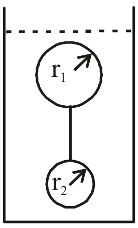
Chapter 7

PROPERTIES OF MATTER & FLUID MECHANICS

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. Two solid spherical balls of radii r_1 and r_2 ($< r_1$) and of density σ are tied up with a long string and released in a viscous liquid column of lesser density ρ with the string just taut as shown. The tension in the string when terminal velocity is attained, is



(a) $\frac{4}{3}\pi\left(\frac{r_2^4 - r_1^4}{r_2 - r_1}\right)(\sigma-\rho)g$

(b) $\frac{2}{3}\pi\ r_2^3 - r_1^3\ (\rho-\sigma)g$

(c) $\frac{4}{3}\pi\ r_2^3 - r_1^3\ (\sigma-\rho)g$

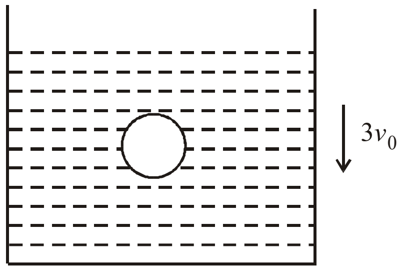
(d) $\frac{4}{3}\pi\left(\frac{r_2^4 - r_1^4}{r_2 \quad r_1}\right)(\sigma-\rho)g$

2. Drops of liquid of density ρ are floating half immersed in a liquid of density σ. If the surface tension of liquid is T, the radius of the drop will be

(a) $\sqrt{\frac{3T}{g(3\rho-\sigma)}}$ (b) $\sqrt{\frac{6T}{g(2\rho-\sigma)}}$

(c) $\sqrt{\frac{3T}{g(2\rho-\sigma)}}$ (d) $\sqrt{\frac{3T}{g(4\rho-3\sigma)}}$

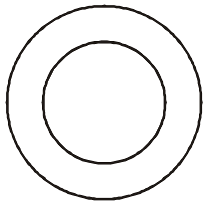
3. A container filled with viscous liquid is moving vertically downwards with constant speed $3v_0$. At the instant shown, a sphere of radius r is moving vertically downwards (in liquid) has speed v_0. The coefficient of viscosity is η. There is no relative motion between the liquid and the container. Then at the shown instant, the magnitude of viscous force acting on sphere is



(a) $6\pi\eta r v_0$ (b) $12\pi\eta r v_0$

(c) $18\pi\eta r v_0$ (d) $24\pi\eta r v_0$

4. A soap bubble of radius R is surrounded by another soap bubble of radius $2R$, as shown. Take surface tension = S. Then the pressure inside the smaller soap bubble, in excess of the atmospheric pressure, will be



Atmosphere

(a) $4S/R$ (b) $3S/R$

(c) $6S/R$ (d) None of these

5. A steel wire is 4.0m long and 2mm in diameter. How much is it elongated by a suspended body of mass 20 kg ? Young's modulus for steel is 1,96,000 Mpa.

(a) 1.273 mm (b) 1.73 mm

(c) 0.123 mm (d) 2.124 mm

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ

6. Water is flowing on a horizontal fixed surface, such that its flow velocity varies with y (vertical direction) as

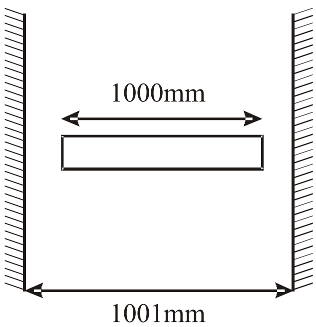
$v = k\left(\frac{2y^2}{a^2} - \frac{y^3}{a^3}\right)$. If coefficient of viscosity for water is η,

what will be shear stress between layers of water at $y = a$.

(a) $\frac{\eta k}{a}$ (b) $\frac{\eta}{ka}$

(c) $\frac{\eta a}{k}$ (d) None of these

7. A rod of length 1000mm and coefficient of linear expansion $\alpha = 10^{-4}$ per degree is placed symmetrically between fixed walls separated by 1001 mm. The Young's modulus of the rod is 10^{11} N/m^2. If the temperature is increased by 20°C, then the stress developed in the rod is (in N/m^2)



(a) 10 (b) 10^8
(c) 2×10^8 (d) cannot be calculated

8. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4 y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to

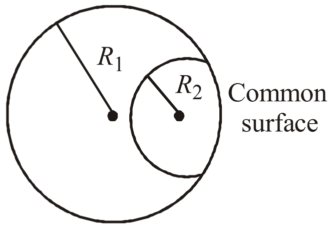
(a) $\frac{L}{\sqrt{2\pi}}$ (b) $2\pi L$ (c) L (d) $\frac{L}{2\pi}$

9. A rain drop of radius 0.3mm falling vertically downwards in air has a terminal velocity of 1m/s. The viscosity of air is 18×10^{-5} poise. The viscous force on the drop is
(a) 101.73×10^{-4} dyne (b) 101.73×10^{-5} dyne
(c) 16.95×10^{-5} dyne (d) 16.95×10^{-4} dyne

10. A vertical capillary tube with inside diameter 0.5mm is submerged into water so that the length of its part protruding over the surface of water is equal to 2.5mm. Find the radius of curvature of the meniscus.
(a) 0.3mm (b) 0.6mm
(c) 0.9mm (d) 1.2mm

11. A platform is suspended by four wires at its corners. The wires are 3m long and have a diameter of 2.0mm. Young's modulus for the material of the wires is 1,80,000 *MPa*. How far will the platform drop (due to elongation of the wires) if a 50 kg load is placed at the centre of the platform ?
(a) 0.25 mm (b) 0.65 mm
(c) 1.65 mm (d) 0.35 mm

12. Two spherical bubbles are in contact with each other internally as shown. The radius of curvature of the common surface is R, then

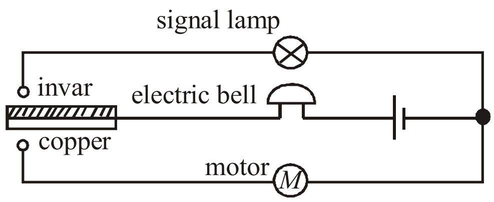


(a) $R > R_1$ (b) $R_1 > R > R_2$
(c) $R < R_2$ (d) $R = R_1$

13. The force exerted by a special compression device is given as function of compression x as $F_x(x) = kx(x - \ell)$ for $0 \le x \le \ell$, where ℓ is maximum possible compression and k is a constant. The force exerted by the device under compression is maximum when compression is –
(a) 0 (b) $\ell/4$
(c) $\ell/\sqrt{2}$ (d) $\ell/2$

14. A steel ball of diameter d = 3.0mm starts sinking with zero initial velocity in olive oil whose viscosity is η = 0.90 P. How soon after the beginning of motion will the velocity of the ball differ from the steady state velocity by n = 1.0%? Density of steel = 7.8×10^3 kg/m^3.
(a) 0.2 sec. (b) 0.8 sec.
(c) 0.6 sec. (d) 1.2 sec.

15. The diagram shows bimetallic strip used as a thermostat in a circuit. The copper expands more than the invar for the same temperature rise.

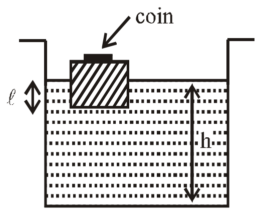


Which will be switched on when the bimetallic strip becomes hot ?
(a) bell only (b) lamp and bell only
(c) motor and bell only (d) lamp, bell and motor

MARK YOUR RESPONSE					
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ

16. What is the minimum diameter of a brass rod if it is to support a 400N load without exceeding the elastic limit ? Assume that the stress for the elastic limit is 379 MPa.
(a) 1.16mm (b) 2.32mm
(c) 0.16mm (d) 1.35mm

17. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance ℓ and h are shown here. After some time the coin falls into the water. Then



(a) ℓ decreases and h increases
(b) ℓ increases and h decreases
(c) both ℓ and h increase (d) both ℓ and h decrease

18. A conical glass capillary tube A of length 0.1m has diameters 10^{-3} and 5×10^{-4}m at the ends. When it is just immersed in a liquid at 0°C with larger diameter in contact with it, the liquid rises to 8×10^{-2}m in the tube. If another cylindrical glass capillary tube B is immersed in the same liquid at 0°C, the liquid rises to 6×10^{-2}m height. The rise of liquid in the tube B is only 5.5×10^{-2}m when the liquid is at 50°C. Find the rate at which the surface tension changes with temperature considering the change to be linear. (The density of the liquid is $(1/14) \times 10^4$ kg/m^3 and angle of contact is zero. Effect of temperature on density of liquid and glass is negligible).
(a) -1.4×10^{-4} N/m°C (b) -2.4×10^{-4} N/m°C
(c) -4.4×10^{-6} N/m°C (d) -5.2×10^{-8} N/m°C

19. Four uniform wires of the same material are stretched by the same force. The dimensions of wire are as given below. The one which has the minimum elongation has –
(a) radius 3mm, length 3m
(b) radius 0.5mm, length 0.5m
(c) radius 2mm, length 2m
(d) radius 3mm, length 2m

20. A long cylinder of radius R_1 is displaced along its axis with a constant velocity v_0 inside a stationary co-axial cylinder of radius R_2. The space between the cylinders is filled with viscous liquid. Find the velocity of the liquid as a function of the distance r from the axis of the cylinders. The flow is laminar.
(a) $2v_0 \dfrac{\ln(r/R_2)}{\ln(R_1/R_2)}$ (b) $\dfrac{v_0}{2} \dfrac{\ln(r/R_2)}{\ln(R_1/R_2)}$
(c) $v_0 \dfrac{\ln(r/R_2)}{\ln(R_1/R_2)}$ (d) $\dfrac{2v_0}{3} \dfrac{\ln(r/R_2)}{\ln(R_1/R_2)}$

21. Following are some statements about buoyant force on a body of certain shape, immersed completely inside ideal fluid. Select the correct statement(s) (Liquid is of uniform density)
(a) Buoyant force depends upon orientation of the concerned body inside the liquid
(b) Buoyant force depends upon the density of the body immersed
(c) Buoyant force depends on the fact whether the system is on moon or on the earth
(d) Buoyant force depends upon the depth of the body inside the liquid

22. Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $a/A = 0.1$ then v^2 is (where v is the velocity of water coming out of the hole)
(a) 50 (b) 51
(c) 48 (d) 51.5

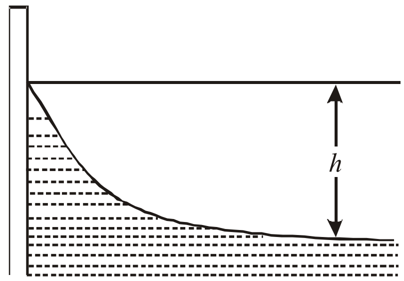
23. A capillary tube with inner cross-section in the form of a square of side 'a' is dipped vertically in a liquid of density ρ and surface tension σ which wet the surface of capillary tube with angle of contact θ. The approximate height to which liquid will be raised in the tube is (Neglect the effect of surface tension at the corners capillary tube)
(a) $\dfrac{2\sigma\cos\theta}{a\rho g}$ (b) $\dfrac{4\sigma\cos\theta}{a\rho g}$
(c) $\dfrac{8\sigma\cos\theta}{a\rho g}$ (d) None of these

24. A U-tube whose ends are open and whose limbs are vertical contains oil of specific gravity 0.8 and surface tension 28 dyne/cm. If one limb has a diameter of 2.2mm and the other of 0.8mm, what is the difference in level of the oil in the two limbs ?
(a) 22.8 mm (b) 6.2 mm
(c) 15.2 mm (d) 11.4 mm

25. The pressure in an explosion chamber is 345 MPa. What would be the percent change in volume of a piece of copper subjected to this pressure ? The bulk modulus for copper is 138 Gpa (= 138×10^9 Pa)
(a) 0.1% (b) 0.5%
(c) 0.25% (d) 0.2%

MARK YOUR RESPONSE					
	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ
	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ

26. Water of density ρ in a clean aquarium forms a meniscus, as illustrated in the figure. Calculate the difference in height h between the centre and the edge of the meniscus. The surface tension of water is γ.



(a) $\sqrt{\frac{2\gamma}{\rho g}}$ (b) $\sqrt{\frac{\gamma}{\rho g}}$

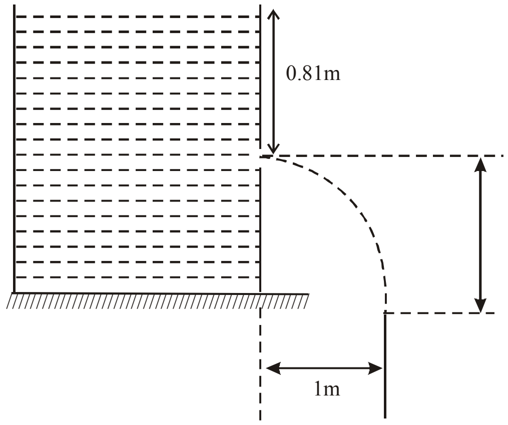
(c) $\frac{1}{2}\sqrt{\frac{\gamma}{\rho g}}$ (d) $2\sqrt{\frac{\gamma}{\rho g}}$

27. A cube of mass $m = 3.2$ kg floats on the surface of water. Water wets it completely. The cube is 0.2m on each edge by what additional distance is it buoyed up or down by surface tension ? Surface tension of water = 0.07 Nm^{-1}.

(a) 2.8×10^{-4} m (b) 1.4×10^{-4} m

(c) 3.2×10^{-6} m (d) 6.1×10^{-2} m

28. For the arrangement shown in the figure, find the time interval in seconds after which the water jet ceases to cross the wall. Area of the cross section of the tank $A \quad \sqrt{5}\,\text{m}^2$ and area of the orifice $a = 4\ \text{cm}^2$. [Assume that the container remaining fixed]



(a) 1000s (b) 2000s

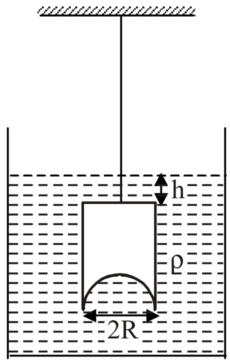
(c) 1500s (d) 500s

29. A right circular cone of density ρ, floats just immersed with its vertex downwards in a vessel containing two liquids of densities σ_1 and σ_2 respectively, the planes of separation of the two liquids cuts off from the axis of the cone a fraction z of its length. Find z.

(a) $\left(\frac{\rho+\sigma_2}{\sigma_1+\sigma_2}\right)^{1/3}$ (b) $\left(\frac{\rho-\sigma_2}{\sigma_1-\sigma_2}\right)^{1/3}$

(c) $\left(\frac{\rho-\sigma_2}{\sigma_1+\sigma_2}\right)^{1/2}$ (d) $\left(\frac{\rho-\sigma_2}{\sigma_1-\sigma_2}\right)$

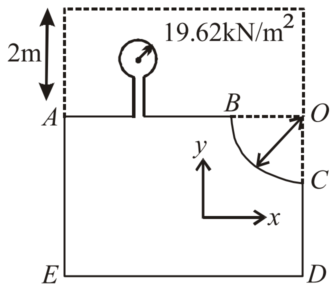
30. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is



(a) Mg (b) $Mg - V\rho g$

(c) $Mg + \pi R^2 h\rho g$ (d) $\rho g(V + \pi R^2 h)$

31. The cross-section of a tank full of water under pressure is shown. Determine the magnitude of the resultant force acting on the quarter-circle curved surface BC if the tank is 2m long.

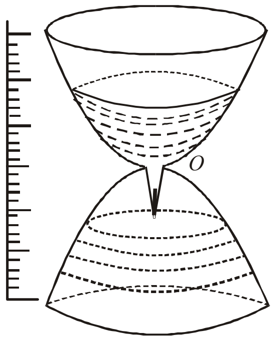


(a) 73.43 kN (b) 55.12 kN

(c) 64.12 kN (d) 43.14 kN

MARK YOUR RESPONSE	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ
	31. ⓐⓑⓒⓓ				

32. A water clock (clepsydra) used in ancient Greece is designed as a vessel with a small orifice O (figure). The time is determined according to the level of the water in the vessel. What should be equation corresponding to the shape of the vessel for the time scale to be uniform ?



(a) $y = kx^4$ where k $\dfrac{\pi^2 v^2}{2ga^2}$

(b) $y = kx^3$ where k $\dfrac{\pi^2 v^2}{2ga^2}$

(c) $y = kx^4$ where k $\dfrac{\pi^2 v^2}{ga}$

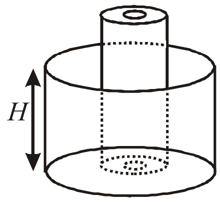
(d) $y = kx^4$ where k $\dfrac{\pi^2 v^3}{2ga^2}$

33. A cylindrical tumbler, half filled with liquid of density ρ : is filled up with a liquid of density ρ' which does not mix with the former one. Find the ratio of the pressure on the base of the tumbler to the whole pressure on its curved surface is (h is the height and r the radius of the base of the tumbler)

(a) $\dfrac{r\,(\rho+\rho')}{h\,(\rho+3\rho')}$ (b) $\dfrac{r\,(\rho+\rho')}{2h\,(\rho+3\rho')}$

(c) $\dfrac{r\,(\rho+\rho')}{2h\,(\rho+2\rho')}$ (d) $\dfrac{2r\,(\rho+\rho')}{h\,(\rho+3\rho')}$

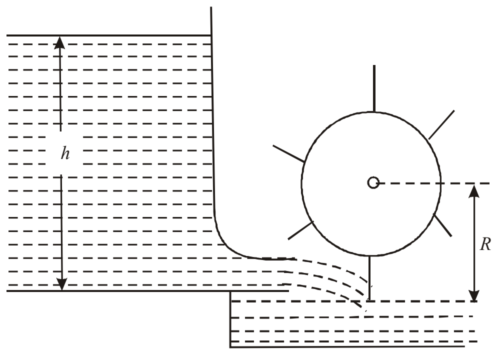
34. A hollow wooden cylinder of height h, inner radius R and outer radius $2R$ is placed in a cylindrical container of radius $3R$. When water is poured into the container, the minimum height H of the container for which cylinder can float inside freely is



(a) $\dfrac{h\rho_{water}}{\rho_{water}+\rho_{wood}}$ (b) $\dfrac{h\rho_{wood}}{\rho_{water}}$

(c) h (d) $\dfrac{h^2}{R}$

35. Find the force acting on the blade of an undershot wheel (figure) if the stream after impinging on the blade continues to move with the velocity of the blade.
The height of the water head is h, the radius of the wheel is R, the angular velocity of the wheel is ω and the cross-section area of the stream is A.



(a) $\rho A\,(\sqrt{2gh}-\omega R)^2$ (b) $\rho A\,(\sqrt{2gh}+\omega R)^2$

(c) $\rho A\,(\sqrt{2gh}+\omega R)$ (d) $\rho A\,(2gh+\omega R)$

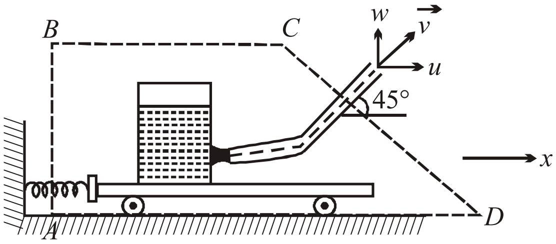
36. A pump is designed as a horizontal cylinder with a piston having an area of A and an outlet orifice having an area of 'a' arranged near the cylinder axis. Determine the velocity of outflow of a liquid from the pump if the piston moves with a constant velocity under the action of a force F. The density of the liquid is ρ. ($a << A$)

(a) $\sqrt{\dfrac{2F}{A\rho}}$ (b) $\sqrt{\dfrac{F}{A\rho}}$

(c) $\sqrt{\dfrac{F}{2A\rho}}$ (d) $\sqrt{\dfrac{2F}{3A\rho}}$

MARK YOUR RESPONSE	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ

37. A tank and a trough are placed on a trolley as shown. Water issues from the tank through a 5cm. diameter nozzle at 5m/s and strikes the trough which turns it by 45° as shown. Determine the compression of the spring of stiffnes 20N cm.

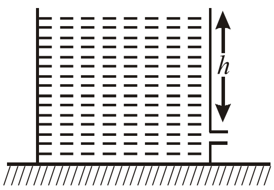


(a) 1.74cm. (b) 1.12 cm.
(c) 0.78cm. (d) 2.12 cm.

38. A right circular cylinder of sp. gr. σ floats in water with its axis vertical, one third begin above the water. If ρ be the sp. gr. of air then choose the correct option

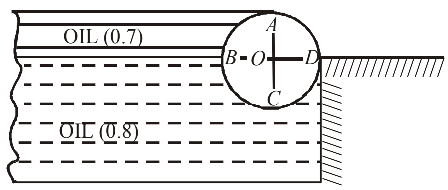
(a) $3\sigma = 2 + \rho$ (b) $3\sigma = 1 + \rho$
(c) $2\sigma = 2 + \rho$ (d) $3\sigma = 2 + 2\rho$

39. In the figure shown, a light container is kept on a horizontal rough surface of coefficient of friction μ $\frac{Sh}{V}$. A very small hole of area S is made at depth h. Water of volume V is filled in the container. The friction is not sufficient to keep the container at rest. The acceleration of the container initially is



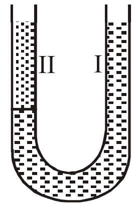
(a) $\frac{V}{Sh}g$
(b) g
(c) zero
(d) $\frac{Sh}{V}g$

40. A cylinder of 1m diameter and 2m length stays in equilibrium as shown. Calculate the specific gravity of the material of the cylinder.



