# **Rational Indices**

Solution 1:  
(1) 
$$(-51)^{0} = \frac{1}{1}$$
  
For any  $a \neq 0$ ,  $a^{0} = 1$   
(2)  $x^{5} \times x^{-4} + x^{2} = \frac{1}{x}$   
 $x^{5} \times x^{-4} + x^{2} = x^{5} \times x^{-4} \times x^{-2} = x^{5-4-2} = x^{-1} = \frac{1}{x}$   
(3)  $(a^{3})^{-4} = \frac{1}{a^{12}}$   
 $(a^{3})^{-4} = a^{3\times(-4)} = a^{-12} = \frac{1}{a^{12}}$   
(4)  $(\sqrt{y})^{5} = \frac{y^{\frac{5}{2}}}{(\sqrt{y})^{5}} = (y^{\frac{1}{2}})^{\frac{5}{2}} = y^{\frac{1}{2}\times5} = y^{\frac{5}{2}}$   
(5)  $4^{-2} \times \frac{1}{4^{-2}} = \frac{1}{1}$   
 $4^{-2} \times \frac{1}{4^{-2}} = 4^{-2} \times 4^{2} = 4^{-2+2} = 4^{0} = 1$   
(6)  $\frac{1}{(3\times 4)^{-1}} = \frac{12}{(12)^{-1}} = 12^{4} = 12$   
(7)  $[(\frac{2}{3})^{2}]^{-2} = 5\frac{1}{16}$   
 $[(\frac{2}{3})^{2}]^{-2} = (\frac{2}{3})^{2\times(-2)} = (\frac{2}{3})^{(-4)} = (\frac{3}{2})^{4} = \frac{31}{16} = 5\frac{1}{16}$   
(8)  $[\frac{16}{81}]^{\frac{1}{4}} = \frac{2}{3}$   
 $[\frac{16}{81}]^{\frac{1}{4}} = [\frac{2^{4}}{3}]^{\frac{1}{4}} = \frac{2^{4\times\frac{1}{4}}}{3^{4\times\frac{1}{4}}} = \frac{2}{3}$ 

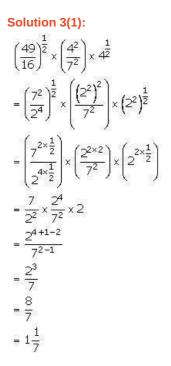
Solution 2(1): 
$$(1)^2 (1)^3 (1)^4$$

$$\begin{pmatrix} \frac{1}{\chi^{2}} \\ \frac{1}{\chi^{3}} \\ \frac{1}{\chi^{3}} \end{pmatrix}^{2} \begin{pmatrix} \frac{1}{\chi^{3}} \\ \frac{1}{\chi^{4}} \\ \frac{1}{\chi^{2}} \end{pmatrix}^{3} \begin{pmatrix} \frac{1}{\chi^{4}} \\ \frac{1}{\chi^{2}} \\ \frac{1}{\chi^{2}} \end{pmatrix}^{4}, (x > 0)$$

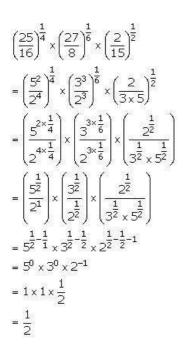
$$= \begin{pmatrix} \frac{1}{\chi^{2}} \\ \frac{1}{\chi^{3}} \\ \frac{1}{\chi^{3}} \\ \frac{1}{\chi^{3}} \\ \frac{1}{\chi^{3}} \\ \frac{1}{\chi^{4}} \\ \frac{1}{\chi^{2}} \\ \frac{1}{\chi^{2}} \\ \frac{1}{\chi^{3}} \\ \frac{1}{\chi$$

