KEY CONCEPTS (INVERSE TRIGONOMETRY FUNCTION)

GENERAL DEFINITION(S):

1. $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ etc. denote angles or real numbers whose sine is x, whose cosine is x and whose tangent is x, provided that the answers given are numerically smallest available. These are also written as arc sinx, arc cosx etc.

If there are two angles one positive & the other negative having same numerical value, then positive angle should be taken.

2. PRINCIPAL VALUES AND DOMAINS OF INVERSE CIRCULAR FUNCTIONS :

(i) $y = \sin^{-1} x$ where $-1 \le x \le 1$; $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $\sin y = x$.

(ii) $y = \cos^{-1} x$ where $-1 \le x \le 1$; $0 \le y \le \pi$ and $\cos y = x$.

(iii) $y = \tan^{-1} x$ where $x \in R$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and $\tan y = x$.

(iv) $y = \text{cosec}^{-1} x$ where $x \le -1$ or $x \ge 1$; $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$ and $\text{cosec}\, y = x$.

(v) $y = \sec^{-1} x$ where $x \le -1$ or $x \ge 1$; $0 \le y \le \pi$; $y \ne \frac{\pi}{2}$ and $\sec y = x$.

(vi) $y = \cot^{-1} x$ where $x \in R$, $0 < y < \pi$ and $\cot y = x$.

NOTE THAT :
(a) 1st quadrant is common to all the inverse functions .
(b) 3rd quadrant is **not used** in inverse functions .
(c) 4th quadrant is used in the **CLOCKWISE DIRECTION** i.e. $-\frac{\pi}{2} \le y \le 0$.

3. PROPERTIES OF INVERSE CIRCULAR FUNCTIONS :

P−1
(i) $\sin(\sin^{-1} x) = x$, $-1 \le x \le 1$
(ii) $\cos(\cos^{-1} x) = x$, $-1 \le x \le 1$
(iii) $\tan(\tan^{-1} x) = x$, $x \in R$
(iv) $\sin^{-1}(\sin x) = x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$
(v) $\cos^{-1}(\cos x) = x$; $0 \le x \le \pi$
(vi) $\tan^{-1}(\tan x) = x$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$

P−2
(i) $\text{cosec}^{-1} x = \sin^{-1} \frac{1}{x}$; $x \le -1$, $x \ge 1$

(ii) $\sec^{-1} x = \cos^{-1} \frac{1}{x}$; $x \le -1$, $x \ge 1$

(iii) $\cot^{-1} x = \tan^{-1} \frac{1}{x}$; $x > 0$

$= \pi + \tan^{-1} \frac{1}{x}$; $x < 0$

P−3
(i) $\sin^{-1}(-x) = -\sin^{-1} x$, $-1 \le x \le 1$
(ii) $\tan^{-1}(-x) = -\tan^{-1} x$, $x \in R$
(iii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$, $-1 \le x \le 1$
(iv) $\cot^{-1}(-x) = \pi - \cot^{-1} x$, $x \in R$

P−4
(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ $\quad -1 \le x \le 1$
(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ $\quad x \in R$
(iii) $\text{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$ $\quad |x| \ge 1$

P–5 $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \dfrac{x+y}{1-xy}$ where $x > 0$, $y > 0$ & $xy < 1$

$= \pi + \tan^{-1} \dfrac{x+y}{1-xy}$ where $x > 0$, $y > 0$ & $xy > 1$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \dfrac{x-y}{1+xy}$ where $x > 0$, $y > 0$

P–6 **(i)** $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$ where $x \ge 0$, $y \ge 0$ & $(x^2 + y^2) \le 1$

Note that : $x^2 + y^2 \le 1 \Rightarrow 0 \le \sin^{-1} x + \sin^{-1} y \le \dfrac{\pi}{2}$

(ii) $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$ where $x \ge 0$, $y \ge 0$ & $x^2 + y^2 > 1$

Note that : $x^2 + y^2 > 1 \Rightarrow \dfrac{\pi}{2} < \sin^{-1} x + \sin^{-1} y < \pi$

(iii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$ where $x \ge 0$, $y \ge 0$

(iv) $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left[xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right]$ where $x \ge 0$, $y \ge 0$

P–7 If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\dfrac{x+y+z-xyz}{1-xy-yz-zx}\right]$ if, $x > 0, y > 0, z > 0$ & $xy + yz + zx < 1$