(a) 0.197 (b) 0.797
(c) 0.517 (d) 0.921

41. A U-tube of uniform cross section (see Fig) is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be

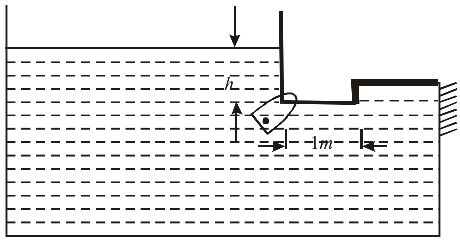


(a) 1.12 (b) 1.1
(c) 1.05 (d) 1.0

42. A solid sphere floats just immersed in heterogeneous liquid composed of three liquids which do not mix and whose densities are as 1 : 2 : 3. If the thickness of the two upper liquids be each one-third of the diameter of the sphere, then choose the correct option

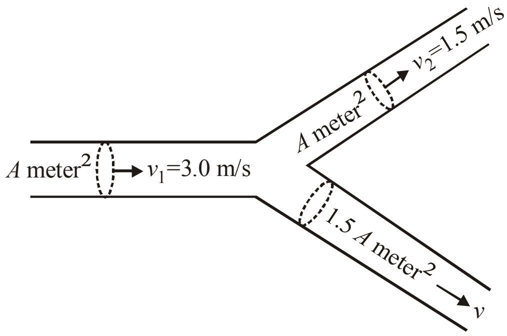
(a) the density of the liquid in the middle is equal to the density of the sphere
(b) the density of the liquid in the middle is half of the density of the sphere
(c) the density of the liquid in the middle is double to the density of the sphere
(d) none of these

43 . A rectangular gate shown should tip automatically when the water rises above a certain level. Determine that level in terms of h.



(a) 2.123m (b) 1.212m
(c) 1.512m (d) 1.732m

44. An incompressible liquid flows through a horizontal tube as shown in the figure.



Then the velocity 'v' of the fluid is

(a) 3.0 m/s (b) 1.5 m/s
(c) 1.0 m/s (d) 2.25 m/s

MARK YOUR RESPONSE					
	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ
	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ		

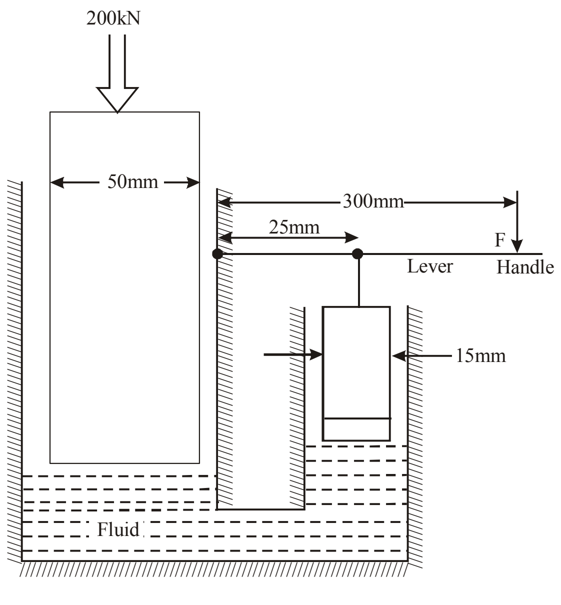
45. A fireman must reach a window 40m above the ground with a water jet, from a nozzle 3cm, diameter discharging 30 kg/s. Assuming the nozzle height of 2m, determine the greatest distance from the building where the fireman can stand and still reach the jet into the window.

(a) 142m. (b) 102m.
(c) 182m. (d) 120m.

46. A broad vessel with water stands on a smooth surface. The level of the water in the vessel is *h*. The vessel together with the water weighs *G*. The side wall of the vessel has at the bottom a plugged hole (with rounded edges) with an area *A*. At what coefficient of friction between the bottom and the surface will the vessel begin to move if the plug is removed ?

(a) $\frac{\rho g h A}{G}$ (b) $\frac{2\rho g h A}{G}$
(c) $\frac{\rho g h A}{2G}$ (d) $\frac{2\rho g h A}{3G}$

47. A hydraulic jack consists of a handle cum lever of 30cm. length and an assembly of given dimensions. In order that a load of 20 kN be supported by the jack, what should be the force exerted on the handle ? The distance between the fulcrum of the lever and the point where the plunger is connected is 25mm.



(a) 2000 N (b) 2500 N
(c) 700 N (d) 1500 N

48. A cylindrical bucket with water in it balances a mass *M* over a pulley. A piece of cork, or mass *m* and sp. gr. σ, is then tied to the bottom of the bucket with the help of string so as to be totally immersed. Find the tension in the string

(a) $\frac{2Mmg}{(2M+m)}\left(\frac{1}{\sigma}-1\right)$ (b) $\frac{Mmg}{(2M+m)}\left(\frac{1}{\sigma}-1\right)$

(c) $\frac{2Mmg}{(2M+m)}\left(\frac{1}{\sigma}\ \ 1\right)$ (d) $\frac{2Mmg}{(M+m)}\left(\frac{1}{\sigma}-1\right)$

49. A hollow weightless hemisphere, filled with liquid is suspended freely from a point in the rim of its base, Find the ratio of the thrust on the plane base to the weight of the contained liquid.

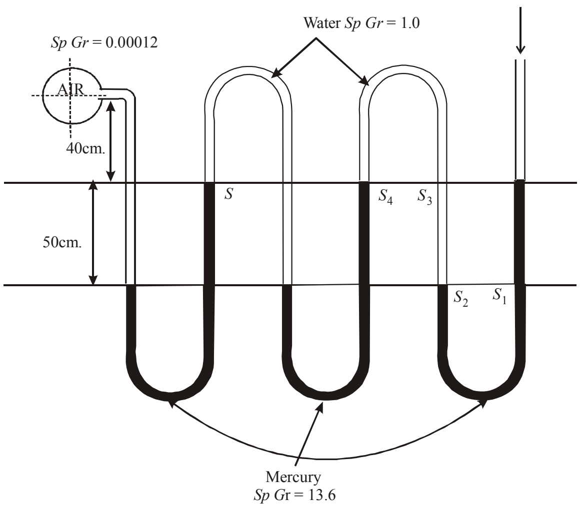
(a) $12:\sqrt{73}$ (b) $12:\sqrt{27}$
(c) $6:\sqrt{73}$ (d) $8:\sqrt{73}$

50. An iceberg is floating in ocean. What fraction of its volume is above the water ? (Given : density of ice = 900kg/m^3 and density of ocean water = 1030 kg/m^3)

(a) $\frac{90}{103}$ (b) $\frac{13}{103}$

(c) $\frac{10}{103}$ (d) $\frac{1}{103}$

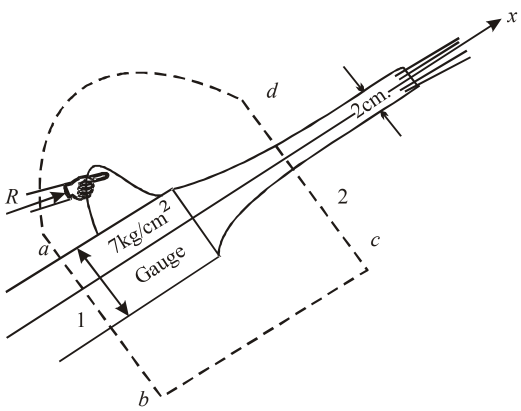
51. A multitube manometer is employed to determine the pressure in a pipe. For the levels in the manometers as shown, compute the pressure in the pipe. What would be the length of a single mercury filled U-tube to record this pressure?



(a) 193cm (b) 165cm
(c) 121cm (d) 153cm

MARK YOUR RESPONSE					
	45. ⓐⓑⓒⓓ	46. ⓐⓑⓒⓓ	47. ⓐⓑⓒⓓ	48. ⓐⓑⓒⓓ	49. ⓐⓑⓒⓓ
	50. ⓐⓑⓒⓓ	51. ⓐⓑⓒⓓ			

52. Find the force exerted by the nozzle on the fireman for the configuration and data in the figure.

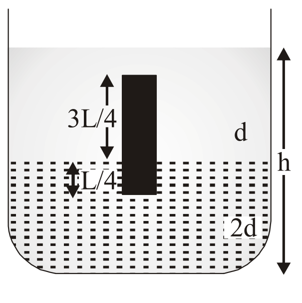


(a) 754 N (b) 1554 N
(c) 1254 N (d) 1712 N

53. A vessel in the shape of a hollow hemisphere surmounted by a cone is held with the axis vertical and vertex uppermost. If it be filled with a liquid so as to submerge half the axis of the cone in the liquid, and the height of the cone be double the radius of its base, the resultant downward thrust of the liquid on the vessel is x times the weight of the liquid that the hemisphere can hold. Find the value of x.

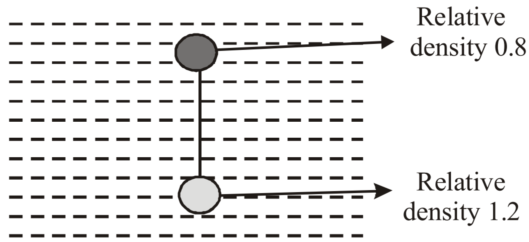
(a) 15/8 (b) 1/8
(c) 5/8 (d) 15/2

54. A homogeneous solid cylinder of length L ($L < H/2$), cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure P_0. Then density D of solid is given by



(a) $\frac{5}{4}d$ (b) $\frac{4}{5}d$
(c) $4d$ (d) $\frac{d}{5}$

55. Two spheres of volume 250cc each but of relative densities 0.8 and 1.2 are connected by a string and the combination is immersed in a liquid in vertical position as shown in figure. The tension in the string is (g = 10 m/s^2)



(a) 5.0N (b) 0.5N
(c) 1.0N (d) 2.0N

56. A solid hemisphere of radius a and weight W is floating in liquid, and at a point on the base at a distance c from the centre rests a weight w, the tangent of the inclination of the axis of the hemisphere to the vertical for the corresponding position of equilibrium, assuming the base of the hemisphere to be entirely out of the fluid, is –

(a) $\frac{4}{3}\frac{c}{a}\frac{w}{W}$ (b) $\frac{2}{3}\frac{c}{a}\frac{w}{W}$
(c) $\frac{8}{3}\frac{c}{a}\frac{w}{W}$ (d) $\frac{8}{5}\frac{c}{a}\frac{w}{W}$

57. A closed tube in the form of an equilateral triangle contains equal volumes of three liquids which do not mix and is placed with its lowest side horizontal. If the densities of the liquids are in A.P., their surface of separation will be at the points of .... of the sides of the triangle.

(a) 1/2 (b) 1/4
(c) 1/3 (d) 1/6

58. A cone full of water, is placed on its side on a horizontal table, the thrust on its base is x times the weight of the contained fluid, where 2α is the vertical angle of the cone. Find the value of x.

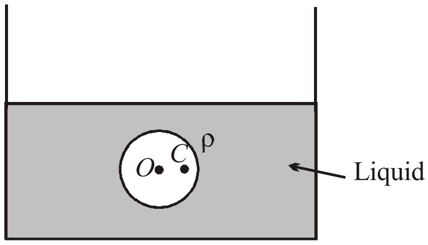
(a) $3\cos\alpha$ (b) $3\sin\alpha$
(c) $2\sin\alpha$ (d) $2\cos\alpha$

59. A balloon of volume V, contains a gas whose density is to that of the air at the earth's surface as 1 : 15. If the envelope of the balloon be of weight w but of negligible volume, find the acceleration with which it will begin to ascend.

(a) $\left(\frac{7Vg\sigma - w}{Vg\sigma + w}\right)\times g$ (b) $\left(\frac{2Vg\sigma - w}{Vg\sigma + w}\right)\times g$
(c) $\left(\frac{14Vg\sigma - w}{Vg\sigma + w}\right)\times g$ (d) $\left(\frac{14Vg\sigma \quad w}{Vg\sigma - w}\right)\times g$

MARK YOUR RESPONSE	52. ⓐⓑⓒⓓ	53. ⓐⓑⓒⓓ	54. ⓐⓑⓒⓓ	55. ⓐⓑⓒⓓ	56. ⓐⓑⓒⓓ
	57. ⓐⓑⓒⓓ	58. ⓐⓑⓒⓓ	59. ⓐⓑⓒⓓ		

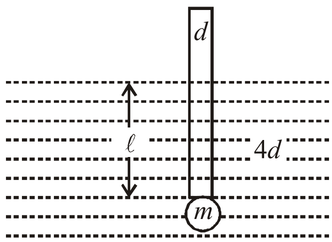
60. An non-homogeneous small sphere having average density same as that of the liquid. It is released from rest, in the position as shown in figure. *C* being its centre of mass and *O* being the centre of sphere.



(a) *O* moves up (b) *O* moves down
(c) *C* moves left (d) None of these

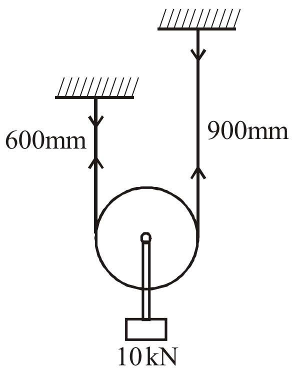
61. A sealed glass bulb containing mercury (incompletely filled) just floats in water at 4°C. If the water and bulb are
(i) cooled to 2°C and (ii) warmed to 8°C, the bulb
(a) (i) sinks and (ii) sinks (b) (i) sinks and (ii) floats
(c) (i) floats and (ii) floats (d) (i) floats and (ii) sinks

62. A uniform wooden stick of length *L*, cross-section area *A* and density *d* is immersed in a liquid of density 4*d*. A small body of mass m and negligible volume is attached at the lower end of the rod so that the stick floats vertically in stable equilibrium then



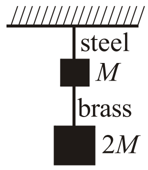
(a) $m > dAL$ (b) $m < dAL$
(c) $m < dAL/2$ (d) $m < dAL/4$

63. A load of 10kN is supported from a pulley which in turn is supported by a rope of sectional area 1×10^3 mm^2 and modulus of elasticity 10^3 N mm^{-2}, as shown in figure. Neglecting the friction at the pulley, determine the deflection of the load.



(a) 2.75 mm (b) 3.75 mm
(c) 5.25 mm (d) 6.50 mm

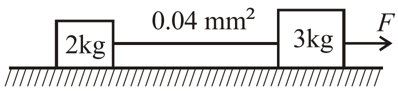
64. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are *a*, *b* and *c* respectively, then the corresponding ratio of increase in their lengths is



(a) $2a^2cb$ (b) $3a/2b^2c$
(c) $2ac/b^2$ (d) $3c/2ab^2$

65. A boat with base area 8m^2 floating on the surface of a still river is intended to move with a constant speed of 2 m/s by the application of a horizontal force. If the river bed is 2m deep find the force needed, assuming a constant velocity gradient. Coefficient of viscosity of water is 0.90×10^{-2} poise.
(a) 720 dyne (b) 620 dyne
(c) 520 dyne (d) 360 dyne

66. Two bodies of masses 2 kg and 3 kg are connected by a metal wire of cross-section 0.04 mm^2. Breaking stress of metal wire is 2.5 GPa. The maximum force *F* that can be applied to 3 kg block so that wire does not break is (Neglect friction)



(a) 100 N (b) 150 N
(c) 200 N (d) 250 N

67. The lower end of a capillary tube of radius 2.00mm is dipped 10.00cm below the surface of water in a beaker. Calculate the pressure within a bubble blown at its end in water, in excess of atmospheric pressure.
[Surface tension of water 72×10^{-3} N/m]
(a) 718 Nm^{-2} (b) 912 Nm^{-2}
(c) 1160 Nm^{-2} (d) 1052 Nm^{-2}

68. The ice storm in the province of Jammu strained many wires to the breaking point. In a particular situation, the transmission pylons are separated by 500 m of wire. The top grounding wire is 15° from the horizontal at the pylons, and has a diameter of 1.5 cm. The steel wire has a density of 7860 kg/m^3. When ice (density 900 kg/m^3) built up on the wire to a total diameter of 10.0 cm, the wire snapped. What was the breaking stress (force/unit area) in N/m^2 in the wire at the breaking point? You may assume the ice has no strength.
(a) 7.4×10^7 N/m^2 (b) 4.5×10^8 N/m^2
(c) 2.6×10^6 N/m^2 (d) 1.15×10^7 N/m^2

MARK YOUR RESPONSE	60. ⓐⓑⓒⓓ	61. ⓐⓑⓒⓓ	62. ⓐⓑⓒⓓ	63. ⓐⓑⓒⓓ	64. ⓐⓑⓒⓓ
	65. ⓐⓑⓒⓓ	66. ⓐⓑⓒⓓ	67. ⓐⓑⓒⓓ	68. ⓐⓑⓒⓓ	

69. Two identical soap bubbles, each of radius x, coalesce to form a bubble of radius y. If P be the atmospheric pressure, and assuming that the process is isothermal, what is the surface tension of soap solution ?

(a) $\dfrac{P[2x^3 \quad y^3]}{4(y^2 - 2x^2)}$ (b) $\dfrac{P[2x^3 - y^3]}{4(y^2 \quad 2x^2)}$

(c) $\dfrac{P[2x^3 - y^3]}{4(y^2 - 2x^2)}$ (d) $\dfrac{P[x^3 - y^3]}{4(y^2 - 2x^2)}$

70. A U-tube is made up of two capillaries of diameters 1.0mm and 1.5mm respectively. The U-tube is kept vertically and partially filled with water of surface tension 0.0075 kg/m and zero contact angle. Calculate the difference in the level of the mencius caused by the capillarity.

(a) 0.68mm (b) 0.12mm

(c) 0.32mm (d) 0.92mm

71. An iron bar of length L cross section A and Young modulus Y is pulled by a force F, from both the ends so as to produce an elongation e. The elastic energy stored in the deformed rod will be

(a) $\dfrac{F^2 L}{AY}$ (b) $\dfrac{F^2 L}{2AY}$

(c) $\dfrac{F^2}{2A^2 Y}$ (d) None of these

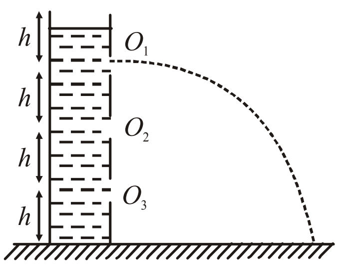
MARK YOUR RESPONSE	69. ⓐⓑⓒⓓ	70. ⓐⓑⓒⓓ	71. ⓐⓑⓒⓓ		

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

Water is filled in a cylinder vessel upto a height 4h. Three small orifices O_1, O_2 and O_3 are made on the wall of the vessel.



1. Let v_1, v_2 and v_3 be the speed of efflux of water from the three orifices then

(a) $v_1 = v_3 < v_2$ (b) $v_1 < v_2 < v_3$

(c) $v_1 = v_3 > v_2$ (d) $v_1 > v_2 > v_3$

2. Let x_1 be the horizontal distance of the point at which the water stream from orifice I strikes the horizontal surface. The distance x_1 is

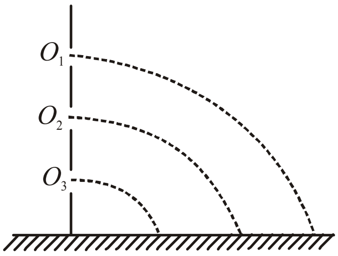
(a) h (b) $2h$

(c) $3\sqrt{2}h$ (d) $2\sqrt{3}h$

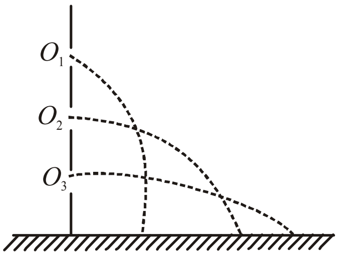
MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ			

3. The correct diagram showing the trajectories is

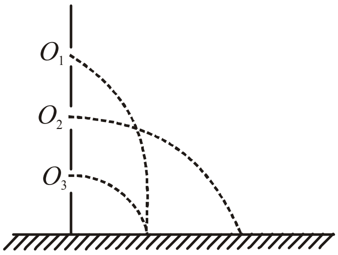
(a)



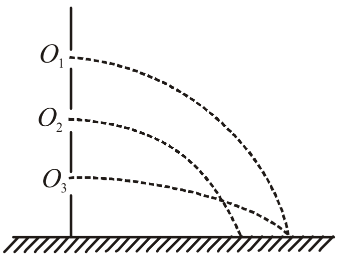
(b)



(c)

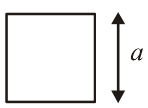


(d)



PASSAGE-2

The property of a surface of liquid to shrink whenever it gets a chance to do so is because of surface tension. The energy that a surface layer has is called as surface energy and is given by product of surface tension and surface area. Consider a solid cube of edge 'a' made of ice. The cube is kept in a gravity free cabin. Now the whole cube is melted. Surface tension of water = T. Viscosity of air = η.



4. The cube will
   (a) remain in cubical shape and fly up
   (b) remain in cubical shape and move down
   (c) become flat and water will stay on
   (d) become spherical
5. The surface energy of liquid thus formed will be equal to
   (a) $(6)(a^2)(T)$ (b) $(5)(a^{3/2})(T)$
   (c) $(3^{2/3})(T.a^2)(4\pi)^{1/3}$ (d) $(12)(a^2)(T)$
6. Now if this melted liquid is dropped in air, in gravity field and viscosity of liquid is η, then
   (a) liquid will move down with a constant speed $\frac{2}{9}\frac{a^2(\rho-\sigma)g}{\eta}\left(\frac{3}{4\pi}\right)^{2/3}$.
   (b) liquid will remain at rest
   (c) liquid will move up with speed $\left(\frac{72}{9}\right)\frac{a^2(\rho-\sigma)g}{\eta}$
   (d) None of above

PASSAGE-3

A tube of length ℓ and radius R carries a steady flow of fluid whose density is ρ and viscosity η. The velocity v of flow is given by $v = v_0\left(\frac{R^2-r^2}{R^2}\right)$, where r is the distance of the flowing fluid from the axis.

