 - 3}$

3. $x^2 - px + q = 0 \begin{cases} \tan\alpha \\ \tan\beta \end{cases}$ ....(1)

$x^2 - \gamma x + s = 0 \begin{cases} \cot\alpha \\ \cot\beta \end{cases}$

$\gamma s = (\cot\alpha + \cot\beta)(\cot\alpha . \cot\beta)$

$\left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)\frac{1}{\tan\alpha . \tan\beta}$

$= \frac{\tan\alpha + \tan\beta}{(\tan\alpha . \tan\beta)^2} = \frac{p}{q^2}$

4. $x^2 + 2bx + 1 = 2ax + 2ab$

$x^2 + 2(b - a)x + 1 - 2ab = 0$

$D < 0$

$4(b - a)^2 - 4(1 - 2ab) < 0$

$a^2 + b^2 < 1$

$\Rightarrow$ Point (a, b) lies inside the circle of radius 1 unit.

$\Rightarrow \boxed{\text{Area of } S = \pi(1)^2 = \pi}$

5. $\because f(2+x) = f(2-x)$

$\Rightarrow$ graph is symmetric to line $x = 2$

Let $x = \lambda_1$ & λ_2 are such that for which

$f(2+\lambda_1) = f(2-\lambda_1) = 0$

$f(2+\lambda_2) = f(2-\lambda_2) = 0$

$\Rightarrow 2 + \lambda_1, 2 - \lambda_1, 2 + \lambda_2$ & $2 - \lambda_2$

are four roots of $f(x) = 0$

$\Rightarrow$ sum of roots

$= (2+\lambda_1) + (2 - \lambda_1) + (2 + \lambda_2) + (2 - \lambda_2)$

$= 8$

6. $ab + bc = 44$ ....(1) &

$ac + bc = 23$ ....(2)

$(a+b)c = 23 \times 1$

$a + b = 23, c = 1$

$[\because a + b \neq 1, a, b \in N]$

$c = 1$ & $a = 23-b$ put in equation (1)

$(23-b)b + b \times 1 = 44$

$b^2 - 24b + 44 = 0$

$\boxed{\begin{matrix} b = 22 \\ a = 1 \\ c = 1 \end{matrix}}$ or $\boxed{\begin{matrix} b = 2 \\ a = 21 \\ c = 1 \end{matrix}}$

$\Rightarrow$ number of triplete = 2

7. $x^2 + ax + 2b = 0$

real roots $D \geq 0 \Rightarrow a^2 - 8b \geq 0$

$\Rightarrow a^2 \geq 8b$ ...(1)

$x^2 + 2bx + a = 0$

roots are real $= D \geq 0 \Rightarrow b^2 \geq a$ ...(2)

squaring (2) $\Rightarrow b^4 \geq a^2$ ...(3)

from equation (3) & (1)

$b^4 \geq a^2 \geq 8b$

$\Rightarrow b^4 \geq 8b \Rightarrow b^3 \geq 8$ $[\because b > 0]$

$\Rightarrow \boxed{b \geq 2} \Rightarrow b_{min} = 2$

$\Rightarrow a^2 \geq 8b \Rightarrow a^2_{min} = 8b_{min}$

$a^2 = 16 \Rightarrow a_{min} = 4$

$\boxed{(a+b)_{min} = 4 + 2 = 6}$

8.

