

BSEH Practice Paper (March 2024)

CLASS: 12th (Senior Secondary)

Code No.: 835

Roll No.

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SET: B

गणित MATHEMATICS

[Hindi and English Medium]

ACADEMIC / OPEN

[Time allowed: 3 hours]

[Maximum Marks: 80]

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- कृपया सुनिश्चित करें कि इस प्रश्न में मुद्रित पृष्ठ पेपर संख्या में 14 हैं और इसमें 38 प्रश्न हैं।
Please make sure that the printed pages in this question paper are 14 in number and it contains 38 questions.
 - प्रश्न पत्र के दायीं ओर दी गयी कोड संख्या को छात्र द्वारा उत्तरपुस्तिका- के पहले पृष्ठ पर लिखा जाना चाहिए।
The Code No. on the right side of the question paper should be written by the candidate on the front page of the answer-book.
 - किसी प्रश्न का उत्तर देना शुरू करने से पहले उसका क्रमांक लिखा जाना चाहिए।
Before beginning to answer a question, its Serial Number must be written.
 - अपनी उत्तर-पुस्तिका में खाली पृष्ठ/पृष्ठ न छोड़ें।
Don't leave blank page/pages in your answer-book.
 - उत्तर-पुस्तिका के अतिरिक्त, कोई अतिरिक्त पत्रक नहीं दिया जायेगा। अतः आवश्यकतानुसार ही लिखें और लिखे उत्तर को न काटे।
Except answer-book, no extra sheet will be given. Write to the point and do not strike the written answer.
 - परीक्षार्थी प्रश्नपत्र पर अपना रोल नंबर अवश्य लिखें।
Candidates must write their Roll Number on the question paper.
 - कृपया प्रश्नों का उत्तर देने से पूर्व, यह सुनिश्चित करें लें कि प्रश्न पत्र पूर्ण व सही हैं, परीक्षा के उपरांत इस संबंध में कोई भी दावा स्वीकार नहीं किया जाएगा।
Before answering the questions, please ensure that you have been supplied the correct and complete question paper, no claim in this regard, will be entertained after examination.

सामान्य निर्देश :

- इस प्रश्न- पत्र में कुल 38 प्रश्न हैं, जो कि पांच खंडों: अ, ब, स, द ल में बांटे गए हैं :
खंड अ : इस खंड में 1 से 20 तक कुल 20 प्रश्न हैं, प्रत्येक प्रश्न 1 अंक का है।
खंड ब : इस खंड में 21 से 25 तक कुल 05 प्रश्न हैं, प्रत्येक प्रश्न 2 अंक का है।
खंड स : इस खंड में 26 से 31 तक कुल 06 प्रश्न हैं, प्रत्येक प्रश्न 3 अंक का है।
खंड द : इस खंड में 32 से 35 तक कुल 04 प्रश्न हैं, प्रत्येक प्रश्न 5 अंक का है।
खंड ल : इस खंड में 36 से 38 तक कुल 03 केस आधारित प्रश्न हैं, प्रत्येक प्रश्न 4 अंक का है।
- सभी प्रश्न अनिवार्य हैं।
- कुछ प्रश्नों में आंतरिक चयन का विकल्प दिया गया है, उनमें से एक ही प्रश्न को चुनना है।
- कैलकुलेटर के प्रयोग की अनुमति नहीं है।

General Instructions:

- This question paper consists of 38 questions in total which are divided into five sections: A, B, C, D, E :
Section A: This section consists of twenty questions from **1 to 20**. Each question carries **1 mark**.
Section B: This section consists of five questions from **21 to 25**. Each question carries **2 marks**.
Section C: This section consists of six questions from **26 to 31**. Each question carries **3 marks**.
Section D: This section consists of four questions from **32 to 35**. Each question carries **5 marks**.
Section E: This section consists of three case based questions from **36 to 38**. Each question carries **4 marks**.
- All questions are compulsory.
- There are some questions where **internal choice** has been provided. Choose only one of them.
- Use of calculator is **not** permitted.

खंड - अ
SECTION - A

इस खंड में प्रत्येक प्रश्न 1 अंक का है।

This section comprises questions of 1 mark each.

1. मान लीजिए कि समुच्चय \mathbf{N} में $R = \{(a, b) : b = a + 1, b > 5\}$ द्वारा दिया गया संबंध है। सही उत्तर का चयन करें।

(A) $(3, 4) \in R$

(B) $(7, 8) \in R$

(C) $(4, 5) \in R$

(D) $(1, 2) \in R$

Let R be the relation in the set \mathbf{N} given by $R = \{(a, b) : b = a + 1, b > 5\}$. Choose the correct answer.

(A) $(3, 4) \in R$

(B) $(7, 8) \in R$

(C) $(4, 5) \in R$

(D) $(1, 2) \in R$

2. $\cos^{-1}(\cos \frac{7\pi}{6})$ का मान

(A) $\frac{7\pi}{6}$ है

(B) $\frac{5\pi}{6}$ है

(C) $\frac{\pi}{3}$ है

(D) $\frac{\pi}{6}$ है

$\cos^{-1}(\cos \frac{7\pi}{6})$ is equal to

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

3. यदि $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, तो $A'A$ है:

(A) I

(B) $\begin{bmatrix} \cos^2 \alpha & \sin^2 \alpha \\ -\cos^2 \alpha & \cos \alpha \end{bmatrix}$

[4]

(C) $\begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix}$ (D) 1

If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then $A'A$ is:

(A) I (B) $\begin{bmatrix} \cos^2 \alpha & \sin^2 \alpha \\ -\cos^2 \alpha & \cos \alpha \end{bmatrix}$

(C) $\begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix}$ (D) 1

4. यदि A और B व्युत्क्रमणीय आव्यूह हैं तब निम्न में से कौन सा सत्य नहीं है?

(A) $\text{adj } A = |A| \cdot A^{-1}$ (B) $\det(A)^{-1} = [\det(A)]^{-1}$

(C) $(AB)^{-1} = B^{-1} A^{-1}$ (D) $(A + B)^{-1} = B^{-1} + A^{-1}$

If A and B are invertible matrices, then which of the following is not correct?

(A) $\text{adj } A = |A| \cdot A^{-1}$ (B) $\det(A)^{-1} = [\det(A)]^{-1}$

(C) $(AB)^{-1} = B^{-1} A^{-1}$ (D) $(A + B)^{-1} = B^{-1} + A^{-1}$

5. यदि एक त्रिभुज के शीर्ष $(-2, -3)$, $(3, 2)$ और $(-1, -8)$ हैं, तो सारणिकों का प्रयोग द्वारा इस त्रिभुज का क्षेत्रफल है

(A) 15 (B) $\frac{15}{2}$

(C) 13 (D) इनमे से कोई नहीं

If the vertices of a triangle are $(-2, -3)$, $(3, 2)$ and $(-1, -8)$, then by using determinants its area is

(A) 15 (B) $\frac{15}{2}$

(C) 13 (D) none of the above

6. यदि $y = \log x^2$, तो $\frac{d^2y}{dx^2}$ बराबर है :

- (A) $\frac{-2}{x^2}$ (B) $\frac{2}{x}$
 (C) $\frac{1}{2x}$ (D) $\frac{1}{x^2}$

If $y = \log x^2$, then $\frac{d^2y}{dx^2}$ is equal to :