7. Volume of fluid, flowing across the section of the tube, in unit time is
   (a) $v_0\pi R^2$ (b) $\frac{\pi v_0 R^2}{2}$
   (c) $\frac{\pi v_0 R^2}{3}$ (d) $\frac{\pi v_0 R^2}{4}$

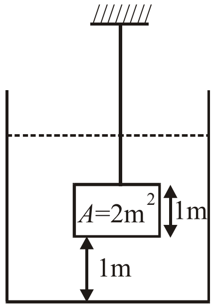
8. Kinetic energy of the fluid within the volume of the tube is
   (a) $\frac{\pi\ell\rho v_0^2 R^2}{2}$ (b) $\frac{\pi\ell\rho v_0^2 R^2}{4}$
   (c) $\frac{\pi\ell\rho v_0^2 R^2}{6}$ (d) $\frac{\pi\ell\rho v_0^2 R^2}{3}$

9. The frictional force exerted on the tube by the fluid is
   (a) $4\pi\eta\ell v_0$ (b) $\pi\eta\ell v_0$
   (c) $2\pi\eta\ell v_0$ (d) $3\pi\eta\ell v_0$

MARK YOUR RESPONSE	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ
	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ			

PASSAGE-4

A tank of base area $4m^2$ is initially filled with water up to height 2m. An object of uniform cross-section $2m^2$ and height 1m is now suspended by wire into the tank, keeping distance between base of tank and that of object 1m. Density of the object is 2000 kg/m^3. Take atmospheric pressure 1×10^5 N/m^2 ; g = 10 m/s^2.



10. The downwards force exerted by the water on the top surface of the object is

(a) 2.0×10^5N (b) 2.1×10^5N
(c) 2.2×10^5N (d) 2.3×10^5N

11. The tension in the wire is

(a) 0.1×10^5N (b) 0.2×10^5N
(c) 0.3×10^5N (d) 0.4×10^5N

12. The buoyant force on the object is

(a) 0.1×10^5N (b) 0.2×10^5N
(c) 0.3×10^5N (d) 0.4×10^5N

PASSAGE-5

A syringe is filled with water. Its volume is $20cm^3$, and the cross-section of its interior part is $4cm^2$. The syringe is held vertically such that its nozzle is at its top, and its 100g piston is pressed by external agent and it moves with a constant speed. The ejected water has an initial upward velocity of 2m/s, and the cross-section of the beam of water at the nozzle is $1mm^2$.

(Neglect the dissipated energy due to friction)

13. Find the speed of the piston

(a) 5 mm/s (b) 5 cm/s
(c) 0.5 m/s (d) 0.5 mm/s

14. What is the total work done by external agent

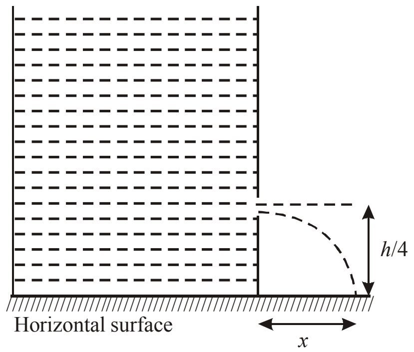
(a) 0.04 J (b) 0.045 J
(c) 0.095 J (d) 4.5 mJ

15. Force exerted by external agent on the piston will

(a) continuously increase
(b) continuously decrease
(c) remain constant
(d) first increase then decrease

PASSAGE-6

Water is filled to a height h in a fixed vertical cylinder placed on horizontal surface. At time $t = 0$ a small hole is drilled at a height $h/4$ from bottom of cylinder as shown. The cross-section area of hole is a and the cross-section area of cylinder is A such that $A >> a$.



16. Let the value of horizontal distance of point where the water fall on horizontal surface from bottom of cylinder be x as shown. Then from time $t = 0$ till water comes out of hole, pick the correct statement

(a) x increases with time
(b) x decreases with time
(c) x first increases and then decreases with time
(d) x first decreases and then increases with time

17. As long as water comes out of hole, the time taken by a water particle starting from hole to reach the horizontal surface

(a) increases
(b) decreases
(c) remains constant
(d) increases and then decreases

18. The duration of time for which water flows out of hole is

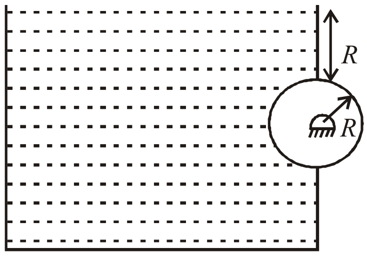
(a) $\frac{A}{a}\sqrt{\frac{3h}{2g}}$ (b) $\frac{a}{A}\sqrt{\frac{3h}{2g}}$

(c) $\frac{A}{a}\sqrt{\frac{2h}{3g}}$ (d) None of these

MARK YOUR RESPONSE					
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	

PASSAGE-7

A cylinder of radius R is kept embedded along the wall of a dam as shown. Take density of water as ρ. Take length as L.



19. The vertical force exerted by water on the cylinder is
(a) $\rho\pi R^2 Lg$ (b) $\rho\pi R^2 Lg/2$
(c) zero (d) None of these

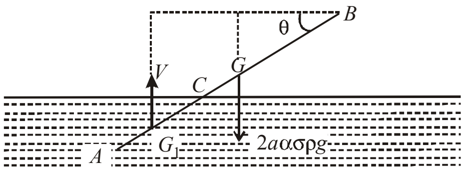
20. The net torque exerted by liquid on the cylinder is
(a) $\frac{2\rho R^3 Lg}{3}$ (b) $\frac{\rho R^3 Lg}{3}$
(c) $\frac{\rho R^3 Lg}{2}$ (d) 0

21. The force exerted by liquid on the cylinder in horizontal direction is [Neglecting atmospheric pressure]
(a) $2R^2\rho gL$ (b) $R^2\rho gL$
(c) $4R^2\rho gL$ (d) $1.61\,R^2\rho gL$

PASSAGE-8

If a body is floating partly immersed in liquid and with a string attached to a point of it. Then to find condition of equilibrium use $\Sigma F_y = 0, \Sigma F_x = 0, \Sigma\tau = 0$

22. A uniform rod capable of turning about one end, which is out of the water, rests inclined to the vertical with one-third of its length in some water. Find the specific gravity of rod .



(a) 5/9 (b) 2/9
(c) 4/9 (d) 1/9

23. A uniform rod 6 ft. long can move about a fulcrum one foot above the surface of the water. In the position of equilibrium four feet of the rod is immersed. Find the specific gravity of rod .
(a) 8/9 (b) 5/9
(c) 2/9 (d) 1/9

24. A uniform rod rests in position inclined to the vertical, with half its length immersed in water, and can turn about a point in it at a distance equal to $\frac{1}{6}$ th of the length of the rod from the extremity below the water. Find the specific gravity of rod .
(a) 5/9 (b) 2/9
(c) 1/8 (d) 1/9

PASSAGE-9

One way of measuring a person's body fat content is by "weighing" them under water. This works because fat tends to float on water as it is less dense than water. On the other hand muscle and bond tend to sink as they are more dense. Knowing your "weight" under water as well as your real weight out of water, the percentage of your body's volume that is made up of fat can easily be estimated. This is only an estimate since it assumes that your body is made up of only two substances, fat (low density) and everything else (high density). The "weight" is measured by spring balance both inside the outside the water. Quotes are placed around weight to indicate that the measurement read on the scale is not your true weight, i.e. the force applied to you body by gravity, but a measurement of the net downward force on the scale.

25. Ram and Shyam are having the same weight when measured outside the water. When measured under water, it is found that weight of Ram is more than that of Shyam, then we can say that
(a) Ram is having more fat content than Shyam
(b) Shyam is having more fat content than Ram
(c) Ram and Shyam both are having the same fat content
(d) None of these

26. A person of mass 165kg having one fourth of his volume consisting of fat (relative density 0.4) and rest of the volume consisting of everything else (average relative density 4/3) is weighed under water by the spring balance. The reading shown by the spring balance is
(a) 15 kg (b) 65 kg
(c) 150 kg (d) None of these

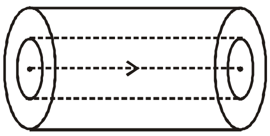
27. Suppose that Ram is floating in water with two-third of his volume immersed. Now the system is taken in a lift which is accelerating upward with acceleration g/3. The new fraction immersed is
(a) one-third (b) half
(c) two-third (d) three-fourth

MARK YOUR RESPONSE					
	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ
	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	

PASSAGE-10

When viscous liquid flows, adjacent layers oppose their relative motion by applying a viscous force given by

$$F = -\eta A \frac{dv}{dz}$$



When η = coefficient of viscosity, A = surface area of adjacent layers in contact.

$$\frac{dv}{dz} = \text{velocity gradient}$$

Now, a viscous liquid having coefficient of viscosity η is flowing through a fixed tube of length ℓ and radius R under a pressure difference P between the two ends of the tube.
Now, consider a cylindrical volume of liquid of radius r. Due to steady flow, net force on the liquid in cylindrical volume should be zero.

$$-\eta 2\pi r\ell \frac{dv}{dr} = P\pi r^2$$

$$-\int_v^0 dv \quad \frac{P}{2\eta\ell}\int_r^R r\, dr$$

($\because$ layer in contact with the tube is stationary)

$$v = v_0\left(1-\frac{r^2}{R^2}\right), \text{ where } v_0 \quad \frac{PR^2}{4\eta\ell}$$

The volume of the liquid per second through the tube,

$$Q = \int_0^R v.2\pi r\, dr = \int_0^R v_0\left(1-\frac{r^2}{R^2}\right)2\pi r\, dr$$

$$= v_0 2\pi\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R$$

$$= v_0 2\pi\left[\frac{R^2}{2}-\frac{R^2}{4}\right] \quad \frac{v_0\pi R^2}{2} \quad \frac{\pi P R^4}{8\eta\ell}$$

This is called Poiseuille's equation

28. Force acting on the tube due to the liquid is .
(a) $\pi\eta\ell v_0$ (b) $2\pi\eta\ell v_0$
(c) $4\pi\eta\ell v_0$ (d) $6\pi\eta\ell v_0$

29. The viscous force on the cylindrical volume of the liquid varies as
(a) $F \quad r^2$ (b) $F \quad r$
(c) $F \quad 1/r$ (d) $F \quad 1/r^2$

30. The momentum of the liquid confined in the tube is
(a) $\rho\pi R^2\ell v_0$ (b) $\rho\pi R^2\ell v_0/2$
(c) $2\rho\pi R^2\ell v_0$ (d) $\rho\pi R^2\ell v_0/4$

MARK YOUR RESPONSE	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ		

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options :
(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. There are two needles of same length, same mass, made of same material but having different radii. The lengths are very large compared to their radii (and radii are not too large) such that buoyancy does not come into picture.
Statement - 1 : If one of the needle floats on water due to surface tension effects, other will also float.
Statement - 2 : Same material ensures that adhesive forces are same.

2. **Statement - 1 :** When two soap bubble's of different radii are brought into contact, the common interface of contact bulges into the bubble of larger radii as shown.
Statement - 2 : Pressure inside a soap bubble of lesser radius is more than pressure inside a soap bubble of larger radius.

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ			

3. Statement 1 : A uniform elastic rod lying on smooth horizontal surface is pulled by constant horizontal force of magnitude F as shown in figure (i). Another identical elastic rod is pulled vertically upwards by a constant vertical force of magnitude F (figure ii). The extension in both rods will be same.

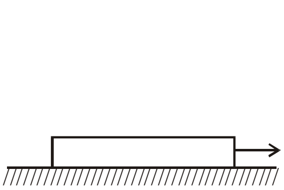


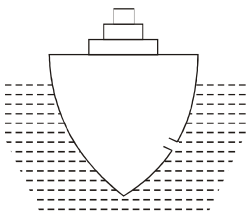
Fig. (i) Fig. (ii)

Statement - 2 : In a uniform elastic rod, the extension depends only on forces acting at the ends of rod.

4. Statement - 1 : A wooden cylinder floats horizontally in a pond of water. By attaching a large cube of ice on one end of cylinder it can be made to float vertically.

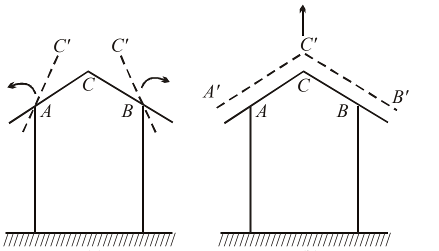
Statement - 2 : To float a cylinder vertically in a liquid its centre of gravity should exist below centre of buoyancy of the submerged part of it.

5. Statement - 1 : A ship gets a large hole in its underwater portion (figure). As a result the motion will begin to retard.



Statement - 2 : The force on the wall is slightly greater than 2pA

6.

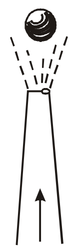


(a) (b)

Statement - 1 : During storms, when the velocity of the wind is very high, it tears off the roofs of buildings. Two cases can be distinguished (1) if the roof is fastened more firmly at points A and B than at ridge C, the wind will break the roof along ridge C and raise both halves up (fig. a) (2) if the roof is secured more firmly at the ridge and less firmly at points A and B, the wind will first lift the roof up and then carry it aside (fig. b).

Statement - 2 : The pressure of the air streaming over the roof is less than of air at rest. It is this surplus pressure of the stationary air under the roof that causes the described phenomena.

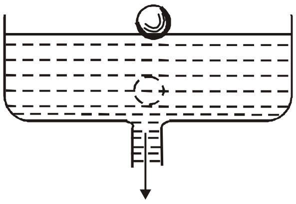
7. Statement - 1 : A light celluloid ball placed in a stream of gas or water issuing at a high velocity from a tube with a narrow neck freely hover in this stream (figure).



Statement - 2 : The gas in the stream has a high velocity, the pressure inside the stream is above atmospheric.

MARK YOUR RESPONSE	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ

8. **Statement - 1 :** The bottom of a broad vessel is provided with a narrow tube through which the water can flow out of the vessel (figure). A screen is placed between the vessel and the tube. If a light ball is submerged to the bottom of the vessel the water flows out of it, and the ball will not rise to the surface. If the outflow of the water is stopped, the ball will immediately rise to the surface.



Statement - 2 : The pressure diminishes in a stream of a flowing liquid with an increase in its velocity. The velocity with which the water flows in the vessel is much smaller than in the tube and, therefore, the pressure of the water in the vessel is greater than in the tube. The velocity increases at the boundary between the vessel and tube, and the pressure drops. For this reason the ball is pressed against the screen and does not rise.

9. **Statement - 1 :** In case of motion of an ideal fluid in a horizontal tube, where the area of cross-section is minimum, pressure is maximum.

Statement - 2 : Hydrostatic pressure in different ideal liquids at points of different depth can be same.

10. **Statement - 1 :** When a rod is pulled by a force acting on one end as shown in diagram then there will be no longitudinal stress in the rod.

Statement - 2 : Longitudinal stress is internal normal force per unit area.

MARK YOUR RESPONSE	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ		

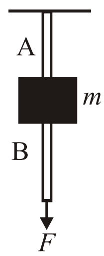
D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. A completely filled closed aquarium is kept on a weighing machine. It can be assumed that the density of the fish is greater than the density of the water. The total mass of the aquarium and its contents put together is M. If now all the fish start accelerating upwards with an acceleration a, then the incorrect option (s) is/are
   (a) the weight recorded will be equal to Mg.
   (b) the weight reading will be less than Mg.
   (c) the weight reading will be more than Mg.
   (d) no conclusion can be drawn from the given information.

2. Two wires A and B have equal lengths and are made of the same material, but the diameter of A is twice that of wire B. Then, for a given load
   (a) the extension of B will be four times that of A
   (b) the extensions of A and B will be equal
   (c) the strain in B is four times that in A
   (d) the strains in A and B will be equal

3. Two light wires A and B shown in the figure are made of the same material and have radii r_A and r_B respectively. The block between them has a mass m.



When the force F is mg/3, one of the wires breaks
   (a) A breaks if $r_A = r_B$
   (b) A breaks if $r_A < 2r_B$
   (c) either A or B may break if $r_A = 2r_B$
   (d) the length of A and B must be known to predict which wire will break.

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ		

4. Four rods, A, B, C and D of the same length and material but of different radii $r, r\sqrt{2}, r\sqrt{3}$ and $2r$ respectively are held between two rigid walls. The temperature of all rods is increased through the same range. If the rods do not bend, then

(a) the stress in the rods A, B, C and D are in the ratio 1 : 2 : 3 : 4

(b) the forces on them exerted by the wall are in the ratio 1 : 2 : 3 : 4

(c) the energy stored in the rods due to elasticity are in the ratio 1 : 2 : 3 : 4

(d) the strains produced in the rods are in the ratio 1 : 2 : 3 : 4

5. An upright open U-tube, which can contain 0.5m mercury in each limb has its limbs 20cm. apart. This is used as an accelerometer in a horizontal flight. From this device acceleration that can be measured are

(a) 13 m/sec^2 (b) 12 m/sec^2

(c) 24 m/sec^2 (d) 20 m/sec^2

6. Two holes with an area of A = 0.2 cm^2 each are drilled one above the other in the wall of vessel filled with water. The distance between the holes H = 50cm. Every second Q = 140cm^3 of water is poured into the vessel. Find the point (x, y) where the streams flowing out of the holes intersect.

(a) $x = 120$ cm (b) $x = 60$cm

(c) $y = 130$cm (d) $y = 75$cm

7. An open rectangular container 2.5m × 1m base and 2m height, half full with water, is accelerated at 4m/s^2 up a 15° incline along its length. Choose the correct options

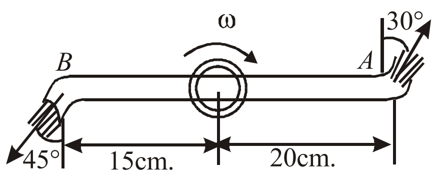
(a) Water will not spill for given acceleration.

(b) At acceleration 5.04 m/s^2 the water begin to spill.

(c) At acceleration 5.04 m/s^2 the water does not spill.

(d) Water will spill for given acceleration.

8. A lawn sprinkler with two nozzles 0.5 cm. diameter each at 20cm. and 15cm. radii is connected across a tap capable of 6 litres/minute discharged. The nozzles discharge water upwards and outwards from the plane of rotation. Choose the correct options



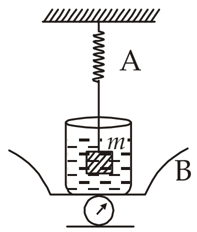
(a) Total torque due to nozzles A and B is 0.355 Nm

(b) If held stationary then angular velocity with which it will rotate freely is 9.72 rad/sec.

(c) If held stationary then angular velocity with which it will rotate freely is 6.14 rad/sec.

(d) Total torque due to nozzles A and B is 0.0355 Nm

9. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation:



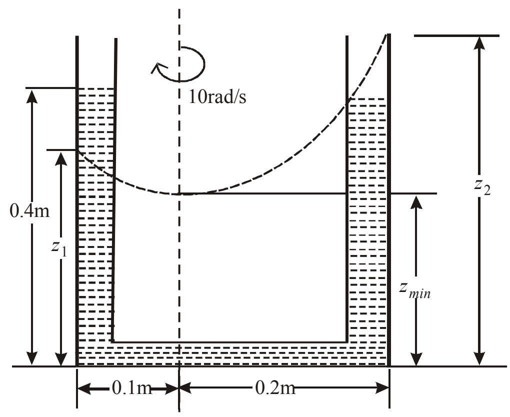
(a) the balance A will read more than 2 kg

(b) the balance B will read more than 5 kg

(c) the balance A will read less than 2 kg and B will read more than 5 kg

(d) the balance A and B will read 2 kg and 5 kg respectively

10. An upright U-tube manometer with its limbs 0.6m high and spaced 0.3m apart contains a liquid to a height of 0.4m in each limb. If the U-tube is rotated at 10 radians/second about a vertical axis at 0.1m from one limb. Choose the correct options

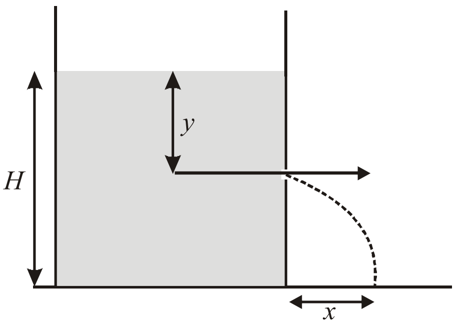


(a) $z_1 = 0.324$ (b) $z_2 = 0.477$

(c) $z_{min} = 0.273$ (b) $z_1 + z_2 = 0.8$

MARK YOUR RESPONSE	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ
	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ			

11. A tank which is open at the top, contains a liquid up to a height H. A small hole is made in the side of the tank at a distance y below the liquid surface. The liquid emerging from the hole lands at a distance x from the tank



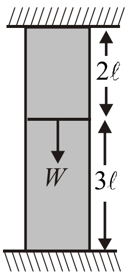
(a) If y is increased from zero to H, x will first increase and then decrease

(b) x is maximum for $y = H/2$

(c) the maximum value of x is H

(d) the maximum value of x will depend on the density of the liquid

12. A vertical bar of uniform section is fixed at both of its ends and a load $W = 5000$ N is applied axially at an intermediate section as shown in figure. Choose the correct options.



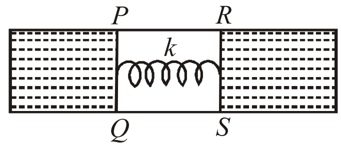
(a) Reaction at the top support is 3000 N

(b) Reaction at the bottom support is 2000 N

(c) Reaction at the top support is 1000 N

(d) Reaction at the bottom support is 3000 N

13. Two identical straight wires PO and RS each of mass m and length ℓ can move smoothly on a fixed rectangular frame. Two thin films of a liquid of surface tension T are formed between each wire and the frame. The two wires are connected by a massless spring of stiffness k and initially in natural length position and released then choose the correct option(s)



(a) maximum elongation of spring, $\Delta x_m \quad \dfrac{2T\ell}{k}$

(b) maximum elongation of spring, $\Delta x_m \quad \dfrac{4T\ell}{k}$

(c) Each wire executes SHM with time period, $T_0 = 2\pi\sqrt{\dfrac{m}{2k}}$

(d) None of these

14. A solid shaft 100mm in diameter, transmits 120kW power at 200 rpm. Choose the correct options

(Modulus of rigidity is 8×10^{10} N/m., Length = 6m)

(a) The maximum intensity of shear stress is 2.92×10^5 Nm^{-2}

(b) The angle of twist is 2°30'

(c) The angle of twist is 1°30'

(d) The maximum intensity of shear stress is 2.92×10^7 Nm^{-2}

MARK YOUR RESPONSE	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

1.

