Quadratic Equations

4

Multiple Choice Questions

Q. 1. The roots of the quadratic equation $x^2 - 0.04 = 0$ are   **[CBSE OD, Set 1, 2020]**

  (a) ± 0.2 (b) ± 0.02

  (c) 0.4 (d) 2

Ans. Given : $x^2 - 0.04 = 0$

$\Rightarrow$ $x^2 - (0.2)^2 = 0$

$\Rightarrow$ $(x + 0.2)(x - 0.2) = 0$

$\Rightarrow$ $x = -0.2, 0.2$

$\therefore$ Option (a) is correct.   **Ans.**

Q. 1. Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.
    [CBSE OD, Set 1, 2019]

Ans. Given, $2x^2 - 4x + 3 = 0$

Comparing it with quadratic equation

$ax^2 + bx + c = 0$

Here, $a = 2$, $b = -4$ and $c = 3$

$\therefore$ $D = b^2 - 4ac$

             $= (-4)^2 - 4 \times (2)(3)$

             $= 16 - 24$

             $= -8 < 0$

$D < 0$ shows that roots will not be real. Hence, roots will be imaginary.

Q. 2. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?
    [CBSE Dehli, Set 1, 2019]

Ans. The given equation is $x^2 + 4x + k = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = 4$ and $c = k$

For real roots, $D \geq 0$

         $b^2 - 4ac \geq 0$

         $16 - 4k \geq 0$

            $k \leq 4$

$\therefore$ For $k \leq 4$, equation $x^2 + 4x + k$ will have real roots.

Q. 3. Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.
    [CBSE Dehli, Set 1, 2019]

Ans. The given equation is $3x^2 - 10x + k = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

       $a = 3$, $b = -10$, $c = k$

Let the roots of the equation are α and $\dfrac{1}{\alpha}$

$\because$ Product of the roots $= \dfrac{c}{a}$

$\therefore$ $\alpha \cdot \dfrac{1}{\alpha} = \dfrac{k}{3}$

           $\dfrac{k}{3} = 1$

or $k = 3$

Q. 4. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.   **[CBSE 2018]**

Ans.

Given quadratic equation is,

     $x^2 - 2kx - 6 = 0$

$x = 3$ is a root of above equation, then

     $(3)^2 - 2k(3) - 6 = 0$

       $9 - 6k - 6 = 0$

         $3 - 6k = 0$

          $3 = 6k$

$\therefore$ $k = \dfrac{3}{6} = \dfrac{1}{2}$

         $k = \dfrac{1}{2}$

Q. 5. If the quadratic equation $px^2 - 2\sqrt{5}\,px + 15 = 0$, has two equal roots then find the value of p.
    [CBSE OD, Term 2, Set 1, 2015]

Ans. The given quadratic equation is,

   $px^2 - 2\sqrt{5}\,px + 15 = 0$

This is of the form

     $ax^2 + bx + c = 0$

Where, $a = p$, $b = -2\sqrt{5}\,p$, $c = 15$

we have, $D = b^2 - 4ac$

           $= (-2\sqrt{5}\,p)^2 - 4 \times p \times 15$

$$= 20p^2 - 60p$$
$$= 20p\,(p-3)$$

For real and equal roots, we must have:
$$D = 0,$$
$$\Rightarrow \qquad 20p\,(p-3) = 0$$
$$\Rightarrow \qquad p = 0,\; p = 3$$

$p = 0$, doesn't satisfy the given equation.

Hence, 3 is the required value of p.

Q. 6. If $x = -\dfrac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.   [CBSE Delhi, Term 2, Set 1, 2015]

Ans. Since $x = \dfrac{-1}{2}$ is a solution of $3x^2 + 2kx - 3 = 0$, it must satisfy the equation.

$$\therefore \qquad 3 \times \left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$
$$\Rightarrow \qquad \frac{3}{4} - k - 3 = 0$$
$$\Rightarrow \qquad k = \frac{3}{4} - 3$$
$$\Rightarrow \qquad k = \frac{-9}{4}$$

✒ Short Answer Type Questions-I ——————— (2 marks each)

Q. 1. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other. [CBSE OD, Term 2, Set 1, 2017]

Ans.

Given equation is $px^2 - 14x + 8 = 0$

Let   one root $= \alpha$,

then   other root $= 6\alpha$

$$\text{Sum of roots} = -\frac{b}{a};$$

$$\alpha + 6\alpha = \frac{-(-14)}{p}$$

$$7\alpha = \frac{14}{p}$$

or $\qquad \alpha = \dfrac{2}{p}$ \qquad ...(i)

$$\text{Product of roots} = \frac{c}{a}$$

$$(\alpha)\,(6\alpha) = \frac{8}{p}$$

$$6\alpha^2 = \frac{8}{p} \qquad ...(ii)$$

Putting value of α from eq. (i),

$$6\left(\frac{2}{p}\right)^2 = \frac{8}{p}$$

$$\Rightarrow \qquad 6 \times \frac{4}{p^2} = \frac{8}{p}$$

$$\Rightarrow \qquad 24p = 8p^2$$
$$\Rightarrow \qquad 8p^2 - 24p = 0$$
$$\Rightarrow \qquad 8p(p-3) = 0$$
$$\Rightarrow \qquad \text{Either } 8p = 0 \Rightarrow p = 0$$
or $\qquad p - 3 = 0 \Rightarrow p = 3$

For $p = 0$, given condition is not satisfied

$$\therefore \qquad p = 3$$

Q. 2. Find the roots of the quadratic equation
$$\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0.$$
[CBSE Delhi, Term 2, Set 1, 2017]

Ans. Given quadratic equation is,
$$\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$$
$$\Rightarrow \qquad \sqrt{2}\,x^2 + 5x + 2x + 5\sqrt{2} = 0$$
[Splitting middle term]
$$\Rightarrow x\left(\sqrt{2}x + 5\right) + \sqrt{2}\left(\sqrt{2}x + 5\right) = 0$$
$$\Rightarrow \qquad \left(\sqrt{2}x + 5\right)\left(x + \sqrt{2}\right) = 0$$
$$\Rightarrow \qquad x = -\frac{5}{\sqrt{2}} \text{ or } x = -\sqrt{2}$$

Hence roots are $\dfrac{-5}{\sqrt{2}}$ and $-\sqrt{2}$.

Q. 3. Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots.   [CBSE Delhi, Term 2, Set 1, 2017]

Ans. Given equation is,
$$x^2 + k(2x + k - 1) + 2 = 0$$
$$\Rightarrow \quad x^2 + 2kx + k(k-1) + 2 = 0$$
Here $a = 1$, $b = 2k$ and $c = k(k-1) + 2$

For real and equal roots
$$b^2 - 4ac = 0$$
$$\Rightarrow (2k)^2 - 4 \cdot 1 \cdot \{k(k-1) + 2\} = 0$$
$$\Rightarrow \quad 4k^2 - 4(k^2 - k + 2) = 0$$
$$\Rightarrow \quad 4k^2 - 4k^2 + 4k - 8 = 0$$
$$\Rightarrow \quad 4k = 8$$
$$\Rightarrow \quad k = \frac{8}{4} = 2$$

Q. 4. Solve for x: $\sqrt{2x+9} + x = 13$

[CBSE OD, Term 2, Set 2, 2016]

Ans. We have, $\sqrt{2x+9} + x = 13$.
$$\Rightarrow \quad \sqrt{2x+9} = 13 - x$$
On squaring both sides,
$$(\sqrt{2x+9})^2 = (13 - x)^2$$
$$\Rightarrow \quad 2x + 9 = 169 + x^2 - 26x$$
$$\Rightarrow \quad x^2 - 28x + 160 = 0$$
$$\Rightarrow \quad x^2 - 20x - 8x + 160 = 0$$
$$\Rightarrow \quad x(x - 20) - 8(x - 20) = 0$$
$$\Rightarrow \quad (x - 8)(x - 20) = 0$$
$$\Rightarrow \quad x = 20 \text{ or } 8$$
$\therefore x = 8$ (As $x = 20$ doesn't satisfy the given equation)

Q. 5. If −5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

[CBSE OD, Term 2, Set 1, 2016]

Ans. Given, −5 is a root of $2x^2 + px - 15 = 0$
then, $f(-5) = 2(-5)^2 + p(-5) - 15 = 0$
$$\Rightarrow \quad 50 - 5p - 15 = 0$$
$$\Rightarrow \quad 35 - 5p = 0$$
$$\Rightarrow \quad 5p = 35$$
$$\therefore \quad p = 7$$
Now, putting the value of p, in, $p(x^2 + x) + k = 0$
we get $\quad 7x^2 + 7x + k = 0$
Now, $\quad D = b^2 - 4ac = 0$
($\because$ the equation has the equal roots)
then, $\quad 49 - 28k = 0$
$$\Rightarrow \quad 28k = 49$$
$$\therefore \quad k = \frac{49}{28} = \frac{7}{4}$$

Q. 6. Solve for x: $\sqrt{6x+7} - (2x - 7) = 0$

[CBSE OD, Term 2, Set 3, 2016]

Ans. We have, $\sqrt{6x+7} - (2x - 7) = 0$
$$\sqrt{6x+7} = (2x - 7)$$
On squaring both sides,
$$(\sqrt{6x+7})^2 = (2x - 7)^2$$
$$\Rightarrow \quad 6x + 7 = 4x^2 + 49 - 28x$$
$$\Rightarrow \quad 4x^2 + 42 - 34x = 0$$
$$\Rightarrow \quad 2x^2 - 17x + 21 = 0$$
$$\Rightarrow \quad 2x^2 - 14x - 3x + 21 = 0$$
$$\Rightarrow \quad 2x(x - 7) - 3(x - 7) = 0$$
$$\Rightarrow \quad (2x - 3)(x - 7) = 0$$
$$\Rightarrow \quad x = \frac{3}{2} \text{ or } 7$$
$\therefore x = 7$ (as $x = 3/2$ doesn't satisfy the given equation)