$$\frac{(x^{5})^{\frac{1}{6}} \times x^{\frac{1}{7}} \times (x^{\frac{2}{3}})^{2}}{(x^{2})^{\frac{2}{3}} \times (x^{\frac{1}{6}})^{5} \times x^{\frac{1}{7}}} = \frac{(x^{5})^{\frac{2}{3}} \times (x^{\frac{1}{6}})^{5} \times x^{\frac{1}{7}}}{(x^{2})^{\frac{2}{3}} \times (x^{\frac{1}{6}})^{\frac{2}{3}} \times (x^{\frac{2}{3} \times 2})} = \frac{(x^{5})^{\frac{2}{3}} \times (x^{\frac{1}{6} \times 5}) \times x^{\frac{1}{7}}}{(x^{\frac{4}{3}}) \times (x^{\frac{5}{6}}) \times x^{\frac{1}{7}}} = (x^{\frac{5}{6}})^{\frac{1}{6}} \times (x^{\frac{5}{7}})^{\frac{1}{7}} \times (x^{\frac{4}{3}})$$
$$= (x^{\frac{5}{6}} - \frac{5}{6}) \times (x^{\frac{1}{7}} - \frac{1}{7}) \times (x^{\frac{4}{3}} - \frac{4}{3})$$
$$= x^{0} \times x^{0} \times x^{0}$$
$$= 1 \times 1 \times 1$$
$$= 1$$

Solution 2(2):



Solution 3(2):



Solution 3(3):

$$\frac{\sqrt[3]{108 \times \sqrt[6]{4}}}{\sqrt[4]{81}}$$

$$= \frac{(108)^{\frac{1}{3} \times (4)^{\frac{1}{6}}}}{(81)^{\frac{1}{4}}}$$

$$= \frac{(27 \times 4)^{\frac{1}{3} \times (4)^{\frac{1}{6}}}}{(81)^{\frac{1}{4}}}$$

$$= \frac{(3^{3} \times 2^{2})^{\frac{1}{3} \times (2^{2})^{\frac{1}{6}}}}{(3^{4})^{\frac{1}{4}}}$$

$$= \frac{3^{3 \times \frac{1}{3} \times 2^{2 \times \frac{1}{3}} \times 2^{2 \times \frac{1}{3}}}{3^{4 \times \frac{1}{4}}}$$

$$= \frac{3^{1} \times 2^{\frac{2}{3} \times 2^{\frac{1}{3}}}}{3^{1}}$$

$$= 3^{1-1} \times 2^{\frac{2}{3} + \frac{1}{3}}$$

$$= 3^{0} \times 2^{\frac{2}{3}}$$

$$= 1 \times 2^{\frac{3}{3}}$$

$$= 1 \times 2^{1}$$

$$= 2$$

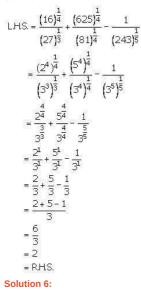
Solution 3(4):

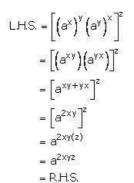
$$\begin{split} &\left(\frac{8}{27}\right)^{\frac{1}{3}} \times \left(\frac{9}{25}\right)^{\frac{1}{2}} \times \left(\frac{2}{5}\right)^{-1} \\ &= \left(\frac{2^3}{3^3}\right)^{\frac{1}{3}} \times \left(\frac{3^2}{5^2}\right)^{\frac{1}{2}} \times \left(\frac{5}{5}\right)^{1} \\ &= \left(\frac{2^{3\times\frac{1}{3}}}{3^{3\times\frac{3}{3}}}\right) \times \left(\frac{3^{2\times\frac{1}{2}}}{5^{2\times\frac{1}{2}}}\right) \times \frac{5}{2} \\ &= \left(\frac{2^1}{3^1}\right) \times \left(\frac{3^1}{5^1}\right) \times \frac{5}{2} \\ &= \frac{2\times3\times5}{3\times5\times2} \\ &= 2^{1-1} \times 3^{1-1} \times 5^{1-1} \\ &= 2^0 \times 3^0 \times 5^0 \\ &= 1 \times 1 \times 1 \\ &= 1 \end{split}$$

Solution 4:

