CHAPTER – 10

VECTOR ALGEBRA

Miscellaneous Exercise

Question 1: Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

Answer:

Given: A unit vector in XY- plane.

Let $\vec{r}$ is a unit vector in the given XY- plane then the value of $\vec{r}$

Will be: $\vec{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

θ is the angle given, which is made by unit vector with positive direction of x-axis.

∴ for θ = 30°

$\vec{r} = \cos 30°\,\hat{i} + \sin 30°\,\hat{j}$

$\vec{r} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

Hence, the required unit vector is $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$.

Question 2: Find the scalar components and magnitude of the vector joining the points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2).

Answer:

Given: points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are given.

The vector obtained by joining the given points P and Q:

$\overrightarrow{PQ} =$ position vector of Q – position vector of P

$= (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$

$\overrightarrow{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Hence the scalar component of the vector obtained by joining the points are

$[(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)]$

And the magnitude of the vector obtained by joining the points is

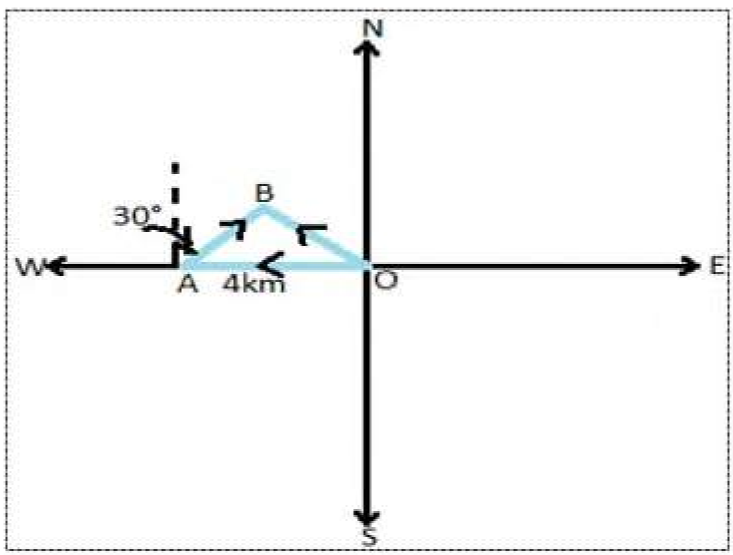
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Question 3: A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Answer:

let O be the initial position and B be the final position of the girl.

Position of girl will be as shown in figure:



Hence, $\overrightarrow{OA} = -4\hat{\imath}$ and $\angle BAO = 60°$

$\overrightarrow{AB} = |\overrightarrow{AB}| \cos 60° \hat{i} + |\overrightarrow{AB}| \sin 60° \hat{j}$

$= 3 \times \frac{1}{2} \hat{i} + 3 \times \frac{\sqrt{3}}{2} \hat{j}$

Now, by using triangle law of vector addition,

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= -4\hat{i} + \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

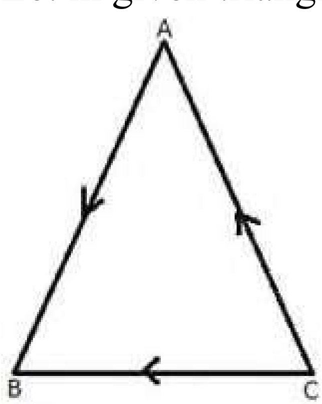
$\Rightarrow \overrightarrow{OB} = \left(\frac{-5}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Hence girls displacement from initial to final position is: $\left(\frac{-5}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Question 4: If $|\vec{a}| = |\vec{b}| + |\vec{c}|$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Answer:

Let in given triangle $\overrightarrow{CB} = \vec{a}, \overrightarrow{CA} = \vec{b}$, and $\overrightarrow{AB} = \vec{c}$,



Now, by using triangle law of vector addition,

$\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB}$,

$\Rightarrow \vec{a} = \vec{b} + \vec{c}$

As we can see that $|\vec{a}|, |\vec{b}|$ and $|\vec{c}|$ represent the sides of triangle.

Also we know that sum of two sides of a triangle must be greater than its third side.

$\Rightarrow |\vec{b}| + |\vec{c}| > |\vec{a}|$

$\Rightarrow |\vec{a}| \neq |\vec{b}| + |\vec{c}|$

$\therefore |\vec{a}| = |\vec{b}| + |\vec{c}|$ is not true.