$(x^2 - x - 1)^{x+2} = 1$	$a^b = 1$
Case I $x + 2 = 0$ & $x^2 - x - 1 \neq 0$	$b = 0, a \neq 0$
$x = -2$	or
	$a = 1$
	or
	$a = -1, b \in$ even

Case II $x^2 - x - 1 = 1$

$x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0$

$x = 2, -1$

Case III $x^2 - x - 1 = -1$ & $x + 2 \in$ Even integer

$x = 0,\ x = 1$ $\quad\Rightarrow x = -2, 2, -1, 0$

at $x = 0$, $x+2$ even

at $x = 1$, $x + 2$ odd reject

9. $4x^2 - 40(x\{x\}) + 51 = 0$

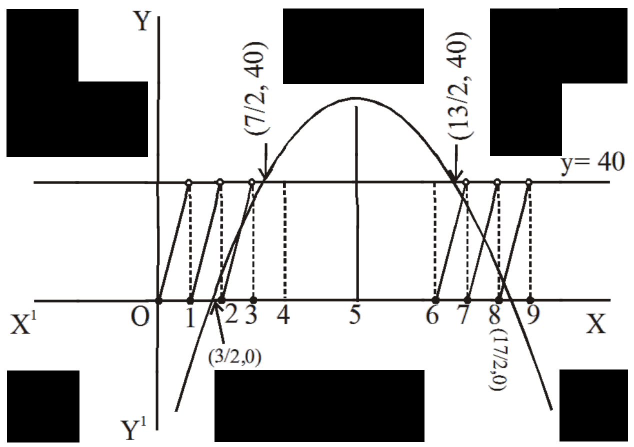
$40\{x\} = -4x^2 + 40x - 51$

$y = 40\{x\}$ (graph green colour)

$y = -4x^2 + 40x - 51$ (Blue colour)

$\Rightarrow$ 4soluion

Ans. See point of intersection



10. **Least integer function :-** The function whose value of any number x is the smallest integer greater than or equal to x is called LIF. It denoted by $\lceil x \rceil$. It is also known as ceiling of x.

$\Rightarrow$ according to question $[x] + \lceil x \rceil = 5$ possible only when

$[x] = 2$ & $\lceil x \rceil = 3$

$2 \le x < 3$ ....(1) & $2 < x \le 3$ ...(2)

$\Rightarrow$ (1) $\cap$ (2)

$\boxed{2 < x < 3}$

11. $A = \dfrac{x+y+z}{3}$ (correct average) ....(1)

Average of x & y $= \dfrac{x+y}{2}$

Average computed by student, $A' = \dfrac{\frac{x+y}{2} + z}{2}$

$A' = \dfrac{x+y+2z}{4}$ ....(2)

Now

$A - A' = \dfrac{x+y+z}{3} - \dfrac{(x+y+2z)}{4}$

$= \dfrac{x+y-2z}{12} = \dfrac{(x-2)+(y-z)}{12} < 0$

$\Rightarrow \boxed{A' > A}$

12. Let degree of polynomial function f(x) = n

$\because f(x^2) = [f(x)]^2 = f(f(x))$ .....(A)

degree 2n degree 2n degree n^2

$\Rightarrow 2n = n^2$

$\Rightarrow \boxed{n = 2}$

Lef $f(x) = ax^n : a \ne 0$
$f(f(x)) = a(f(x))^n$
$= a(ax^n)^n$
$= a^{n+1} x^{n^2}$

Here coeff of x^2 will be 1 from $f(x^2) = [f(x)]^2$

degree of polynomial = 2

Now let $f(x) = x^2 + bx + c$

$\Rightarrow f(x^2) = (x^2)^2 + bx^2 + c$

$= x^4 + bx^2 + c$ ....(1)

Now $[f(x)]^2 = (x^2 + bx + c)^2$

$= x^4 + b^2x^2 + c^2 + 2bx^3 + 2bcx + 2cx^2$

$[f(x)]^2 = x^4 + 2bx^3 + (b^2 + 2c)x^2 + 2bcx + c^2$ .....(2)

Now $f(f(x)) = (f(x))^2 + b(f(x)) + c = (f(x))^2$ form equation (A)

$\Rightarrow b\ f(x) + c = 0$ ...(3)

from equaiton (1) & (2) comparing coeff of x^3 & constant term

$0 = b$ & $c = c^2$

$\Rightarrow c = 0, 1$ ....(4)

put $b = 0$ in equation (3) $\Rightarrow c = 0$ ....(5)