Q. 7. If $x = \dfrac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. The given polynomial is,
$$p(x) = ax^2 + 7x + b$$
$$\therefore \quad p\left(\frac{2}{3}\right) = a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$
$$= \frac{4a}{9} + \frac{14}{3} + b = 0 \quad \text{...(i)}$$
and, $\quad p(-3) = a(-3)^2 + 7(-3) + b = 0$
$$\Rightarrow \quad 9a - 21 + b = 0 \quad \text{...(ii)}$$
Solving equation (i) and (ii), we get
$$4a + 42 + 9b = 0$$
$$81a - 189 + 9b = 0$$
$$\underline{\quad - \quad + \quad - \quad\quad}$$
$$-77a + 231 = 0$$
$$\therefore \quad a = \frac{231}{77} = 3$$
Putting $a = 3$ in equation (ii), we get
$$9(3) - 21 + b = 0$$
$$\Rightarrow \quad b = -6$$
$$\therefore \quad a = 3 \text{ and } b = -6$$

Q. 8. Solve the following quadratic equation for x:
$$4x^2 + 4bx - (a^2 - b^2) = 0$$
[CBSE OD, Term 2, Set 1, 2015]

Ans. The given equation is

$$4x^2 + 4bx - (a^2 - b^2) = 0 \qquad \ldots(i)$$

Comparing equation (i) with quadratic equation

$$Ax^2 + Bx + C = 0, \text{ we get}$$

$$A = 4, B = 4b, C = -(a^2 - b^2)$$

By quadratic formula

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\Rightarrow \quad x = \frac{-4b \pm \sqrt{16b^2 + 4 \times 4 \times (a^2 - b^2)}}{2 \times 4}$$

$$\Rightarrow \quad x = \frac{-4b \pm \sqrt{16b^2 + 16a^2 - 16b^2}}{8}$$

$$\Rightarrow \quad x = \frac{-4b \pm 4a}{8}$$

$$\Rightarrow \quad x = \frac{-b \pm a}{2}$$

Therefore, $\quad x = \dfrac{-b-a}{2} \quad = -\left(\dfrac{a+b}{2}\right)$

or $\qquad x = \dfrac{-b+a}{2} \quad = \dfrac{a-b}{2}$

Hence, $x = -\left(\dfrac{a+b}{2}\right)$ and $x = \dfrac{a-b}{2}$.

Q. 9. Solve the following quadratic equation for x:

$$x^2 - 2ax - (4b^2 - a^2) = 0$$

[CBSE OD, Term 2, Set 3, 2015]

Ans. We have, $x^2 - 2ax - (4b^2 - a^2) = 0$

$$\Rightarrow \qquad x^2 - 2ax + a^2 - 4b^2 = 0$$

$$\Rightarrow \qquad (x - a)^2 - (2b)^2 = 0$$

$$\therefore \qquad (x - a + 2b)(x - a - 2b) = 0$$

$$\Rightarrow \qquad x = a - 2b \text{ or } a + 2b$$

Hence, $\quad x = a - 2b \text{ or } x = a + 2b$

Q. 10. Solve the following quadratic equation for x:

$$4x^2 - 4a^2x + (a^4 - b^4) = 0.$$

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. We have $\quad 4x^2 - 4a^2x + (a^4 - b^4) = 0$

$$\Rightarrow \qquad (4x^2 - 4a^2x + a^4) - b^4 = 0$$

$$\Rightarrow \qquad (2x - a^2)^2 - (b^2)^2 = 0$$

$$\Rightarrow \quad (2x - a^2 + b^2)(2x - a^2 - b^2) = 0$$

$$\therefore \qquad x = \frac{a^2 - b^2}{2} \text{ or } \frac{a^2 + b^2}{2}$$

Q. 11. Solve the following quadratic equation for x:

$$9x^2 - 6b^2x - (a^4 - b^4) = 0$$

[CBSE Delhi, Term 2, Set 3, 2015]

Ans. We have, $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$$(9x^2 - 6b^2x + b^4) - a^4 = 0$$

$$(3x - b^2)^2 - (a^2)^2 = 0$$

$$(3x - b^2 + a^2)(3x - b^2 - a^2) = 0$$

$$x = \frac{b^2 - a^2}{3} \text{ or } \frac{b^2 + a^2}{3}$$

📝 Short Answer Type Questions-II ──────────────── (3 marks each)

Q. 1. In a flight of 600 km, an aircraft was slowed down due to bad weather. The average speed of the trip was reduced by 200 km/hr and the time of flight was increased by 30 minutes. Find the duration of flight.

[CBSE OD, SET 1, 2020]

Ans. Let the original speed of the aircraft be x km/hr.

So, time taken to cover 6000 km $= \dfrac{600}{x}$ hours

Now, Reduced speed $= (x - 200)$ km/hr.

So, time taken to cover 600 km at reduced speed

$$= \frac{600}{(x - 200)} \text{ hours}$$

∴ According to the question,

$$\Rightarrow \qquad \frac{600}{x - 200} - \frac{600}{x} = \frac{1}{2}$$

$$\Rightarrow \qquad \frac{1}{x - 200} - \frac{1}{x} = \frac{1}{1200}$$

$$\Rightarrow \qquad \frac{x - (x - 200)}{x(x - 200)} = \frac{1}{1200}$$

$$\Rightarrow \qquad \frac{200}{x^2 - 200x} = \frac{1}{1200}$$

$\Rightarrow \qquad\qquad x^2 - 200x = 240000$

$\Rightarrow \qquad\qquad x^2 - 200x - 240000 = 0$

$\Rightarrow \qquad x^2 - 600x + 400x - 240000 = 0$

$\Rightarrow \qquad x(x - 600) + 400\,(x - 600) = 0$

$\Rightarrow \qquad\qquad (x - 600)\,(x + 400) = 0$

$\Rightarrow \qquad\qquad\qquad x - 600 = 0$

or $\qquad\qquad\qquad\qquad x + 400 = 0$

$\Rightarrow \qquad\qquad\qquad\qquad x = 600 \text{ or } -400$

$\Rightarrow \qquad\qquad\qquad\qquad x = 600$

[∵ speed cannot be negative]

∴ Original speed of aircraft = 600 km/hr

∴ Original duration of flight $= \dfrac{600}{600} = 1$ hour

And, Increased duration of flight

$$= \left(1 + \dfrac{1}{2}\right) \text{ hours}$$

$= 1.5$ hours. **Ans.**

Q. 2. Solve for x : $\dfrac{1}{x+4} - \dfrac{1}{x+7} = \dfrac{11}{30}$, $x \ne -4, 7$.

[CBSE OD, Set 1, 2020]

Solution: $\qquad\qquad \dfrac{1}{x+4} - \dfrac{1}{x+7} = \dfrac{11}{30}$

$\Rightarrow \qquad \dfrac{x+7-(x+4)}{(x+4)(x+7)} = \dfrac{11}{30}$

$\Rightarrow \qquad \dfrac{3}{x^2 + 11x + 28} = \dfrac{11}{30}$

$\Rightarrow \qquad 11x^2 + 121x + 218 = 0$

$\Rightarrow \qquad x = \dfrac{-121 \pm \sqrt{(121)^2 - 4 \times 11 \times 218}}{2 \times 11}$

$\Rightarrow \qquad x = \dfrac{-121 \pm \sqrt{5049}}{22}$

Ans.

Q. 3. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/h less than that of the fast train, find the speed of each train. [CBSE OD, Set 2, 2020]

Ans. Let the speed of the slow train be x km/hr.

∴ $\qquad$ Speed of fast train $= (x + 10)$ km/hr

We know, $\qquad$ Time $= \dfrac{\text{Distance}}{\text{Speed}}$

∴ Time taken by slow train to cover 600 km

$= \dfrac{600}{x}$

Time taken by fast rain to cover 600 km

$= \dfrac{600}{x + 10}$

Given,

Time taken by slow train – Time taken by fast train = 3 hours

$\Rightarrow \qquad \dfrac{600}{x} - \dfrac{600}{x + 10} = 3$

$\Rightarrow \qquad \dfrac{600x + 6000 - 600x}{x(x + 10)} = 3$

$\Rightarrow \qquad \dfrac{6000}{x(x + 10)} = 3$

$\Rightarrow \qquad \dfrac{2000}{x(x + 10)} = 1$

$\Rightarrow \qquad x^2 + 10x - 2000 = 0$

$\Rightarrow \qquad x^2 + (50 - 40)x - 2000 = 0$

$\Rightarrow \qquad x^2 + 50x - 40x - 2000 = 0$

$\Rightarrow \qquad x(x + 50) - 40\,(x + 50) = 0$

$\Rightarrow \qquad (x - 40)\,(x + 50) = 0$

$\Rightarrow x = 40$ or $x = -50$ (Rejected, as speed cannot be nagative)

∴ Speed of slow train = 40 km/hr

And, Speed of fast train = 50 km/hr **Ans.**

Q. 4. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

[CBSE Delhi, Set 2, 2020]

Ans. Let the original speed of the train

$= x$ km/h

Total distance to be covered = 480 km.