- (A) $\frac{-2}{x^2}$ (B) $\frac{2}{x}$
 (C) $\frac{1}{2x}$ (D) $\frac{1}{x^2}$

7. $(1-x)\sqrt{x}$ का प्रतिअवकलज है:

- (A) $\frac{3}{2}x^{\frac{3}{2}} - \frac{5}{2}x^{\frac{5}{2}} + C$ (B) $\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$
 (C) $\frac{1}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} + C$ (D) $\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C$

The antiderivative of $(1-x)\sqrt{x}$ equals:

- (A) $\frac{3}{2}x^{\frac{3}{2}} - \frac{5}{2}x^{\frac{5}{2}} + C$ (B) $\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$
 (C) $\frac{1}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} + C$ (D) $\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C$

8. $\int e^x \sec x (1 + \tan x) dx$ बराबर है:

- (A) $e^x \cos x + C$ (B) $e^x \sec x + C$
 (C) $e^x \sin x + C$ (D) $e^x \tan x + C$

$\int e^x \sec x (1 + \tan x) dx$ equals

- (A) $e^x \cos x + C$ (B) $e^x \sec x + C$
 (C) $e^x \sin x + C$ (D) $e^x \tan x + C$

9. $\int_{-\pi/2}^{\pi/2} \tan^5 x dx$ का मान है:

- (A) 1 (B) -1
 (C) 0 (D) 2

The value of $\int_{-\pi/2}^{\pi/2} \tan^5 x dx$ is

- (A) 1 (B) - 1
(C) 0 (D) 2

10. अवकल समीकरण $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ की घात है:

- (A) 3 (B) 2
(C) 1 (D) परिभाषित नहीं

The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is :

- (A) 3 (B) 2
(C) 1 (D) not defined

11. चार कोटि वाले किसी अवकल समीकरण के व्यापक हल में उपस्थित स्वेच्छ अचरों की संख्या कितनी होती है ?

How many number of arbitrary constants are there in the general solution of a differential equation of fourth order?

12. यदि फलन $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ बिंदु $x = 0$ पर संतत है, तो k का मान ज्ञात कीजिये।

The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then find the value of k

13. यदि एक रेखा x , y और z -अक्ष के साथ क्रमशः 90° , 135° , 45° के कोण बनाती है तो इसकी दिक् कोसाइन ज्ञात कीजिए।

If a line makes angles 90° , 135° , 45° with the x , y and z -axes respectively, find its direction cosines.

14. यदि $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$ और A तथा B स्वतंत्र घटनाएँ हैं तो $P(A \cap B)$ ज्ञात कीजिए।

If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

15. \vec{a} तथा $-\vec{a}$ संरेख हैं। (सत्य / असत्य)

\vec{a} and $-\vec{a}$ are collinear. (True / False)

16. यदि पासों का एक जोड़ा उछाला जाता है तो प्रत्येक पासे पर सम अभाज्य संख्या प्राप्त करने की प्रायिकता $\frac{6}{36}$ है। (सत्य / असत्य)

The probability of obtaining an even prime number on each die, when a pair of dice is rolled is $\frac{6}{36}$. (True / False)

17. यदि A और B ऐसी दो घटनाएँ हैं कि $P(A) + P(B) - P(A \cap B) = P(A)$, तब $P(A|B)$ _____.

If A and B are any two events such that $P(A) + P(B) - P(A \cap B) = P(A)$, then $P(A|B)$ _____.

18. सदिश $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ का, सदिश $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$ पर प्रक्षेप है _____.

The projection vector of $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ on $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$ is _____.

प्रश्न संख्या 19 और 20 अभिकथन और तर्क आधारित प्रश्न हैं, जिनमें से प्रत्येक प्रश्न 1 अंक का है। दो कथन दिए गए हैं, एक को अभिकथन (A) और दूसरे को तर्क (R) अंकित किया गया है। इन प्रश्नों के सही उत्तर निचे दिए गए कोडो (A), (B), (C) और (D) में से चुनकर दीजिये।

- (A) अभिकथन (A) और तर्क (R) दोनों सही है और तर्क (R), अभिकथन (A) की सही व्याख्या है।
- (B) अभिकथन (A) और तर्क (R) दोनों सही है, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या **नहीं** करता है।
- (C) अभिकथन (A) सही है तथा तर्क (R) गलत है।
- (D) अभिकथन (A) गलत है तथा तर्क (R) सही है।

Question number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labeled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A)
- (C) Assertion (A) is true and Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is true.

19. अभिकथन (A): यदि सम्मूच्य $A = \{1, 2\}$ तथा $B = \{3, 4\}$ तो A से B तक संबंधों की संख्या 16 है।
 तर्क (R) : यदि $n(A) = p$ तथा $n(B) = q$, तो संबंधों की संख्या 2^{pq} है।

Assertion (A): Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Then, number of relations from A to B is 16.

Reason (R): If $n(A) = p$ and $n(B) = q$, then number of relations is 2^{pq} .

20. अभिकथन (A): रेखा $\frac{x-5}{3} = \frac{y+4}{2} = \frac{z+8}{1}$ के दिक् कोसाइन $\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}$ है।

तर्क (R): दो रेखाओं $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ तथा $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ के बीच की दुरी $d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$ द्वारा प्रदत्त है।

Assertion (A): The direction cosines of line $\frac{x-5}{3} = \frac{y+4}{2} = \frac{z+8}{1}$ is $\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}$.

Reason (R): The distance between two parallel lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}$ is given by $d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$.

खंड- ब

SECTION - B

इस खंड में प्रत्येक प्रश्न 2 अंक का है।

This section comprises questions of 2 marks each.

21. सिद्ध कीजिए कि $f(x) = x^2$, द्वारा परिभाषित फलन $f: \mathbb{R} \rightarrow \mathbb{R}$, न तो एकैकी है और न आच्छादक है।

Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$, defined as $f(x) = x^2$, is neither one-one nor onto.

अथवा /OR

$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ का मान ज्ञात कीजिये।

Find the value of: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

22. एक ऐसे 3×2 आव्यूह की रचना कीजिए, जिसके अवयव $a_{ij} = \frac{1}{2} |i - 3j|$ द्वारा प्रदत्त हैं।

Construct a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2} |i - 3j|$.

23. k के मानों को ज्ञात कीजिए ताकि प्रदत्त फलन बिंदु $x = 1$ पर संतत हो:

$$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x = 1 \\ k & x \neq 1 \end{cases}$$

Find the value of k so that the function is continuous at $x = 1$.

$$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x = 1 \\ k & x \neq 1 \end{cases}$$

24. सत्यापित कीजिए कि फलन $y = a \cos x + b \sin x$, जिसमें $a, b \in \mathbf{R}$, अवकल समीकरण $\frac{d^2y}{dx^2} + y = 0$ का हल है।

Verify that the function $y = a \cos x + b \sin x$, where $a, b \in \mathbf{R}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$

अथवा /OR

अवकल समीकरण $y \log y dx - x dy = 0$ का व्यापक हल ज्ञात कीजिये।

Find the general solution of the differential equation $y \log y dx - x dy = 0$

25. एक कलश में 10 काली और 5 सफ़ेद गेंदें हैं। दो गेंद एक के बाद एक निकाली जाती हैं और पहली गेंद दूसरे के निकालने से पहले वापस नहीं रखी जाती हैं। तो दोनों काले गेंद निकलने की क्या प्रायिकता है?

An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

खंड- स

SECTION - C

इस खंड में प्रत्येक प्रश्न 3 अंक का है।

This section comprises questions of 3 marks each.

