Chapter 20

Trigonometric Functions-II

Solutions

[Elementary Trigonometric Functions and Transformation Formulae]

1. If the maximum value of cos(cos$x$) is a and minimum value is b, then

(1) $b = \cos a$ (2) $a = \cos b$ (3) $a = 0$ (4) $b = 0$

Sol. Answer (1)

$y = \cos(\cos x)$

$y_{max} = 1 = a$, when $x = \dfrac{\pi}{2}$

$y_{min} = \cos 1 = b$ when $x = 0$

$\therefore \quad b = \cos a$

2. If $\dfrac{2\sin A}{1+\sin A+\cos A} = k$, then $\dfrac{1+\sin A-\cos A}{1+\sin A}$ is

(1) $\dfrac{k}{2}$ (2) k (3) $2k$ (4) $\dfrac{1}{k}$

Sol. Answer (2)

$\dfrac{2\sin A}{1+\sin A+\cos A} = k$, $\dfrac{1+\sin A-\cos A}{1+\sin A} = ?$

$$k = \frac{2\sin A.(1+\sin A-\cos A)}{(1+\sin A)^2-\cos^2 A}$$

$$\Rightarrow k = \frac{2\sin A.(1+\sin A-\cos A)}{(1+\sin A)^2-(1-\sin^2 A)}$$

$$= \frac{2\sin A.(1+\sin A-\cot A)}{(1+\sin A).\{(1+\sin A)-(1-\sin A)\}}$$

$$= \frac{1+\sin A-\cos A}{1+\sin A}$$

3. If $\sin x+\sin^2 x=1$, then the value of $\cos^{12} x+3\cos^{10} x+3\cos^8 x+\cos^6 x-1$ is equal to

(1) 0 (2) 1 (3) –1 (4) 2

Sol. Answer (1)

$\sin x+\sin^2 x=1$

$\Rightarrow \sin x=\cos^2 x$

$=\cos^6 x.(\cos^6 x+3\cos^4 x+3\cos^2 x+1)-1$

$=\{\cos^2 x(1+\cos^2 x)\}^3-1$

$=\{\sin x(1+\sin x)\}^3-1$

$=\{\sin x+\sin^2 x\}^3-1=(1)^3-1=0$

4. The maximum value of $\cos^2(\cos(33\pi+\theta))+\sin^2(\sin(45\pi+\theta))$ is

(1) $1+\sin^2 1$ (2) 2 (3) $1+\cos^2 1$ (4) $\cos^2 2$

Sol. Answer (1)

$\cos^2(\cos(33\pi+\theta))+\sin^2(\sin(45\pi+\theta))$

$=\cos^2(-\cos\theta)+\sin^2(-\sin\theta)$

$=\cos^2(-\cos\theta)+\sin^2(\sin\theta)$

$\because \cos\le\cos(\cos\theta)\le 1$

$\cos^2 1\le\cos^2(\cos\theta)\le 1$

and $0\le\sin^2(\sin\theta)\le\sin^2 1$

$\because \cos^2 1\le\cos^2(\cos\theta)+\sin^2(\sin\theta)\le 1+\sin^2 1$

$\therefore$ Maximum value = $1+\sin^2 1$ $\left(\text{At } \theta=\frac{\pi}{2}\right)$

5. In a ΔABC, if $\sin A-\cos B=\cos C$, then the measure of $\angle B$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Sol. Answer (1)

$\sin A=\cos C+\cos B$

$$\sin A=2\cos\frac{B+C}{2}.\cos\frac{B-C}{2}$$

$$\sin A=2\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)$$

$$\sin A=2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$$

$$2\sin\frac{A}{2}\cos\frac{A}{2}-2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)=0$$

$$2\sin\frac{A}{2}\left(\cos\frac{A}{2}-\cos\frac{B-C}{2}\right)=0$$

$$\Rightarrow\quad \cos\frac{A}{2}-\cos\left(\frac{B-C}{2}\right)=0\ \left[\because A\neq 0 \text{ or } \frac{A}{2}\neq 0\ \Rightarrow \sin\frac{A}{2}\neq 0\right]$$

$$\frac{A}{2}=\frac{B-C}{2}$$

$A - B + C = 0$

$\Rightarrow$ $A + C = B$

Also, $A + B + C = 180°$

$2B = 180°$

$\Rightarrow$ $B = 90°$ or $\frac{\pi}{2}$

6. The value of cos10° cos20° cos40° is

(1) $\frac{1}{8}\cot 10°$ (2) $\frac{1}{8}\tan 10°$ (3) $\frac{1}{4}\sec 10°$ (4) $\frac{1}{4}\text{cosec}\,10°$

Sol. Answer (1)

Multiply and divide by $\frac{1}{2}\sin 10°$

7. The value of the expression

$$2\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{9\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right) \text{ is}$$

(1) 0 (2) –1 (3) 1 (4) 2

Sol. Answer (1)

$$2\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{9\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)$$

$$=\cos\left(\frac{10\pi}{13}\right)+\cos\left(\frac{8\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)$$

$$=\cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)$$

$$=-\cos\left(\frac{3\pi}{13}\right)-\cos\left(\frac{5\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)$$

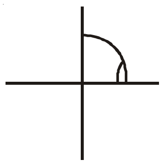
$= 0$

8. If $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then $\theta + \phi$ belongs to, where $0 < \theta,\ \phi < \frac{\pi}{2}$

(1) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (2) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (3) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$ (4) $\left(\frac{5\pi}{6}, \pi\right)$

Sol. Answer (2)

$\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$



$\theta = \frac{\pi}{6}$, $\phi = \frac{\pi}{3}$

$\therefore \quad \theta + \phi = \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

9. The range of

$f(\theta) = 3\cos^2\theta - 8\sqrt{3}\cos\theta\cdot\sin\theta + 5\sin^2\theta - 7$ is given by

(1) [–7, 7] (2) [–10, 4] (3) [–4, 4] (4) [–10, 7]

Sol. Answer (2)

$f(\theta) = 3\cos 2\theta - 8\sqrt{3}\cos\theta\sin\theta + 5\sin 2\theta - 7$

$= 3(\cos^2\theta + \sin^2\theta) + 2\sin^2\theta - 4\sqrt{3}\sin 2\theta - 7$

$= 3 + 1 - \cos 2\theta - 4\sqrt{3}\sin 2\theta - 7$

$= -\cos 2\theta - 4\sqrt{3}\sin 2\theta - 3$

Maximum value = $\sqrt{1+48} - 3 = 7 - 3 = 4$

Minimum value = – 7 – 3 = – 10

R_f = [– 10, 4]

10. The maximum value of $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2\cos\left(\frac{\pi}{4} - \theta\right)$ for real values of θ is

(1) 3 (2) 5 (3) 4 (4) 2

Sol. Answer (3)

$$1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2\cos\left(\frac{\pi}{4} - \theta\right)$$

$$= 1 + \frac{1}{\sqrt{2}}(\sin\theta + \cos\theta) + 2\left(\frac{1}{2}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right)$$

$$= 1 + \sin\theta\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) + \cos\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)$$

$$= 1 + \frac{3}{\sqrt{2}}\sin\theta + \frac{3}{\sqrt{2}}\cos\theta$$

Maximum value = $\sqrt{\frac{9}{2} + \frac{9}{2}} + 1 = 3 + 1 = 4$

11. The value of $\cos\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{8}\right)\cdot\cos\left(\frac{\pi}{16}\right)........\cos\left(\frac{\pi}{2^n}\right)$ equals

