DETERMINANTS & MATRICES

DETERMINANTS

1. INTRODUCTION TO DETERMINANTS

1.1 Definition

(i) The determinant consisting two rows and two columns is $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$. Its value is given by :

$D = a_1 b_2 - a_2 b_1$

(ii) A determinant which consists of 3 rows and 3 columns is called a 3rd-order-determinant and is of the following form.

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

It's value is :

$$D = a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{21} a_{32} a_{13} - a_{13} a_{22} a_{31} - a_{23} a_{32} a_{11} - a_{12} a_{21} a_{33}$$

1.2 Minors and Cofactors

$$\text{Let } \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Here a_{ij} = Element in i^{th} row and j^{th} column of Δ.

Minor of a_{ij} :

It is defined as the value of the determinant obtained by eleminating the i^{th} row and j^{th} column of Δ.

We donote the minor of a_{ij} by M_{ij}.

e.g. M_{11} = minor of $a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22} a_{33} - a_{32} a_{23}$

Cofactor of a_{ij} :

Denoted by C_{ij}

Cofactor of a_{ij} $(C_{ij}) = (-1)^{i+j}$ Minor of a_{ij}

e.g. Cofactor of a_{11} $(C_{11}) = (-1)^{1+1} M_{11} = M_{11}$

1.3 Evaluation of Determinant

Value of any determinant can be obtain by adding product of all elements of a row (or column) to their corresponding cofactors.

e.g. $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\begin{aligned} \Delta &= a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13} \\ &= a_{21} C_{21} + a_{22} C_{22} + a_{23} C_{23} \\ &= a_{31} C_{31} + a_{32} C_{32} + a_{33} C_{33} \\ &= a_{11} C_{11} + a_{21} C_{21} + a_{31} C_{31} \\ &= a_{12} C_{12} + a_{22} C_{22} + a_{32} C_{32} \\ &= a_{13} C_{13} + a_{23} C_{23} + a_{33} C_{33} \end{aligned}$$

2. PROPERTIES OF DETERMINANTS

(i) The value of a determinant remains unaltered; if the rows and columns are interchanged,

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(ii) If any two rows (or columns) of a determinant be interchanged, the value of determinant changes in sign only.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and } D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$D' = -D$.

(iii) If a determinant has all the elements zero in any row (or column) then its values is zero.

$$D = \begin{vmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

(iv) If a determinant has any two rows (or columns) identical or proportional, then its values is zero.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ ka_1 & kb_1 & kc_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

(v) If all the elements of any row (or column) are multiplied by the same number, then the determinant is multiplied by that number.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{; and } D' = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Then $D' = KD$

(vi) If each element of any row (or column) can be expressed as a sum of two terms then the determinant can be expressed as the sum of two determinants, i.e.

$$\begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(vii) The value of determinant is not altered by adding to the elements of any row (or column) a constant multiple of the corresponding elements of any other row (or column).

e.g.

$R_1 \to R_1 + mR_2$ (change R_1 as sum of R_1 and m (R_2)).

$R_3 \to R_3 + nR_2$ (change R_3 as sum of R_3 and n (R_2)).

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and}$$

$$D' = \begin{vmatrix} a_1+ma_2 & b_1+mb_2 & c_1+mc_2 \\ a_2 & b_2 & c_2 \\ a_3+na_2 & b_3+nb_2 & c_3+nc_2 \end{vmatrix}$$

Then $D' = D$.

3. CRAMER'S RULE

(Not in CBSE Syllabus)

(i) Two Variables

If $a_1x + b_1y = c_1$ ... (i)

$a_2x + b_2y = c_2$ ... (ii)

then $x = \frac{D_x}{D}, y = \frac{D_y}{D}$

values of x, y are unique, if $D \neq 0$. where, $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$,

$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

Similarly 'n' equations in 'n' variables can be solved.

(ii) Three Variables

Let, $a_1x + b_1y + c_1z = d_1$ ....(i)

$a_2x + b_2y + c_2z = d_2$ ....(ii)

$a_3x + b_3y + c_3z = d_3$ ....(iii)

Then, $x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{D_z}{D}$

Where, $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$; $D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$;

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \text{ and } D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Consistency of a System of Equations

(i) If $D \neq 0$ then the given system of equations are consistent and have unique solution.

(ii) If $D = 0$ but at least one of D_x, D_y, D_z is not zero then the equations are inconsistent and have no solution.

(iii) If $D = D_x = D_y = D_z = 0$ then the given system of equations are consistent and have infinite solution except the case of parallel planes when there is no solution.

(iv) If $d_1 = d_2 = d_3 = 0$ then system of equation is called Homogenous system of equations.

(v) Solution of Homogenous Equations is always consistent, as x = 0 = y = z is always a solution. This is known as TRIVIAL solution.

(vi) For Homogenous Equations, if $D \neq 0$. Then $x = 0 = y = z$ is the only solution.

(vii) For Homogenous Equations, if D = 0, then there exists non zero solutions [NON TRIVIAL SOLUTONS] also.

4. APPLICATION OF DETERMINANTS

Following examples of short hand writing of large expressions are:

(i) Area of triangle whose vertices are (x_r, y_r) ; r = 1, 2, 3, is:

$$D = \left| \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right|$$

If D = 0 then the three points are collinear.

(ii) Equation of straight line passing through (x_1, y_1) &

$$(x_2, y_2) \text{ is } \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

(iii) The lines :

$a_1x + b_1y + c_1 = 0$ ....(1)

$a_2x + b_2y + c_2 = 0$............(2)

$a_3x + b_3y + c_3 = 0$............(3)

$$\text{are concurrent if, } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

This is condition for the consistency of simultaneous linear equation in two variables.

(iv) $ax^2 + 2\,hxy + by^2 + 2\,gx + 2\,fy + c = 0$ represents a pair of straight lines if :

$$abc + 2\,fgh - af^2 - bg^2 - ch^2 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

(v) To find the variable (x, y, z etc) in linear equations (Cramer's rule)

5. SOME MORE PROPERTIES OF DETERMINANT

(i) Determinant of a skew-symmetric matrix of odd order is zero.

$$\textbf{e.g. } D = \begin{vmatrix} 0 & 2 & 9 \\ -2 & 0 & \log_a b \\ -9 & \log_a\left(\frac{1}{b}\right) & 0 \end{vmatrix} = 0$$

(ii) Determinant of a skew-symmetric matrix of even order is always a perfect square.

$$\textbf{e.g. } D = \begin{vmatrix} 0 & 5 \\ -5 & 0 \end{vmatrix}_{2\times2} = 25$$

(iii) Δ^{n-1}, where n is order of the determinant is equal to the determinant made from cofactors of elements of Δ.

$$\textbf{e.g. } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}_{3\times3}^{3-1} = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$$

Where A_i's are co-factors of a_i's

(iv) Determinant of a diagonal matrix is product of its diagonal elements

$$\textbf{e.g. } D = \begin{vmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{vmatrix} = 5 \times 2 \times 6 = 60$$

(v) If a determinant considered as a polynomial becomes zero when x = a, then x – a is factor of this. (This is an application of Factor Theorem)

$$D = \begin{vmatrix} x & a & a^2 \\ a & x & x^2 \\ a & x & a \end{vmatrix}$$

Because D = 0 when x = a so x – a is factor of D.

(vi) The sum of the products of the elements of the i^{th} row/column with the co-factor of the corresponding elements of k^{th} row/column is zero provided $i \neq k$.

i.e. (i) $\sum_{j=1}^{n} a_{ij}C_{kj} = 0$; if $i \neq k$

(ii) $\sum_{i=1}^{n} a_{ij}C_{ik} = 0$ if $j \neq k$

(vii) $|AB| = |A||B|$

i.e. $A = \begin{vmatrix} 5 & -2 \\ -1 & -1 \end{vmatrix}$; $B = \begin{vmatrix} -2 & -3 \\ -4 & 1 \end{vmatrix}$; $AB = \begin{vmatrix} -2 & -17 \\ 6 & 2 \end{vmatrix}$

$|A| = -7 \quad |B| = -14$

$|A||B| = -7 \times -14 = 98$,

$|AB| = -4 + 102 \Rightarrow |AB| = 98$.

(viii) Determinant of a triangular matrix is product of its diagonal elements only.

$$D = \begin{vmatrix} 3 & 2 & 1 \\ 0 & 4 & 3 \\ 0 & 0 & -1 \end{vmatrix} = 3 \times 4 \times -1 = -12$$

$$D' = \begin{vmatrix} 5 & 0 & 0 \\ 4 & 9 & 0 \\ 3 & 2 & 1 \end{vmatrix} = 5 \times 9 \times 1 = 45$$

6. SPECIAL DETERMINANTS

(i) Circulant Determinants :

The elements of the rows (or columns) are in cyclic arrangement

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc).$$

$$= -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$

(iii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)$

(iv) $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(ab + bc + ca)$

7. MULTIPLICATION OF TWO DETERMINANTS

(a) $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} \ell_1 & m_1 \\ \ell_2 & m_2 \end{vmatrix} = \begin{vmatrix} a_1\ell_1 + b_1\ell_2 & a_1m_1 + b_1m_2 \\ a_2\ell_1 + b_2\ell_2 & a_2m_1 + b_2m_2 \end{vmatrix}$

(b) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \\ \ell_3 & m_3 & n_3 \end{vmatrix}$

$$= \begin{vmatrix} a_1\ell_1 + b_1\ell_2 + c_1\ell_3 & a_1m_1 + b_1m_2 + c_1m_3 & a_1n_1 + b_1n_2 + c_1n_3 \\ a_2\ell_1 + b_2\ell_2 + c_2\ell_3 & a_2m_1 + b_2m_2 + c_2m_3 & a_2n_1 + b_2n_2 + c_2n_3 \\ a_3\ell_1 + b_3\ell_2 + c_3\ell_3 & a_3m_1 + b_3m_2 + c_3m_3 & a_3n_1 + b_3n_2 + c_3n_3 \end{vmatrix}$$

8. SUMMATION OF DETERMINANTS

Let $\Delta_r = \begin{vmatrix} f(r) & a & \ell \\ g(r) & b & m \\ h(r) & c & n \end{vmatrix}$

Where a, b, c, *l*, m and n are constants independent of r, then

$$\sum_{r=1}^{n} \Delta_r = \begin{vmatrix} \sum_{r=1}^{n} f(r) & a & \ell \\ \sum_{r=1}^{n} g(r) & b & m \\ \sum_{r=1}^{n} h(r) & c & n \end{vmatrix}$$

Here functions of r can be the elements of only one row or column. None of the elements other than row or column should be dependent on r.

9. DIFFERENTIATION AND INTEGRATION OF DETERMINANT

$$\Delta(x)=\begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$$

Then

$$\Delta'(x)=\begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}+\begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1'(x) & g_2'(x) & g_3'(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$$

$$+\begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1'(x) & h_2'(x) & h_3'(x) \end{vmatrix}$$

Integration of determinant

If $\Delta(x)=\begin{vmatrix} f(x) & g(x) \\ \lambda_1 & \lambda_2 \end{vmatrix}$,

then $\int_a^b \Delta(x)\, dx=\begin{vmatrix} \int_a^b f(x)\,dx & \int_a^b g(x)\,dx \\ \lambda_1 & \lambda_2 \end{vmatrix}$.

Here f (x) and g (x) and functions of x and λ_1, λ_2 are constants.

NOTES:

This formula is only applicable if there is a variable only in one row or column, otherwise expand the determinant and then integrate.

Example : If $f(x)=\begin{vmatrix} x^3 & \cos^2 x & 2^{x^4} \\ \tan^5 x & 1 & \sec 2x \\ \sin^3 x & x^4 & 5 \end{vmatrix}$ then $\int_{-\pi/2}^{\pi/2} f(x)dx=$

(a) 2 (b) –2 (c) 0 (d) none of these

Sol. We have

$$f(-x)=\begin{vmatrix} -x^3 & \cos^2 x & 2^{x^4} \\ -\tan^5 x & 1 & \sec 2x \\ -\sin^3 x & x^4 & 5 \end{vmatrix}=-f(x)$$

$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx=0$. (since f(x) is an odd function)

Example : If $f(x)=\begin{vmatrix} x^3 & \sin x \\ 1 & 2 \end{vmatrix}$, then $\int_{-a}^{a} f(x)\,dx$ is

(a) 0 (b) $\frac{1}{2}$ (c) 3 (d) $-\frac{1}{2}$

Sol. $\int_{-a}^{a} f(x)\,dx=\begin{vmatrix} \int_{-a}^{a} x^3 dx & \int_{-a}^{a} \sin x dx \\ 1 & 2 \end{vmatrix}=\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}=0$

Hence (a) is correct answer.

MATRICES

1. INTRODUCTION TO MATRICES

A set of (m × n) numbers arranged in the form of an ordered set of m rows and n columns is called a matrix of order m × n.

$A=[a_{ij}]_{m\times n}$

or $A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ is a matrix of order m×n.

NOTES:

The matrix is not a number. It has no numerical value. But it is an arrangement of numbers.

2. TYPES OF MATRICES

2.1 Row Matrix

A matrix having only one row is called a row matrix or a row vector.

e.g. $A = [1 \quad 2 \quad -1 \quad -2]$ is a row matrix of order 1×4.

2.2 Column Matrix

A matrix having only one column is called a column matrix or a column vector.

e.g. $A = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 2 \\ 5 \\ 4 \end{bmatrix}$ are column matrices or order 3×1 and 4×1 respectively.

2.3 Square Matrix

A matrix in which the number of rows is equal to the number of column, say $(n \times n)$ is called a square matirx of order n.

e.g. the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 3 & -2 & 5 \\ 1 & 5 & -3 \end{bmatrix}$ is square matrix of order 3.

Sum of diagonal elements of a square matrix is called its trace (tr (A)). Here tr(A) = 2-2-3= -3

2.4 Diagonal Matrix

A square matix is called a diagonal matrix if all the elements, except those in the leading diagonal, are zero.

$A = [a_{ij}]_{n\times n}$, $a_{ij} = 0$ for all $i \neq j$

2.5 Scalar Matrix

A diagonal matrix in which all the diagonal elements are equal is called the scalar matrix.

A square matrix $A = [a_{ij}]_{n\times n}$ is called a scalar matrix if.

(i) $a_{ij} = 0$ for all $i \neq j$ and

(ii) $a_{ii} = C$ for all $i \in \{1, 2, ..., n\}$

2.6 Identity or Unit Matrix

A square matrix each of whose diagonal element is unity and each of whose non diagonal element is equal to zero is called an identity or unit matrix.

A square matrix $A = [a_{ij}]_{n\times n}$ is called identity or unit matrix if

(i) $a_{ij} = 0$ for all $i \neq j$, and

(ii) $a_{ii} = 1$ for all $i \in \{1, 2, ..., n\}$

The identity matrix of order n is denoted by I_n.

e.g. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2.7 Null Matrix

A matrix whose all elements are zero is called a null matrix or a zero matrix, represented by O.

e.g. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2.8 Upper Triangular Matrix

A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0 \ \ \forall \ i > j$.

e.g $A = \begin{bmatrix} 5 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix}$

2.9 Lower Triangular Matrix

A square matrix $A = [a_{ij}]$ is called lower triangular if $a_{ij} = 0 \ \forall \ i < j$.

e.g $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 5 & 3 \end{bmatrix}$

2.10 Singular Matrix

A square matrix with zero determinant is called a singular matrix.

3. EQUALITY OF MATRICES

Two matrices $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{r\times s}$ are equal if

(i) $m = r$, i.e., the number of rows in A equals the number of rows in B.

(ii) $n = s$, i.e., the number of columns in A equals the number of columns in B.

(iii) $a_{ij} = b_{ij}$ for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$.

If two matrices A and B are equal, we write $A = B$, otherwise we write $A \neq B$.

4. SUM OF MATRICES

Let $A = [a_{ij}], B = [b_{ij}]$ be matrices of the same order $m \times n$.

Then $C = A + B = [c_{ij}]$, is a matrix of order $m \times n$.

Where, $[c_{ij}] = [a_{ij} + b_{ij}]$

e.g. $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 & 2 \\ 5 & 1 & 9 \end{bmatrix}$

$$A + B = \begin{bmatrix} 1+7 & 2+3 & 4+2 \\ 0+5 & 5+1 & 3+9 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 6 \\ 5 & 6 & 12 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 1-7 & 2-3 & 4-2 \\ 0-5 & 5-1 & 3-9 \end{bmatrix} = \begin{bmatrix} -6 & -1 & 2 \\ -5 & 4 & -6 \end{bmatrix}$$

Properties of Matrix Addition

(i) Matrix addition is commutative

$A + B = B + A$

(ii) Matrix addition is associative

$A + (B + C) = (A + B) + C.$

5. SCALAR MULTIPLE OF A MATRIX

If A be a given matrix and k is any scalar number real or complex.

Then matrix kA is a matrix of same order, where all the elements of kA are k times of the corresponding elements of A.

e.g. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 2 & 4 \end{bmatrix}$

Then $3A = \begin{bmatrix} 3.2 & 3.3 & 3.1 \\ 3.5 & 3.2 & 3.4 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 3 \\ 15 & 6 & 12 \end{bmatrix}$

Properties of Multiplication by a Scalar

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrix of the same order and α and β are any scalars, then

(i) $\alpha(A + B) = \alpha A + \alpha B$

(ii) $(\alpha + \beta) A = \alpha A + \beta A$

(iii) $\alpha(\beta A) = (\alpha\beta) A.$

(iv) If A is a square matrix of order 'n'

Then $|kA| = k^n |A|$

6. MATRIX MULTIPLICATION

If $A = [a_{ij}]_{m \times p}$ and $B = [b_{jk}]_{p \times n}$

Then $A_{m \times p} \times B_{p \times n} = (AB)_{m \times n}$

or $C = AB = [c_{ik}]_{m \times n}$ where $c_{ik} = \sum_{j=1}^{p} a_{ij} b_{jk}$

For multiplication number of columns of first matrix should be equal to number of rows of second matrix.

i.e. $c_{ik} = a_{i1} b_{1k} + a_{i2} b_{2k} + ... a_{ip} b_{pk}$

In other words c_{ik} = Sum of the products of i^{th} row of A (having p elements) with k^{th} column of B (having p elements).

e.g. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2\times3}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}_{3\times2}$

Compute AB and show that $AB \neq BA$. A is 2×3 type and B is 3×2 type and hence both AB and BA are defined because the number of columns in pre factor is equal to the number of rows in post factor.

Sol.

$$AB = \begin{bmatrix} 1.2 - 2.4 + 3.2 & 1.3 - 2.5 + 3.1 \\ -4.2 + 2.4 + 5.2 & -4.3 + 2.5 + 5.1 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}_{2\times2}$$

$$BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}_{3\times2} \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2\times3} = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}_{3\times3}$$

Hence $AB \neq BA$.

7. PROPERTIES OF MATRIX MULTIPLICATION

(i) Multiplication of matrices is distributive with respect to a addition of matrices.

$A(B + C) = AB + AC$ and $(A + B) C = AC + BC$

(ii) Matrix multiplication is associative if conformability is assured.

i.e. $A(BC) = (AB) C.$

(iii) The multiplication of matrices is not always commutative. i.e. AB is not always equal to BA.

(iv) Multiplication of a matrix A by a null matrix conformable with A for multiplication is a null matrix i.e. AO = O.

In particular if A be a square matrix and O be square null matrix of the same order, then OA = AO = O.

(v) If AB = O then it does not necessarily mean that A = O or B = O.

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

None of the matrices on the left is a null matrix whereas their products is a null matrix.

(vi) Multiplication of matrix A by a unit matrix I : Let A be a m × n matrix.

Then $AI_n = A$ and $I_m A = A$..

(vii) If A and B are square matrices of order 'n'

Then |AB| = |A| |B|

(viii) Positive Integral Powers of Matrix

Let A be any square matrix of order n.

Then $A^2 = A.A$

$A^3 = A.A.A$

$A^m = A.A.A$ ... m times

All are square matrix of order n.

(i) $A^m . A^n$ = (A.A.A ... m times) (A.A.A ... n times)

= A.A.A.... (m + n) times

= A^{m+n}

(ii) $(A^m)^n = A^{mn}$

Also, we define $A^0 = I$

8. TRANSPOSE OF A MATRIX

If A be a given matrix of the order m × n then the matrix obtained by changing the rows of A into columns and columns of A into rows is called Transpose of matrix A and is denoted by A' or A^T. Hence the matrix A' is of order n × m.

e.g. $A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 5 & 9 \end{bmatrix}_{3\times 2}$ then $A^T = A' = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 9 \end{bmatrix}_{2\times 3}$

Properties of Transpose

(i) (A')' = A.

(ii) (kA)' = kA'. k being a scalar.

(iii) (A + B)' = A' + B'.

(iv) (AB)' = B'A'.

(v) (ABC)' = C'B'A'.

9. SYMMETRIC AND SKEW SYMMETRIC MATRICES

(i) A square matrix $A = [a_{ij}]$ will be called **symmetric** if $A^T = A$.

i.e. every ij^{th} element = ji^{th} element.

e.g. $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}_{3\times 3}$

(ii) A square matrix $A = [a_{ij}]$ will be called **skew symmetric** if $A^T = -A$.

i.e. every ij^{th} element = -(ji^{th} element).