Question 5: Find the value of *x* for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

Answer:

Given: $x(\hat{i} + \hat{j} + \hat{k})$ as a unit vector.

Now, if $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector then

$|x(\hat{i} + \hat{j} + \hat{k})| = 1$

$\Rightarrow \sqrt{(x-0)^2 + (x-0)^2 + (x-0)^2} = 1$

$\Rightarrow \sqrt{(x)^2 + (x)^2 + (x)^2} = 1$

$\Rightarrow \sqrt{3(x)^2} = 1$

$\Rightarrow x\sqrt{3} = 1$

$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

Question 6: Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

Answer:

Given: $\vec{a} = \left(2\hat{\imath} + 3\hat{\jmath} - \hat{k}\right)$, and $\vec{b} = \left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$

Let the resultant of $\vec{a}$ and $\vec{b}$ is $\vec{c}$

Then $\vec{c} = \vec{a} + \vec{b}$

$\Rightarrow \vec{c} = (2+1)\hat{\imath} + (3-2)\hat{\jmath} + (-1+1)\hat{k}$

$\Rightarrow \vec{c} = (3)\hat{\imath} + (1)\hat{\jmath} + (0)\hat{k}$

$\Rightarrow \vec{c} = 3\hat{\imath} + \hat{\jmath}$

Then

$|\vec{c}| = \sqrt{(3)^2 + (1)^2}$

$\Rightarrow |\vec{c}| = \sqrt{9+1}$

$\Rightarrow |\vec{c}| = \sqrt{10}$

$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{3\hat{\imath}+\hat{\jmath}}{\sqrt{10}}$

Now the vector of magnitude 5 units and parallel to $\hat{c}$ is:

$\pm 5\hat{c} = \pm 5.\frac{3\hat{\imath}+\hat{\jmath}}{\sqrt{10}}$

$= \pm\sqrt{5}\sqrt{5}.\frac{3\hat{\imath}+\hat{\jmath}}{\sqrt{2}\sqrt{5}}$

$= \pm\sqrt{5}.\frac{3\hat{\imath}+\hat{\jmath}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \pm\sqrt{10}.\frac{3\hat{\imath}+\hat{\jmath}}{2}$

$= \pm.\frac{3\sqrt{10}\hat{\imath}}{2} \pm \frac{\sqrt{10}}{2}\hat{\jmath}$

Question 7: If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

Answer:

Given: $\vec{a} = (\hat{i} + \hat{j} + \hat{k})$, $\vec{b} = (2\hat{i} - \hat{j} + 3\hat{k})$, and $\vec{c} = (\hat{i} - 2\hat{j} + \hat{k})$

Let the resultant of $\vec{a}$ and $\vec{b}$ is $\vec{c}$

Then $\overrightarrow{2a} - \vec{b} + 3\vec{c} = 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$

$= (2 - 2 + 3)\,\hat{i} + (2 + 1 - 6)\hat{j} + (2 - 3 + 3)\hat{k}$

$= (3)\,\hat{i} + (-3)\hat{j} + (2)\hat{k}$

$\Rightarrow \overrightarrow{2a} - \vec{b} + 3\vec{c} = 3\hat{i} - 3\hat{j} + 2\hat{k}$

Then

$\left|\overrightarrow{2a} - \vec{b} + 3\vec{c}\right| = \sqrt{(3)^2 + (-3)^2 + (2)^2}$

$\Rightarrow \left|\overrightarrow{2a} - \vec{b} + 3\vec{c}\right| = \sqrt{9 + 9 + 4}$

$\Rightarrow \left|\overrightarrow{2a} - \vec{b} + 3\vec{c}\right| = \sqrt{22}$

$\therefore$ unit vector along $\overrightarrow{2a} - \vec{b} + 3\vec{c} = \frac{\overrightarrow{2a} - \vec{b} + 3\vec{c}}{\left|\overrightarrow{2a} - \vec{b} + 3\vec{c}\right|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}}$

$= \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$

Question 8: Show that the points A (1, – 2, – 8), B (5, 0, –2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

Answer:

Given: A (1, – 2, – 8), B (5, 0, –2) and C (11, 3, 7)

Then

$\overrightarrow{AB} = (5-1)\hat{i} - \big(0-(-2)\big)\hat{j} + \big(-2-(-8)\big)\hat{k}$