Note : (i) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$ then $x + y + z = xyz$

(ii) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \dfrac{\pi}{2}$ then $xy + yz + zx = 1$

P–8 $2\tan^{-1} x = \sin^{-1} \dfrac{2x}{1+x^2} = \cos^{-1} \dfrac{1-x^2}{1+x^2} = \tan^{-1} \dfrac{2x}{1-x^2}$

Note very carefully that :

$$\sin^{-1} \frac{2x}{1+x^2} = \begin{cases} 2\tan^{-1} x & \text{if } |x| \le 1 \\ \pi - 2\tan^{-1} x & \text{if } x > 1 \\ -\left(\pi + 2\tan^{-1} x\right) & \text{if } x < -1 \end{cases}$$

$$\cos^{-1} \frac{1-x^2}{1+x^2} = \begin{cases} 2\tan^{-1} x & \text{if } x \ge 0 \\ -2\tan^{-1} x & \text{if } x < 0 \end{cases}$$

$$\tan^{-1} \frac{2x}{1-x^2} = \begin{cases} 2\tan^{-1} x & \text{if } |x| < 1 \\ \pi + 2\tan^{-1} x & \text{if } x < -1 \\ -\left(\pi - 2\tan^{-1} x\right) & \text{if } x > 1 \end{cases}$$

REMEMBER THAT :

(i) $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \dfrac{3\pi}{2} \Rightarrow x = y = z = 1$

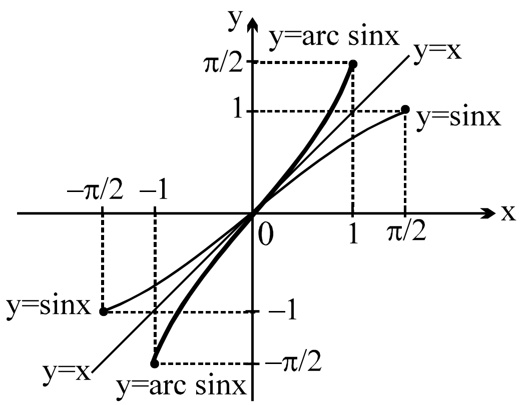
(ii) $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi \Rightarrow x = y = z = -1$

(iii) $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$ and $\tan^{-1} 1 + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \frac{\pi}{2}$

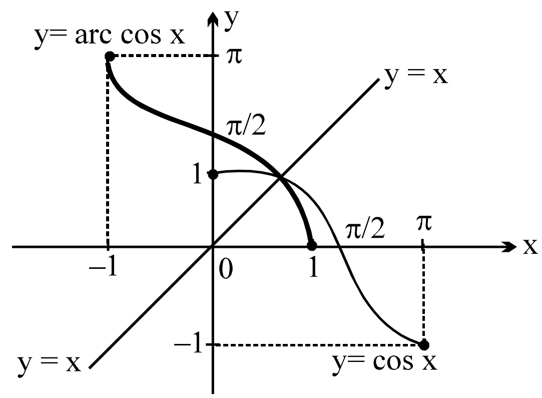
INVERSE TRIGONOMETRIC FUNCTIONS

Some Useful Graphs

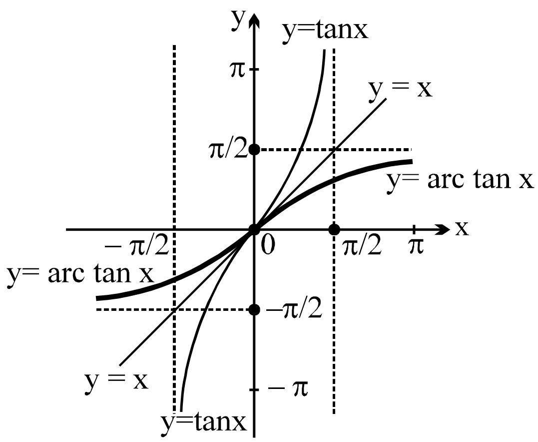
1. $y = \sin^{-1} x$, $|x| \le 1$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



