Conic Sections

11

MCQs with One Correct Answer

1. The line $4x+3y-4=0$ divides the circumference of the circle centered at (5, 3), in the ratio 1 : 2. Then the equation of the circle is

(a) $x^2+y^2-10x-6y-66=0$

(b) $x^2+y^2-10x-6y+100=0$

(c) $x^2+y^2-10x-6y+66=0$

(d) $x^2+y^2-10x-6y-100=0$

2. Let $A(-4, 0)$ and $B(4, 0)$. Then the number of points $C=(x, y)$ on the circle $x^2+y^2=16$ lying in first quadrant such that the area of the triangle whose vertices are A, B and C is a integer is

(a) 14 (b) 15

(c) 16 (d) None of these

3. If (α, β) is a point on the circle whose centre is on the x-axis and which touches the line $x+y=0$ at (2, –2), then the greatest value of α is

(a) $4-\sqrt{2}$ (b) 6

(c) $4+2\sqrt{2}$ (d) $4+\sqrt{2}$

4. The set of values of 'c' so that the equations $y=|x|+c$ and $x^2+y^2-8|x|-9=0$ have no solution is

(a) $(-\infty, -3)\cup(3, \infty)$

(b) $(-3, 3)$

(c) $(-\infty, -\sqrt{2})\cup(5\sqrt{2}, \infty)$

(d) $(5\sqrt{2}-4, \infty)$

5. Tangents are drawn from O (origin) to touch the circle $x^2+y^2+2gx+2fy+c=0$ at points P and Q. The equation of the circle circumscribing triangle OPQ is

(a) $2x^2+2y^2+gx+fy=0$

(b) $x^2+y^2+gx+fy=0$

(c) $x^2+y^2+2gx+2fy=0$

(d) None of these

6. A ray of light incident at the point (– 2, – 1) gets reflected from the tangent at (0, –1) to the circle $x^2+y^2=1$. The reflected ray touches the circle. The equation the line along which the incident ray moved, is

(a) $4x-3y+11=0$ (b) $4x+3y+11=0$

(c) $3x+4y+11=0$ (d) $4x+3y+7=0$

7. If the line $y=mx+1$ meets the circle $x^2+y^2+3x=0$ in two points equidistant from and on opposite sides of x-axis, then

(a) $3m+2=0$ (b) $3m-2=0$

(c) $2m+3=0$ (d) $2m-3=0$

8. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is

(a) $2ax-2by-(a^2+b^2+4)=0$

(b) $2ax+2by-(a^2+b^2+4)=0$

(c) $2ax-2by+(a^2+b^2+4)=0$

(d) $2ax+2by+(a^2+b^2+4)=0$

9. The set of all real values of λ for which exactly two common tangents can be drawn to the circles $x^2 + y^2 - 4x - 4y + 6 = 0$ and $x^2 + y^2 - 10x - 10y + \lambda = 0$ is the interval:

(a) $(12, 32)$ (b) $(18, 42)$
(c) $(12, 24)$ (d) $(18, 48)$

10. A circle bisects the circumference of the circle $x^2 + y^2 - 2y - 3 = 0$ and touches the line $x = y$ and the point (1, 1). Its radius is :

(a) $\frac{3}{\sqrt{2}}$ (b) $\frac{9}{\sqrt{2}}$ (c) $4\sqrt{2}$ (d) $3\sqrt{2}$

11. Let L_1 be the length of the common chord of the curves $x^2 + y^2 = 9$ and $y^2 = 8x$, and L_2 be the length of the latus rectum of $y^2 = 8x$, then:

(a) $L_1 > L_2$ (b) $L_1 = L_2$
(c) $L_1 < L_2$ (d) $\frac{L_1}{L_2} = \sqrt{2}$

12. If the tangent at the point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x + b)$ at Q and R, then the mid-point of QR is

(a) $(x_1 + b, y_1 + b)$ (b) $(x_1 - b, y_1 - b)$
(c) (x_1, y_1) (d) $(x_1 + b, y_1 - b)$

13. Tangent to the curve $y = x^2 + 6$ at a point (1, 7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then the coordinates of Q are

(a) $(-6, -11)$ (b) $(-9, -13)$
(c) $(-10, -15)$ (d) $(-6, -7)$

14. A circle is drawn with centre at the focus S of the parabola $y^2 = 4x$ so that a common chord of the parabola and the circle is equidistant from the focus and the vertex. Then the equation of the circle is

(a) $(x-1)^2 + y^2 = \frac{9}{4}$ (b) $(x-1)^2 = \frac{9}{16} - y^2$
(c) $(x-1)^2 + x^2 = \frac{9}{4}$ (d) $(y-1)^2 + x^2 = \frac{9}{16}$