(1) $\frac{1}{2^n}\text{cosec}\left(\frac{\pi}{2^n}\right)$ (2) $\frac{1}{2^{n-1}}\text{cosec}\left(\frac{\pi}{2^{n-1}}\right)$ (3) $\frac{1}{2^n}\text{cosec}\left(\frac{\pi}{2^{n-1}}\right)$ (4) $\frac{1}{2^{n-1}}\text{cosec}\left(\frac{\pi}{2^n}\right)$

Sol. Answer (4)

$$\cos\left(\frac{\pi}{4}\right).\cos\left(\frac{\pi}{8}\right).\cos\left(\frac{\pi}{16}\right)........\cos\left(\frac{\pi}{2^n}\right)$$

$$= \cos A.\cos 2A.\cos 2^n A \ \ldots\ldots\ldots \ \cos 2^{n-2}A.,\ A = \frac{\pi}{2^n}$$

$$= \frac{1}{2^n \sin A}\times \sin 2^{n-1}A$$

$$= \frac{1}{2^n \sin\theta}\times \sin\left(\frac{2^n A}{2}\right)$$

$$= \frac{1}{2^n \sin\theta}\times \sin\left(\frac{\pi}{2}\right)$$

$$= \frac{1}{2^n\cdot\sin\left(\frac{\pi}{2^n}\right)}$$

$$= \frac{1}{2^n}\text{cosec}\left(\frac{\pi}{2^n}\right)$$

12. The value of $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + ...$ to n terms is equal to

(1) 1 (2) 0 (3) $\frac{n}{2}$ (4) $\frac{n-1}{2}$

Sol. Answer (2)

$$\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + ...$$

$$= \frac{\sin\left(n.\frac{2\pi}{2n}\right)}{\sin\left(\frac{2\pi}{2n}\right)}\times \sin\left(\frac{\pi}{n} + (n-1)\times\frac{2\pi}{2n}\right) = 0$$

13. $\sum_{r=1}^{n-1}\cos^2\frac{r\pi}{n}$ is equal to

(1) $\frac{n}{2}$ (2) $\frac{n-1}{2}$ (3) $\frac{n}{2}-1$ (4) $\frac{n+1}{2}$

Sol. Answer (3)

$$\sum_{r=1}^{n-1}\cos^2\frac{r\pi}{n}$$

$$= \cos^2\frac{\pi}{n}\times\cos^2\frac{2\pi}{n}+\cos^2\frac{3\pi}{n}+.........+\cos^2\frac{(n-1)\pi}{n}$$

$$=\frac{1}{2}\left[(n-1)+\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}+........\right]\text{upto } n-1 \text{ terms}$$

$$=\frac{n-1}{2}+\frac{1}{2}\frac{\sin\left((n-1)\frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)}\cdot\cos\left(\frac{2\pi}{n}+(n-2)\cdot\frac{\pi}{n}\right)$$

$$=\frac{n-1}{2}-\frac{1}{2}=\frac{n}{2}-1$$

14. If $\tan\theta+\tan\left(\theta+\frac{\pi}{3}\right)+\tan\left(\theta-\frac{\pi}{3}\right)=K\tan3\theta$, then K is equal to

(1) 1 (2) 3 (3) $\frac{1}{3}$ (4) 2

Sol. Answer (2)

$$\tan\theta+\tan\left(\theta+\frac{\pi}{3}\right)+\tan\left(\theta-\frac{\pi}{3}\right)=K\tan3\theta$$

$K = 3$

15. If $4n\alpha = \pi$, then $\cot\alpha\cdot\cot2\alpha\cdot\cot3\alpha$ ..... $\cot(2n-1)\alpha$ $n \in Z$ is equal to

(1) 1 (2) –1 (3) 2 (4) Zero

Sol. Answer (1)

$4n\alpha = \pi \Rightarrow 2n\alpha = \frac{\pi}{2}$

$\cot\alpha.\cot2\alpha.\cot3\alpha$ …….$\cot(2n-1)\alpha$

$= \cot\alpha.\cot2\alpha.\cot3\alpha$ …….. $\cot\left(\frac{\pi}{2}-3\alpha\right).\cot\left(\frac{\pi}{2}-\alpha\right).\cot\left(\frac{\pi}{2}-\alpha\right)$

= 1

16. The value of $\tan10°\cdot\tan50°\cdot\tan70°$ is

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$ (3) 1 (4) –1

Sol. Answer (2)

$\tan30° = \frac{1}{\sqrt{3}}$

17. The expression 2sin2° + 4sin4° + 6sin6° + ... 180sin180° equals

(1) cot1° (2) 90cot1° (3) sin1° (4) 90cos1°

Sol. Answer (2)

2sin2° + 4sin4° + 6sin6° +............+ 178sin178° + 180sin180°

= (2 + 178)sin2° + (4 + 176)sin4° + (6 + 174)sin6° + ...+ sin88°

= 180°(sin2° + sin4° + sin6° + ...) + 90sin90°

$$= 180 \times \frac{\sin\left(44 \cdot \frac{2}{2}\right)}{\sin\left(\frac{2}{2}\right)} \times \sin\left(2 + (44-1).\frac{2}{2}\right) + 90°$$

$$= 180 \times \frac{\sin 44°}{\sin 1°} \times \sin 45° + 90$$

$$= \frac{180}{\sqrt{2}} \times \frac{\sin(45° - 1°)}{\sin 1} + 90$$

$$= \frac{180}{\sqrt{2}}\left[\frac{\frac{1}{\sqrt{2}}\cos 1° - \frac{1}{\sqrt{2}}\sin 1°}{\sin 1°}\right] + 90$$

$$= \frac{180°}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\cot 1° - \frac{1}{\sqrt{2}}\right] + 90°$$

$$= 90° \cot 1° - \frac{180}{2} + 90$$

= 90cot1°

18. The value of $\tan\frac{\theta}{2}$ (1 + secθ)(1 + sec2θ) (1 + sec2^2θ) .... (1 + sec2^nθ) is

(1) $\tan 2^n\theta$ (2) $\tan 2^{n-1}\theta$ (3) $\tan 2^{n+1}\theta$ (4) $\tan 2^{n-2}\theta$

Sol. Answer (1)

$$\tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)........(1+\sec 2^n\theta)$$

$$= \tan\frac{\theta}{2} \times \frac{1+\tan\theta}{\cos\theta} \times \frac{1+\tan 2\theta}{\cos 2\theta} \times \frac{1+\tan 4\theta}{\cos 4\theta}$$

$$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} \times \frac{2\cos^2\frac{\theta}{2}}{\cos\theta} \times \frac{2\cos^2\theta}{\cos 2\theta} \times \frac{2\cos^2 2\theta}{\cos 4\theta}$$

$$= 2^3 \cdot \sin\frac{\theta}{2}.\cos\frac{\theta}{2}.\frac{\cos\theta.\cos 2\theta}{\cos 4\theta}$$

$= 2^2 \cdot \sin\theta.\cos\theta.\dfrac{\cos 2\theta}{\cos 4\theta}$

$= 2\dfrac{\sin 2\theta \cos 2\theta}{\cos 4\theta}$

= tan4θ

= $\tan 2^2\theta$

$\therefore \quad \tan\dfrac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta).........(1+\sec 2^{2n}\theta)$

