Rational Indices

Solution 1:

(1) $(-51)^0 =$ 1

For any $a \neq 0$, $a^0 = 1$

(2) $x^5 \times x^{-4} \div x^2 =$ $\frac{1}{x}$

$x^5 \times x^{-4} \div x^2 = x^5 \times x^{-4} \times x^{-2} = x^{5-4-2} = x^{-1} = \frac{1}{x}$

(3) $\left(a^3\right)^{-4} =$ $\frac{1}{a^{12}}$

$\left(a^3\right)^{-4} = a^{3\times(-4)} = a^{-12} = \frac{1}{a^{12}}$

(4) $\left(\sqrt{y}\right)^5 =$ $y^{\frac{5}{2}}$

$\left(\sqrt{y}\right)^5 = \left(y^{\frac{1}{2}}\right)^5 = y^{\frac{1}{2}\times 5} = y^{\frac{5}{2}}$

(5) $4^{-2} \times \frac{1}{4^{-2}} =$ 1

$4^{-2} \times \frac{1}{4^{-2}} = 4^{-2} \times 4^2 = 4^{-2+2} = 4^0 = 1$

(6) $\frac{1}{(3\times 4)^{-1}} =$ 12

$\frac{1}{(3\times 4)^{-1}} = \frac{1}{(12)^{-1}} = 12^1 = 12$

(7) $\left[\left(\frac{2}{3}\right)^2\right]^{-2} =$ $5\frac{1}{16}$

$\left[\left(\frac{2}{3}\right)^2\right]^{-2} = \left(\frac{2}{3}\right)^{2\times(-2)} = \left(\frac{2}{3}\right)^{(-4)} = \left(\frac{3}{2}\right)^4 = \frac{81}{16} = 5\frac{1}{16}$

(8) $\left[\frac{16}{81}\right]^{\frac{1}{4}} =$ $\frac{2}{3}$

$\left[\frac{16}{81}\right]^{\frac{1}{4}} = \left[\frac{2^4}{3^4}\right]^{\frac{1}{4}} = \frac{2^{4\times\frac{1}{4}}}{3^{4\times\frac{1}{4}}} = \frac{2}{3}$

Solution 2(1):

$$\left(\frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right)^2\left(\frac{x^{\frac{1}{3}}}{x^{\frac{1}{4}}}\right)^3\left(\frac{x^{\frac{1}{4}}}{x^{\frac{1}{2}}}\right)^4,(x>0)$$

$$=\left(\frac{x^{\frac{1}{2}\times 2}}{x^{\frac{1}{3}\times 2}}\right)\times\left(\frac{x^{\frac{1}{3}\times 3}}{x^{\frac{1}{4}\times 3}}\right)\times\left(\frac{x^{\frac{1}{4}\times 4}}{x^{\frac{1}{2}\times 4}}\right)$$

$$=\left(\frac{x^1}{x^{\frac{2}{3}}}\right)\times\left(\frac{x^1}{x^{\frac{3}{4}}}\right)\times\left(\frac{x^1}{x^2}\right)$$

$$=\left(x^{1-\frac{2}{3}}\right)\times\left(x^{1-\frac{3}{4}}\right)\times\left(x^{1-2}\right)$$

$$=\left(x^{\frac{3-2}{3}}\right)\times\left(x^{\frac{4-3}{4}}\right)\times\left(x^{-1}\right)$$

$$=\left(x^{\frac{1}{3}}\right)\times\left(x^{\frac{1}{4}}\right)\times\left(x^{-1}\right)$$

$$=(x)^{\frac{1}{3}+\frac{1}{4}-1}$$

$$=(x)^{\frac{4+3-12}{12}}$$

$$=(x)^{\frac{-5}{12}}$$

$$=\frac{1}{x^{\frac{5}{12}}}$$

Solution 2(2):

$$\frac{\left(x^5\right)^{\frac{1}{6}}\times x^{\frac{1}{7}}\times\left(x^{\frac{2}{3}}\right)^2}{\left(x^2\right)^{\frac{2}{3}}\times\left(x^{\frac{1}{6}}\right)^5\times x^{\frac{1}{7}}}$$