15. Locus of all such points so that sum of its distances from (2, – 3) and (2, 5) is always 10, is

(a) $\frac{(x-2)^2}{25} + \frac{(y-1)^2}{9} = 1$

(b) $\frac{(x-2)^2}{25} + \frac{(y-1)^2}{16} = 1$

(c) $\frac{(x-2)^2}{16} + \frac{(y-1)^2}{25} = 1$

(d) $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{25} = 1$

16. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having its centre at (0, 3) is

(a) 4 (b) 3 (c) $\sqrt{\frac{1}{2}}$ (d) $\frac{7}{2}$

17. Equation of the line passing through the points of intersection of the parabola $x^2 = 8y$ and the ellipse $\frac{x^2}{3} + y^2 = 1$ is :

(a) $y - 3 = 0$ (b) $y + 3 = 0$
(c) $3y + 1 = 0$ (d) $3y - 1 = 0$

18. Equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$, is

(a) $2x^2 + 2y^2 - 4x + 7 = 0$

(b) $x^2 + y^2 - 2x + 5 = 0$

(c) $3x^2 + 3y^2 - 6x - 8 = 0$

(d) None of these

19. The normal at $\left(2, \frac{3}{2}\right)$ to the ellipse, $\frac{x^2}{16} + \frac{y^2}{3} = 1$ touches a parabola, whose equation is

(a) $y^2 = -104x$ (b) $y^2 = 14x$
(c) $y^2 = 26x$ (d) $y^2 = -14x$

20. The angle subtended by the common tangent of the two ellipse $\frac{(x-4)^2}{25} + \frac{y^2}{4} = 1$ and $\frac{(x+1)^2}{1} + \frac{y^2}{4} = 1$ at the origin is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Numeric Value Answer

21. Two equal chords AB and AC of the circle $x^2 + y^2 - 6x - 8y - 24 = 0$ are drawn from the point $A\left(\sqrt{33}+3,0\right)$. Another chord PQ is drawn intersecting AB and AC at points R and S, respectively given that $AR = SC = 7$ and $RB = AS = 3$. The value of PR/QS is

22. If p and q be the longest and the shortest distance respectively of the point (–7, 2) from any point (α, β) on the curve whose equation is $x^2+y^2-10x-14y-51=0$ and G.M. of p and q is $2\sqrt{k}$, then value k is _______.

23. The straight line $y = mx + c$ $(m > 0)$ touches the parabolas $y^2 = 8(x+2)$ then the minimum value taken by c is

24. Two tangents are drawn from a point (–2, –1) to the curve, $y^2 = 4x$. If α is the angle between them, then $|\tan\alpha|$ is equal to:

25. Tangents are drawn to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ at ends of latus rectum. The area of quadrilateral so formed is

26. A trapezium is inscribed in the parabola $y^2 = 4x$ such that its diagonal pass through the point (1, 0) and each has length $\frac{25}{4}$. If the area of trapezium be P then $\left[\frac{P}{4}\right]$ is equal to

27. S_1 and S_2 be the foci of the hyperbola whose transverse axis length is 4 and conjugate axis length is 6, S_3 and S_4 be the foci of the conjugate hyperbola. If the area of the quadrilateral $S_1 S_3 S_2 S_4$ is A, then find $\frac{A}{13}$.

28. If the ratio of the area of equilateral triangles made of the common chord of the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x + 4 = 0$ and their respective pairs of tangents drawn from points on the positive x- axis is $57+24\sqrt{3}:k$ then k is ________.

29. $P(a,b)$ is a points in the first quadrant. Circles are drawn through P touching the coordinate axes, such that the length of common chord of these circle is maximum. If possible values of a/b is $k_1 \pm k_2\sqrt{2}$ then k_1+k_2 is equal to______.

30. C is the centre of the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$, and '$A$' is any point on it. The tangent at A to the hyperbola meets the line $x-2y=0$ and $x+2y=0$ at Q and R respectively. The value of $CQ.CR$ is equal to

ANSWER KEY

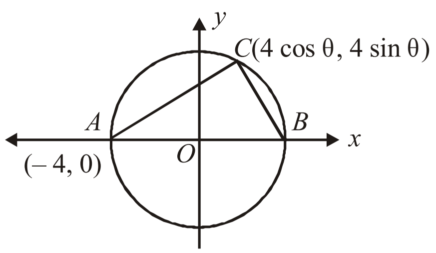
1	(a)	**5**	(b)	**9**	(b)	**13**	(d)	**17**	(d)	**21**	(1)	**25**	(27)	**29**	(5)		
2	(b)	**6**	(b)	**10**	(b)	**14**	(a)	**18**	(c)	**22**	(11)	**26**	(4)	**30**	(5)		
3	(c)	**7**	(b)	**11**	(c)	**15**	(d)	**19**	(a)	**23**	(4)	**27**	(2)				
4	(d)	**8**	(b)	**12**	(c)	**16**	(a)	**20**	(a)	**24**	(3)	**28**	(9)				