= $\tan 2^n\theta$

19. The value of [100(*x* – 1)] is where [*x*] is the greatest integer less than or equal to *x* and $x = \dfrac{\sum_{n=1}^{44°} \cos n°}{\sum_{n=1}^{44°} \sin n°}$

(1) 140 (2) 141 (3) 142 (4) 144

Sol. Answer (2)

$\therefore \quad A = \sum_{n=1}^{44} \cos n°$

= cos1° + cos2° + cos3° + …….. + cos44°

B = sin1° + sin2° + sin3° + ……. + sin44°

$x = \dfrac{A}{B} = \dfrac{\cos 1° + \cos 2° + \cos 3° + ... + \cos 44°}{\sin 1° + \sin 2° + \sin 3° + ... + \sin 44°}$

$= \dfrac{\sin 89° + \sin 88° + \sin 87° + ... + \sin 46°}{\sin 1° + \sin 2° + \sin 3° + ... + \sin 44°}$

$$= \dfrac{\dfrac{\sin\left(44.\dfrac{1}{2}\right)}{\sin\left(\dfrac{1}{2}\right)} \times \cos\left(1 + (44-1)\cdot\dfrac{1}{2}\right)}{\dfrac{\sin\left(44.\dfrac{1}{2}\right)}{\sin\dfrac{1}{2}} \times \sin\left(1 + (44-1)\dfrac{1}{2}\right)}$$

$$= \dfrac{\cos\left(22 + \dfrac{1}{2}\right)}{\sin\left(22 + \dfrac{1}{2}\right)} = \cot 22\dfrac{1}{2}°$$

$= \sqrt{2} + 1$

$\therefore \quad \left[100\left(\sqrt{2} + 1 - 1\right)\right] = [100 \times 1.41] = 141$

20. If $f(\alpha, \beta) = \cos^2\alpha + \sin^2\alpha \cdot \cos 2\beta$, then which of the following is incorrect?

(1) $f\left(\frac{\pi}{5}, \frac{2\pi}{5}\right) \neq f\left(\frac{2\pi}{5}, \frac{\pi}{5}\right)$

(2) $f\left(\frac{\pi}{12}, \frac{\pi}{3}\right) = f\left(\frac{\pi}{3}, \frac{\pi}{12}\right)$

(3) $3f\left(\frac{\pi}{5}, \frac{\pi}{3}\right) \neq f\left(\frac{\pi}{3}, \frac{\pi}{5}\right)$

(4) $f\left(\frac{\pi}{4}, \frac{\pi}{18}\right) \neq 3f\left(\frac{\pi}{18}, \frac{\pi}{4}\right)$

Sol. Answer (1)

$f(\alpha, \beta) = \cos^2\alpha + \sin^2\alpha.\cos_2\beta$

$= 1 - \sin^2\alpha + \sin^2\alpha.\cos^2\beta$

$= 1 - (1 - \cos 2\beta)\sin^2\alpha$

$= 1 - (1 - \cos^2\beta)\sin^2\alpha$

$= 1 - 2\sin^2\alpha \sin^2\beta$

$$f\left(\frac{\pi}{12}, \frac{\pi}{3}\right) = 1 - 2\sin^2\frac{\pi}{12}\sin^2\frac{\pi}{3}$$

$$= 1 - 2 \cdot \frac{\left(\sqrt{3}-1\right)^2}{8} \times \frac{3}{4}$$

$$= 1 - \frac{3\left(4 - 2\sqrt{3}\right)}{16}$$

$$= 1 - \frac{3\left(2 - \sqrt{3}\right)}{8} = \frac{8 - 6 + 3\sqrt{3}}{8} = \frac{2 + 3\sqrt{3}}{8}$$

21. Let $f(x) = \cos 10x + \cos 8x + 3\cos 4x + 3\cos 2x$ and $g(x) = 8\cos x \cdot \cos^3 3x$, then for all x we have

(1) $f(x) = g(x)$ (2) $2f(x) = 3g(x)$ (3) $f(x) = 2g(x)$ (4) $2f(x) = g(x)$

Sol. Answer (1)

$f(x) = \cos 10x + \cos 8x + 3\cos 4x + 3\cos 2x$

$g(x) = 8\cos x.\cos^3 3x$

$= 8\cos x(4\cos^3 x - 3\cos x)^3$

$= 8\cos x\ ((4\cos^3 x)^3 - 3.(4\cos^3 x)^2.3\cos x + 3.(4\cos^3 x)(3\cos x)^2 - (3\cos x)^3)$

$= 8\cos x(64\cos^9 x - 144\cos^7 x + 108\cos^5 x - 27\cos^3 x)$

$= 8\cos^4 x(64\cos^6 x - 144\cos^4 x + 108\cos^2 x - 27)$

$\therefore$ $f(x) = \cos 10x + \cos 8x + 3\cos 4x + 3\cos 2x$

22. $\sum_{r=1}^{n}\left(\frac{1}{\cos\theta + \cos(2r+1)\theta}\right)$, $n \in N$ is equal to

(1) $\frac{\sin(n+1)\theta}{\sin\theta \cdot \cos n\theta}$ (2) $\frac{\sin n\theta}{\sin 2\theta \cdot \cos(n+1)\theta}$ (3) $\frac{\tan(n+1)\theta}{\sin n\theta}$ (4) $\frac{\sin(n-1)\theta}{\sin\theta \cdot \cos n\theta}$

Sol. Answer (2)

$$\sum_{r=1}^{n}\left(\frac{1}{\cos\theta+\cos(2r+1)\theta}\right)$$

$$=\frac{1}{\cos\theta+\cos3\theta}+\frac{1}{\cos\theta+\cos5\theta}+.........+\frac{1}{\cos\theta+\cos(2n+1)\theta}$$

$$=\frac{1}{2\cos2\theta\cdot\cos\theta}+\frac{1}{2\cos3\theta\cdot\cos2\theta}+.........+\frac{1}{2\cos(n+1)\theta.\cos n\theta}$$

$$=\frac{1}{2\sin\theta}\left[\frac{\sin(2\theta-\theta)}{\cos2\theta.\cos\theta}+\frac{\sin(3\theta-2\theta)}{\cos3\theta\cos2\theta}+........\right]$$

$$=\frac{1}{2\sin\theta}\left[\frac{\sin2\theta\cos\theta-\cos2\theta\sin\theta}{\cos2\theta.\cos\theta}+\frac{\sin3\theta\cos2\theta-\cos3\theta\sin2\theta}{\cos3\theta\cdot\cos2\theta}+........\right]$$

$$=\frac{1}{2\sin\theta}\ [\tan2\theta-\tan\theta+\tan3\theta-\tan2\theta+.........\tan(n+1)\theta-\tan n\theta]$$

$$=\frac{1}{2\sin\theta}[\tan(n+1)\theta-\tan\theta]$$

$$=\frac{1}{2\sin\theta}\times\frac{\sin(n+1-1)\theta}{\cos(n+1)\theta\cos\theta}$$

$$=\frac{\sin n\theta}{\cos(n+1)\theta.\sin2\theta}$$

23. The minimum value of $27^{\cos 3x}.81^{\sin 3x}$ is

(1) 1 (2) $\frac{1}{81}$ (3) $\frac{1}{243}$ (4) $\frac{1}{27}$

Sol. Answer (3)