(iii) For any square matrix A, $A + A^T$ is symmetric and $A - A^T$ is skew - symmetric.

(iv) Any square matrix can be uniquely expressed as a sum of a symmetric matrix and a skew-symmetric matrix.

$\frac{1}{2}\left(A + A^T\right)$ is symmetric

and $\frac{1}{2}\left(A - A^T\right)$ is skew-symmetric

(v) Let A and B be symmetric matrices of the same order. Then the following hold:

1. A^n is symmetric for all positive integers n.

2. AB is symmetric if and only if AB = BA.

3. AB + BA is symmetric.

4. AB – BA is skew - symmetric

NOTES:

For a skew symmetric matrix :

$a_{ii} = -a_{ii}$ for all values of i

[i = j when elements are diagonals].

$2a_{ii} = 0 \quad \therefore \quad a_{ii} = 0$

Hence the diagonal elements of skew symmetric matrix are zero.

e.g. $\begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix}$ is a skew symmetric matrix

10. ADJOINT

If A is a square matrix, then transpose of a matrix made from cofactors of elements of A is called adjoint matrix of A. It's denoted by **adj A**.

Properties of Adjoint Matrix

(i) $A.(\text{Adj } A) = |A| I_n = (\text{adj } A).A$

(ii) $|\text{adj } A| = |A|^{n-1}$

(iii) $\text{adj}(\text{adj } A) = |A|^{n-2} A$

(iv) $(\text{adj } A)^T = \text{adj}(A^T)$

(v) $\text{adj}(AB) = (\text{adj } B).(\text{adj } A)$

(iv) $\text{Adj}(A^{-1}) = (\text{adj } A)^{-1}$

(vii) $|(\text{adj}(\text{adj}(A))| = |A|^{(n-1)^2}$

11. INVERSE OF MATRIX (A)

11.1

A square matrix A of order n is said to be invertible or non-singular if there exists a square matrix B of order n such that

$$AB = I_n = BA$$

where I_n is the identity matrix of order n, B is called inverse of A and is denoted by A^{-1}.

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

11.2 Properties of Inverse Matrices

(i) $(A^T)^{-1} = (A^{-1})^T$

(ii) $(AB)^{-1} = B^{-1} A^{-1}$

(iii) $(A^{-1})^{-1} = A$

(iv) $|A^{-1}| = \frac{1}{|A|}$

(v) $(kA)^{-1} = \frac{1}{k} A^{-1}$ if $k \neq 0$.

(vi) Let A, B, C be square matrix of the same order n. If A is non–singular matrix then

(a) $AB = AC \Rightarrow B = C$ (left cancellation law)

(b) $BA = CA \Rightarrow B = C$ (right cancellation law)

(vii) If A is non singular matrix such that A is symmetric then A^{-1} is also symmetric.

12. ELEMENTARY TRANSFORMATIONS

Any one of the following operations on a matrix is called an elementary transformation.

(i) Interchanging any two rows (or column).

(ii) Multiplication of the elements of any row (or column) by a non zero scalar quantity.

(iii) Addition of constant multiple of the elements of any row (or column) to the corresponding element of any other row (or column).

Two matrices are said to be equivalent if one is obtained from the other by elementary transformation. The sysmbol $\approx$ is used for equivalence.

Method to Find Inverse by Elementary Transformations :

Row Transformation

(i) A^{-1} exists if $|A| \neq 0$.

(ii) To find A^{-1} by row transformation, then we write $IA = A$.

(iii) Apply row transformations to the pre-factor I on L.H.S. & to A on R.H.S. such that A becomes a unit matrix.

(iv) Now equation becomes $BA = I$, so $B = A^{-1}$

13. SOLUTION OF A SYSTEM OF LINEAR EQUATION BY MATRIX METHOD

Consider a system of linear equation

$a_{11} x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots a_{1n} x_n = b_1$

$a_{21} x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots a_{2n} x_n = b_2$

....

....

$a_{n1} x_1 + a_{n2}x_2 + \ldots\ldots\ldots\ldots a_{nn} x_n = b_n$

We can express these equations as a single matrix equation.

$$\begin{bmatrix} a_{11} & a_{12}\ldots\ldots & a_{1n} \\ a_{21} & a_{22}\ldots\ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2} & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \ldots\ldots \\ \ldots\ldots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \ldots\ldots \\ \ldots\ldots \\ b_n \end{bmatrix}$$

A X B

Let $|A| \neq 0$ so that A^{-1} exists uniquely

pre-multiplying both sides of $AX = B$ by A^{-1} we get

$A^{-1}(AX) = A^{-1}B \quad \Rightarrow \quad (AA^{-1})X = A^{-1}B$

$\Rightarrow \quad IX = A^{-1}B \quad \Rightarrow \quad X = A^{-1}B$

Criterion of Consistency

Let $AX = B$ be a system of n linear equation in n variables.

(i) If $|A| \neq 0$ then the system of the equations is consistent and has a unique solution given by $X = A^{-1}B$.

(ii) If $|A| = 0$ and (adj A) B = O then the system of equations is consistent and has infinitely many solutions except the case of parallel planes when there is no solution.

(iii) If $|A| = 0$ and (adj A) B ≠ O then the system of equations is inconsistent i.e. it has no solution.

Homogeneous Equation

The system of equations $AX = B$ is said to be homogeneous if the constants $b_1, b_2, b_3 \ldots\ldots b_n$ are all zero. That is if the matrix B is a zero matrix and the system is of the form

$AX = O$

Where O is the null matrix of order $n \times 1$.

(i) If $|A| \neq 0$ then its only solution $X = 0$ is called trivial solution. $(x = y = z = 0)$

(ii) If $|A| = 0$ then $AX = O$ have both trivial and non trivial type solutions. In this case number of solutions will be Infinite.

(iii) The condition for

$a_1x + b_1y + c_1z = 0;$

$a_2x + b_2y + c_2z = 0$ and

$a_3x + b_3y + c_3z = 0$

to have non-zero or non-trivial solutions is :

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

14. SOME OTHER TYPES OF MATRICES

(a) **Orthogonal Matrix :** A square matrix A is called an orthogonal matrix if the product of the matrix A and its transpose A′ is identity matrix.

$AA' = I$

NOTES:

(i) If $AA' = I$ then $A^{-1} = A'$

(ii) If A and B are orthogonal then AB is also orthogonal.

(iii) All above properties are defined for square matrix only.

(iv) Elements of all 3 rows (or columns) of orthogonal matrix of order 3 × 3 represent unit vectors.

(b) **Idempotent Matrix :** A matrix 'A' such that $A^2 = A$ is called idempotent..

- Only square matrix can be idempotent matrix.
- Identity matrix (unit matrix) is also idempotent matrix.

Example : $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(c) **Periodic Matrix :.** A matrix 'A' will be called a periodic matrix if $A^{k+1} = A$ where k is +ve integer and k is least positive integer for which $A^{k+1} = A$, then k is said to be the period of A.

(d) **Nilpotent Matrix :.** A matrix 'A' will be called nilpotent matrix if $A^k = 0$ (null matrix), k is least positive integer and k is called index of the nilpolent matrix.

(e) **Involutary Matrix :.** A matrix 'A' will be called an involutary matrix if $A^2 = I$ (Unit Matrix). Unit matrix is also involuntary matrix.

15. SOME IMPORTANT APPLICATIONS

15.1

Let $f(x) = a_0 + a_1x + a_2x^2 + ... + a_mx^m$ be a polynomial in x where $a_0, a_1, a_2, ..., a_m$ are real numbers, such that $a_0 \neq 0$. If A is a non-singular matrix such that f(A) = 0, then

$$A^{-1} = \frac{-1}{a_0}\left(a_1 + a_2A + a_3A^2 + ... + a_mA^{m-1}\right)$$

15.2

To find the inverse of a square matrix A, or to express A^{-1} in terms of A, the concept of a characteristic polynomial of a square matrix and the much known Cayley-Hamilton Theorem are useful, especially for 2×2 and 3×3 matrices.

15. 3

If A is a square matrix and I is the corresponding unit matrix, then the polynomial |A – xI| in x is called characteristic polynomial of A and the equation |A – xI| = 0 is called the characteristic equation of the matrix A.

15.4

(Cayley-Hamilton) Every square matrix satisfies its characteristic equation; that is, if A is a square matrix of order n and

$$f(x) = |A - xI| = a_0 + a_1x + a_2x^2 + ... + a_nx^n = 0$$

is its characteristic equation, then

$$f(A) = a_0I_n + a_1A + a_2A^2 + ... + a_nA^n = O$$

Also if $a_0 \neq 0$, then

$$A^{-1} = \frac{-1}{a_0}\left(a_1I + a_2A + a_3A^2 + ... + a_nA^{n-1}\right)$$

Note that A^{-1} exists if and only if the constant term of the characteristic of A is non-zero.

SOLVED EXAMPLES

Example – 1

Find minors and cofactors of elements of the following determinants

(i) $\begin{vmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & 3 & 0 \end{vmatrix}$ (ii) $\begin{vmatrix} -1 & 0 & 4 \\ -2 & 1 & 3 \\ 0 & -4 & 2 \end{vmatrix}$

Sol. (i) $D = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & 3 & 0 \end{vmatrix}$

Minors	**Cofactors**
$M_{11} = \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix}$;	$A_{11} = (-1)^{1+1} M_{11}$
$= 0 - 3 = -3$	$= -3$
$M_{12} = \begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix}$;	$A_{12} = (-1)^{1+2} M_{12}$
$= 0 - 2 = -2$	$= 2$
$M_{13} = \begin{vmatrix} 3 & 0 \\ 2 & 3 \end{vmatrix}$;	$A_{13} = (-1)^{1+3} M_{13}$
$= 9 - 0 = 9$	$= 9$
$M_{21} = \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}$;	$A_{21} = (-1)^{2+1} M_{21}$
$= 0 - 6 = -6$	$= 6$
$M_{22} = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix}$;	$A_{22} = (-1)^{2+2} M_{22}$
$= 0 - 4 = -4$	$= -4$
$M_{23} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix}$;	$A_{23} = (-1)^{2+3} M_{23}$
$= 0 - 2 = -2$	$= 2$
$M_{31} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$;	$A_{31} = (-1)^{3+1} M_{31}$
$= 1 - 0 = 1$	$= 1$
$M_{32} = \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix}$;	$A_{32} = (-1)^{3+2} M_{32}$
$= 0 - 6 = -6$	$= 6$
$M_{33} = \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix}$;	$A_{33} = (-1)^{3+3} M_{33}$
$= 0 - 3 = -3$	$= -3$

(ii) $D = \begin{vmatrix} -1 & 0 & 4 \\ -2 & 1 & 3 \\ 0 & -4 & 2 \end{vmatrix}$

Minors	**Cofactors**
$M_{11} = \begin{vmatrix} 1 & 3 \\ -4 & 2 \end{vmatrix}$;	$A_{11} = (-1)^{1+1} M_{11}$
$= 2 + 12 = 14$	$= 14$
$M_{12} = \begin{vmatrix} -2 & 3 \\ 0 & 2 \end{vmatrix}$;	$A_{12} = (-1)^{1+2} M_{12}$
$= -4 - 0 = -4$	$= 4$
$M_{13} = \begin{vmatrix} -2 & 1 \\ 0 & -4 \end{vmatrix}$;	$A_{13} = (-1)^{1+3} M_{13}$
$= 8 - 0 = 8$	$= 8$
$M_{21} = \begin{vmatrix} 0 & 4 \\ -4 & 2 \end{vmatrix}$;	$A_{21} = (-1)^{2+1} M_{21}$
$= 0 + 16 = 16$	$= -16$
$M_{22} = \begin{vmatrix} -1 & 4 \\ 0 & 2 \end{vmatrix}$;	$A_{22} = (-1)^{2+2} M_{22}$

$= -2 + 0 = -2 \qquad = -2$

$M_{23} = \begin{vmatrix} -1 & 0 \\ 0 & -4 \end{vmatrix}; \qquad A_{23} = (-1)^{2+3} M_{23}$

$= 4 - 0 = 4 \qquad = -4$

$M_{31} = \begin{vmatrix} 0 & 4 \\ 1 & 3 \end{vmatrix}; \qquad A_{31} = (-1)^{3+1} M_{31}$

$= 0 - 4 = -4 \qquad = -4$

$M_{32} = \begin{vmatrix} -1 & 4 \\ -2 & 3 \end{vmatrix}; \qquad A_{32} = (-1)^{3+2} M_{32}$

$= -3 + 8 = 5 \qquad = -5$

$M_{33} = \begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix}; \qquad A_{33} = (-1)^{3+3} M_{33}$

$= -1 + 0 = -1 \qquad = -1$

Example – 2

Show that $\begin{vmatrix} x-y & y-z & z-x \\ y-z & z-x & x-y \\ z-x & x-y & y-z \end{vmatrix} = 0$

Sol. $D = \begin{vmatrix} x-y & y-z & z-x \\ y-z & z-x & x-y \\ z-x & x-y & y-z \end{vmatrix}$

$C_3 \to C_3 + C_2 + C_1$ gives

$D = \begin{vmatrix} x-y & y-z & 0 \\ y-z & z-x & 0 \\ z-x & x-y & 0 \end{vmatrix}$

$= 0$

[$\because$ all elements of C_3 are zero.]

Example – 3

Evaluate

(i) $\begin{vmatrix} 16 & 29 & 35 \\ 50 & 100 & 110 \\ 82 & 158 & 180 \end{vmatrix}$

(ii) $\begin{vmatrix} 5 & 13 & 17 \\ 30 & 68 & 105 \\ 25 & 66 & 84 \end{vmatrix}$

Sol. (i) $D = \begin{vmatrix} 16 & 29 & 35 \\ 50 & 100 & 110 \\ 82 & 158 & 180 \end{vmatrix}$

Using $\left(\frac{1}{10} R_2\right)$,

$D = 10 \begin{vmatrix} 16 & 29 & 35 \\ 5 & 10 & 11 \\ 82 & 158 & 180 \end{vmatrix}$

$R_1 = R_1 - 3R_2,\ R_3 \to R_3 - 16\,R_2$

$D = 10 \begin{vmatrix} 1 & -1 & 2 \\ 5 & 10 & 11 \\ 2 & -2 & 4 \end{vmatrix} = 0$

$\because \quad (R_3 \equiv 2 \times R_1)$

(ii) $D = \begin{vmatrix} 5 & 13 & 17 \\ 30 & 68 & 105 \\ 25 & 66 & 84 \end{vmatrix}$

$R_2 \to R_2 - 6R_1,\ R_3 \to R_3 - 5R_1$

$\therefore \quad D = \begin{vmatrix} 5 & 13 & 17 \\ 0 & -10 & 3 \\ 0 & 1 & -1 \end{vmatrix}$

$= 5\,(10 - 3)$

(Expanding along 1^{st} column) $= 35$

Example – 4

Show that $\begin{vmatrix} 10 & 24 & 36 \\ 36 & 10 & 24 \\ 24 & 36 & 10 \end{vmatrix}$ is divisible by 35.

Sol. $D = \begin{vmatrix} 10 & 24 & 36 \\ 36 & 10 & 24 \\ 24 & 36 & 10 \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$\therefore \quad D = \begin{vmatrix} 70 & 70 & 70 \\ 36 & 10 & 24 \\ 24 & 36 & 10 \end{vmatrix}$

Using $\frac{1}{35} R_1$, we get

$D = 35 \begin{vmatrix} 2 & 2 & 2 \\ 36 & 10 & 24 \\ 24 & 36 & 10 \end{vmatrix}$

All elements in determinant are natural numbers

$\therefore$ Its value will be whole number

$\therefore$ D is multiple of 35 i.e. it is divisible by 35.

Example – 5

If x, y, z are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$,

then show that $1 + xyz = 0$

Sol. We have

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$$

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} \text{ (Using Property 6)}$$

Take common x, y, z from R_1, R_2, R_3 resp.

$$= (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

(Using $C_3 \leftrightarrow C_2$ and then $C_1 \leftrightarrow C_2$)

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (1 + xyz)$$

$$= (1 + xyz) \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$$

(Using $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$)

Taking out common factor $(y - x)$ from R_2 and $(z - x)$ from R_3, we get

$$\Delta = (1 + xyz)(y - x)(z - x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$$

$= (1 + xyz)(y - x)(z - x)(z - y)$ (on expanding along C_1)

Since $\Delta = 0$ and x, y, z are all different, i.e., $x - y \neq 0$, $y - z \neq 0$, $z - x \neq 0$, we get $1 + xyz = 0$

Example – 6

If l, m, n are p^{th}, q^{th} and r^{th} terms of an arithmetic progression respectively, prove that $\begin{vmatrix} l & p & 1 \\ m & q & 1 \\ n & r & 1 \end{vmatrix} = 0$

Sol. In arithmetic progression

$T_1 = a$

$T_2 = a + d$

(where d is constant difference)

$T_3 = a + 2d$

$T_n = a + (n - 1) d$

$\therefore \quad l = T_p = a + (p - 1) d = a + pd - d$

$m = T_q = a + (q-1)d = a + qd - d$

$n = T_r = a + (r-1)d = a + rd - d$

$$\therefore \quad L.H.S = \begin{vmatrix} l & p & 1 \\ m & q & 1 \\ n & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a+pd-d & p & 1 \\ a+qd-d & q & 1 \\ a+rd-d & r & 1 \end{vmatrix}$$

gives $C_1 \to C_1 - d \times C_2$

$$L.H.S. = \begin{vmatrix} a-d & p & 1 \\ a-d & q & 1 \\ a-d & r & 1 \end{vmatrix}$$

Using $\left(\frac{1}{a-d}\right) C_1$, we get

$$L.H.S. = (a-d)\begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix}$$

$= 0 \quad (\because C_1 \equiv C_3)$

Example – 7

Without expanding the determinants, show that

(i) $\begin{vmatrix} 1/x & x & yz \\ 1/y & y & zx \\ 1/z & z & xy \end{vmatrix} = 0$ (ii) $\begin{vmatrix} 0 & b & c \\ -b & 0 & a \\ -c & -a & 0 \end{vmatrix} = 0$

Sol. (i) $D = \begin{vmatrix} 1/x & x & yz \\ 1/y & y & zx \\ 1/z & z & xy \end{vmatrix}$

using xR_1, yR_2 and zR_3, we get

$$D = \frac{1}{xyz}\begin{vmatrix} 1 & x^2 & xyz \\ 1 & y^2 & xyz \\ 1 & z^2 & xyz \end{vmatrix}$$

Using $\left(\frac{1}{xyz}\right) C_3$, we get

$$D = \frac{xyz}{xyz}\begin{vmatrix} 1 & x^2 & 1 \\ 1 & y^2 & 1 \\ 1 & z^2 & 1 \end{vmatrix}$$

$= 0 \ (\because C_1 \equiv C_3)$

(ii) $D = \begin{vmatrix} 0 & b & c \\ -b & 0 & a \\ -c & -a & 0 \end{vmatrix}$

$= (-1)^3 \begin{vmatrix} 0 & -b & -c \\ b & 0 & -a \\ c & a & 0 \end{vmatrix}$ (Taking –1 common from R_1, R_2 & R_3)

So $D = -\begin{vmatrix} 0 & -b & -c \\ b & 0 & -a \\ c & a & 0 \end{vmatrix}$

$= -\begin{vmatrix} 0 & b & c \\ -b & 0 & a \\ -c & -a & 0 \end{vmatrix}$ (Interchanging rows with columns & vice-versa).

$= -D$

$\therefore 2D = 0 \qquad \Rightarrow D = 0.$

Example – 8

Show that

$$\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = (x-y)(y-z)(z-x)$$

Sol. $D = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$, gives

$$D=\begin{vmatrix}0 & x-z & -y(x-z)\\0 & y-z & -x(y-z)\\1 & z & xy\end{vmatrix}$$

Using $\left(\frac{1}{z-x}\right)R_1$ and $\left(\frac{1}{y-z}\right)R_2$, we get

$$D=(z-x)(y-z)\begin{vmatrix}0 & -1 & y\\0 & 1 & -x\\1 & z & xy\end{vmatrix}$$

$R_1 \to R_1 + R_2$ gives

$$D=(z-x)(y-z)\begin{vmatrix}0 & 0 & y-x\\0 & 1 & -x\\1 & z & xy\end{vmatrix}$$

Expanding along R_1

$$\therefore \quad D=(z-x)(y-z)(y-x)\begin{vmatrix}0 & 1\\1 & z\end{vmatrix}$$

$=(z-x)(y-z)(y-x)(0-1)$

$=(x-y)(y-z)(z-x)$

$=$R.H.S.

Example – 9

Show that

$$\begin{vmatrix}a+b+c & -c & -b\\-c & a+b+c & -a\\-b & -a & a+b+c\end{vmatrix}$$

$=2(a+b)(b+c)(c+a)$

Sol. $$D=\begin{vmatrix}a+b+c & -c & -b\\-c & a+b+c & -a\\-b & -a & a+b+c\end{vmatrix}$$

$C_1 \to C_1 + C_3$; $C_2 \to C_2 + C_3$ gives

$$D=\begin{vmatrix}a+c & -(b+c) & -b\\-(a+c) & b+c & -a\\a+c & b+c & a+b+c\end{vmatrix}$$

Using $\left(\frac{1}{a+c}\right)C_1$ and $\left(\frac{1}{b+c}\right)C_2$ we get

$$D=(a+c)(b+c)\begin{vmatrix}1 & -1 & -b\\-1 & 1 & -a\\1 & 1 & a+b+c\end{vmatrix}$$

$R_1 \to R_1 + R_2$ gives

$$D=(b+c)(c+a)\begin{vmatrix}0 & 0 & -(a+b)\\-1 & 1 & -a\\1 & 1 & a+b+c\end{vmatrix}$$

Expanding along R_1

$=(b+c)(c+a)\times[-(a+b)(-1-1)]$

$=2(a+b)(b+c)(c+a)$

$=$R.H.S.