Hints & Solutions

CHAPTER 11 Conic Sections

1. **(a)** Since $4x + 3y - 4 = 0$ is dividing the circumference in the ratio 1 : 2, angle subtended at the centre $= 2\pi/3$.
Also the perpendicular distance from the centre of the given line is 5
$\Rightarrow$ Radius = 10 $\Rightarrow$ Equation of the circle is
$x^2 + y^2 - 10x - 6y - 66 = 0$.

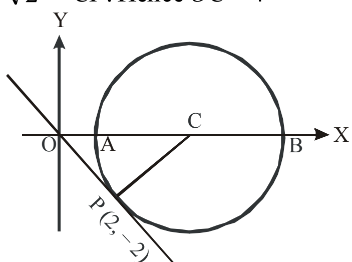
2. **(b)**



$A = \frac{1}{2} . 8 . 4 \sin\theta = |16 \sin \theta|$

Now, $\sin\theta$ can be equal to $\frac{1}{16}, \frac{2}{16}, ... \frac{15}{16}$
i.e. there are 15 points in each quadrant.

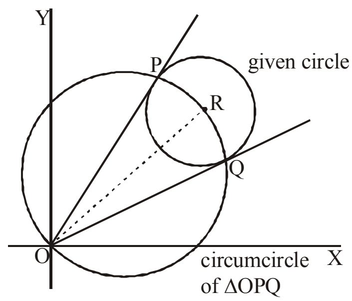
3. **(c)** If $(a, 0)$ is the centre C and P is $(2, -2)$, then $\angle COP = 45°$.
Since the equation of OP is $x + y = 0$.
$\therefore$ $OP = 2\sqrt{2} = CP$. Hence $OC = 4$



The point on the circle with the greatest x coordinate is B.
$\alpha = OB = OC + CB = 4 + 2\sqrt{2}$.

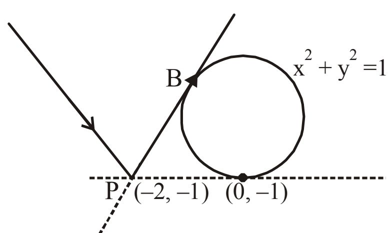
4. **(d)** Since $y = |x| + c$ and $x^2 + y^2 - 8|x| - 9 = 0$ both are symmetrical about y-axis we consider the case $x > 0$, when the equations become $y = x + c$ and $x^2 + y^2 - 8x - 9 = 0$. Equation of tangent to circle $x^2 + y^2 - 8x - 9 = 0$ parallel to $y = x + c$ is $y = (x - 4) + 5\sqrt{1+1}$
$\Rightarrow$ $y = x + (5\sqrt{2} - 4)$
For no solution $c > 5\sqrt{2} - 4$
$\therefore$ $c \in (5\sqrt{2} - 4, \infty)$.

5. **(b)** Equation of the given circle is
$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad ......(i)$$



Equation of the chord of contact PQ, drawn from the origin (0, 0) to the given circle will be
$$gx + fy + c = 0 \quad .....(ii)$$
Eq. of any circle passing through the intersection points of the given circle and the chord PQ can be written as
$$(x^2 + y^2 + 2gx + 2fy + c) + \lambda(gx + fy + c) = 0 \quad ....(iii)$$
If this circle passes through the origin, then we have,
$$c + \lambda c = 0 \text{ gives } \lambda = -1$$
Putting the above value of λ in equation (iii) gives the equation of the required circle as
$$x^2 + y^2 + gx + fy = 0$$

6. **(b)**



Any line through $(-2, -1)$ is $y + 1 = m(x + 2)$

It touches the circle if $\left|\frac{2m-1}{\sqrt{1+m^2}}\right| = 1$

$\Rightarrow$ $m = 0, \frac{4}{3}$

$\therefore$ Equation of PB is $y+1=\frac{4}{3}(x+2)$

$\Rightarrow 4x-3y+5=0$

A point on PB is (– 5, – 5), (we can choose some other point as well)

Its image by the line $y = -1$ is P′ (– 5 , 3).