$$=\frac{\left(x^{5\times\frac{1}{6}}\right)\times x^{\frac{1}{7}}\times\left(x^{\frac{2}{3}\times 2}\right)}{\left(x^{2\times\frac{2}{3}}\right)\times\left(x^{\frac{1}{6}\times 5}\right)\times x^{\frac{1}{7}}}$$

$$=\frac{\left(x^{\frac{5}{6}}\right)\times x^{\frac{1}{7}}\times\left(x^{\frac{4}{3}}\right)}{\left(x^{\frac{4}{3}}\right)\times\left(x^{\frac{5}{6}}\right)\times x^{\frac{1}{7}}}$$

$$=\left(x^{\frac{5}{6}-\frac{5}{6}}\right)\times\left(x^{\frac{1}{7}-\frac{1}{7}}\right)\times\left(x^{\frac{4}{3}-\frac{4}{3}}\right)$$

$$=x^0\times x^0\times x^0$$

$$=1\times 1\times 1$$

$$=1$$

Solution 3(1):

$$\left(\frac{49}{16}\right)^{\frac{1}{2}} \times \left(\frac{4^2}{7^2}\right) \times 4^{\frac{1}{2}}$$

$$= \left(\frac{7^2}{2^4}\right)^{\frac{1}{2}} \times \left(\frac{\left(2^2\right)^2}{7^2}\right) \times \left(2^2\right)^{\frac{1}{2}}$$

$$= \left(\frac{7^{2\times\frac{1}{2}}}{2^{4\times\frac{1}{2}}}\right) \times \left(\frac{2^{2\times 2}}{7^2}\right) \times \left(2^{2\times\frac{1}{2}}\right)$$

$$= \frac{7}{2^2} \times \frac{2^4}{7^2} \times 2$$

$$= \frac{2^{4+1-2}}{7^{2-1}}$$

$$= \frac{2^3}{7}$$

$$= \frac{8}{7}$$

$$= 1\frac{1}{7}$$

Solution 3(2):

$$\left(\frac{25}{16}\right)^{\frac{1}{4}} \times \left(\frac{27}{8}\right)^{\frac{1}{6}} \times \left(\frac{2}{15}\right)^{\frac{1}{2}}$$

$$= \left(\frac{5^2}{2^4}\right)^{\frac{1}{4}} \times \left(\frac{3^3}{2^3}\right)^{\frac{1}{6}} \times \left(\frac{2}{3\times 5}\right)^{\frac{1}{2}}$$

$$= \left(\frac{5^{2\times\frac{1}{4}}}{2^{4\times\frac{1}{4}}}\right) \times \left(\frac{3^{3\times\frac{1}{6}}}{2^{3\times\frac{1}{6}}}\right) \times \left(\frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}\right)$$

$$= \left(\frac{5^{\frac{1}{2}}}{2^1}\right) \times \left(\frac{3^{\frac{1}{2}}}{2^{\frac{1}{2}}}\right) \times \left(\frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}\right)$$

$$= 5^{\frac{1}{2}-\frac{1}{1}} \times 3^{\frac{1}{2}-\frac{1}{2}} \times 2^{\frac{1}{2}-\frac{1}{2}-1}$$

$$= 5^0 \times 3^0 \times 2^{-1}$$

$$= 1 \times 1 \times \frac{1}{2}$$

$$= \frac{1}{2}$$

Solution 3(3):

$$\frac{\sqrt[3]{108}\times\sqrt[6]{4}}{\sqrt[4]{81}}$$

$$=\frac{(108)^{\frac{1}{3}}\times(4)^{\frac{1}{6}}}{(81)^{\frac{1}{4}}}$$

$$=\frac{(27\times4)^{\frac{1}{3}}\times(4)^{\frac{1}{6}}}{(81)^{\frac{1}{4}}}$$

$$=\frac{\left(3^3\times2^2\right)^{\frac{1}{3}}\times\left(2^2\right)^{\frac{1}{6}}}{\left(3^4\right)^{\frac{1}{4}}}$$

$$=\frac{3^{3\times\frac{1}{3}}\times2^{2\times\frac{1}{3}}\times2^{2\times\frac{1}{6}}}{3^{4\times\frac{1}{4}}}$$