Example – 10

Show that

$$\begin{vmatrix}1+a & 1 & 1\\1 & 1+b & 1\\1 & 1 & 1+c\end{vmatrix}=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=abc+bc+ca+ab$$

Sol. Taking out factors a,b,c common from R_1, R_2 and R_3, we get

$$\text{L.H.S.}=abc\begin{vmatrix}\frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a}\\\frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b}\\\frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1\end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have

$$\Delta=abc\begin{vmatrix}1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\\\frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b}\\\frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1\end{vmatrix}$$

Taking $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ common from R_1

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

Now applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we get

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$

Expanding along C_3

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\left[1(1-0)\right]$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc + bc + ca + ab = \text{R.H.S.}$$

Example – 11

Prove that

$$\Delta = \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = \left(1-x^2\right)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

Sol. Applying $R_1 \to R_1 - x\,R_2$ to Δ, we get

$$\Delta = \begin{vmatrix} a\left(1-x^2\right) & c\left(1-x\right)^2 & p\left(1-x^2\right) \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

Taking $1-x^2$ common from R_1

$$= \left(1-x^2\right)\begin{vmatrix} a & c & p \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

Applying $R_2 \to R_2 - x\,R_1$, we get

$$\Delta = \left(1-x^2\right)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

$= \text{RHS}$

Example – 12

Find x, if

(i) $\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 4 & 6 & 9 \end{vmatrix} = 0$ (ii) $\begin{vmatrix} 1 & 2x & 4x^2 \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0$

Sol. (i) $\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 4 & 6 & 9 \end{vmatrix} = 0$

Expanding along R_1

$\therefore$ $1(18-24) - x(9-16) + x^2(6-8) = 0$

$\therefore$ $-6 + 7x - 2x^2 = 0$

$\therefore$ $2x^2 - 7x + 6 = 0$

$\therefore$ $(2x-3)(x-2) = 0$

$\therefore$ $2x - 3 = 0$ or $x - 2 = 0$

$\therefore$ $x = \frac{3}{2}$ or $x = 2$

(ii) Given $\begin{vmatrix} 1 & 2x & 4x^2 \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0$

Method I: Here $a_{13} = (a_{12})^2$ (element in third column is equal to square of respective element in 2nd column)

We know, when two rows are identical then the determinant is zero.

$\therefore$ for $R_1 \equiv R_2$, $2x = 4$ i.e. $x = 2$

$\therefore$ for $R_1 \equiv R_3$, $2x = 1$ i.e. $x = \frac{1}{2}$

$\therefore$ solution is $x = \frac{1}{2}, 2$

Method II: if we expand the given determinant by R_1, we get

$1(4-16)-2x(1-16)+4x^2(1-4)=0$

$\therefore$ $-12-2x+32x-12x^2=0$

$\therefore$ $-12x^2+30x-12=0$

$\therefore$ $4x^2-10x+4=0$

$\therefore$ $(4x-2)(x-2)=0$

$\therefore$ $4x-2=0$ or $x-2=0$

$\therefore$ $x=\frac{1}{2}$ or $x=2$

Example – 13

Solve the following equations using Cramer's Rule

$x+2y-z=3, 3x-y+2z=1,$

$2x-2y+3z=2$

Sol. The given equations are

$x+2y-z=3$

$3x-y+2z=1$

$2x-2y+3z=2$

$$D=\begin{vmatrix}1 & 2 & -1\\ 3 & -1 & 2\\ 2 & -2 & 3\end{vmatrix}$$

Expanding along R_1

$=1(-3+4)-2(9-4)-1(-6+2)$

$=1-10+4=-5$

$$D_x=\begin{vmatrix}3 & 2 & -1\\ 1 & -1 & 2\\ 2 & -2 & 3\end{vmatrix}$$

Expanding along R_1

$=3(-3+4)-2(3-4)-1(-2+2)$

$=5$

$$D_y=\begin{vmatrix}1 & 3 & -1\\ 3 & +1 & 2\\ 2 & 2 & 3\end{vmatrix}$$

Expanding along R_1

$=1(3-4)-3(9-4)-1(6-2)$

$=-20$

$$D_z=\begin{vmatrix}1 & 2 & 3\\ 3 & -1 & 1\\ 2 & -2 & 2\end{vmatrix}$$

$=1(-2+2)-2(6-2)+3(-6+2)$

$=-20$

By Cramer's Rule

$$x=\frac{Dx}{D}=\frac{5}{-5}=-1$$

$$y=\frac{Dy}{D}=\frac{-20}{-5}=4$$

$$z=\frac{Dz}{D}=\frac{-20}{-5}=4$$

$\therefore$ Solution is $x=-1, y=4, z=4$

Example – 14

Show that the following equations are consistent

$2x+3y+1=0, x+2y+1=0, x+y=0$

Sol. Given

$2x+3y+1=0$

$x+2y+1=0$

$x+y=0$

By condition of consistency

$$\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$$

$$\text{Now, }\begin{vmatrix}2 & 3 & 1\\ 1 & 2 & 1\\ 1 & 1 & 0\end{vmatrix}$$

$R_1 \to R_1-R_2$ gives

$$=\begin{vmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 1 & 1 & 0\end{vmatrix}=0 \qquad (\because R_1 \equiv R_3)$$

$\therefore$ The given equations are consistent.

Example – 15

Find k, if the following equations are consistent

$(k-2)x+(k-1)y=17,$

$(k-1)x+(k-2)y=18,$

$x+y=5$

Sol. The given equation are

$(k-2)x+(k-1)y-17=0$

$(k-1)x+(k-2)y-18=0$

$x+y-5=0$

$\because$ The equations are consistent

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} k-2 & k-1 & -17 \\ k-1 & k-2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ gives

$$= \begin{vmatrix} -1 & 1 & 1 \\ k-1 & k-2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

$C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$ gives

$$\begin{vmatrix} -1 & 0 & 0 \\ k-1 & 2k-3 & k-19 \\ 1 & 2 & -4 \end{vmatrix}$$

Expanding along R_1

$\therefore$ $-1(-8k+12-2k+38)=0$

$\therefore$ $10k-50=0$

$\therefore$ $k=5$

Example – 16

Find k if the area of the triangle ABC is 35 sq. units, where $A=(2,6), B=(5,4)$ and $C=(k,4)$

Sol. $$\text{Area} = \frac{1}{2}\begin{vmatrix} 2 & 6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = 35 \text{ sq. units}$$

$\Rightarrow$ $\frac{1}{2}(k-5)\,2 = \pm 35$

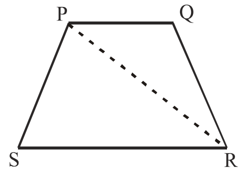
$\Rightarrow$ $k-5=\pm 35$

$\Rightarrow$ $k=40, -30.$

Example – 17

Find the area of the quadrilateral whose vertices are $P(-3,1), Q(1,-1), R(2,1), S(0,3)$.

Sol.



$A(PQRS) = A(\Delta PQR) + A(\Delta PRS)$

$$A(\Delta PQR)\ \frac{1}{2} = \begin{vmatrix} -3 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\,[-3(-1-1)-1(1-2)+1(1+2)]$$

$$= \frac{1}{2}\,(6+1+3)$$

$= 5$ Sq. units

$$A(\Delta PRS) = \frac{1}{2}\begin{vmatrix} -3 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\,[-3(1-3)-1(2-0)+1(6-0)]$$

$$= \frac{1}{2}\,[6-2+6]$$

$= 5$ sq. units

$\therefore$ $A(PQRS) = 5+5 = 10$ Sq. units.

Example – 18

The sum of first and second numbers is greater than the third number by 5. The sum of first and third numbers is more than the second number by 7. The sum of second and third numbers is greater than the first number by 2. Find such three numbers.

Sol. Let the numbers be x, y, z respectively. We get the following equations.

$x + y - z = 5$

$x - y + z = 7$

$-x + y + z = 2$

$\therefore \quad D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix}$

Expanding along R_1

$= 2(-1-1)$

$= -4$

$D_x = \begin{vmatrix} 5 & 1 & -1 \\ 7 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 12 & 0 & 0 \\ 7 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$

Expanding along R_1

$= 12(-1-1)$

$= -24$

$D_y = \begin{vmatrix} 1 & 5 & -1 \\ 1 & 7 & 1 \\ -1 & 2 & 1 \end{vmatrix}$

Expanding along R_1

$= 1(7-2) - 5(1+1) - 1(2+7)$

$= -14$

$D_z = \begin{vmatrix} 1 & 1 & 5 \\ 1 & -1 & 7 \\ -1 & 1 & 2 \end{vmatrix}$

Expanding along R_1

$= 1(-2-7) - 1(+2+7) + 5(1-1)$

$= -18$

By Cramer's Rule

$x = \frac{D_x}{D} = \frac{-24}{-4} = 6$

$y = \frac{D_y}{D} = \frac{-14}{-4} = \frac{7}{2}$

$z = \frac{D_z}{D} = \frac{-18}{-4} = \frac{9}{2}$

$\therefore$ The numbers are 6, $\frac{7}{2}$ and $\frac{9}{2}$ respectively.

Example – 19

If $A = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$, find matrix C such that A + B + C = 0, where 0 is the zero matrix.

Sol. Given, A + B + C = 0

$\therefore \quad C = -[A + B]$

$A + B = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$

$C = -[A + B] = -\begin{bmatrix} 8 & 4 \\ -2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ 2 & -4 \\ -4 & -4 \end{bmatrix}$

Example – 20

Find matrices A and B, where

$2A - B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ and $A + 3B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Sol. Given $2A - B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ ... (i)

$A + 3B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ ... (ii)

From 3 × (i) + (ii), we get

$$7A = \begin{bmatrix} 3 & -3 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & -3+1 \\ 0-1 & 3+0 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\therefore \quad A = \frac{1}{7}\begin{bmatrix} 3 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 3/7 & -2/7 \\ -1/7 & 3/7 \end{bmatrix}$$

$$\therefore \quad B = 2A - \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad \text{... (i)}$$

$$\therefore \quad B = 2\begin{bmatrix} 3/7 & -2/7 \\ -1/7 & 3/7 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6/7 & -4/7 \\ -2/7 & 6/7 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/7 & 3/7 \\ -2/7 & -1/7 \end{bmatrix}$$

Example – 21

If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that $A_\alpha . A_\beta = A_{\alpha+\beta}$

Sol. $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

then $A_\beta = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$$A_\alpha . A_\beta = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \cos\alpha\sin\beta + \sin\alpha\cos\beta \\ -\sin\alpha\cos\beta - \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$$

$$= A_{(\alpha+\beta)}$$

Example – 22

If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$, verify that $|AB| = |A|\,|B|$.

Sol. $AB = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 2+3 & 4-2 \\ 0+9 & 0-6 \end{bmatrix}$$

$$\therefore \quad AB = \begin{bmatrix} 5 & 2 \\ 9 & -6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 6 - 0 = 6$$

$$|B| = \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -2 - 6 = -8$$

$$|AB| = \begin{vmatrix} 5 & 2 \\ 9 & -6 \end{vmatrix} = -30 - 18 = -48$$

Also, $|A| . |B| = 6(-8) = -48$

Hence, $|AB| = |A| . |B|$ is verified.

Example – 23

If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$

Sol. $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$\therefore \quad A^2 = A . A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$\therefore \quad A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25+0 \\ -20+20-0 & 24-10-14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore \quad A^2 - 4A + 3I = 0$

Example – 24

If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

show that $(A+B).(A-B) \neq A^2 - B^2$.

Sol. $(A+B)(A-B) = A^2 - AB + BA - B^2$

$$AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

And, $BA = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\because \quad AB \neq BA$

$\therefore \quad -AB + BA \neq 0$

$\therefore \quad (A+B)(A-B) \neq A^2 - B^2$

Example – 25

If $A = \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & x \\ y & 0 \end{bmatrix}$, and $(A+B).(A-B) = A^2 - B^2$,

find x and y

Sol. Condition given

$(A+B)(A-B) = A^2 - B^2$

$\therefore \quad A^2 - AB + BA - B^2 = A^2 - B^2$

$\therefore \quad -AB + BA = 0$

$\therefore \quad AB = BA$

$$\therefore \quad \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} 1 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 1 & x \\ y & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} -3+2y & -3x+0 \\ 2-4y & 2x+0 \end{bmatrix} = \begin{bmatrix} -3+2x & 2-4x \\ -3y+0 & 2y+0 \end{bmatrix}$$

Comparing corresponding elements.

$-3x = 2 - 4x$ and $-3y = 2 - 4y$

$\therefore \quad x = 2 \quad y = 2$

$[\therefore x = y = 2]$

Example – 26

If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, find A^3

Sol. $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$$\therefore \quad A^2 = A.A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\therefore \quad A^3 = A.A^2 = A.I = A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

Example – 27

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$,

show that AB and BA are both singular matrices.

Sol. $AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1-6-6 & -1+4+3 & 1-2+0 \\ 2-12-12 & -2+8+6 & 2-4+0 \\ 1-6-6 & -1+4+3 & 1-2+0 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{vmatrix} = 0 \quad (\because R_1 = R_3)$$

Similarly,

$$BA = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+1 & 2-4+2 & 3-6+3 \\ -3+4-1 & -6+8-2 & -9+12-3 \\ -2+2+0 & -4+4+0 & -6+6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $|BA| = 0$ ($\because$ it is zero matrix)

Hence, AB and BA both are singular.

Example – 28

Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Sol. Here

$$B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

Let $P = \frac{1}{2}(B+B') = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix}$

Now $P' = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} = P$

Thus $P = \frac{1}{2}(B+B')$ is a symmetric matrix.

Also, let

$$Q = \frac{1}{2}(B-B') = \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

Then $Q' = \begin{bmatrix} 0 & \frac{1}{2} & \frac{5}{3} \\ \frac{-1}{2} & 0 & -3 \\ \frac{-5}{2} & 3 & 0 \end{bmatrix} = -Q$

Thus $Q = \frac{1}{2}(B-B')$ is a skew symmetric matrix.

Now

$$P+Q = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = B$$

Thus, B is represented as the sum of a symmetric and a skew symmetric matrix.

Example – 29

The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Sol. Let first, second and third numbers be denoted by x, y and z, respectively. Then, according to given conditions, we have

$x+y+z=6$

$y+3z=11$

$x+z=2y$ or $x-2y+z=0$

This system can be written as A X = B, where

$$A=\begin{bmatrix}1&1&1\\0&1&3\\1&-2&1\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix}\text{ and }B=\begin{bmatrix}6\\11\\0\end{bmatrix}$$

Here $|A|=1(1+6)-(0-3)+(0-1)=9\neq 0$. Now we find adj A

$A_{11}=1(1+6)=7, A_{12}=-(0-3)=3, A_{13}=-1$

$A_{21}=-(1+2)=-3, A_{22}=0, A_{23}=-(-2-1)=3$

$A_{31}=(3-1)=2, A_{32}=-(3-0)=-3, A_{33}=(1-0)=1$

$$\text{Hence adj}A=\begin{bmatrix}7&-3&2\\3&0&-3\\-1&3&1\end{bmatrix}$$

$$\text{Thus } A^{-1}=\frac{1}{|A|}\text{adj}(A)=\frac{1}{9}\begin{bmatrix}7&-3&2\\3&0&-3\\-1&3&1\end{bmatrix}$$

Since $X=A^{-1}B$

$$X=\frac{1}{9}\begin{bmatrix}7&-3&2\\3&0&-3\\-1&3&1\end{bmatrix}\begin{bmatrix}6\\11\\0\end{bmatrix}$$

or $$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{9}\begin{bmatrix}42-33+0\\18+0+0\\-6+33+0\end{bmatrix}=\frac{1}{9}\begin{bmatrix}9\\18\\27\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

Thus $x=1, y=2, z=3$

Example – 30

Use product $\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}$ to solve the system of equations

$x-y+2z=1$

$2y-3z=1$

$3x-2y+4z=2$

Sol. Consider the product $\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}$

$$=\begin{bmatrix}-2-9+12&0-2+2&1+3-4\\0+18-18&0+4-3&0-6+6\\-6-18+24&0-4+4&3+6-8\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\text{Hence }\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}^{-1}=\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}$$

Now, given system of equations can be written, in matrix form, as follows

$$\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\1\\2\end{bmatrix}$$

or $$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}^{-1}\begin{bmatrix}1\\1\\2\end{bmatrix}=\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix}$$

$$=\begin{bmatrix}-2+0+2\\9+2-6\\6+1-4\end{bmatrix}=\begin{bmatrix}0\\5\\3\end{bmatrix}$$

Hence $x=0, y=5$ and $z=3$

Example – 31

Given $A=\begin{bmatrix}1&1&1\\2&4&1\\2&3&1\end{bmatrix}, B=\begin{bmatrix}2&3\\3&4\end{bmatrix}$.

Find P such that $BPA=\begin{bmatrix}1&0&1\\0&1&0\end{bmatrix}$.

Sol. Given $BPA=\begin{bmatrix}1&0&1\\0&1&0\end{bmatrix}$.

Pre-multiplying both sides by B^{-1}

$$B^{-1}BPA=B^{-1}\begin{bmatrix}1&0&1\\0&1&0\end{bmatrix}$$

$$\Rightarrow \quad IPA=B^{-1}\begin{bmatrix}1&0&1\\0&1&0\end{bmatrix}$$

$$\Rightarrow \quad PA=B^{-1}\begin{bmatrix}1&0&1\\0&1&0\end{bmatrix} \quad \ldots(i)$$

To find B^{-1}.

Now $B=\begin{bmatrix}2&3\\3&4\end{bmatrix}$

$$|B|=\begin{vmatrix}2&3\\3&4\end{vmatrix}=8-9=-1\neq 0. \text{ As } |B|\neq 0$$

so it is non-singular matrix and hence inverse of B exists.

$$\Rightarrow \quad B^{1}=\frac{Adj.B}{|B|}=\begin{bmatrix}-4&3\\3&-2\end{bmatrix}$$

NOTES :

For a 2×2 matrix, adjoint can be obtained by swapping diagonal elements and changing the sign of non-diagonal elements.

Now from (i),

$$PA=\begin{bmatrix}-4&3\\3&-2\end{bmatrix}\times\begin{bmatrix}1&0&1\\0&1&0\end{bmatrix}$$

$$\Rightarrow \quad PA=\begin{bmatrix}-4&3&-4\\3&-2&3\end{bmatrix}$$

Post-multiplying both sides by A^{-1}

$$PAA^{-1}=\begin{bmatrix}-4&3&-4\\3&-2&3\end{bmatrix}A^{-1}$$

$$\Rightarrow \quad PI=\begin{bmatrix}-4&3&-4\\3&-2&3\end{bmatrix}A^{-1}$$

$$\therefore \quad P=\begin{bmatrix}-4&3&-4\\3&-2&3\end{bmatrix}A^{-1} \quad \ldots(ii)$$

For A^{-1} :

since $A=\begin{bmatrix}1&1&1\\2&4&1\\2&3&1\end{bmatrix}$. Now $|A|=-1\neq 0$

$\Rightarrow$ it is non-singular matrix and hence A^{-1} exists

$$adj.(A)=\begin{bmatrix}1&2&-3\\0&-1&1\\-2&-1&2\end{bmatrix}$$

$$A^{-1}=\frac{Adj.A}{|A|}=\begin{bmatrix}-1&-2&3\\0&1&-1\\2&1&-2\end{bmatrix}$$

Now From (ii),

$$P=\begin{bmatrix}-4&3&-4\\3&-2&3\end{bmatrix}\times\begin{bmatrix}-1&-2&3\\0&1&-1\\2&1&-2\end{bmatrix}$$

$$\Rightarrow \quad P=\begin{bmatrix}-4&7&-7\\3&-5&5\end{bmatrix}$$

Example – 32

Solve the system of equations :

$x+2y+z=2$

$2x-3y+4z=1$

$3x+6y+3z=6$

Sol. 1^{st} and 3^{rd} equations are integral mutiple of each other.

(dependent equations)

$\Rightarrow$ $D=D_1=D_2=D_3=0$

$\Rightarrow$ infinite solutions

consider $x+2y+z=2$

$2x-3y+4z=1$

Let $z=k$

$$\Rightarrow \begin{cases} x+2y=2-k \\ 2x-3y=1-4k \end{cases}$$

$$\Rightarrow \quad y=\frac{3+2k}{7} \text{ and } x=\frac{8-11k}{7}$$

$$\text{Hence : } \left[x=\frac{8-11k}{7},\ y=\frac{3+2k}{7} \text{ and } z=k\right]$$

where k is an arbitrary constant.