26. मान लीजिए कि समुच्चय A में धन पूर्णाकों के क्रमित युग्मों (ordered pairs) का एक संबंध R , $(x, y) R (u, v)$, यदि और केवल यदि, $xv = yu$ द्वारा परिभाषित है। सिद्ध कीजिए कि R एक तुल्यता संबंध है।

Let R be a relation on the set A of ordered pairs of positive integers defined by $(x, y) R (u, v)$ if and only if $xv = yu$. Show that R is an equivalence relation.

अथवा / OR

सिद्ध कीजिये : $\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$

Prove that: $\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$

27. यदि $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, दिखाइए कि $F(x) \cdot F(y) = F(x + y)$

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x) \cdot F(y) = F(x + y)$.

28. प्रदत्त फलन $(\cos x)^y = (\cos y)^x$ के लिए $\frac{dy}{dx}$ ज्ञात कीजिये।

Find $\frac{dy}{dx}$ of the function $(\cos x)^y = (\cos y)^x$.

29. अंतराल ज्ञात कीजिये जिनमें $f(x) = -2x^3 - 9x^2 - 12x + 1$ द्वारा प्रदत्त फलन f वर्धमान या ह्रासमान है।

Find the intervals in which the function f is given by $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing or strictly decreasing.

30. समाकलन कीजिये: $\int \frac{4x+1}{\sqrt{2x^2+x-3}} dx$

Integrate: $\int \frac{4x+1}{\sqrt{2x^2+x-3}} dx$

अथवा /OR

$\int_2^8 |x - 5| dx$ का मान ज्ञात कीजिये

Evaluate: $\int_2^8 |x - 5| dx$

31. एक त्रिभुज का क्षेत्रफल ज्ञात कीजिए जिसके शीर्ष बिंदु $A(1, 1, 1)$, $B(1, 2, 3)$ और $C(2, 3, 1)$ हैं।

Find the area of a triangle having points $A(1, 1, 1)$, $B(1, 2, 3)$ and $C(2, 3, 1)$ as its vertices.

खंड- द

SECTION - D

इस खंड में प्रत्येक प्रश्न 5 अंक का है।

This section comprises questions of 5 marks each.

32. आव्यूहों के गुणनफल $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ का प्रयोग करते हुए निम्नलिखित

समीकरण निकाय को हल कीजिए:

$$x - y + 2z = 1$$

$$2y - 3z = 1$$

$$3x - 2y + 4z = 2$$

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$$x - y + 2z = 1$$

$$2y - 3z = 1$$

$$3x - 2y + 4z = 2$$

33. रेखाओं $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ और $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ के बीच की न्यूनतम दुरी ज्ञात कीजिये।

Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

अथवा /OR

बिंदु $(1, -2, -3)$ से जाने वाली और दोनो रेखाओं $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{3}$ और $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$ पर लंब रेखा का सदिश समीकरण ज्ञात कीजिये।

Find the vector equation of the line passing through the point $(1, -2, -3)$ and perpendicular to the two lines : $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{3}$ and $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$.

34. दिए गए वक्र $y = x^2$, और दी गई रेखाओं $x = 1, x = 2$ एवं x -अक्ष से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Find the area under the given curve $y = x^2$ and the given lines $x = 1, x = 2$ and x -axis.

अथवा /OR

दीर्घवृत्त $\frac{x^2}{4} + \frac{y^2}{9} = 1$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

35. आलेखीये विधि से निम्न समस्या को हल कीजिये :

निम्न व्यवरोधो के अंतर्गत $x + 2y \leq 120$

$$x + y \geq 60$$

$$x - 2y \geq 0$$

$$x \geq 0, y \geq 0$$

$Z = 5x + 10y$ का न्यूनतम और अधिकतम मान ज्ञात कीजिये।

Solve the following problem graphically:

Minimise and Maximise $Z = 5x + 10y$

Subject to the constraints: $x + 2y \leq 120$

$$x + y \geq 60$$

$$x - 2y \geq 0$$

$$x \geq 0, y \geq 0$$

खंड- इ

SECTION – E

इस खंड में प्रत्येक प्रश्न 4 अंक का है।

This section comprises questions of 4 marks each.

Case Study – 1

36. $P(x) = -6x^2 + 120x + 25000$ (₹ में) एक कंपनी का कुल लाभ फलन है, जहां x कंपनी के उत्पादन को दर्शाता है।

उपरोक्त जानकारी के आधार पर निम्नलिखित का उत्तर दें:

(i) जब उत्पादन 3 इकाई हो तो कंपनी का लाभ ज्ञात कीजिए। (1)

(ii) $P'(5)$ खोजें। (1)

(iii) लाभ अधिकतम होने पर उत्पादन ज्ञात कीजिए। (2)



$P(x) = -6x^2 + 120x + 25000$ (in ₹) is the total profit function of a company, where x denotes the production of the company.

Based on the above information answer the following:

(i) Find the profit of the company when the production is 3 units. (1)

(ii) Find $P'(5)$. (1)

(iii) Find the production, when the profit is maximum. (2)

Case Study – 2

37. एक रैखिक अवकल समीकरण $\frac{dy}{dx} + Py = Q$ के रूप का हो, जहाँ P और Q, x के फलन हों, तो ऐसे समीकरण को रैखिक अवकल समीकरण कहते हैं। इसका समाधान $y \cdot (\text{I.F.}) = \int Q(\text{I.F.}) dx + c$, जहाँ I.F.(समाकलन गुणक) = $e^{\int P dx}$

अब, मान लीजिए दिया गया समीकरण $x \frac{dy}{dx} + 2y = x^2$ है

उपरोक्त जानकारी के आधार पर, निम्नलिखित प्रश्नों के उत्तर दें:

- (i) क्रमशः P और Q के मान क्या हैं? (1)
- (ii) I.F का मान क्या है? (1)
- (iii) दिए गए समीकरण का हल ज्ञात कीजिए। (2)

A linear differential equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by $y \cdot (\text{I.F.}) = \int Q(\text{I.F.}) dx + c$, where I.F.(Integrating Factor) = $e^{\int P dx}$

Now, suppose the given equation is $x \frac{dy}{dx} + 2y = x^2$

Based on the above information, answer the following questions:

- (i) What are the values of P and Q respectively? (1)
- (ii) What is the value of I.F.? (1)
- (iii) Find the Solution of given equation. (2)

Case Study – 3

38. एक कार्यालय में तीन कर्मचारी विनय, सोनिया और इकबाल एक निश्चित फॉर्म की आने वाली प्रतियों को संसाधित करते हैं। विनय 50 फीसदी फॉर्म प्रोसेस करते हैं। सोनिया 20% और इकबाल शेष 30% फॉर्म प्रोसेस करते हैं। विनय की त्रुटि दर 0.06 है, सोनिया की त्रुटि दर 0.04 है और इकबाल की त्रुटि दर 0.03 है।

उपरोक्त जानकारी के आधार पर निम्नलिखित प्रश्नों के उत्तर दीजिये :

- (i) फॉर्म को संसाधित करने में त्रुटि होने की कुल संभावना। (2)

(ii) कंपनी का प्रबंधक गुणवत्ता जांच करना चाहता है। निरीक्षण के दौरान वह संसाधित प्रपत्रों के आउटपुट के दिनों में से यादृच्छिक रूप से एक प्रपत्र का चयन करता है। यदि यादृच्छिक रूप से चुने गए फॉर्म में कोई त्रुटि है, तो संभावना है कि फॉर्म विनय द्वारा संसाधित नहीं किया गया है। (2)



In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information answer the following questions:

- (i) The total probability of committing an error in processing the form. (2)
- (ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vinay. (2)

BSEH Practice Paper (March 2024)

(2023-24)

Marking Scheme

MATHEMATICS

SET-B

CODE: 835

- ⇒ Important Instructions: • All answers provided in the Marking scheme are SUGGESTIVE
• Examiners are requested to accept all possible alternative correct answer(s).