$$=\frac{3^1\times2^{\frac{2}{3}}\times2^{\frac{1}{3}}}{3^1}$$

$$=3^{1-1}\times2^{\frac{2}{3}+\frac{1}{3}}$$

$$=3^0\times2^{\frac{2+1}{3}}$$

$$=1\times2^{\frac{3}{3}}$$

$$=1\times2^1$$

$$=2$$

Solution 3(4):

$$\left(\frac{8}{27}\right)^{\frac{1}{3}}\times\left(\frac{9}{25}\right)^{\frac{1}{2}}\times\left(\frac{2}{5}\right)^{-1}$$

$$=\left(\frac{2^3}{3^3}\right)^{\frac{1}{3}}\times\left(\frac{3^2}{5^2}\right)^{\frac{1}{2}}\times\left(\frac{5}{2}\right)^1$$

$$=\left(\frac{2^{3\times\frac{1}{3}}}{3^{3\times\frac{3}{3}}}\right)\times\left(\frac{3^{2\times\frac{1}{2}}}{5^{2\times\frac{1}{2}}}\right)\times\frac{5}{2}$$

$$=\left(\frac{2^1}{3^1}\right)\times\left(\frac{3^1}{5^1}\right)\times\frac{5}{2}$$

$$=\frac{2\times3\times5}{3\times5\times2}$$

$$=2^{1-1}\times3^{1-1}\times5^{1-1}$$

$$=2^0\times3^0\times5^0$$

$$=1\times1\times1$$

$$=1$$

Solution 4:

$$\text{L.H.S.}=\left(\frac{2^{\frac{1}{3}}}{2^{\frac{-1}{3}}}\right)^3+\frac{3^{\frac{1}{2}}}{3^{\frac{-1}{2}}}$$

$$=\left(2^{\frac{1}{3}-\left(\frac{-1}{3}\right)}\right)^3+3^{\frac{1}{2}-\left(\frac{-1}{2}\right)}$$

$$=\left(2^{\frac{2}{3}}\right)^3+3^{\frac{2}{2}}$$

$$=2^{\frac{2}{3}\times3}+3^1$$

$$=2^2+3^1$$

$$=4+3$$

$$=7$$

$$=\text{R.H.S.}$$

Solution 5:

$$\text{L.H.S.} = \frac{(16)^{\frac{1}{4}}}{(27)^{\frac{1}{3}}} + \frac{(625)^{\frac{1}{4}}}{(81)^{\frac{1}{4}}} - \frac{1}{(243)^{\frac{1}{5}}}$$

$$= \frac{(2^4)^{\frac{1}{4}}}{(3^3)^{\frac{1}{3}}} + \frac{(5^4)^{\frac{1}{4}}}{(3^4)^{\frac{1}{4}}} - \frac{1}{(3^5)^{\frac{1}{5}}}$$

$$= \frac{2^{\frac{4}{4}}}{3^{\frac{3}{3}}} + \frac{5^{\frac{4}{4}}}{3^{\frac{4}{4}}} - \frac{1}{3^{\frac{5}{5}}}$$

$$= \frac{2^1}{3^1} + \frac{5^1}{3^1} - \frac{1}{3^1}$$

$$= \frac{2}{3} + \frac{5}{3} - \frac{1}{3}$$

$$= \frac{2+5-1}{3}$$

$$= \frac{6}{3}$$

$$= 2$$

$$= \text{R.H.S.}$$

Solution 6:

$$\text{L.H.S.} = \left[\left(a^x\right)^y \left(a^y\right)^x\right]^z$$

$$= \left[\left(a^{xy}\right)\left(a^{yx}\right)\right]^z$$

$$= \left[a^{xy+yx}\right]^z$$

$$= \left[a^{2xy}\right]^z$$

$$= a^{2xy(z)}$$

$$= a^{2xyz}$$

$$= \text{R.H.S.}$$

Solution 7:

$$\text{L.H.S.} = \left[\left(\frac{x^a}{x^b}\right)^{\frac{1}{a}}\right]^{\frac{1}{b}} \left[\left(\frac{x^b}{x^c}\right)^{\frac{1}{b}}\right]^{\frac{1}{c}} \left[\left(\frac{x^c}{x^a}\right)^{\frac{1}{c}}\right]^{\frac{1}{a}}$$