Example – 33

Obtain the inverse of the following matrix using elementary operations

$$A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Sol. Write $A = IA$, i.e.,

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}A$$

or $$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}A \text{ (appplying } R_1 \leftrightarrow R_2)$$

or $$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}A \text{ (applying } R_3 \to R_3-3R_1)$$

or $$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix}=\begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}A \text{ (applying } R_1 \to R_1-2R_2)$$

or $$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}=\begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix}A \text{ (applying } R_3 \to R_3+5R_2)$$

or $$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}A \text{ (applying } R_3 \to \frac{1}{2}R_3)$$

or $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}A \text{ (applying } R_1 \to R_1+R_3)$$

or $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}A \text{ (applying } R_2 \to R_2-2R_3)$$

$$\text{Hence } A^{-1}=\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$

Example – 34

If M is a 3 × 3 matrix, where $M^TM = I$ and det (M) = I, then prove that det (M – I) = 0.

Sol. $(M-I)^T = M^T - I = M^T - M^TM = M^T(I-M)$

$\Rightarrow$ $|(M-I)^T| = |M-I| = |M^T||I-M|$

$= |I-M| \Rightarrow |M-I| = 0.$

Alternate Method :

$\det(M-I) = \det(M-I)\det(M^T)$

$= \det(MM^T - M^T)$

$= \det(I-M^T) = -\det(M^T - I)$

$= -\det(M-I)^T = -\det(M-I)$

$\Rightarrow$ $\det(M-I) = 0.$

Example – 35

If S is a skew-symmertric matrix of order n and I + S is non-singular, then prove that

$A = (I-S)(I+S)^{-1}$ is an orthogonal matrix of order n.

Sol. $A^T = \left[(I+S)^T\right]^{-1}[I-S]^T$

$= (I-S)^{-1}(I+S),$

[since $S^T = -S$; S being skew symmertric].

$\therefore$ $A^TA = (I-S)^{-1}(I+S)(I-S)(I+S)^{-1}$

$= (I-S)^{-1}(I-S)(I+S)(I+S)^{-1},$

since $(I+S)(I-S) = (I-S)(I+S)$

$= I$

$\therefore$ A is orthogonal, I – S is a square matrix of order n.

$\therefore$ $A = (I-S)(I+S)^{-1}$ is a square matrix of order n.

Example – 36

If A and B are n-rowed non-zero square matrix such that AB = 0, then show that both A and B are singular. If both A and B are singular and AB = 0, does it follows that BA = 0.

Justify your answer.

Sol. (i) AB = 0 and A non-singular implies

$A^{-1}AB = A^{-1}(0) = 0$

$\Rightarrow$ $(A^{-1}A)B = 0$

$\Rightarrow$ $IB = 0 \Rightarrow B = 0$ [Note true]

(ii) AB = 0 and B non-singular implies

$ABB^{-1} = 0(B)^{-1} = 0 \Rightarrow AI = 0$

$\Rightarrow$ A = 0 [Not true]

$\therefore$ Both A and B are singular.

(iii) Consider the counter example

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\text{whereas } BA = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq 0$$

Example – 37

Let A and B be matrix of order n. Prove that if (I – AB) is invertible, then (I – BA) is also invertible and $(I-BA)^{-1} = I + B(I-AB)^{-1}A$.

Sol. $I - BA = BIB^{-1} - BABB^{-1}$

$= B(I-AB)B^{-1}$ ...(i)

Hence, $|I-BA| = |B||I-AB||B^{-1}|$

$= |I-AB||B||B^{-1}|$

$= |I-AB||B||B^{-1}|$

$= |I-AB|$ ...(ii)

Since $|B||B^{-1}| = |BB^{-1}| = |I| = I$

If I – AB is invertible, |I – AB| has to be non-zero.

Hence, $|I-BA| \neq 0$ and therefore I – BA is also invertible

Now $(I-BA)\{I + B(I-AB)^{-1}A\}$

$= (I-BA) + (I-BA)B(I-AB)^{-1}A$

$= (I-BA) + \{B(I-AB)B^{-1}\}B(I-AB)^{-1}A$

(Using (i))

$= (I-BA) + B(I-AB)(I-AB)^{-1}A$

$= I - BA + BA = I$

Hence, $(I-BA)^{-1} = I + B(I-AB)^{-1}A$

Example – 38

If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, prove that

$$A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix} \text{ for all } n\in N$$

Sol. Consider $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$

for $n=1, A^1=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$,

it is true as given

$\therefore$ A^n is true for $n = 1$

Let A^n is true for $n = r$, where $r\in N$

$$\therefore\quad A^r=\begin{bmatrix}\cos r\theta & \sin r\theta\\ -\sin r\theta & \cos r\theta\end{bmatrix}$$

then,

$A^{r+1} = A\,.\,A^r$

$$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos r\theta & \sin r\theta\\ -\sin r\theta & \cos r\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos\theta\cos r\theta-\sin\theta\sin r\theta & \cos\theta\sin r\theta+\sin\theta\sin r\theta\\ -\sin\theta.\cos r\theta-\cos\theta\sin r\theta & -\sin\theta\sin r\theta+\cos\theta\cos r\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos(\theta+r\theta) & \sin(\theta+r\theta)\\ -\sin(\theta+r\theta) & \cos(\theta+r\theta)\end{bmatrix}$$

$$=\begin{bmatrix}\cos(r+1)\theta & \sin(r+1)\theta\\ -\sin(r+1)\theta & \cos(r+1)\theta\end{bmatrix}$$

i.e. A^n is true for $n = r + 1$,

$\therefore$ A^n is true for $n = 1$

And A^n is true for $n = r + 1$, if it is true for $n = r$

$\therefore$ A^n is true for all $n\in N$

$$\therefore\quad A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix} \text{ for all } n\in N.$$

Example – 39

$$A=\begin{bmatrix}a & 0 & 1\\ 1 & c & b\\ 1 & d & b\end{bmatrix}, B=\begin{bmatrix}a & 1 & 1\\ 0 & d & c\\ f & g & h\end{bmatrix}, U=\begin{bmatrix}f\\ g\\ h\end{bmatrix}, V=\begin{bmatrix}a^2\\ 0\\ 0\end{bmatrix}, ab\neq 1$$

If there is vector matrix X, such that AX = U has infinitely many solutions, then prove that BX = V cannot have unique solution. If $afd\neq 0$ then prove that BX = V has no solution.

Sol. AX = U has infinite solutions

$\Rightarrow$ $|A| = 0$

$$\begin{vmatrix}a & 0 & 1\\ 1 & c & b\\ 1 & d & b\end{vmatrix}=0$$

$\Rightarrow$ $ab = 1$ or $c = d$

and $|A_1|=\begin{vmatrix}a & 0 & f\\ 1 & c & g\\ 1 & d & h\end{vmatrix}=0$

$\Rightarrow$ $g = h$; [Here A_1 is actually D_1 for A : Cramer's Rule in Determinants section]

$$|A_2|=\begin{vmatrix}a & f & 1\\ 1 & g & b\\ 1 & h & b\end{vmatrix}=0 \Rightarrow g=h$$

$$\Rightarrow\quad |A_3|=\begin{vmatrix}f & 0 & 1\\ g & c & b\\ h & d & b\end{vmatrix}=0 \Rightarrow g=h, c=d$$

$\Rightarrow$ $c = d$ and $g = h$

So, for infinite solutions $c = d$ and $g = h$

BX=V

$$|B|=\begin{vmatrix}a & 1 & 1\\ 0 & d & c\\ f & g & h\end{vmatrix}=0 \text{ (Since } C_2 \text{ and } C_3 \text{ are equal)}$$

$\Rightarrow$ BX = V has no unique solution

and $|B_1|=\begin{vmatrix}a^2 & 1 & 1\\ 0 & d & c\\ 0 & g & h\end{vmatrix}=0$ (since $c = d$, $g = h$)

$$|B_2| = \begin{vmatrix} a & a^2 & 1 \\ 0 & 0 & c \\ f & 0 & h \end{vmatrix} = a^2cf = a^2df \quad (\text{since } c = d)$$

$$|B_3| = \begin{vmatrix} a & 1 & a^2 \\ 0 & d & 0 \\ f & g & 0 \end{vmatrix} = a^2df$$

Since if $adf \neq 0$ then $|B_2| = |B_3| \neq 0$. Hence no solution exists.

Example – 40

Prove that the inverse of $\begin{pmatrix} A & 0 \\ B & C \end{pmatrix}$ is $\begin{pmatrix} A^{-1} & 0 \\ -C^{-1}BA^{-1} & C^{-1} \end{pmatrix}$ where A, C are non-singular matrix and hence find the inverse of $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

Sol. First part :

As $\begin{pmatrix} A & 0 \\ B & C \end{pmatrix}\begin{pmatrix} A^{-1} & 0 \\ -C^{-1}BA^{-1} & C^{-1} \end{pmatrix}$

$$\begin{pmatrix} AA^{-1} & 0 \\ BA^{-1} - CC^{-1}BA^{-1} & CC^{-1} \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

and $\begin{pmatrix} A^{-1} & 0 \\ -C^{-1}BA^{-1} & C^{-1} \end{pmatrix}\begin{pmatrix} A & 0 \\ B & C \end{pmatrix}$

$$\begin{pmatrix} A^{-1}A & 0 \\ -C^{-1}B + C^{-1}B & C^{-1}C \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

Hence $\begin{pmatrix} A^{-1} & 0 \\ C^{-1}BA^{-1} & C^{-1} \end{pmatrix}$ is the inverse of $\begin{pmatrix} A & 0 \\ B & C \end{pmatrix} = I.$

Second Part :

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} A & 0 \\ B & C \end{pmatrix}$$

where $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

Inverse of $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$

since $C^{-1}BA^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Properties of determinants

1. If $P = \begin{vmatrix} 2 & 1 & 0 \\ 3 & 1 & 2 \\ 5 & 2 & 3 \end{vmatrix}$, then $\begin{vmatrix} 2 & 2 & 0 \\ 9 & 6 & 6 \\ 5 & 4 & 3 \end{vmatrix}$ is equal to

(a) 2P (b) 3P

(c) 5P (d) 6P

2. $\begin{vmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 15 \end{vmatrix}$ is equal to

(a) 1 (b) 0

(c) –1 (d) 67

3. If every element of a third order determinant of value Δ is multiplied by 5, then the value of new determinant is

(a) Δ (b) 5 Δ

(c) 25 Δ (d) 125 Δ

4. $\Delta = \begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix}$ is equal to

(a) 1 (b) – 1

(c) zero (d) 2

5. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$, ω being a cube root of unity, is

(a) 0 (b) 1

(c) ω^2 (d) ω

6. $\Delta = \begin{vmatrix} 0 & p-q & a-b \\ q-p & 0 & x-y \\ b-a & y-x & 0 \end{vmatrix}$ is equal to

(a) 0 (b) a + b

(c) x + y (d) p + q

7. If α, β & γ are the roots of the equation $x^3 + px + q = 0$ then the value of the determinant

$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} =$

(a) p (b) q

(c) $p^2 - 2q$ (d) none

8. Given a, b, c are in A.P. Then determinant

$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ in its simplied form is :

(a) $x^3 + 3ax + 7c$ (b) 0

(c) 15 (d) $10x^2 + 5x + 2c$

9. If a + b + c = 0, one root of :

$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is

(a) x = 1 (b) x = 2

(c) $x = a^2 + b^2 + c^2$ (d) x = 0

10. The determinant $\begin{vmatrix} b_1+c_1 & c_1+a_1 & a_1+b_1 \\ b_2+c_2 & c_2+a_2 & a_2+b_2 \\ b_3+c_3 & c_3+a_3 & a_3+b_3 \end{vmatrix} =$

(a) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (b) $2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

(c) $3\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (d) none of these

11. If $\begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix}$, then the value of $\sum_{r=1}^{n-1} \Delta r$:

(a) depends only on a

(b) depends only on n

(c) depends both on a and n

(d) is independent of both a and n.

Algebra of matrices

12. A matrix $A = [a_{ij}]$ of order 2×3 whose elements are such that $a_{ij} = i + j$ is -

(a) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 5 & 3 \end{bmatrix}$ (d) none of these

13. If A and B are 3×3 matrices, then AB = O implies :

(a) A = O and B = O

(b) |A| = 0 and |B| = 0

(c) either |A| = 0 or |B| = 0

(d) A = O or B = O

14. If X and Y two matrices are such that $X - Y = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $X + Y = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ then Y is given by

(a) $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix}$ (d) None of these

15. If $A + B = \begin{bmatrix} 7 & 4 \\ 8 & 9 \end{bmatrix}$ and $A - B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ then the value of A is-

(a) $\begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ 4 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 2 \\ 8 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 6 \\ 8 & 6 \end{bmatrix}$

16. If $A = [a_{ij}]$ is a square matrix of order $n \times n$ and k is a scalar, then $|kA| =$

(a) $k^n |A|$ (b) $k|A|$

(c) $kn^{-1}|A|$ (d) none of these

17. If A, B, C are matrices of order 1×3, 3×3 and 3×1 respectively, the order of ABC will be -

(a) 3×3 (b) 1×3

(c) 1×1 (d) 3×1

18. The order of $[xyz]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is

(a) 3×1 (b) 1×1

(c) 1×3 (d) 3×3

19. If $[1 \ x \ 2]\begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 2 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ 1 \\ -1 \end{bmatrix} = O$, then the value of x is

(a) –1 (b) 0

(c) 1 (d) 2

20. If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ then

(a) x = 2, y = 1 (b) x = 1, y = 2

(c) x = 3, y = 2 (d) x = 2, y = 3

21. If $A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$ then element a_{21} of A^2 is -

(a) 22 (b) –15

(c) –10 (d) 7

22. If $E(\theta)=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then value of $E(\alpha).E(\beta)$ is -

(a) $E(0°)$ (b) $E(90°)$

(c) $E(\alpha+\beta)$ (d) $E(\alpha-\beta)$

23. The root of the equation $[x\ 1\ 2]\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ -1\\ 1\end{bmatrix}=O$ is

(a) 1/3 (b) -1/3

(c) 0 (d) 1

24. If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, the $A^4=$

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

25. If $A=\begin{bmatrix}p & q\\ -q & p\end{bmatrix}$, $B=\begin{bmatrix}r & s\\ -s & r\end{bmatrix}$ then

(a) $AB = BA$ (b) $AB \neq BA$

(c) $AB = -BA$ (d) none of these

26. If $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ and a and b are arbitrary constants then $(aI + bA)^2 =$

(a) $a^2I + abA$ (b) $a^2I + 2abA$

(c) a^2I+b^2A (d) none of these

27. If $A=\begin{bmatrix}1 & 2\\ 3 & 0\end{bmatrix}$ and $B=\begin{bmatrix}3 & 4\\ 1 & 6\end{bmatrix}$ then $(AB)^T$ equals -

(a) $\begin{bmatrix}5 & 16\\ 9 & 16\end{bmatrix}$ (b) $\begin{bmatrix}5 & 9\\ 16 & 12\end{bmatrix}$

(c) $\begin{bmatrix}5 & 9\\ 4 & 3\end{bmatrix}$ (d) none of these

28. If $A=\begin{bmatrix}2 & -1\\ -7 & 4\end{bmatrix}$ and $B=\begin{bmatrix}4 & 1\\ 7 & 2\end{bmatrix}$ then B^TA^T is equal to –

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$

29. If $A=\begin{bmatrix}1 & 2\\ 3 & 0\end{bmatrix}$; $B=\begin{bmatrix}3 & 4\\ 1 & 6\end{bmatrix}$ then which of the following statements is true -

(a) $AB = BA$ (b) $A^2 = B$

(c) $(AB)^T=\begin{bmatrix}5 & 9\\ 16 & 12\end{bmatrix}$ (d) none of these

30. If $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then which one of the following holds for all $n ^3 1$, by the principle of mathematical induction?

(a) $A^n = 2^{n-1}A-(n-1)I$ (b) $A^n = nA-(n-1)I$

(c) $A^n = 2^{n-1}A+(n-1)I$ (d) $A^n = nA+(n-1)I$

Type of matrices

31. In the following, singular matrix is -

(a) $\begin{bmatrix}2 & 3\\ 1 & 3\end{bmatrix}$ (b) $\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}$ (d) $\begin{bmatrix}2 & 6\\ 4 & 12\end{bmatrix}$

32. If $A=\begin{bmatrix}1 & -3 & 2\\ 2 & k & 5\\ 4 & 2 & 1\end{bmatrix}$ is a singular matrix, then k is equal to

(a) –1 (b) 8

(c) 4 (d) –8

33. If $A = [a_{ij}]$ is a skew-symmetric matrix of order n, then $a_{ii} =$

(a) 0 for some i (b) 0 for all i = 1, 2, ... n

(c) 1 for some i (d) 1 for all i = 1, 2, ..., n.

34. Matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is a –

(a) diagonal matrix (b) upper triangular matrix

(c) skew–symmetric matrix (d) symmetric matrix

35. If A and B are square matrices of same order, then which of the following is skew–symmetric –

(a) $\frac{A+A^T}{2}$ (b) $\frac{A^T+B^T}{2}$

(c) $\frac{A^T-B^T}{2}$ (d) $\frac{B-B^T}{2}$

36. If A is symmetric as well as skew symmetric matrix, then -

(a) A is a diagonal matrix (b) A is a null matrix

(c) A is a unit matrix (d) A is a triangular matrix

37. If $A=\begin{bmatrix} -1 & 7 \\ 2 & 3 \end{bmatrix}$, then skew–symmetric part of A is –

(a) $\begin{bmatrix} -1 & 9/2 \\ -9/2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -9/2 \\ 9/2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$

Adjoint of matrix and its properties

38. If $A=\begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 1 \\ 5 & 1 & 3 \end{bmatrix}$, then adj. A is equal to -

(a) $\begin{bmatrix} 14 & -4 & -22 \\ -4 & -22 & 14 \\ -22 & 14 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} -14 & 4 & 22 \\ 4 & 22 & -14 \\ 22 & -14 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 14 & 4 & -22 \\ 4 & -22 & -14 \\ -22 & -14 & -4 \end{bmatrix}$ (d) none of these

39. If $A=\begin{bmatrix} 1 & 2 & 3 \\ 5 & 0 & 4 \\ 2 & 6 & 7 \end{bmatrix}$ then adj A is equal to -

(a) $\begin{bmatrix} -24 & 4 & 8 \\ 4 & 1 & 2 \\ 8 & 11 & -11 \end{bmatrix}$ (b) $\begin{bmatrix} -24 & 4 & 8 \\ 4 & 1 & 11 \\ 30 & -2 & -10 \end{bmatrix}$

(c) $\begin{bmatrix} -24 & 4 & 8 \\ -27 & 1 & 11 \\ 30 & -2 & -10 \end{bmatrix}$ (d) none of these

40. If $A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}$, then A (adj A) equals -

(a) $\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ (b) $-\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 9 \\ 0 & 9 & 0 \\ 9 & 0 & 0 \end{bmatrix}$ (d) none of these

41. If A is an invertible matrix of order n, then the determinant of Adj. A =

(a) $|A|^n$ (b) $|A|^{n+1}$

(c) $|A|^{n-1}$ (d) $|A|^{n+2}$

Inverse of a matrix and Its properties

42. If A and B are invertible matrices of the order n, then $(AB)^{-1}$ is equal to

(a) AB^{-1} (b) $A^{-1}B$

(c) $B^{-1}A^{-1}$ (d) $A^{-1}B^{-1}$

43. If $A^2 - A + I = 0$, then the inverse of A is

(a) A (b) A + I

(c) I – A (d) A – I

44. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then $19A^{-1}$ is equal to

(a) A' (b) $2A$

(c) $\frac{1}{2}A$ (d) A

45. $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}^{-1} =$

(a) $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & -5 \\ 7 & -6 \end{bmatrix}$

46. Inverse matrix of $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$ is -

(a) $-\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (b) $-\frac{1}{8}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

(c) $\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$

47. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $M = AB$, then M^{-1} is equal to –

(a) $\begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 1/6 \end{bmatrix}$

(c) $\begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 1/6 \end{bmatrix}$ (d) $\begin{bmatrix} 1/3 & -1/3 \\ -1/3 & 1/6 \end{bmatrix}$

48. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and X is a matrix such that $A = BX$, then X equals -

(a) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (d) none of these

49. Matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is not invertible if -

(a) $\lambda = -15$ (b) $\lambda = -17$

(c) $\lambda = -16$ (d) $\lambda = -18$

50. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ then A^{-n} is equal to -

(a) $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ -n & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$ (d) none of these

51. $\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^{-1}$ is equal to

(a) $\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ (b) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (d) none of these

Consistency of simultaneous Equations

52. The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution if

(a) $k \neq 0$ (b) $-1 < k < 1$

(b) $-2 < k < 2$ (d) $k = 0$

53. If the system of equations $x+y+z=6$, $x+2y+3z=10$ and $x+2y+\lambda z=\mu$ has no solution, then the values of λ and μ are

(a) $\lambda=3, \mu=10$ (b) $\lambda=3, \mu\neq 10$

(c) $\lambda\neq 3, \mu=10$ (d) $\lambda\neq 3, \mu\neq 10$

54. Consider the system of equations $a_1x+b_1y+c_1z=0$, $a_2x+b_2y+c_2z=0$, $a_3x+b_3y+c_3z=0$ if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0,$$ then the system has

(a) more than two solutions

(b) only non trivial solutions

(c) no solution

(d) only trivial solution (0, 0,0).