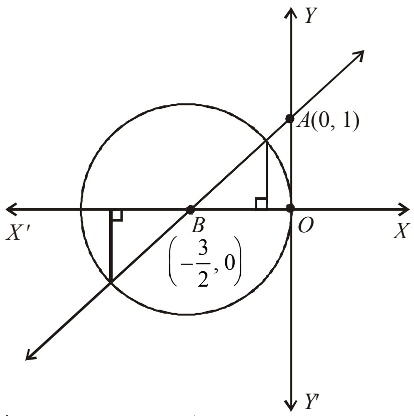
Hence equation of incident ray PP′ is

$y-3=\frac{3+1}{-5+2}(x+5) \Rightarrow 4x+3y+11=0$

7. **(b)** Circle : $x^2+y^2+3x=0$,

Centre, $B=\left(-\frac{3}{2},0\right)$

Radius $=\frac{3}{2}$ units.



Line : $y=mx+1$

y-intercept of the line = 1

$\therefore$ A = (0, 1)

Slope of line, $m=\tan\theta=\frac{OA}{OB}$

$\Rightarrow m=\frac{1}{\frac{3}{2}}=\frac{2}{3} \Rightarrow 3m-2=0$

8. **(b)** Let the variable circle is

$x^2+y^2+2gx+2fy+c=0$ ...(i)

It passes through (a, b)

$\therefore a^2+b^2+2ga+2fb+c=0$ ...(ii)

(i) cuts $x^2+y^2=4$ orthogonally

$\therefore 2(g\times 0+f\times 0)=c-4 \Rightarrow c=4$

$\therefore$ from (ii) $a^2+b^2+2ga+2fb+4=0$

$\therefore$ Locus of centre $(-g,-f)$ is

$a^2+b^2-2ax-2by+4=0$

or $2ax+2by=a^2+b^2+4$

9. **(b)** The equations of the circles are

$x^2+y^2-10x-10y+\lambda=0$ ...(i)

and $x^2+y^2-4x-4y+6=0$ ...(ii)

C_1 = centre of (i) = (5, 5)

C_2 = centre of (ii) = (2, 2)

d = distance between centres = C_1C_2

$=\sqrt{9+9}=\sqrt{18}$

$r_1=\sqrt{50-\lambda}$, $r_2=\sqrt{2}$

For exactly two common tangents we have

$r_1-r_2<C_1C_2<r_1+r_2$

$\Rightarrow \sqrt{50-\lambda}-\sqrt{2}<3\sqrt{2}<\sqrt{50-\lambda}+\sqrt{2}$

$\Rightarrow \sqrt{50-\lambda}-\sqrt{2}<3\sqrt{2}$ or $3\sqrt{2}<\sqrt{50-\lambda}+\sqrt{2}$

$\Rightarrow \sqrt{50-\lambda}<4\sqrt{2}$ or $2\sqrt{2}<\sqrt{50-\lambda}$

$\Rightarrow 50-\lambda<32$ or $8<50-\lambda$

$\Rightarrow \lambda>18$ or $\lambda<42$

Required interval is (18, 42)

10. **(b)** Equation of required circle :

$S:(x-1)^2+(y-1)^2+\lambda(x-y)=0$

$S':x^2+y^2-2y-3=0$

Common chord of $S=0$ and $S'=0$ is $S-S'=0$

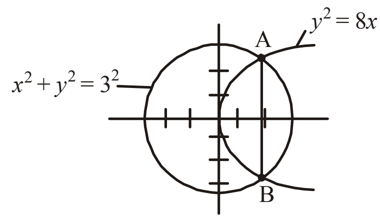
$(\lambda-2)x-(\lambda+4)y+5=0$

Centre of S' : (0, – 1) lies on common chord

$\Rightarrow \lambda=-9$

$S:(x-1)^2+(y-1)^2-9(x-y)=0 \Rightarrow r=\frac{9}{\sqrt{2}}$

11. **(c)**



We have : $x^2 + (8x) = 9$

$\Rightarrow x^2 + 9x - x - 9 = 0$

$\Rightarrow x(x + 9) - 1(x + 9) = 0$

$\Rightarrow (x + 9)(x - 1) = 0 \Rightarrow x = -9, 1$

for $x = 1$, $y = \pm 2\sqrt{2x} = \pm 2\sqrt{2}$

L_1 = Length of AB

$=\sqrt{(2\sqrt{2}+2\sqrt{2})^2+(1-1)^2}=4\sqrt{2}$

L_2 = Length of latus rectum = $4a = 4 \times 2 = 8$

$L_1 < L_2$

12. **(c)** Equation of the tangent at $P(x_1, y_1)$ to $y^2 = 4ax$ is $yy_1-2ax-2ax_1=0$ ...(i)

Equation of the chord of $y^2 = 4a(x + b)$ whose mid-point is (x', y') is

$yy' - 2ax - 2ax' - 4ab$
$= y'^2 - 4a x' - 4ab$ or $yy' - 2ax - (y'^2 - 2a x') = 0$ ...(ii)

Equation (i) and (ii) represent the same line.