Column I	**Column II**
(A) Temperature increases	(p) Surface tension decreases of liquids
(B) Temperature increases.......	(q) Modulus of increases elasticity of metals
(C) Hooke's law is related to	(r) Viscosity of liquid
(D) .... is independent of shape and size of substance (liquid, gas or solid)	(s) Viscosity of gas

2. Bucket A contains only water, an identical bucket B contains water, but also contains a solid object in the water. Consider the following four situations. Which bucket weighs more

Column I	**Column II**
(A) The object floats in bucket B, and the buckets have the same water level.	(p) Bucket A
(B) The object floats in bucket B, and the buckets have the same volume of water.	(q) Bucket B
(C) The object sinks completely in bucket B, and the buckets have the same water level.	(r) Both buckets have the same weight
(D) The object sinks completely in bucket B, and the buckets have the same volume of water.	(s) The answer cannot be determined from the information given.

3. Match the columns correctly.

Column I	**Column II**
(A) Bernoulli's theorem	(p) Elasticity
(B) Stoke's law	(q) Speed of efflux
(C) Torricelli's theorem	(r) Venturimeter
(D) Hooke's law	(s) Viscosity
	(t) Conservation of energy

4. Column II depends on physical quantity/law given in column I. Match the column correctly.

Column I	**Column II**
(A) Stoke's law	(p) radius
(B) Terminal velocity	(q) density of the material of body
(C) Excess pressure inside mercury drop	(r) coefficient of viscosity
(D) Viscous force	(s) surface tension
	(t) velocity gradient

5. Match the columns correctly.

Column I	**Column II**
(A) With rise in temperature forces that decreases	(p) Elastic force
(B) Forces involved in capillary action	(q) Force due to surface tension
(C) Water flows in a continuous stream down a vertical pipe whereas it breaks into drops when falling freely because of	(r) Frictional force
(D) Terminal velocity of rain drop	(s) Viscous force
	(t) Gravitational force

MARK YOUR RESPONSE

1.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

2.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

3.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

4.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

5.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

NUMERIC/INTEGER ANSWER TYPE

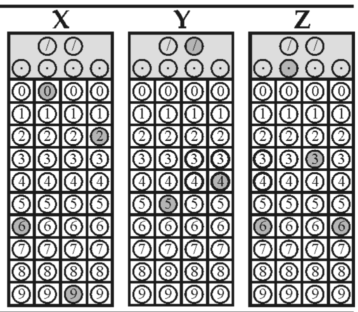
F

The answer to each of the questions is either numeric (eg. 304, 40, 3010, 3 etc.) or a fraction (2/3, 23/7) or a decimal (2.35, 0.546).

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to question X, Y & Z are 6092, 5/4 & 6.36 respectively then the correct darkening of bubbles will look like the following.

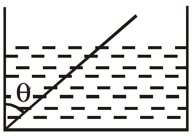
For single digit integer answer darken the extreme right bubble only.



1. A column of mercury of 10 cm length is contained in the middle of a narrow horizontal $1m$ long tube which is closed at both the ends. Both the halves of the tube contain air at a pressure of 76 cm of mercury. By what distance (in cm) will the column of mercury be displaced if the tube is held vertically?

2. A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube rises by 2cm. What is the side of the cube (in cm)?

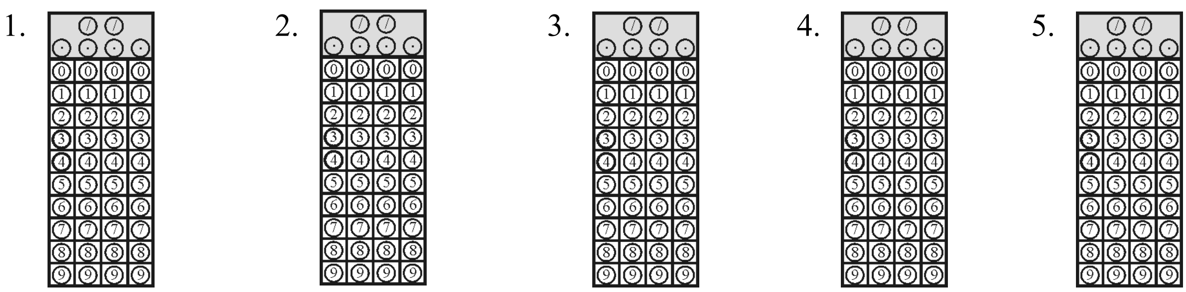
3. A wooden plank of length 1m and uniform cross-section is hinged at one end to the bottom of a tank as shown in fig. The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ (in degree) that the plank makes with the vertical in the equilibrium position. (Exclude the case $\theta = 0°$)



4. A thin tube of uniform cross-section is sealed at both ends. It lies horizontally, the middle 5 cm containing mercury and the two equal end containing air at the same pressure P. When the tube is held at an angle of 60° with the vertical direction, the length of the air column above and below the mercury column are 46cm and 44.5 cm respectively. Calculate the pressure P in centimeters of mercury. (The temperature of the system is kept at 30°C).

5. A cubical block of wood has density ρ_1 = 500 kg/m^3 and side ℓ 30 cm. It is floating in rectangular tank partially filled with water of density ρ_2 = 1000 kg/m and having base area, A = 45 cm × 60 cm. Calculate the work done (in joule) to press the block slowly so that it is just immersed in water. (g = 10 m/s^2) (Express your answer in J)

MARK YOUR RESPONSE



Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(d)	**13**	(d)	**25**	(c)	**37**	(a)	**49**	(a)	**61**	(a)
2	(c)	**14**	(a)	**26**	(a)	**38**	(a)	**50**	(b)	**62**	(a)
3	(b)	**15**	(b)	**27**	(b)	**39**	(d)	**51**	(d)	**63**	(b)
4	(c)	**16**	(a)	**28**	(a)	**40**	(b)	**52**	(b)	**64**	(b)
5	(a)	**17**	(d)	**29**	(b).	**41**	(b)	**53**	(a)	**65**	(a)
6	(a)	**18**	(a)	**30**	(d)	**42**	(a)	**54**	(a)	**66**	(d)
7	(b)	**19**	(d)	**31**	(a)	**43**	(d)	**55**	(b)	**67**	(d).
8	(a)	**20**	(c)	**32**	(a)	**44**	(c)	**56**	(c)	**68**	(b)
9	(a)	**21**	(c)	**33**	(d)	**45**	(a)	**57**	(c)	**69**	(c)
10	(b)	**22**	(a)	**34**	(b)	**46**	(b)	**58**	(b)	**70**	(a)
11	(b)	**23**	(b)	**35**	(a).	**47**	(d)	**59**	(c)	**71**	(b)
12	(c).	**24**	(d)	**36**	(a)	**48**	(a)	**60**	(a)		

B COMPREHENSION TYPE

1	(b)	**6**	(a)	**11**	(b)	**16**	(b)	**21**	(c)	**26**	(a)
2	(d)	**7**	(b)	**12**	(b)	**17**	(c)	**22**	(a)	**27**	(c)
3	(c)	**8**	(c)	**13**	(a)	**18**	(a)	**23**	(a)	**28**	(c)
4	(d)	**9**	(a)	**14**	(c)	**19**	(b)	**24**	(c)	**29**	(a)
5	(c)	**10**	(b)	**15**	(b)	**20**	(d)	**25**	(b)	**30**	(b)

C REASONING TYPE

1	(b)	**3**	(c)	**5**	(a)	**7**	(c)	**9**	(d)
2	(a)	**4**	(d)	**6**	(a)	**8**	(a)	**10**	(d)

D MULTIPLE CORRECT CHOICE TYPE

1	(a, b, d)	**4**	(b, c)	**7**	(a, b)	**10**	(a, b, c, d)	**13**	(b, c)
2	(a, c)	**5**	(a, b, c, d)	**8**	(b, d).	**11**	(a, b, c)	**14**	(b, d)
3	(a, b, c)	**6**	(a, c)	**9**	(b,c)	**12**	(a, b)		

E MATRIX-MATCH TYPE

1. **A-p, q, r; B-s; C-q; D-p, q, r, s**

2. **A-r; B-q; C-q; D-q**

3. **A-q, r, t; B-s; C-q, t; D-p**

4. **A-p, r; B-p, q, r; C-p, s; D-r, t**

5. **A-p, q, r, s; B-q, t; C-q; D-s, t**

F NUMERIC/INTEGER ANSWER TYPE

1	3	**2**	10	**3**	45	**4**	75.4	**5**	6.75

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(d)** $\frac{4}{3}\pi\ r_1^3 + r_2^3\ \ \sigma - \rho\ \ g = 6\pi\eta(r_1\ \ r_2)v$

and $\frac{4}{3}\pi r_2^3(\sigma-\rho)g\ \ \ T\ \ \ 6\pi\eta r_1 v$

$\Rightarrow T = \frac{4}{3}\pi\frac{(r_2^4 - r_1^4)(\sigma-\rho)g}{(r_1\ \ \ r_2)}$

2. **(c)** Balancing the forces acting on the drop, we get

$\frac{4}{3}\pi r^3\rho g = 2\pi r T + \frac{1}{2}.\frac{4}{3}\pi r^3\sigma \Rightarrow r\ \ \ \sqrt{\frac{3T}{(2\rho-\sigma)g}}$

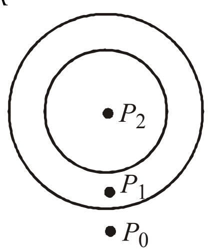
3. **(b)** Relative to liquid, the velocity of sphere is $2v_0$ upwards.

$\therefore$ Viscous force on sphere $= 6\pi\eta\ \mathrm{r}\ 2\mathrm{v}_0$ downwards

$= 12\pi\eta\ r\ v_0$ downwards

4. **(c)** $P_1 - P_0\ \ \ \frac{4S}{2R}$ .... (1)

$P_2 - P_1\ \ \ \frac{4S}{R}$ .... (2)



Add (1) and (2), $P_2 - P_0\ \ \ \frac{6S}{R}$

5. **(a)** Let ΔL be the elongation. Then, by Hooke's law,

$\frac{F}{A}\ \ \ Y\frac{\Delta L}{L}$ where Y is Young's modulus.

The elongation is $\Delta L\ \ \ \frac{1}{Y}\frac{F}{A}L\ \ \ \frac{mgL}{YA}$

$= \frac{(20)\ (9.8)\ (4.0)}{(196\times10^9)\ \pi\ (0.001)^2}$

$= 1.273\times10^{-3}\ \mathrm{m} = 1.273\ \mathrm{mm}$

6. **(a)** Newton's law of viscosity, $F = \eta A\frac{dv}{dy}$

Stress $= \frac{F}{A} = \eta\left(\frac{dv}{dy}\right) = \eta k\left(\frac{4y}{a^2} - \frac{3y^2}{a^3}\right)$

At $y = a$, stress $= \eta k\left(\frac{4}{a} - \frac{3}{a}\right)\ \ \ \frac{\eta k}{a}$

7. **(b)** The change in length of rod due to increase in temperature in absence of walls is

$\Delta\ell = \ell\ \ \ \Delta T = 1000\times10^{-4}\times20\mathrm{mm} = 2\mathrm{mm}$.

But the rod can expand upto 1001mm only.

At that temperature its natural length = 1002 mm.

$\therefore$ Compression = 1mm

$\therefore$ Mechanical stress $= Y\frac{\Delta\ell}{\ell} = 10^{11}\times\frac{1}{1000}$

$10^8\ \mathrm{N/m^2}$

8. **(a)** Equating the rate of flow, we have

$\sqrt{(2gy)}\times L^2 = \sqrt{(2g\times4y)}\ \pi R^2$

[Flow = (area) × (velocity), velocity = $\sqrt{2gx}$]

where x = height from top

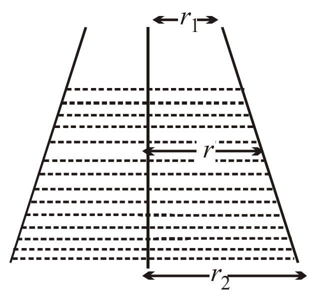
$\Rightarrow\ \ L^2 = 2\pi R^2$

$\Rightarrow R = \frac{L}{\sqrt{2\pi}}$

9. **(a)** Viscous force = $6\pi\eta\mathrm{rv}$

$= 6\pi\times18\times10^{-5}\times0.03\times100 = 101.73\times10^{-4}$ dyne

10. **(b)** Let the liquid rise to a height h.



$T\ \ \ \frac{h\rho rg}{2}\ \ \therefore\ \ hr\ \ \ \frac{2T}{\rho g}$

If the tube is of height $h_1 < h$

$h_1 r_1 = h\mathrm{r} = \frac{2T}{\rho g}$

$\therefore\ r_1\ \ \ \frac{2T}{h_1\rho g}\ \ \ \frac{2\times0.073}{\frac{25}{1000}\times1000\times9.8}\ \ \left\{25\mathrm{mm}\ \ \ \frac{25}{1000}\mathrm{m}\right\}$

$= \frac{2\times0.073}{9.8\times25}\ \ \ 0.0006\mathrm{m}\ \ \ 0.6\mathrm{mm}$

11. (b) $\Delta L \quad \frac{LF}{AY}$, where $L = 3$m,

$A = \pi\,(1.0 \times 10^{-3}\ \text{m})^2 = 3.14 \times 10^{-6}\ \text{m}^2$,

and since each wire supports one-quarter of the load,

$$F \quad \frac{(50\ \text{kg})\ (9.8\ \text{m/s}^2)}{4} \quad 123\ \text{N}$$

$$\Delta L \quad \frac{(3\text{m})\ (123\text{N})}{(3.14 \times 10^{-6}\text{m}^2)\ (1.8 \times 10^{11}\ \text{N/m}^2)}$$

$= 65 \times 10^{-5}$ m or 0.65 mm

12. (c) Pressure difference is largest between atmosphere and smaller bubbles. Hence radius of curvature (R) is smallest.

13. (d) $\frac{dF}{dx} \quad 0$ at $\ell/2$

14. (a) Net force on the ball $= W - F_u - F_v$

W = weight of ball, F_u = upthrust force,

F_v = viscous force

$$m\frac{dv}{dt} = W - F_u - 6\pi\eta r v$$

(where η is coefficient of viscosity)

$$m\frac{dv}{dt} = A - Bv, \text{ Here } A = W - F_u,\ B = 6\,\pi\eta r$$

$$\int_0^{(1-n)\,v_s} \frac{m}{A - Bv}dv \quad \int_0^t dt \qquad (v_s = \text{terminal velocity})$$

$$-\frac{m}{B}\ln\left[\frac{A - B\,(1-n)\,v_s}{A}\right] \quad t$$

At steady state, the net force is zero.

$$\therefore\ A - Bv_s = 0,\ v_s = \frac{A}{B}$$

$$t = -\frac{m}{6\pi\eta r}\ln\left[1 - \frac{1}{v_s}(1-n)\,v_s\right]$$

$$= -\frac{m}{6\pi\eta r}\ln(n) = -\frac{\frac{4}{3}\pi r^3\rho}{6\pi\eta r}\ln\,(0.01)$$

$$= \frac{2}{9} \times \frac{(1.5)^2 \times 10^{-6} \times 7.8 \times 10^3}{0.9 \times 10^{-1}}\ln 100$$

$$= \frac{2 \times 2.25 \times 7.8}{8.1} \times 10^{-2}\ln 100 \quad 0.2\ \text{sec}.$$

15. (b) As copper expands more, it will bend and strip comes in contact with lamp & bell.

16. (a) To find the minimum diameter, and hence minimum cross-sectional area, we assume that the force $F = 400$ N brings us to the elastic limit. Then from the stress, $F/A = 379 \times 10^6$ Pa, we get

$$A \quad \frac{400\text{N}}{379 \times 10^6\ \text{Pa}} \quad 1.0554 \times 10^{-6}\ \text{m}^2.$$

Then, $A \quad \frac{\pi D^2}{4}$

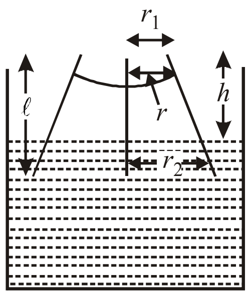
$$D^2 \quad \frac{4A}{\pi} \quad \frac{4\,(1.0554 \times 10^{-6}\ \text{m}^2)}{\pi} = 1.344 \times 10^{-6}$$

and $D = \sqrt{1.344 \times 10^{-6}\ \text{m}^2} = 1.16 \times 10^{-3}$ m $= 1.16$ mm

17. (d) ℓ decreases as the block moves up. h will also decreases because when the coin is in water it will displace a volume of water, equal to its own volume, whereas when it is on the block it displaces more volume than to own volume (because density of coin is greater than density of water).

18. (a) In tube A, the radius of the capillary tube

$$r = r_2 - \left(\frac{r_2 - r_1}{\ell}\right)x$$



At a height of 8cm,

$$r = 0.5 \times 10^{-3} - \frac{(0.5 - 0.25) \times 10^{-3}}{0.1} \times 8 \times 10^{-2}$$

$= 0.3 \times 10^{-3}$ m

Surface tension of liquid at 0°C

$$T_0 \quad \frac{hr\rho g}{2} \quad \frac{8 \times 10^{-2} \times 0.3 \times 10^{-3} \times 10^4 \times 9.8}{2 \times 14} \quad 0.084\ \text{N/m}$$

In an uniform tube B of radius r' at 0°C , the surface tension of liquid $T_0 = \frac{r'\rho h_0 g}{2}$

At 50°C, $T_{50} \quad \frac{r'\rho h_{50} g}{2}$

$$\frac{T_{50}}{T_0} \quad \frac{h_{50}}{h_0} \quad \frac{5.5 \times 10^{-2}}{6 \times 10^{-2}} \quad \frac{11}{12}$$

$$T_{50} \quad \frac{11}{12}T_0 \quad \frac{11}{12} \times 0.084 \quad 0.077\ \text{N/m}$$

Hence rate of change of surface tension with temperature

$$\frac{\Delta T}{\Delta\theta} \quad \frac{T_{50}-T_0}{\Delta\theta} \quad \frac{7.7\times10^{-2}-8.4\times10^{-2}}{50}$$

$= -1.4 \times 10^{-4}$ N/m°C

19. **(d)** $\Delta\ell = \frac{F\ell}{\pi r^2 y} \Rightarrow \Delta\ell \quad \frac{\ell}{r^2}$

Only option radius 3mm, length 2m is satisfying the given condition.

20. **(c)** Viscous force $F = -\eta\,(2\pi r\ell)\frac{dv}{dr}$,

where ℓ = length of the cylinder

$$\frac{dr}{r} = -\left(\frac{2\pi\eta\ell}{F}\right)dv$$

$$\int_{R_1}^{R_2}\frac{dr}{r} = -\int_{v_0}^{0}\frac{2\pi\eta\ell}{F}\,dv \Rightarrow \ln\left(\frac{R_2}{R_1}\right) \quad \frac{2\pi\eta\ell}{F}v_0 \quad .... (1)$$

Suppose velocity of liquid at a distance r is v,

$$\int_{R_1}^{r}\frac{dr}{r} = -\int_{v_0}^{v}\frac{2\pi\eta\ell}{F}\,dv \Rightarrow \ln\frac{r}{R_1} = \frac{2\pi\eta\ell}{F}(v_0 - v) .... (2)$$

$$\ln\left(\frac{R_2}{r}\right) \quad \frac{2\pi\eta\ell}{F}v$$

On solving eq. (1) and (2),

$$v = v_0\frac{\ln\,(R_2/r)}{\ln\,(R_2/R_1)} \Rightarrow v \quad v_0\frac{\ln\,(r/R_2)}{\ln\,(R_1/R_2)}$$

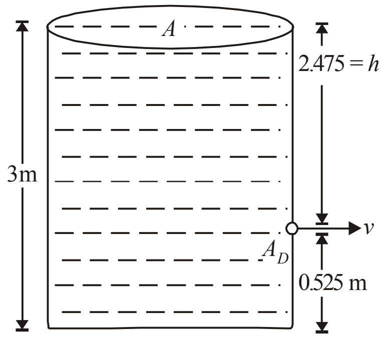
21. **(c)** Buoyant force = Effective weight of the displaced liquid and weight of displaced liquid depends on the acceleration due to gravity which is different for different planets.

22. **(a)** The square of the velocity of efflux

$$v^2 = \frac{2gh}{\sqrt{1-\left(\frac{a}{A}\right)^2}}$$

h = 3 – 0.525

or, $v^2 = \frac{2\times10\times2.475}{\sqrt{1-(0.1)^2}} = 50\ \text{m}^2/\text{s}^2$



23. **(b)** Upward force by capillary tube on top surface of liquid is $f_{up} = 4\sigma a\cos\theta$.

If liquid is raised to a height h then we use

$4\sigma a\cos = ha^2\rho g$ or $h \quad \frac{4\sigma\cos\theta}{a\rho g}$

24. **(d)** Pressure just below the meniscus of the left and the right limbs are $P_0 - \frac{2T}{r_1}$ and $P_0 - \frac{2T}{r_2}$

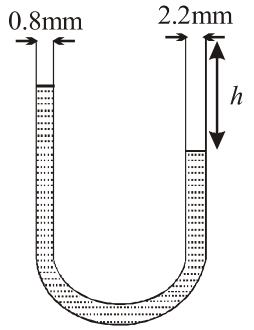
Given T = 28 dynes/cm = 28×10^{-3} N/m

$\rho = 0.8 \times 10^3$ kg/m^3

Hence pressure difference is $2T\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = \rho gh$

$$h = \frac{2\times28\times10^3}{0.8\times10^{-3}\times9.8}\left(\frac{1}{0.4\times10^{-3}}-\frac{1}{1.1\times10^{-3}}\right)$$

$= 11.36 \times 10^{-3}$ m

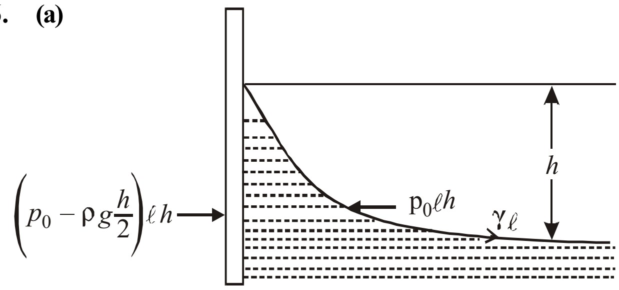


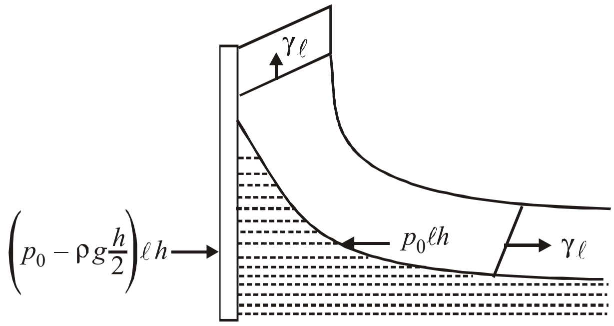
or difference in level of the oil in the two limbs, h = 11.4mm.