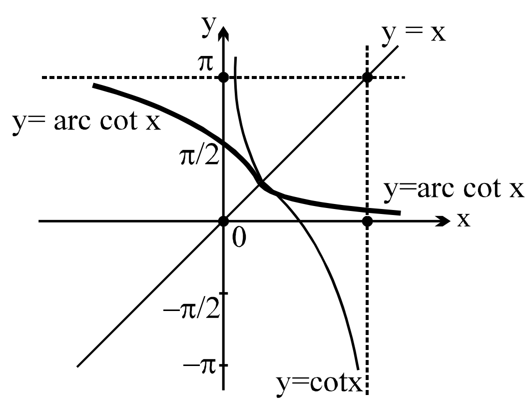
2. $y = \cos^{-1} x$, $|x| \le 1$, $y \in [0, \pi]$



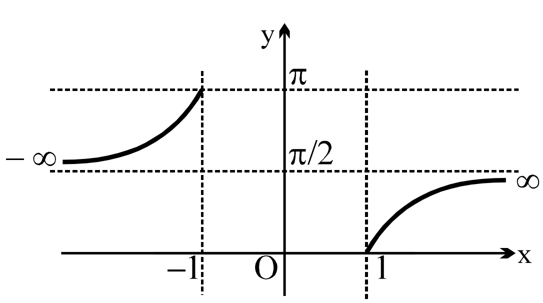
3. $y = \tan^{-1} x$, $x \in R$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



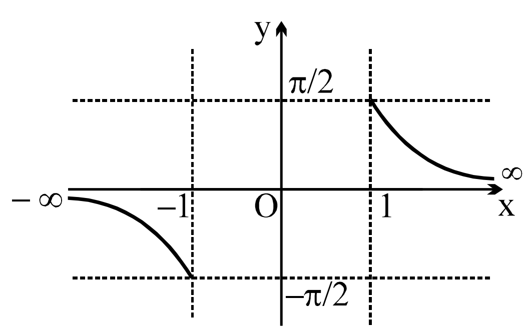
4. $y = \cot^{-1} x$, $x \in R$, $y \in (0, \pi)$



5. $y = \sec^{-1} x$, $|x| \ge 1$, $y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

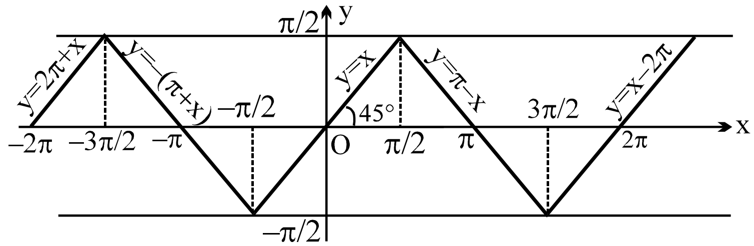


6. $y = \text{cosec}^{-1} x$, $|x| \ge 1$, $y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



7. (a) $y = \sin^{-1}(\sin x)$, $x \in R$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$,

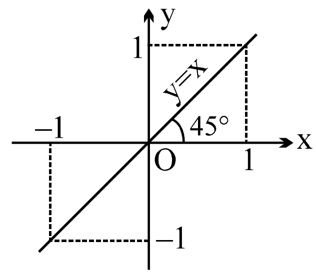
Periodic with period 2π



7.(b) $y = \sin(\sin^{-1}x)$,

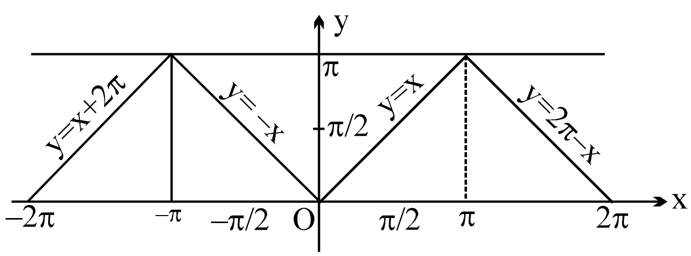
$= x$

$x \in [-1, 1]$, $y \in [-1, 1]$, y is aperiodic



8. (a) $y = \cos^{-1}(\cos x)$, $x \in R, y \in [0, \pi]$, periodic with period 2π