According to the question,

$$\dfrac{480}{(x-8)} - \dfrac{480}{x} = 3$$

$\Rightarrow \qquad \dfrac{480x - 480(x-8)}{x(x-8)} = 3$

$\Rightarrow \qquad \dfrac{480x - 480x + 3840}{x^2 - 8x} = 3$

$\Rightarrow \qquad\qquad 3840 = 3(x^2 - 8x)$

$\Rightarrow \quad 3x^2 - 24x - 3840 = 0$

$\Rightarrow \quad x^2 - 8x - 1280 = 0$

$\Rightarrow \quad x^2 - 40x + 32x - 1280 = 0$

$\Rightarrow \quad (x - 40)(x + 32) = 0$

$\Rightarrow (x - 40) = 0, \text{ or } (x + 32) = 0$

$\Rightarrow x = 40 \text{ or } x = -32 \text{ (Neglected)}$

∴ Original speed of the train is 40 km/h. **Ans.**

Q. 5. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

[CBSE Delhi, Set 3, 2020]

Ans. Let speed of the stream be x km/hr

speed of the boat in still water be y km/hr.

Speed of the boat upstream = $(y - x)$ km/hr

speed of the boat downstream = $(y + x)$ km/hr

For downstream motion :

$$\frac{20}{y + x} = 2$$

or $\quad\quad\quad y + x = 10 \quad\quad\quad$ ...(i)

For upstream motion :

$$\frac{4}{y - x} = 2$$

or $\quad\quad\quad y - x = 2 \quad\quad\quad$ ...(ii)

On adding equation (i) and (ii), we get

$$2y = 12$$

$\Rightarrow \quad\quad\quad\quad y = 6$

Putting $y = 6$ in equation (i), we get

$$6 + x = 10$$

$$x = 4$$

∴ Speed of the stream = 4 km/hr

Speed of the boat in still water = 6 km/hr

Ans.

Q. 6. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

[CBSE OD, Set 1, 2019]

Ans. Given, equation is $x^2 + px + 16 = 0$

This is of the form $ax^2 + bx + c = 0$

where, $a = 1, b = p$ and $c = 16$

∴ $\quad\quad\quad\quad D = b^2 - 4ac$

$$= p^2 - 4 \times 1 \times 16$$

$$= p^2 - 64$$

for equal roots, we have

$$D = 0$$

$$p^2 - 64 = 0$$

$$p^2 = 64$$

$$p = \pm 8$$

Putting $p = 8$ in given equation we have,

$$x^2 + 8x + 16 = 0$$

$$(x + 4)^2 = 0$$

$$x + 4 = 0$$

$$x = -4, -4$$

Now, putting $p = -8$ in the given equation, we get

$$x^2 - 8x + 16 = 0$$

$$(x - 4)^2 = 0$$

$$x = 4, 4$$

∴ Required roots are -4 and -4 or 4 and 4.

Q. 7. Solve for x:

$$x^2 + 5x - (a^2 + a - 6) = 0$$

[CBSE OD, Set 2, 2019]

Ans. Taking $(a^2 + a - 6)$

$$= a^2 + 3a - 2a - 6$$

$$= a(a + 3) - 2(a + 3)$$

$$= (a + 3)(a - 2)$$

∴ $\quad\quad x^2 + 5x - (a + 3)(a - 2) = 0$

$x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$

$x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$

$$(x - a + 2)(x + a + 3) = 0$$

Hence, $\quad\quad x - a + 2 = 0$ and $x + a + 3 = 0$

$$x = a - 2 \quad \text{ and } \quad x = -(a + 3)$$

Required values of x are $(a - 2), -(a + 3)$

Q. 8. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed. [CBSE 2018]

16) Given, distance is 1500 km.

Usual speed = s.

We know, speed = $\frac{distance}{time}$ → time = $\frac{distance}{speed}$.

→ From question, $\frac{1500}{100+s} + \frac{1}{2} = \frac{1500}{s}$. [half an hour late]. (30 min = 0.5 hr).

$$\frac{1500}{100+s} = \frac{1500}{s} - \frac{1}{2}.$$

$$\frac{1500}{100+s} = \frac{3000-s}{2s}.$$

Cross multiplying,

$3000s = 300000 - 100s + 3000s - s^2$.

$s^2 + 100s - 300000 = 0$.

$s^2 + 600s - 500s - 300000 = 0$.

$s(s+600) - 500(s+600) = 0$

$(s-500)(s+600) = 0$.

⇒ $s - 500 = 0$ or $s + 600 = 0$.

→ $s = 500$ km/h   → $s = -600$ km/h.

⇒ $s = 500$ or -600 km/hr.

But speed cannot be negative.

⇒ The usual speed of the plane is 500 km/hr.

Ans.

Let the usual speed of plane be x km/h.

Increased speed = $(x + 100)$ km/h.

∴ Distance to cover = 1500 km.

Time taken by plane with usual speed

$$= \frac{1500}{x} \text{hr}$$

Time taken by plane with increased speed

$$= \frac{1500}{(100 + x)} \text{hr}$$

According to the question,

$$\frac{1500}{x} - \frac{1500}{(100+x)} = \frac{30}{60} = \frac{1}{2}$$

$$1500 \left[\frac{1}{x} - \frac{1}{x+100} \right] = \frac{1}{2}$$

$$1500 \left[\frac{x+100-x}{(x)(x+100)} \right] = \frac{1}{2}$$

$$\frac{1500 \times 100}{x^2 + 100x} = \frac{1}{2}$$

$$x^2 + 100x = 300000$$

$$x^2 + 100x - 300000 = 0$$

$$x^2 + 600x - 500x - 300000 = 0$$

$$x(x + 600) - 500(x + 600) = 0$$

$$(x + 600)(x - 500) = 0$$

Either $x + 600 = 0$

$$x = -600 \ \text{(Rejected)}$$

or $x - 500 = 0$

$$x = 500$$

∴ Usual speed of plane = 500 km/hr.

Q. 9. If $ad \neq bc$, then prove that the equation $(a^2 + b^2) x^2 + 2 (ac + bd) x + (c^2 + d^2) = 0$ has no real roots.

[CBSE OD, Term 2, Set 1, 2017]

Ans. Given, $ad \neq bc$

$$(a^2 + b^2) x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$$

$$D = b^2 - 4ac$$

$$= [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$$
$$= 4[a^2c^2 + b^2d^2 + 2abcd]$$
$$\qquad - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$$
$$= 4[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 -$$
$$b^2c^2 - b^2d^2]$$
$$= 4[-a^2d^2 - b^2c^2 + 2abcd]$$
$$= -4[a^2d^2 + b^2c^2 - 2abcd]$$
$$= -4[ad - bc]^2$$
$$\Rightarrow \quad D \text{ is negative}$$

Hence given equation has no real roots.

Hence Proved.

Q. 10. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ in x are equal, then show that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$. **[CBSE OD, Term 2, Set 2, 2017]**

Ans. $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$

For equal roots, $\qquad\qquad D = 0$

$$\Rightarrow \quad [-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$$
$$\Rightarrow 4[a^4 + b^2c^2 - 2a^2bc] - 4[b^2c^2 - ac^3 - ab^3$$
$$+ a^2bc] = 0$$
$$\Rightarrow 4[a^4 + b^2c^2 - 2a^2bc - b^2c^2 + ac^3 + ab^3$$
$$- a^2bc] = 0$$
$$\Rightarrow \qquad 4a[a^3 - 3abc + c^3 + b^3] = 0$$

Either $\quad 4a = 0$ or $\quad a^3 + b^3 + c^3 - 3abc = 0$

$a = 0$ or $a^3 + b^3 + c^3 = 3abc$. **Hence Proved.**

Q. 11. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\dfrac{a}{b} = \dfrac{c}{d}$.

[CBSE OD, Term 2, Set 3, 2017]

Ans. $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$

For equal roots, $D = 0$

$$[-2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$$
$$\Rightarrow 4(ac + bd)^2 - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$$
$$\Rightarrow 4[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2$$
$$+ b^2c^2 + b^2d^2] = 0$$
$$\Rightarrow -4[a^2d^2 + b^2c^2 - 2abcd] = 0$$
$$\Rightarrow \qquad\qquad -4(ad - bc)^2 = 0$$
$$\therefore \quad -4 \neq 0 \text{ so, } (ad - bc)^2 = 0$$
$$\Rightarrow \qquad\qquad ad - bc = 0$$
$$\Rightarrow \qquad\qquad ad = bc$$
$$\therefore \qquad\qquad \frac{a}{b} = \frac{c}{d}$$

Hence Proved.

Q. 12. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1 + m^2)$.
[CBSE Delhi, Term 2, Set 1, 2017]

Ans. The given equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots

Here, $A = 1 + m^2$, $B = 2mc$, $C = c^2 - a^2$

For equal roots, $D = 0 = B^2 - 4AC$

$$\Rightarrow \qquad (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$$
$$\Rightarrow 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$$
$$\Rightarrow \qquad m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$$
$$\Rightarrow \qquad\qquad -c^2 + a^2(1 + m^2) = 0$$
$$\Rightarrow \qquad\qquad c^2 = a^2(1 + m^2)$$

Hence Proved.