$\Rightarrow \overrightarrow{AB} = (4)\hat{i} - (2)\hat{j} + (6)\hat{k}$

$\overrightarrow{BC} = (11-5)\hat{i} - (3-0)\hat{j} + \big(7-(-2)\big)\hat{k}$

$\Rightarrow \overrightarrow{BC} = (6)\hat{i} - (3)\hat{j} + (9)\hat{k}$

$\overrightarrow{AC} = (11-1)\hat{i} - \big(3-(-2)\big)\hat{j} + \big(7-(-8)\big)\hat{k}$

$\Rightarrow \overrightarrow{BC} = 10\hat{i} + 5\hat{j} + 15\hat{k}$

Then

$\left|\overrightarrow{AB}\right| = \sqrt{(4)^2 + (2)^2 + (6)^2}$

$= \sqrt{16 + 4 + 36}$

$\Rightarrow \left|\overrightarrow{AB}\right| = \sqrt{56} = 2\sqrt{14}$

$\left|\overrightarrow{BC}\right| = \sqrt{(6)^2 + (3)^2 + (9)^2}$

$= \sqrt{36 + 9 + 81}$

$\Rightarrow \left|\overrightarrow{BC}\right| = \sqrt{126} = 3\sqrt{14}$

$\left|\overrightarrow{AC}\right| = \sqrt{(10)^2 + (5)^2 + (15)^2}$

$= \sqrt{100 + 25 + 225}$

$\Rightarrow \left|\overrightarrow{AC}\right| = \sqrt{350} = 5\sqrt{14}$

$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{BC}\right| + \left|\overrightarrow{AB}\right|$

Thus the given points are collinear.

Now to find the ratio in which B divides AC. Let it be λ :1

$\overrightarrow{OB} = \frac{\lambda.\overrightarrow{OC} + 1.\overrightarrow{OA}}{\lambda+1}$

$\Rightarrow \overrightarrow{OB} = \frac{\lambda.\overrightarrow{OC} + 1.\overrightarrow{OA}}{\lambda+1}$

$\Rightarrow 5\hat{i} + 2\hat{k} = \frac{\lambda.(11\hat{i} + 3\hat{j} + 7\hat{k}) + 1.(\hat{i} - 2\hat{j} - 8\hat{k})}{\lambda+1}$

$\Rightarrow 5\hat{i} + 2\hat{k}(\lambda + 1) = \lambda.(11\hat{i} + 3\hat{j} + 7\hat{k}) + 1.(\hat{i} - 2\hat{j} - 8\hat{k})$

$\Rightarrow (5(\lambda + 1)\hat{i} - 2(\lambda + 1)\hat{k}) = (11\lambda + 1)\hat{i} + (3\lambda - 2)\hat{j} + (7\lambda - 8)\hat{k}$

On equating the terms, we get:

$5(\lambda + 1) = 11\lambda + 1$

$\Rightarrow 5\lambda + 5 = 11\lambda + 1$

$\Rightarrow 4 = 6\lambda$

$\Rightarrow \lambda = 4/6 = 2/3$

Hence, B divides AC in the ratio 2:3.

Question 9: Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1 : 2. Also, show that P is the midpoint of the line segment RQ.

Answer:

Given: pointsP$(2\vec{a} + \vec{b})$ and Q$(\vec{a} - 3\vec{b})$ are given.

Point R is given which divides P and Q in the ratio 1:2.

Then

$\overrightarrow{OR} = \frac{2.(2\vec{a}+\vec{b}) - 1.(\vec{a} - 3\vec{b})}{2-1}$

$\Rightarrow \overrightarrow{OR} = \frac{(4\vec{a}+2\vec{b})-(\vec{a}-3\vec{b})}{1}$

$\Rightarrow \overrightarrow{OR} = \left(3\vec{a} + 5\vec{b}\right)$

$\therefore$ position vector of R is $\left(3\vec{a} + 5\vec{b}\right)$

And position vector of mid-point of RQ = $\frac{(\overrightarrow{OQ}+\overrightarrow{OR})}{2}$

$\frac{\left((\vec{a}-3\vec{b})+(3\vec{a}+5\vec{b})\right)}{2} = \frac{1}{2}\left(4\vec{a} + 2\vec{b}\right) = \left(2\vec{a} + \vec{b}\right) = \overrightarrow{OP}$

Hence, P is mid-point of the line segment RQ.