55. Consider the system of linear equations

$x_1+2x_2+x_3=3$

$2x_1+3x_2+x_3=3$

$3x_1+5x_2+2x_3=1$

The system has

(a) Infinite number of solutions

(b) Exactly 3 solutions

(c) A unique solution

(d) No solution

56. If the trivial solution is the only solution of the system of equations

$x-ky+z=0$

$kx+3y-kz=0$

$3x+y-z=0$

Then, the set of all values of k is

(a) $\{2,-3\}$ (b) $R-\{2,-3\}$

(c) $R-\{2\}$ (d) $R-\{-3\}$

57. The number of values of k, for which the system of equations

$(k+1)x+8y=4k$

$kx+(k+3)y=3k-1$

has no solution, is

(a) infinite (b) 1

(c) 2 (d) 3

58. If a, b, c are non-zero real numbers and if the system of equations

$(a-1)x=y+z,$

$(b-1)y=z+x,$

$(c-1)z=x+y,$

has a non-trivial soltuion, then ab + bc + ca equals:

(a) $a+b+c$ (b) abc

(c) 1 (d) -1

59. With the help of matrices, the solution of the equations

$3x+y+2z=3,$ $2x-3y-z=-3,$

$x+2y+z=4$ is given by

(a) $x=1, y=2, z=-1$ (b) $x=-1, y=2, z=1$

(c) $x=1, y=-2, z=-1$ (d) $x=-1, y=-2, z=1$

60. Solution of

$x+3y-2z=0$

$2x-y+4z=0$

$x-11y+14z=0$ is

(a) $\frac{x}{8}=\frac{y}{-10}=\frac{z}{7}=\lambda$ (b) $\frac{x}{-10}=\frac{y}{8}=\frac{z}{7}=\lambda$

(c) $\frac{x}{7}=\frac{y}{8}=\frac{z}{-10}=\lambda$ (d) None of these

Numerical Value Type Questions

61. Let $f(\theta)=\begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$ then $f\left(\frac{\pi}{6}\right)=$

62. If f(x) = tan x and A, B, C are the angles of

$$\Delta \begin{vmatrix} f(A) & f(\pi/4) & f(\pi/4) \\ f(\pi/4) & f(B) & f(\pi/4) \\ f(\pi/4) & f(\pi/4) & f(C) \end{vmatrix}$$

then is equal to

63. $\begin{vmatrix} 109 & 102 & 95 \\ 6 & 13 & 20 \\ 1 & -6 & -13 \end{vmatrix}$ is equal to

64. If ω (≠ 1) is a cube root of unity, then

$$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} =$$

65. If $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = k\,(xyz)$, then k is equal to

66. If $ax^3 + bx^2 + cx + d = \begin{vmatrix} 3x & x+1 & x-1 \\ x-3 & -2x & x+2 \\ x+3 & x-4 & 5x \end{vmatrix}$

be an identity in x, where a, b, c are constants, then the value of –d is

67. If l, m, n are the p^{th}, q^{th} and r^{th} term of a G.P. all positive, then

$\begin{vmatrix} \log \ell & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals

68. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $A^2 - 4A - nI = 0$, then –n is equal to -

69. If $\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$, then find the value of

$x + \frac{y}{2} + \frac{z}{3}$

70. If p and q are real so that the system of equations px + 4y + z= 0, 2y + 3z = 1 and 3x – qz = –2 has infinite solutions then $\sqrt{q^2 - p^2}$ is equal to -

71. The system of equations λx + y + z = 1, x + λy + z = λ and x + y + λz = λ^2 have no solution. Then the value of λ^4 is

72. The system of equations kx + y + z = 1, x + ky + z = k and x + y + kz = k^2 have no solution if –k equals

73. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that x = cy + bz, y = az + cx and z = bx + ay. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

74. The number of values of k for which the linear equations 4x + ky + 2z =0, kx + 4y + z = 0 and 2x + 2y + z = 0 posses a non-zero solution is

75. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and det $(A^n - I) = 1 - \lambda^n$, n∈N, then λ is equal to

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. The system of linear equations **(2016)**

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for :

(a) exactly one value of λ.

(b) exactly two values of λ.

(c) exactly three values of λ.

(d) infinitely many values of λ.

2. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = AA^T, then 5a + b is equal to: **(2016)**

(a) 5 (b) 4

(c) 13 (d) –1

3. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP^T, then P^T Q^{2015} P is **(2016/Online Set–1)**

(a) $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$

(c) $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

4. The number of distinct real roots of the equation,

$\begin{vmatrix} \cos x & -\sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$ in the intervals $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is:

(2016/Online Set–1)

(a) 4 (b) 3

(c) 2 (d) 1

5. Let A be a 3 × 3 matrix such that A^2-5A+7I=O.

Statement – I : $A^{-1} = \frac{1}{7}$ (5I-A).

Statement – II : The Polynomial A^3-2A^2-3A+I can be reduced to 5 (A – 4I). Then : **(2016/Online Set–2)**

(a) Statement-I is true, but Statement-II is false.

(b) Statement-I is false, but Statement-II is true.

(c) Both the statements are true.

(d) Both the statements are false.

6. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is :

(2016/Online Set–2)

(a) 2014 (b) –175

(c) 2016 (d) –25

7. If S is the set of distinct values of 'b' for which the following system of linear equations **(2017)**

x + y + z = 1

x + ay + z = 1

ax + by + z = 0

has no solution, then S is :

(a) an empty set

(b) an infinite set

(c) a finite set containing two or more elements

(d) a singleton

8. Let ω be a complex number such that 2ω + 1= z where $z = \sqrt{-3}$. If **(2017)**

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to:

(a) –z (b) z

(c) –1 (d) 1

9. Let A be any 3×3 invertible matrix. Then which one of the following is **not** always true ? **(2017)**

(a) adj A) = |A|. A^{-1}

(b) adj (adjA))= |A|. A

(c) adj (adjA)) = $|A|^2$. $(adj(A))^{-1}$

(d) adj (adj(A)) = |A|. $(adj(A))^{-1}$

10. If $S=\left\{x\in[0,\ 2\pi]:\begin{vmatrix}0 & \cos x & -\sin x\\ \sin x & 0 & \cos x\\ \cos x & \sin x & 0\end{vmatrix}=0\right\}$, then

$\sum_{x\in S}\tan\left(\frac{\pi}{3}+x\right)$ is equal to : **(2017)**

(a) $4+2\sqrt{3}$ (b) $-2+\sqrt{3}$

(c) $-2-\sqrt{3}$ (d) $-4-2\sqrt{3}$

11. If $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$, then adj $(3A^2+12A)$ is equal to: **(2017)**

(a) $\begin{bmatrix}72 & -84\\ -63 & 51\end{bmatrix}$ (b) $\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$

(c) $\begin{bmatrix}51 & 84\\ 63 & 72\end{bmatrix}$ (d) $\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$

12. For two 3 × 3 matrices A and B, let $A + B = 2B'$ and $3A + 2B = I_3$, where B' is the transpose of B and I_3 is 3 × 3 identity matrix. Then : **(2017)**

(a) $5A+10B=2I_3$ (b) $10A+5B=3I_3$

(c) $B+2A=I_3$ (d) $3A+6B=2I_3$

13. The number of real values of λ for which the system of linear equations

$2x+4y-\lambda z=0$

$4x+\lambda y+2z=0$

$\lambda x+2y+2z=0$

has infinitely many solutions, is **(2017/Online Set–1)**

(a) 0 (b) 1

(c) 2 (d) 3

14. If the system of linear equations

$x+ky+3z=0$

$3x+ky-2z=0$

$2x+4y-3z=0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to : **(2018)**

(a) 30 (b) -10

(c) 10 (d) -30

15. If $\begin{vmatrix}x-4 & 2x & 2x\\ 2x & x-4 & 2x\\ 2x & 2x & x-4\end{vmatrix}=(A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to : **(2018)**

(a) (4, 5) (b) (-4, -5)

(c) (-4, 3) (d) (-4, 5)

16. Let A be matrix such that $A\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is a scalar matrix and |3A| = 108. Then A^2 equals : **(2018/Online Set–1)**

(a) $\begin{bmatrix}4 & -32\\ 0 & 36\end{bmatrix}$ (b) $\begin{bmatrix}36 & 0\\ -32 & 4\end{bmatrix}$

(c) $\begin{bmatrix}4 & 0\\ -32 & 36\end{bmatrix}$ (d) $\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$

17. If $f(x)=\begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ \tan x & x & 1\end{vmatrix}$, then $\lim_{x\to 0}\frac{f'(x)}{x}$ **(2018/Online Set–1)**

(a) does not exist

(b) exists and is equal to 2

(c) exists and is equal to 0

(d) exists and is equal to -2

18. Let S be the set of all real values of k for which the system of linear equations

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

has a unique solution. Then S is :

(2018/Online Set–1)

(a) an empty set (b) equal to $\{0\}$

(c) equal to R (d) equal to R - $\{0\}$

19. Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$.

If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to :

(2018/Online Set–2)

(a) 8 (b) 7

(c) 13 (d) 12

20. If the system of linear equations

$x + ay + z = 3$

$x + 2y + 2z = 6$

$x + 5y + 3z = b$

has no solution, then : **(2018/Online Set–2)**

(a) $a = -1, b = 9$ (b) $a = -1, b \neq 9$

(c) $a \neq -1, b = 9$ (d) $a = 1, b \neq 9$

21. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is :

(2018/Online Set–3)

(a) 210 (b) 211

(c) 231 (d) 251

22. The number of values of k for which the system of linear equations,

$(k+2)x + 10y = k$

$kx + (k+3)y = k-1$

has no solution, is : **(2018/Online Set–3)**

(a) 1 (b) 2

(c) 3 (d) infinitely many

23. The greatest value of $c \in R$ for which the system of linear equations

x-cy-cz = 0

cx-y+cz=0

cx+cy-z=0

has a non-trivial solution, is : **(2019-04-08/Shift-1)**

(a) -1 (b) $\frac{1}{2}$

(c) 2 (d) 0

24. Let the numbers 2, b, c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det(A) $\in$ [2, 16], then c lies in the interval : **(2019-04-08/Shift-2)**

(a) [2, 3) (b) $(2 + 2^{3/4}, 4)$

(c) [4, 6] (d) $[3, 2 + 2^{3/4}]$

25. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x, y, z), z $\neq$ 0, then (x, y) lies on the straight line whose equation is : **(2019-04-08/Shift-2)**

(a) $3x - 4y - 1 = 0$ (b) $4x - 3y - 4 = 0$

(c) $4x - 3y - 1 = 0$ (d) $3x - 4y - 4 = 0$

26. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdots\cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$ then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is **(2019-04-09/Shift-1)**

(a) $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$

27. The total number of matrics $A=\begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$

$(x, y \in R, x \neq y)$ for which $A^T A = 3I_3$ is:

(2019-04-09/Shift-2)

(a) 2 (b) 3

(c) 6 (d) 4

28. If the system of equations $2x+3y-z=0$, $x+ky-2z=0$ and $2x-y+z=0$ has a non-trivial solution (x,y,z), then

$\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to **(2019-04-09/Shift-2)**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $-\frac{1}{4}$ (d) -4

29. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}, x \neq 0$$

then for all $\Delta_1 = \theta \in \left(0, \frac{\pi}{2}\right)$: **(2019-04-10/Shift-1)**

(a) $\Delta_1 - \Delta_2 = -2x^3$

(b) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

(c) $\Delta_1 \times \Delta_2 = -2\left(x^3 + x - 1\right)$

(d) $\Delta_1 + \Delta_2 = -2x^3$

30. Let λ be a real number for which the system of linear equations:

$x+y+z=6$

$4x+\lambda y-\lambda z=\lambda-2$

$3x+2y-4z=-5$

has infinitely many solutions. Then λ is a root of the quadratic equation: **(2019-04-10/Shift-2)**

(a) $\lambda^2+3\lambda-4=0$ (b) $\lambda^2-3\lambda-4=0$

(c) $\lambda^2+\lambda-6=0$ (d) $\lambda^2-\lambda-6=0$

31. The sum of the real roots of the equation

$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to: **(2019-04-10/Shift-2)**

(a) 6 (b) 0

(c) 1 (d) -4

32. If A is a symmetric matrix and B is a skew-symmetric matrix such that $A+B=\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to :

(2019-04-12/Shift-1)

(a) $\begin{bmatrix} -4 & -1 \\ -1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

33. If $B=\begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3 x 3 matrix A,

then the sum of all values of α for which det (A) + 1 = 0, is **(2019-04-12/Shift-1)**

(a) 0 (b) –1

(c) 1 (d) 2

34. A value of $\theta \in \left(0, \frac{\pi}{3}\right)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0 \text{ is } \_\_\_\_\_.$$

(2019-04-12/Shift-2)

(a) $\frac{\pi}{9}$ (b) $\frac{\pi}{18}$

(c) $\frac{7\pi}{24}$ (d) $\frac{7\pi}{36}$

35. The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$ **(2019-01-09/Shift-1)**

(a) is inconsistent when $a = 4$

(b) has a unique solution for $|a| = \sqrt{3}$

(c) has infinitely many solutions for $a = 4$

(d) is inconsistent when $|a| = \sqrt{3}$

36. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$ is equal to **(2019-01-09/Shift-1)**

(a) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

37. If the system of linear equations

$x - 4y + 7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then : **(2019-01-09/Shift-2)**

(a) $g + 2h + k = 0$ (b) $g + h + 2k = 0$

(c) $2g + h + k = 0$ (d) $g + h + k = 0$

38. If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$

then A is: **(2019-01-09/Shift-2)**

(a) invertible for all $t \in R$

(b) invertible only if $t = \pi$

(c) not invertible for any $t \in R$

(d) invertible only if $t = \frac{\pi}{2}$

39. If the system of equations

$x + y + z = 5$

$x + 2y + 3z = 9$

$x + 3y + \alpha z = \beta$

has infinitely many solutions, then $\beta - \alpha$ equals:

(2019-01-10/Shift-1)

40. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$ has a non-trivial solution, is **(2019-01-10/Shift-2)**

(a) three (b) two

(c) four (d) one

41. Let $A=\begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$ where $b>0$. Then the minimum value of $\frac{\det(A)}{b}$ is: **(2019-01-10/Shift-2)**

(a) $2\sqrt{3}$ (b) $-2\sqrt{3}$

(c) $-\sqrt{3}$ (d) $\sqrt{3}$

42. Let $a_1, a_2, a_3, \ldots, a_{10}$ be in G.P. with $a_i > 0$ for $i=1,2,\ldots,10$ and S be the set of pairs (r, k), r, k $\in$ N (the set of natural numbers) for which

$$\begin{vmatrix}\log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k\\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k\\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k\end{vmatrix}=0$$

Then the number of elements in S, is :

(2019-01-10/Shift-2)

(a) 4 (b) infinitely many

(c) 2 (d) 10

43. If the system of linear equations

2x + 2y + 3z = a

3x -y + 5z = b

x-3y+ 2z = c

where, a, b, c are non-zero real numbers, has more than one solution, then : **(2019-01-11/Shift-1)**

(a) b – c + a = 0 (b) b – c – a = 0

(c) a + b + c = 0 (d) b + c – a = 0

44. If $\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$

$=(a+b+c)(x+a+b+c)^2, x\neq 0$ and $a+b+c\neq 0$,

then x is equal to : **(2019-01-11/Shift-2)**

(a) abc (b) $-(a+b+c)$

(c) $2(a+b+c)$ (d) $-2(a+b+c)$

45. Let A and B be two invertible matrices of order 3×3. If $\det(ABA^T)=8$ and $\det(AB^{-1})=8$ then $\det(BA^{-1}B^T)$ is equal to : **(2019-01-11/Shift-2)**

(a) $\frac{1}{4}$ (b) 1

(c) $\frac{1}{16}$ (d) 16

46. An ordered pair (α, β) for which the system of linear equations

$(1+\alpha)x+\beta y+z=2,$

$\alpha x+(1+\beta)y+z=3,$

$\alpha x+\beta y+2z=2$

has a unique solution, is: **(2019-01-12/Shift-1)**

(a) $(2,4)$ (b) $(-3,1)$

(c) $(-4,2)$ (d) $(1,-3)$

47. If $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$; then for all $\theta\in\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$, det(A) lies in the interval :

(2019-01-12/Shift-2)

(a) $\left(1,\frac{5}{2}\right]$ (b) $\left[\frac{5}{2},4\right)$

(c) $\left(0,\frac{3}{2}\right]$ (d) $\left(\frac{3}{2},3\right]$

48. The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solution : **(2019-01-12/Shift-2)**

(a) is a singleton

(b) contains exactly two elements

(c) is an empty set

(d) contains more than two elements

49. Let A be a 2 × 2 real matrix with entries from $\{0,1\}$ and $|A| \neq 0$. Consider the following two statements :

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $tr(A) = 2$,

Where I_2 denotes 2 × 2 identity matrix and tr (A) denotes the sum of the diagonal entries of A. Then :

(2020-09-02/Shift-1)

(a) Both (P) and (Q) are false

(b) (P) is true and (Q) is false

(c) Both (P) and (Q) are true

(d) (P) is false and (Q) is true

50. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

has no solution. Then the set S **(2020-09-02/Shift-1)**

(a) is an empty set.

(b) is a singleton.

(c) contains more than two elements.

(d) contains exactly two elements.

51. Let $A = \{X = (x,y,z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1\}$, where

$P\begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$, then the set A : **(2020-09-02/Shift-2)**

(a) contains more than two elements

(b) is a singleton.

(c) contains exactly two elements

(d) is an empty set.

52. Let a, b, c $\in$ R be all non-zero and satisfy

$a^3 + b^3 + c^3 = 2$ If the matrix $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ satisfies

$A^T A = I$, then a value of abc can be :

(2020-09-02/Shift-2)

(a) $\frac{2}{3}$ (b) 3

(c) a – b (d) $\frac{1}{3}$

53. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$,

then B + C is equal to : **(2020-09-03/Shift-1)**

(a) 1 (b) –1

(c) –3 (d) 9

54. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then

a_{22} is equal to .... **(2020-09-03/Shift-1)**

55. Let A be a 3×3 matrix such that $adj\ A=\begin{bmatrix}2&-1&1\\-1&0&2\\1&-2&-1\end{bmatrix}$

and $B=adj(adjA)$. If $|A|=\lambda$ and $\left|\left(B^{-1}\right)^{T}\right|=\mu$ then the ordered pair, $(|\lambda|,\mu)$ is equal to : **(2020-09-03/Shift-2)**

(a) $\left(9,\frac{1}{81}\right)$ (b) $\left(9,\frac{1}{9}\right)$

(c) $\left(3,\frac{1}{81}\right)$ (d) $(3,81)$

56. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x-2y+5z=0$

$-2x+4y+z=0$

$-7x+14y+9z=0$

such that $15\le x^2+y^2+z^2\le150$. Then, the number of elements in the set S is equal to ....

(2020-09-03/Shift-2)

57. If $A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix},\left(\theta=\frac{\pi}{24}\right)$ and $A^5=\begin{bmatrix}a&b\\c&d\end{bmatrix}$,

where $i=\sqrt{-1}$ then which one of the following is not true ? **(2020-09-04/Shift-1)**

(a) $a^2-d^2=0$ (b) $a^2-c^2=1$

(c) $0\le a^2+b^2\le1$ (d) $a^2-b^2=\frac{1}{2}$

58. If the system of equations

$x-2y+3z=9$

$2x+y+z=b$

$x-7y+az=24$

has infinitely many solutions, then $a-b$ is equal to......

(2020-09-04/Shift-1)

59. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax = b when the vector b on the right side is equal to b_1, b_2 and b_3 respectivly.