Q. 13. Solve for x:

$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1, 2, 3$$

[CBSE OD, Term 2, Set 1, 2016]

Ans. We have,

$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1, 2, 3$$
$$3(x - 3) + 3(x - 1) = 2(x - 1)(x - 2)(x - 3)$$
$$3x - 9 + 3x - 3 = 2(x - 1)(x - 2)(x - 3)$$
$$6x - 12 = 2(x - 1)(x - 2)(x - 3)$$
$$6(x - 2) = 2(x - 1)(x - 2)(x - 3)$$
$$3 = (x - 1)(x - 3)$$
$$3 = x^2 - 3x - x + 3$$
$$x^2 - 4x = 0$$
$$x(x - 4) = 0$$
$$\therefore \qquad\qquad x = 0 \text{ or } 4$$

Q. 14. If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$.
[CBSE OD, Term 2, Set 2, 2016]

Ans. By comparing the given equation with $ax^2 + bx + c = 0$

$A = a - b$, $B = b - c$, $C = c - a$

Since the roots of the given quadratic equation are equal.

then, $D = (b - c)^2 - 4(c - a)(a - b) = 0$

$$\Rightarrow \quad b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$$

$\Rightarrow b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$

$\Rightarrow \quad (b^2 + c^2 + 2bc) - 4a(b+c) + 4a^2 = 0$

$\Rightarrow \quad (b+c)^2 - 4a(b+c) + (2a)^2 = 0$

$\Rightarrow \quad\quad\quad\quad\quad ((b+c) - 2a)^2 = 0$

$\Rightarrow \quad\quad\quad\quad\quad\quad b + c - 2a = 0$

i.e. $\quad\quad\quad\quad\quad\quad\quad 2a = b + c$

Hence Proved.

Q. 15. **Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers.**

[CBSE OD, Term 2, Set 3, 2016]

Ans. Let the three consecutive natural numbers be x, $x+1$ and $x+2$.

According to the given condition,

$$(x+1)^2 - [(x+2)^2 - x^2] = 60$$

$\Rightarrow \quad\quad x^2 + 2x + 1 - 4 - 4x = 60$

$\Rightarrow \quad\quad\quad\quad\quad x^2 - 2x - 63 = 0$

$\Rightarrow \quad\quad\quad\quad x^2 - 9x + 7x - 63 = 0$

$\Rightarrow \quad\quad\quad\quad x(x-9) + 7(x-9) = 0$

$\Rightarrow \quad\quad\quad\quad\quad (x+7)(x-9) = 0$

$\therefore \quad\quad\quad\quad\quad\quad\quad x = 9 \text{ or } -7$

$\therefore \quad\quad\quad\quad\quad\quad\quad x = 9$

$\quad\quad\quad\quad\quad\quad\quad (\text{neglect } x = -7)$

$\therefore$ Numbers are 9, 10, 11.

Q. 16. **Solve for x:**

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -3/2$$

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. We have,

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -3/2$$

$\Rightarrow \quad 2x(2x+3) + (x-3) + (3x+9) = 0$

$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0$

$\Rightarrow \quad\quad\quad\quad\quad 4x^2 + 10x + 6 = 0$

$\Rightarrow \quad\quad\quad\quad\quad 2x^2 + 5x + 3 = 0$

$\Rightarrow \quad\quad\quad 2x^2 + 2x + 3x + 3 = 0$

$\Rightarrow \quad\quad\quad 2x(x+1) + 3(x+1) = 0$

$\Rightarrow \quad\quad\quad\quad\quad (2x+3)(x+1) = 0$

$\Rightarrow \quad\quad\quad x = -1, \dfrac{-3}{2}$

$\therefore \quad\quad\quad\quad x = -1 \quad [\because \text{Given } x \neq -3/2]$

Q. 17. **Solve for x:**

$$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; x \neq 1, -2, 2$$

[CBSE Delhi, Term 2, Set 2, 2016]

Ans. We have,

$$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; \ x \neq 1, -2, 2$$

$\Rightarrow \dfrac{(x+1)(x+2) + (x-2)(x-1)}{(x-1)(x+2)}$

$\quad\quad\quad = \dfrac{4(x-2) - (2x+3)}{x-2}$

$\Rightarrow \quad (x-2)[x^2 + x + 2x + 2 + x^2 - 2x - x + 2]$

$\quad\quad = [4x - 8 - 2x - 3](x^2 + x - 2)$

$\Rightarrow (x-2)(2x^2 + 4) = (2x - 11)(x^2 + x - 2)$

$\Rightarrow \quad 2x^3 + 4x - 4x^2 - 8 = 2x^3 + 2x^2 - 4x$

$\quad\quad\quad\quad\quad\quad\quad - 11x^2 - 11x + 22$

$\Rightarrow \quad\quad 4x - 4x^2 - 8 = -9x^2 - 15x + 22$

$\Rightarrow \quad\quad\quad\quad 5x^2 + 19x - 30 = 0$

$\Rightarrow \quad 5x^2 + 25x - 6x - 30 = 0$

$\Rightarrow 5x(x+5) - 6(x+5) = 0$

$\Rightarrow \quad\quad\quad\quad (5x-6)(x+5) = 0$

$\Rightarrow \quad\quad\quad\quad\quad x = -5, \dfrac{5}{6}$

$\therefore \quad\quad\quad\quad x = -5 \text{ or } x = \dfrac{5}{6}$

Q. 18. **Solve the following quadratic equation for x:**

$$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$$

[CBSE Delhi, Term 2, Set 3, 2016]

Ans. We have,

$$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$$

$\Rightarrow \quad\quad x^2 + \dfrac{a}{a+b}x + \dfrac{a+b}{a}x + 1 = 0$

$\Rightarrow \ x\left(x + \dfrac{a}{a+b}\right) + \dfrac{a+b}{a}\left(x + \dfrac{a}{a+b}\right) = 0$

$\Rightarrow \quad\quad \left(x + \dfrac{a+b}{a}\right)\left(x + \dfrac{a}{a+b}\right) = 0$

$\Rightarrow \quad\quad x = -\dfrac{a}{a+b}, -\dfrac{(a+b)}{a}$

$\therefore \quad\quad x = -\dfrac{a}{a+b} \text{ or } x = -\dfrac{(a+b)}{a}$

Q. 19. Solve for x:

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$

[CBSE OD, Term 2, Set 1, 2015]

Ans. We have, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \qquad \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \qquad \sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6}) = 0$

$\Rightarrow \qquad (\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) = 0$

$\Rightarrow \qquad x = -\sqrt{\dfrac{2}{3}}$ or $\sqrt{6}$

Q. 20. Solve for x:

$$2x^2 + 6\sqrt{3}x - 60 = 0$$

[CBSE OD, Term 2, Set 2, 2015]

Ans. Consider the given equation

$2x^2 + 6\sqrt{3}x - 60 = 0$

$\Rightarrow \qquad x^2 + 3\sqrt{3}x - 30 = 0 \qquad \qquad ...(i)$

Comparing equation (i) with

$ax^2 + bx + c = 0$

We get $\qquad a = 1, b = 3\sqrt{3}, c = -30.$

By quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3\sqrt{3} \pm \sqrt{27 + 120}}{2}$$

$$x = \frac{-3\sqrt{3} \pm \sqrt{147}}{2}$$

Hence value for $x = \dfrac{-3\sqrt{3} \pm \sqrt{147}}{2}$

Q. 21. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k - 1)$ $x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. The given equation can be written as

$(k + 1)x^2 - 2(k - 1)x + 1 = 0$

Since the equation has equal roots

$\therefore \qquad 4(k - 1)^2 - 4(k + 1) = 0$

$\Rightarrow \qquad 4(k^2 + 1 - 2k) - 4(k + 1) = 0$

$\Rightarrow \qquad 4k^2 + 4 - 8k - 4k - 4 = 0$

$\Rightarrow \qquad 4k^2 - 12k = 0$

$\Rightarrow \qquad 4k(k - 3) = 0$

$\Rightarrow \qquad k = 0, 3$

$\therefore$ Non-zero value of k is 3.

And the equation becomes,

$4x^2 - 4x + 1 = 0$

$\Rightarrow \qquad (2x - 1)^2 = 0$

$\Rightarrow \qquad x = \dfrac{1}{2}$

$x = \dfrac{1}{2}, \dfrac{1}{2}$ which are the required roots of the given equation.

Long Answer Type Questions ——————— (4 marks each)

Q. 1. It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for four hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately? **[CBSE OD, Set 1, 2020]**

Solution : Let the time taken by pipe of larger diameter be x hours and the time taken by the pipe of smaller diameter by y hours.

So, amount of water filled by pipe of larger diameter in 1 hr = $\dfrac{1}{x}$

and amount of water filled by pipe of smaller diameter in 1 hr = $\dfrac{1}{y}$

$\therefore$ According to the question,

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{12} \qquad \qquad ...(i)$$

Now, amount of water filled by pipe of larger diameter in 4 hr = $\dfrac{4}{x}$

and amount of water filled by pipe of smaller diameter in 9 hr = $\dfrac{9}{y}$

$\therefore \qquad \dfrac{4}{x} + \dfrac{9}{y} = \dfrac{1}{2} \qquad \qquad ...(ii)$

Consider $\dfrac{1}{x} = a$ and $\dfrac{1}{y} = b$

So, by equation (i)

$$a + b = \frac{1}{12}$$

$$12a + 12b = 1 \qquad \text{...(iii)}$$

and, by equation (ii)

$$4a + 9b = \frac{1}{2}$$

$$\Rightarrow \qquad 8a + 18b = 1 \qquad \text{...(iv)}$$

Multiplying equation (iii) by 2 and equation (iv) by 3 and then subtracting them, we get

$$24a + 24b = 2$$
$$24a + 54b = 3$$
$$\underline{- \quad - \qquad -}$$
$$-30b = -1$$

$$\Rightarrow \qquad b = \frac{1}{30}$$

Putting the value of b in equation (iii), we get

$$12a + 12\left(\frac{1}{30}\right) = 1$$

$$\Rightarrow \qquad 12a + \frac{2}{5} = 1$$

$$\Rightarrow \qquad 12a = 1 - \frac{2}{5}$$

$$\Rightarrow \qquad 12a = \frac{3}{5}$$

$$\Rightarrow \qquad a = \frac{1}{20}$$

$$\because \qquad a = \frac{1}{x} \Rightarrow x = 20 \text{ hr}$$

and $\qquad b = \frac{1}{y} \Rightarrow y = 30 \text{ hr}$

Hence, the larger diameter pipe would take 20 hours and the smaller diameter pipe would take 30 hours to fill the pool separately. **Ans.**

Q. 2. In a class test, the sum of Arun's marks in Hindi and English is 30. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects. [CBSE OD, Set 1, 2019]

Ans. Let Arun marks in Hindi be x and marks in English be y.