SECTION – A (1Mark × 20Q)		
Q. No.	EXPECTED ANSWERS	Marks
Question 1.	Let R be the relation in the set N given by $R = \{(a, b) : b = a + 1, b > 5\}$. Choose the correct answer.	
Solution:	(B) $(7, 8) \in R$	1
Question 2.	$\cos^{-1}(\cos \frac{7\pi}{6})$ is equal to	
Solution:	(B) $\frac{5\pi}{6}$	1
Question 3.	If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then $A'A$ is:	
Solution:	(A) I	1
Question 4.	If A and B are invertible matrices, then which of the following is not correct	
Solution:	(D) $(A + B)^{-1} = B^{-1} + A^{-1}$	1
Question 5.	If the vertices of a triangle are $(-2, -3)$, $(3, 2)$ and $(-1, -8)$, then by using determinants its area is	
Solution:	(A) 15	1
Question 6.	If $y = \log x^2$, then $\frac{d^2y}{dx^2}$ is equal to :	
Solution:	(A) $\frac{-2}{x^2}$	1
Question 7.	The antiderivative of $(1 - x)\sqrt{x}$ equals:	
Solution:	(B) $\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$	1
Question 8.	$\int e^x \sec x (1 + \tan x) dx$ equals	
Solution:	(C) $e^x \sec x + C$	1
Question 9.	The value of $\int_{-\pi/2}^{\pi/2} \tan^5 x dx$ is	
Solution:	(C) 0	1
Question 10.	The degree of the differential equation $(\frac{d^2y}{dx^2})^3 + (\frac{dy}{dx})^2 + \sin(\frac{dy}{dx}) + 1 = 0$ is :	

Solution:	(D) not defined	1
Question 11.	How many number of arbitrary constants are there in the general solution of a differential equation of fourth order?	
Solution:	4	1
Question 12.	The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k & , \text{ if } x = 0 \end{cases}$ is continuous at $x = 0$, then find the value of k	
Solution:	$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} + \cos x \right)$ $= 1 + 1$ $= 2$ <p>Since $f(x)$ is continuous at $x = 0$</p> $\therefore \lim_{x \rightarrow 0} f(x) = f(0)$ $\Rightarrow 2 = k$	1
Question 13.	If a line makes angles $90^\circ, 135^\circ, 45^\circ$ with the x, y and z-axes respectively, find its direction cosines.	
Solution:	<p>Line makes angles $90^\circ, 135^\circ, 45^\circ$ with the x,y and z-axes respectively</p> <p>\therefore Direction Cosines are</p> $l = \cos 90^\circ, m = \cos 135^\circ, n = \cos 45^\circ$ $l = 0, m = \frac{-1}{\sqrt{2}}, n = \frac{1}{\sqrt{2}}$ <p>\Rightarrow D.C.'s are $\langle 0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$</p>	1
Question 14.	If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.	
Solution:	<p>Since A and B are independent therefore $P(A \cap B) = P(A) \cdot P(B)$</p> $\therefore P(A \cap B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$	1
Question 15.	\vec{a} and $-\vec{a}$ are collinear. (True / False)	
Solution:	True	1
Question 16.	The probability of obtaining an even prime number on each die, when a pair of dice is rolled is $\frac{6}{36}$. (True / False)	
Solution:	False	1
Question 17.	If A and B are any two events such that $P(A) + P(B) - P(A \cap B) = P(A)$, then $P(A B)$ _____.	
Solution:	1	1
Question 18.	The projection vector of $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ on $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$ is _____.	

Solution:	$\frac{60}{\sqrt{114}}$	
Question 19.	Assertion (A): Let $A = \{1,2\}$ and $B = \{3,4\}$. Then, number of relations from A to B is 16. Reason (R): If $n(A) = p$ and $n(B) = q$, then number of relations is 2^{pq} .	
Solution:	(A). Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)	1
Question 20.	Assertion (A): The direction cosines of line $\frac{x-5}{3} = \frac{y+4}{2} = \frac{z+8}{1}$ is $\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}$. Reason (R): The distance between two parallel lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}$ is given by $d = \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} }$.	
Solution:	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is <i>not</i> the correct explanation of the Assertion (A)	1
SECTION – B (2Marks × 5Q)		
Question 21.	Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$, defined as $f(x) = x^2$, is neither one-one nor onto.	
Solution:	$f(x) = x^2$ Checking for ONE-ONE let x_1 and x_2 are be any two real numbers. $f(x_1) = x_1^2$ and $f(x_2) = x_2^2$ Now $f(x_1) = f(x_2)$ $\Rightarrow x_1^2 = x_2^2$ $\Rightarrow x_1 = x_2, x_1 = -x_2$ \Rightarrow Since x_1 does not have a unique image so $f(x)$ is not one-one. Checking for ONTO Let $f(x) = y$ such that $y \in \mathbb{R}$ $\Rightarrow x^2 = y$ $\Rightarrow x = \pm\sqrt{y}$ Note that y is a real number, so it can be negative also $\Rightarrow f(x)$ is not an onto function. (Note: Students can also use some illustrations to show $f(x)$ neither one-one nor onto.)	1 1
OR Question 21.	Find the value of: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$	
Solution:	Let $\tan^{-1}\sqrt{3} = x$. Then $\tan x = \sqrt{3} = \tan(\pi/3)$	

	<p>We know that the range of the principal value branch of \tan^{-1} is $(-\frac{\pi}{2}, \frac{\pi}{2})$ $\therefore \tan^{-1}\sqrt{3} = \pi/3$</p> <p>Let $\sec^{-1}(-2) = y$. Then, $\sec y = -2 = -\sec(\pi/3)$ $\sec y = -2 = \sec(\pi - \frac{\pi}{3})$</p> <p>We know that the range of the principal value branch of \sec^{-1} is $[0, \pi] - \{\frac{\pi}{2}\}$ $\therefore \sec^{-1}(-2) = 2\pi/3$</p> <p>Now $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \pi/3 - 2\pi/3$ $= -\pi/3$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>
Question 22.	Construct a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2} i - 3j $.	
Solution:	<p>Since it is 3×2 Matrix It has 3 rows and 2 columns Let the matrix be A</p> <p>Where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$</p> <p>Now it is given that $a_{ij} = \frac{1}{2} i - 3j$</p> <p>Hence the required matrix is</p> <p>$a_{11} = 1$ $a_{12} = 5/2$ $a_{21} = 1/2$ $a_{22} = 2$ \Rightarrow $A = \begin{bmatrix} 1 & 5/2 \\ 1/2 & 2 \\ 0 & 3/2 \end{bmatrix}$ $a_{31} = 0$ $a_{32} = 3/2$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>
Question 23.	<p>Find the value of k so that the function is continuous is at $x = 1$.</p> <p>$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ k & x = 1 \end{cases}$</p>	
Solution:		

