ACTIVITY 11

To measure the shortest distance between two skew lines and verify it analytically.

Material Required

A piece of cardboard of size 40 cm × 30 cm, a squared paper, three wooden block of size 2 cm × 2 cm × 2 cm each and one wooden blocks of size 2 cm × 2 cm × 4 cm, some wires of different lengths, set squares, gum, pen/pencil, etc.

Method of Construction

1. Paste a squared paper on a piece of cardboard.
2. On the squared paper, draw two lines *OA* and *OB* to represent *X*-axis and *Y*-axis, respectively.
3. Name the three blocks of size 2 cm × 2 cm × 2 cm as I, II and III. Name the other wooden block of size 2 cm × 2 cm × 4 cm as IV.
4. Place blocks I, II, III such that their base centres are at the points (2, 2), (1, 6) and (7, 6), respectively, and block IV with its base centre at (6, 2). Other wooden block of size 2 cm × 2 cm × 4 cm as IV.
5. Place a wire joining the points *C* and *D*, the centres of the bases of the blocks I and III and another wire joining the centres *E* and *F* of the tops of blocks II and IV as shown in given below fig.
6. The lines represented by these two lines and skew lines.
7. Take a wire and join it perpendicularly with the skew lines and calculate the actual distance.

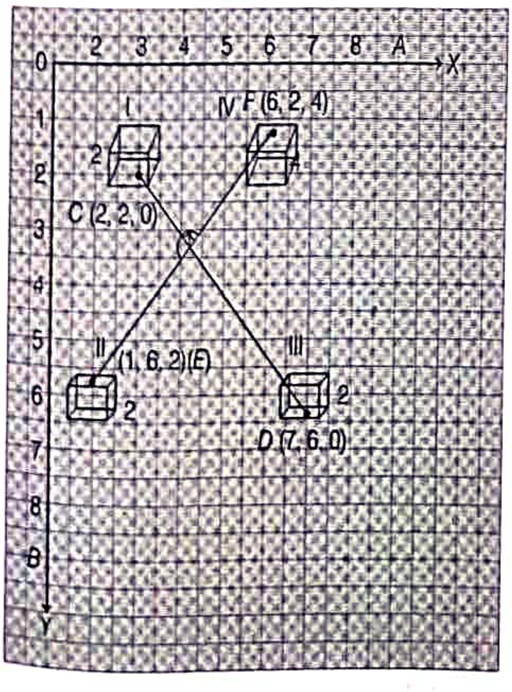

Figure 12.1

Demonstration

1. A set-square is placed in such a way that its one perpendicular side is along the wire *CD*
2. Move the set-square along *CD* till its other perpendicular side touches the other wire.
3. Using set-squares, measure the distance between the two lines in this position. This is known as the shortest distance between two skew lines.
4. Analytically, find the equation of line joining $C(2,2,0)$ and $D(7,6,0)$ and other line joining $E(1,6,2)$ and $F(6,2,4)$ and find S.D. using $\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$. The two distances obtained will be equal.

Observation

1. Coordinates of point *C* are ....
2. Coordinates of point *D* are ....
3. Coordinates of point *E* are ....
4. Coordinates of point *F* are ....
5. Equation of line *CD* is ....
6. Equation of line *EF* is ....

Shortest distance between *CD* and *EF* analytically = ...

Shortest distance by actual measurement = ...

The results so obtained are ....

Application

This activity can be used to explain the concept of skew lines and of shortest distance between two lines in space.

The activity explains the concept of skew lines and describe how to measure shortest distance between two lines in space.

VIVA-VOCE

1 Define skew lines.

Ans Two straight line in space which are neither parallel nor intersecting are called skew lines.

2 Explain line of shortest distance between two lines.

Ans If l_1 and l_2 are two skew lines, then there is one and only one line perpendicular to each of lines l_1 and l_2 which is known as the line of shortest distance.

3 Write the condition for two given lines to intersect.

Ans If the lines $\bar{r} = \bar{a}_1 + \lambda \bar{b}_1$ and $\bar{r} = \bar{a}_2 + \mu \bar{b}_2$ intersect, then the shortest distance between them is zero

$$\therefore \quad \left| \frac{(\bar{b}_1 \times \bar{b}_2) \cdot (\bar{a}_2 - \bar{a}_1)}{|\bar{b}_1 \times \bar{b}_2|} \right| = 0$$

$$\Rightarrow \quad (\bar{b}_1 \times \bar{b}_2).(\bar{a}_2 - \bar{a}_1) = 0$$

4 Write the formula for shortest distance between the lines $\bar{r} = \bar{a}_1 + \lambda \bar{b}$ and $\bar{r}_2 = \bar{a}_2 + \mu \bar{b}$

Ans Since, the given lines are parallel, therefore,

$$SD = \left| \frac{|(\bar{a}_2 - \bar{a}_1) \times \bar{b}|}{|\bar{b}|} \right|$$

5 Write the equation of a line in cartesian form which passing through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$.

Ans $$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

6 Write the formula for angle between lines,
$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$

Ans $$\cos \theta = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| \cdot |\vec{b}_2|} \right|$$