Then, according to question, we have

$$x + y = 30 \qquad \text{...(i)}$$
$$(x + 2)(y - 3) = 210 \qquad \text{...(ii)}$$

from equation (i), put $x = 30 - y$ in equation (ii)

$$(30 - y + 2)(y - 3) = 210$$
$$(32 - y)(y - 3) = 210$$
$$32y - 96 - y^2 + 3y = 210$$
$$y^2 - 35y + 306 = 0$$
$$y^2 - 18y - 17y + 306 = 0$$
$$y(y - 18) - 17(y - 18) = 0$$
$$(y - 18)(y - 17) = 0$$
$$y = 18, 17$$

Put $y = 18$ and 17 in equation (i), we get

$$x = 12, 13$$

Hence his marks in Hindi can be 12 or 13 and in English his marks can be 18 or 17 respectively.

Q. 3. The total cost of a certain length of a piece of cloth is ₹ 200. If the piece was 5 m longer and each metre of cloth costs ₹ 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre? [CBSE OD, Set 2, 2019]

Ans. Let the original length of piece of cloth is x m and rate of cloth is ₹ y per metre.

Then according to question, we have

$$x \times y = 200 \qquad \text{(i)}$$

and if length be 5 m longer and each metre of cloth be ₹ 2 less then

$$(x + 5)(y - 2) = 200$$
$$xy - 2x + 5y - 10 = 200 \qquad \text{...(ii)}$$

On equating equations (i) and (ii), we have

$$xy = xy - 2x + 5y - 10$$

$$\Rightarrow \qquad 2x - 5y = -10 \qquad \text{...(iii)}$$

$$\Rightarrow \qquad 2x - 5 \times \frac{200}{x} = -10$$

$$\left(\because y = \frac{200}{x} \text{from equation (i)}\right)$$

$$\Rightarrow \qquad 2x - \frac{1000}{x} = -10$$

$$\Rightarrow \qquad 2x^2 - 1000 = -10x$$

$$\Rightarrow \qquad 2x^2 + 10x - 1000 = 0$$

$$\Rightarrow \qquad x^2 + 5x - 500 = 0$$

$$\Rightarrow \qquad x^2 + 25x - 20x - 500 = 0$$

$$\Rightarrow \quad x(x + 25) - 20(x + 25) = 0$$

$$(x + 25)(x - 20) = 0$$
$$x = 20$$

$(x \neq -25$, length of cloth can never be negative)

$\therefore \qquad x \times y = 200$
$$20 \times y = 200$$
$$y = 10$$

Thus, length of the piece of cloth is 20 m and original price per metre is ₹ 10.

Q. 4. **Two water taps together can fill a tank in $1\dfrac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.**

[CBSE Delhi, Set 1, 2019]

Ans. Let the tap A with longer diameter take x hours and the tap B with smaller diameter take $(x + 2)$ hours to fill the tank.

$\therefore$ Portion of tank filled by the tap A in 1 hr.
$$= \frac{1}{x}$$

and Portion of tank filled by the tap B in 1 hr.
$$= \frac{1}{x + 2}$$

Portion of the tank filled by both taps in 1 hr.
$$= \frac{1}{x} + \frac{1}{x + 2}$$
$$= \frac{x + 2 + x}{x(x + 2)}$$

Time taken by both taps to fill the tank
$$= 1\frac{7}{8} \text{ hrs} = \frac{15}{8} \text{ hrs}$$

$\therefore$ Portion of the tank filled by both in 1 hr.
$$= \frac{8}{15}$$

According to question,
$$\frac{2x + 2}{x(x + 2)} = \frac{8}{15}$$
$$\frac{2(x + 1)}{x(x + 2)} = \frac{8}{15}$$
$$15x + 15 = 4x^2 + 8x$$
$$4x^2 - 7x - 15 = 0$$
$$4x^2 - 12x + 5x - 15 = 0$$

$$4x(x - 3) + 5(x - 3) = 0$$
$$(4x + 5)(x - 3) = 0$$
$$\Rightarrow \quad 4x + 5 = 0 \text{ or } x - 3 = 0$$
$$\Rightarrow x = \frac{-5}{4}, 3$$

Since, time can not be negative hence, $x = 3$

Hence, the time taken with longer diameter tap = 3 hours

and the time taken with smaller diameter tap = 5 hours.

Q. 5. **A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.**

[CBSE Delhi, Set 1, 2019]

Ans. Let the speed of the boat in still water be x km/h and the speed of the stream be y km/h

Then the speed of the boat downstream
$$= (x + y) \text{ km/h}$$

and the speed of the boat upstream
$$= (x - y) \text{ km/h}$$

Given, $\dfrac{30}{x - y} + \dfrac{44}{x + y} = 10$ \qquad ...(i)

and $\dfrac{40}{x - y} + \dfrac{55}{x + y} = 13$ \qquad ...(ii)

Let $\dfrac{1}{x - y} = u$ and $\dfrac{1}{x + y} = v$ \qquad ...(iii)

$\therefore$ From equations (i) and (ii), we get
$$30u + 44v = 10$$
$$40u + 55v = 13$$

On solving, we get
$$u = \frac{1}{5} \qquad \text{and} \qquad v = \frac{1}{11}$$

From equation (iii), we get
$$\frac{1}{x - y} = \frac{1}{5} \quad \text{and} \quad \frac{1}{x + y} = \frac{1}{11}$$

i.e., $\quad x - y = 5 \quad$ and $\quad x + y = 11$

On solving, we get
$$x = 8 \quad \text{and} \quad y = 3$$

Hence, the speed of the boat in still water
$$= 8 \text{ km/h}$$

and the speed of the stream = 3 km/h

Q. 6. **A motor boat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.** **[CBSE 2018]**

Ans. Given, speed of motor boat in still water = 18 km/hr.

Let speed of stream = x km/hr.

∴ Speed of boat downstream = $(18 + x)$ km/hr.

And speed of boat upstream = $(18 - x)$ km/hr.

Time of the upstream journey = $\dfrac{24}{(18 - x)}$

Time of the downstream journey = $\dfrac{24}{(18 + x)}$

According to the question,

$$\frac{24}{(18 - x)} - \frac{24}{(18 + x)} = 1$$

$$\frac{24(18 + x) - 24(18 - x)}{(18 - x)(18 + x)} = 1$$

$$\frac{24 \times 18 + 24x - 24 \times 18 + 24x}{324 - x^2} = 1$$

$$\frac{48x}{324 - x^2} = 1$$

$\Rightarrow$ $48x = 324 - x^2$

$\Rightarrow$ $x^2 + 48x - 324 = 0$

$\Rightarrow$ $x^2 + 54x - 6x - 324 = 0$

$\Rightarrow$ $x(x + 54) - 6(x + 54) = 0$

$\Rightarrow$ $(x + 54)(x - 6) = 0$

Either $x + 54 = 0$

             $x = -54$

Rejected, as speed cannot be negative

or $x - 6 = 0$

             $x = 6$

Thus, the speed of the stream is 6 km/hr.

Q. 7. **A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed?** **[CBSE 2018]**

Ans. Let original average speed of train be x km/hr.

∴ Increased speed of train = $(x + 6)$ km/hr.

Time taken to cover 63 km with average speed

$$= \frac{63}{x} \text{ hr.}$$

Time taken to cover 72 km with increased speed

$$= \frac{72}{(x + 6)} \text{ hr.}$$

According to the question,

$$\frac{63}{x} + \frac{72}{x + 6} = 3$$

$\Rightarrow$ $\dfrac{63(x + 6) + 72(x)}{(x)(x + 6)} = 3$

$\Rightarrow$ $\dfrac{63x + 378 + 72x}{x^2 + 6x} = 3$

$\Rightarrow$ $135x + 378 = 3(x^2 + 6x)$

$\Rightarrow$ $135x + 378 = 3x^2 + 18x$

$\Rightarrow$ $3x^2 + 18x - 135x - 378 = 0$

$\Rightarrow$ $3x^2 - 117x - 378 = 0$

$\Rightarrow$ $3(x^2 - 39x - 126) = 0$

$\Rightarrow$ $x^2 - 39x - 126 = 0$

$\Rightarrow$ $x^2 - 42x + 3x - 126 = 0$

$\Rightarrow$ $x(x - 42) + 3(x - 42) = 0$

$\Rightarrow$ $(x - 42)(x + 3) = 0$

Either $x - 42 = 0$

             $x = 42$

or $x + 3 = 0$

             $x = -3$

Rejected (as speed cannot be negative)

Thus, average speed of train is 42 km/hr.