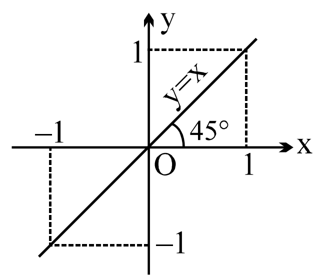
$= x$



8. (b) $y = \cos(\cos^{-1}x)$,

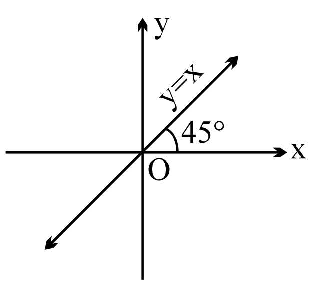
$= x$

$x \in [-1, 1]$, $y \in [-1, 1]$, y is aperiodic



9. (a) $y = \tan(\tan^{-1}x)$, $x \in R$, $y \in R$, y is aperiodic

$= x$

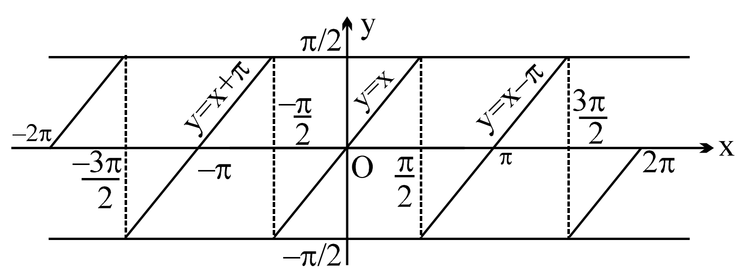


9. (b) $y = \tan^{-1}(\tan x)$,

$= x$

$x \in R - \left\{(2n-1)\frac{\pi}{2} \;\; n \in I\right\}$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

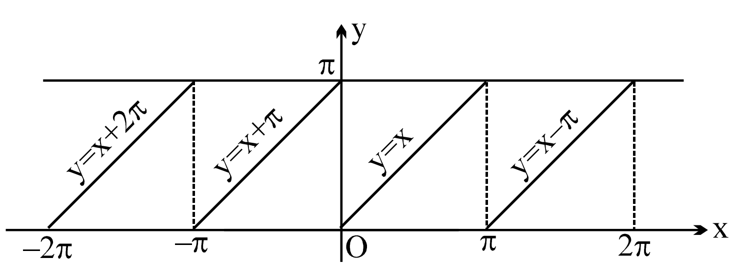
periodic with period π



10. (a) $y = \cot^{-1}(\cot x)$,

$= x$

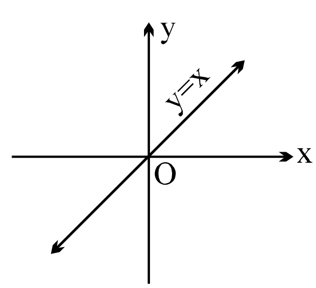
$x \in R - \{n\pi\}$, $y \in (0, \pi)$, periodic with π



10. (b) $y = \cot(\cot^{-1}x)$,

$= x$

$x \in R$, $y \in R$, y is aperiodic

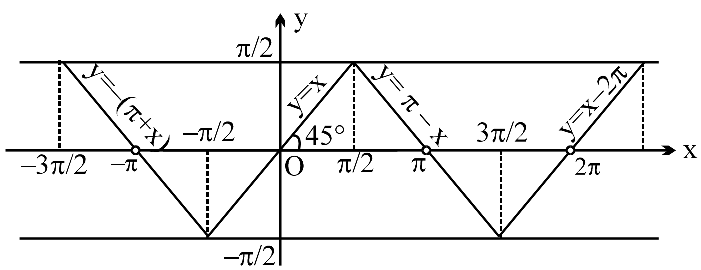


11. (a) $y = \text{cosec}^{-1}(\text{cosec}\, x)$,

$= x$

$x \in R - \{n\pi,\ n \in I\}$, $y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