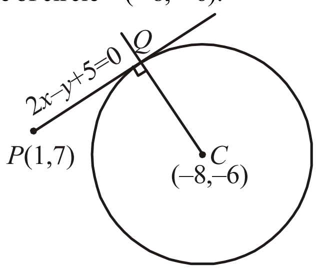
$\therefore \quad \frac{y_1}{y'} = \frac{2a}{2a} = -\frac{2ax}{y'^2 - 2ax'}$

This gives $y' = y_1$ and then $2ax_1 = y'^2 - 2ax'$
$= y_1^2 - 2ax' = 4ax_1 - 2ax' \therefore x' = x_1$
$\therefore$ mid-point $(x', y') = (x_1, y_1)$.

13. **(d)** The given curve is $y = x^2 + 6$
Equation of tangent at (1, 7) is

$$\frac{1}{2}(y+7) = x.1 + 6 \Rightarrow 2x - y + 5 = 0 \qquad \text{...(i)}$$

As given this tangent (1) touches the circle
$x^2 + y^2 + 16x + 12y + c = 0$ at Q
Centre of circle = (– 8, – 6).



Then equation of CQ which is perpendicular to (1) and passes through (– 8, – 6) is

$$y + 6 = -\frac{1}{2}(x+8) \Rightarrow x + 2y + 20 = 0 \qquad \text{...(ii)}$$

Now Q is pt. of intersection of (i) and (ii)
$\therefore$ Solving eqs (i) & (ii) we get ; $x = -6, y = -7$
$\therefore$ Req. pt. is (– 6, – 7).

14. **(a)** Since $S = (a, 0) = (1, 0)$, the circle is of the form $(x-1)^2 + y^2 = r^2$
Suppose AB is a common chord. Since this is equidistant from the focus and the vertex. $M(1/2, 0)$ lies on AB and AB is double ordinate of the parabola, let $A = (1/2, y)$ so that

$$y^2 = 4\left(\frac{1}{2}\right) \Rightarrow y = \pm\sqrt{2}$$

$$\Rightarrow A = \left(\frac{1}{2}, \sqrt{2}\right) \text{ and } B = \left(\frac{1}{2}, -\sqrt{2}\right)$$

Since ΔAMS is right-angled triangle, we have

$$SA^2 = SM^2 + MA^2 = \frac{1}{4} + 2 = \frac{9}{4} = (\text{Radius})^2$$

Hence, the equation of the circle is

$$(x-1)^2 + y^2 = \frac{9}{4}$$

15. **(d)** As per the definition, the locus must be an ellipse, with given points as foci and 10 as its major axis. Since the line segment joining (2, –3) and (2, 5) is parallel to y-axis, therefore, ellipse is vertical.

$\therefore \quad 2\,be = 8$ and $2b = 10 \Rightarrow b = 5$ and $e = \frac{4}{5}$

$\therefore \quad a^2 = b^2(1 - e^2) = 9$ and centre of the ellipse is (2, 1)

$\therefore$ Equation of the required ellipse is

$$\frac{(x-2)^2}{9} + \frac{(y-1)^2}{25} = 1$$

16. **(a)** For ellipse $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$
$a = 4, b = 3$

$$\Rightarrow e = \sqrt{1 - \left(\frac{3}{4}\right)^2} = \frac{\sqrt{7}}{4}$$

$\therefore$ Foci are $\left(\sqrt{7}, 0\right)$ and $\left(-\sqrt{7}, 0\right)$

Centre of circle is at (0, 3) and it passes through $\left(\pm\sqrt{7}, 0\right)$, therefore radius of circle

$$= \sqrt{\left(\sqrt{7}\right)^2 + (3)^2} = 4$$

17. **(d)** $x^2 = 8y$ ...(i)

$$\frac{x^2}{3} + y^2 = 1 \qquad \text{...(ii)}$$

From (i) and (ii), $\frac{8y}{3} + y^2 = 1 \Rightarrow y = -3, \frac{1}{3}$

When $y = -3$, then $x^2 = -24$, which is not possible.