$x_1=\begin{bmatrix}1\\1\\1\end{bmatrix},x_2=\begin{bmatrix}0\\2\\1\end{bmatrix},x_3=\begin{bmatrix}0\\0\\1\end{bmatrix},$

$b_1=\begin{bmatrix}1\\0\\0\end{bmatrix},b_2=\begin{bmatrix}0\\2\\0\end{bmatrix}\text{and }b_3=\begin{bmatrix}0\\0\\2\end{bmatrix},$

If then the determinant of A is equal to:

(2020-09-04/Shift-2)

(a) 2 (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) 4

60. If the system of equations

$x+y+z=2$

$2x+4y-z=6$

$3x+2y+\lambda z=\mu$

has infinitely many solutions, then

(2020-09-04/Shift-2)

(a) $\lambda-2\mu=-5$ (b) $2\lambda+\mu=14$

(c) $\lambda+2\mu=14$ (d) $2\lambda-\mu=5$

61. If the minimum and the maximum values of the function $f:\left[\frac{\pi}{4},\frac{\pi}{2}\right]\to R$, defined by

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$

are m and M respectively, then the ordered pair (m,M) is equal to:

(2020-09-05/Shift-1)

(a) (0,4) (b) (-4,0)

(c) (-4,4) (d) $(0,2\sqrt{2})$

62. Let $\lambda \in R$. The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$

$$x_1 - 6x_2 + x_3 = 2$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

is inconsistent for: **(2020-09-05/Shift-1)**

(a) exactly two values of λ

(b) exactly one negative value of λ

(c) every value of λ

(d) exactly one positive value of λ

63. $a + x = b + y = c + z + 1$, where a,b,c,x,y,z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to:

(2020-09-05/Shift-2)

(a) y(a-b) (b) 0

(c) y(b-a) (d) y(a-c)

64. If the system of linear equations

$$x + y + 3z = 0$$

$$x + 3y + k^2 z = 0$$

$$3x + y + 3z = 0$$

has a non-zero solution (x,y,z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to: **(2020-09-05/Shift-2)**

(a) –9 (b) 9

(c) –3 (d) 3

65. The values of λ and μ for which the system of linear equations

$$x + y + z = 2$$

$$x + 2y + 3z = 5$$

$$x + 3y + \lambda z = \mu$$

has infinitely many solutions are, respectively:

(2020-09-06/Shift-1)

(a) 6 and 8 (b) 5 and 8

(c) 5 and 7 (d) 4 and 9

66. Let m and M be respectively the minimum and maximum values of $\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$

Then the ordered pair (m, M) is equal to:

(2020-09-06/Shift-1)

(a) (-3, -1) (b) (-4, -1)

(c) (1, 3) (d) (-3, 3)

67. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ If $B = A + A^4$, then det (B): **(2020-09-06/Shift-2)**

(a) is one (b) lies in (1, 2)

(c) lies in (2, 3) (d) is zero

68. The sum of distinct values of λ for which the system of equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$, has non-zero solutions, is __________ **(2020-09-06/Shift-2)**

69. Let α be the root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to, **(2020-01-07/Shift-1)**

(a) A (b) A^2

(c) A^3 (d) I_3

70. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + c = 0$

Where $a, b, c \in R$ are non-zero and distinct; has non-zero solution, then **(2020-01-07/Shift-1)**

(a) $a + b + c = 0$ (b) a,b,c are A.P.

(c) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. (d) a, b, c are in G. P.

71. $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is :

(2020-01-07/Shift-2)

(a) $\frac{1}{9}$ (b) $\frac{1}{81}$

(c) $\frac{1}{3}$ (d) 3

72. If system of linear equations

$x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

has more than two solutions, then $\mu - \lambda^2$ is equal to ________. **(2020-01-07/Shift-2)**

73. For which of the following ordered pairs (μ, δ), the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent? **(2020-01-08/Shift-1)**

(a) (4, 6) (b) (3, 4)

(c) (1, 0) (d) (4, 3)

74. The number of all 3×3 matrices A, with entries from the set {-1, 0, 1} such that the sum of the diagonal elements of (AA^T) is 3, is_____. **(2020-01-08/Shift-1)**

75. If $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $10A^{-1}$ is equal to:

(2020-01-08/Shift-2)

(a) $6I - A$ (b) $A - 6I$

(c) $4I - A$ (d) $A - 4I$

76. The system of linear equations

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has: **(2020-01-08/Shift-2)**

(a) no solution when $\lambda = 2$

(b) infinitely many solutions when $\lambda = 2$

(c) no solution when $\lambda = 8$

(d) a unique solution when $\lambda = -8$

77. If for some α and β in R, the intersection of the following three planes

$x + 4y - 2z = 1$

$x + 7y - 5z = \beta$

$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to:

(2020-01-09/Shift-1)

(a) 0 (b) 10

(c) –10 (d) 2

78. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = adj\ A$ and $C = 3A$, then $\frac{|adj\ B|}{|C|}$ is equal to: **(2020-01-09/Shift-1)**

(a) 16 (b) 2

(c) 8 (d) 72

79. Let $a-2b+c=1$. If $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then:

(2020-01-09/Shift-2)

(a) $f(-50)=501$ (b) $f(-50)=10$

(c) $f(50)=1$ (d) $f(50)=-501$

80. The following system of linear equtions

$7x+6y-2z=0,$

$3x+4y+2z=0,$

$x-2y-6z=0$, has **(2020-01-09/Shift-2)**

(a) infinitely many solutions, (x,y,z) satisfying y=2z

(b) infinitely many solutions (x,y,z) satisfying x=2z

(c) no solution

(d) only the trivial solution

81. Let $A=[a_{ij}]$ be a 3×3 matrix,

where $a_{ij}=\begin{cases} 1, & \text{if } i=j \\ -x, & \text{if } |i-j|=1 \\ 2x+1, & \text{otherwise} \end{cases}$

Let a function $f:R\to R$ be defined as f(x) = det (A). Then the sum of maximum and minimum values of f on R is equal to: **(2021-07-20/Shift-1)**

(a) $\frac{20}{27}$ (b) $-\frac{88}{27}$

(c) $-\frac{20}{27}$ (d) $\frac{88}{27}$

82. Let $A=\begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a\in R$ be written as P + Q, where P is a symmetric matrix and Q is a skew-symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to:

(2021-07-20/Shift-1)

(a) 24 (b) 18

(c) 45 (d) 36

83. Let a,b,c,d be in arithmetic progression with common difference λ.

If $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix}=2$, then value of λ^2 is equal to _____. **(2021-07-20/Shift-1)**

84. Let $A=\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B=7A^{20}-20A^{7}+2I$, where I is an identity matrix of order 3×3. If $B=[b_{ij}]$, then b_{13} is equal to _____. **(2021-07-20/Shift-1)**

85. The value of $k\in R$, for which the following system of linear equations

$3x-y+4z=3,$

$x+2y-3z=-2,$

$6x+5y+kz=-3,$

has infinitely many solutions, is: **(2021-07-20/Shift-2)**

(a) 3 (b) –3

(c) 5 (d) –5

86. Let $A=\{a_{ij}\}$ be a 3×3 matrix,

where $a_{ij}=\begin{cases} (-1)^{j-i} & \text{if } i<j, \\ 2 & \text{if } i=j, \\ (-1)^{i+j} & \text{if } i>j \end{cases}$ then $\det\left(3\operatorname{Adj}\left(2A^{-1}\right)\right)$ is equal to ________ ? **(2021-07-20/Shift-2)**

87. Let $S=\left\{n\in N \,\middle|\, \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a,b,c,d\in R\right\}$,

where $i=\sqrt{-1}$. Then the number of 2-digit numbers in the set S is ____________ ? **(2021-07-25/Shift-1)**

88. The values of a and b, for which the system of equations

$2x+3y+6z=8$

$x+2y+az=5$

$3x+5y+9z=b$

Has no solution, are ? **(2021-07-25/Shift-1)**

(a) $a=3, b=13$ (b) $a\neq 3, b\neq 13$

(c) $a\neq 3, b=3$ (d) $a=3, b\neq 13$

89. $f(x)=\begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x\in[0,\pi]$

Then the maximum value of f(x) is equal to _____.

(2021-07-27/Shift-1)

90. For real numbers and consider the following system of linear equations:

$x+y-z=2,\ x+2y+\alpha z=1, 2x-y+z=\beta$.

If the system has infinite solutions, then $\alpha+\beta$ is equal to ______. **(2021-07-27/Shift-1)**

91. Let $A=\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1}=\alpha I+\beta A, \alpha,\beta\in R$, I is a 2×2 identity matrix, then $4(\alpha-\beta)$ is equal to:

(2021-07-27/Shift-1)

(a) 5 (b) 4

(c) 2 (d) $\frac{8}{3}$

92. Let A and B be two 3×3 real matrices such that (A^2-B^2) is invertible matrix. If $A^5=B^5$ and $A^3B^2=A^2B^3$, then the value of the determinant of the matrix A^3+B^3 is equal to:

(2021-07-27/Shift-2)

(a) 0 (b) 2

(c) 1 (d) 4

93. If $A=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M=A+A^2+A^3+....+A^{20}$, then the sum of the all the elements of the matrix M is equal to _______. **(2021-07-27/Shift-2)**

94. Let $A=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3×3 matrices B with entries from the set $\{1,2,3,4,5\}$ and satisfying AB=BA is _______. **(2021-07-22/Shift-2)**

95. Let $A=[a_{ij}]$ be a real matrix of order 3×3 such that $a_{i1}+a_{i2}+a_{i3}=1$, for $i=1,2,3$. Then, the sum of all the entries of the matrix A^3 is equal to:

(2021-07-22/Shift-2)

(a) 1 (b) 3

(c) 2 (d) 9

96. The value of λ and μ such that the system of equations $x+y+z=6$, $3x+5y+5z=26$, $x+2y+\lambda z=\mu$ has no solution, are: **(2021-07-22/Shift-2)**

(a) $\lambda=3,\mu\neq 10$ (b) $\lambda\neq 2,\mu=10$

(c) $\lambda=3,\mu=5$ (d) $\lambda=2,\mu\neq 10$

97. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}=0$ in the interval $-\frac{\pi}{4}\le x\le\frac{\pi}{4}$ is:

(2021-07-25/Shift-2)

(a) 1 (b) 2

(c) 3 (d) 4

98. If $P=\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$, then P^{50} is: **(2021-07-25/Shift-2)**

(a) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

99. Consider the system of linear equations

$-x+y+2z=0$

$3x-ay+5z=1$

$2x-2y-az=7$

Let S_1 be the set of all $a\in R$ for which the system is inconsistent and S_2 be the set of all $a\in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then **(2021-09-01/Shift-2)**

(a) $n(S_1)=0, n(S_2)=2$

(b) $n(S_1)=2, n(S_2)=2$

(c) $n(S_1)=2, n(S_2)=0$

(d) $n(S_1)=1, n(S_2)=0$

100. Let $J_{n,m}=\int_0^{1/2}\frac{x^n}{x^m-1}dx, \forall n>m$ and $n,m\in N$. Consider a matrix $A=[a_{ij}]_{3\times3}$ where $a_{ij}=\begin{cases}J_{6+i,3}-J_{i+3,3}, & i\le j\\ 0, & i>j\end{cases}$.

Then $\left|adjA^{-1}\right|$ is **(2021-09-01/Shift-2)**

(a) $(15)^2\times2^{34}$ (b) $(105)^2\times2^{38}$

(c) $(15)^2\times2^{42}$ (d) $(105)^2\times2^{36}$

101. Two fair dice are thrown. The number on them are taken as λ and μ and a system of linear equations

$x+y+z=5$

$x+2y+3z=\mu$

$x+3y+\lambda z=1$

Is constructed. If p is true probability that the system has a unique solution and q is the probability that the system has no solution, then: **(2021-08-26/Shift-2)**

(a) $p=\frac{5}{6}$ and $q=\frac{5}{36}$ (b) $p=\frac{1}{6}$ and $q=\frac{1}{36}$

(c) $p=\frac{1}{6}$ and $q=\frac{5}{36}$ (d) $p=\frac{5}{6}$ and $q=\frac{1}{36}$

102. Let A be a 3×3 real matrix.

If $\det\left(2Adj\left(2Adj\left(Adj(2A)\right)\right)\right)=2^{41}$, then the value of $\det\left(A^2\right)$ equals _______. **(2021-08-26/Shift-2)**

103. Let $A=\begin{pmatrix}1&0&0\\0&1&1\\1&0&0\end{pmatrix}$. Then $A^{2025}-A^{2020}$ is equal to:

(2021-08-26/Shift-2)

(a) A^5 (b) A^6

(c) A^6-A (d) A^5-A

104. If the matrix $A=\begin{pmatrix}0&2\\k&-1\end{pmatrix}$ satisfies $A\left(A^3+3I\right)=2I$, then the value of k is: **(2021-08-27/Shift-1)**

(a) 1 (b) -1

(c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

105. If the system of linear equations

$2x+y-z=3$

$x-y-z=\alpha$

$3x+3y+\beta z=3$

has infinitely many solutions, then $\alpha+\beta-\alpha\beta$ is equal to _______. **(2021-08-27/Shift-1)**

106. Let $\theta\in\left(0,\frac{\pi}{2}\right)$. If the system of linear equations,

$\left(1+\cos^2\theta\right)x+\left(\sin^2\theta\right)y+\left(4\sin3\theta\right)z=0$

$\left(\cos^2\theta\right)x+\left(1+\sin^2\theta\right)y+\left(4\sin3\theta\right)z=0$

$\left(\cos^2\theta\right)x+\left(\sin^2\theta\right)y+\left(1+4\sin3\theta\right)z=0$

Has a non-trivial solution, then the value of θ is:

(2021-08-26/Shift-1)

(a) $\frac{4\pi}{9}$ (b) $\frac{\pi}{18}$

(c) $\frac{5\pi}{18}$ (d) $\frac{7\pi}{18}$

107. If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$ and

$Q = A^T BA$, then the inverse of the matrix $A\ Q^{2021}\ A^T$ is equal to : **(2021-08-26/Shift-1)**

(a) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

108. Let $[\lambda]$ be the greatest integer less than or equal to λ. The set of all value of λ for which the system of linear equations

$x+y+z=4,\ 3x+2y+5z=3,\ 9x+4y+(28+[\lambda])z=[\lambda]$

has a solution is **(2021-08-27/Shift-2)**

(a) $(-\infty,-9)\cup[-8,\infty)$ (b) $(-\infty,-9)\cup(-9,\infty)$

(c) $[-9,-8]$ (d) R

109. Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where $[x]$ denotes the greatest integer less than or equal to x. If $\det(A) = 192$, then the set of values of x is in the interval: **(2021-08-27/Shift-2)**

(a) [62, 63] (b) [60, 61]

(c) [68, 69] (d) [65, 66]

110. If the following system of linear equations

$2x+y+z=5$

$x-y+z=3$

$x+y+az=b$

Has no solution, then ? **(2021-08-31/Shift-1)**

(a) $a \neq \frac{1}{3}, b = \frac{7}{3}$ (b) $a \neq -\frac{1}{3}, b = \frac{7}{3}$

(c) $a = \frac{1}{3}, b \neq \frac{7}{3}$ (d) $a = -\frac{1}{3}, b \neq \frac{7}{3}$

111. The number of elements in the set

$\left\{A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}; a,b,d \in \{-1,0,1\} \text{ and } (I-A)^3 = I - A^3\right\}$,

where I is 2×2 identity matrix, is ___

(2021-08-31/Shift-2)

112. If $\alpha+\beta+\gamma = 2\pi$, then the system of equations

$x+(\cos\gamma)y+(\cos\beta)z=0$

$(\cos\gamma)x+y+(\cos\alpha)z=0$

$(\cos\beta)x+(\cos\alpha)y+z=0$

has: **(2021-08-31/Shift-2)**

(a) exactly two solutions (b) a unique solution

(c) no solution (d) infinitely many solutions

113. Let A be the set of all points (α,β) such that the area of triangle formed by the points (5,6), (3, 2) and (α,β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is:

(2021-08-31/Shift-2)

(a) $\frac{16}{\sqrt{5}}$ (b) $\frac{12}{\sqrt{5}}$

(c) $\frac{8}{\sqrt{5}}$ (d) $\frac{4}{\sqrt{5}}$

114. Consider the following system of equations:

$x+2y-3z=a$

$2x+6y-11z=b$

$x-2y+7z=c,$

where a,b and c are real constants. Then the system of equations: **(2021-02-26/Shift-2)**

(a) has a unique solution when $5a = 2b + c$

(b) has no solution for all a, b and c

(c) has infinite number of solutions when $5a = 2b + c$

(d) has a unique solution for all a, b and c

115. If the matrix $A=\begin{bmatrix}1&0&0\\0&2&0\\3&0&-1\end{bmatrix}$ satisfies the equation

$A^{20}+\alpha A^{19}+\beta A=\begin{bmatrix}1&0&0\\0&4&0\\0&0&1\end{bmatrix}$ for some real numbers α

and β, then $\beta-\alpha$ is equal to ______.

(2021-02-26/Shift-2)

116. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is :

(2021-02-26/Shift-1)

(a) 12 (b) 4

(c) 6 (d) 1

117. The value of $\begin{vmatrix}(a+1)(a+2)&a+2&1\\(a+2)(a+3)&a+3&1\\(a+3)(a+4)&a+4&1\end{vmatrix}$ is :

(2021-02-26/Shift-1)

(a) $(a+2)(a+3)(a+4)$ (b) 0

(c) –2 (d) $(a+1)(a+2)(a+3)$

118. If $A=\begin{bmatrix}0&-\tan\left(\frac{\theta}{2}\right)\\\tan\left(\frac{\theta}{2}\right)&0\end{bmatrix}$ and

$(I_2+A)(I_2-A)^{-1}=\begin{bmatrix}a&-b\\b&a\end{bmatrix}$, then 13 (a^2+b^2) is equal to ____. **(2021-02-25/Shift-1)**

119. Let , where and are real numbers such that and . If , then the value of is ______. **(2021-02-25/Shift-1)**

120. If the system of equations

$kx+y+2z=1$

$3x-y-2z=2$

$-2x-2y-4z=3$

has infinitely many solutions, then k is equal to ______.

(2021-02-25/Shift-1)

121. If for the matrix, $A=\begin{bmatrix}1&-\alpha\\\alpha&\beta\end{bmatrix}$, $AA^T=I_2$, then the value of $\alpha^4+\beta^4$ is: **(2021-02-25/Shift-2)**

(a) 3 (b) 1

(c) 4 (d) 2

122. The following system of linear equations

$2x+3y+2z=9$

$3x+2y+2z=9$

$x-y+4z=8$ **(2021-02-25/Shift-2)**

(a) does not have any solution

(b) has infinitely many solutions

(c) has a unique solution

(d) has a solution (α,β,γ) satisfying $\alpha+\beta^2+\gamma^3=12$

123. Let A be a 3×3 matrix with $\det(A)=4$. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2\to 2R_2+5R_3$ on 2A, then det (B) is equal to: **(2021-02-25/Shift-2)**

(a) 80 (b) 128

(c) 64 (d) 16

124. For the system of linear equations:

$x-2y=1, x-y+kz=-2, ky+4z=6, k\in R$

consider the following statements :

(A) The system has unique solution if $k\neq 2, k\neq -2$.

(B) The system has unique solution if $k=-2$.

(C) The system has unique solution if $k=2$.

(D) The system has no-solution if $k=2$.

(E) The system has infinite number of solutions if $k\neq -2$.

Which of the following statements are correct ?

(2021-02-24/Shift-2)

(a) (B) and (E) only (b) (A) and (E) only

(c) (A) and (D) only (d) (C) and (D) only

125. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2-B^2A^2)X=O$, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × 1 null matrix, has **(2021-02-24/Shift-2)**

(a) a unique solution (b) exactly two solutions

(c) no solution (d) infinitely many solutions

126. The system of linear equations

$3x-2y-kz=10$

$2x-4y-2z=6$

$x+2y-z=5m$

is inconsistent if: **(2021-02-24/Shift-1)**

(a) $k=3, m=\frac{4}{5}$ (b) $k\neq 3, m\in R$

(c) $k\neq 3, m\neq\frac{4}{5}$ (d) $k=3, m\neq\frac{4}{5}$

127. Let M be any 3 × 3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is ________. **(2021-02-24/Shift-1)**

128. Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in R$. Suppose $Q=[q_{ij}]$ is a matrix satisfying $PQ=kI_3$ for some non-zero $k\in R$. If $q_{23}=-\frac{k}{8}$ and $|Q|=\frac{k^2}{2}$, then α^2+k^2 is equal to ______. **(2021-02-24/Shift-1)**

129. The solutions of the equation

$$\begin{vmatrix}1+\sin^2 x & \sin^2 x & \sin^2 x\\ \cos^2 x & 1+\cos^2 x & \cos^2 x\\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x\end{vmatrix}=0, (0<x<\pi), \text{ are :}$$

(2021-03-18/Shift-1)

(a) $\frac{7\pi}{12}, \frac{11\pi}{12}$ (b) $\frac{\pi}{12}, \frac{\pi}{6}$

(c) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (d) $\frac{\pi}{6}, \frac{5\pi}{6}$

130. Let $A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix}$ and $2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$.

If T, (A) denotes the sum of all diagonal elements of the matrix A, then T,(A) – T,(B) has value equal to :

(2021-03-18/Shift-1)

(a) 3 (b) 1

(c) 2 (d) 0

131. Let α, β, γ be the real roots of the equation $x^3+ax^2+bx+c=0$, $(a, b, c\in R \text{ and } a, b\neq 0)$. If the system of equation (in u, v, w) given by $\alpha u+\beta v+\gamma w=0;\ \beta u+\gamma v+\alpha w=0;$

$\gamma u+\alpha v+\beta w=0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is : **(2021-03-18/Shift-1)**

(a) 5 (b) 1

(c) 3 (d) 0

132. Let the system of linear equations

$4x + \lambda y + 2z = 0$

$2x - y + z = 0$

$\mu x + 2y + 3z = 0, \lambda, \mu \in R.$

has a non-trivial solution. Then which of the following is true ? **(2021-03-18/Shift-2)**

(a) $\lambda = 3, \mu \in R$ (b) $\lambda = 2, \mu \in R$

(c) $\mu = -6, \lambda \in R$ (d) $\mu = 6, \lambda \in R$

133. Define a relation R over a class of $n \times n$ real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$".

Then which of the following is true? **(2021-03-18/Shift-2)**

(a) R is reflexive, symmetric but not transitive

(b) R is an equivalence relation

(c) R is symmetric, transitive but not reflexive,

(d) R is reflexive, transitive but not symmetric

134. Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$.