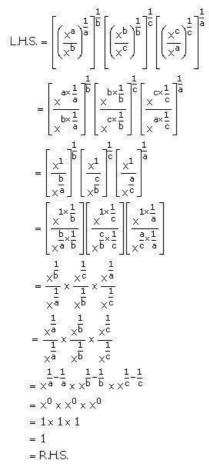
LHS. = 
$$\left(\frac{2^{\frac{1}{3}}}{2^{\frac{-1}{3}}}\right)^3 + \frac{3^{\frac{1}{2}}}{3^{\frac{-1}{2}}}$$
  
=  $\left(2^{\frac{1}{3}} - \left(\frac{-1}{3}\right)\right)^3 + 3^{\frac{1}{2}} - \left(\frac{-1}{2}\right)^3$   
=  $\left(2^{\frac{2}{3}}\right)^3 + 3^{\frac{2}{2}}$   
=  $2^{\frac{2}{3}\times3} + 3^1$   
=  $2^2 + 3^1$   
=  $4 + 3$   
= 7  
= RHS.

Solution 5:





Solution 7:



Practice 1

Solution 1:

### Solution 2(3):

 $(-3)^4 \times (-3) = (-3)^4 \times (-3)^1 = (-3)^{4+1} = (-3)^5$ 

Solution 2(2):

$$(2^{7})^{3} \times (2^{4})^{6}$$
  
=  $(2^{7\times3}) \times (2^{4\times6})$   
=  $2^{21} \times 2^{24}$   
=  $2^{21+24}$   
=  $2^{45}$ 

Solution 2(1):

$$(1) (-10)^{6} \times (-10)^{6} = (-10)^{12} 
(-10)^{6} \times (-10)^{6} = (-10)^{6+6} = (-10)^{12} 
(2) 4^{5} + 4^{3} = 4^{2} 
4^{5} + 4^{3} = 4^{5-3} = 4^{2} 
(3)  $\frac{x^{3}}{x^{6}} = \frac{1}{x^{3}} = \frac{1}{x^{3}} = \frac{1}{x^{3}} = \frac{1}{x^{3}} = \frac{1}{x^{3}} = \frac{1}{x^{6-3}} = \frac{1}{x^{3}} = \frac{1}{x^{3}} = \frac{1}{x^{6}} = \frac{1}{x^{3}} = \frac{1$$$

#### Practice 2

Solution 2(6):  

$$\frac{(2a^{2}b^{3})^{5} \times (2a^{2}b^{2})^{3}}{(5a^{4}b)^{6}}$$

$$= \frac{\left[(2)^{5} \cdot (a^{2})^{5} \cdot (b^{3})^{5}\right] \times \left[(2)^{3} \cdot (a^{2})^{3} \cdot (b^{2})^{3}\right]}{(5)^{6} \cdot (a^{4})^{6} \cdot (b)^{6}}$$

$$= \frac{(2^{5} \times a^{2\times5} \times b^{3\times5}) \times (2^{3} \times a^{2\times3} \times b^{2\times3})}{5^{6} \times a^{4\times6} \times b^{6}}$$

$$= \frac{2^{5} \times a^{10} \times b^{15} \times 2^{3} \times a^{6} \times b^{6}}{5^{6} \times a^{24} \times b^{6}}$$

$$= \frac{2^{5+3} \times a^{10+6} \times b^{15+6}}{5^{6} \times a^{24} \times b^{6}}$$

$$= \frac{2^{6} \times a^{16} \times b^{21}}{5^{6} \times a^{24} \times b^{6}}$$

$$= \frac{2^{8} \times a^{16} \times b^{21}}{5^{6} \times a^{24} \times b^{6}}$$

$$= \frac{2^{8} \times a^{10} \times b^{21-6}}{5^{6} a^{8}}$$

$$\left(\frac{1}{3} \times x\right)^3$$
$$= \left(\frac{x}{3}\right)^3$$
$$= \frac{x^3}{3^3}$$

Solution 2(5):