Question 10: The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

Given: Two adjacent sides of a parallelogram are

$\vec{a} = \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 2\hat{j} - 3\hat{k}\right)$

Then the diagonal of parallelogram is given by the resultant of $\vec{a}$ and $\vec{b}$.

Let the diagonal is $\vec{c}$

Then $\vec{c} = \vec{a} + \vec{b}$

$\Rightarrow \vec{c} = (2 + 1)\hat{i} + (-4 - 2)\hat{j} + (5 - 3)\hat{k}$

$\Rightarrow \vec{c} = (3)\hat{i} + (-6)\hat{j} + (2)\hat{k}$

$\Rightarrow \vec{c} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

Then

$|\vec{c}| = \sqrt{(3)^2 + (-6)^2 + (2)^2}$

$\Rightarrow |\vec{c}| = \sqrt{9 + 36 + 4}$

$\Rightarrow |\vec{c}| = \sqrt{49} = 7$

$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$

$\therefore$ unit vector parallel to its diagonal is $\hat{c} = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$

And area of parallelogram ABCD is $|\vec{a} \times \vec{b}|$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$$

$= \hat{i}(12 + 10) - \hat{j}(-6 - 5) + \hat{k}(-4 + 4)$

$= 22\hat{i} + 11\hat{j}$

$= 11(2\hat{i} + \hat{j})$

$\therefore |\vec{a} \times \vec{b}| = 11\sqrt{(2)^2 + (1)^2} = 11\sqrt{5}$

Hence, area of parallelogram ABCD is $11\sqrt{5}$

Question 11: Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Answer:

let the vector equally inclined to OX, OY and OZ at angle α.

Then the direction cosine of the vectors is cos α, cos α and cos α.

Because,

$\text{Cos}^2\alpha + \text{Cos}^2\alpha + \text{Cos}^2\alpha = 1$

$\Rightarrow 3\ \text{Cos}^2\alpha = 1$

$\Rightarrow \text{Cos}^2\alpha = 1/3$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

Hence, the direction cosines of the vector which are equally inclined to the axis are

$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Question 3: Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$

Answer:

Given: $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

Let $\vec{d} = (d_1\hat{i} + d_2\hat{j} + d_3\hat{k})$

Because $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

$\Rightarrow \vec{d}.\vec{a} = 0$ and $\vec{d}.\vec{b} = 0$

Then $\vec{d}.\vec{a} = (d_1\hat{i} + d_2\hat{j} + d_3\hat{k}).(\hat{i} + 4\hat{j} + 2\hat{k}) = 0$

$d_1 + 4d_2 + 2d_3 = 0$ ....(1)

And $\vec{d}.\vec{b} = (d_1\hat{i} + d_2\hat{j} + d_3\hat{k}).(3\hat{i} - 2\hat{j} + 7\hat{k}) = 0$

$3d_1 - 2d_2 + 7d_3 = 0$ ......(2)

And $\vec{c}.\vec{d} = 15$(given)

$\Rightarrow \vec{c}.\vec{d} = (d_1\hat{i} + d_2\hat{j} + d_3\hat{k}).(2\hat{i} - \hat{j} + 4\hat{k}) = 15$

$2d_1 - d_2 + 4d_3 = 15$ ....(3)

So we have the equations to solve as, $d_1 + 4d_2 + 2d_3 = 0$
$3d_1 - 2d_2 + 7d_3 = 0$
$2d_1 - d_2 + 4d_3 = 15$
From equation (1), $d_1 = -4d_2 - 2d_3$ ....(4)

Putting this value in equation 2 we get,$-12d_2 - 6d_3 - 2d_2 + 7d_3 = 0$

Therefore,$14d_2 = d_3$ ....(5)

Now putting the value of d_3 in equation 4 we get,

$d_1 = -4d_2 - 28d_2$

$d_1 = -32d_2$........(6)

Putting (5) and (6) in equation (3) we get,$-64d_2 - d_2 + 56d_2 = 15$

$-9d_2 = 15$

$d_2 = -5/3$

Now we can find other values as well,

$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3}$ and $d_3 = -\frac{70}{3}$

Hence $\vec{d} = \left(\frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k}\right)$

$\vec{d} = \frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$

Hence, the required vector is$\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$

Question 13: The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

Given: given vectors are $\vec{a} = \left(2\hat{i} + 4\hat{j} - 5\hat{k}\right)$ and $\vec{b} = \left(\lambda\hat{i} + 2\hat{j} + 3\hat{k}\right)$,

Then sum of vector is given by the resultant of $\vec{a}$ and $\vec{b}$.