When $y = \frac{1}{3}$, then $x = \pm\frac{2\sqrt{6}}{3}$

Point of intersection are

$$\left(\frac{2\sqrt{6}}{3}, \frac{1}{3}\right) \text{ and } \left(-\frac{2\sqrt{6}}{3}, \frac{1}{3}\right)$$

Required equation of the line, $y - \frac{1}{3} = 0$

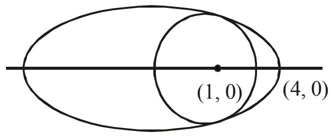
$\Rightarrow 3y - 1 = 0$

18. **(c)** Given ellipse is $\frac{x^2}{16} + \frac{y^2}{4} = 1$ ... (i)

Equation of a circle centered at (1, 0) can be written as $(x-1)^2 + y^2 = r^2$ ... (ii)

The abscissae of the intersection points of the circle and the ellipse is given by the equation

$$(x-1)^2 + \frac{16-x^2}{4} = r^2$$



i.e. $4(x^2 - 2x + 1) + 16 - x^2 = 4r^2$

i.e. $3x^2 - 8x + 20 - 4r^2 = 0$

If the circle lies inside the ellipse, then the roots of the above equation must be imaginary or equal

i.e. $D \le 0$ i.e. $64 + 12(4r^2 - 20) \le 0$

$\Rightarrow \; r = \le \sqrt{\frac{11}{3}}$

Hence, greatest value of $r = \sqrt{\frac{11}{3}}$ and the equation of required circle is

$(x-1)^2 + y^2 = \frac{11}{3}$

i.e. $3(x^2 + y^2) - 6x - 8 = 0.$

19. **(a)** Ellipse is $\frac{x^2}{16} + \frac{y^2}{3} = 1$

Now, equation of normal at (2, 3/2) is

$\frac{16x}{2} - \frac{3y}{3/2} = 16 - 3$

$\Rightarrow \; 8x - 2y = 13 \Rightarrow y = 4x - \frac{13}{2}$

Let $y = 4x - \frac{13}{2}$ touches a parabola

$y^2 = 4ax.$

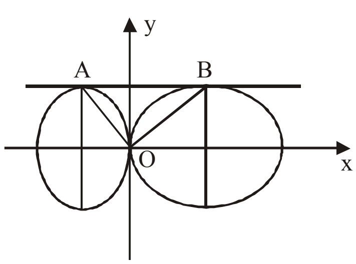
We know, a straight line $y = mx + c$ touches a parabola $y^2 = 4ax$ if $a - mc = 0$

$\therefore \; a - (4)\left(-\frac{13}{2}\right) = 0 \Rightarrow a = -26$

Hence, required equation of parabola is

$y^2 = 4(-26)x = -104x$

20. **(a)**



See figure $B(4, 2)$ is one end of the minor axis of the ellipse $\frac{(x-4)^2}{25} + \frac{y^2}{4} = 1$ and $(-1, 2)$ is one end of the major axis of the second ellipse.

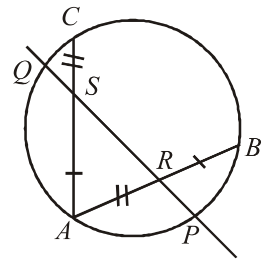
Therefore,

$AB = 5, OB = \sqrt{16+4} = \sqrt{20}$, $OA = \sqrt{1+4} = \sqrt{5}$

We have $(OA)^2 + (OB)^2 = 25 = (AB)^2$

Therefore $\angle AOB = \frac{\pi}{2}$

21. **(1)**



Point $A(\sqrt{33} + 3, 0)$ lies on the given circle,

$x^2 + y^2 - 6x - 8y - 24 = 0$

PQ and AB intersect inside the circle.

Let $PR = a, RS = b, QS = c$

Since $PR \times RQ = AR \times RB \Rightarrow a(b+c) = 3 \times 7$

Also, $QS \times SP = 3 \times 7 \Rightarrow c(a+b) = 3 \times 7$

$\Rightarrow a = c \; \therefore \; PR/QS = 1$

22. **(11)** The centre C of the circle = (5,7) and the radius

$= \sqrt{5^2 + 7^2 + 51} = 5\sqrt{5}$

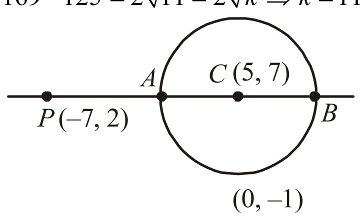
PC $= \sqrt{12^2 + 5^2} = 13 \Rightarrow q = PA = 13 - 5\sqrt{5}$

and $p = PB = 13 + 5\sqrt{3}$

$\therefore$ G.M. of p and q

$= \sqrt{pq} = \sqrt{(13 - 5\sqrt{5}(13 + 5\sqrt{5}))}$

$= \sqrt{169 - 125} = 2\sqrt{11} = 2\sqrt{k} \Rightarrow k = 11.$



23. **(4)** Tangent to $y^2 = 8(x+2)$ is

$y = m(x+2) + \frac{2}{m}$

$c = 2m + \frac{2}{m} \Rightarrow \frac{c}{2} = \left(m + \frac{1}{m}\right)$

$\because \quad m+\frac{1}{m} \geq 2 \;\Rightarrow\; \frac{c}{2} \geq 2 \;\Rightarrow c \geq 4$

$\Rightarrow$ The minimum value of $c = 4$.