Solution 2(4):  

$$\frac{(y^7)^3}{(y^6)^5}; (y \neq 0)$$

$$= \frac{y^{7\times3}}{y^{6\times5}}$$

$$= \frac{y^{21}}{y^{30}}$$

$$= \frac{1}{y^{30-21}}$$

$$= \frac{1}{y^9}$$

$$\frac{(a^{14})^4 \times (a^2)^3}{(a^7)^3}; (a \neq 0)$$
  
=  $\frac{a^{14 \times 4} \times a^{2 \times 3}}{a^{7 \times 3}}$   
=  $\frac{a^{56} \times a^6}{a^{21}}$   
=  $\frac{a^{56+6}}{a^{21}}$   
=  $\frac{a^{62}}{a^{21}}$   
=  $a^{62-21}$   
=  $a^{41}$ 

Solution 1:

(1) c 
$$\frac{1}{64}$$
  
 $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$   
(2) a  $5^{-1}$   
 $\frac{1}{5} = \frac{1}{5^1} = 5^{-1}$   
(3) c  $64$   
 $(8^{-1})^{-2} = 8^{(-1)\times(-2)} = 8^2 = 64$   
(4) a  $a^0$   
 $a^{-2} \times \frac{1}{a^{-2}} = a^{-2-(-2)} = a^{-2+2} = a^0$ 

(5) b. 
$$2^2$$
  
$$\frac{1}{\left(2^{-1}\right)^2} = \frac{1}{2^{-1\times 2}} = \frac{1}{2^{-2}} = 2^2$$

(6) b. 1  
$$(\sqrt{5})^3 + (\sqrt{5})^3 = (\sqrt{5})^{3-3} = (\sqrt{5})^0 = 1$$

$$\frac{1}{\left(2^{-1}\right)^2} = \frac{1}{2^{-1\times 2}} = \frac{1}{2^{-2}} = 2^2$$
  
(6) b. 1

5) b. 
$$2^2$$
  
$$\frac{1}{(2^{-1})^2} = \frac{1}{2^{-1\times 2}} = \frac{1}{2^{-2}} = 2^2$$

Solution 2:

= 2<sup>-22</sup>

 $=\frac{1}{2^{22}}$ 

(1)  $2^{-3} \times \left(\frac{1}{4}\right)^5 \times 8^{-3}$ 

(4)=  $2^{-3}x \left(\frac{1}{2^2}\right)^5 x(2^3)^{-3}$ =  $2^{-3}x \frac{1}{2^{10}} x 2^{-9}$ =  $\frac{2^{-3} x 2^{-9}}{2^{10}}$ =  $\frac{2^{(-12)}}{2^{10}}$ =  $2^{-12-10}$  $2^{-22}$ 

 $(2)\left(\frac{a}{b}\right)^{m+n}\times\left(\frac{a}{b}\right)^{m-n}\times\left(ab\right)^{m}$ 

 $=\frac{a^{m+n}}{b^{m+n}}\times\frac{a^{m-n}}{b^{m-n}}\times a^m\times b^m$ 

 $=\frac{a^{m+n+m-n+m}}{b^{m+n+m-n-m}}$ 

 $=\frac{a^{3m}}{b^m}$ 

= a<sup>3m</sup> ÷ b<sup>m</sup>

$$\frac{1}{\left(2^{-1}\right)^2} = \frac{1}{2^{-1\times 2}} = \frac{1}{2^{-2}} = 2^2$$
) b. 1

$$\frac{1}{\left(2^{-1}\right)^2} = \frac{1}{2^{-1\times 2}} = \frac{1}{2^{-2}} = 2^2$$

$$\frac{1}{2} = \frac{1}{2^{-1 \times 2}} = \frac{1}{2^{-2}} = \frac{1}{2^{-2}} = \frac{1}{2^{-2}}$$