25. **(c)** The bulk modulus is defined as $B = -\frac{\Delta p}{\Delta V/V}$, where the minus sign is inserted because ΔV is negative when Δp is positive.

$$100\left|\frac{\Delta V}{V}\right| \quad 100\frac{\Delta p}{B} = 100\frac{345\times10^6}{138\times10^9} \quad 0.25\%$$

26. (a)





Balancing forces in horizontal direction

$$\left(p_0 - \rho g\frac{h}{2}\right)\ell h + \gamma\ell \quad p_0\ell h \Rightarrow h \quad \sqrt{\frac{2\gamma}{\rho g}}$$

27. (b) If surface tension is neglected, by the law of floatation
$mg = (a^2 x\rho)\, g$
where a is the side of cube, ρ is the density of water and m is the mass of cube.
$3.2\text{ g} = 0.2^2 \times (10^3)\text{g}$
The height to which the cube is immersed

$$x \quad \frac{3.2}{(0.2)^2 \times 10^3} \quad 0.08\text{m}.$$

Since water wets the cube, the angle of contact is zero and the force of surface tension acts vertically downwards. So it is buoyed down by surface tension
$mg + 4aT = a^2 x'\rho g$, where T is surface tension of water
$3.2\text{g} + 4\,(0.2)\,(0.07) = (0.2)^2\, x'\,(10^3)\,\text{g}$

$$x' \quad 0.08 \quad \frac{4\,(0.2)\,(0.07)}{(0.2)^2(10^3)\,(9.8)} = (0.08 + 1.4 \times 10^{-4})\text{ m}$$

The additional distance it is buoyed down by surface tension $= 1.4 \times 10^{-4}$ m

28. (a) $v \quad \sqrt{2gx} \quad \frac{\ell}{t} \quad \frac{1}{t}$

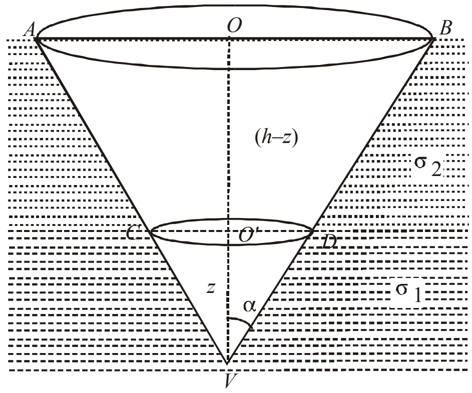
For vertical motion $x \quad \frac{1}{2}gt^2$

$\Rightarrow x = 0.25\text{m}$ (i.e. level goes down from 0.81m to 0.25m.)
Using equation of continuity

$$\sqrt{5}.\frac{dx}{dt} = \sqrt{2gx}.4 \times 10^{-4}$$

On solving $t = 1000$s

29. (b) VAB is the given cone. Let its height be h and semi-vertical angle α. Let the base AB of the cone be in the surface. CD is the surface of separation of two liquids, O and O' are the centres of the base AB and surface of separation CD.



Let $VO' = z$ then $OO' = h - z$
$\therefore$ The weight of the cone = (vol. of the cone) ρg

$$= \frac{1}{3}\pi h^3 \tan^2\alpha\, \rho\, g$$

Volume of liquid (of density σ_1) displaced

$$= \text{volume of cone } (VCD) = \frac{1}{3}\pi z^3 \tan^2\alpha$$

and volume of liquid of density σ_2 displaced = volume of the frustum $ABDC = \left(\frac{1}{3}\pi h^3 \tan^2 - \alpha\right) - \left(\frac{1}{3}\pi z^3 \tan^2 - \alpha\right)$

$\therefore$ For equilibrium,
weight of the cone = (weight of liquid of density σ_1 displaced) + (weight of liquid of density σ_2 displaced)

$$\text{or } \frac{1}{3}\pi h^3 \tan^2\alpha\rho g = \frac{1}{3}\pi z^3 \tan^2\alpha\sigma_1 g$$

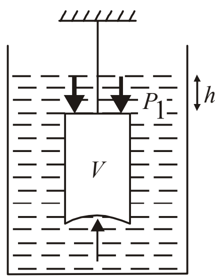
$$+ \frac{1}{3}\pi\,(h^3 - z^3)\tan^2\alpha.\sigma_2 g$$

$$\text{or } h^3\rho = z^3\sigma_1 + (h^3 - z^3)\,\sigma_2$$

$$\text{or } h^3(\rho - \sigma_2) = z^3(\sigma_1 - \sigma_2)$$

$$\text{or } z \quad h\left(\frac{\rho - \sigma_2}{\sigma_1 - \sigma_2}\right)^{1/3}.$$

30. (d) According to Archimedes principle
Upthrust = Wt. of fluid displaced



$F_{bottom} - F_{top} = V\rho g$
$\therefore$ $F_{bottom} = F_{top} + V\rho g$
$= P_1 \times A + V\rho g$
$= (h\rho g) \times (\pi R^2) + V\rho g$
$= \rho g\,[\pi R^2 h + V]$

31. (a) The horizontal force on the curved surface BC equals the force on the projected area normal to x-axis, i.e. $OC \times 2 = 1 \times 2 = 2\text{m}^2$

The depth of the centroid of this projected area is 0.5m below the top edge of water. But the pressure of 19.62 kN/m^2 is equivalent to a head of

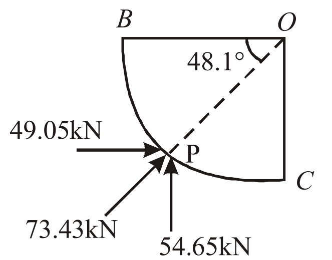
$$\frac{19.62 \times 1000}{1000 \times 9.81} = 2\text{m of water}$$

which makes $\bar{z}$ to be $(0.5 + 2)$, i.e., 2.5m.

The horizontal force is, therefore,

$F_x = 1000 \times 9.81 \times 2.5 \times 2 = 49050\,\text{N} = 49.05\,\text{kN}$

The vertical force on the curved surface BC equals the weight of the liquid virtually supported by it.



$F_z = (\pi \times 1^2/4 + 1 \times 2) \times 2 \times 1000 \times 9.81$

$= (0.785 + 2) \times 19620 = 54649.5\,\text{N}$

$= 54.65$ kN upwards

The second term in the parenthesis represents the column of water equivalent to the pressure of 19.62 kN/m^2 acting on the water.

The resultant force on the curved surface is

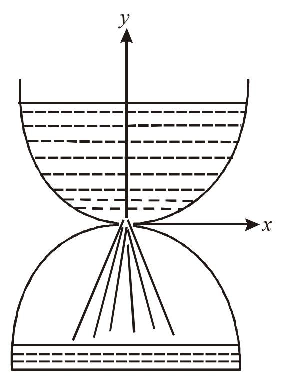
$$F \quad \sqrt{F_x^2 \quad F_z^2} \quad \sqrt{49.05^2 \quad 54.65^2} \quad 73.43\text{ kN}$$

indicated at $\alpha = \tan^{-1}(54.65/49.05)$

$= \tan^{-1}(1.114) = 48.1°$ with the horizontal

The resultant force must pass through the centre O since this is a circular curved surface.

32. (a) Let us introduce the coordinate system depicted in figure. According to Torricelli's formula, the outflow velocity of a liquid is $V \quad \sqrt{2gy}$, where y is the thickness of the water layer in the upper vessel. Since water is incompressible, $aV = Av$, where v is the velocity with which the upper layer on the water lowers, A is its area, and a is the area of the orifice.



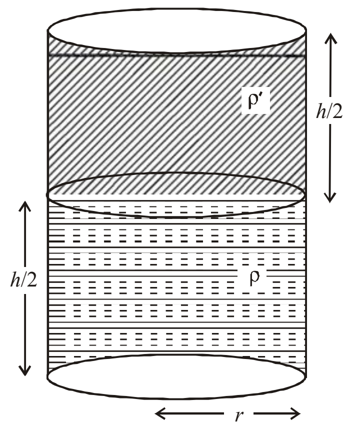
If we assume that the vessel is axially symmetrical, then $A = \pi x^2$, where x is the horizontal coordinate of the vessel wall. Therefore, $\dfrac{\pi x^2}{\sqrt{2gy}} \quad \dfrac{a}{v}$ const.

since in conformity with the initial condition, the water level should lower with a constant velocity. Hence, the shape of the vessel can be determined from the equation $y = kx^4$.

where $k \quad \dfrac{\pi^2 v^2}{2ga^2}$

33. (d) Pressure at C.G. of the base

$$= g\rho'.\frac{1}{2}h + g.\rho\frac{1}{2}h \qquad \text{.......(1)}$$



$\therefore$ Pressure on the base of the cylinder

= (pressure at the C.G. of the base) × area of the base

$$= \left(g\rho'.\frac{1}{2}h + g\rho.\frac{1}{2}h\right).\pi r^2 = \frac{1}{2}\pi r^2(\rho' + \rho)\,gh$$

The whole pressure on the curved surface of the cylinder = the whole pressure on the upper half + the whole pressure on the lower half of the curved surface.

The whole pressure on the upper half of the curved surface

= (pressure at the C.G. of the upper half) × area of the upper half

$$= \left(g\rho'.\frac{1}{4}h\right) \times \left(2\pi r.\frac{1}{2}h\right) = \frac{1}{4}\pi r h^2 g\rho' \text{ .... (2)}$$

The whole pressure on the lower half of the curved surface = (pressure at the C.G. of the lower half) × area of this lower half

$$\left(g\rho'.\frac{1}{2}h + g\rho.\frac{1}{4}h\right) \times \left(2\pi r.\frac{1}{2}h\right)$$

$$= \frac{1}{4}\pi r h^2(2\rho' + \rho)\,g \qquad \text{.... (3)}$$

$\therefore$ From (2) and (3), the whole pressure on the curved surface

$$= \frac{1}{4}\pi rh^2\rho' g + \frac{1}{4}\pi rh^2(2\rho' + \rho)g$$

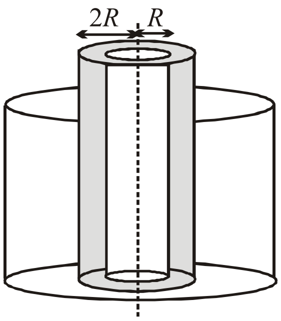
$$= \frac{1}{4}\pi rh^2(3\rho' + \rho)g \qquad \text{... (4)}$$

$\therefore$ From (1) and (4),

$$\frac{\text{The whole pressure on the base}}{\text{The whole pressure on the curved surface}}$$

$$= \frac{\frac{1}{2}\pi rh^2(\rho' + \rho)gh}{\frac{1}{4}\pi rh^2(3\rho' + \rho)g} \quad \frac{2r\,(\rho' + \rho)}{h(3\rho' + \rho)}$$

34. (b)



$$mg = v\rho_{wood}\, g = (3\pi R^2 h)\,\rho_{wood}\, g$$

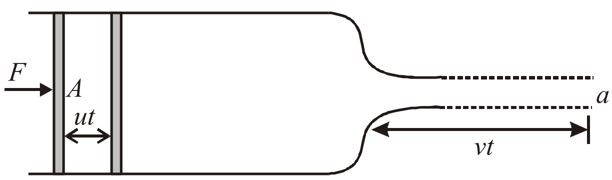
$$(3\pi R^2 h)\,\rho_{wood}\, g = (3\pi R^2 H)\,\rho_w g$$

$$\frac{h\rho_{wood}}{\rho_{water}} \quad H$$

35. (a) The initial velocity of the water with respect to the blade is $v = \sqrt{2gh} - \omega R$. Therefore, a mass of water $m = \rho A\,(\sqrt{2gh} - \omega R)$ impinges on the blade in a unit of time. After the impact, the velocity of the water with reference to the blade is zero, and for this reason the change in the momentum of the water in a unit time is mv. According to Newton's second law, the sought force is

$$F = \rho A\,(\sqrt{2gh} - \omega R)^2.$$

36. (a) The piston will cover the distance ut during the time t (figure). The force F will perform the work $W = Fut$. The mass of the liquid flowing out during the time t is ρAut.



The outflow velocity of the liquid v can be found from the equation $Au = av$. The change in the kinetic energy of the liquid during the time t is

$$\rho Aut\left(\frac{v^2}{2} - \frac{u^2}{2}\right)$$

This change should be equal to the work performed by the force F.

$$F\,ut = \rho Aut\left(\frac{v^2}{2} - \frac{u^2}{2}\right)$$

Upon eliminating u, we find that

$$v^2 = \frac{2F}{A\rho} \times \frac{1}{1 - \frac{a^2}{A^2}}\ .$$

If $a << A$, then $v \quad \sqrt{\frac{2F}{A\rho}}$

37. (a) Let the control volume be $ABCD$. The only external force acting on it is the spring reaction R_x and the mass the momentum leave normal to CD.

The mass flow rate $m = 1000 \times \frac{\pi}{4} \times 0.05^2 \times 5$

$= 9.82$ kg/s

$v_1 = 0,\ v_2 \cos\theta = 5 \cos 45° = 3.54$ m/s

Applying the momentum equation in the x-direction,

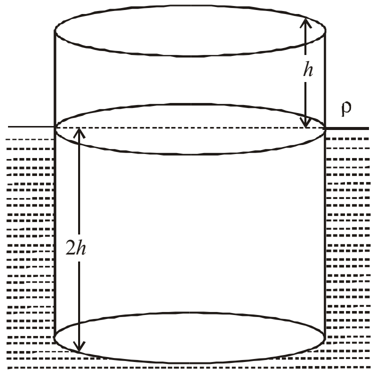
$R_x = 9.82 \times 3.54 = 34.7$ N

Force acting on the spring $= -R_x = -34.7$ which could tend to compress it.

Compression of the spring $= \frac{34.7}{20} \quad 1.74$ cm.

38. (a) Let A be the area of the cross-section of the cylinder and $3h$ its height. Let ρ_1 be the density of water.

Then the weight of the cylinder $= A.3h\sigma\rho_1$g.



Then weight of air displaced $= Ah.\rho\rho_1 g$, since the length of cylinder in air is $\frac{1}{3}(3h)$ i.e. h.

The weight of water displaced $= A.2h\rho_1 g$

For equilibrium, $A.3h\sigma\rho_1 g = Ah\rho_1 g + A.2h\rho_1 g$

or $3\sigma = \rho + 2$.

39. **(d)** Let the density of water be ρ, then the force by escaping liquid on container = $\rho S(\sqrt{2gh})^2$

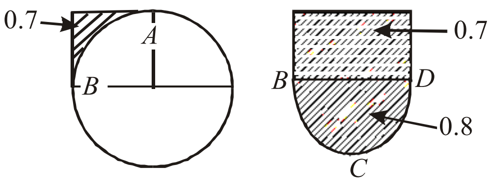
$\therefore$ Acceleration of container

$$a \quad \frac{2\rho Sgh - \mu\rho Vg}{\rho V} \quad \left(\frac{2Sh}{V} - \mu\right) g$$

Now, $\mu \quad \frac{Sh}{V} \qquad \therefore \; a \quad \frac{Sh}{V} g$

40. **(b)** In order to determine its specific gravity we are only concerned with the vertical force on the cylinder.

On surface AB, the vertical force equals the weight of the oil actually supported by it.



$$F_1 = \left(0.5 \times 0.5 - \pi \times 1 \times \frac{1}{16}\right) \times 2 \times \rho g$$

$F_1 = 0.054 \times 2 \times 1000 \times 0.7 \times 9.81 = 741.64$ N downwards.

On surface BCD, the vertical force equals the virtual weight of oil supported by it

$$F_2 = \left(1 \times 0.5 \times 0.7 + \pi \times 1\frac{1}{8} \times 0.8\right) \times 2 \times \rho_{water} \times g$$

$F_2 = 0.664 \times 2 \times 1000 \times 9.81 = 13027.7$ N

Net hydrostatic force on the cylinder

$F_2 - F_1 = 13027.7 - 741.64 = 12286$ N upwards

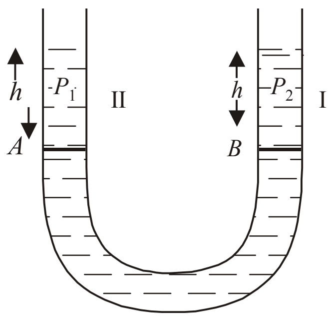
This must be equal to the weight of the cylinder

$$\rho_c \times 9.81 \times \pi \times 1 \times \frac{1}{4} \times 2 = 12236,$$

where, $\rho_c = 797.3 \text{ kg/m}^3$

The specific gravity of the material should, therefore, be 0.797.

41. **(b)** The level of liquid is same in both the limbs. Pressure in limb I at B = Pressure in limb II at A



$h\rho_1 g = h\rho_2 g$

$\Rightarrow \; \rho_1 = \rho_2$

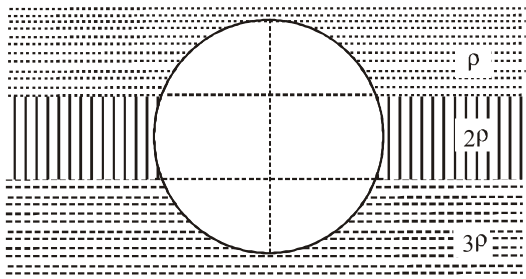
42. **(a)** Let 'a' be the radius of the sphere and σ its density. Let the densities of three liquids be ρ, 2ρ and 3ρ.

$\therefore$ The weight of the sphere = $\frac{4}{3}\pi a^3 \sigma g$.

The volume of the uppermost liquid displaced = the volume of the lowermost liquid displaced, by symmetry

$$= \frac{1}{3}\pi(d_2 - d_1)\,[3r^2 - (d_1^2 \quad d_1 d_2 \quad d_2^2)]$$

$$= \frac{1}{3}\pi\left(a - \frac{a}{3}\right) \times \left[3a^2 - \left(\frac{a^2}{9} \quad \frac{a^2}{3} \quad a^2\right)\right]$$



[since the distances of the two parallel faces from the centre are a and $a/3$ in this case]

$$= \left(\frac{1}{3}\pi - \frac{2a}{3}\right) \times \left[3a^2 - \frac{13}{9}a^2\right] \quad \frac{28\pi a^3}{81}$$

$\therefore$ The volume of the middle liquid (of density 2ρ) displaced

= volume of the whole sphere – 2 (volume of the upper most liquid displaced)

$$= \frac{4}{3}\pi a^3 - \frac{56}{81}\pi a^3 = \frac{52}{81}\pi a^3$$

$\therefore$ The weight of liquid of density ρ displaced

$$= \frac{23}{81}\pi a^3 \rho g,$$

The weight of liquid of density 2ρ displaced

$$= \frac{52}{81}\pi a^3 2\rho g$$

and the weight of liquid of density 3ρ displaced

$$= \frac{28}{81}\pi a^3 3\rho g$$

For equilibrium, the weight of the sphere = the sum of the weights of liquids displaced

$$\text{or} \quad \frac{4}{3}\pi a^3 \sigma g = \frac{28}{81}\pi a^3 \rho g + \frac{52}{81}\pi a^3 .2\rho g + \frac{28}{81}\pi a^3 .3\rho g$$

or $\quad 108\sigma = 8\rho + 104\rho + 84\rho = 216\rho$

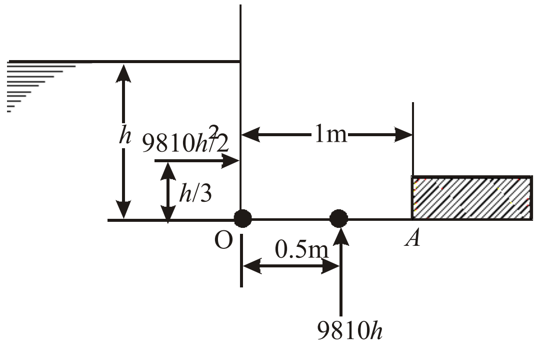
or $\sigma = 2\rho$.

43 . (d) Consider the forces per unit width of the gate. For the level of water, h metre above the hinge,
Force on horizontal part OA equals
$1000 \times 9.81 \times h \times (1 \times 1) = 9810h$ N vertically upwards.
This force acts at a distance of 1/2 m from the hinge.
Force on the vertical part of the gate equals

$$1000 \times 9.81 \times \frac{h}{2} \times (h \times 1) = 9810\,\frac{h^2}{2}.$$

This force acts at $\frac{h}{3}$ from the hinge.

Tipping would occur when the overturning clockwise moment just exceeds the resisting anticlockwise moment about the hinge O.