$$= \left[\frac{x^{a\times\frac{1}{a}}}{x^{b\times\frac{1}{a}}}\right]^{\frac{1}{b}} \left[\frac{x^{b\times\frac{1}{b}}}{x^{c\times\frac{1}{b}}}\right]^{\frac{1}{c}} \left[\frac{x^{c\times\frac{1}{c}}}{x^{a\times\frac{1}{c}}}\right]^{\frac{1}{a}}$$

$$= \left[\frac{x^1}{x^{\frac{b}{a}}}\right]^{\frac{1}{b}} \left[\frac{x^1}{x^{\frac{c}{b}}}\right]^{\frac{1}{c}} \left[\frac{x^1}{x^{\frac{a}{c}}}\right]^{\frac{1}{a}}$$

$$= \left[\frac{x^{1\times\frac{1}{b}}}{x^{\frac{b}{a}\times\frac{1}{b}}}\right] \left[\frac{x^{1\times\frac{1}{c}}}{x^{\frac{c}{b}\times\frac{1}{c}}}\right] \left[\frac{x^{1\times\frac{1}{a}}}{x^{\frac{a}{c}\times\frac{1}{a}}}\right]$$

$$= \frac{x^{\frac{1}{b}}}{x^{\frac{1}{a}}} \times \frac{x^{\frac{1}{c}}}{x^{\frac{1}{b}}} \times \frac{x^{\frac{1}{a}}}{x^{\frac{1}{c}}}$$

$$= \frac{x^{\frac{1}{a}}}{x^{\frac{1}{a}}} \times \frac{x^{\frac{1}{b}}}{x^{\frac{1}{b}}} \times \frac{x^{\frac{1}{c}}}{x^{\frac{1}{c}}}$$

$$= x^{\frac{1}{a}-\frac{1}{a}} \times x^{\frac{1}{b}-\frac{1}{b}} \times x^{\frac{1}{c}-\frac{1}{c}}$$

$$= x^0 \times x^0 \times x^0$$

$$= 1 \times 1 \times 1$$

$$= 1$$

$$= \text{R.H.S.}$$

Practice 1

Solution 1:

(1) $(-10)^6 \times (-10)^6 = (-10)^{\underline{12}}$

$(-10)^6 \times (-10)^6 = (-10)^{6+6} = (-10)^{12}$

(2) $4^5 \div 4^3 = 4^{\underline{2}}$

$4^5 \div 4^3 = 4^{5-3} = 4^2$

(3) $\frac{x^3}{x^6} = \frac{1}{x^{\underline{3}}}$

$\frac{x^3}{x^6} = \frac{1}{x^{6-3}} = \frac{1}{x^3}$

(4) $\left(21^3\right)^3 = 21^{\underline{9}}$

$\left(21^3\right)^3 = 21^{3\times3} = 21^9$

(5) $\left(\sqrt{3}\right)^0 = \underline{1}$

For any $a \neq 0$, $a^0 = 1$

(6) $[(-3) \times 4]^2 = (-3)^{\underline{2}} \times \underline{4^2}$

(7) $\left(\frac{5}{x}\right)^4 = \frac{5^{\underline{4}}}{x^{\underline{4}}}$

(8) $\left(\frac{1}{2}\right)^3 \div \left(\frac{1}{2}\right)^3 = \underline{1}$

$\left(\frac{1}{2}\right)^3 \div \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{3-3} = \left(\frac{1}{2}\right)^0 = 1$

(9) $12x^4 \div 3x^6 = \underline{\frac{4}{x^2}}$

$12x^4 \div 3x^6 = \frac{12x^4}{3x^6} = \frac{12 \div 3}{x^{6-4}} = \frac{4}{x^2}$

(10) $a^{10} \div \underline{a^2} = a^8$

$a^{10} \div a^2 = a^{10-2} = a^8$

Solution 2(1):

$\left(2^7\right)^3 \times \left(2^4\right)^6$

$= \left(2^{7\times3}\right) \times \left(2^{4\times6}\right)$

$= 2^{21} \times 2^{24}$

$= 2^{21+24}$

$= 2^{45}$

Solution 2(2):