$\Rightarrow f(x) = x^2 + bx + c = x^2$

$\Rightarrow \boxed{f(x) = x^2}$ be the required polynomial (only are polynomial)

13. Case I $\quad x \ge 0, y \ge 0 \qquad x + |y| - y = 12$

$|x| + x + y = 10 \qquad x + y - y = 12$

$2x + y = 10 \quad \& \quad x = 12$

$y = -14$ reject

Case II $\quad x < 0, y \ge 0 \quad \& \quad x + |y| - y = 12$

$|x| + x + y = 10 \qquad x + y - y = 12$

$\Rightarrow -x + x + y = 10 \qquad x = 12 > 0$ reject

$y = 10$

Case III $\quad x \ge 0, y < 0 \quad \& \quad x + |y| - y = 12$

$|x| + x + y = 10 \qquad x - y - y = 12$

$\Rightarrow 2x + y = 10 \qquad x - 2y = 12$

Slove $x = 32/5, y = -14/5$

$\Rightarrow \boxed{x + y = \frac{18}{5}}$

Case IV $x < 0, y < 0 \Rightarrow |x| + x + y = 10$

$y = 10$ reject

$\Rightarrow \boxed{x + y = \frac{18}{5}}$ Ans.

14. $$\frac{a^2x^2(bx+ay)+b^2y^2(bx+ay)+bx2a^2y^2+ay\cdot 2b^2x^2}{(bx+ay)}$$

$$a^2x^2+b^2y^2+\frac{2abxy(ay+bx)}{(bx+ay)}$$

$= (ax)^2 + (by)^2 + 2\ (ax)\ (by)$

$= (ax + by)^2$

15. In a triangle the sum of any two sides is greater than the third.

$\Rightarrow 10 + 24 > x \Rightarrow x < 34 \qquad ...(1)$

$10 + x > 24 \Rightarrow x > 14 \qquad ...(2)$

$24 + x < 10 \Rightarrow x > -14 \qquad ...(3)$

also $x > 0$

$(1) \cap (2) \cap (3)$

$14 < x < 34$

Now $\cos A = \dfrac{b^2+c^2-a^2}{2bc} > 0$

for acute angle

$\Rightarrow a^2 < b^2 + c^2$ where $a \ge b$

$a \ge c$

$\Rightarrow a = 24,\ b = 10,\ \boxed{c = x = 22, 23, 24}$

when $b = 10,\ c = 24,\ \boxed{a = x = 25}$

$\Rightarrow \boxed{\text{value of } x = 22, 23, 24, 25}$]

16. Substituting n for 2014, we get

$\sqrt{n}x^3 - (1+2n)x^2 + 2 = \sqrt{n}x^3 - x^2 - 2nx^2 + 2$

$= x^2(\sqrt{n}x - 1) - 2(nx^2 - 1) = 0$

Noting that $nx^2 - 1$ factors as a difference of squares to $(\sqrt{n}x-1)(\sqrt{n}x+1)$, we can factor the left side as $(\sqrt{n}x-1)(x^2-2(\sqrt{n}x+1))$.

This means that $\frac{1}{\sqrt{n}}$ is a root, and the other two roots are the roots of $x^2 - 2\sqrt{nx} - 2$. NOte that the constant term of the quadratic is negative, so one of the two roots is positive and the other is negative. In addition, by Vieta's Formulas, the roots sum to $2\sqrt{n}$, so the positive root must be greater than $2\sqrt{n}$ in order to produce this sum when added to a negative value. Since $0 < \frac{1}{\sqrt{2014}} < 2\sqrt{2014}$ is clearly true, $x_2 = \frac{1}{\sqrt{2014}}$ and $x_1 + x_3 = 2\sqrt{2014}$. Multiplying these values together, we find that $x_2(x_1 + x_3) = \boxed{002}$