Let $y = 27^{\cos 3x}.81^{\sin 3x}$

$= 3^{3\cos 3x}.3^{4\sin 3x}$

$= 3^{3\cos 3x + 4\sin 3x}$

Then minimum value of y is

$3^{-5} = \frac{1}{243}$

24. Given that $\sum_{k=1}^{35}\sin5k^\circ=\tan\left(\frac{m}{n}\right)^\circ$, where m and n are relatively prime positive integers that satisfy $\left(\frac{m}{n}\right)^\circ<90^\circ$, then $m + n$ is equal to

(1) 173 (2) 175 (3) 177 (4) 179

Sol. Answer (3)

$$\sum_{k=1}^{35} \sin 5k = \tan\frac{m^\circ}{n^\circ}$$

$$\Rightarrow \sin 5 + \sin 10 + ... + \sin 5.35 = \tan\frac{m^\circ}{n^\circ}$$

$$\Rightarrow \frac{\sin\left(35\cdot\frac{5}{2}\right)}{\sin\left(\frac{5}{2}\right)}\times\sin\left(5+(35-1)\times\frac{5}{2}\right) = \tan\frac{m^\circ}{n^\circ}$$

$$\Rightarrow \frac{\sin\left(\frac{175^\circ}{2}\right)}{\sin\left(\frac{5^\circ}{2}\right)}\times\sin\left(\frac{175+5}{2}\right) = \tan\frac{m^\circ}{n^\circ}$$

$$\Rightarrow \frac{\sin\left(\frac{175^\circ}{2}\right)}{\sin\left(\frac{5^\circ}{2}\right)} = \tan\frac{m}{n}$$

$$\Rightarrow \frac{\sin\left(\frac{175}{2}\right)}{\cos\left(\frac{175}{2}\right)} = \frac{\sin\left(\frac{m}{n}\right)}{\cos\left(\frac{m}{n}\right)}$$

$$\therefore \frac{m}{n} = \frac{175}{2}$$

$\therefore$ $m + n = 175 + 2 = 177$

25. For $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$ lies in the interval

(1) $(-\infty, \infty)$ (2) $(-2, 2)$ (3) $(0, \infty)$ (4) $(-1, 1)$

Sol. Answer (1)

$$\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$$

$$= \frac{\sin\theta(1 + 2\cos\theta)}{\cos\theta(1 + 2\cos\theta)}$$

$= \tan\theta$; $\tan\theta \in (-\infty, \infty)$

26. If $A + B + C = \pi$ and $\sin\left(A + \frac{C}{2}\right) = K\sin\frac{C}{2}$, then $\tan\frac{A}{2}\cdot\tan\frac{B}{2}$ is equal to

(1) $\frac{K-1}{K+1}$ (2) $\frac{K+1}{K-1}$ (3) $\frac{K}{K+1}$ (4) $\frac{K+1}{K}$

Sol. Answer (1)

$A + B + C = \pi$

$$\sin\left(A+\frac{C}{2}\right) = K\sin\frac{C}{2}$$

$$\Rightarrow \frac{\sin\left(A+\frac{C}{2}\right)}{\sin\frac{C}{2}} = \frac{K}{1}$$

$$\Rightarrow \frac{\sin\left(A+\frac{C}{2}\right)+\sin\frac{C}{2}}{\sin\left(A+\frac{C}{2}\right)-\sin\frac{C}{2}} = \frac{K+1}{K-1}$$

$$\Rightarrow \frac{2\sin\left(\frac{A+C}{2}\right)\cos\frac{A}{2}}{2\cos\left(\frac{A+C}{2}\right)\sin\frac{A}{2}} = \frac{K+1}{K-1}$$

$$\Rightarrow \frac{\cos\frac{B}{2}}{\sin\frac{B}{2}}.\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{K+1}{K-1}$$

$$\Rightarrow \sin\frac{A}{2}.\sin\frac{B}{2} = \frac{K-1}{K+1}$$

[Trigonometric Equations]

27. The number of the solutions of the equation $3\sin x + 4\cos x - x^2 - 16 = 0$ is

(1) 3 (2) 2 (3) 1 (4) 0

Sol. Answer (4)

$3\sin x + 4\cos x = x^2 + 16$

Range of LHS = [–5, 5]

Range of RHS = [16, ∞)

∴ No solution exists.

28. The solution set of $\sin^4 x - \tan^8 x = 1$ is given by

(1) $x = 2n\pi + \frac{\pi}{8}, n \in I$ (2) $x = n\pi + \frac{\pi}{12}, n \in I$

(3) $x = 2n\pi + \frac{\pi}{24}, n \in I$ (4) None of these

Sol. Answer (4)

$\sin^4 x - \tan^8 x = 1$

$\Rightarrow \sin^4 x = 1 + \tan^8 x$

Range of LHS : [0, 1]

Range of RHS : [1, ∞]

Only solution exists when LHS and RHS are both equal to 1.

$\sin^4 x = 1,\ 1 + \tan^8 x = 1$

$\sin^2 x = 1,\ \tan^8 x = 0$

$\Rightarrow\ x = n\pi + \frac{\pi}{2}$ but at these value $\tan^8 x$ 0.

29. The general solution of the equation tan2θ tan3θ = 1 is

(1) $\theta = (4n + 1)\frac{\pi}{10},\ n \neq 4k - 2,\ k \in I$

(2) $\theta = (2n + 3)\frac{\pi}{10},\ n \neq 5k + 2,\ k \in I$

(3) $\theta = (4n + 3)\frac{\pi}{10},\ n \neq 5k + 2,\ k \in I$

(4) $\theta = (2n + 1)\frac{\pi}{10},\ n \neq 5k - 3,\ k \in I$

Sol. Answer (4)

30. If $m, n \in N (n > m)$, then number of solutions of the equation $n|\sin x| = m|\sin x|$ in $[0, 2\pi]$ is

(1) m (2) n (3) mn (4) 3

Sol. Answer (4)

$n|\sin x| = m|\sin x|$

$\Rightarrow\ (n - m)\,|\sin x| = 0$

Since $n > m\ \therefore\ |\sin x| = 0$

$\Rightarrow\ x = n\pi$

In $[0, 2\pi]$, $x = 0, \pi, 2\pi$

31. If $\sin x + \cos x = \sqrt{y + \frac{1}{y}},\ x \in (0, \pi)$, then

(1) $x = \frac{\pi}{4},\ y = 1$ (2) $y = 0$ (3) $y = 2$ (4) $x = \frac{3\pi}{4}$

Sol. Answer (1)

$\sin x + \cos x = \sqrt{y + \frac{1}{y}}$ $\qquad x \in (0, \pi)$

Comparing range on both sides,

LHS = $\left[-\sqrt{2}, \sqrt{2}\right]$

RHS = $\left[\sqrt{2}, \infty\right)$

Only solution exists $\sin x + \cos x = \sqrt{2}$

$\sin\left(x + \frac{\pi}{4}\right) = 1$

$\Rightarrow\ x + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{2}$

$\Rightarrow\ x = \frac{\pi}{4}$

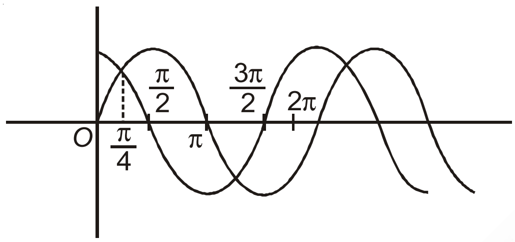
Also, $\sqrt{y + \frac{1}{y}} = \sqrt{2} \Rightarrow y = 1$