Let the sum is $\vec{c}$.

Then $\vec{c} = \vec{a} + \vec{b}$

$\Rightarrow \vec{c} = (2 + \lambda)\hat{i} + (4 + 2)\hat{j} + (-5 + 3)\hat{k}$

$\Rightarrow \vec{c} = (2 + \lambda)\hat{i} + (6)\hat{j} + (-2)\hat{k}$

$\Rightarrow \vec{c} = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

Then

$|\vec{c}| = \sqrt{(2+\lambda)^2 + (6)^2 + (-2)^2}$

$\Rightarrow |\vec{c}| = \sqrt{4 + (\lambda)^2 + 4\lambda + 36 + 4}$

$\Rightarrow |\vec{c}| = \sqrt{(\lambda)^2 + 4\lambda + 44}$

$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(\lambda)^2+4\lambda+44}}$

Scalar product of $\hat{i}+\hat{j}+\hat{k}$ and $\hat{c}$ is 1.

$\Rightarrow (\hat{i}+\hat{j}+\hat{k}).\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(\lambda)^2+4\lambda+44}} = 1$

$\Rightarrow \frac{(2+\lambda)+6-2}{\sqrt{(\lambda)^2+4\lambda+44}} = 1$

$\Rightarrow (2+\lambda) + 4 = \sqrt{(\lambda)^2 + 4\lambda + 44}$

$\Rightarrow (\lambda + 6) = \sqrt{(\lambda)^2 + 4\lambda + 44}$

Square on both sides:

$\Rightarrow (\lambda + 6)^2 = (\lambda)^2 + 4\lambda + 44$

$\Rightarrow (\lambda)^2 + 12\lambda + 36 = (\lambda)^2 + 4\lambda + 44$

$\Rightarrow 8\lambda = 8$

$\Rightarrow \lambda = 1$

Question 14: If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$, and $\vec{c}$

Answer:

Given: vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are mutually perpendicular to each other and are of equal magnitude.

$\Rightarrow \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

Let the vector $\vec{a} + \vec{b} + \vec{c}$ be inclined to $\vec{a}, \vec{b}$, and $\vec{c}$ at angles α, β and γ respectively.

Then, we have

$$\cos\alpha = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{a}|}$$

$$= \frac{\left(\vec{a}.\vec{a}+\overrightarrow{b.\vec{a}}+\overrightarrow{c.\vec{a}}\right)}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{a}|}$$

$$= \frac{\left(|\vec{a}|^2+0+0\right)}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{a}|} = \frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|} \qquad \text{….(1)}$$

$$\cos\beta = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{b}}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{b}|}$$

$$= \frac{\left(\vec{a}.\vec{b}+\overrightarrow{b.\vec{b}}+\overrightarrow{c.\vec{b}}\right)}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{b}|}$$

$$= \frac{\left(0+|\vec{b}|^2+0\right)}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{b}|} = \frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|} \qquad \text{….(2)}$$

$$\cos\gamma = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{a}|}$$

$$= \frac{\left(\vec{a}.\vec{c}+\overrightarrow{b.\vec{c}}+\overrightarrow{c.\vec{c}}\right)}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{a}|}$$

$$= \frac{\left(0+0+|\vec{c}|^2\right)}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{a}|} = \frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|} \qquad \text{….(3)}$$

From (1), (2) and (3)

As $|\vec{a}| = |\vec{b}| = |\vec{c}|$

Hence, $\cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \alpha = \beta = \gamma$

Hence, the vector $\vec{a} + \vec{b} + \vec{c}$ are equal inclined to $\vec{a}$, $\vec{b}$, and $\vec{c}$.

Question 15: Prove that $(\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$, if and only if are perpendicular, given $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$.

Answer:

given: $(\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$

To prove: vectors $\vec{a}$ and $\vec{b}$ are mutually perpendicular to each other.