17. $\sqrt{a-\sqrt{a+x}} = x \ : x \ge 0$

$\sqrt{a-\sqrt{a+x}} = x^2$ & $a^2 > a + x$

$a - x^2 = \sqrt{a+x}$

$(a - x^2)^2 = a + x$

$a^2 + x^4 - 2ax^2 = a + x$

$a^2 - (2x^2 + 1)a + x^4 - x = 0$

$$a = \frac{2x^2+1 \pm \sqrt{(2x^2+1)^2 - 4(x^4-x)}}{2}$$

$$= \frac{2x^2+1 \pm \sqrt{4x^2+4x+1}}{2}$$

$$a = \frac{2x^2+1 \pm (2x+1)}{2}$$

$a = x^2 + x + 1$ or $a = x^2 - x$

when $x^2 + x + 1 - a = 0$

$$x = \frac{-1 \pm \sqrt{4a-3}}{2}$$

$$x = \frac{\sqrt{4a-3}-1}{2}, \underbrace{\frac{-1-\sqrt{4a-3}}{2} < 0}_{\text{reject}}$$

when $x^2 - x - a = 0$

$$x = \frac{1 \pm \sqrt{4a+1}}{2}$$

$$x = \underbrace{\frac{1+\sqrt{4a+1}}{2}}_{\text{reject}}, \quad \underbrace{\frac{1-\sqrt{4a+1}}{2} < 0}_{\text{reject}}$$

at $a=1$

$x = \frac{1+\sqrt{5}}{2}$

here $a - \sqrt{a+x} < 0$

which is not possible

$$\Rightarrow \text{sum of solutions} = \frac{\sqrt{4a-3}-1}{2}$$

18. $ax^2 + bx + c = 0$: a, b, c $\in$ I & $a \neq 0$

$D = b^2 - 4ac$

$\Rightarrow b^2 - D = 4ac$ .....(1)

4ac is multiple of 4 now for L.H.S.

Case I when b is even integer

Let $b = 2r : r \in I$

$\Rightarrow b^2 = 4r^2$

$\Rightarrow b^2 - D$ multiple of 4 il D is multide of 4

$\Rightarrow$ D can be 24 or 28

Case II When b is odd integer

let $b = (2r+1) \Rightarrow b^2 = (4r^2 + 4r + 1) = (4k+1)$ ...(2)

or if $b = (2r-1) \Rightarrow b^2 = (4K'+1)$

where K, K'$\in$I

$\Rightarrow$ if b is odd $\Rightarrow b^2 = 4k + 1$ Type

[From equation (2) & (3)]

$\Rightarrow b^2 - D$ will be multiple of 4

if D is odd (only (4n+1) type)

$\Rightarrow$ D can not be 23

19. $f_{1981}(x) = f_{1982}(x)$

$f_{1981}(x) = f_{1982}(x) = |1 - f_{1981}(x)|$

$$\Rightarrow f_{1981}(x) = \frac{1}{2}$$

$$\Rightarrow f_{1981}(x) = \frac{1}{2} = |1 - f_{1980}(x)|$$

$$\Rightarrow f_{1980}(x) = \frac{1}{2}, \frac{3}{2} \text{ (2 values)}$$

$$f_{1879}(x) = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \text{ (3 values)}$$

Similarily $f_1(x) = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}........\frac{3961}{2}$ (1981 values)

$|1 - x| = \frac{1}{2}, \frac{3}{2}........$ (1981) values

$\Rightarrow$ number of values of x = 3962

20. $\log_8 3 = p$ .....(1)

$\log_8 5 = q$ .....(2)

(1) $\times$ (2)

$$\frac{\log 3}{\log 8} \times \frac{\log 5}{\log 3} = pq$$

$\log 5 = 3pq \log 2$ .....(3)

$$\frac{\log 5}{\log 2} + 1 = 3pq + 1 \text{ from equation (3)}$$

$$\frac{\log 5+\log 2}{\log 2} = 1 + 3pq$$

$$\Rightarrow \frac{\log 10}{1+3pq} = \log 2 \Rightarrow \log 2 = \frac{1}{1+3pq}$$

put in equaiton (3)

$$\Rightarrow \boxed{\log 5 = \frac{3pq}{1+3pq}} \text{ here base of log is 10}$$