32. The solution of the inequality $\log_{1/2}\sin x > \log_{1/2}\cos x$ is

(1) $x \in \left(0, \frac{\pi}{2}\right)$ (2) $x \in \left(0, \frac{\pi}{8}\right)$ (3) $x \in \left(0, \frac{\pi}{4}\right)$ (4) $x \in \left[0, \frac{\pi}{4}\right]$

Sol. Answer (3)

$\log_{1/2} \sin x > \log_{1/2} \cos x \Rightarrow \sin x < \cos x$



From graph $\sin x < \cos x$ in $\left(0, \frac{\pi}{4}\right)$

33. The solution set of the equation $4\sin^4 x + \cos^4 x = 1$ is

(1) $x = n\pi + \frac{\pi}{6}$

(2) $x = n\pi \pm \theta$ where $\cos^2\theta = \frac{3}{5}$ and $x = n\pi$, $n \in I$

(3) $x = (2n+1)\frac{\pi}{2}$

(4) $x = 2n\pi \pm \frac{\pi}{3}$

Sol. Answer (2)

$4\sin^4 x + \cos^4 x = 1$

$\Rightarrow\ 4\sin^4 x + (1 - \sin^2 x)^2 = 1$

$\Rightarrow\ 4\sin^4 x + 1 + \sin^4 x - 2\sin^2 x = 1$

$\Rightarrow\ 5\sin^4 x - 2\sin^2 x = 0$

$\Rightarrow\ \sin^2 x(5\sin^2 x - 2) = 0$

$\Rightarrow\ \sin^2 x = 0, \sin^2 x = \frac{2}{5}$

$\Rightarrow\ x = n\pi$ or $x = n\pi \pm \theta$, where $\sin^2\theta = \frac{2}{5}, \cos^2\theta = \frac{3}{5}$, $n \in I$

34. The number of values of $\theta \in [0, 2\pi]$ satisfying $r\sin\theta = \sqrt{3}$ and $r + 4\sin\theta = 2(\sqrt{3}+1)$ is

(1) 4 (2) 5 (3) 6 (4) 7

Sol. Answer (1)

$r\sin\theta = \sqrt{3}$

$r + 4\sin\theta = 2(\sqrt{3}+1)$

$$r+\frac{4\sqrt{3}}{r}=2\sqrt{3}+2$$

$$\Rightarrow r^2-2(\sqrt{3}+1)r+4\sqrt{3}=0$$

$$r(r-2\sqrt{3})-2(r-2\sqrt{3})=0$$

$$r=2 \text{ or } r=2\sqrt{3}$$

$$\sin\theta=\frac{\sqrt{3}}{2},\ \sin\theta=\frac{1}{2}$$

$\therefore$ 4 solutions exist in $[0, 2\pi]$

35. How many solutions does the equation $\sec x-1=(\sqrt{2}-1)\tan x$ have in the interval $(0, 6\pi]$?

(1) 6 (2) 5 (3) 10 (4) 9

Sol. Answer (1)

$$\sec x-1=(\sqrt{2}-1)\tan x$$

$$\frac{1-\cos x}{\cos x}=\frac{(\sqrt{2}-1)\sin x}{\cos x}$$

$$\Rightarrow (\sqrt{2}-1)\sin x+\cos x=1$$

Using auxiliary form, 6 solutions exist.

36. The general solution of the equation $\tan(x+20^\circ)\tan(x-40^\circ)=\tan(x-20^\circ)\tan(x+40^\circ)$ is

(1) $x = n\pi,\ n \in Z$

(2) $x=n\pi+\frac{\pi}{4},\ n\in Z$

(3) $x=\frac{n\pi}{2}, n\in Z$

(4) $x=2n\pi+\frac{\pi}{4},\ n\in Z$

Sol. Answer (3)

$$\frac{\tan(x+20^\circ)}{\tan(x-20^\circ)}=\frac{\tan(x+40^\circ)}{\tan(x-40^\circ A)}$$

$$\Rightarrow \frac{\sin(x+20^\circ)\cos(x-20^\circ)}{\sin(x-20^\circ)\cos(x+20^\circ)}=\frac{\sin(x+40^\circ)\cdot\cos(x-40)}{\cos(x+40)\sin(x-40)}$$

Applying components-dividends

$$\Rightarrow \frac{\sin 2x}{\sin 40^\circ}=\frac{\sin(2x)}{80^\circ}$$

$\Rightarrow \sin 2x = 0$

$\Rightarrow 2x = n\pi$

$$\Rightarrow x=\frac{n\pi}{2},\ n\in Z$$

37. Let $x \in [0, 2\pi]$. The curve $y = \sec x \tan x + 2\tan x - \sec x$ and the line $y = 2$ intersect in

(1) No point (2) 2 points (3) 3 points (4) 4 points

Sol. Answer (4)

$\sec x \tan x + 2\tan x - \sec x - 2 = 0$

$\Rightarrow (\sec x + 2)(\tan x - 1) = 0$

$\Rightarrow \tan x = 1, \sec x = -2$

$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, x = \frac{2\pi}{3}, \frac{4\pi}{3}$ (in $[0, 2\pi]$)

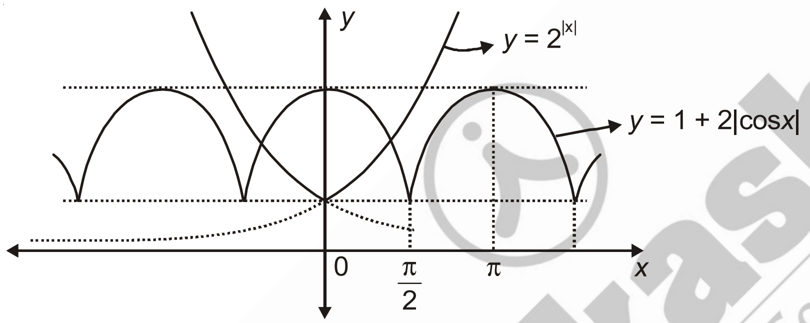
The curve and the line intersect is 4 points

38. The number of solutions of the equation $2^{|x|} = 1 + 2|\cos x|$ is

(1) 0 (2) 2 (3) 4 (4) Infinite

Sol. Answer (2)

Clearly from graph these are two solution of the given equation



39. If the equation $1 + \sin^2 x\theta = \cos\theta$ has a non-zero solution in θ, then x must be

(1) An integer

(2) A rational number

(3) An irrational number

(4) None of these

Sol. Answer (2)

Given $1 + \sin^2 x\theta = \cos\theta$

OR $\cos\theta = 1 + \sin^2 x\theta$

It is possible iff

$\sin^2 x\theta = 0$ and $\cos\theta = 1$

$x\theta = n\pi$ and $\theta = 2m\pi$ where $n; m \in I$

$\therefore \quad x = \frac{n\pi}{2m\pi}$

$x = \frac{n}{2m}$

40. The smallest positive values of x (in degrees) such that $\tan(x + 100°) = \tan(x + 50°)\tan x \cdot \tan(x - 50°)$ is equal to zero