$$\frac{1}{2^{-1\times 2}} = \frac{1}{2^{-1}} = \frac{1}{2^{-2}} = 2^2$$

$$\frac{1}{2} = \frac{1}{2^{-1 \times 2}} = \frac{1}{2^{-2}} = 2^2$$

$$\frac{1}{1\times 2} = \frac{1}{2^{-2}} = 2^2$$

$$\frac{1}{\sqrt{2}} = \frac{1}{2^{-2}} = 2^2$$

$$8^{(-1)\times(-2)} = 8^2 = 64$$

$$(3) 4a^{-2} \times \left(\frac{a}{2}\right)^{5} + (2a)^{2}$$
$$= 2^{2} \times a^{-2} \times \frac{a^{5}}{2^{5}} \times \frac{1}{2^{2} a^{2}}$$
$$= \frac{2^{2} \times a^{-2+5}}{2^{5+2} \times a^{2}}$$
$$= \frac{2^{2} \times a^{3}}{2^{7} \times a^{2}}$$
$$= \frac{a^{3-2}}{2^{7-2}}$$
$$= \frac{a}{2^{5}}$$
$$= \frac{a}{32}$$

Solution 3: (1)  $2^2 \times 2^{-3} \times 2^{-1}$ =  $2^{2+(-3)+(-1)}$ 

$$= 2^{2 + (-3) + (-1)}$$

$$= 2^{2-3-1}$$

$$= 2^{2^{-4}}$$

$$= 2^{-2}$$

$$= \frac{1}{2^{2}}$$

$$= \frac{1}{4}$$
(2)  $\left(\frac{1}{3}\right)^{3} \times 3^{-2} \times 3^{5}$ 

$$= \frac{1^{3}}{3^{3}} \times 3^{-2+5}$$

$$= \frac{1}{3^{3}} \times 3^{3}$$

$$= 1 \times 3^{3-3}$$

$$= 1 \times 3^{0}$$

$$= 1 \times 1$$

$$= 1$$
(3)  $\left(8^{-2} \times 12^{4}\right) + 27^{2}$ 

$$= \left[(2^{3})^{-2} \times (4 \times 3)^{4}\right] + (3^{3})^{2}$$

$$= \left[(2^{3})^{-2} \times (2^{2} \times 3)^{4}\right] + (3^{3})^{2}$$

$$= 2^{-6} \times 2^{8} \times 3^{4} + 3^{6}$$

$$= \frac{2^{8-6}}{3^{6-4}}$$

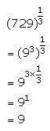
$$= \frac{2^{2}}{3^{2}}$$

$$= \frac{4}{9}$$

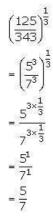
## Practice 3

Solution 1(1):

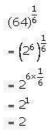
Solution 1(2):



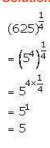
Solution 1(3):



Solution 1(4):



Solution 1(5):

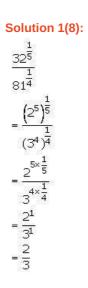


Solution 1(6):

$$\left(\frac{81}{144}\right)^{\frac{1}{2}} = \left(\frac{9^2}{12^2}\right)^{\frac{1}{2}} = \frac{9^{2\times\frac{1}{2}}}{12^{2\times\frac{1}{2}}} = \frac{9^{1}}{12^{1}} = \frac{9^{1}}{12} = \frac{9}{12} = \frac{3}{4}$$

Solution 1(7):

$$\begin{pmatrix} 81\\\overline{625} \end{pmatrix}^{-3} = \begin{pmatrix} 34\\\overline{54} \end{pmatrix}^{-3} = \begin{pmatrix} 34\\\overline{54} \end{pmatrix}^{-3} = \frac{4}{4} \begin{pmatrix} -3\\\overline{54} \end{pmatrix}^{-3} = \frac{4}{5} \begin{pmatrix} -3\\\overline{54} \end{pmatrix}^{-3} = \frac{3}{5} \begin{pmatrix} -3\\\overline{5} \end{pmatrix}^{-3} = \frac{3}{5} \begin{pmatrix} -3\\\overline{5} \end{pmatrix}^{-3} = \frac{5}{3} \\ = \frac{5}{3} \\ = \frac{125}{27}$$