Q. 8. **Solve for x:**

$$\frac{1}{x + 1} + \frac{3}{5x + 1} = \frac{5}{x + 4}, x \neq -1, -\frac{1}{5}, -4$$

[CBSE OD, Term 2, Set 1, 2017]

Ans. Given, $\dfrac{1}{x + 1} + \dfrac{3}{5x + 1} = \dfrac{5}{x + 4}$

$\Rightarrow$ $\dfrac{1}{x + 1} - \dfrac{5}{x + 4} = \dfrac{-3}{5x + 1}$

$\Rightarrow$ $\dfrac{(x + 4) - 5(x + 1)}{(x + 1)(x + 4)} = \dfrac{-3}{5x + 1}$

$\Rightarrow$ $\dfrac{x + 4 - 5x - 5}{x^2 + 5x + 4} = \dfrac{-3}{5x + 1}$

$\Rightarrow \quad \dfrac{(-4x-1)}{x^2+5x+4} = \dfrac{-3}{5x+1}$

$\Rightarrow \quad (4x+1)(5x+1) = 3(x^2+5x+4)$

$\Rightarrow \quad 20x^2+4x+5x+1 = 3x^2+15x+12$

$\Rightarrow \quad 17x^2-6x-11 = 0$

$\Rightarrow \quad 17x^2-17x+11x-11 = 0$

$\Rightarrow \quad 17x(x-1)+11(x-1) = 0$

$\Rightarrow \quad (x-1)(17x+11) = 0$

$\Rightarrow \quad x = 1 \text{ or } x = \dfrac{-11}{17}$

Q. 9. Two taps running together can fill a tank in $3\dfrac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank? [CBSE OD, Term 2, Set 1, 2017]

Ans. Let tank be filled by one tap in x hrs

and other tap in $(x+3)$ hrs

Together they fill in $3\dfrac{1}{13}$ hrs $= \dfrac{40}{13}$ hrs

Now, $\quad \dfrac{1}{x}+\dfrac{1}{x+3} = \dfrac{13}{40}$

$\Rightarrow \quad \dfrac{x+3+x}{(x)(x+3)} = \dfrac{13}{40}$

$\Rightarrow \quad \dfrac{2x+3}{x^2+3x} = \dfrac{13}{40}$

$\Rightarrow \quad 13x^2+39x = 80x+120$

$\Rightarrow \quad 13x^2-41x-120 = 0$

$\Rightarrow \quad 13x^2-65x+24x-120 = 0$

$\Rightarrow \quad 13x(x-5)+24(x-5) = 0$

$\Rightarrow \quad (x-5)(13x+24) = 0$

Either $x-5 = 0$ or $13x+24 = 0$

$x = 5, x = -24/13$ (Rejected)

One tap fill the tank in 5 hrs

And other tap fill the tank in $5+3 = 8$ hrs

Q. 10. A train covers a distance of 300 km at a uniform speed. If the speed of the train is increased by 5 km/hour, it takes 2 hours less in the journey. Find the original speed of the train. [CBSE OD, Term 2, Set 2, 2017]

Ans. Let original speed of train $= x$ km/hr

Increased speed of train $= (x+5)$ km/hr

Distance $= 300$ km

According to the question,

$\dfrac{300}{x}-\dfrac{300}{x+5} = 2$

$\Rightarrow \quad \dfrac{300(x+5-x)}{(x)(x+5)} = 2$

$\Rightarrow \quad 1500 = 2(x^2+5x)$

$\Rightarrow \quad 1500 = 2x^2+10x$

$\Rightarrow \quad 2x^2+10x-1500 = 0$

$\Rightarrow \quad x^2+5x-750 = 0$

$\Rightarrow \quad x^2+30x-25x-750 = 0$

$\Rightarrow \quad x(x+30)-25(x+30) = 0$

$\Rightarrow \quad (x+30)(x-25) = 0$

Either $x+30 = 0$ or $x-25 = 0$

$\Rightarrow \quad x = -30$ (Rejected),

so $\quad x = 25$

Original speed of train is 25 km/hr.

Q. 11. Solve for x:

$\dfrac{x-1}{2x+1}+\dfrac{2x+1}{x-1} = 2$, where $x \neq -\dfrac{1}{2}, 1$

[CBSE OD, Term 2, Set 3, 2017]

Ans. $\dfrac{(x-1)^2+(2x+1)^2}{(2x+1)(x-1)} = 2$

$\Rightarrow \quad \dfrac{x^2+1-2x+4x^2+1+4x}{2x^2-x-1} = \dfrac{2}{1}$

$\Rightarrow \quad 5x^2+2x+2 = 2(2x^2-x-1)$

$\Rightarrow \quad 5x^2+2x+2 = 4x^2-2x-2$

$\Rightarrow \quad x^2+4x+4 = 0$

$\Rightarrow \quad (x+2)^2 = 0$

$\therefore \quad x = -2, -2$

Q. 12. A takes 6 days less than B to do a work. If both A and B working together can do it in 4 days, how many days will B take to finish it? [CBSE OD, Term 2, Set 3, 2017]

Ans. Let B can finish a work in x days

so, A can finish work in $(x-6)$ days

Together they finish work in 4 days

Now,

$\dfrac{1}{x}+\dfrac{1}{x-6} = \dfrac{1}{4}$

$\Rightarrow \quad \dfrac{x-6+x}{(x)(x-6)} = \dfrac{1}{4}$

$\Rightarrow \quad 4(2x-6) = x^2-6x$

$\Rightarrow \quad 8x-24 = x^2-6x$

$\Rightarrow \qquad x^2 - 14x + 24 = 0$

$\Rightarrow \qquad x^2 - 12x - 2x + 24 = 0$

$\Rightarrow \qquad x(x - 12) - 2(x - 12) = 0$

$\Rightarrow \qquad (x - 12)(x - 2) = 0$

Either $\qquad x - 12 = 0$ or $x - 2 = 0$

$\qquad x = 12$ or $x = 2$ (Rejected)

B can finish work in 12 days

A can finish work in 6 days

Q. 13. Speed of a boat in still water is 15 km/h. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

[CBSE Delhi, Term 2, Set I, 2017]

Ans. Let speed of stream be x.

According to question,

$$\frac{30}{x + 15} + \frac{30}{15 - x} = 4 + \frac{30}{60}$$

$\Rightarrow \qquad 30\dfrac{(15 - x + x + 15)}{(15)^2 - (x)^2} = \dfrac{9}{2}$

$\Rightarrow \qquad \dfrac{900}{225 - x^2} = \dfrac{9}{2}$

$\Rightarrow \qquad \dfrac{2}{9} \times 900 = 225 - x^2$

$\Rightarrow \qquad 200 = 225 - x^2$

$\Rightarrow \qquad x^2 = 25$

$\Rightarrow \qquad x = 5 \text{ km/hr}$

Speed of stream is 5 km/hr.

Q. 14. A boat goes 30 km upstream and 44 km downstream in 10 hours. The same boat goes 40 km upstream and 55 km downstream in 13 hours. On this information some student guessed the speed of the boat in still water as 8.5 km/h and speed of the stream as 3.8 km/h. Do you agree with their guess? Explain what do we learn from the incident?

[CBSE Term 1, 2016]

Ans. Let the speed of boat = x km/hr.

Let the speed of stream = y km/hr.

Speed of boat in upstream = $(x - y)$ km/hr.

Speed of boat in downstream = $(x + y)$ km/hr.

Time taken to cover 30 km upstream

$$= \frac{30}{x - y} \text{ hrs.}$$

Time taken to cover 40 km downstream

$$= \frac{44}{x + y} \text{ hrs.}$$

According to the question,

Total time taken = 10 hrs.

$\Rightarrow \qquad \dfrac{30}{x - y} + \dfrac{44}{x + y} = 10 \qquad$ ...(i)

Now, Time taken to cover 55 km downstream

$$= \frac{55}{x + y} \text{ hrs}$$

Time taken to cover 40 km upstream

$$= \frac{40}{x - y} \text{ hrs.}$$

Total time taken = 13 hrs.

$$\frac{40}{x - y} + \frac{55}{x + y} = 13 \qquad \text{...(ii)}$$

Solving equation (i) and equation (ii),

Let $\qquad \dfrac{1}{x - y} = u, \quad \dfrac{1}{x + y} = v.$

Then, $\qquad 30u + 44v = 10$

and $\qquad 40u + 55v = 13$

or $\qquad 15u + 22v = 5 \qquad$ ....(iiii)

and $\qquad 8u + 11v = \dfrac{13}{5} \qquad$ ....(iv)

Multiplying equation (iii) by 8 and equation (iv) by 15, we get

$\qquad 120u + 176v = 40$

$\qquad 120u + 165v = 39$

$\qquad \overline{ - - }$ [on subtracting]

$\qquad \qquad 11v = 1$

$\Rightarrow \qquad \qquad v = \dfrac{1}{11}$

Putting the value of v in equation (iii),

$\qquad 15u + 22v = 5$

$\Rightarrow \qquad 15u + 22 \times \dfrac{1}{11} = 5$

$\Rightarrow \qquad 15u + 2 = 5$

$\Rightarrow \qquad 15u = 3$

$\Rightarrow \qquad u = \dfrac{3}{15}$

or $\qquad u = \dfrac{1}{5}$

Now, $\qquad v = \dfrac{1}{11}$

$\Rightarrow \qquad \dfrac{1}{x + y} = \dfrac{1}{11}$

$$\Rightarrow \qquad x + y = 11 \qquad \qquad ...(v)$$

And $\qquad u = \dfrac{1}{5}$

$$\Rightarrow \qquad \dfrac{1}{x-y} = \dfrac{1}{5}$$

$$\Rightarrow \qquad x - y = 5 \qquad \qquad ...(vi)$$

On solving equation (v) and (vi),

$$x + y = 11$$
$$x - y = 5$$
$$\underline{+ \quad - \qquad +}$$
$$2x = 16$$

or $\qquad\qquad x = 8$

Put the value of x in equation (v),

$$8 + y = 11$$

$$\Rightarrow \qquad\qquad y = 11 - 8$$

$$\therefore \qquad\qquad y = 3$$

The speed of boat in still water = 8 km/hr.