(1) 60° (2) 30° (3) 40° (4) 50°

Sol. Answer (2)

$$\frac{\tan(x+100°)}{\tan(x-50°)} = \tan(x+50°).\tan x$$

$$\Rightarrow \frac{2\sin(x+100°)\cos(x-50°)}{2\sin(x-50°)\cos(x+100°)} = \frac{2\sin(x+50)\sin x}{2\cos(x+50°)\cos x}$$

$$\Rightarrow \frac{\sin(2x+50°)+\sin 150°}{\sin(2x+50°)-\sin 150°} = \frac{\cos 50° - \cos(2x+50)}{\cos(2x+50°)+\cos 50°}$$

$$\Rightarrow \frac{2\sin(2x+50°)}{1} = \frac{2\cos 50°}{-2\cos(2x+50°)}$$

$\Rightarrow$ $2\sin(2x + 50°) \cos(2x + 50°) = -\cos 50°$

$\Rightarrow$ $\text{Sin}(4x + 100°) = -\sin 40° = \sin 220°, \sin 320°$

$\Rightarrow$ $4x + 100 = 220, 320$

$\Rightarrow$ $4x = 120, 220$

$\Rightarrow$ $x = 30, 55$

[Height and Distance]

41. An aeroplane flying at a height of 300 m above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Then the height of the lower plane from the ground is

(1) $100\sqrt{3}$ m (2) $\frac{100}{\sqrt{3}}$ m (3) 50 m (4) $150(\sqrt{3}+1)$ m

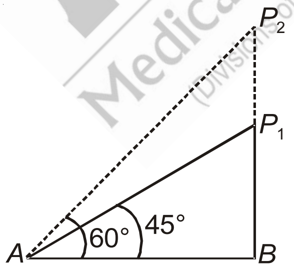
Sol. Answer (1)

Given BP_2 = 300 m

In ΔABP_2

$$\frac{BP_2}{AB} = \tan 60° = \sqrt{3}$$

$$\therefore \quad AB = \frac{300}{\sqrt{3}} = 100\sqrt{3} \text{ m}$$



Also in ΔABP_1

$$\frac{P_1B}{AB} = \tan 45° = 1$$

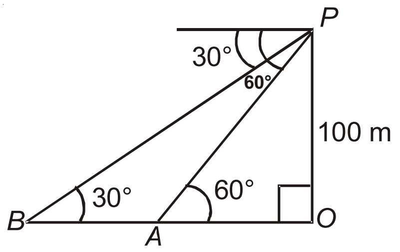
$$\therefore \quad P_1B = 100\sqrt{3} \text{ m}$$

42. A man from the top of a 100 m high tower sees a car moving towards the tower at an angle of depression 30°. After sometime, the angle of depression becomes 60°. The distance travelled by the car during this time is

(1) $100\sqrt{3}$ m (2) $\frac{200}{\sqrt{3}}$ m (3) $200\sqrt{3}$ m (4) $\frac{100}{\sqrt{3}}$ m

Sol. Answer (2)

In ΔPOA



$$\frac{PO}{OA} = \tan 60° = \sqrt{3}$$

$$OA = \frac{100}{\sqrt{3}} \text{ m}$$

Also, in ΔPOB

$$\frac{PO}{OB} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \quad OB = PO\sqrt{3} = 100\sqrt{3}$$

$$\therefore \quad AB = OB - OA = 100\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{200}{\sqrt{3}}$$

43. ABC is a triangular park with $AB = AC = 100$ m. A clock tower is situated at the midpoint of BC. The angles of elevation of top of the tower at A and B are $\cot^{-1}(3.2)$ and $\text{cosec}^{-1}(2.6)$ respectively. The height of tower is

(1) $\frac{25}{2}$ m (2) 25 m (3) 50 m (4) None of these

Sol. Answer (2)

Let $OP = h$ be the tower of height given that $AB = AC = 100$ m

$BO = OC$

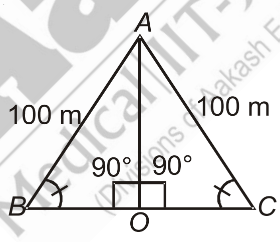
Clearly

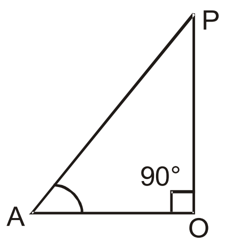
So $OA = h\cot(\angle PAO)$

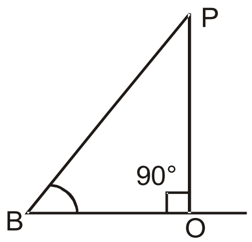
$= 3.2\,h$

$OB = h\cot(\angle PBO)$

Hence in right angle triangle AOB







$AO^2 + OB^2 = AB^2$

$(3.2h)^2 + [h\cot(PBO)]^2 = 100^2$

$h^2[3.2^2 + \text{cosec}^2(PBO) - 1] = 100^2$

$h^2\,[10.24 + 6.76 - 1] = 100^2$

$\therefore \quad h^2\,[16] = 100^2$

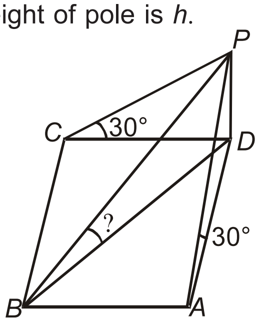
$$h = \frac{100}{4} = 25 \text{ m}$$

44. $ABCD$ is a square plot. The angle of elevation of the top of a pole standing on D from A or C is 30° and that from B is θ, then $\tan\theta$ is equal to

(1) $\sqrt{6}$ (2) $\frac{1}{\sqrt{6}}$ (3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{2}}{\sqrt{3}}$

Sol. Answer (2)

Let the side of square be x, then $BD = x\sqrt{2}$. Let the height of pole is h.



Then,

$$\frac{PD}{DA} = \frac{h}{x} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{x}{\sqrt{3}} \qquad ....(i)$$

$$\frac{PD}{DB} = \frac{h}{x\sqrt{2}} = \tan\theta \qquad ....(ii)$$

$$\therefore \quad \tan\theta = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{6}}$$

45. The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is

(1) 20 m (2) 40 m (3) 60 m (4) 80 m

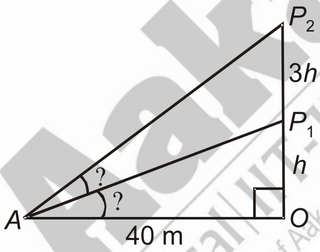
Sol. Answer (2)

As in adjacent diagram

$$\tan\theta = \frac{3}{5}$$

Let angle $P_1AO = \alpha$

In ΔP_1AO



$$\frac{h}{40} = \tan\alpha$$

$h = 40\tan\alpha$

Also $\tan(\theta + \alpha) = \frac{4h}{40}$

$$\Rightarrow \quad \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha} = \frac{4h}{40} = \frac{h}{10}$$

$$\Rightarrow \quad \frac{\frac{3}{5} + \frac{h}{40}}{1 - \frac{3h}{200}} = \frac{h}{10} \Rightarrow \frac{120 + 5h}{200 - 3h} = \frac{h}{10}$$

$\Rightarrow$ $1200 + 50h = 200h - 3h^2$

$\Rightarrow$ $3h^2 - 150h + 1200 = 0$

$\Rightarrow$ $h^2 - 50h + 400 = 0$

$\Rightarrow$ $(h - 40)(h - 10) = 0$

$h = 10$ m or $h = 40$ m

Hence height of pole is $4h = 40$ m or 160 m

[Properties of Triangle]

46. If $a = 16$, $b = 24$ and $c = 20$, then the value of $\cos\left(\frac{B}{2}\right)$ is

(1) $\frac{3}{4}$ (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

Sol. Answer (1)

Find $\cos B$ and then $\cos 2\theta = 2\cos^2\theta - 1$

47. If $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$, then ΔABC is

(1) Isosceles (2) Equilateral (3) Right angled (4) Scalene

Sol. Answer (2)

Given,

$$\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$$

Using sine rule,

$\tan A = \tan B = \tan C$

i.e., $A = B = C$

$\therefore$ Triangle is equilateral.