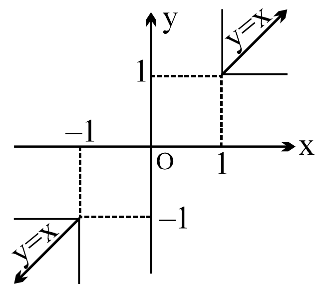
y is periodic with period 2π



11. (b) $y = \text{cosec}(\text{cosec}^{-1} x)$,

$= x$

$|x| \geq 1,\ |y| \geq 1,$ y is aperiodic

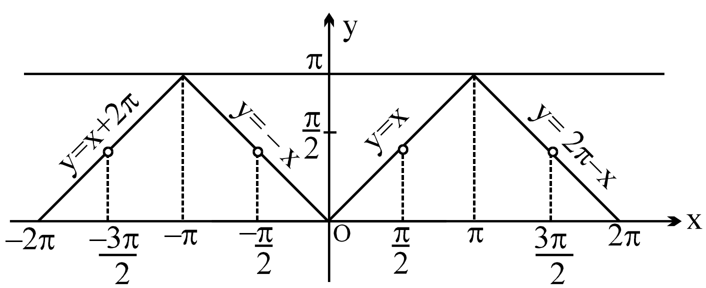


12. (a) $y = \sec^{-1}(\sec x)$,

$= x$

y is periodic with period 2π;

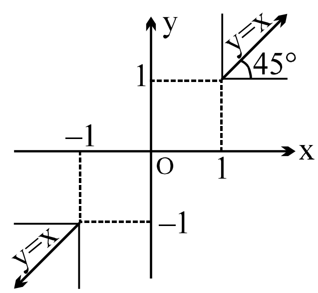
$x \in R - \left\{(2n-1)\frac{\pi}{2}\ \ n \in I\right\}$ $y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



12. (b) $y = \sec(\sec^{-1} x)$,

$= x$

$|x| \geq 1$; $|y| \geq 1]$, y is aperiodic



1. Domain of f(x) = $\cos^{-1} x + \cot^{-1} x + \text{cosec}^{-1} x$ is
(A) [–1 , 1] (B) R
(C) (–∞,–1] ∪ [1,∞) (D) {–1,1}

2. Range of f(x) = $\sin^{-1} x + \tan^{-1} x + \sec^{-1} x$ is
(A) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$
(C) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (D) None of these

3. If $x \ge 0$ and $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$, then
(A) $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$ (B) $0 < \theta \le \frac{\pi}{4}$
(C) $0 < \theta \le \frac{\pi}{2}$ (D) $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

4. $\text{cosec}^{-1}(\cos x)$ is real if
(A) x ∈ [–1,1]
(B) x ∈ R
(C) x is an odd multiple of π / 2
(D) x is a multiple of π

5. If $\cos[\tan^{-1}\{\sin(\cot^{-1}\sqrt{3})\}] = y$, then :
(A) y = 4/5 (B) $y = \frac{2}{\sqrt{5}}$
(C) $y = -\frac{2}{\sqrt{5}}$ (D) $y^2 = 10/11$

6. The value of $\tan\left[\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is
(A) 6/17 (B) 7/16
(C) 5/7 (D) 17/6

7. The value of $\sin^{-1}[\cos\{\cos^{-1}(\cos x) + \sin^{-1}(\sin x)\}]$, where $x \in \left(\frac{\pi}{2}, \pi\right)$ is
(A) π/2 (B) π/4
(C) –π/4 (D) –π/2

8. $\tan^{-1}a + \tan^{-1} b$, where a > 0, b > 0, ab > 1, is equal to
(A) $\tan^{-1}\left(\frac{a+b}{1-ab}\right)$
(B) $\tan^{-1}\left(\frac{a+b}{1-ab}\right) - \pi$
(C) $\pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right)$
(D) $\pi - \tan^{-1}\left(\frac{a+b}{1-ab}\right)$

9. The value of
$$\cot^{-1}\left\{\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1+\sin x} - \sqrt{1+\sin x}}\right\}, \frac{\pi}{2} < x < \pi, \text{ is}$$
(A) $\pi - \frac{\pi}{2}$ (B) $\frac{\pi}{2} + \frac{x}{2}$
(C) $\frac{x}{2}$ (D) $2\pi - \frac{x}{2}$

10. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to
(A) 2π/3 (B) π/3
(C) π/6 (D) π

11. If $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \frac{\pi}{45}$, then
(A) x = tan2° (B) x = tan 4°
(C) x = tan (1/4)° (D) x = tan 8°

12. $\cos^{-1}\left\{\frac{1}{2}x^2 + +\sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right\} = \cos^{-1}\frac{x}{2} - \cos^{-1} x$ holds for
(A) |x| ≤ 1 (B) x ∈ R
(C) 0 ≤ x ≤ 1 (D) –1 ≤ x ≤ 0