$(-3)^4 \times (-3)$

$= (-3)^4 \times (-3)^1$

$= (-3)^{4+1}$

$= (-3)^5$

Solution 2(3):

$$\frac{(a^{14})^4 \times (a^2)^3}{(a^7)^3}; (a \neq 0)$$

$$= \frac{a^{14\times4} \times a^{2\times3}}{a^{7\times3}}$$

$$= \frac{a^{56} \times a^6}{a^{21}}$$

$$= \frac{a^{56+6}}{a^{21}}$$

$$= \frac{a^{62}}{a^{21}}$$

$$= a^{62-21}$$

$$= a^{41}$$

Solution 2(4):

$$\frac{(y^7)^3}{(y^6)^5}; (y \neq 0)$$

$$= \frac{y^{7\times3}}{y^{6\times5}}$$

$$= \frac{y^{21}}{y^{30}}$$

$$= \frac{1}{y^{30-21}}$$

$$= \frac{1}{y^9}$$

Solution 2(5):

$$\left(\frac{1}{3} \times x\right)^3$$

$$= \left(\frac{x}{3}\right)^3$$

$$= \frac{x^3}{3^3}$$

Solution 2(6):

$$\frac{(2a^2b^3)^5 \times (2a^2b^2)^3}{(5a^4b)^6}$$

$$= \frac{\left[(2)^5.(a^2)^5.(b^3)^5\right] \times \left[(2)^3.(a^2)^3.(b^2)^3\right]}{(5)^6.(a^4)^6.(b)^6}$$

$$= \frac{\left(2^5 \times a^{2\times5} \times b^{3\times5}\right) \times \left(2^3 \times a^{2\times3} \times b^{2\times3}\right)}{5^6 \times a^{4\times6} \times b^6}$$

$$= \frac{2^5 \times a^{10} \times b^{15} \times 2^3 \times a^6 \times b^6}{5^6 \times a^{24} \times b^6}$$

$$= \frac{2^5 \times 2^3 \times a^{10} \times a^6 \times b^{15} \times b^6}{5^6 \times a^{24} \times b^6}$$

$$= \frac{2^{5+3} \times a^{10+6} \times b^{15+6}}{5^6 \times a^{24} \times b^6}$$

$$= \frac{2^8 \times a^{16} \times b^{21}}{5^6 \times a^{24} \times b^6}$$

$$= \frac{2^8}{5^6} \times \frac{1}{a^{24-16}} \times b^{21-6}$$

$$= \frac{2^8 b^{15}}{5^6 a^8}$$

Practice 2

Solution 1:

(1) c. $\frac{1}{64}$

$4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

(2) a. 5^{-1}

$\frac{1}{5} = \frac{1}{5^1} = 5^{-1}$

(3) c. 64

$\left(8^{-1}\right)^{-2} = 8^{(-1)\times(-2)} = 8^2 = 64$

(4) a. a^0

$a^{-2} \times \frac{1}{a^{-2}} = a^{-2-(-2)} = a^{-2+2} = a^0$

(5) b. 2^2

$\frac{1}{\left(2^{-1}\right)^2} = \frac{1}{2^{-1\times 2}} = \frac{1}{2^{-2}} = 2^2$

(6) b. 1

$\left(\sqrt{5}\right)^3 \div \left(\sqrt{5}\right)^3 = \left(\sqrt{5}\right)^{3-3} = \left(\sqrt{5}\right)^0 = 1$

Solution 2:

(1) $2^{-3} \times \left(\frac{1}{4}\right)^5 \times 8^{-3}$

$= 2^{-3} \times \left(\frac{1}{2^2}\right)^5 \times (2^3)^{-3}$

$= 2^{-3} \times \frac{1}{2^{10}} \times 2^{-9}$

$= \frac{2^{-3} \times 2^{-9}}{2^{10}}$

$= \frac{2^{(-12)}}{2^{10}}$

$= 2^{-12-10}$

$= 2^{-22}$

$= \frac{1}{2^{22}}$

(2) $\left(\frac{a}{b}\right)^{m+n} \times \left(\frac{a}{b}\right)^{m-n} \times (ab)^m$