Then the value of $n \in N$ for which $P^n = 5I - 8P$ is equal to ________. **(2021-03-18/Shift-2)**

135. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, i = \sqrt{-1}$. Then, the system of linear equations $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has: **(2021-03-16/Shift-1)**

(a) A unique solution

(b) Exactly two solutions

(c) Infinitely many solutions

(d) No solution

136. The total number of 3×3 matrices A having entries from the set {0, 1, 2, 3} such that the sum of all the diagonal entries of AA^T is 9, is equal to ________.

(2021-03-16/Shift-1)

137. Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$

where $\omega = \frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $\left(P^{-1}AP - I_3\right)^2$ is $\alpha\omega^2$, then the value of α is equal to ________.

(2021-03-16/Shift-1)

138. The maximum value of

$$f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R \text{ is}$$

(2021-03-16/Shift-2)

(a) $\sqrt{5}$ (b) 5

(c) $\frac{3}{4}$ (d) $\sqrt{7}$

139. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real numbers such that $A = XB$, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ and $\left(k^2+1\right)b_2^2 \neq -2b_1b_2$ then the value of k is ____.

(2021-03-16/Shift-2)

140. The system of equations $kx + y + z = 1, x + ky + z = k$ and $x + y + zx = k^2$ has no solution is k is equal to :

(2021-03-17/Shift-1)

(a) 1 (b) – 2

(c) – 1 (d) 0

141. If $A=\begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2-\frac{1}{2}I\right)=0$, then a possible value of α is : **(2021-03-17/Shift-1)**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

142. If $A=\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of

$\det(A^4)+\det(A^{10}-(\text{Adj}(2A))^{10})$ is equal to ....

(2021-03-17/Shift-1)

143. If x, y, z are in arithmetic progression with common difference d, $x \neq 3d$, and the determinant of the matrix

$\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is :

(2021-03-17/Shift-2)

(a) 36 (b) 12

(c) 6 (d) 72

144. Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B=\begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and $a+d=2021$, then the value of $ad-bc$ is equal to .... **(2021-03-17/Shift-2)**

145. If 1, $\log_{10}(4^x-2)$ and $\log_{10}\left(4^x+\frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant $\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to :

(2021-03-17/Shift-2)

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. If $A = \text{diag}(d_1, d_2, d_3, \ldots\ldots d_n)$, then A^n is equal to

(a) $\text{diag}\ (d_1^{n-1}, d_2^{n-1}, d_3^{n-1}, \ldots.. d_n^{n-1})$

(b) $\text{diag}\ (d_1^n, d_2^n, d_3^n, \ldots d_n^n)$

(c) A

(d) none of these

2. If f(x), g (x) and h (x) are three polynomials of degree 2, then

$$\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$$ is a polynomial of degree

(a) 2 (b) 3

(c) 4 (d) none of these

3. If n is not a multiple of 3 and $1, \omega, \omega^2$ are the cube roots of unity, then

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$$ is equal to

(a) 0 (b) ω

(c) ω^2 (d) 1

4. $D = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is :

(a) – 1 (b) 0

(c) 1 (d) none of these

5. If $\begin{vmatrix} x^2-2x+3 & 7x+2 & x+4 \\ 2x+7 & x^2-x+2 & 3x \\ 3 & 2x-1 & x^2-4x+7 \end{vmatrix} =$

$ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$ the value of g is

(a) 2 (b) 1

(c) –2 (d) none of these

6. Which one of the following is correct ? If A is non-singular matrix, then, :

(a) $\det(A^{-1}) = \det(A)$ (b) $\det(A^{-1}) = \frac{1}{\det(A)}$

(c) $\det(A^{-1}) = 1$ (d) none of these

7. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then the value of $|A^TA^{-1}|$ is

(a) cos 4x (b) $\sec^2 x$

(c) – cos4x (d) 1

8. For what value of x, the matrix

$$\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$ is singular.

(a) x = 1, 2 (b) x = 0, 2

(c) x = 0, 1 (d) x = 0, 3.

9. If m is a positive integer and

$$\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$$

Then the value of $\sum_{r=0}^{m} \Delta_r$ is

(a) 0 (b) $m^2 - 1$

(c) 2^m (d) $2^m \sin^2(2^m)$

10. Let $D_r = \begin{vmatrix} a & 2^r & 2^{16}-1 \\ b & 3(4^r) & 2(4^{16}-1) \\ c & 7(8^r) & 4(8^{16}-1) \end{vmatrix}$, then the value of $\sum_{k=1}^{16} D_k$ is

(a) 0 (b) a + b + c

(c) ab + bc + ca (d) none of these

11. If $A=\begin{bmatrix}4 & x+2\\ 2x-3 & x+1\end{bmatrix}$ is symmetric, then x =

(a) 3 (b) 5

(c) 2 (d) 4

12. If $\begin{vmatrix}x & 3 & 6\\ 3 & 6 & x\\ 6 & x & 3\end{vmatrix}=\begin{vmatrix}2 & x & 7\\ x & 7 & 2\\ 7 & 2 & x\end{vmatrix}=\begin{vmatrix}4 & 5 & x\\ 5 & x & 4\\ x & 4 & 5\end{vmatrix}=0$ then x is equal to

(a) 9 (b) –9

(c) 0 (d) None of these

13. $\Delta=\begin{vmatrix}p & 2-i & i+1\\ 2+i & q & 3+i\\ 1-i & 3-i & r\end{vmatrix}$ is always (where $p,q,r \in R$)

(a) real (b) imaginary

(c) zero (d) none of these

14. If $\Delta=\begin{vmatrix}1 & a & bc\\ 1 & b & ca\\ 1 & c & ab\end{vmatrix}=\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$. then

(a) $\Delta=(a-b)(b-c)(c-a)$

(b) a, b, c are in G.P.

(c) b, c, a are in G.P.

(d) a, c, b are in G.P.

15. If $E(\theta)=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and θ and φ differ by an odd multiple of π/2, then E(θ). E(φ) is a

(a) Null matrix (b) Unit matrix

(c) Diagonal matrix (d) none of these.

16. If $U_n=\begin{vmatrix}n & 1 & 5\\ n^2 & 2N+1 & 2N+1\\ n^3 & 3N^2 & 3N+1\end{vmatrix}$, then $\sum_{n=1}^{N} U_n$ is equal to

(a) $2\sum_{n=1}^{N} n$ (b) $2\sum_{n=1}^{N} n^2$

(c) $\frac{1}{2}\sum_{n=1}^{N} n^2$ (d) 0

17. If $a^2+b^2+c^2=-2$ and

$$f(x)=\begin{vmatrix}1+a^2x & (1+b^2)x & (1+c^2)x\\ (1+a^2)x & 1+b^2x & (1+c^2)x\\ (1+a^2)x & (1+b^2)x & 1+c^2x\end{vmatrix}$$

Then f (x) is a polynomial of degree :

(a) 2 (b) 3

(c) 0 (d) 1

18. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ and

$G(y)=\begin{bmatrix}\cos y & 0 & \sin y\\ 0 & 1 & 0\\ -\sin y & 0 & \cos y\end{bmatrix}$ then $[F(x)G(y)]^{-1}$ is equal to

(a) $F(-x)G(-y)$ (b) $F(x^{-1})G(y^{-1})$

(c) $G(-y)F(-x)$ (d) $G(y^{-1})F(x^{-1})$

19. If the system of linear equations

$x+2ay+az=0$

$x+3by+bz=0$

$x+4cy+cz=0$

has a non-zero solution, then a, b, c :

(a) are in AP (b) are in GP

(c) are in HP (d) satisfy $a+2b+3c=0$

20. The system of equations

$\alpha x+y+z=\alpha-1$

$x+\alpha y+z=\alpha-1$

$x+y+\alpha z=\alpha-1$

has no solution, if α is :

(a) 1 (b) not – 2

(c) either –2 or 1 (d) –2

21. The system of equation $-2x+y+z=1$, $x-2y+z=-2$, $x+y+\lambda z=4$ will have no solution if

(a) $\lambda=-2$ (b) $\lambda=-1$

(c) $\lambda=3$ (d) none of these

22. The system of equations $x+2y+3z=4$, $2x+3y+4z=5$, $3x+4y+5z=6$ has

(a) Infinitely many solutions

(b) No solution

(c) A unique solution

(d) None of these

23. If a, b, c > 0 & x, y, z ∈ R then the determinant

$$\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^y+b^{-y})^2 & (b^y-b^{-y})^2 & 1 \\ (c^z+c^{-z})^2 & (c^z-c^{-z})^2 & 1 \end{vmatrix} =$$

(a) $a^x b^y c^z$ (b) $a^{-x} b^{-y} c^{-z}$

(c) $a^{2x} b^{2y} c^{2z}$ (d) zero

24. If $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$, then the value of $\frac{p}{x}+\frac{q}{y}+\frac{r}{z}$ is

(a) 0 (b) 1

(c) 2 (d) 4qpr

25. Suppose $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and

$$D' = \begin{vmatrix} a_1+pb_1 & b_1+qc_1 & c_1+ra_1 \\ a_2+pb_2 & b_2+qc_2 & c_2+ra_2 \\ a_3+pb_3 & b_3+qc_3 & c_3+ra_3 \end{vmatrix}.$$ Then

(a) $D' = D$ (b) $D' = D(1-pqr)$

(c) $D' = D(1+p+q+r)$ (d) $D' = D(1+pqr)$

26. Let $ax^7+bx^6+cx^5+dx^4+ex^3+fx^2+gx+h$

$$= \begin{vmatrix} (x+1) & (x^2+2) & (x^2+x) \\ (x^2+x) & (x+1) & (x^2+2) \\ (x^2+2) & (x^2+x) & x+1 \end{vmatrix}.$$ Then

(a) g = 3 and h = – 5 (b) g = –3 and h = – 5

(c) g = – 3 and h = – 9 (d) None of these

Objective Questions II

[One or more than one correct option]

27. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax+B$, where A and B are constants, then

(a) $A+B=12$ (b) $A-B=36$

(c) $A^2+B^2=720$ (d) $A+2B=0$

28. Let A be a symmetric matrix such that $A^5 = O$ and $B = I+A+A^2+A^3+A^4$, then B is

(a) symmetric (b) singular

(c) non-singular (d) skew symmetric

29. The determinant $\Delta = \begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$ is divisible by

(a) x (b) x^2

(c) x^3 (d) x^4

30. If $f(\theta) = \begin{vmatrix} 1 & 1 & -1 \\ 1 & e^{i\theta} & 1 \\ 1 & -1 & -e^{-i\theta} \end{vmatrix}$ then

(a) $\int_{-\pi/2}^{\pi/2} f(\theta)d\theta = 2\int_{0}^{\pi/2} f(\theta)d\theta$

(b) $f(\theta)$ is purely real

(c) $f(\pi/2)=2$

(d) None of these

31. The value of the determinant

$$\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$$, where $i=\sqrt{-1}$, is

(a) complex number (b) real number

(c) irrational number (d) rational number

32. If $a > b > c$ and the system of equations $ax + by + cz = 0$, $bx + cy + az = 0$, $cx + ay + bz = 0$ has a non trivial solution, then both the roots of the quadratic equation $at^2 + bt + c = 0$ are

(a) real (b) of opposite sign

(c) positive (d) complex

33. If $\Delta = \begin{vmatrix} e^x & \sin x & 1 \\ \cos x & \log_e(1+x^2) & 1 \\ x & x^2 & 1 \end{vmatrix} = a + bx + cx^2 ....$ then

(a) $a = 0$ (b) $a = 1$

(c) $b = -1$ (d) $b = -2$

34. If $f(x)$ and $g(x)$ are functions such that $f(x + y) = f(x)\,g(y) + g(x)\,f(y)$, then

$\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha+\theta) \\ f(\beta) & g(\beta) & f(\beta+\theta) \\ f(\gamma) & g(\gamma) & f(\gamma+\theta) \end{vmatrix}$ is independent of

(a) α (b) β

(c) γ (d) θ

35. $\begin{vmatrix} 1 & a & a^2 \\ \cos(xz-yz) & \cos xz & \cos(xz-yz) \\ \sin(xz-yz) & \sin xz & \sin(xz-yz) \end{vmatrix}$ depends on

(a) x (b) y

(c) z (d) a

36. If $x \in N$ and ${}^{x}C_i, {}^{x^2}C_i$ and ${}^{x^3}C_i, (i = 1, 2, 3)$ are binomial coefficients, then

$12\begin{vmatrix} {}^{x}C_1 & {}^{x}C_2 & {}^{x}C_3 \\ {}^{x^2}C_1 & {}^{x^2}C_2 & {}^{x^2}C_3 \\ {}^{x^3}C_1 & {}^{x^3}C_2 & {}^{x^3}C_3 \end{vmatrix}$ is divisible by

(a) x^3 (b) x^6

(c) x^9 (d) x^{12}

37. The digits A, B, C are such that the three digit numbers A88, 6B8, 86C are divisible by 72, then the determinant

$\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$ is divisible by

(a) 72 (b) 144

(c) 288 (d) 216

38. Let $f(a, b) = \begin{vmatrix} a & a^2 & 0 \\ 1 & (2a+b) & (a+b)^2 \\ 0 & 1 & (2a+3b) \end{vmatrix}$, then

(a) $(a + b)$ is a factor of $f(a, b)$

(b) $(a + 2b)$ is a factor of $f(a, b)$

(c) $(2a + b)$ is a factor of $f(a, b)$

(d) a is a factor of $f(a, b)$

39. a, b, c are non-zero real numbers. Then

$\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$, if

(a) $\frac{1}{a} + \frac{1}{b\omega} + \frac{1}{c\omega^2} = 0$ (b) $\frac{1}{a} + \frac{1}{b\omega^2} + \frac{1}{c\omega} = 0$

(c) $\frac{1}{a\omega} + \frac{1}{b\omega^2} + \frac{1}{c} = 0$ (d) none of these

40. Let $f(x) = \begin{vmatrix} 1/x & \ln x & x^n \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$, then $\frac{d^n}{dx^n} f(x)$ at $x = 1$ is

(a) independent of a (b) independent of n

(c) independent of a and n (d) zero

41. If $a^2 + b^2 + c^2 = 1$, then

$\begin{vmatrix} a^2+(b^2+c^2)\cos\phi & ab(1-\cos\phi) & ac(1-\cos\phi) \\ ba(1-\cos\phi) & b^2+(c^2+a^2)\cos\phi & bc(1-\cos\phi) \\ ca(1-\cos\phi) & cb(1-\cos\phi) & c^2+(a^2+b^2)\cos\phi \end{vmatrix}$

is independent of

(a) a (b) b

(c) c (d) ϕ

42. Let A and B be two matrices different from I such that $AB = BA$ and $A^n - B^n$ is invertible for some positive integer n. If $A^n - B^n = A^{n+1} - B^{n+1} = A^{n+2} - B^{n+2}$, then

(a) $I - A$ is singular

(b) $I - B$ is singular

(c) $A = B$

(d) $(I - A)(I - B)$ is non-singular

43. Let $f_1(x) = x + a$, $f_2(x) = x^2 + bx + c$ and

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ f_1(x_1) & f_1(x_2) & f_1(x_3) \\ f_2(x_1) & f_2(x_2) & f_2(x_3) \end{vmatrix}, \text{ then}$$

(a) Δ is independent of a

(b) Δ is independent of b and c

(c) Δ is independent of x_1, x_2, x_3

(d) none of the above

44. Let $0 < \theta < \pi/2$ and

$$\Delta(x, \theta) = \begin{vmatrix} x & \tan\theta & \cot\theta \\ -\tan\theta & -x & 1 \\ \cot\theta & 1 & x \end{vmatrix}$$

then

(a) $\Delta(0, \theta) = 0$

(b) $\Delta(x, \pi/4) = x^2 + 1$

(c) $\underset{0<\theta<\pi/2}{\text{Min}} \Delta(1, \theta) = 2$

(d) $\Delta(x, \theta)$ is independent of x

45. Let A, B and C be 2×2 matrices with entries from the set of real numbers. Define * as follows :

$$A * B = \frac{1}{2}(AB' + A'B)$$

(a) $A * B = B * A$

(b) $A * A = A^2$

(c) $A * (B + C) = A * B + A * C$

(d) $A * I = A + A'$

46. If $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$,

f (a) and f (b) be the least and greatest value of f (x), then

(a) $f(a) = 2, f(b) = 6$

(b) $f(a) = -2, f(b) = 6$

(c) $f(a) = 2, f(b) = -6$

(d) period of f (x) is π

47. Eliminating a, b, c from $x = \dfrac{a}{b-c}, y = \dfrac{b}{c-a}, z = \dfrac{c}{a-b}$, we get

(a) $\begin{vmatrix} 1 & -x & x \\ -y & 1 & y \\ 1 & -z & z \end{vmatrix} = 0$

(b) $\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix} = 0$

(c) $\begin{vmatrix} 1 & -x & x \\ y & 1 & -y \\ -z & z & 1 \end{vmatrix} = 0$

(d) none of these

Numerical Value Type Questions

48. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$,

then x^2 is equal to ....

49. The value of the determinant $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$ is ....

50. If $x \neq 0, y \neq 0, z \neq 0$ and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$, then

$-(x^{-1} + y^{-1} + z^{-1})$ is equal to

51. If $f(x) = \begin{vmatrix} x & \sin x & \cos x \\ x^2 & \tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix}$, then $\lim\limits_{x \to 0} \dfrac{f'(x)}{x}$ is ....

Assertion & Reason

(A) If both assertion and reason are correct and reason is the correct explanation of assertion.

(B) If both assertion and reason are true but reason is not the correct explanation of assertion.

(C) If assertion is true but reason is false.

(D) If assertion is false but reason is true.

52. **Assertion :** If $\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$, then

$\Delta'(x) \neq \begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$

Reason : $\frac{d}{dx}\{f(x)g(x)\} \neq \frac{d}{dx}f(x)\frac{d}{dx}g(x)$

(a) A (b) B

(c) C (d) D

53. **Assertion :** The system of equations possess a non trivial solution over the set of rationals $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$, then the value of k is 31/2,

Reason : For non trivial solution $\Delta = 0$.

(a) A (b) B

(c) C (d) D

54. Let $A(\theta) = \begin{pmatrix} \cos\theta + \sin\theta & \sqrt{2}\sin\theta \\ -\sqrt{2}\sin\theta & \cos\theta - \sin\theta \end{pmatrix}$

Assertion : $A(\pi/3)^3 = -I$

Reason : $A(\theta)A(\phi) = A(\theta + \phi)$

(a) A (b) B

(c) C (d) D

55. **Assertion :** $\begin{vmatrix} a^2+x^2 & ab-cx & ac+bx \\ ab+cx & b^2+x^2 & bc-ax \\ ac-bx & bc+ax & c^2+x^2 \end{vmatrix} = \begin{vmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{vmatrix}^2$

Reason : $\Delta^c = \Delta^{n-1}$ where n is order of determinant, and Δ^c is the determinant of cofactors of Δ.

(a) A (b) B

(c) C (d) D

56. **Assertion :** $f(x) = \begin{vmatrix} (1+x)^{21} & (1+x)^{22} & (1+c)^{23} \\ (1+x)^{31} & (1+x)^{32} & (1+x)^{33} \\ (1+x)^{41} & (1+x)^{42} & (1+x)^{43} \end{vmatrix}$, then coefficient of x in f (x) is zero.

Reason : If $F(x) = A_0 + A_1x + A_2x^2 + ... + A_nx^n$, then $A_1 = F'(0)$, when dash denotes the differential coefficient.

(a) A (b) B

(c) C (d) D

57. **Assertion :** $\begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ is independent of θ.

Reason : If $f(\theta) = c$, then $f(\theta)$ is independent of θ.

(a) A (b) B

(c) C (d) D

58. Suppose a, b, c are distinct real numbers.

Let $f(x) = \begin{vmatrix} x^3+a^3 & x^2-a^2 & x+a \\ x^3+b^3 & x^2-b^2 & x+b \\ x^3+c^3 & x^2-c^2 & x+c \end{vmatrix}$

Assertion : f (x) is a polynomial of degree 3.

Reason : a, b, c are zeroes of f(x).

(a) A (b) B

(c) C (d) D

59. Suppose $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfies the equation $X^2 - 4X + 3I = O$.

Assertion : If $a + d \neq 4$, then there are just two such matrix X.

Reason : There are infinite number of matrices X, satisfying $X^2 - 4X + 3I = O$.

(a) A (b) B

(c) C (d) D

60. Let $a_i, b_i, c_i \in N$ for i = 1, 2, 3 and let

$$\Delta = \begin{vmatrix} \frac{1+a_1^3b_1^3}{1+a_1b_1} & \frac{1+a_1^3b_2^3}{1+a_1b_2} & \frac{1+a_1^3b_3^3}{1+a_1b_3} \\ \frac{1+a_2^3b_1^3}{1+a_2b_1} & \frac{1+a_2^3b_2^3}{1+a_2b_2} & \frac{1+a_2^3b_3^3}{1+a_2b_3} \\ \frac{1+a_3^3b_1^3}{1+a_3b_1} & \frac{1+a_3^3b_2^3}{1+a_3b_2} & \frac{1+a_3^3b_3^3}{1+a_3b_3} \end{vmatrix}$$

Assertion : $\Delta = 0$

Reason : Δ can be written as product of two determinants.