24. **(3)** The locus of the point of intersection of tangents to the parabola $y^2 = 4ax$ inclined at an angle α to each other is $\tan^2\alpha\,(x + a)^2 = y^2 - 4ax$

Given equation of Parabola $y^2 = 4x$ $\{a = 1\}$

Point of intersection $(-2, -1)$

$\tan^2\alpha\,(-2 + 1)^2 = (-1)^2 - 4 \times 1 \times (-2)$

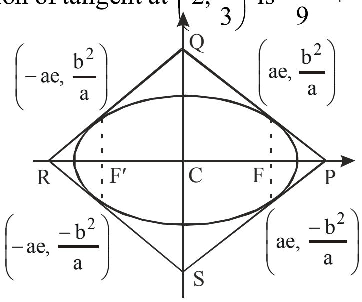
$\Rightarrow \tan^2\alpha = 9 \;\Rightarrow \tan\alpha = \pm 3 \Rightarrow |\tan\alpha| = 3$

25. **(27)** $\frac{x^2}{9} + \frac{y^2}{5} = 1 \Rightarrow e^2 = 1 - \frac{5}{9} = \frac{4}{9}$

$\Rightarrow e = \frac{2}{3}$

One end of latusrectum is $\left(2, \frac{5}{3}\right)$

Equation of tangent at $\left(2, \frac{5}{3}\right)$ is $\frac{2x}{9} + \frac{y}{3} = 1$



Area of $\Delta CPQ = \frac{1}{2} \times \frac{9}{2} \times 3 = \frac{27}{4}$

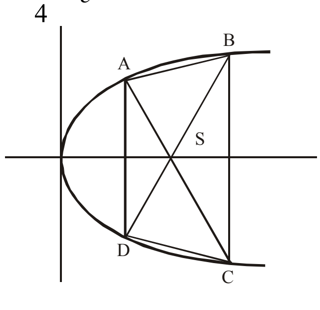
$\therefore$ Area of quadrilateral $PQRS = 4 \times \frac{27}{4} = 27$.

26. **(4)** Focus of the parabola $y^2 = 4x$ is $(1, 0)$

So diagonals are focal chord.

$AS = 1 + t^2 = c$ (say)

$\because \frac{1}{c} + \frac{1}{\frac{25}{4} - c} = 1 \qquad \left[\because \frac{1}{AS} + \frac{1}{CS} = \frac{1}{a}\right]$



$\frac{25}{4} = \frac{25}{4}c - c^2 \;\Rightarrow\; 4c^2 - 25c + 25 = 0$

$\Rightarrow c = \frac{5}{4}, 5$

For $c = \frac{5}{4}, 1 + t^2 = \frac{5}{4} \Rightarrow t^2 = \frac{1}{4} \Rightarrow t = \pm\frac{1}{2}$

For $c = 5, 1 + t^2 = 5 \Rightarrow t = \pm 2$

$A \equiv \left(\frac{1}{4}, 1\right)$, $B \equiv (4, 4)$, $C \equiv (4, -4)$ and $D \equiv \left(\frac{1}{4}, -1\right)$

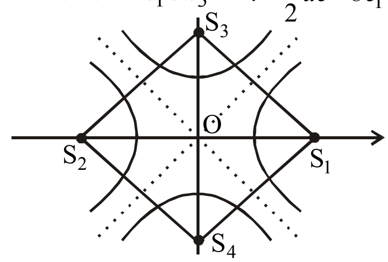
$AD = 2$ and $BC = 8$, distance between AD and $BC = \frac{15}{4}$

$\therefore$ Area of trapezium $ABCD$

$= \frac{1}{2}(2 + 8) \times \frac{15}{4} = \frac{75}{4}$ sq. units.

27. **(2)** Due to symmetry the desired area

$= 4 \times$ area of $\Delta S_1OS_3 = 4 \times \frac{1}{2} ae \times be_1$



Where e_1 is eccentricity of conjugate hyperbola

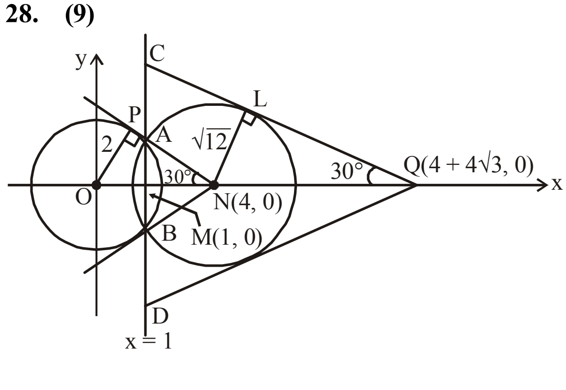
$= 2 \times 2e \times 3e_1 = 12ee_1$

Now $b^2 = a^2(e^2 - 1) \Rightarrow e^2 = 13/4$

and $\frac{1}{e^2} + \frac{1}{e_1^2} = 1 \Rightarrow e_1^2 = \frac{13}{9}$

$\therefore$ Required area $= 12 \times \frac{\sqrt{13}}{2} \times \frac{\sqrt{13}}{3} = 26$