$= \frac{a^{m+n}}{b^{m+n}} \times \frac{a^{m-n}}{b^{m-n}} \times a^m \times b^m$

$= \frac{a^{m+n+m-n+m}}{b^{m+n+m-n-m}}$

$= \frac{a^{3m}}{b^m}$

$= a^{3m} \div b^m$

(3) $4a^{-2} \times \left(\frac{a}{2}\right)^5 \div (2a)^2$

$= 2^2 \times a^{-2} \times \frac{a^5}{2^5} \times \frac{1}{2^2 a^2}$

$= \frac{2^2 \times a^{-2+5}}{2^{5+2} \times a^2}$

$= \frac{2^2 \times a^3}{2^7 \times a^2}$

$= \frac{a^{3-2}}{2^{7-2}}$

$= \frac{a}{2^5}$

$= \frac{a}{32}$

Solution 3:

(1) $2^2 \times 2^{-3} \times 2^{-1}$

$= 2^{2+(-3)+(-1)}$

$= 2^{2-3-1}$

$= 2^{2-4}$

$= 2^{-2}$

$= \frac{1}{2^2}$

$= \frac{1}{4}$

(2) $\left(\frac{1}{3}\right)^3 \times 3^{-2} \times 3^5$

$= \frac{1^3}{3^3} \times 3^{-2+5}$

$= \frac{1}{3^3} \times 3^3$

$= 1 \times 3^{3-3}$

$= 1 \times 3^0$

$= 1 \times 1$

$= 1$

(3) $\left(8^{-2} \times 12^4\right) \div 27^2$

$= \left[(2^3)^{-2} \times (4 \times 3)^4\right] \div (3^3)^2$

$= \left[(2^3)^{-2} \times (2^2 \times 3)^4\right] \div (3^3)^2$

$= 2^{-6} \times 2^8 \times 3^4 \div 3^6$

$= \frac{2^{8-6}}{3^{6-4}}$

$= \frac{2^2}{3^2}$

$= \frac{4}{9}$

Practice 3

Solution 1(1):

$\sqrt[5]{243}$

$= (243)^{\frac{1}{5}}$

$= (3^5)^{\frac{1}{5}}$

$= 3^{5 \times \frac{1}{5}}$

$= 3^1$

$= 3$

Solution 1(2):

$(729)^{\frac{1}{3}}$

$= (9^3)^{\frac{1}{3}}$

$= 9^{3 \times \frac{1}{3}}$

$= 9^1$

$= 9$

Solution 1(3):

$\left(\frac{125}{343}\right)^{\frac{1}{3}}$

$= \left(\frac{5^3}{7^3}\right)^{\frac{1}{3}}$

$= \frac{5^{3 \times \frac{1}{3}}}{7^{3 \times \frac{1}{3}}}$

$= \frac{5^1}{7^1}$

$= \frac{5}{7}$

Solution 1(4):

$(64)^{\frac{1}{6}}$

$= \left(2^6\right)^{\frac{1}{6}}$

$= 2^{6 \times \frac{1}{6}}$

$= 2^1$

$= 2$

Solution 1(5):

$(625)^{\frac{1}{4}}$

$= \left(5^4\right)^{\frac{1}{4}}$

$= 5^{4 \times \frac{1}{4}}$

$= 5^1$

$= 5$

Solution 1(6):

$$\left(\frac{81}{144}\right)^{\frac{1}{2}}$$

$$=\left(\frac{9^2}{12^2}\right)^{\frac{1}{2}}$$

$$=\frac{9^{2\times\frac{1}{2}}}{12^{2\times\frac{1}{2}}}$$

$$=\frac{9^1}{12^1}$$

$$=\frac{9}{12}$$

$$=\frac{3}{4}$$

Solution 1(7):

$$\left(\frac{81}{625}\right)^{\frac{-3}{4}}$$

$$=\left(\frac{3^4}{5^4}\right)^{\frac{-3}{4}}$$

$$=\frac{3^{4\times\left(\frac{-3}{4}\right)}}{5^{4\times\left(\frac{-3}{4}\right)}}$$