21. $x^3 - x^2 + x - 2 = 0 \begin{matrix} \nearrow p \\ \rightarrow q \\ \searrow r \end{matrix}$

$p^3 + q^3 + r^3 = (p+q+r)^3 - 3(p+q)\ (q+r)\ (r+p)$

$= (1)^3 - 3\ (1 - r)\ (1 - p)\ (1 - q)$

$p^3 + q^3 + r^3 = 1 - 3\ (1 - p)\ (1 - q)\ (1 - r)$ ....(2)

[$\because$ using (1) $p + q = 1 - r$]

$x^3 - x^2 + x - 2 = (x-p)\ (x-q)\ (x-r)$

put $x = 1$

$-1 = (1-p)\ (1-q)\ (1-r)$ put in equation (2)

$$\Rightarrow \boxed{p^3+q^3+r^3 = 1-3(-1) = 4}$$

22. $x(y + 4z + 2) = 22$ ...(2)

$x(x + 2y + 4z + 2 - x - y) = 22$

from (1)

$x(12 + 2 - x - y) = 2$

$$\Rightarrow 14 - x - y = \frac{24}{x}$$

$$\Rightarrow y = \left(14 - x - \frac{24}{x}\right) \quad ...(4)$$

put in equation (1)

$4z = 12 - x - 2y$

$$= 12 - x - 2\left(14 - x - \frac{24}{x}\right)$$

$$4z = x - 16 + \left(\frac{48}{x}\right) \quad ....(5)$$

$xyz = 6$

$\Rightarrow xy(4z) = 24$

$$x\left(14 - x - \frac{24}{x}\right)\left(x - 16 + \frac{48}{x}\right) = 24$$

$(14x - x^2 - 24)\ (x^2 - 16x + 48) = 24\ x$

$\Rightarrow x^4 - 32x^3 + 296x^2 - 1032\ x + 1152 = 0$

$(x - 6)\ (x^3 - 26x^2 + 140x - 192) = 0$

$\Rightarrow$ 4 different value of x statisfy the equation

$\Rightarrow$ 4 triplets statisly the given system of equaiton

23. $g(x) = (x^5 + x^4) + (x^3 + x^2) + (x + 1)$

$= x^4\ (x + 1) + x^2\ (x + 1) + 1\ (x + 1)$

$= (x + 1)\ (x^4 + x^2 + 1)$

$g(x^{12}) = (x + 1)\ (x^4 + x^2 + 1)\ Q(x)$ + remainder

$(x^{12})^5 + (x^{12})^4 + (x^{12})^3 + (x^{12})^2 + (x^{12}) + 1$

$= (x + 1)\ (x^4 + x^2 + 1)Q(x)$ + Remainder ...(A)

Put $x = -1$

$\Rightarrow$ 6 = Ramainder

Now put $x^2 = w$

$\Rightarrow x^4 + x^2 + 1 = w^2 + w + 1 = 0$

ω is cube root of unity & $w^3 = 1$

6 = Remainder

Now check option by puting $x = -1$, $x^2 = w$

Option (1) is correct

24. **Case I**

when $x < 0$

$\Rightarrow y - x < \sqrt{x^2}$

$y - x < |x|$ [$\because x < 0 \Rightarrow |x| = -x$]

$y - x < |x|$

$\Rightarrow y - x < -x$

$\Rightarrow y < 0$

Case II

When $x \geq 0$

$\Rightarrow y - x < \sqrt{x^2}$ [$\because x \geq 0\ |x| = -x$]