13. All possible values of p and q for which $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$ holds, is
(A) $p = 1, q = \frac{1}{2}$ (B) $p > 1, p = \frac{1}{2}$
(C) $0 \le p \le 1, q = \frac{1}{2}$ (D) None of these

14. The set of values of "x" for which the formula $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ is true is
(A) (–1,0) (B) [0,1]
(C) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ (D) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

15. The value of x satisfying $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1}x$ are
(A) $0, \frac{1}{2}$ (B) 0
(C) 1,–1 (D) None of these

16. The number of solutions of the equation

$\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is

(A) 0 (B) 1
(C) 2 (D) 3

17. Which one of the following correct ?
(A) $\tan 1 > \tan^{-1} 1$ (B) $\tan 1 < \tan^{-1} 1$
(C) $\tan 1 = \tan^{-1} 1$ (D) None of these

18. If $\sum_{i=1}^{n}\cos^{-1}\alpha_1 = 0$, then $\sum_{i=1}^{n}\alpha_1 =$

(A) n (B) – n
(C) 0 (D) None of these

19. If $\sum_{i=1}^{2n}\sin^{-1} = n\pi$ then $\sum_{i=1}^{2n}x_i$ is equal to

(A) n (B) 2n
(C) $\frac{n(n+1)}{2}$ (D) None of these

20. If $u = \cot^{-1}\sqrt{\tan\alpha} - \tan^{-1}\sqrt{\tan\alpha}$, then $\tan\left(\frac{\pi}{4}-\frac{u}{2}\right)$ is equal to

(A) $\sqrt{\tan\alpha}$ (B) $\sqrt{\cot\alpha}$
(C) $\tan\alpha$ (D) $\cot\alpha$

21. If $[\cot^{-1} x] + [\cos^{-1} x] = 0$ then complete set of values of 'x' is
(where [ ] denotes the greatest integer function)
(A) (cos 1, 1] (B) (cot 1, cos 1)
(C) (cot 1, 1] (D) None of these

22. The solution of the equation

$\sin^{-1}\left(\tan\frac{\pi}{4}\right)-\sin^{-1}\left(\sqrt{\frac{3}{x}}\right)-\frac{\pi}{6}=0$ is

(A) x = 2 (B) x = –4
(C) x = 4 (D) None of these

23. The number of solution (s) of the equation, $\sin^{-1} x + \cos^{-1}(1 - x) = \sin^{-1}(-x)$, is/ are
(A) 0 (B) 1
(C) 2 (D) more than 2

24. The complete solution set of the inequality $[\cot^{-1} x]^2 - 6[\cot^{-1}x] + 9 \le 0$ is
(where [] denotes the greatest integer function
(A) $(-\infty, \cot 3]$ (B) [Cot 3, cot 2]
(C) $[\cot 3, \infty]$ (D) None of these

25. The value of $\tan^{-1}\left(\frac{a}{b+c}\right)+\tan^{-1}\left(\frac{b}{c+a}\right)$, if $\angle C = 90°$ in triangle ABC, is
(A) $\pi/4$ (B) $\pi/3$
(C) $\pi/2$ (D) π

26. If $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to **[AIEEE - 2005]**
(A) $2\sin 2\alpha$ (B) 4
(C) $4\sin^2\alpha$ (D) $-4\sin^2\alpha$

27. If $\sin^{-1}\left(\frac{x}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$ then a value of x is **[AIEEE -2007]**
(A) 1 (B) 3
(C) 4 (D) 5

28. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is **AIEEE-2008]**
(A) 3/17 (B) 1/17
(C) 2/17 (D) 6/17

29. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is : **[AIEEE-2015]**

(A) $\frac{3x-x^3}{1+3x^2}$ (B) $\frac{3x+x^3}{1+3x^2}$

(C) $\frac{3x-x^3}{1-3x^2}$ (D) $\frac{3x+x^3}{1-3x^2}$

30. The value of $\sin^{-1}(-\sqrt{3}/2)$ is -
(A) $-\pi/3$
(B) $-2\pi/3$
(C) $4\pi/3$
(D) $5\pi/3$

31. $\cos\left(2\tan^{-1}\left(\frac{1}{7}\right)\right)$ equals -

(A) $\sin(4\cot^{-1} 3)$ (B) $\sin(3\cot^{-1} 4)$
(C) $\cos(3\cot^{-1} 4)$ (D) $\cos(4\cot^{-1} 4)$