$$=\frac{3^{-3}}{5^{-3}}$$

$$=\frac{5^3}{3^3}$$

$$=\frac{125}{27}$$

Solution 1(8):

$$\frac{32^{\frac{1}{5}}}{81^{\frac{1}{4}}}$$

$$=\frac{\left(2^5\right)^{\frac{1}{5}}}{\left(3^4\right)^{\frac{1}{4}}}$$

$$=\frac{2^{5\times\frac{1}{5}}}{3^{4\times\frac{1}{4}}}$$

$$=\frac{2^1}{3^1}$$

$$=\frac{2}{3}$$

Solution 2:

$$\text{L.H.S.} = 5^{\frac{1}{3}} \times \left(\frac{2}{5}\right)^{\frac{1}{3}} \times \frac{64^{\frac{1}{3}}}{3^{\frac{1}{3}}} \times \frac{9^{\frac{1}{6}}}{2^{\frac{1}{3}}}$$

$$= 5^{\frac{1}{3}} \times \frac{2^{\frac{1}{3}}}{5^{\frac{1}{3}}} \times \frac{\left(2^6\right)^{\frac{1}{3}}}{3^{\frac{1}{3}}} \times \frac{\left(3^2\right)^{\frac{1}{6}}}{2^{\frac{1}{3}}}$$

$$= 5^{\frac{1}{3}} \times \frac{2^{\frac{1}{3}}}{5^{\frac{1}{3}}} \times \frac{2^{6\times\frac{1}{3}}}{3^{\frac{1}{3}}} \times \frac{3^{2\times\frac{1}{6}}}{2^{\frac{1}{3}}}$$

$$= \frac{5^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^2 \times 3^{\frac{1}{3}}}{5^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times 2^{\frac{1}{3}}}$$

$$= 5^{\frac{1}{3}-\frac{1}{3}} \times 2^{\frac{1}{3}+2-\frac{1}{3}} \times 3^{\frac{1}{3}-\frac{1}{3}}$$

$$= 5^0 \times 2^2 \times 3^0$$

$$= 1 \times 4 \times 1$$

$$= 4$$

$$= \text{R.H.S.}$$

Hence, proved.

Solution 3:

Given : $x = 243$

$$\therefore x^{\frac{1}{5}} \times x^{\frac{-1}{5}} = (243)^{\frac{1}{5}} \times (243)^{\frac{-1}{5}}$$

$$= (243)^{\frac{1}{5}+\left(-\frac{1}{5}\right)}$$

$$= (243)^{\frac{1}{5}-\frac{1}{5}}$$

$$= 243^0$$

$$= 1$$

Solution 4(1):

$$\sqrt{625} - 5\sqrt[3]{27} - (100)^{\frac{1}{2}} = 0$$

$$\text{L.H.S.} = \sqrt{625} - 5\sqrt[3]{27} - (100)^{\frac{1}{2}}$$

$$= (625)^{\frac{1}{2}} - 5(27)^{\frac{1}{3}} - (100)^{\frac{1}{2}}$$

$$= \left(25^2\right)^{\frac{1}{2}} - 5\left(3^3\right)^{\frac{1}{3}} - \left(10^2\right)^{\frac{1}{2}}$$

$$= 25^{2\times\frac{1}{2}} - 5\left(3^{3\times\frac{1}{3}}\right) - 10^{2\times\frac{1}{2}}$$

$$= 25 - 5 \times 3 - 10$$

$$= 25 - 15 - 10$$

$$= 0$$

$$= \text{R.H.S.}$$

Solution 4(2):

$$\left[(81)^{\frac{1}{2}}+1\right]\left[(81)^{\frac{1}{4}}-1\right]=20$$

L.H.S. $=\left[(81)^{\frac{1}{2}}+1\right]\left[(81)^{\frac{1}{4}}-1\right]$

$=\left[\left(3^4\right)^{\frac{1}{2}}+1\right]\left[\left(3^4\right)^{\frac{1}{4}}-1\right]$

$=\left[3^{4\times\frac{1}{2}}+1\right]\left[3^{4\times\frac{1}{4}}-1\right]$

$=\left(3^2+1\right)(3-1)$

$=(9+1)(2)$

$=(10)(2)$

$=20$

= R.H.S.