Solution 2:

L.H.S. = 
$$5^{\frac{1}{3}} \times \left(\frac{2}{5}\right)^{\frac{1}{3}} \times \frac{64^{\frac{1}{3}}}{3^{\frac{1}{3}}} \times \frac{9^{\frac{1}{6}}}{2^{\frac{1}{3}}}$$
  
=  $5^{\frac{1}{3}} \times \frac{2^{\frac{1}{3}}}{5^{\frac{1}{3}}} \times \frac{(2^{6})^{\frac{1}{3}}}{3^{\frac{1}{3}}} \times \frac{(3^{2})^{\frac{1}{6}}}{2^{\frac{1}{3}}}$   
=  $5^{\frac{1}{3}} \times \frac{2^{\frac{1}{3}}}{5^{\frac{1}{3}}} \times \frac{2^{6\times\frac{1}{3}}}{3^{\frac{1}{3}}} \times \frac{3^{2\times\frac{1}{6}}}{2^{\frac{1}{3}}}$   
=  $\frac{5^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{2}{3}} \times 2^{2\times\frac{1}{3}}}{5^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times 2^{\frac{1}{3}}}$   
=  $5^{\frac{1}{3} - \frac{1}{3}} \times 2^{\frac{1}{3} + 2 - \frac{1}{3}} \times 3^{\frac{1}{3} - \frac{1}{3}}$   
=  $5^{0} \times 2^{2} \times 3^{0}$   
=  $1 \times 4 \times 1$   
=  $4$   
= R.H.S.

Hence, proved.

## Solution 3:

Given: 
$$x = 243$$
  

$$x^{\frac{1}{5}} \times x^{\frac{-1}{5}} = (243)^{\frac{1}{5}} \times (243)^{\frac{-1}{5}}$$

$$= (243)^{\frac{1}{5} + (-\frac{1}{5})}$$

$$= (243)^{\frac{1}{5} - \frac{1}{5}}$$

$$= 243^{0}$$

$$= 1$$

#### Solution 4(1):

Solution 4(1):  

$$\sqrt{625} - 5\sqrt[3]{27} - (100)^{\frac{1}{2}} = 0$$
  
L.H.S. =  $\sqrt{625} - 5\sqrt[3]{27} - (100)^{\frac{1}{2}}$   
=  $(625)^{\frac{1}{2}} - 5(27)^{\frac{1}{3}} - (100)^{\frac{1}{2}}$   
=  $(25^2)^{\frac{1}{2}} - 5(3^3)^{\frac{1}{3}} - (10^2)^{\frac{1}{2}}$   
=  $25^{2\times\frac{1}{2}} - 5(3^{3\times\frac{1}{3}}) - 10^{2\times\frac{1}{2}}$   
=  $25 - 5 \times 3 - 10$   
=  $25 - 15 - 10$   
=  $0$   
= R.H.S.

Solution 4(2):

$$\begin{bmatrix} (81)^{\frac{1}{2}} + 1 \end{bmatrix} \begin{bmatrix} (81)^{\frac{1}{4}} - 1 \end{bmatrix} = 20$$
  
L.H.S. =  $\begin{bmatrix} (81)^{\frac{1}{2}} + 1 \end{bmatrix} \begin{bmatrix} (81)^{\frac{1}{4}} - 1 \end{bmatrix}$   
=  $\begin{bmatrix} (3^4)^{\frac{1}{2}} + 1 \end{bmatrix} \begin{bmatrix} (3^4)^{\frac{1}{4}} - 1 \end{bmatrix}$   
=  $\begin{bmatrix} 3^{4\times\frac{1}{2}} + 1 \end{bmatrix} \begin{bmatrix} 3^{4\times\frac{1}{4}} - 1 \end{bmatrix}$   
=  $(3^2 + 1)(3 - 1)$   
=  $(9 + 1)(2)$   
=  $(10)(2)$   
=  $20$   
= R.H.S.