The speed of stream = 3 km/hr.

Q. 15. Solve for x:

$$\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}, x \neq -1, -2, -4$$

[CBSE OD, Term 2, Set 1, 2016]

Ans. We have,

$$\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}, x \neq -1, -2, -4$$

$$\Rightarrow (x+2)(x+4) + 2(x+1)(x+4)$$
$$= 4(x+1)(x+2)$$

$$\Rightarrow x^2 + 2x + 4x + 8 + 2(x^2 + x + 4x + 4)$$
$$= 4(x^2 + x + 2x + 2)$$

$$\Rightarrow x^2 + 6x + 8 + 2x^2 + 10x + 8 = 4x^2 + 12x + 8$$

$$\Rightarrow \qquad 3x^2 + 16x + 16 = 4x^2 + 12x + 8$$

$$\Rightarrow \qquad x^2 - 4x - 8 = 0$$

$$\Rightarrow \qquad x = \dfrac{4 \pm \sqrt{16+32}}{2}$$

$$\Rightarrow \qquad x = \dfrac{4 \pm \sqrt{48}}{2} = \dfrac{4 \pm 4\sqrt{3}}{2}$$

$$\therefore \qquad x = 2 \pm 2\sqrt{3}$$

Q. 16. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses preceeding the house numbered X is equal to sum of the numbers of houses following X.

[CBSE OD, Term 2, Set 1, 2016]

Ans. Given, the houses in a row numbered consecutively from 1 to 49.

Now, sum of numbers preceeding the number X

$$= \dfrac{X(X-1)}{2}$$

And, sum of numbers following the number X

$$= \dfrac{49(50)}{2} - \dfrac{X(X-1)}{2} - X$$

$$= \dfrac{2450 - X^2 + X - 2X}{2}$$

$$= \dfrac{2450 - X^2 - X}{2}$$

According to the given condition,

Sum of no's preceeding X = Sum of no's following X

$$\dfrac{X(X-1)}{2} = \dfrac{2450 - X^2 - X}{2}$$

$$\Rightarrow \qquad X^2 - X = 2450 - X^2 - X$$

$$\Rightarrow \qquad 2X^2 = 2450$$

$$\Rightarrow \qquad X^2 = 1225$$

$$\therefore \qquad X = 35$$

Hence, at $X = 35$, sum of no. of houses preceeding the house no. X is equal to sum of the no. of houses following X.

Q. 17. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream. **[CBSE OD, Term 2, Set 1, 2016]**

Ans. Let the speed of the stream be x km/hr. Then, speed upstream = $(24 - x)$ km/hr and speed downstream = $(24 + x)$ km/hr.

Time taken to cover 32 km upstream = $\dfrac{32}{24-x}$ hrs.

Time taken to cover 32 km downstream = $\dfrac{32}{24+x}$ hrs.

$$\therefore \quad \text{Time difference} = \dfrac{32}{24-x} - \dfrac{32}{24+x} = 1$$

$$\Rightarrow 32[(24+x) - (24-x)] = (24-x)(24+x)$$

$$\Rightarrow \quad 32(24 + x - 24 + x) = 576 - x^2$$

$$\Rightarrow \qquad\qquad 64x = 576 - x^2$$

$$\Rightarrow \qquad\qquad x^2 + 64x - 576 = 0$$

$\Rightarrow$ $x^2 + 72x - 8x - 576 = 0$

$\Rightarrow$ $x(x + 72) - 8(x + 72) = 0$

$\Rightarrow$ $(x + 72)(x - 8) = 0$

$\Rightarrow$ $x = 8$ or -72

$\therefore$ $x = 8$

(As speed can't be negative)

$\therefore$ Speed of the stream is 8 km/h.

Q. 18. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metre more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

[CBSE OD, Term 2, Set 2, 2016]

Ans. Let the length of the rectangular park be x m then, breadth be $(x - 3)$ m.

$\therefore$ Area of rectangular park = $x(x - 3)$ m^2

Area of isosceles triangular park

$$= \frac{1}{2}(x - 3) \times 12 \text{ m}^2$$

$$= 6(x - 3) \text{ m}^2$$

Acording to the given condition,

$x(x - 3) - 6(x - 3) = 4$

$\Rightarrow$ $x^2 - 3x - 6x + 18 = 4$

$\Rightarrow$ $x^2 - 9x + 14 = 0$

$\Rightarrow$ $x^2 - 7x - 2x + 14 = 0$

$\Rightarrow$ $x(x - 7) - 2(x - 7) = 0$

$\Rightarrow$ $(x - 2)(x - 7) = 0$

$\Rightarrow$ $x = 2$ or 7

$\therefore$ $x = 7$ m

(As breadth can't be negative)

and $x - 3 = (7 - 3)$ m $= 4$ m

Hence, length and breadth of the rectangular park is 7 m and 4 m respectively.

Q. 19. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

[CBSE OD, Term 2, Set 3, 2016]

Ans. Let the time taken by the one tap to fill the tank be x minutes.

then, other pipe takes $(x + 5)$ minutes to fill the tank.

According to the question,

$$\frac{1}{x} + \frac{1}{x + 5} = \frac{1}{100/9}$$

$\Rightarrow$ $$\frac{x + 5 + x}{x(x + 5)} = \frac{9}{100}$$

$\Rightarrow$ $100(5 + 2x) = 9x(x + 5)$

$\Rightarrow$ $500 + 200x = 9x^2 + 45x$

$\Rightarrow$ $9x^2 - 155x - 500 = 0$

$\Rightarrow$ $9x^2 - 180x + 25x - 500 = 0$

$\Rightarrow$ $9x(x - 20) + 25(x - 20) = 0$

$\Rightarrow$ $(9x + 25)(x - 20) = 0$

$\Rightarrow$ $x = 20$ or $-\frac{25}{9}$ (Neglect)

$\therefore$ $x = 20$

$\therefore$ Time in which each pipe would fill the tank separately is 20 mins and 25 mins, respectively.

Q. 20. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane.

What value is depicted in this question?

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Let the usual speed of the plane be x km/h.

$\therefore$ Time taken by plane to reach 1500 km away = $\frac{1500}{x}$ and the time taken by plane to reach 1500 km with increased speed = $\frac{1500}{x + 250}$

Now, $\frac{1500}{x} - \frac{1500}{x + 250} = \frac{1}{2}$ (Given)

$\Rightarrow$ $1500 \frac{(x + 250 - x)}{x(x + 250)} = \frac{1}{2}$

$\Rightarrow$ $3000 \times 250 = x^2 + 250x$

$\Rightarrow$ $x^2 + 250x - 750000 = 0$

$\Rightarrow$ $x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow$ $x(x + 1000) - 750(x + 1000) = 0$

$\Rightarrow \quad (x + 1000)(x - 750) = 0$

$\Rightarrow \qquad x = -1000 \text{ or } x = 750$

$\therefore \qquad\qquad\qquad x = 750$

(As speed can't be negative)

∴ Speed of plane is 750 km/h.

Value: it shows his responsibility towards mankind and his work.

Q. 21. Find x in terms of a, b and c:

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c$$

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. We have,

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a,b,c$$

$\Rightarrow a(x-b)(x-c) + b(x-a)(x-c)$
$$= 2c(x-a)(x-b)$$

$\Rightarrow a(x^2 - bx - cx + bc) + b(x^2 - ax - cx + ac)$
$$= 2c(x^2 - ax - bx + ab)$$

$\Rightarrow ax^2 - abx - acx + abc + bx^2 - abx - bcx + abc$
$$= 2cx^2 - 2acx - 2bcx + 2abc$$

$\Rightarrow ax^2 + bx^2 - 2abx - acx - bcx + 2abc$
$$= 2cx^2 - 2acx - 2bcx + 2abc$$

$\Rightarrow ax^2 + bx^2 - 2cx^2 - 2abx - acx - bcx + 2acx$
$$+ 2bcx = 0$$

$\Rightarrow \quad (a + b - 2c)x^2 + (-2ab + ac + bc)\,x = 0$

$\Rightarrow \quad x[(a + b - 2c)x + (ac + bc - 2ab)] = 0$

$\therefore \qquad x = 0, -\dfrac{(ac + bc - 2ab)}{a + b - 2c}$

Q. 22. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction.

[CBSE Delhi, Term 2, Set 3, 2016]

Ans. Let the speed while going be x km/h

So, speed while returning $= (x + 10)$ km/h

Time taken by a person to cover 150 km

$$= \frac{150}{x} \text{ hours}$$

Time taken by a person in return journey

$$= \frac{150}{x + 10} \text{ hours}$$

Now, according to the given condition,

$$\frac{150}{x} - \frac{150}{x+10} = \frac{5}{2}$$

$\Rightarrow \qquad \dfrac{150(x + 10 - x)}{x(x+10)} = \dfrac{5}{2}$

$\Rightarrow \qquad 300 \times 10 = 5x(x + 10)$

$\Rightarrow \qquad 3000 = 5x^2 + 50x$

$\Rightarrow \qquad 5x^2 + 50x - 3000 = 0$

$\Rightarrow \qquad x^2 + 10x - 600 = 0$

$\Rightarrow \quad x^2 + 30x - 20x - 600 = 0$

$\Rightarrow x(x + 30) - 20(x + 30) = 0$

$\Rightarrow \qquad (x - 20)(x + 30) = 0$

$\Rightarrow \qquad\qquad x = 20$

or $\qquad\qquad\qquad x = -30$ (neglect)

Hence, the speed while going is 20 km/h and the speed while returning is 30 km/h

Q. 23. The diagonal of a rectangular field is 16 m more than the shorter side. If the longer side is 14 m more than the shorter side, then find the lengths of the sides of the field. **[CBSE OD, Term 2, Set 1, 2015]**

Ans.
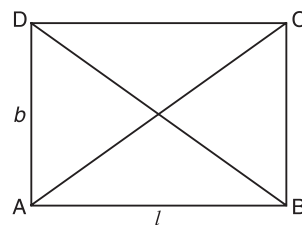
Let l be the length of the longer side and b be the length of the shorter side.