$y - x < |x|$

$y - x < x$

$y < 2x$

$\Rightarrow y < 0$ or $y < 2x$ or both inequalities hold

25. $x^8 = \left(x+\frac{1}{2}\right)q_1(x) + r_1$ ...(1)

put $x = -\frac{1}{2} \Rightarrow r_1 = \left(\frac{1}{2}\right)^8$ put in (1)

$\left(x+\frac{1}{2}\right)q_1(x) = x^8 - \left(\frac{1}{2}\right)^8$

$\left(x+\frac{1}{2}\right)q_1(x)$

$= \left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)\left(x^2+\frac{1}{4}\right)\left(x^4+\frac{1}{16}\right)$

$q_1(x) = \left(x-\frac{1}{2}\right)\left(x^2+\frac{1}{4}\right)\left(x^4+\frac{1}{16}\right)$

$r_2 = q_1\left(-\frac{1}{2}\right)$

$= \left(-\frac{1}{2}-\frac{1}{2}\right)\left(\frac{1}{4}+\frac{1}{4}\right)\left(\frac{1}{16}+\frac{1}{16}\right)$

$= -1\left(\frac{1}{2}\right)\left(\frac{1}{8}\right)$

$r_2 = -\frac{1}{16}$

26. $f(x) + f(y) = f(x+y) - xy - 1$ ....(1)

put $y = 1$ & $f(1) = 1$

$f(x) + f(1) = f(x+1) - x - 1$

$f(x+1) - f(x) = x + 2$

put $x = 0$, $f(1) - f(0) = 2 \Rightarrow f(0) = -1$

$x = -1$, $f(0) - f(-1) = 1 \Rightarrow f(-1) = -2$

$x = -2$, $f(-1) - f(-2) = 0 \Rightarrow$ **$f(-2) = -2$**

$x = -3$, $f(-2) - f(-3) = -1 \Rightarrow f(-3) = -1$

$f(2) = 4$, $f(3) = 8$,.....

$\Rightarrow$ only one value for which $f(n) = n$

27. $f(x) = \dfrac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}+2\right)}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$

$= \dfrac{\left[\left(x+\frac{1}{x}\right)^3\right]^2 - \left(x^3+\frac{1}{x^3}\right)^2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$

$= \left(x+\frac{1}{x}\right)^3 - \left(x^3+\frac{1}{x^3}\right)\left[\because \frac{a^2-b^2}{a+b} = a-b\right]$

$f(x) = 3\left(x+\frac{1}{x}\right)$ $\left[\begin{array}{l}\because x>0 \\ \Rightarrow x+\frac{1}{x}\geq 2\end{array}\right]$

$f(x)_{min} = 3 \times 2 = 6$

28. $f(f(x) = \dfrac{c(f(x)}{2f(x)+3} = \dfrac{c\left(\frac{cx}{2x+3}\right)}{2\left(\frac{cx}{2x+3}\right)+3}$

$f(fx)) = \dfrac{c^2x}{2cx+3(2x+3)}$ ....(1)

$f(fx)) = x$ given ....(2)

$\Rightarrow \dfrac{c^2x}{2cx+6x+9} = x$ (1) = (2)

$c^2x = 9x + (2c+6)x^2$

comparing coeff. $c^2 = 9 \Rightarrow c = \pm 3$ ...(A)

& $2c + 6 = 0$ ...(B)

(A) $\cap$ (B) $\Rightarrow c = -3$

29. $8x^3 - 4x^2 + 45 = (x-r)^2\left(8x+\frac{45}{r^2}\right)$

$= (x^2 - 2xr + r^2)\left(8x+\frac{45}{r^2}\right)$

comparing coeff of x^2

$-4 = \frac{45}{r^2} - 16r$

$\Rightarrow 16r^3 - 4r^2 - 45 = 0$

$(2r-3)(8r^2+10r+15) = 0$

$\Rightarrow \boxed{r = \frac{3}{2}}$

30. $x^2 + x + 1 = 0 \begin{smallmatrix} \nearrow w \\ \searrow w^2 \end{smallmatrix}$ is