	<p>Given function is $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ k & x = 1 \end{cases}$</p> <p>Now</p> $\lim_{x \rightarrow 1} f(x) \Rightarrow \lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$ $\lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x \rightarrow 1} (x+1) = 2 \quad \dots(1)$ <p>Since function is continuous, therefore</p> $\lim_{x \rightarrow 1} f(x) = f(1)$ $k = 2$	<p>1</p> <p>1</p>
Question 24.	<p>Verify that the function $y = a \cos x + b \sin x$, where $a, b \in \mathbf{R}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$</p>	
Solution:	<p>Given: $y = a \cos x + b \sin x \quad \dots(1)$ Diff. w.r.t. 'x', and we get $\frac{dy}{dx} = -a \sin x + b \cos x$</p> <p>Again differentiate (1) w.r.t. 'x', we get $\frac{d^2y}{dx^2} = -a \cos x - b \sin x \quad \dots(2)$</p> <p>Now, substitute (1) and (2) in the given differential equation, and we get the following:</p> $\begin{aligned} \text{L.H.S} &= \frac{d^2y}{dx^2} + y \\ &= (-a \cos x - b \sin x) + (a \cos x + b \sin x) \\ &= -a \cos x - b \sin x + a \cos x + b \sin x \\ &= 0 = \text{R.H.S} \end{aligned}$ <p>As L.H.S = R.H.S, the given function is the solution of the corresponding differential equation.</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>
OR Question 24.	<p>Find the general solution of the differential equation $y \log y \, dx - x \, dy = 0$</p>	
Solution:	<p>Since $y \log y \, dx - x \, dy = 0$, therefore separating the variables, the given differential equation can be written as</p>	

	$\frac{dy}{y \log y} = \frac{dy}{x}$(1) Integrating both sides of equation (1), we get $\int \frac{dy}{y \log y} = \int \frac{dy}{x}$ $\log \log y = \log x + C$ which is the general solution of equation (1)	$\frac{1}{2}$ $1\frac{1}{2}$
Question 25.	An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?	
Solution:	Total number of balls = 10 black balls + 5 red balls = 15 balls Let A be the event of drawing a black ball in first draw and B be the events of drawing a black ball in second draw. $P(A) = \text{Probability of getting a black ball in the first draw} = \frac{10}{15} = \frac{2}{3}$ As the ball is not replaced after the first throw, $\therefore P(B/A) = \text{Probability of getting another black ball in the second draw} = \frac{8}{14} = \frac{4}{7}$ Since the two balls are drawn without replacement, the two draws are not independent. $P(\text{both balls are black}) = P(A) \times P(B/A)$ Now, the probability of getting both balls red = $\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$	$\frac{1}{2}$ $\frac{1}{2}$ 1
	SECTION – C (3Marks × 8Q)	
Question 26.	Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v) if and only if xv = yu. Show that R is an equivalence relation.	
Solution:	Clearly, (x, y) R (x, y), $\forall (x, y) \in A$	

	<p>Since $xy = yx$ This shows that R is reflexive.</p> <p>Further, $(x, y) R (u, v)$ $\Rightarrow xv = yu$ $\Rightarrow uy = vx$ $\Rightarrow (u, v) R (x, y)$ $\forall (x, y), (u, v) \in A$ This shows that R is symmetric.</p> <p>Similarly, $(x, y) R (u, v)$ and $(u, v) R (a, b)$ $\Rightarrow \quad xv = yu \quad \text{and} \quad ub = va$ $\Rightarrow \quad \frac{x}{y} = \frac{u}{v} \quad \text{and} \quad \frac{u}{v} = \frac{a}{b}$ $\Rightarrow \quad \frac{x}{y} = \frac{a}{b}$ $\Rightarrow \quad xb = ya$ Hence $(x, y) R (a, b)$ $\forall (x, y), (u, v) (a, b) \in A$ Thus, R is transitive. Thus, R is an equivalence relation.</p>	<p>1</p> <p>1</p> <p>1</p>
OR Question 26.	<p>Prove that: $\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$</p>	
Solution:	<p>$\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$</p> <p>We know $\cos^{-1} x = \sin^{-1} \sqrt{1 - x^2}$ where $x < 1$</p> <p>$\Rightarrow \cos^{-1} \frac{12}{13} = \sin^{-1} \sqrt{1 - \left(\frac{12}{13}\right)^2}$</p> <p>$\Rightarrow \quad \quad \quad = \sin^{-1} \sqrt{\frac{25}{169}}$</p> <p>$\Rightarrow \cos^{-1} \frac{12}{13} = \sin^{-1} \frac{5}{13}$</p> <p>Now taking L.H.S. $= \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{3}{5}$</p> <p>We know that,</p> <p>$\sin^{-1} x + \sin^{-1} y = \sin^{-1} [x\sqrt{1 - y^2} + y\sqrt{1 - x^2}]$ if $xy < 1$</p> <p>$\therefore \quad \quad \quad = \sin^{-1} \left[\frac{5}{13} \sqrt{1 - \left(\frac{3}{5}\right)^2} + \right.$</p> <p>$\left. \frac{3}{5} \sqrt{1 - \left(\frac{5}{13}\right)^2} \right]$</p> <p>$\quad \quad \quad = \sin^{-1} \left[\frac{5}{13} \sqrt{\frac{16}{25}} + \frac{3}{5} \sqrt{\frac{144}{169}} \right]$</p> <p>$\quad \quad \quad = \sin^{-1} \left[\frac{5}{13} \left(\frac{4}{5}\right) + \frac{3}{5} \left(\frac{12}{13}\right) \right] = \sin^{-1} \frac{56}{65}$</p> <p style="text-align: center;">L.H.S. = R.H.S.</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>

Question 27.	If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x).F(y) = F(x + y)$.	
Solution:	$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\text{and } F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $F(x + y) = \begin{bmatrix} \cos(x + y) & -\sin(x + y) & 0 \\ \sin(x + y) & \cos(x + y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ <p>$F(x).F(y) = F(x + y)$</p> $F(x).F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} \cos(x + y) & -\sin(x + y) & 0 \\ \sin(x + y) & \cos(x + y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ <p>$\Rightarrow F(x).F(y) = F(x + y)$</p>	<p>$\frac{1}{2}$</p> <p>$1\frac{1}{2}$</p> <p>1</p>
Question 28.	Find $\frac{dy}{dx}$ of the function $(\cos x)^y = (\cos y)^x$.	
Solution:	<p>Given: $(\cos x)^y = (\cos y)^x$</p> <p>$(\cos x)^y = (\cos y)^x$</p> <p>Taking log on both sides</p> <p>$\log((\cos x)^y) = \log((\cos y)^x)$</p> <p>$y \cdot \log(\cos x) = x \cdot \log(\cos y)$</p> <p>Diff. on both sides w.r.t. 'x'</p> <p>$\frac{d}{dx} (y \cdot \log(\cos x)) = \frac{d}{dx} (x \cdot \log(\cos y))$</p>	<p>1</p>