Given that the length of the diagonal of the rectangular field is 16 m more than shorter side.

Thus, Diagonal $= 16 + b$

Since longer side is 14 m more than shorter side,

$\therefore \qquad\qquad\qquad l = 14 + b$

We know

$(\text{Diagonal})^2 = (\text{Length})^2 + (\text{Breadth})^2$

[By Pythagoras theorem]

$(16 + b)^2 = (14 + b)^2 + b^2$

$\Rightarrow \quad 256 + b^2 + 32b = 196 + b^2 + 28b + b^2$

$\Rightarrow \qquad b^2 - 4b - 60 = 0$

$\Rightarrow \quad b^2 - 10b + 6b - 60 = 0$

$\Rightarrow b(b - 10) + 6(b - 10) = 0$

$\Rightarrow \qquad (b + 6)(b - 10) = 0$

$\Rightarrow \qquad\qquad b = -6 \text{ or } + 10$

As breadth cannot be negative

∴   Breadth (b) = 10 m.

Now, length of rectangular field= $(14 + b)$ m

           $= (14 + 10)$ m

           $= 24$ m

Thus, length of rectangular field is 24 cm and breadth is 10 m.

Q. 24. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

[CBSE OD, Term 2, Set 1, 2015]

Ans. Let the average speed of the train be x km/hr.

Then, new average speed of the train = $(x + 6)$ km/hr

Time taken by train to cover 54 km = $\dfrac{54}{x}$ hrs

And time taken by train to cover 63 km = $\dfrac{63}{(x+6)}$ hrs

According to the question,

$$\frac{54}{x} + \frac{63}{x+6} = 3$$

$\Rightarrow \qquad \dfrac{54(x+6)+63x}{x(x+6)} = 3$

$\Rightarrow \qquad 54x + 324 + 63x = 3x(x+6)$

$\Rightarrow \qquad 324 + 117x = 3x^2 + 18x$

$\Rightarrow \qquad 3x^2 - 99x - 324 = 0$

$\Rightarrow \qquad x^2 - 33x - 108 = 0$

$\Rightarrow \qquad x^2 - 36x + 3x - 108 = 0$

$\Rightarrow \qquad x(x - 36) + 3(x - 36) = 0$

$\Rightarrow \qquad (x + 3)(x - 36) = 0$

∴ $\qquad\qquad x = -3$ or 36

Since, speed cannot be negative

∴ $\qquad\qquad x = 36$

so, First speed of train = 36 km/hr

Q. 25. A bus travels at a certain average speed for a distance of 75 km and then travels a distance of 90 km at an average speed of 10 km/h more than the first speed. If it takes 3 hours to complete the total journey, find its first speed.

[CBSE OD, Term 2, Set 2, 2015]

Ans. Let x be the initial speed of the bus.

As $\qquad\qquad$ Time = $\dfrac{\text{Distance}}{\text{Speed}}$

Thus, we have

$$3 = \frac{75}{x} + \frac{90}{x+10}$$

$\Rightarrow \qquad 3 = \dfrac{75(x+10)+90x}{x(x+10)}$

$\Rightarrow \qquad 3(x)(x + 10) = 75x + 750 + 90x$

$\Rightarrow \qquad 3x^2 + 30x = 75x + 750 + 90x$

$\Rightarrow \qquad 3x^2 - 135x - 750 = 0$

$\Rightarrow \qquad x^2 - 45x - 250 = 0$

$\Rightarrow \qquad x^2 - 50x + 5x - 250 = 0$

$\Rightarrow \qquad x(x - 50) + 5(x - 50) = 0$

$\Rightarrow \qquad (x + 5)(x - 50) = 0$

$\Rightarrow \qquad\qquad x = -5$ or $x = 50$

Since, speed cannot be negative

So, $\qquad\qquad x = 50$

Hence, the initial speed of bus is 50 km/hr.

Q. 26. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck.

[CBSE OD, Term 2, Set 3, 2015]

Ans. Let the average speed of the truck be x km/hr.

Then, new average speed of truck

$\qquad\qquad = (x + 20)$ km/hr.

Time taken by truck to cover 150 km = $\dfrac{150}{x}$ hrs.

and time taken by truck to cover 200 km

$\qquad\qquad = \dfrac{200}{x+20}$ hrs.

∴ $\qquad \dfrac{150}{x} + \dfrac{200}{x+20} = 5$

$\Rightarrow \qquad \dfrac{150(x+20)+200x}{x(x+20)} = 5$

$\Rightarrow \qquad 150x + 3000 + 200x = 5x(x + 20)$

$\Rightarrow \qquad 350x + 3000 = 5x^2 + 100x$

$\Rightarrow \qquad 5x^2 - 250x - 3000 = 0$

$\Rightarrow \qquad x^2 - 50x - 600 = 0$

$\Rightarrow \qquad x^2 - 60x + 10x - 600 = 0$

$\Rightarrow \qquad x(x - 60) + 10(x - 60) = 0$

$\Rightarrow \qquad (x + 10)(x - 60) = 0$

∴ $\qquad\qquad x = -10$ or 60

Since speed cannot be negative.

So, $\qquad\qquad x = 60$

∴ First speed of truck = 60 km/hr.

Q. 27. **The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\dfrac{29}{20}$. Find the original fraction.**

Ans. Let the denominator of the fraction be x then numerator is $x - 3$ and fraction is $\dfrac{x-3}{x}$

If 2 is added to both numerator and denominator then New fraction is

$$\frac{x-3+2}{x+2} = \frac{x-1}{x+2}$$

According to the question

$$\therefore \qquad \frac{x-3}{x} + \frac{x-1}{x+2} = \frac{29}{20}$$

$$\Rightarrow \qquad \frac{(x-3)(x+2) + x(x-1)}{x(x+2)} = \frac{29}{20}$$

$\Rightarrow 20(x^2 - 3x + 2x - 6 + x^2 - x) = 29(x^2 + 2x)$

$\Rightarrow \qquad 40x^2 - 40x - 120 = 29x^2 + 58x$

$\Rightarrow \qquad 11x^2 - 98x - 120 = 0$

$\Rightarrow \qquad 11x^2 - 110x + 12x - 120 = 0$

$\Rightarrow \qquad 11x(x - 10) + 12(x - 10) = 0$

$\Rightarrow \qquad (11x + 12)(x - 10) = 0$

$$x = 10 \text{ or } -\frac{12}{11} \text{ (neglect)}$$

Hence, the fraction is $\dfrac{10-3}{10}$ i.e., $\dfrac{7}{10}$.

Q. 28. **Solve for x:**

$$\frac{2}{x+1} + \frac{3}{2(x-2)} = \frac{23}{5x}, x \neq 0, -1, 2$$

Ans. We have, $\qquad \dfrac{2}{x+1} + \dfrac{3}{2(x-2)} = \dfrac{23}{5x}$

$\Rightarrow 2(10x)(x-2) + 3(5x)(x+1)$
$$= 23(2)(x+1)(x-2)$$

$\Rightarrow 20x(x-2) + 15x(x+1)$
$$= 46(x+1)(x-2)$$

$\Rightarrow 20x^2 - 40x + 15x^2 + 15x$
$$= 46(x^2 + x - 2x - 2)$$

$\Rightarrow 20x^2 - 40x + 15x^2 + 15x = 46x^2 - 46x - 92$

$\Rightarrow \qquad 11x^2 - 21x - 92 = 0$

$\Rightarrow \qquad x = \dfrac{21 \pm \sqrt{441 + 4048}}{22}$

$\Rightarrow \qquad x = \dfrac{21 \pm \sqrt{4489}}{22}$

$\Rightarrow \qquad x = \dfrac{21 \pm 67}{22}$

$\Rightarrow \qquad x = \dfrac{21 + 67}{22} \text{ or } \dfrac{21 - 67}{22}$

$\Rightarrow \qquad x = \dfrac{88}{22} \text{ or } -\dfrac{46}{22}$

$\therefore \qquad x = 4 \text{ or } -\dfrac{23}{11}$

Q. 29. **Solve for x:**

$$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}; x \neq 1, -1, \frac{1}{4}$$

Ans. We have,

$$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}; x \neq 1, -1, \frac{1}{4}$$

$\Rightarrow 3(x-1)(4x-1) + 4(x+1)(4x-1)$
$$= 29(x+1)(x-1)$$

$\Rightarrow 3(4x^2 - 4x - x + 1) + 4(4x^2 + 4x - x - 1)$
$$= 29(x^2 - 1)$$

$\Rightarrow 12x^2 - 15x + 3 + 16x^2 + 12x - 4 = 29x^2 - 29$

$\Rightarrow \qquad 28x^2 - 3x - 1 = 29x^2 - 29$

$\Rightarrow \qquad x^2 + 3x - 28 = 0$

$\Rightarrow \qquad x^2 + 7x - 4x - 28 = 0$

$\Rightarrow \qquad x(x+7) - 4(x+7) = 0$

$\Rightarrow \qquad (x-4)(x+7) = 0$

$\Rightarrow \qquad x = -7 \text{ or } 4.$