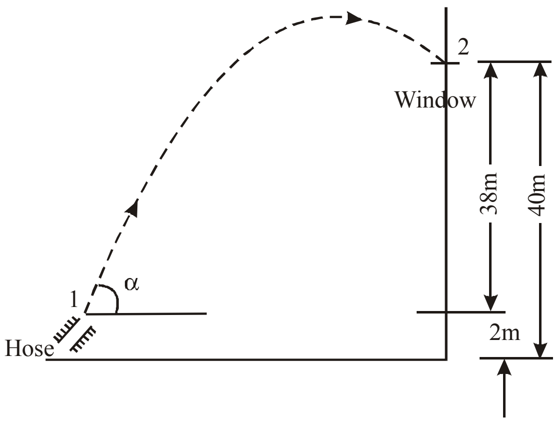
$$9810 \times \frac{h^2}{2} \times \frac{h}{3} = 9810 \times h \times \frac{1}{2}$$

or $h \quad \sqrt{3} = 1.732$m

44. (c) $A_1v_1 = A_2v_2 + A_3v_3$
$A \times 3 = A \times 1.5 + 1.5A \times v_3$
$\Rightarrow v_3 = 1$ m/s

45. (a) $v_1 \quad \frac{m}{\rho A} \quad \frac{30}{1000 \times \pi / 4 \times (0.03)^2} \quad 42.4$ m/s

($m \rightarrow$ mass flow rate)



(Note that $v_1^2/2g$ should be more than the height of the window)
$v_1^2/2g = 92\text{m} > 40\text{m}$)
The question requires an optimisation of α for the maximisation of x to achieve the desired height of z of 38m.

$$38 = x\tan\alpha - \frac{9.81}{2 \times 42.5^2}\frac{x^2}{\cos^2\alpha} \quad \text{...... (1)}$$

Differentiating each w.r.t. α

$$0 = x\sec^2\alpha + \tan\alpha\frac{dx}{d\alpha} - 0.0027x^2.2\sec^2\alpha\tan\alpha$$

$$-\frac{0.0027}{\cos^2\alpha}.2x\frac{dx}{d\alpha}$$

Maximum x for α demands that $\frac{dx}{d\alpha} \quad 0$

hence $x\sec^2\alpha - 2 \times 0.0027x^2\sec^2\alpha\tan\alpha = 0$

or $x\tan\alpha \quad \frac{1}{0.0054} \quad 185$ .... (2)

Solving (1) and (2) simultaneously

$$38 = 1.85 - \frac{0.0027x^2}{\cos^2\alpha}$$

$$\frac{x^2}{\cos^2\alpha} \quad 54513 \qquad \text{or} \quad \frac{x}{\cos\alpha} \quad 233$$

But $x\tan\alpha \quad \frac{x\sin\alpha}{\cos\alpha} \quad 185$

So, $\sin\alpha \quad \frac{185}{233}$, $\alpha = 52.4°$ and $x = 142$m.

46. (b) The velocity of water outflow from a hole is $v \quad \sqrt{2gh}$.
The impulse of the force acting from the side of the vessel on the outflowing water $F\Delta t = \Delta mv$, where $\Delta m = \rho Av\Delta t$ is the mass of the water flowing out during the time Δt. Hence, $F = \rho v^2A = 2\rho ghA$. The pressure at the bottom $p = \rho gh$ and therefore $F = 2pA$. The same force acts on the vessel from the side of the stream.
Thus, the water acts on the wall with the hole with a force 2pA smaller than that acting on the opposite wall, and net with a force smaller by pA as might be expected. This is due to a reduction in the pressure acting on the wall with the hole, since the water flows faster at this wall.
The vessel will begin to move if $\mu G < 2pA$ or

$\mu \quad \frac{2\rho ghA}{G}$ ($\mu \rightarrow$ coefficient of friction)

47. (d) Area of cross section of the large piston

$$A = \pi \times \frac{(0.05)^2}{4} = 0.00196\text{m}^2$$

Pressure of the hydraulic fluid under the piston should be such as to balance the force applied on the piston

$$p \quad \frac{200 \times 1000}{0.00196} \quad 10.2 \times 10^6 \text{ N/m}^2$$

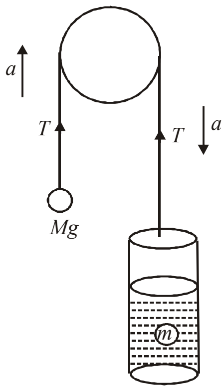
By Pascal's law, pressure is transmitted undiminished in all directions. The pressure at the bottom of the small piston must be 10.2×10^6 N/m^2. The force exerted on the small piston must be

$$10.2 \times 10^6 \times \pi \times \frac{(0.015)^2}{4} = 18000\,\text{N}$$

The force desired to be exerted at the handle of the level should be

$$F = 18000 \times \frac{25}{300} = 1500\,\text{N}$$

48. (a) Since mass M over the pulley balances the bucket with water, hence the mass of bucket with water is M, which becomes $(M + m)$ when a cork of mass m is tied to the bottom of the bucket.



Let in this case the acceleration of the system be a, then for the bucket with cork we have the equation of motion

$(M + m)\,a = (M + m)\,g - T$ .... (1)

And for the mass M, $Ma = T - Mg$ .... (2)

where T is the tension in the string round the pulley.

Adding (1) and (2), $a \quad \frac{mg}{(m \quad 2M)}$ .... (3)

The forces acting on the cork are :

(i) its weight mg acting vertically downwards.

(ii) the force of buoyancy, acting vertically upwards.

(iii) the tension T' in the string tied to the cork which also acts downwards in the vertical direction as the other two forces are vertical.

Now the mass of the cork = m and its sp. gr. = σ. Let ρ be the density of water, then the mass of the liquid displaced by the cork = (its volume) × ρ = $\frac{m}{\rho\sigma}.\rho \quad \frac{m}{\sigma}$

But this mass is descending with acceleration a, and if P be the upwards force on this mass, we have

$$\frac{m}{\sigma}.a = \frac{m}{\sigma}g - P \qquad \text{or} \quad P = \frac{m}{\sigma}(g - a)$$

The upward thrust on the cork = $P = \frac{m}{\sigma}(g - a)$

For the motion of the cork in space

$$m \times a = mg + T' - \frac{m}{\sigma}(g - a)$$

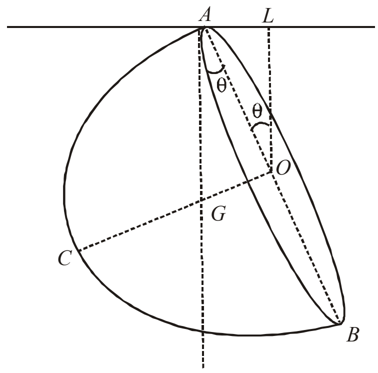
$$T' = ma - mg + \frac{m}{\sigma}(g - a)$$

$$= m\,(g - a)\left(\frac{1}{\sigma} - 1\right) = m\left(g - \frac{mg}{m + 2M}\right)\left(\frac{1}{\sigma} - 1\right)$$

[using eq. (3)

$$= m\left(\frac{2Mg}{m + 2M}\right)\left(\frac{1}{\sigma} - 1\right)$$

49. (a)



Let G be the C.G. of the hemisphere full of liquid (solid hemisphere) and O the centre of the base of the hemisphere. Then $OG \quad \frac{3}{8}a$, where a is the radius of the hemisphere. Let the hemisphere be suspended from the point A. Then the hemisphere (full of liquid) is in equilibrium under the action of two force viz. wt. of liquid acting at G and the force at A. Hence A and G must be in the same vertical line. Let OA i.e. the base of the hemisphere be inclined to the vertical at an angle θ.

Then, $\tan\theta \quad \frac{OG}{OA} \quad \frac{3a/8}{a} \quad \frac{3}{8}$

or $\tan\theta \quad \frac{3}{8}$ .... (1)

Also the depth of C.G. of the base = $OL = AO$ $\cos\theta$ = $a\cos\theta$.

$\therefore$ The thrust on the base of the hemisphere $= w.z.S = w.OL.\pi a^2 = w.a\cos\theta\;\pi a^2$,

where w is the weight per unit volume of the liquid.

Also the weight of the liquid contained = $\frac{2}{3}\pi a^3.w$

And from (1),

$$\cos\theta \quad \frac{1}{\sqrt{\sec^2\theta}} \quad \frac{1}{\sqrt{1 + \tan^2\theta}} \quad \frac{1}{\sqrt{1 \quad \frac{9}{64}}} \quad \frac{8}{\sqrt{73}}$$

$$\therefore \frac{\text{The thrust on the plane base}}{\text{wt. of the contained liquid}} \quad \frac{w.a\cos\theta.\pi a^2}{\frac{2}{3}\pi a^3 w} \quad \frac{3\cos\theta}{2}$$

$$= \frac{3 \times 8}{2\sqrt{73}} \quad \frac{12}{\sqrt{73}}$$

50. (b) Let V be the volume of iceberg and let x be the fraction of volume above water.
Using law of floatation, weight of floating body = weight of liquid displaced by part of the floating body inside the liquid.
Therefore, $V\rho_{ice}g = (1-x)\,V\rho_{water}g$.
Using the value of ρ_{ice} and ρ_{water}, we get $x = (13/103)$.

51. (d) $p_{s_1} = 50$ cm. mercury $= \dfrac{50}{100} \times 13.6 = 6.8$m WG
($WG \to$ water gauge)

$p_{s_2} \quad p_{s_1} = 6.8$m WG

$p_{s_3} = 6.8 - 0.5 = 6.3$m WG

Equating p_{s_4} to p_{s_3} etc.
$p_s = 6.3 \times 2 = 12.6$m WG

$$p_{pipe} = 12.6 + \frac{50}{100} \times 13.6 - 0.0012 \times \frac{90}{100} = 19.4 \text{ m WG}$$
$= 19.4 \times 1000 \times 9.81 = 190 \times 10^3$ N/m^2
$= 190$ kN/m^2 (gauge)
The pressure, 19.4m WG equals 1.43m mercury gauge. Allowing, say, 10 cm. to stay in the bottom U-space, the single U-tube mercury manometer would be 153cm. long.

52. (b) Let the control volume be bounded by a surface abcd cutting through the hand of the fireman. Take the x-axis along the flow and neglect the weight of the nozzle assembly and the water it contains. Applying the Bernouli equation between stations 1 and 2, neglecting their datum difference,

$$\frac{p_1}{\rho} \quad \frac{v_1^2}{2} \quad \frac{p_2}{\rho} \quad \frac{v_2^2}{2}$$

Since $(p_1 - p_2) = p_{1\,gauge} = 7 \times 9.81 \times 100^2 = 686700$ N/m^2

$$\frac{p_1 - p_2}{\rho} \quad \frac{686700}{1000} \quad 686.7 \text{ m}^2/\text{s}^2$$

$$v_2^2 - v_1^2 = 2 \times 686.7 \quad 1373.4 \text{ m}^2/\text{s}^2$$

But by continuity $\dfrac{v_2}{v_1} \quad \left(\dfrac{6}{2}\right)^2 \quad 9$

Hence $(81 - 1)\,v_1^2 = 1373.4$, $v_1^2 = 17.17$ m^2/s^2
$v_1 = 4.14$ m/s, $v_2 = 37.26$ m/s
Mass flow rate,

$$m = 1000 \times \frac{\pi}{4} \times 0.06^2 \times 4.14 \quad 11.71 \text{ kg/s}$$

Applying the momentum equation (i.e. R_x = mass flow rate $(v_2 - v_1) - (p_1 - p_2)\,A$) between stations 1 and 2, reaction R_x along the x-direction,

$$R_x = 11.71 \times (37.26 - 4.14) - 686700 \times \frac{\pi}{4} \times 0.06^2$$
$= -1554$ N
The force F_x exerted by the nozzle on the fireman is equal and opposite to it.
$F_x = 1554$ N along the flow.

53. (a) Let r be the radius of the base of the hemisphere or cone then the height of the base $= 2r$ (given)
i.e. $VK = 2r$ or $VL = LK = r$
Also from similar triangles VLF and VKB, we have

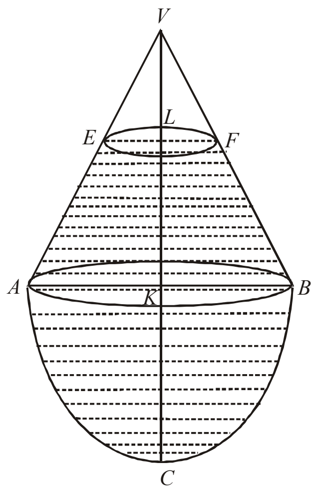
$$\frac{LF}{KB} \quad \frac{VL}{VK} \quad \frac{r}{2r} \quad \frac{1}{2} \quad \text{or } LF = \frac{1}{2}KB = \frac{1}{3}r$$

$\therefore$ The volume of the frustum

$$ABFE = \frac{\pi}{3}h\,(r_1^2 \quad r_2^2 \quad r_1 r_2)$$

$$= \frac{1}{3}\pi r\left[r^2 \quad \left(\frac{1}{2}r\right)^2 \quad r\left(\frac{1}{2}r\right)\right] \quad \frac{7}{12}\pi r^3$$

And the volume of the hemisphere $= \dfrac{2}{3}\pi r^3$



$\therefore$ Weight of the liquid contained in the vessel = (volume of the frustum + volume of the hemisphere) g ρ

$$= \left(\frac{7}{12}\pi r^3 + \frac{2}{3}\pi r^3\right) g\rho = \frac{5}{4}\pi r^3 g\rho\text{, where } \rho \text{ is the}$$

density of the liquid.
Now the resultant vertical thrust on the vessel, which is partly pressed upwards and partly downwards
= weight of the liquid contained

$$= \frac{5}{4}\pi r^3 g\rho = \frac{15}{8}\left(\frac{2}{3}\pi r^3 g\rho\right) = \frac{15}{8} \text{ (the weight of the}$$

liquid that the hemisphere can hold)

54. (a) Weight of cylinder = Upthrust due to upper liquid + Upthrust due to lower liquid.

$$D\left(\frac{A}{5} \times L \times g\right) \quad d\left(\frac{A}{5}\right)\left(\frac{3}{4}L\right)g \quad 2d\left(\frac{A}{5}\right)\left(\frac{L}{4}\right) \times g$$

$$\therefore \quad D = \frac{5d}{4}$$

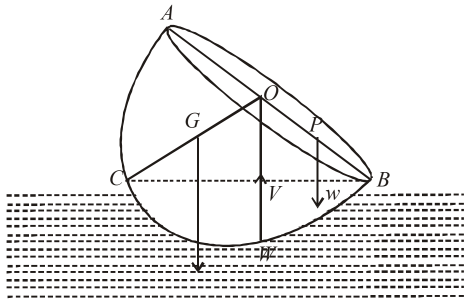
55. **(b)** $T + 0.8 \times 250 \times 10^{-3} g = 250\, d_\ell g$

$T + 250\, d_\ell g = 1.2 \times 250 \times 10^{-3} g$

Solving, $T = 0.5$ N

56. **(c)** The weight w is placed at P, such that $OP = c$ (given)

The fluid thrust at each point of the curved surface being normal to the surface passes through the centre O of the base.



Hence the resultant fluid thrust V on the curved surface in contact with the fluid will be passing through O.

The hemisphere is in equilibrium under the action of the following forces,

(i) Its weight W acting vertically downwards through G, the C.G. of the hemisphere where $OG = 3a/8$.

(ii) The force of buoyancy V, acting vertically upwards through O, and

(iii) The weight w placed at P, acting vertically downwards.

Let θ be the inclination of the axis OC of the hemisphere to the vertical.

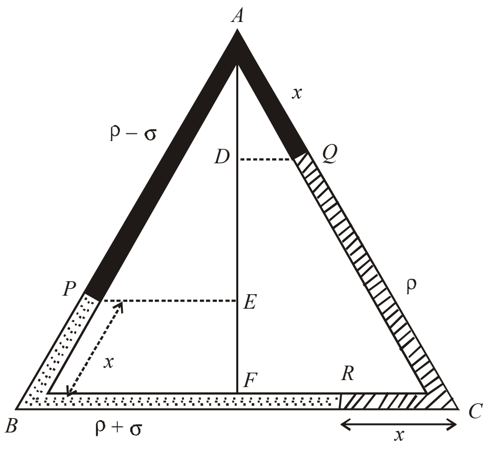
Taking moment of these forces about O, we get

$W.OG \sin\theta = w.OP \sin(90° - \theta)$

or $W.\dfrac{3a}{8}\sin\theta = w.c.\cos\theta$

or $\tan\theta \quad \dfrac{8cw}{3aW}$

57. **(c)** ABC is the given closed tube. Let a be the length of each side of the tube. P, Q and R are the points on the surfaces of separation of the liquids. Let the densities of the liquids be $(\rho - \sigma)$, ρ and $(\rho + \sigma)$. The portions PAQ, QCR and RBP be filled with liquids of densities $\rho - \sigma$, ρ and $\rho + \sigma$ respectively.



Let $AQ = x = CR = BP$,

then $AP = BR = CQ = a - x$

Pressure at B, due to the liquids in the tube

$APB = g(\rho - \sigma) AE + g(\rho + \sigma).EF$

$= g(\rho - \sigma) AP \cos 30° + g(\rho + \sigma) BP \sin 60°$

$= g(\rho - \sigma)(a - x)\dfrac{1}{2}\sqrt{3} + g(\rho + \sigma) x \dfrac{1}{2}\sqrt{3}$

And pressure at C, due to the liquids in the tube AQC

$= g(\rho - \sigma) AQ \cos 30° + g\rho\, QC \sin 60°$

$= g(\rho - \sigma) x \dfrac{1}{2}\sqrt{3} + g\rho(a - x)\dfrac{1}{2}\sqrt{3}$

Since B and C are in the same horizontal line, so the pressure at B = pressure at C

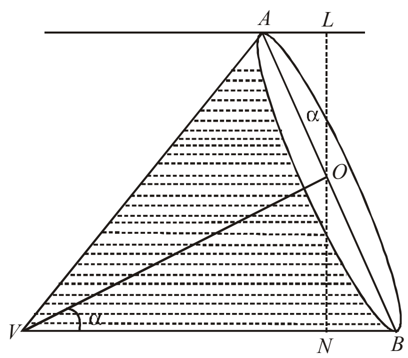
$$g(\rho - \sigma)(a - x)\frac{1}{2}\sqrt{3} + g(\rho + \sigma) x \frac{1}{2}\sqrt{3}$$

$$= g(\rho - \sigma) x \frac{1}{2}\sqrt{3} + g\rho(a - x)\frac{1}{2}\sqrt{3}$$

or $(\rho - \sigma)(a - x) + (\rho + \sigma)x = (\rho - \sigma)x + \rho(a - x)$

or $3\sigma x = \sigma a$ or $x = \dfrac{1}{3}a$

58. **(b)** V is the vertex of the cone and VB is the generator in contact with the horizontal table.



A is the highest point of the cone and the horizontal plane passing through A is the level of the free-surface. Let α be the semi-vertical angle of the cone, then $\angle AOL = \alpha$. Let h be the height of the cone, then the radius of the base of the cone = $h \tan\alpha$.

Also the depth of O, the C.G. of the base of the cone below the free surface

$= OL = AO \cos\alpha = h \tan\alpha \cos\alpha = h \sin\alpha$

Thrust on the base of the cone

$= wzs = w.OL.\pi(h \tan\alpha)^2$

$= \pi w h \sin\alpha\, h^2 \tan^2\alpha = 3\sin\alpha \left[\dfrac{1}{3}\pi h^3 \tan^2\alpha.w\right]$

$= 3\sin\alpha$ [weight of the liquid contained in the cone]

59. (c) Let σ be the density of the gas, then that of the air is 15σ. Then the weight of the balloon = weight of the gas + weight of the envelope = $Vg\sigma + w$.
If f be the required acceleration of the balloon acting vertically upward and then from "mass × acceleration = forces acting in the sense of acceleration" we get

$\frac{(Vg\sigma \quad w)}{g} \times a$ = force of buoyance – wt. of the balloon with gas $= V\,15\sigma g - (Vg\sigma + w)$

or $a = \left(\frac{14Vg\sigma - w}{Vg\sigma + w}\right) \times g$

60. (a) As net force on the sphere is zero therefore centre of mass does not move. But as there is a torque on sphere due to mg and buoyant force in clockwise direction there sphere rotates clockwise and hence equilibrium is achieved when O comes directly above C.

61. (a) Density is maximum at 4°C.

62. (a) For equilibrium
$4dA\ell g = (dAL + m)\,g$

and $\frac{\ell}{2} \quad Y_{cm}$ (for rotational equilibrium)

$$Y_{cm} \quad \frac{m \times 0 \quad dAL(L/2)}{m \quad dAL} \quad \frac{dAL^2}{2\,(m \quad dAL)}$$

$$\ell > \frac{AL^2 d}{(ALd \quad m)} \Rightarrow \frac{m \quad dAL}{4dA} \quad \frac{AL^2 d}{(ALd \quad m)}$$

$m > ALd$

63. (b) If T be the tension in the rope, then
$2T = 10$kN
$T = 5$kN.
∴ Longitudinal stress in the rope

$$\sigma \quad \frac{T}{A} \quad \frac{5\text{kN}}{10^3\,\text{mm}^2} \quad 5\ \text{Nmm}^{-2}$$

∴ Extension in the rope

$$= \frac{\text{Stress}}{Y} \times L = \frac{5\ \text{Nmm}^{-2}}{10^3\ \text{Nmm}^{-2}} \times 1500\text{mm} = 7.5\text{mm}$$

∴ Deflection of the load $\delta = \frac{7.5}{2} = 3.75$mm

64. (b) $$\frac{\Delta\ell_{steel}}{\Delta\ell_{brass}} \quad \frac{\frac{3Mg}{A_s Y_s}\ell_s}{\frac{2Mg}{A_b Y_b}\ell_b} \quad \frac{3}{2}\frac{\ell_s}{\ell_b}\frac{A_b Y_b}{A_s Y_s} \quad \frac{3}{2}\frac{a}{b^2 c}$$

65. (a) For low velocity, the water layer in contact with the river bed can be assumed to be stationary.
Since the velocity of water layers, increase from 0 to 2 m/s over a vertical height of 2m so, the velocity gradient

$$= \frac{\Delta v}{\Delta y} \quad \frac{2-0}{2} \quad 1\,s^{-1}$$

From viscous drag $F = \eta A\left(\frac{\Delta v}{\Delta y}\right)$ (numerically),

we have $F = 0.90 \times 10^{-2} \times 8 \times 10^4 \times 1 = 720$ dyne
To maintain, uniform speed, an external force equal to the viscous drag F should be applied. So, the required force is 720 dyne.