$w^2 + w + 1 = 0$ & $w^3 = 1$

if polynomial $x^{2n} + 1\ (x + 1)^{2n}$ is divisible by $x^2 + x + 1$ then polynomial must have w, w^2 as the solution

Now $2^{2x} + 1\ ((x+1)^2)^4 = 2^{2n} + 1 + \underbrace{(x^2 + x + 1)^n}$

$= x^{2n} + 1 + (0 + x)^n$

$= x^{2n} + x^n + 1$

$= w^{2n} + w^n + 1 = \begin{cases} 3; & \text{if n is multiple of 3} \\ 0; & \text{if n is not multiple 3} \end{cases}$

n = 21

31. $x^2 - 3xy + 2y^2 - z^2 = 31$ ...(1)

$-x^2 + 6yz + 2z^2 = 44$ ...(2)

$x^2 + xy + 8z^2 = 100$ ...(3)

---

adding

$(x^2 - 2xy + y^2) + y^2 + 6yz + 9z^2 = 175$

$(x - y)^2 + (y + 3z)^2 = 175$

$\because$ x,y,z are integers

sum of squares of two integers can not be 175

$\Rightarrow$ no sol^n.

32. $(2x)^{\log_h 2} = (3x)^{\log_h 3}$

taking log on both sides

$\log_h 2\ \log(2x) = \log_h 3\ \log(3x)$

$\frac{\log 2}{\log h}$ [log2 + logx] = $\frac{\log 3}{\log h}$ [log3 + logx]

$(\log 2)^2 - (\log 3)^2 = \log x\ (\log 3 - \log 2)$

$(\log 2 + \log 3)(\log 2 - \log 3) = \log x \left(\log \frac{3}{2}\right)$

$(\log 6) \left(-\log \frac{3}{2}\right) = \log x \log \frac{3}{2}$

$\Rightarrow \log x = -\log 6 = \log \frac{1}{6}$

$x = \frac{1}{6}$

33. $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ ....(1)

replace $x \xrightarrow{\text{by}} \frac{1}{x}$

$f\left(\frac{1}{x}\right) + 2(x) = \frac{3}{x}$ ....(2)

(1) – (2) × 2

$-3f(x) = 3x - \frac{6}{x}$

$f(x) = \frac{2}{x} - x = \frac{2 - x^2}{x}$

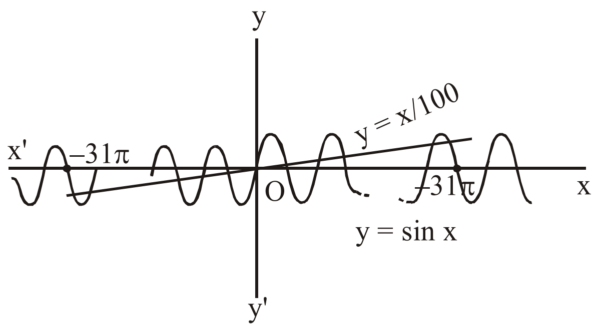
now $f(x) = f(-x)$

$\frac{2 - x^2}{x} = \frac{2 - x^2}{-x}$

$(2 - x^2) = 0$

$x = \pm\sqrt{2}$

34.



Draw graph y = sinx & y = $\frac{x}{100}$ and find point of intersections

Here number of solutions of $\frac{x}{100}$ = sin x is 63

35. $a^b = b^a$ & $b = 9a$

$\Rightarrow a^{9a} = (9a)^a$

$\left(\frac{a^9}{9a}\right)^a = 1 \Rightarrow \frac{a^9}{9a} = 1$ [$\because$ a ≠ 0]

$\Rightarrow a^9 = 9a \Rightarrow a^8 = 9$

$a = 9^{\frac{1}{8}} = 3^{\frac{2}{8}} = 3^{\frac{1}{4}}$

$\boxed{a = \sqrt[4]{3}}$