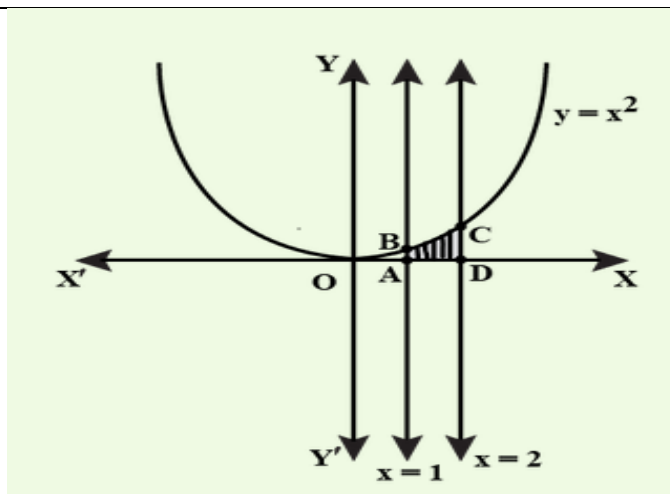
	$y \cdot \frac{1}{\cos x}(-\sin x) + \log(\cos x) \cdot \frac{dy}{dx} = x \cdot \frac{1}{\cos y}(-\sin y) \cdot \frac{dy}{dx} + \log(\cos y) \cdot 1$ $-y \cdot (\tan x) + \log(\cos x) \cdot \frac{dy}{dx} = -x \cdot (\tan y) \cdot \frac{dy}{dx} + \log(\cos y)$ $(\log(\cos x) + x(\tan y)) \cdot \frac{dy}{dx} = \log(\cos y) - y \cdot (\tan x)$ $\frac{dy}{dx} = \frac{\log(\cos y) - y \cdot (\tan x)}{\log(\cos x) + x \cdot (\tan y)}$	$\frac{1}{2}$ $\frac{1}{2}$
Question 29.	Find the intervals in which the function f is given by $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing or strictly decreasing.	
Solution:	<p>Given function: $f(x) = -2x^3 - 9x^2 - 12x + 1$</p> $f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2)$ $f'(x) = -6(x + 2)(x + 1) \quad \dots\dots(1)$ <p>Now for increasing or decreasing, $f'(x) = 0$ $-6(x + 2)(x + 1) = 0$ $x + 2 = 0$ or $x + 1 = 0$ $x = -2$ or $x = -1$</p> <p>Therefore, we have sub-intervals are $(-\infty, -2)$, $(-2, -1)$ and $(-1, \infty)$</p> <p>For interval $(-\infty, -2)$, picking $x = -3$, from equation (1), $f'(x) = (-ve)(-ve)(-ve) = (-ve) < 0$ Therefore, f is strictly decreasing in $(-\infty, -2)$</p> <p>For interval $(-2, -1)$, picking $x = -1.5$, from equation (1), $f'(x) = (-ve)(+ve)(-ve) = (+ve) > 0$ Therefore, f is strictly increasing in $(-2, -1)$.</p> <p>For interval $(-1, \infty)$, picking $x = 4$, from equation (1), $f'(x) = (-ve)(+ve)(+ve) = (-ve) < 0$ Therefore, f is strictly decreasing in $(-1, \infty)$. So, f is strictly decreasing in $(-\infty, -2)$ and $(-1, \infty)$. f is strictly increasing in $(-2, -1)$.</p>	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$
Question 30.	Integrate: $\int \frac{4x+1}{\sqrt{2x^2+x-3}} dx$	
Solution:	It is given that $I = \int \frac{4x+1}{\sqrt{2x^2+x-3}} dx$	

	<p>Here form of integral is $\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$</p> <p>$\therefore 4x + 1 = A \frac{d}{dx}(2x^2 + x - 3) + B$</p> <p>$4x + 1 = A(2x + 1) + B \quad \dots(1)$</p> <p>On comparing the like terms , we have</p> <p>$2A = 4$ and $A + B = 1$</p> <p>$\Rightarrow A = 2$ and $B = -1$</p> <p>$\Rightarrow 4x + 1 = 2(4x + 1) - 1 \quad \dots\text{from (1)}$</p> <p>$I = \int \frac{2(4x+1)-1}{\sqrt{2x^2+x-3}} dx$</p> <p>$I = 2 \int \frac{4x+1}{\sqrt{2x^2+x-3}} dx - \int \frac{1}{\sqrt{2x^2+x-3}} dx$</p> <p>Put $2x^2 + x - 3 = t \Rightarrow (4x + 1) dx = dt$</p> <p>$I = 2 \int \frac{1}{\sqrt{t}} dt - \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 + \frac{1}{2}x - \frac{3}{2}}} dx$</p> <p>$I = 4\sqrt{t} - \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 + \frac{1}{2}x - \frac{3}{2}}} dx$</p> <p>$I = 4\sqrt{2x^2 + x - 3} - \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 + \frac{1}{2}x + \frac{1}{4} - \frac{1}{4} - \frac{3}{2}}} dx \quad (\text{completing the square})$</p> <p>$I = 4\sqrt{2x^2 + x - 3} - \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{(x + \frac{1}{2})^2 - (\frac{\sqrt{7}}{4})^2}} dx$</p> <p>$I = 4\sqrt{2x^2 + x - 3} - \frac{1}{\sqrt{2}} \log \left \left(x + \frac{1}{2}\right) + \sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{4}\right)^2} \right + C$</p> <p>$I = 4\sqrt{2x^2 + x - 3} - \frac{1}{\sqrt{2}} \log \left \frac{(2x+1) + \sqrt{2x^2+x-3}}{2} \right + C$</p>	<p>1</p> <p>1</p> <p>1</p>
<p>OR Question 30.</p>	<p>Evaluate: $\int_2^8 x - 5 dx$</p>	
<p>Solution:</p>	<p>$I = \int_2^8 x - 5 dx$</p> <p>We know $x - 5 = \begin{cases} -(x - 5), & x \leq 5 \\ (x - 5), & x > 5 \end{cases}$</p> <p>$I = \int_2^5 x - 5 dx + \int_5^8 x - 5 dx$</p> <p>$I = \int_2^5 -(x - 5) dx + \int_5^8 (x - 5) dx$</p>	<p>$\frac{1}{2}$</p>

	$I = \left \frac{-(x-5)^2}{2} \right _2^5 + \left \frac{(x-5)^2}{2} \right _5^8$ $I = \left(\frac{-(0)^2}{2} - \frac{-(-3)^2}{2} \right) + \left(\frac{(3)^2}{2} - \frac{(0)^2}{2} \right)$ $I = \frac{9}{2} + \frac{9}{2}$ $I = 9$	$1\frac{1}{2}$ 1
Question 31.	Find the area of a triangle having points A(1, 1, 1), B(1, 2, 3) and C(2, 3,1) as its vertices.	
Solution:	<p>We have $\vec{AB} = (1-1)\hat{i} + (2-1)\hat{j} + (3-1)\hat{k} = \hat{j} + 2\hat{k}$ and $\vec{AC} = (2-1)\hat{i} - (3-1)\hat{j} - (1-1)\hat{k} = \hat{i} - 2\hat{j}$</p> <p>Area of the given triangle is $\frac{1}{2} \vec{AB} \times \vec{AC}$.</p> <p>Now $\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix}$ $= -4\hat{i} + 2\hat{j} - \hat{k}$</p> <p>Therefore, $\vec{AB} \times \vec{AC} = \sqrt{16 + 4 + 1} = \sqrt{21}$</p> <p>Thus the required area is $\frac{1}{2}\sqrt{21}$.</p>	 1/2 1/2 1 1
SECTION – C (5Marks × 4Q)		
Question 32.	Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - y + 2z = 1; 2y - 3z = 1; 3x - 2y + 4z = 2$	
Solution:	$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = =$ $\begin{bmatrix} -2 - 9 + 12 & -2 + 2 & 1 + 3 - 4 \\ 18 - 18 & 4 - 3 & -6 + 6 \\ -6 - 18 + 24 & -4 + 4 & 3 + 6 - 8 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	