$\Rightarrow \vec{a}.\vec{b} = 0$

$\because (\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$

$\Rightarrow (\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$

$\Rightarrow (\vec{a}.\vec{b} = \vec{b}.\vec{a})$ scalar product is commutative.

$\Rightarrow \left(|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}\right) = |\vec{a}|^2 + |\vec{b}|^2$

$\Rightarrow (2\vec{a}.\vec{b}) = 0$

$\Rightarrow \vec{a}.\vec{b} = 0$

Hence, $\vec{a}$ and $\vec{b}$ are mutually perpendicular to each other.as $\vec{a} \neq 0, \vec{b} \neq 0$ is given.

Question 16: If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} \geq 0$ only when

A. $0 < \theta < \frac{\pi}{2}$

B. $0 \leq \theta < \frac{\pi}{2}$

C. $0 < \theta < \pi$

D. $0 \leq \theta < \pi$

Answer:

let θ is the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then $\vec{a}$ and $\vec{b}$ are non-zero vectors so that $|\vec{a}|$and $|\vec{b}|$are positive.

As we know $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$

For $\vec{a}.\vec{b} > 0$

$\Rightarrow |\vec{a}||\vec{b}| \cos\theta > 0$

As $|\vec{a}\ |$and $|\vec{b}|$ are positive.

$\Rightarrow \cos\theta > 0$

$\Rightarrow 0 \leq \theta < \frac{\pi}{2}$

Hence, $\vec{a}.\vec{b} > 0$ when $0 \leq \theta < \frac{\pi}{2}$

The correct answer is (B).

Question 17: Let $\vec{a}$ and $\vec{b}$ be two unit vectors and q is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if

A. $\theta = \frac{\pi}{4}$

B. $\theta = \frac{\pi}{3}$

C. $\theta = \frac{\pi}{2}$

D. $\theta = \frac{2\pi}{3}$

Answer:

let the two unit vectors are $\vec{a}$ and $\vec{b}$ and θ is the angle between.

Then $|\vec{a}| = |\vec{b}| = 1$

Then this is $(\vec{a} + \vec{b})$ is unit vector if $|\vec{a} + \vec{b}| = 1$

$|\vec{a} + \vec{b}| = 1$

$\Rightarrow |\vec{a} + \vec{b}|^2 = (1)^2$

$\Rightarrow (\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = 1$

$\Rightarrow (\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b}) = 1$

$\Rightarrow (\vec{a}.\vec{b} = \vec{b}.\vec{a})$ scalar product is commutative.

$\Rightarrow \left(|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}\right) = 1$

$\Rightarrow (1 + 1 + 2\vec{a}.\vec{b}) = 1$

$\Rightarrow 2 + 2\left(|\vec{a}|^2|\vec{b}|^2 \cos\theta\right) = 1$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = \frac{2\pi}{3}$

Hence, $(\vec{a} + \vec{b})$ is unit vector if $\theta = \frac{2\pi}{3}$

The correct answer is (D).

Question 18: The value of $\hat{i}(\hat{j} \times \hat{k}) + \hat{j}(\hat{i} \times \hat{k}) + \hat{k}(\hat{i} \times \hat{j})$is

A. 0

B. –1

C. 1

D. 3

Answer:

given: $\hat{i}(\hat{j} \times \hat{k}) + \hat{j}(\hat{i} \times \hat{k}) + \hat{k}(\hat{i} \times \hat{j})$

$\hat{i}(\hat{j} \times \hat{k}) + \hat{j}(\hat{i} \times \hat{k}) + \hat{k}(\hat{i} \times \hat{j}) = \hat{i}.\hat{i} + \hat{j}.(-\hat{j}) + \hat{k}.\hat{k}$

$=1 - \hat{j}.\hat{j} + 1$

=1 – 1 + 1

= 1

The correct answer is (C).

Question 19: If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$when θ is equal to

A. 0

B. π/4

C. π/2

D. π

Answer:

let θ is the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then $\vec{a}$ and $\vec{b}$ are non-zero vectors so that are positive.

$\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$

$\Rightarrow |\vec{a}|\left|\vec{b}\right| \cos\theta = |\vec{a}|\left|\vec{b}\right| \sin\theta$

$\Rightarrow \cos\theta = \sin\theta$

$\Rightarrow 1 = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$

Thus $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ when $\theta = \frac{\pi}{4}$.

The correct answer is (B).