32. The value of

$$\sec\left[\sin^{-1}\left(-\sin\frac{50\pi}{9}\right)+\cos^{-1}\cos\left(-\frac{31\pi}{9}\right)\right]$$

is equal to -

(A) $\sec\frac{10\pi}{9}$ (B) $\sec\frac{\pi}{9}$

(C) 1 (D) –1

33. $(\sin^{-1}x)^2+(\sin^{-1}y)^2+2(\sin^{-1}x)(\sin^{-1}y)=\pi^2$, then x^2+y^2 is equal to -

(A) 1 (B) 3/2

(C) 2 (D) 1/2

34. $\cot^{-1}[(\cos\alpha)^{1/2}]-\tan^{-1}[(\cos\alpha)^{1/2}]=x$, then sin x =

(A) $\tan^2\left(\frac{\alpha}{2}\right)$ (B) $\cot^2\left(\frac{\alpha}{2}\right)$

(C) tan α (D) $\cot\left(\frac{\alpha}{2}\right)$

35. $\tan(\cos^{-1}x)$ is equal to

(A) $\frac{x}{1+x^2}$ (B) $\frac{\sqrt{1+x^2}}{x}$

(C) $\frac{\sqrt{1-x^2}}{x}$ (D) $\sqrt{1-2x}$

36. If $x=2\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\tan^{-1}\left(\sqrt{3}\right)$ and $y=\cos\left(\frac{1}{2}\sin^{-1}\left(\sin\frac{x}{2}\right)\right)$ then which of the following statements holds good ?

(A) $y=\cos\frac{3\pi}{16}$ (B) $y=\cos\frac{5\pi}{16}$

(C) $x=4\cos^{-1}y$ (D) none of these

37. $\tan^{-1}2+\tan^{-1}3=\operatorname{cosec}^{-1}x$, then x is equal to -

(A) 4 (B) $\sqrt{2}$

(C) $-\sqrt{2}$ (D) none of these

38. If $\tan(\cos^{-1}x)=\sin(\cot^{-1}1/2)$ then x is equal to -

(A) $1/\sqrt{5}$ (B) $2/\sqrt{5}$

(C) $3/\sqrt{5}$ (D) $\sqrt{5}/3$

39. $\sin^{-1}(2x\sqrt{1-x^2})=2\sin^{-1}x$ is true if -

(A) $x\in[0,1]$ (B) $\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$

(C) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (D) $\left[-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right]$

40. Domain of the explicit form of the function y represented implicitly by the equation $(1+x)\cos y-x^2=0$ is -

(A) (–1,1] (B) $\left(-1,\frac{1-\sqrt{5}}{2}\right]$

(C) $\left[\frac{1-\sqrt{5}}{2},\frac{1+\sqrt{5}}{2}\right]$ (D) $\left[0,\frac{1+\sqrt{5}}{2}\right]$

41. If $\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$, then $4x^2-4xy\cos\alpha+y^2$ is equal to -

(A) $-4\sin^2\alpha$ (B) $4\sin^2\alpha$

(C) 4 (D) 2 sin 2α

42. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, then -

(A) $x^2+y^2+z^2+xyz=0$ (B) $x^2+y^2+z^2+xyz=1$

(C) $x^2+y^2+z^2+2xyz=0$ (D) $x^2+y^2+z^2+2xyz=1$

43. If $\tan^{-1}\frac{x}{\pi}<\frac{\pi}{3}$, $x\in N$, then the maximum value of x is -

(A) 2 (B) 5

(C) 7 (D) none of these

44. The solution of the inequality $(\tan^{-1}x)^2-3\tan^{-1}x+2\geq 0$ is -

(A) $(-\infty,\tan 1]\cup[\tan 2,\infty)$

(B) $(-\infty,\ \tan 1]$

(C) $(-\infty,-\tan 1]\cup[\tan 2,\infty)$

(D) $[\tan 2,\ \infty)$

45. The set of values of x, satisfying the equation $\tan^2(\sin^{-1}x)>1$ is -

(A) [–1,1]

(B) $\left[-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right]$

(C) $(-1,1)-\left[-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right]$

(D) $[-1,1]-\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

(2 Question multiple (46 to 47))