66. (d) $F - T = 3a$; $T = 2a$
$T = 2.5 \times 10^9 \times 4 \times 10^{-8}$
$T = 100$ N, $T = 2$a
$100 = 2a$; $a = 50$ N
$F = 5 \times 50 = 250$ N

67. (d) The pressure at a point just outside the bubble
$p_0 = P_{atm} + h\rho g$
$= P_{atm} + 10 \times 10^{-2} \times 10^3 \times 9.8$
$= P_{atm} + 980\ \text{Nm}^{-2}$
Now excess pressure within the bubble compared to a point just outside

$$= \frac{2T}{R} \quad \frac{2 \times 72 \times 10^{-3}}{2 \times 10^{-3}} \quad 72\ \text{Nm}^{-2}$$

∴ Inside pressure = excess pressure + outside pressure
$= P_{atm} + 980 + 72$
$= P_{atm} + 1052\ \text{Nm}^{-2}$
∴ Pressure within the bubble in excess of atmospheric pressure = 1052 Nm^{-2}.

68. (b) The vertical component of the tension in the two ends of the wire, $2T\sin(15°)$, must equal the total loaded weight W. The volume of the wire is $\pi R_w^2 L$, where R_w is the radius of the wire, and the total volume of ice is $\pi(R_i^2 - R_w^2)L$, where R_i is the radius of the ice-covered wire. Multiplying by the respective densities, the total mass of the 500m-long ice-covered wire is 694.5 + 3531.5 = 4226 kg, having a total weight of 41415N. The tension in the wire is thus 41415/(2sin15°) = 80000 N, and dividing by the cross-sectional area of 1.77 x 10^{-4} m^3, the stress is 4.52 x 10^8 N/m^2

69. (c) Let P_1 and P_2 be the internal pressure of air, in the two identical bubbles and the bubble resulting from the combination of the two respectively,

∴ $P_1 - P = \frac{4T}{x}$ and $P_2 - P = \frac{4T}{y}$

Since, the total number of moles remain constant,
∴ 2 (no. of moles in bubble of radius x) = (no. of moles in bubble of radius y)

i.e., $2\left[\frac{P_1 V_1}{R\theta}\right] \quad \frac{P_2 V_2}{R\theta}$ [where θ = temperature]

i.e., $2P_1\left(\frac{4}{3}\pi x^3\right) = P_2\left(\frac{4}{3}\pi y^3\right)$

$$\Rightarrow \frac{P_1}{P_2} \quad \frac{y^3}{2x^3} \qquad \text{........ (1)}$$

Since $P_1 = P \quad \frac{4T}{x}$ and $P_2 = P \quad \frac{4T}{y}$

So, $\frac{P_1}{P_2} \quad \frac{(P \quad 4T/x)}{(P \quad 4T/y)}$ .... (2)

Equating eqns. (1) and (2), we get

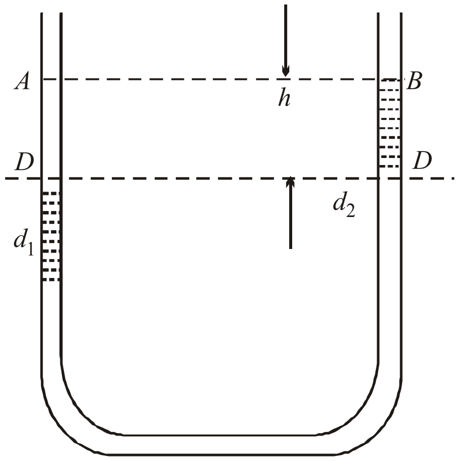
$$\frac{y^3}{2x^3} \quad \frac{P \quad 4T/x}{P \quad 4T/y}$$

$$\Rightarrow \quad Py^3 + 4Ty^2 = 2Px^3 + 8Tx^2$$

$$\Rightarrow \quad 4T[y^2 - 2x^2] = P[2x^3 - y^3]$$

$$\Rightarrow T \quad \frac{P[2x^3 - y^3]}{4(y^2 - 2x^2)}$$

70. **(a)**



For the columns of water in the two vertical limbs of the U-tube referred to datum.
Surface tension force at $A = \pi d_1 T$ upwards
and surface tension force at $B = \pi d_2\ T$ upwards
The difference in the two surface tension forces supports the weight of the column of water in the smaller tube above the datum level.

$$\pi d_2 T - \pi d_1 T = \frac{\pi d_2^2}{4} h\rho g$$

whence, $h \quad \frac{4(d_2 - d_1)T}{d_2^2 \rho g}$

$$= \frac{4(1.5 - 1.0) \times 0.0075}{(1.5/1000)^2 \times 1000 \times 9.81} \quad 0.68 \text{ mm}$$

71. **(b)** $\frac{F}{A} \quad y\frac{e}{L}$; $e \quad \frac{LF}{AY}$

Energy stored $= \frac{1}{2} \times$ stress $\times$ strain $\times$ volume

$$= \frac{1}{2}\frac{F}{A}\frac{e}{L}AL \quad \frac{1}{2}Fe \quad \frac{F^2 L}{2AY}$$

B COMPREHENSION TYPE

1. **(b)** The speed of efflux $\quad \sqrt{\text{depth of orifice from free surface}}$

2. **(d)** The distance $x_1 = v_1 t$

$$= \sqrt{2gh} \times \sqrt{\frac{2.(3h)}{g}} = 2\sqrt{3h}$$

3. **(c)** The horizontal distance

$$x = \sqrt{2gy}.\sqrt{\frac{2(H-y)}{g}}$$

where H : total height, y : depth of hole

$$= 2\sqrt{y(H-y)}$$

x_{max} for $y = \frac{H}{2}$ and same from y and $H - y$.

4. **(d)** The cube will become spherical because of surface tension.

5. **(c)** $\frac{4}{3}\pi r^3 \quad a^3$

$$(r) = \left[\frac{3a^3}{4\pi}\right]^{1/3} \quad \left[\frac{3}{4\pi}\right]^{1/3} \cdot (a)$$

$$U = (T)(4\pi r^2) = (T).4\pi \frac{3^{2/3}a^2}{(4\pi)^{2/3}}$$

$$U = (T)(4\pi)^{1/3}.(3)^{2/3}(a^2)$$

6. **(a)** $v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$; where $r = \left(\frac{3a^3}{4\pi}\right)^{1/3}$

7. (b) Volume of fluid crossing the section of the tube of length ℓ and radius R in unit time

$$V = \int 2\pi r v dr = \int_0^R 2\pi v_0 r\left(\frac{R^2 - r^2}{R^2}\right) dr$$

$$= 2\pi v_0\left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \pi\frac{v_0 R^2}{2}$$

8. (c) Kinetic energy of the fluid within the volume of the tube

$$\int \frac{1}{2}(dm)\, v^2 \quad \int \frac{1}{2}(2\pi r dr \ell \rho)\, v^2$$

$$= \int_0^R \pi r \ell \rho v_0^2 \left(\frac{R^2 - r^2}{R^2}\right)^2 dr$$

$$= \pi\rho v_0^2 \ell \int_0^R \left(r + \frac{r^5}{R^4} - \frac{2r^3}{R^2}\right) dr$$

$$= \pi\rho v_0^2 \ell \left(\frac{R^2}{2} + \frac{R^2}{6} - \frac{R^2}{2}\right) \quad \frac{\pi \ell \rho v_0^2 R^2}{6}$$

9. (a) $v = v_0\left(1 - \frac{r^2}{R^2}\right), \frac{dv}{dr} = -v_0\left(\frac{2r}{R^2}\right)$

Friction force due to viscosity

$$= \eta A\frac{dv}{dr} = \eta\,(2\pi R\ell)\left(\frac{dv}{dr}\right)_{r \quad R}$$

$$= \eta\,(2\pi R\ell)\, v_0\left(\frac{2R}{R^2}\right) = 4\pi\eta\ell v_0$$

10. (b), 11. (b), 12. (b)

By conservation of volume

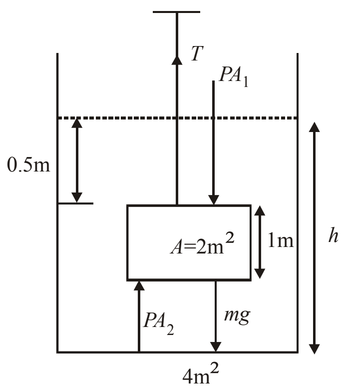
$4 \times h = 4 \times 2 + 2 \times 1 = 10$

$h = 2.5$m.

Pressure at top of the object

$= P_0 + 0.5 \times 1000 \times 10 = 1.05 \times 10^5$ N/m^2.

$F = P_1 A = 1.05 \times 10^5 \times 2 = 2.1 \times 10^5$ N



By F.B.D. $T + P_2 A = mg = P_1 A$

$T = mg + (P_1 - P_2)\,A = mg - (P_2 - P_1)\,A$

$= 2 \times 2000 \times 10 - (0.2 \times 10^5)$

$= 0.4 \times 10^5 - 0.2 \times 10^5 = 0.2 \times 10^5$ N

$F_b = V.\rho_w g = 2 \times 1000 \times 10 = 0.2 \times 10^5$ N

It is also equal to net contact force by the liquid

$= P_2 A - P_1 A$

$= 0.2 \times 10^5$ N

Note : Net contact force and buoyant force are same.

13. (a) $A_1 V_1 = A_2 V_2$

$4\text{cm}^2 \times 1 = 1\text{mm}^2 \times 2\text{m/s}$

$400\text{mm}^2 \times \text{V} = 1\ \text{mm}^2 \times 2\text{m/s}$

$$V \quad \frac{2}{400} \quad \frac{20}{4}\text{mm/s} \quad 5\text{mm/s}$$

14. (c) Height of spring = V/A = 5cm

Work done = Δ KE of water + gain in PE of water + gain in PE of piston

$$= \frac{1}{2}\,(V\rho)v^2 + m_1 g h_1 + m_2 g h_2$$

$$= \frac{1}{2}\,[(20 \times 10^{-6})\,1000]2^2 + [20 \times 10^{-3} \times 10 \times 2.5 \times 10^{-2}] + [0.1 \times 10 \times 0.05]$$

$= 0.04 + 0.005 + 0.05 = 0.095$ J

15. (b) As amount of water reduces continuously force also reduces continuously.

16. (b), 17. (c), 18. (a)

The initial velocity of water coming out of hole is horizontal and hole is at a height $h/4$ from ground. Hence time taken by water to reach ground is $t \quad \sqrt{\frac{2(h/4)}{g}}$

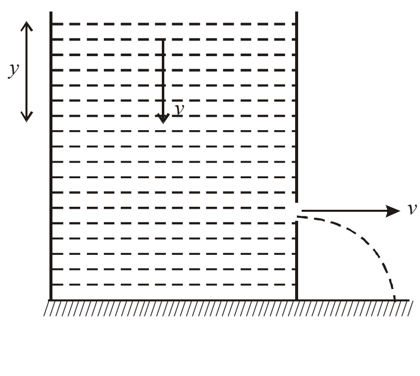
which remains constant.

$\therefore$ $x = vt$, where v is velocity of efflux.

Since v decreases with time x will decrease.

Let y be the height of water surface above hole

$$\therefore \quad -\frac{dy}{dt} \quad \frac{av}{A} \quad \frac{a\sqrt{2gy}}{A}$$

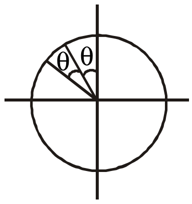


$$\therefore \int_{3h/4}^{0} \frac{dy}{\sqrt{2gy}} = \int_0^t -\frac{a}{A}dt \quad \therefore\ t \quad \frac{A}{a}\sqrt{\frac{3h}{2g}}$$

19. (b), 20. (d), 21. (c).

$$F_v = \int_0^h [\rho g\ (2R - R\cos\theta)] \times [Rd\theta \times L]\cos\theta$$

$$F_v = \rho g R^2 L \int_0^{\pi} \cos\theta\ (2 - \cos\theta)\ d\theta$$



$$= \rho g R^2 L \int_0^{\pi} (2\cos\theta - \cos^2\theta)\ d\theta = -\ \rho g R^2 L \frac{\pi}{2}$$

$\therefore$ Force $= \rho g R^2 L \dfrac{\pi}{2}$ upwards

As all forces are radial and all pass through axis and hence torque is zero.

$$F_x = \int_0^{\pi} \rho g\ (2R - R\cos\theta)\ (Rd\theta \times L)\ \sin\theta = 4R^2 \rho g L$$

22. (a) AB is the rod of length $2a$ say, with end B fixed and length $AC = \frac{1}{3}(2a)$ immersed in water.

Let α be the area of cross-section of the rod and σ its sp gr.

Let ρ be the density of water.

The rod AB is in equilibrium under the action of the following forces :

(i) The weight $2a\alpha\sigma\rho g$ acting vertically downwards through G, the C.G. of the rod.

(ii) The force of buoyancy V or the weight of the water displaced $= AC.\alpha\rho g = \frac{1}{3}(2a)\ \alpha\rho g$, acting vertically upwards through G_1, the centre of buoyancy or C.G. of the water displaced.

(iii) The reaction at the end B, which is fixed.

If θ be the inclination of the rod with the horizontal, then taking moment of these forces about B, we have

$$2a\alpha\sigma\rho g\ BG\cos\theta = \frac{1}{3}\ (2a)\ \alpha\rho g.BG_1 \cos\theta$$

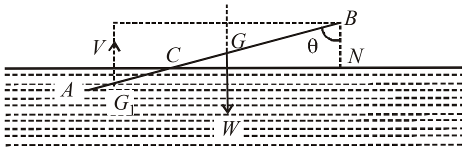
$$\text{or } BG\sigma = \frac{1}{3} BG_1$$

$$\text{or } \sigma \quad \frac{BG_1}{3BG} \quad \frac{BC \quad \frac{1}{2}(AC)}{3.\frac{1}{2}(2a)}$$

$$\text{or } \sigma \quad \frac{\frac{2}{3}(2a) \quad \frac{1}{2}\left(\frac{2}{3}a\right)}{3a} \quad \frac{5}{9}$$

23. (a) Given that the rod $AB = 6$ ft.

The immersed length $AC = 4$ft.



$\because$ The length outside the water $= BC = 6 - 4 = 2'$

$\therefore$ If the rod makes an angle θ with the vertical, then

$$\cos\theta \quad \frac{BN}{BC} \quad \frac{1}{2} \qquad \text{or} \quad \sin\theta \quad \frac{\sqrt{3}}{2}$$

The forces acting on the rod are :

(i) The weight W of the rod $= 6 \times \alpha\sigma\rho g$ acting through G, the C.G. of the rod, where α, σ and ρ are the area of cross-section of the rod, sp. gr. of the rod and density of water respectively.

(ii) The force of buoyancy $V = 4 \times \alpha\rho g$, acting vetically upwards through G_1, the C.G. of the water displaced and

(iii) The reaction at the fulcrum at B.

Taking moments of these forces about B, we have

$W.BG \sin\theta = V.BG_1 \sin\theta$.

or $6\alpha\sigma g\rho.BG = 4\alpha\rho g.BG_1$

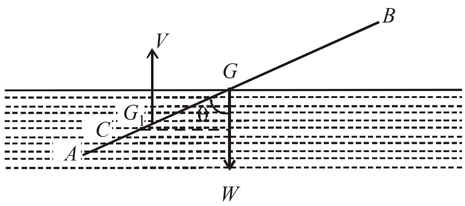
or $3\sigma BG = 2BG_1$

$$\text{or} \quad \sigma \quad \frac{2}{3}.\frac{BG_1}{BG} \quad \frac{2\left(BC \quad \frac{1}{2}AC\right)}{3.\left(\frac{1}{2}AB\right)}$$

$$\frac{2\,(2 \quad 2)}{3(3)} \quad \frac{8}{9}$$

24. (c) $AB = 2a$ is the rod whose middle point G is in the surface and the point C on the rod, such that AC $= \frac{1}{6}(AB) = \frac{1}{3}a$, is fixed and the rod can turn about C.

Let α be the area of cross-section of the rod and σ be its sp. gr. Let ρ be the density of the water.



The forces acting on the rod are :

(i) its weight $W = 2a\alpha\sigma\rho g$, acting downwards through G, the C.G. of the rod :

(ii) the force of buoyancy $V = a\alpha\rho g$, acting vertically upwards through G_1, the C.G. of the water displaced and

(iii) the reaction at C.

Let θ be the inclination of the rod to the vertical. Taking moments of the forces about C, we have

$W.CG \sin\theta = V.CG_1 \sin\theta$.

or $W.CG = V.CG_1$

or $2a\alpha\sigma\rho g\,(AG - AC) = a\alpha\rho g\,(AG_1 - AC)$

or $2\sigma\left(a - \frac{1}{3}a\right) = \left(\frac{1}{2}a - \frac{1}{3}a\right)$

or $\sigma\left(\frac{4}{3}a\right) \quad \frac{1}{6}a$ or $\sigma \quad \frac{1}{8}$

25. **(b)** Weight of Ram is more than that of Shyam in water means upthrust on Ram is less hence less volume and less fat content.

26. **(a)** Let Fat mass = m_1, Other mass = m_2.
Total volume = V

Given : $\frac{m_1}{0.4d_w} \quad \frac{V}{4}, \frac{m_2}{(4/3)d_w} \quad \frac{3V}{4}$ and

$m_1 + m_2 = 165$

Solving, $V \quad \frac{1650}{11d_w}$

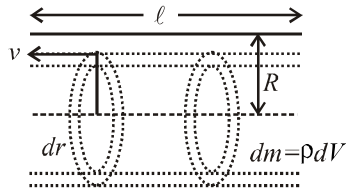
Spring balance reading = $165 - \frac{1650}{11d_w}d_w \quad 15$ kg

27. **(c)** Upthrust and effective weight changes by same factor hence fraction immersed remains same.

28. (c) $f = 2\eta\pi R\ell\left(\frac{dv}{dr}\right) = -2\eta\pi R\ell\left[\frac{-2Rv_0}{R^2}\right]$; $F = 4\pi\eta\ell v_0$

29. (a) $F = -\eta 2\pi r\ell\left[\frac{-2rv_0}{R^2}\right] \Rightarrow F \quad r^2$

30. (b)



$dm = \rho\, dV = \rho\, dA.\ell$; $dm = \rho\,\ell\, 2\pi r\, dr$

so momentum of mass dm

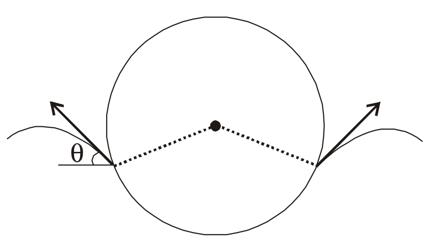
$dp = v\, dm$; $\int_0^p dp \quad \int_0^R v\, dm$

$p = 2\pi\rho\ell v_0 \int_0^R r\left(1 - \frac{r^2}{R^2}\right)dr$; $p = 2\pi\rho\ell v_0\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$

$= 2\pi\rho\ell v_0\left[\frac{R^2}{2} - \frac{R^2}{4}\right] \quad \frac{2\pi\rho\ell v_0 R^2}{4}$; $p \quad \frac{\rho\pi R^2\ell v_0}{2}$

C REASONING TYPE

1. (b)



$mg = 2T\,\ell\sin\theta$

Radius does not affect the process.

2. **(a)** Since pressure inside bubble of smaller radius is more the common interface should bulge inside bubble of larger radii. Hence statement-2 is correct explanation of statement-1.

3. **(c)** Tension at a point on rod of (length L) at a distance x from point of application of force is $T = F\left(1 - \frac{x}{L}\right)$ in both cases. Hence weight has no effect on tension in situation of figure (ii). Extension in rod occurs due to force acting at any point on the rod. in certain cases when net force acts at the centre of rod like weight, extension due to this force may not occur like the given case.

4. **(d)** For stable equilibrium centre of gravity should exist below centre of buoyancy of submerged part. Hence statement-1 is false.

5. **(a)** At the first moment the ship will begin to move to the right, since the pressure on the starboard side diminishes by $2pA$, where p is the pressure at the depth h of the hole, and A is its area. As soon as the stream of water reaches the opposite wall, this wall will be acted upon by the force F = ρAv^2, where v is the velocity of the stream with respect to the ship. The force F is slightly greater than $2pA$, since $v \quad \sqrt{2gh}$ because the ship moves towards the stream. As a result, the motion will begin to retard.

6. **(a)** According to Bernoulli's theorem, the pressure decreases in the region where velocity of the wind is high. The pressure difference is enough to lift the roof up and carry it aside.

7. **(c)** Since the gas in the stream has a high velocity, the pressure inside the stream is below atmospheric. The ball will be supported from the bottom by the thrust of the stream, and on the sides by the static atmospheric pressure.

8. **(a)** Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

9. **(d)** Applying Principle of continuity & Bernoullies theorem

$A\downarrow v\uparrow P\downarrow,\ h_1d_1 = h_2d_2$

10. **(d)** At distance x from where force is applied

$T \quad \frac{F}{L}x$

D MULTIPLE CORRECT CHOICE TYPE

1. **(a, b, d)** Since acceleration of centre of mass is upwards therefore N must be greater than mg.

2. **(a, c)** Area of cross-section, $A \quad \frac{\pi d^2}{4}$

where d is the diameter of the wire. Therefore,

$$\ell \quad \frac{4FL}{\pi d^2 Y}$$

Since, F, L and Y are the same for wires A and B

$$\therefore \quad \ell \quad \frac{1}{d^2}$$

i.e., the extension is inversely proportional to the square of the diameter.