28. **(9)**



Common chord of both the circles is $x=1$.
Now, we have to find the ratio of areas of equilateral triangles ANB and CQD.
Now in triangle OPN,
$ON = OP\ \text{cosec}\ 30° = 2\times 2 = 4$.
Area of triangle NAB

$$\frac{1}{2}MN.AB = MN.AM = MN.\ MN\tan 30°$$
$$=(ON-OM)^2\tan 30°$$
$$=(4-1)^2\frac{1}{\sqrt{3}}=\frac{9}{\sqrt{3}}\text{ sq. units.}$$

Now in triangle NLQ,
$NQ = NL\ \text{cosec}\ 30° = 4\sqrt{3}$.
Since area of triangle $CQD=\frac{1}{2}QM.CD$
$=QM.CM$

QM. QM tan 30° $=(MN+NQ)^2\tan 30°$
$$=(3+4\sqrt{3})^2\frac{1}{\sqrt{3}}=\frac{57+24\sqrt{3}}{\sqrt{3}}\text{ sq. units.}$$

So, ratio of area of trianlges $=\dfrac{57+24\sqrt{3}}{9}$.

29. **(5)** Let r be the radius of the circle. Its equation is $x^2+y^2-2r(x+y)+r^2=0$. Since it passes through $P(a,b)$

$$a^2+b^2-2r(a+b)+r^2=0$$

Solving $r_1=a+b+\sqrt{2ab}$ ...(1)
$r_2=a+b-\sqrt{2ab}$

Now, the equations of two circles are
$x^2+y^2-2r_1(x+y)+r_1^2=0$ and
$x^2+y^2-2r_2(x+y)+r_2^2=0$
The common chord is $S_1-S_2=0$
$$\Rightarrow 2(r_2-r_1)(x+y)+r_1^2-r_2^2=0$$
$$\Rightarrow 2(x+y)=r_1+r_2$$

For maximum length of the common chord, it must pass through the centre of the smaller circle (r_2,r_2), so

$$4r_2=r_1+r_2\Rightarrow\frac{r_1}{r_2}=3$$
$$\Rightarrow\frac{a+b+\sqrt{2ab}}{a+b-\sqrt{2ab}}=3\Rightarrow 2(a+b)=4\sqrt{2ab}$$
$$\Rightarrow (a+b)^2=8ab\Rightarrow\quad a^2-6ab+b^2=0$$
$$\Rightarrow a=\frac{6b\pm\sqrt{36b^2-4b^2}}{2}=(3\pm 2\sqrt{2})b$$
$$\Rightarrow\frac{a}{b}=3\pm 2\sqrt{2}$$

30. **(5)** The tangent at any point $A(2\sec\theta,\tan\theta)$ is given by $\dfrac{x\sec\theta}{2}-\dfrac{y\tan\theta}{1}=1$.
It meets the line $x-2y=0$
$$\Rightarrow\frac{x\sec\theta}{2}-\frac{x\tan\theta}{2}=1\Rightarrow x=\frac{2}{\sec\theta-\tan\theta}$$
$$\Rightarrow Q\equiv\left(\frac{2}{\sec\theta-\tan\theta},\frac{1}{\sec\theta-\tan\theta}\right)\qquad ...(1)$$

Also, the tangent meets the line $x+2y=0$ at R, so
$$\Rightarrow\frac{x}{2}\sec\theta+\frac{x}{2}\tan\theta=1$$
$$\Rightarrow x=\frac{2}{\sec\theta+\tan\theta}$$
$$\Rightarrow R\equiv\left(\frac{2}{\sec\theta+\tan\theta},\frac{-1}{\sec\theta+\tan\theta}\right)\qquad ...(2)$$

Now,
$$CQ.CR=\sqrt{\frac{2^2+1^2}{(\sec\theta-\tan\theta)^2}}\sqrt{\frac{2^2+1^2}{(\sec\theta+\tan\theta)^2}}$$
$$=2^2+1^2$$
$$\Rightarrow CQ.CR=5$$