46. $\cos^{-1}x = \tan^{-1}x$ then -

(A) $x^2 = \left(\frac{\sqrt{5}-1}{2}\right)$

(B) $x^2 = \left(\frac{\sqrt{5}+1}{2}\right)$

(C) $\sin(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$

(D) $\tan(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$

47. The value of

$\sin\left(\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right)\right) + \cos\left(\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right)\right)$ is/are equal to -

(A) 1 (B) $\frac{3\sqrt{2}}{\sqrt{10}}$

(C) $\sqrt{2}\sin\left(\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right) + \cot^{-1}(1)\right)$

(D) $\sqrt{2}\sin\left(\pi - \tan^{-1}(1) - \frac{1}{2}\tan^{-1}\frac{4}{3}\right)$

48. The value of

$\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$

for $0 < A < (\pi/4)$ is -

(A) $4\tan^{-1}(1)$ (B) $2\tan^{-1}(2)$

(C) 0 (D) none

49. Which of the following, satisfy the equation

$2\cos^{-1}x = \cot^{-1}\left(\frac{2x^2-1}{\sqrt{4x^2-4x^4}}\right)$

(A) (–1, 0) (B) (0, 1)

(C) $\left(-\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ (D) [–1, 1]

50. The solution set of the equation

$\sin^{-1}\sqrt{1-x^2} + \cos^{-1}x = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) - \sin^{-1}x$ is -

(A) [–1, 1]–{0} (B) (0,1] U {–1}

(C) [–1,0) U {1} (D) [–1,1]

51. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is -

[JEE 99]

(A) zero (B) one

(C) two (D) infinite

52. Value of k for which the point $(\alpha, \sin^{-1}\alpha)(\alpha > 0)$ lies inside the triangle formed by $x + y = k$ with co-ordinate axes is -

(A) $\left(1+\frac{\pi}{2}, \infty\right)$ (B) $\left(-\left(1+\frac{\pi}{2}\right), \left(1+\frac{\pi}{2}\right)\right)$

(C) $\left(-\infty, 1+\frac{\pi}{2}\right)$ (D) (–1–sin1, 1+sin1)

53. Solution set of the inequality $\sin^{-1}\left(\sin\frac{2x^2+3}{x^2+1}\right) \le \pi - \frac{5}{2}$ is -

(A) $(-\infty, 1) \cup (1, \infty)$ (B) [–1, 1]

(C) (–1, 1) (D) $(-\infty, -1] \cup [1, \infty)$

54. Consider two geometric progressions $a_1, a_2, a_3, \ldots\ldots a_n$ & $b_1, b_2, b_3, \ldots\ldots b_n$ with $a_r = \frac{1}{b_r} = 2^{r-1}$ and another sequence $t_1, t_2, t_3, \ldots\ldots t_n$ such that $t_r = \cot^{-1}(2a_r + b_r)$ then $\lim_{n\to\infty}\sum_{r=1}^{n} t_r$ is -

(A) 0 (B) $\pi/4$

(C) $\tan^{-1}2$ (D) $\pi/2$

55. The sum of the infinite terms of the series -

$\cot^{-1}\left(1^2+\frac{3}{4}\right) + \cot^{-1}\left(2^2+\frac{3}{4}\right) + \cot^{-1}\left(3^2+\frac{3}{4}\right) + \ldots\ldots\ldots$

is equal to -

(A) $\tan^{-1}(1)$ (B) $\tan^{-1}(2)$

(C) $\tan^{-1}(3)$ (D) $\frac{3\pi}{4} - \tan^{-1}3$

ANSWER KEY

1. D **2.** C **3.** D **4.** D **5.** B **6.** D **7.** D **8.** C **9.** B **10.** B **11.** D **12.** C **13.** C
14. D **15.** A **16.** B **17.** A **18**. A **19.** B **20.** A **21.** C **22**. C **23.** B **24.** A **25.** A **26**. C
27. B **28.** D **29.** C **30.** A **31.** A **32.** D **33.** C **34.** A **35.** C **36.** A **37.** D **38.** D **39.** B
40. C **41.** B **42.** D **43.** B **44.** A **45.** C **46.** AC **47.** BCD **48.** A **49.** B **50.** C **51.** C **52.** A
53. B **54.** B **55.** D