The strain is $\frac{\ell}{L} \quad \frac{4F}{\pi d^2 Y}$. Thus, strain $\quad \frac{1}{d^2}$

3. **(a, b, c)** $T_B \quad \frac{mg}{3}$ and $T_A \quad mg \quad \frac{mg}{3} \quad \frac{4mg}{3}$

$\therefore \; T_A = 4T_B$

Stress, $S \quad \frac{T}{\pi r^2}$

(a) For $r_A = r_B$, $S_A = 4S_B$
$\therefore$ A breaks

(b) For $r_A < 2r_B$, $S_A > S_B$
$\therefore$ A breaks

(c) For $r_A = 2r_B$, $S_A = S_B$
$\therefore$ either may break

4. **(b, c)** Thermal force $= YA\alpha\, d\theta = Y\pi r^2 \alpha\, d\theta$

$r_1 \quad r, r_2 \quad r\sqrt{2}, r_3 \quad r\sqrt{3}, r_4 \quad 2r$,

$F_1 : F_2 : F_3 : F_4 = 1 : 2 : 3 : 4$

Thermal stress $= Y\alpha\, d\theta$

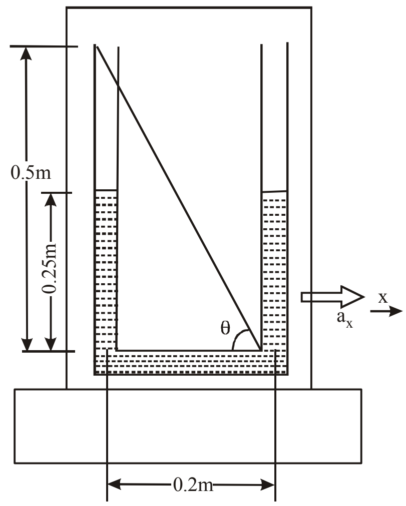
As Y and α are same for all the rods, hence stress developed in each rod will be same.

As strain $= \alpha\, d$, so strain will also be same.

$$E = \text{Energy stored} = \frac{1}{2} Y\,(\text{strain})^2 \times A \times L$$

$\therefore \; E_1 : E_2 : E_3 : E_4 = 1 : 2 : 3 : 4$

5. **(a, b, c, d)** When subjected to horizontal acceleration, the fluid in one limb rises by the amount equal to the depression in the other limb. Thus, for maximum possible acceleration, i.e. for a maximum value of tan θ, the fluid should be initially filled to 50% height in the instrument i.e. 0.25m in each limb, and should be at the average of spilling over when accelerated.



$z_{max} = 0.5\text{m} = 0.2 \tan\theta$ hence $\tan\theta = 2.5$

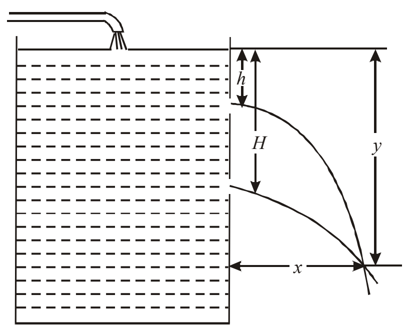
But, $\tan\theta = \frac{a_x}{g} = 2.5$

Hence, $a_x = 2.5\text{g} = 24.5 \text{ m/s}^2$

The range of operation of this accelerometer is 0 to 24.5 m/s^2.

6. **(a, c)** Let us denote the distance from the level of the water to the upper hole by h, the sought distance from the vessel to the point where the streams intersect by x, and the distance from the level of the water in the vessel to this point by y (figure). The point of intersection will remain at the same place if the level of the water in the vessel does not change. This will occur if $Q = Av_1 + Av_2$, where

$v_1 \quad \sqrt{2gh}$ and $v_2 \quad \sqrt{2g\,(H \quad h)}$ are the outflow velocities of the streams from the holes. On the basis of the law of kinematics,



$$x \quad v_1 t_1 \quad v_2 t_2 \text{ and } y \quad h \quad \frac{g t_1^2}{2} \quad h \quad H \quad \frac{g t_2^2}{2}$$

where t_1 and t_2 are the times during which the water falls from the holes to the point of intersection.

Hence, $x = \frac{1}{2}\left(\frac{Q^2}{2gA^2} - H^2 \frac{2gA^2}{Q^2}\right) = 120$ cm.

$y \quad \frac{1}{2}\left(\frac{Q^2}{2gA^2} \quad H^2 \frac{2gA^2}{Q^2}\right) = 130$ cm.

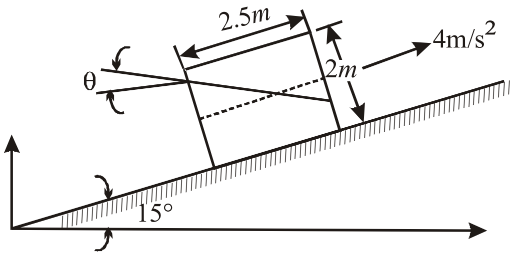
7. **(a, b)** Let us compute the slope of the free surface of the water

$$\tan\theta \quad \frac{a_x}{g \quad a_x} \quad \frac{4\cos 15}{9.81 \quad 4\sin 15} \quad -0.35$$

whence $\theta = 19.8°$ backwards

The maximum slope which the surface can make with the horizontal, without spilling, is

$\theta_{max} = \tan^{-1}(2/2.5) - 15° = 23.66°$ backwards



The slope, 19.2°, made by the surface is less than this value, the water will not spill.

Let the acceleration at which water begins to spill be 'a' then,

$$\tan\theta_{max} = -\left(\frac{a\cos 15}{g \quad a\sin 15}\right) = \tan(-23.66)$$

$= -0.438$

whence $a\cos 15° - a\sin 15° \times 0.438$

$= +0.438 \times 9.81$

and $a = 5.04$ m/s^2.

8. **(b, d)** Assume the discharge to be equally divided between the two nozzles.

$$Q_A = Q_B = \frac{3\times10^{-3}}{60} = 50\times10^{-6} \text{ m}^3/\text{s}$$

$$v_A = v_B = \frac{50\times10^{-6}}{\pi/4\times0.005^2} \quad 2.54 \text{ m/s}$$

$v_{\theta A} = 2.54\cos 30° = 2.2$ m/s

$v_{\theta B} = 2.54\cos 45° = 1.8$ m/s

Assume that the water entering the sprinkler through a tap does not involve any angular momentum.

When stationary, the torque due to nozzles action,

for nozzle A, $\tau_A = 1000 \times 50 \times 10^{-6} \times 2.2 \times 0.20$

$= 0.0220$ Nm

for nozzle B, $\tau_B = 1000 \times 50 \times 10^{-6} \times 1.8 \times 1.5$

$= 0.0135$ Nm

Total torque due to nozzles A and B,

$\tau = 0.0355$ Nm

When rotating free, let the angular velocity be ω.

Now the absolute velocities of the nozzle-discharge in the circumferential direction are

for nozzle A, $v_{\theta A} = (2.2 - 0.2\,\omega)$ m/s

for nozzle B, $v_{\theta B} = (1.8 - 0.15\,\omega)$ m/s

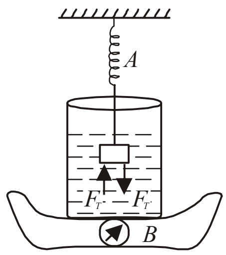
There being no external moment, the angular momentum should be conserved,

$\rho\, Q_A (2.2 - 0.2\omega) \times -0.2 + \rho\, Q_B (1.8 - 0.15\omega)$

$\times 0.15 = 0$

Cancelling ρ and using $Q_A = Q_B$, $\omega = 9.72$ rad/s.

9. **(b,c)** When the block of mass m is arranged as shown in the figure, an upthrust F_T will act on the mass which will decrease the reading on A.



According to Newton's third law, to each and every action, there is equal and opposite reaction.

So F_T will act on the liquid of the beaker which will increase the reading in B.

10. **(a, b, c, d)** Let z_{min} be the minimum reference level of the dotted parabola and z_1 and z_2 the liquid levels above the base.

$$z_1 \quad \frac{r_1^2\omega^2}{2g} \quad z_{min} \quad \frac{0.1^2\times10^2}{2\times9.81} \quad z_{min},$$

$$z_2 \quad \frac{r_2^2\omega^2}{2g} \quad z_{min} \quad \frac{0.2^2\times10^2}{2\times9.81} \quad z_{min}$$

But $z_1 + z_2 = 2 \times 0.4 = 0.8$m

$$\text{Hence, } 0.8 = \frac{0.2^2\times10^2 + 0.1\times10^2}{2\times9.81} \quad 2z_{min}$$

whence, $z_{min} = 0.273$m

Consequently,

$$z_1 \quad 0.273 \quad \frac{0.1^2\times10^2}{2\times9.81} \quad 0.324\text{m}$$

$$z_2 \quad 0.273 \quad \frac{0.2^2\times10^2}{2\times9.81} \quad 0.477\text{m}$$

11. **(a, b, c)** $x \quad vt \quad \sqrt{2gy}\sqrt{\frac{2(H-y)}{g}}$

$$x = 2\sqrt{y(H-y)} \Rightarrow \text{Max. when } y \quad \frac{H}{2}$$

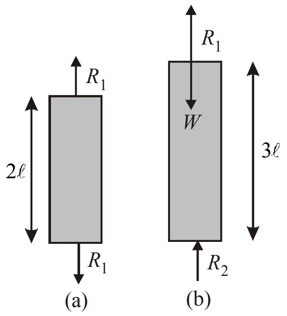
$$\text{as } y = \frac{H}{2} \Rightarrow x \quad H$$

12. **(a, b)** Figure (a) and (b) shows the F.B.D. of two partitions of the bar. Considering the vertical equilibrium of the part of length 3ℓ.

$R_1 + R_2 = W = 5000$ N ..... (1)

Now, stress in the two portions are

$\sigma_1 = \dfrac{R_1}{A}$ and $\sigma_2 \quad \dfrac{R_2}{A}$



$\therefore$ Elongation in the portion of length 2ℓ

$$\delta\ell_1 = \frac{\sigma_1}{Y} \times 2\ell = \frac{R_1}{AY} \times 2\ell$$

and comparison in the portion of length 3ℓ

$$\delta\ell_2 = \frac{\sigma_2}{Y} \times 3\ell = \frac{R_2}{AY} \times 3\ell$$

Since there is no net change in the length of the bar so, elongation in the upper portion = compression in the lower portion.

$$\therefore \delta\ell_1 = \delta\ell_2 \Rightarrow \frac{2R_1\ell}{AY} = \frac{3R_2\ell}{AY} \Rightarrow R_1 \quad \frac{3R_2}{2} \quad ...(2)$$

Solving eq. (1) and (2),

$R_1 = 3000$ N, $R_2 = 2000$ N

13. **(b, c)** $2 \times 2T\ell x \quad \dfrac{1}{2}k(2x)^2$; $x = \dfrac{2T\ell}{k}$, $\Delta x_m \quad \dfrac{4T\ell}{k}$

$F_{net} = 2T\ell - 2kx$; $T_0 = 2\pi\sqrt{\dfrac{m}{2k}}$

14. **(b, d)** Power transmitted is given by $P = \tau\omega$

$$\text{or } \tau \quad \frac{P}{\omega} \quad \frac{120 \times 10^{-3}}{\left(\dfrac{200}{60} \times 2\pi\right)} = 5.73 \times 10^3 \text{ N-m}$$

$$\text{From, } \tau \quad \frac{\pi\eta r^4\theta}{2\ell}, \text{ we have } \tau \quad \frac{\pi r^3}{2}\left(\frac{r\theta\eta}{\ell}\right)$$

But maximum intensity of stress is at the periphery

σ_{max} = shear strain × modulus of rigidity

$$= \left(\frac{r\theta}{\ell}\right)\eta$$

$$\therefore \tau = \frac{\pi r^3}{2}\sigma_{\max} \Rightarrow \sigma_{\max} \quad \frac{2\tau}{\pi r^3}$$

$$\therefore \sigma_{\max} = \frac{2 \times 5.73 \times 10^3}{3.14 \times (5 \times 10^{-2})^3} = 2.92 \times 10^7 \text{ Nm}^{-2}$$

$$\text{Again, from } \tau \quad \frac{\pi r^4}{2}\frac{\theta\eta}{\ell}$$

We have

$$\theta \quad \frac{2\tau\ell}{\pi r^4\eta} \quad \frac{2 \times 5.73 \times 10^3 \times 6}{3.14 \times (5 \times 10^{-2})^4 \times 8 \times 10^{10}}$$

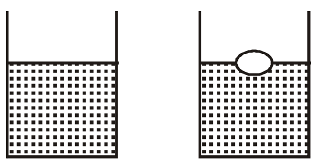
$= 4.37 \times 20^{-2}$ radian $= 2°30'$

E MATRIX-MATCH TYPE

1. **A-p, q, r; B-s; C-q; D-p, q, r, s**

2. **A-r; B-q; C-q; D-q**

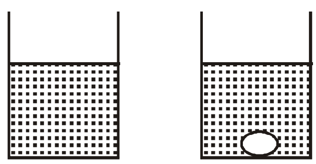
In A : Same water level implies



Wt. of fluid displaced is the same as that of object hence both buckets have equal weight.

In B, C : Mass of water in both buckets is equal and B has additional mass of solid object hence B is heavier.

In D : Same water level and object sinks $\rho_0 > \rho_\omega$ i.e. some volume of ρ_ω is replaced by same volume of ρ_0 mass increases.



$\therefore$ B is heavier.

3. **A-q, r, t; B-s; C-q, t; D-p**

Bernoulli's theorem can be used to calculate speed of efflux; it is based on conservation of energy and a device venturimeter works reading on basis of Bernoulli's theorem.

Stoke's law is based on viscosity.

Torricelli's theorem can be used to calculate speed of efflux and is based on conservation of energy.

Hooke's law is used to determine elastic limit.

4. A-p, r; B-p, q, r; C-p, s; D-r, t

Stoke's law :

$$F = 6\pi\eta r v$$

Terminal velocity :

$$v \quad \frac{2}{9}\frac{r^2 g\,(\rho-\sigma)}{\eta}$$

Excess pressure inside mercury drop :

$$P \quad \frac{2T}{r}$$

Viscous force :

$$F = -\eta A\frac{dv}{dx}$$

5. A-p, q, r, s; B-q, t; C-q; D-s, t

Elastic force, Force due to surface tension, Frictional force and Viscous force decrease with rise in temperature.

Forces involved in capillary action are surface tension and gravitational force.

Water flows in a continuous stream down a vertical pipe whereas it breaks into drops when falling freely because of surface tension.

Terminal velocity of rain drop depends on viscous force and gravitational force.

F NUMERIC/INTEGER ANSWER TYPE

1. **3**

M is the mid-point of tube AB at equilibrium

$$p_1 \times A + mg = p_2 \times A$$

$$p_1 \times A + 10 \times A \times d_{Hg} g = p_2 \times A$$

$$\Rightarrow \quad p_1 + 10 d_{Hg} \times g = p_2 \qquad \text{... (i)}$$

For air present in column AP

$$p \times 45 \times A = p_1 \times (45 + x) \times A$$

$$\Rightarrow \quad p_1 = \frac{45}{45 \quad x} \times 76 d_{Hg} \times g \qquad \text{... (ii)}$$

For air present in column QB

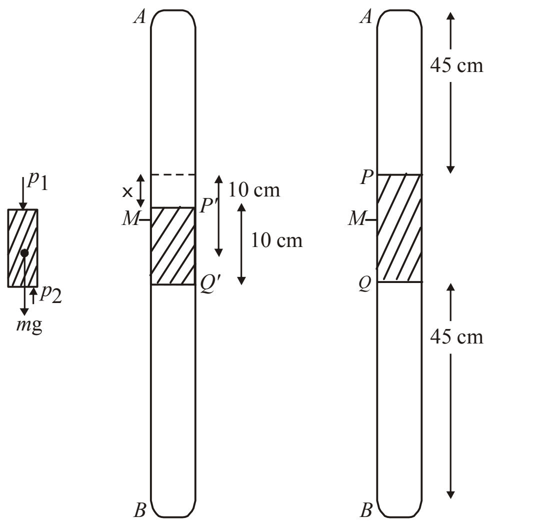
$$p \times 45 \times A = p_2 \times (45 - x) \times A$$

$$\Rightarrow \quad p_2 = \frac{45}{45 - x} \times 76 d_{Hg} \times g \qquad \text{... (iii)}$$

From (i), (ii) and (iii)

$$\frac{45 \times 76 \times d_{Hg}}{45 + x} + 10 d_{Hg} \times g = \frac{45}{45 - x} \times 76 \times d_{Hg} \times g$$

$$\Rightarrow \quad \frac{45 \times 76}{45 + x} \quad 10 \quad \frac{45 \times 76}{45 - x}$$



$$\Rightarrow \quad 10 = 45 \times 76 \left[\frac{1}{45 - x} - \frac{1}{45 \quad x}\right]$$

$$= 45 \times 76 \left[\frac{45 + x - 45 \quad x}{(45)^2 - x^2}\right]$$

$$\Rightarrow \quad 10 = \frac{45 \times 76[2x]}{2025 - x^2} \Rightarrow 2025 - x^2 = 684x$$

$$\Rightarrow \quad x^2 + 684x - 2025 = 0$$

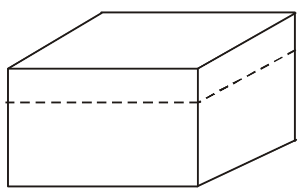
$$\Rightarrow \quad x = -684 \pm \frac{\sqrt{(684)^2 - 4 \times 1 \times (-2025)}}{2}$$

$$= -684 \pm \frac{\sqrt{475956}}{4}$$

$$= -342 \pm 345 = \mathbf{3\ cm.}$$

2. **10**

Let the edge of cube be ℓ. When mass is on the cube of wood



$$200\text{g} \quad \ell^3 d_{wood}\,\text{g} \quad \ell^3 d_{H_2O}\,\text{g}$$

$$\Rightarrow \quad 200 \quad \ell^3 d_{wood} \quad \ell^3 d_{H_2O}$$

$$\Rightarrow \quad \ell^3 d_{wood} = \ell^3 d_{H_2O} - 200 \qquad \text{... (i)}$$

when the mass is removed

$$\ell^3 d_{wood} = (\ell - 2)\,\ell^2 d_{H_2O} \qquad \text{... (ii)}$$

From (i) and (ii)

$$\ell^3 d_{H_2O} - 200 = (\ell - 2)\ell^2 \, d_{H_2O}$$

But $d_{H_2O} \quad 1$

$\therefore \quad \ell^3 - 200 = \ell^2(\ell - 2)$

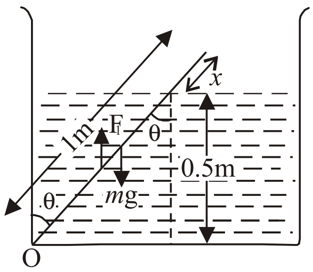
$\therefore \quad \ell^3 - 200 = \ell^3 - 2\ell^2 \;\Rightarrow\; \ell \quad$ **10 cm**

3. **45**

For equilibrium

$F_{net} = 0$

$\tau_{net} = 0$



Taking moment about O

$$mg \times \frac{\ell}{2}\sin\theta = F_T\left(\frac{\ell - x}{2}\right)\sin\theta \qquad \text{... (i)}$$

Also F_T = weight of fluid displaced.

$$F_T = \; (\ell - x)A \;\times \rho g \qquad \text{... (ii)}$$

and $m = (\ell A)\,0.5\rho_w$ ... (iii)

where A is the area of cross section of the rod.

From (i), (ii) and (iii)

$$(\ell A)\,0.5\rho_w g \times \frac{\ell}{2}\sin\theta = [(u - x)A]\rho_w g \times \left(\frac{\ell - x}{2}\right)\sin\theta$$

Here, $\ell \quad 1$

$\therefore \quad (1 - x)^2 = 0.5$

$\therefore \quad 1 - x = 0.707$

$\Rightarrow \quad x = 0.293$ m

From the diagram

$$\cos\theta \quad \frac{0.5}{1 - x} \quad \frac{0.5}{0.707}$$

$\Rightarrow \quad \theta$ = **45°**

4. **75.4**

Let A be the area of cross-section of the tube.

Since temperature is the same, applying Boyle's law on the side AB

$$P \times (x \times A) = P_2 \times (x_2 \times A) \qquad \text{... (i)}$$

Applying Boyle's law in section CD

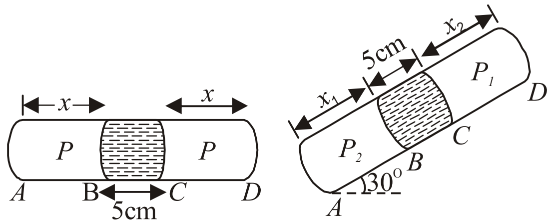
$$P \times (x \times A) = P_1 \times (x_1 \times A) \qquad \text{... (ii)}$$

From (i) and (ii)

$$P_1 \times (x_1 \times A) = P_2 \times (x_2 \times A)$$

$$\Rightarrow \quad P_1 x_1 = P_2 x_2$$

where $P_2 = P_1$ + Pressure due to mercury column



Pressure due to mercury column

$$P = \frac{F}{A} \quad \frac{mg\sin 30}{A} \quad \frac{Vdg\sin 30}{A}$$

$$= \frac{(A \times 5) \times dg\sin 30}{A} = 5\sin 30° \text{ cm of Hg}$$

$P_2 = P_1 + 5\sin 30° = P_1 + 2.5$

Substituting this value in (iii)

$P_1 \times x_1 = [P_1 + 2.5] \times x_2$

$P_1 \times 46 = [P_1 + 2.5] \times 44.5$

$$\therefore \quad P_1 = \frac{44.5 \times 2.5}{1.5}$$

Substituting this value in (ii)

$$P \times r = \frac{44.5 \times 2.5}{1.5} \times 46$$

$$\Rightarrow \quad P \times \left[\frac{46 + 44.5}{2}\right] = \frac{44.5 \times 2.5}{1.5} \times 46$$

$$\therefore \quad x = \frac{x_1 \quad x_2}{2} \;\Rightarrow\; P = \textbf{75.4 cm of Hg}$$

5. **6.75**

$\frac{\rho_1}{\rho_2} \quad \frac{1}{2}$, *i.e.*, in equilibrium, block is half submerged in water.

Let h' be increase in level if block is pressed by an amount h. Then

$$(A - \ell^2)h' \quad \ell^2 h \;\Rightarrow h' \quad 0.5h$$

We have to immerse further $\ell/2$.

$$\Rightarrow \ell/2 \quad h \quad h' \quad 1.5h$$

Extra thrust upon depressing block by h

$$F \quad (h \quad h')\rho_2 \ell^2 g = 1.5\ell^2 \rho_2 g h$$

$$W \quad \int_0^{\ell/3} F\,dh \quad \int_0^{\ell/3} (1.5\ell^2 \rho_2 g h)\,dh \quad \textbf{6.75 J}$$