	$\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}, \quad \text{and} \quad \vec{b}_1 = 7\hat{i} - 6\hat{j} + \hat{k}$ $\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k} \quad \text{and} \quad \vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$ <p>Therefore $\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 6\hat{j} + 8\hat{k}$</p> <p>And $\vec{b}_1 \times \vec{b}_2 = (7\hat{i} - 6\hat{j} + \hat{k}) \times (\hat{i} - 2\hat{j} + \hat{k})$</p> $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{i} - 6\hat{j} - 8\hat{k}$ $ \vec{b}_1 \times \vec{b}_2 = \sqrt{16 + 36 + 64} = \sqrt{116}$ <p>Hence, the shortest distance between the given lines is given by</p> $D = \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 } = \frac{ (4\hat{i} + 6\hat{j} + 8\hat{k}) \cdot (-4\hat{i} - 6\hat{j} - 8\hat{k}) }{\sqrt{116}}$ $\frac{ -16 - 36 - 64 }{\sqrt{116}} = \frac{116}{\sqrt{116}} = \sqrt{116} = 2\sqrt{29}$	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>
<p>OR Question 33.</p>	<p>Find the vector equation of the line passing through the point $(1, -2, -3)$ and perpendicular to the two lines : $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{3}$ and $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$.</p>	<p>1</p>
<p>Solution:</p>	<p>The vector equation of a line passing through a point with position vector \vec{a} and parallel to \vec{b} is $\vec{r} = \vec{a} + \lambda \vec{b}$.</p> <p>It is given that, the line passes through $(1, -2, -3)$ So, $\vec{a} = 1\hat{i} - 2\hat{j} - 3\hat{k}$</p> <p>Given lines are $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{3}$ and $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$</p> <p>It is also given that, line is perpendicular to both given lines. So we can say that the line is perpendicular to both parallel vectors of two given lines.</p>	<p>1</p>

	<p>We know that, $\vec{a} \times \vec{b}$ is perpendicular to both \vec{a} & \vec{b}, so let \vec{b} is cross product of parallel vectors of both lines i.e. $\vec{b} = \vec{b}_1 \times \vec{b}_2$ where $\vec{b}_1 = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$</p> <p>and Required Normal</p> $\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & 1 & 2 \end{vmatrix}$ $= \hat{i}(-2 - 3) - \hat{j}(2 - 6) + \hat{k}(1 + 2)$ $\vec{b} = -5\hat{i} + 4\hat{j} + 3\hat{k}$ <p>Now, by substituting the value of \vec{a} & \vec{b} in the formula $\vec{r} = \vec{a} + \lambda \vec{b}$, we get</p> $\vec{r} = (1\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(-5\hat{i} + 4\hat{j} + 3\hat{k})$	<p>2</p> <p>1</p> <p>1</p>
<p>Question 34.</p>	<p>Find the area under the given curve $y = x^2$ and the given lines $x = 1$, $x = 2$ and x-axis.</p>	
<p>Solution:</p>	<p>Equation of the curve is $y = x^2$. It is an upward parabola having vertex at origin and symmetrical about y-axis. $x = 1$ and $x = 2$ are two straight lines parallel to y-axis. $y = x^2$(1) $x = 1$ and $x = 2$</p> <p>Points of intersections of given curves At $x = 1$, $y = 1$ points are (1, 1) At $x = 2$, $y = 4$ points are (2, 4) ∴ Points in first quadrant A(1, 1) B(2, 4) Points on x- axis with given lines are (1, 0) and (2, 0)</p> <p>Make a rough hand sketch of given curves by taking some corresponding values of x and y.</p>	<p>$\frac{1}{2}$</p> <p>$1\frac{1}{2}$</p>



1

Required area is shaded region ABCD:

$$|\int_1^2 y \, dx| = |\int_1^2 x^2 \, dx| \quad [\text{From equation (1)}]$$

2

$$= \left| \frac{x^3}{3} \right|_1^2$$

$$= \frac{1}{3} |(2^3 - 1^3)|$$

$$= \frac{1}{3} |(8 - 1)| = \frac{1}{3} (7) = \frac{7}{3} \text{ sq. units}$$

OR
Question 34.

Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Solution:

$$\text{Here } \frac{x^2}{4} + \frac{y^2}{9} = 1 \quad \dots(1)$$

It is a vertical ellipse having center at origin and is symmetrical about both axes (if we change y to -y or x to -x, equation remain same).

$$\text{Standard equation of an ellipse is } \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

By comparing, $a = 3$ and $b = 2$

From equation (1)

$$\Rightarrow y^2 = \frac{9}{4} (4 - x^2)$$

$$\Rightarrow y = \frac{3}{2} \sqrt{4 - x^2} \quad \dots\dots(2)$$

Points of Intersections of ellipse (1) with x-axis ($y = 0$)

Put $y = 0$ in equation (1), we have

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1

$$x^2/4 = 1 \Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

Therefore, Intersections of ellipse(1) with x-axis are (0, 2) and (0, -2).

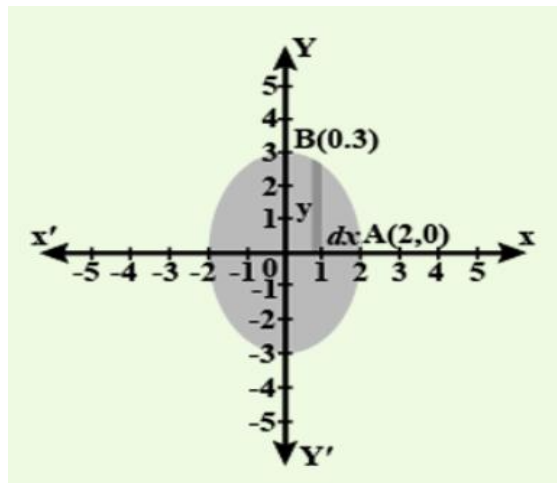
Points of Intersections of ellipse (1) with y-axis (x = 0)

$$\text{Putting } x = 0 \text{ in equation (1), } y^2/9 = 1 \Rightarrow y^2 = 9$$

$$\Rightarrow y = \pm 3.$$

Therefore, Intersections of ellipse (1) with y-axis are (0, 3) and (0,-3).

for arc of ellipse in first quadrant.



Now,

Area of region bounded by ellipse (1)

Total shaded area = 4 x Area OAB of ellipse in first quadrant

$$= 4 \left| \int_0^2 y \cdot dx \right| \quad [\because \text{at end B of arc AB of ellipse: } x=0 \text{ and at end A of arc AB; } x=2]$$

$$= 4 \left| \int_0^2 \frac{3}{2} \sqrt{4 - x^2} \cdot dx \right| = 6 \left| \int_0^2 \sqrt{2^2 - x^2} \cdot dx \right|$$

$$= 6 \left[\frac{x}{2} \sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \quad [\because \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}]$$

$$= 6 \left[\left(\frac{2}{2} \sqrt{4 - 4} + 2 \sin^{-1} 1 \right) - (0 + 2 \sin^{-1} 0) \right] = 6 \left[0 + \left(2 \frac{\pi}{2} \right) \right]$$

$$= 6\pi \text{ sq. Units}$$

1

1/2

1 1/2

1/2

Question 35.