48. In triangle ABC, if $\tan\left(\frac{A-B}{2}\right) = \frac{3}{11}$ and $\frac{a}{b} = \frac{7}{4}$ then the value of angle C is

(1) 30° (2) 60° (3) 90° (4) 45°

Sol. Answer (3)

Given that $\tan\left(\frac{A-B}{2}\right) = \frac{3}{11}$ and $\frac{a}{b} = \frac{7}{4} \Rightarrow \frac{a-b}{a+b} = \frac{3}{11}$

We know that $\frac{A-B}{2} = \frac{a-b}{a+b} \cdot \cot\frac{C}{2}$

$$\frac{3}{11} = \frac{3}{11}.\cot\frac{C}{2} \Rightarrow \cot\frac{C}{2} = 1 \Rightarrow \boxed{C = 90^\circ}$$

49. In a triangle, the length of two larger sides are 24 and 22 respectively. If the angles are in AP, then the third side is

(1) $12 + 2\sqrt{13}$ (2) $12 - 3\sqrt{3}$ (3) $2\sqrt{3} + 2$ (4) $2\sqrt{3} - 2$

Sol. Answer (1)

Given that, angles A, B and C are in $A.P$

$\Rightarrow$ $B = 60^\circ$

We know that $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \frac{1}{2} = \frac{(24)^2 + c^2 - (22)^2}{2 \times 24 \times c} \Rightarrow c^2 + 92 = 24c$

$\Rightarrow c^2 - 24c + 92 = 0$

$\Rightarrow c = 12 + 2\sqrt{13}$

50. In a triangle ABC, $a = 4$, $b = 3$, $\angle A = 60^\circ$, then c is the root of the equation

(1) $c^2 - 3c - 7 = 0$ (2) $c^2 + 3c + 7 = 0$ (3) $c^2 - 3c + 7 = 0$ (4) $c^2 + 3c - 7 = 0$

Sol. Answer (1)

We know that

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$\Rightarrow \frac{1}{2} = \frac{9 + c^2 - 16}{2 \times 3 \times c} \Rightarrow c^2 - 3c - 7 = 0$

51. In triangle ABC, point D lies on BC such that $\angle BAD = 45°$, $\angle DAC = 30°$. If $BD : DC = 2 : 1$, then $3\cot\angle ADC$ is equal to

(1) $2 + \sqrt{3}$ (2) $2 - \sqrt{3}$ (3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

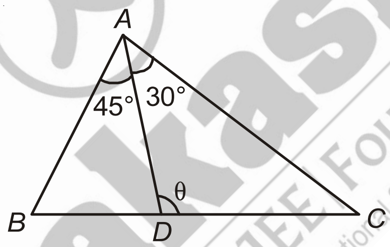
Sol. Answer (2)

Given that $BD : DC = 2 : 1$

So from (m, n) theorem

$(2 + 1)\cot\theta = 2\cot 45 - 1.\cot 30$

$3\cot\theta = 2 - \sqrt{3}$



52. In ΔABC, $\tan\frac{A}{2} = \frac{5}{6}$, $\tan\frac{C}{2} = \frac{2}{5}$, then

(1) a, b, c are in A.P. (2) a, c, b are in A.P.

(3) b, a, c are in A.P. (4) a, b, c are in G.P.

Sol. Answer (1)

Given that $\tan\frac{A}{2} = \frac{5}{6}, \tan\frac{c}{2} = \frac{2}{5}$

So, $\tan\frac{A}{2}, \tan\frac{C}{2} = \frac{5}{6} \times \frac{2}{5}$

$$\sqrt{\frac{(s-b)(s-c)}{s(s-a)} \times \frac{(s-a)(s-b)}{s(s-c)}} = \frac{1}{3}$$

$\Rightarrow \frac{s-b}{s} = \frac{1}{3} \Rightarrow \frac{a+c-b}{a+b+c} = \frac{1}{3}$

$\Rightarrow 2b = a + c$

Hence a, b, c are in A.P.

53. If in a triangle ABC, $\Delta = a^2 - (b - c)^2$, then $\tan A$ is equal to

(1) $\frac{15}{16}$ (2) $\frac{8}{15}$ (3) $\frac{8}{17}$ (4) $\frac{1}{2}$

Sol. Answer (2)

Given that $\Delta = a^2 - (b - c)^2$

$$\sqrt{s(s-a)(s-b)(s-c)} = 4(s-b)(s-c)$$

$$\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{1}{4} = \tan\frac{A}{2} \quad \left[\because \tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\right]$$

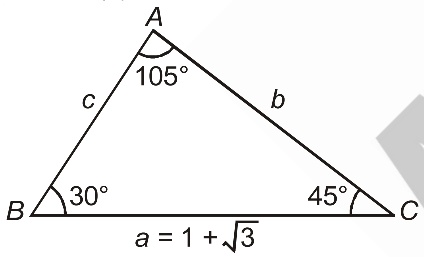
$$\text{Now, } \tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{8}{15}$$

$$\Rightarrow \quad \tan A = \frac{8}{15}$$

54. If the angles of a triangle are 30° and 45° and the included side is $\left(1+\sqrt{3}\right)$ cm, then the area of the triangle is (in square cm)

(1) $2\left(1+\sqrt{3}\right)$ (2) $\frac{1}{2}\left(1+\sqrt{3}\right)$ (3) $2\left(\sqrt{3}-1\right)$ (4) $\frac{1}{2}\left(\sqrt{3}-1\right)$

Sol. Answer (2)



From Sine Rule

$$\frac{1+\sqrt{3}}{\sin 105} = \frac{c}{\sin 45} \Rightarrow c = 2$$

$$\text{Hence Area } = \frac{1}{2}ac\sin B = \frac{1}{2}(1+\sqrt{3})$$

55. In ΔABC, if $ar(\Delta ABC) = 8$. Then, $a^2\sin(2B) + b^2\sin(2A)$ is equal to

(1) 2 (2) 16 (3) 32 (4) 128

Sol. Answer (3)

Given that $\Delta = 8$

Now, $a^2\sin^2B + b^2\sin^2A = 2a^2\sin B\cos B + 2b^2\sin A\cos A$

$$= 2a^2\cdot\frac{b}{2R}.\cos B + 2b^2\cdot\frac{a}{2R}\cdot\cos A$$

$= \dfrac{ab}{R}(a\cos B + b\cos A)$

$= \dfrac{abc}{R} = 4\left(\dfrac{abc}{4R}\right) = 4\Delta = 32$

$\Rightarrow \boxed{a^2 \sin 2B + b^2 \sin 2A = 32}$

56. With usual notations, if in a triangle *ABC*,

$\dfrac{b+c}{11} = \dfrac{c+a}{12} = \dfrac{a+b}{13}$, then cos*A* : cos*B* : cos*C* is equal to

(1) 7 : 19 : 25 (2) 19 : 7 : 25 (3) 12 : 14 : 20 (4) 19 : 25 : 20

Sol. Answer (1)