Solve the following problem graphically:
 Minimise and Maximise $Z = 5x + 10y$
 Subject to the constraints: $x + 2y \leq 120$
 $x + y \geq 60$
 $x - 2y \geq 0$
 $x \geq 0, y \geq 0$

Solution:

$Z = 5x + 10y. \dots(1)$
 $x + 2y \leq 120 \dots(2)$
 $x + y \geq 60. \dots(3)$
 $x - 2y \geq 0 \dots(4)$
 $x \geq 0, y \geq 0 \dots(5)$

First of all, let us graph the feasible region of the system of linear inequalities (2) to (5).

Let $Z = 5x + 10y \dots(1)$

Converting inequalities to equalities

$x + 2y = 120$

X	0	120
Y	60	0

Points are (0, 60), (120,0)

Now put (0, 0) in inequation (2),
 we find $0 \leq 120$, which is true.

Therefore area lies towards the origin from this line.

$x + y = 60$

x	0	60
y	60	0

Points are (0, 60), (60, 0)

Now put (0, 0) in inequation (3),
 we find $0 \geq 60$, which is False.

Therefore area lies away from the origin from this line.

$x - 2y = 0$

X	0	20	40
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$\frac{1}{2}$

y	0	10	20
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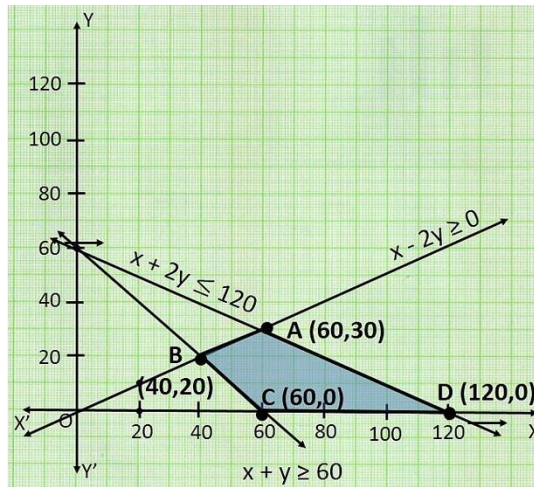
Points are (0,0),(20,10),(40,20)

Now put (1, 0) in inequation (4),

we find $1 \geq 0$, which is true.

Therefore area lies towards (1, 0) origin from this line.

Plot the graph for the set of points



To find maximum and minimum

The feasible region ABCD is shown in the figure. Note that the region is bounded. The coordinates of the corner points A, B, C and D are (60, 30), (40, 20), (60, 0) and (120, 0) respectively.

Corner Point	Corresponding Value of $Z = 5x + 10y$
A (60, 30)	600 ← Maximum
B (40, 20)	400
C (60, 0)	300 ← Minimum
D (120, 0)	600 ← Maximum (Multiple optimal solutions)

We now find the minimum and maximum value of Z.

From the table, we find that the minimum value of Z is 300 at the point B (60, 0) of the feasible region.

$\frac{1}{2}$

1

$\frac{1}{2}$

$1\frac{1}{2}$

	The maximum value of Z on the feasible region occurs at the two corner points C (60, 30) and D (120, 0) and it is 600 in each case.	$\frac{1}{2}$
	SECTION – E (4Marks × 3Q)	
Question 36.	<p>$P(x) = -6x^2 + 120x + 25000$ (in ₹) is the total profit function of a company, where x denotes the production of the company.</p> <p>Based on the above information answer the following:</p> <p>(i) Find the profit of the company when the production is 3units. (1)</p> <p>(ii) Find $P'(5)$. (1)</p> <p>(iii) Find the production, when the profit is maximum. (2)</p>	
Solution:	<p>(i) When $x = 3$</p> $P(3) = -6(3)^2 + 120(3) + 25000$ $= -54 + 360 + 25000$ $= ₹ 25306$	1
	<p>(ii) We have, $P(x) = -6x^2 + 120x + 25000$... (1)</p> <p>Differentiating equation (1) w.r.t. x</p> $P'(x) = -12x + 120$... (2) <p>$\therefore P'(5) = -12(5) + 120 = 60$</p>	1
	<p>(iii) We have, $P(x) = -6x^2 + 120x + 25000$... (1)</p> <p>Differentiating equation (1) w.r.t. x</p> $P'(x) = -12x + 120$... (2) <p>For maximum or minimum value of $P(x)$, $P'(x) = 0$ we have</p> $-12x + 120 = 0$ $-12x = -120$ <p>i.e. $x = 10$</p> <p>Differentiating equation (2) w.r.t. x</p> $P''(x) = -12$ <p>Now ,</p> <p>At $x = 10$ $P''(x) = -12 = -ve$</p> <p>$\Rightarrow P(x)$ has maximum value at $x = 10$</p>	2
Question 37.	<p>A linear differential equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by</p> $y \cdot (\text{I.F.}) = \int Q(\text{I.F.}) dx + c,$ <p>where I.F. (Integrating Factor) = $e^{\int P dx}$</p> <p>Now, suppose the given equation is $x \frac{dy}{dx} + 2y = x^2$</p> <p><i>Based on the above information, answer the following questions:</i></p>	

	<p>(i)What are the values of P and Q respectively? (1)</p> <p>(ii)What is the value of I.F.? (1)</p> <p>(iii)Find the Solution of given equation. (2)</p>	
Solution:	<p>(i) Given equation is $x \frac{dy}{dx} + 2y = x^2$ Dividing on both side by x, we have $\frac{dy}{dx} + \frac{2}{x}y = x$ $\Rightarrow P = \frac{2}{x}, Q = x$</p>	1
	<p>(ii) I.F.(Integrating Factor) = $e^{\int Pdx}$ $= e^{\int \frac{2}{x}dx}$ $= e^{2\log x}$ $= x^2$</p>	1
	<p>(iii) Solution of given equation is $y.(IF.) = \int Q(IF.) dx + c$ $y(x^2) = \int x(x^2) dx + c$ $x^2y = \int x^3 dx + c$ $x^2y = \frac{x^4}{4} + c$</p>	2
Question 38.	<p>In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03. <i>Based on the above information answer the following questions:</i></p>	

	<p>(i) The total probability of committing an error in processing the form. (2)</p> <p>(ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vinay. (2)</p>	
<p>Solution:</p>	<p>(i) Let E_1 = Event of processing form by Vinay. E_2 = Event of processing form by Soniya. E_3 = Event of processing form by Iqbal.</p> <p>$P(E_1) = \frac{50}{100} = \frac{5}{10}$, $P(E_2) = \frac{20}{100} = \frac{2}{10}$, $P(E_3) = \frac{30}{100} = \frac{3}{10}$</p> <p>Also $P(A/E_1) = 0.06$, $P(A/E_2) = 0.04$, $P(A/E_3) = 0.03$</p> <p>Required Probability</p> $P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)$ $= \frac{5}{10} (0.06) + \frac{2}{10} (0.04) + \frac{3}{10} (0.03)$ $= 0.03 + 0.008 + 0.009 = 0.047$	<p>2</p>
	<p>(ii) Probability that the form is not processed by Vinay = $P(\bar{E}_1 A)$ $P(\bar{E}_1 A) = 1 - P(E_1 A)$</p> <p>By Bayes' Theorem</p> $P(E_1 A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$ $P(E_1 A) = \frac{\frac{5}{10} (0.06)}{0.047}$ $P(E_1 A) = \frac{0.03}{0.047} = \frac{30}{47}$ <p>$P(\bar{E}_1 A) = 1 - P(E_1 A)$</p> $= 1 - \frac{30}{47} = \frac{17}{47}$	<p>2</p>