Let $\dfrac{b+c}{11} = \dfrac{c+a}{12} = \dfrac{a+b}{13} = k$

$\therefore$ $b + c = 11\ k$

$c + a = 12\ k$

$a + b = 13\ k$

$2(a + b + c) = 36\ k$

$a + b + c = 18\ k$

$\therefore$ $a = 7\ k,\ b = 6\ k,\ c = 5\ k$

cos*A* : cos*B* : cos*C*

$\dfrac{b^2+c^2-a^2}{2bc} : \dfrac{a^2+c^2-b^2}{2ac} : \dfrac{a^2+b^2-c^2}{2ab}$

Put the values of *a, b* & *c*

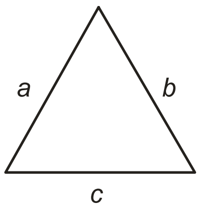
7 : 19 : 25

So option (1) is correct

57. If the perimeter of a triangle is 6, then its maximum area is

(1) $\sqrt{3}$ (2) 3 (3) 4 (4) 2

Sol. Answer (1)



$a + b + c = 6$

$\dfrac{(s-a)+(s-b)+(b-c)}{3} \ge \{(s-a)(s-b)(s-c)\}^{1/3}$

$\dfrac{s}{3} \ge \left(\dfrac{s(s-a)(s-b)(s-c)}{s}\right)^{1/3}$

$$\frac{s}{3} \geq \left(\frac{A^2}{s}\right)^{1/3}$$

$$\frac{s}{3} \geq \frac{A^{2/3}}{s^{1/3}}$$

$$\frac{s^{4/3}}{3} \geq A^{2/3}$$

$$\left(\frac{s^{4/3}}{3}\right)^{3/2} \geq A$$

$$\frac{s^2}{3\sqrt{3}} \geq A$$

$\therefore$ max $A = \sqrt{3}$

Hence, option(1) is correct.

58. In a triangle ABC, if $A - B = 120°$ and $R = 8r$, then the value of $\cos C$ is

(1) $\frac{7}{8}$ (2) $\frac{3}{4}$ (3) $\frac{4}{5}$ (4) $\frac{3}{5}$

Sol. Answer (1)

$A - B = 120°$

$R = 8r$

$$r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$\frac{R}{8} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$\frac{1}{16} = \left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right)\sin\frac{C}{2}$$

$$\frac{1}{16} = \left(\frac{1}{2} - \cos\frac{A+B}{2}\right)\sin\frac{C}{2}$$

$$\frac{1}{16} = \left(\frac{1}{2} - \sin\frac{C}{2}\right)\sin\frac{C}{2}$$

$$\frac{1}{16} = \left(\frac{1}{2} - x\right)x$$

Let $\sin\frac{C}{2} = x$

$\Rightarrow$ $16x^2 - 8x + 1 = 0$

$\Rightarrow\ x = \dfrac{1}{4}$

$\Rightarrow\ \sin\dfrac{C}{2} = \dfrac{1}{4}$

$\Rightarrow\ \cos C = 1 - 2\sin^2 C/2 = \dfrac{7}{8}$

Hence option (1) is correct.

59. If in a triangle $2(a\cos B + b\cos C + c\cos A) = a + b + c$, then

(1) The triangle is isosceles
(2) The triangle is equilateral
(3) The triangle is isosceles right angled
(4) The triangle is right angled

Sol. Answer (1)

$2[a\cos B + b\cos C + C\cos A] = a + b + c$

$\Rightarrow 2[2R\sin A\cos B + 2R\sin B\cos C + 2R\sin C\cos A] = a + b + c$

$\Rightarrow 2R[2\sin A\cos B + 2\sin B\cos C + 2\sin C\cos A] = 2R(\sin A + \sin B + \sin C)$

$\Rightarrow \sin[A + B) + \sin(A - B) + \sin(B + C) + \sin(B - C) + \sin(C - A) + \sin(C + A) = \sin A + \sin B + \sin C$

$\Rightarrow \sin(A - B) + \sin(B - C) + \sin(S - A) = 0$

$\Rightarrow\ \sin\dfrac{A-C}{2}\sin\dfrac{C-B}{2}\sin\dfrac{B-A}{2} = 0$

which is only possible when $A = B$ or $B = C$ or $A = C$

$\therefore$ Triangle is isosceles.

Hence, option (1) is correct.

60. In ΔABC, if $8R^2 - a^2 - b^2 = c^2$ then the triangle must be

(1) Equilateral
(2) Right angle
(3) Scalene
(4) Isoceles angle

Sol. Answer (2)

61. In ΔABC, the expression $\dfrac{b^2-c^2}{a\sin(B-C)} + \dfrac{c^2-a^2}{b\sin(C-A)} + \dfrac{a^2-b^2}{c\sin(A-B)}$ is equal to

(1) $\dfrac{3R}{2}$
(2) $3R$
(3) R
(4) $6R$

Sol. Answer (4)

$$\frac{b^2-c^2}{a\sin(B-C)} = \frac{(2R\sin B)^2 - (2R\sin C)^2}{2R\sin A\sin(B-C)}$$

$$= \frac{4R^2(\sin^2 B - \sin^2 C)}{2R\sin A\sin(B-C)}$$

$$= \frac{4R^2\sin(B+C)\sin(B-C)}{2R\sin A\sin(B-C)}$$

$= 2R$ other terms are also $2R$

$\therefore\ 2R + 2R + 2R = 6R$

Hence, option (4) is correct.

62. In a triangle ABC, if $\Delta^2 = \dfrac{a^2b^2c^2}{2(a^2+b^2+c^2)}$, where Δ is the area of the triangle, then the triangle is

(1) Isosceles but not right angled
(2) Right angled
(3) Isosceles right angled
(4) Equilateral

Sol. Answer (2)

Given that $\Delta^2 = \dfrac{a^2b^2c^2}{2(a^2+b^2+c^2)}$

But we know that $\dfrac{abc}{4\Delta} = R \Rightarrow abc = 4\Delta R$

$\Delta^2 = \dfrac{16\Delta^2R^2}{2(a^2+b^2+c^2)} \Rightarrow a^2+b^2+c^2 = 8R^2$

$\Rightarrow \sin^2A + \sin^2B + \sin^2C = 2$ ...(i)

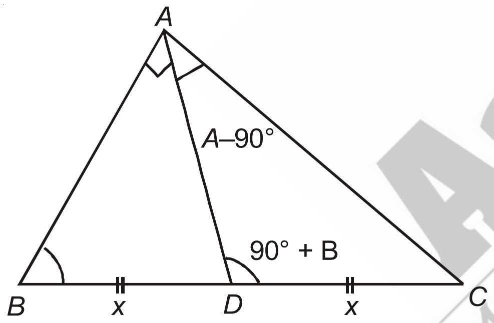
From (i) it is clear that the triangle will be right-angled triangle.

So, option (2) is correct.

63. If the median of triangle ABC through A is perpendicular AB, then the value of $\sin A \cos B + 2\sin B \cos A$ is equal to

(1) 0
(2) 1
(3) $\dfrac{1}{2}$
(4) $\dfrac{1}{3}$

Sol. Answer (1)



From $(m_1\ n)$ theorem

$2x \cot(90 + B) = x\cot 0 - x\cot(A - 90)$

$\Rightarrow\ -2\tan B = \tan A$

$\Rightarrow\ -2\sin B \cos A = \sin A \cos B$

$\Rightarrow\ \sin A\cos B + 2\sin B\cos A = 0$

$\Rightarrow$ Hence option (1) is correct

❑ ❑ ❑