(a) A (b) B

(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

61. Match the following List-I and List-II

Column–I	Column–II
(P) If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then the value of $|A^T A^{-1}|$ is	**(1)** 10
(Q) If x, y, z are cube root of unity and $D = \begin{vmatrix} x^2+y^2 & z^2 & z^2 \\ x^2 & y^2+z^2 & x^2 \\ y^2 & y^2 & z^2+x^2 \end{vmatrix}$ then real part of ID is	**(2)** 1
(R) If any triangle the area $A_1 \le \dfrac{b^2+c^2}{\lambda}$, then largest possible numerical quantity λ is	**(3)** 4
(S) The equation $x^4 - 4x + c$ has no real roots, then minimum integral value of c^2 can be	**(4)** 0

Codes :

	P	Q	R	S
(A)	2	4	1	3
(B)	1	2	3	4
(C)	4	3	2	1
(D)	2	4	3	1

62. Match the following List-I and List-II

Column–I	Column–II
(A) If 2 is the root of the equation $|A-xI|=0$, (where A is a non singular matrix), $\dfrac{|A|}{2}$ a root of $|B-xI|=0$, then B can be	**(P)** e
(B) If $e^{i\alpha}$ is the root of $|A-yI| = 0$ then a root of $|\bar{A}'\text{-}xI| = 0$ is (where A is a non singular matrix)	**(Q)** adj(A)
(C) Let A_{ij} be a 2×2 non singular matrix where $i, j \in N$ and $B = \begin{vmatrix} |A_{11}| & |A_{12}| & \ldots & |A_{1n}| \\ 0 & |A_{22}| & \ldots & |A_{2n}| \\ 0 & 0 & |A_{33}| \ldots & |A_{3n}| \\ \ldots & & & \\ \ldots & & & \\ 0 & \ldots & & |A_{nn}| \end{vmatrix}$ then $|B - \lambda I| = 0$ has root as	**(R)** $\cos\alpha - i\sin\alpha$
(D) Consider a matrix such that $\bar{A}'=A$ then the equation $|A - xI| = 0$ can have root as (where A is a non singular matrix)	**(S)** $|A_{11}|$

The correct matching is

(a) A–Q; B–R; C–S; D–P

(b) A–R; B–Q; C–S; D–P

(c) A–Q; B–S; C–R; D–P

(d) A–Q; B–R; C–P; D–S

Paragraph Type Questions

Use the following passage, solve Q. 63 to Q. 67

Passage

Let $\Delta = 0$ and Δ^c denotes the determinant of cofactors, then $\Delta^c = \Delta^{n-1}$, where n (> 0) is the order of Δ.

On the basis of above information, answer the following questions :

63. If a, b, c are the roots of the equation $x^3 - px^2 + r = 0$, then the value of

$$\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} \text{ is}$$

(a) p^2 (b) p^4

(c) p^6 (d) p^9

64. If l_1, m_1, n_1 ; l_2, m_2, n_2 ; l_3, m_3, n_3 are real quantities satisfying the six relations :

$$\ell_1^2 + m_1^2 + n_1^2 = \ell_2^2 + m_2^2 + n_2^2 = \ell_3^2 + m_3^2 + n_3^2 = 1$$

$$l_2 l_3 + m_2 m_3 + n_2 n_3 = l_3 l_1 + m_3 m_1 + n_3 n_1 =$$

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$, then the value of

$$\begin{vmatrix} \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \\ \ell_3 & m_3 & n_3 \end{vmatrix} \text{ is}$$

(a) 0 (b) ± 1

(c) ± 2 (d) ± 3

65. If a, b, c are the roots of the equation $x^3 - 3x^2 + 3x + 7 = 0$, then the value of

$$\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} \text{ is}$$

(a) 9 (b) 27

(c) 81 (d) 0

66. If $a^2 + b^2 + c^2 = \lambda^2$, then the value of

$$\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix} \times \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix} \text{ is}$$

(a) $8\lambda^6$ (b) $27\lambda^9$

(c) $8\lambda^9$ (d) $27\lambda^6$

67. Suppose a, b, c $\in$ R, $a + b + c > 0$, $A = bc - a^2$ $B = ca - b^2$ and $C = ab - c^2$ and

$$\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix} = 49, \text{ then } \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \text{ equals}$$

(a) –7 (b) 7

(c) –2401 (d) 2401

Use the following passage, solve Q. 68 to Q. 70

Passage

Elementary Transformation of a matrix :

The following operation on a matrix are called elementary operations (transformations)

1. The interchange of any two rows (or columns)
2. The multiplication of the elements of any row (or column) by any non zero number

   The addition to the elements of any row (or column) the corresponding elements of any other row (or column) the corresponding elements of any other row (or column) multiplied by any number

Echelon form of matrix :

A matrix A is said to be in echelon from if

(i) every row of A which has all its elements 0, occurs below row, which has a non-zero elements

(ii) The first non zero element in each non-zero row is 1.

(iii) the number of zeros before the first non zero elements in a row is less than the number of such zeroes in the next row.

[A row of matrix is said to be a zero row if all its elements are zero]

Note : Rank of a matrix does not change by application of any elementary operations.

For example $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are echelon forms

The number of non-zero rows in the echelon form of a matrix is defined as its RANK.

For example we can reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$ into echelon form using following elementary row transformation.

(i) $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(ii) $R_2 \to R_2 - 2R_1$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

This is the echelon form of matrix A

Number of non zero rows in the echelon form = 2

$\Rightarrow$ Rank of the matrix A is 2

68. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ is

(a) 1 (b) 2

(c) 3 (d) 0

69. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$ is

(a) 1 (b) 2

(c) 3 (d) 4

70. The echelon form of the matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 & 4 & -\frac{3}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \text{ in the interval } -\frac{\pi}{4} \le x \le \frac{\pi}{4} \text{ is}$$

(2001)

(a) 0 (b) 2

(c) 1 (d) 3

2. The number of values of k for which the system of equations

$(k+1)x + 8y = 4k$

$kx + (k+3)y = 3k - 1$

has infinitely many solution, is **(2002)**

(a) 0 (b) 1

(c) 2 (d) infinite

3. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which

$A^2 = B$, is **(2003)**

(a) 1 (b) –1

(c) 4 (d) no real values

4. If the system of equations $x + ay = 0$, $az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is

(2003)

(a) –1 (b) 1

(c) 0 (d) no real values

5. Given $2x - y - z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ then the vaue of λ such that the given system of equation has no solution, is **(2004)**

(a) 3 (b) – 2

(c) 0 (d) – 3

6. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$ then the value of α is

(2004)

(a) ± 1 (b) ± 2

(c) ± 3 (d) ± 5

7. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $6A^{-1} = A^2 + cA + dI$, then (c, d) is

(2005)

(a) (–11, 6) (b) (–6, 11)

(c) (6, 11) (d) (11, 6)

8. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then

$P^T (Q^{2005}) P$ is equal to **(2005)**

(a) $\begin{bmatrix} 1 & \frac{\sqrt{3}}{2} \\ 0 & 2005 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2005 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2005 \\ \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$

9. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is **(2010)**

(a) 0 (b) $2^9 - 1$

(c) 168 (d) 2

10. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b and c is either ω or ω^2. Then, the number of distinct matrices in the set S is **(2011)**

(a) 2 (b) 6

(c) 4 (d) 8

11. How many 3×3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of M^TM is 5 ? **(2017)**

(a) 126 (b) 198

(c) 162 (d) 135

12. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$, where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers and I is the 2×2 identity matrix.

If α^* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$ and β^* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$, then the value of $\alpha^* + \beta^*$ is **(2019)**

(a) $\frac{-37}{16}$ (b) $\frac{-29}{16}$

(c) $\frac{-31}{16}$ (d) $\frac{-17}{16}$

Objective Questions II

[One or more than one correct option]

13. Let M and N be two 3×3 non-singular skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to **(2011)**

(a) M^2 (b) $-N^2$

(c) $-M^2$ (d) MN

14. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is/are **(2012)**

(a) –2 (b) –1

(c) 1 (d) 2

15. For 3×3 matrices M and N, which of the following statement(s) is (are) not correct ? **(2013)**

(a) N^TMN is symmetric or skew-symmetric, according as M is symmetric or skew-symmetric

(b) MN – NM is skew symmetric for all symmetric matrices M and N

(c) M N is symmetric for all symmetric matrices M and N

(d) (adj M) (adj N) = adj (MN) for all invertible matrices M and N

16. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then **(2014)**

(a) determinant of $(M^2 + MN^2)$ is 0

(b) there is a 3 × 3 non-zero matrix U such that $(M^2 + MN^2)$ U is the zero matrix

(c) determinant of $(M^2 + MN^2) \geq 1$

(d) for a 3 × 3 matrix U, if $(M^2 + MN^2)$ U equals the zero matrix then U is the zero matrix

17. Let M be a 2 × 2 symmetric matrix with integer entries. Then M is invertible if **(2014)**

(a) the first column of M is the transpose of the second row of M

(b) the second row of M is the transpose of the first column of M

(c) M is a diagonal matrix with nonzero entries in the main diagonal

(d) the product of entries in the main diagonal of M is not the square of an integer

18. Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha\ ?$$ **(2015)**

(a) –4 (b) 9

(c) –9 (d) 4

19. Let X and Y be two arbitrary, 3 × 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 × 3, non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric ? **(2015)**

(a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$

(c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} - Y^{23}$

20. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and det $(Q) = \frac{k^2}{2}$, then **(2016)**

(a) $\alpha = 0, k = 8$ (b) $4\alpha - k + 8 = 0$

(c) $\det(P \operatorname{adj}(Q)) = 2^9$ (d) $\det(Q \operatorname{adj}(P)) = 2^{13}$

21. Which of the following is(are) **NOT** the square of a 3×3 matrix with real entries? **(2017)**

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

22. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in R$ and the system of equations (in real variables)

$-x + 2y + 5z = b_1$

$2x - 4y + 3z = b_2$

$x - 2y + 2z = b_3$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$? **(2018)**

(a) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(b) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(c) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(d) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

23. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj $M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ Where a and b are real numbers. Which of the following options is /are correct? **(2019)**

(a) det (adj M^2)=81

(b) $a + b = 3$

(c) If $M \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

(d) $(\operatorname{adj} M)^{-1} + \operatorname{adj} M^{-1} = -M$

24. Let $x \in R$ and let

$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$ and $R = PQP^{-1}$

Then which of the following is/are correct **(2019)**

(a) There exists a real number x such that PQ= QP

(b) $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$ for all $x \in R$

(c) For $x = 1$ there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which are $R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(d) For $x = 0$ if $R \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6 \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$ then $a + b = 5$

25. Let

$$P_1 = I = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}, P_2 = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}, P_3 = \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$$

$$P_4 = \begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}, P_5 = \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}, P_6 = \begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$$

and $X = \sum_{k=1}^{6} P_k \begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix} P_k^T$.

Where P_K^T denotes the transpose of matrix P_k. Then which of the following options is/are correct? **(2019)**

(a) X is a symmetric matrix

(b) The sum of diagonal entries of X is 18.

(c) if $X\begin{bmatrix}1\\1\\1\end{bmatrix} = \alpha\begin{bmatrix}1\\1\\1\end{bmatrix}$, then $\alpha=30$

(d) $X - 30I$ is an invertible matrix

26. Let M be a 3 × 3 invertible matrix with real entries and let I denote the 3 × 3 identity matrix. If M^{-1} = adj (adj M), then which of the following statement is/are ALWAYS TRUE ? **(2020)**

(a) M = I (b) det M = 1

(c) $M^2 = I$ (d) $(\text{adj } M)^2 = I$

27. For any 3 × 3 matrix M, let |M| denote the determinant of M.

Let $E = \begin{bmatrix}1&2&3\\2&3&4\\8&13&18\end{bmatrix}, P = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$

and $F = \begin{bmatrix}1&3&2\\8&18&13\\2&4&3\end{bmatrix}$

If Q is a non-singular matrix of order 3 × 3, then which of the following statements is(are) TRUE? **(2021)**

(a) $F = PEP$ and $P^2 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

(b) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

(c) $|(EF)^3| > |EF|^2$

(d) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

28. For any 3 × 3 matrix M, let |M| denote the determinant of M. Let I be the 3 × 3 identify matrix. Let E and F be two 3 × 3 matrices such that (I – EF) is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) TRUE? **(2021)**

(a) $|FE| = |I - FE||FGE|$

(b) $(I - FE)(I + FGE) = I$

(c) $EFG = GEF$

(d) $(I - FE)(I - FGE) = I$

Numerical Value Type Questions

29. If matrix $A = \begin{bmatrix}a&b&c\\b&c&a\\c&a&b\end{bmatrix}$ where a, b, c are real positive numbers, abc = 1 and $A^T A = I$, then find value of $a^3 + b^3 + c^3$. **(2003)**

30. Let k be a positive real number and let

$$A = \begin{bmatrix}2k-1&2\sqrt{k}&2\sqrt{k}\\2\sqrt{k}&1&-2k\\-2\sqrt{k}&2k&-1\end{bmatrix} \text{ and } B = \begin{bmatrix}0&2k-1&\sqrt{k}\\1-2k&0&2\sqrt{k}\\-\sqrt{k}&-2\sqrt{k}&0\end{bmatrix}$$

If det (adj A) + det (adj B) = 10^6, then [k] is equal to.... **(2010)**

31. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$(y+z)\cos 3\theta = (xyz)\sin 3\theta$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

and $(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$ have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is ..... **(2010)**

32. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex number z satisfying

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$ is equal to..... **(2010)**

33. Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \text{and } M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

Then, the sum of the diagonal entries of M is..... **(2011)**

34. Let $z = \frac{-1+\sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1, 2, 3\}$.

Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is **(2016)**

35. The total number of distinct $x \in R$ for which

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$$ is **(2016)**

36. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ -z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$ of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$ **(2017)**

37. The trace of a square matrix is defined to be the sum of its diagonal entries. If A is a 2 × 2 matrix such that the trace of A is 3 and the trace of A^3 is -18, then the value of the determinant of A is _____ **(2020)**

38. Let α, β and γ be real numbers such that the system of linear equations

$x + 2y + 3z = \alpha$

$4x + 5y + 6z = \beta$

$7x + 8y + 9z = \gamma - 1$

is consistent. Let |M| represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

The value of |M| is ____. **(2021)**

Assertion & Reason

(A) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.

(B) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.

(C) If ASSERTION is true, REASON is false

(D) If ASSERTION is false, REASON is true

39. Consider the system of equations

$x - 2y + 3z = -1, -x + y - 2z = k, x - 3y + 4z = 1$

Assertion : The system of equations has no solution for $k \neq 3$.

Reason : The determinant. $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$

(1997)

(a) A (b) B

(c) C (d) D

Paragraph Type Questions

Using the following passage, solve Q.40 to Q.42

Passage

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, If U_1, U_2 are U_3 are columns matrices

satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and U is

3 × 3 matrix when columns are U_1, U_2, U_3 then answer the following questions.

40. The value of |U| is **(2006)**

(a) 3 (b) – 3

(c) 3/2 (d) 2

41. The sum of the elements of U^{-1} is **(2006)**

(a) – 1 (b) 0

(c) 1 (d) 3

42. The value of $[3\ 2\ 0]\ U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is **(2006)**

(a) 5 (b) 5/2

(c) 4 (d) 3/2

Using the following passage, solve Q.43 to Q.45

Passage

Let p be an odd prime number and T_p be the following set of 2×2 matrices

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}; a, b, c \in \{0, 1, 2, \dots p-1\} \right\}$$ **(2010)**

43. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det (A) is divisible by p is

(a) $(p-1)^2$ (b) $2(p-1)$

(c) $(p-1)^2+1$ (d) $2p-1$

44. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is

[Note : The trace of a matrix is the sum of its diagonal entries]

(a) $(p-1)(p^2-p+1)$ (b) $p^3-(p-1)^2$

(c) $(p-1)^2$ (d) $(p-1)(p^2-2)$

45. The number of A in T_p such that det (A) is not divisible by p, is

(a) $2p^2$ (b) p^3-5p

(c) p^3-3p (d) p^3-p^2

Answer Key

CHAPTER -1 | DETERMINANTS & MATRICES

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (d)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (a)
6. (a)	**7.** (d)	**8.** (b)	**9.** (d)	**10.** (b)
11. (d)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (b)
16. (a)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (b)
21. (b)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (a)
26. (b)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (b)
31. (d)	**32.** (d)	**33.** (b)	**34.** (c)	**35.** (d)
36. (b)	**37.** (d)	**38.** (a)	**39.** (c)	**40.** (b)
41. (c)	**42.** (c)	**43.** (c)	**44.** (d)	**45.** (a)
46. (a)	**47.** (c)	**48.** (b)	**49.** (b)	**50.** (c)
51. (c)	**52.** (a)	**53.** (b)	**54.** (a)	**55.** (d)
56. (b)	**57.** (b)	**58.** (b)	**59.** (a)	**60.** (b)
61. (1)	**62.** (2)	**63.** (0)	**64.** (0)	**65.** (4)
66. (6)	**67.** (0)	**68.** (3)	**69.** (3)	**70.** (4)
71. (16)	**72.** (2)	**73.** (1)	**74.** (2)	**75.** (2)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. (c)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (c)
6. (d)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (d)
11. (b)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (d)
16. (d)	**17.** (d)	**18.** (d)	**19.** (a)	**20.** (b)
21. (c)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (b)
26. (b)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (d)
31. (b)	**32.** (b)	**33.** (c)	**34.** (a)	**35.** (d)
36. (c)	**37.** (c)	**38.** (a)	**39.** (8.00)	**40.** (b)
41. (a)	**42.** (b)	**43.** (b)	**44.** (d)	**45.** (c)
46. (a)	**47.** (d)	**48.** (a)	**49.** (d)	**50.** (d)
51. (c)	**52.** (d)	**53.** (c)	**54.** (10.00)	**55.** (c)
56. (8.00)	**57.** (d)	**58.** (5.00)	**59.** (a)	**60.** (b)
61. (b)	**62.** (b)	**63.** (a)	**64.** (c)	**65.** (b)
66. (a)	**67.** (b)	**68.** (3.00)	**69.** (c)	**70.** (c)
71. (a)	**72.** (13.00)	**73.** (d)	**74.** (672.00)	
75. (b)	**76.** (a)	**77.** (b)	**78.** (c)	**79.** (c)
80. (b)	**81.** (b)	**82.** (d)	**83.** (1.00)	
84. (910.00)		**85.** (d)	**86.** (108.00)	
87. (11.00)	**88.** (d)	**89.** (6.00)	**90.** (5.00)	**91.** (b)
92. (a)	**93.** (2020.00)		**94.** (3125.00)	
95. (b)	**96.** (d)	**97.** (a)	**98.** (b)	**99.** (c)
100. (b)	**101.** (a)	**102.** (4.00)	**103.** (c)	**104.** (d)
105. (5.00)	**106.** (d)	**107.** (c)	**108.** (d)	**109.** (a)
110. (c)	**111.** (8.00)	**112.** (d)	**113.** (c)	**114.** (c)
115. (4.00)	**116.** (b)	**117.** (c)	**118.** (13.00)	**119.** (7.00)
120. (21.00)		**121.** (b)	**122.** (c)	**123.** (c)
124. (a)	**125.** (d)	**126.** (d)	**127.** (540.00)	
128. (17.00)		**129.** (a)	**130.** (c)	**131.** (c)
132. (d)	**133.** (b)	**134.** (6.00)	**135.** (d)	
136. (766.00)		**137.** (36.00)		**138.** (a)
139. (1.00)	**140.** (b)	**141.** (a)	**142.** (16.00)	**143.** (d)
144. (2020.00)		**145.** (2.00)		

ANSWER KEY

CHAPTER -1 | DETERMINANTS & MATRICES

EXERCISE - 3 :
ADVANCED OBJECTIVE QUESTIONS

1. (b)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (d)
6. (b)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (a)
11. (b)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (a)
16. (b)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (d)
21. (a)	**22.** (a)	**23.** (d)	**24.** (c)	**25.** (d)
26. (d)	**27.** (a,b,c,d)		**28.** (a,c)	
29. (a,b,c,d)		**30.** (a,b,c)	**31.** (b,d)	**32.** (a,d)
33. (a,c)	**34.** (a,b,c,d)		**35.** (b,c,d)	
36. (a,b,c)	**37.** (a,b,c)	**38.** (a,b,d)	**39.** (a,b,c)	
40. (a,b,c)	**41.** (a,b,c)	**42.** (a,b,c)	**43.** (a,b)	**44.** (a)
45. (c)	**46.** (b,d)	**47.** (b,c)	**48.** (81)	**49.** (0)
50. (3)	**51.** (4)	**52.** (a)	**53.** (d)	**54.** (a)
55. (a)	**56.** (b)	**57.** (b)	**58.** (c)	**59.** (b)
60. (d)	**61.** (d)	**62.** (a)	**63.** (c)	**64.** (b)
65. (d)	**66.** (c)	**67.** (a)	**68.** (b)	**69.** (c)
70. (a)				

EXERCISE - 4 :
PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (c)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)
6. (c)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (a)
11. (b)	**12.** (b)	**13.** (c)	**14.** (a,d)	**15.** (c,d)
16. (a,b)	**17.** (c,d)	**18.** (b,c)	**19.** (c,d)	**20.** (b,c)
21. (a,b)	**22.** (a,c,d)	**23.** (b,c,d)	**24.** (b,d)	
25. (a,b,c)	**26.** (b,c,d)		**27.** (a,b,d)	
28. (a,b,c)	**29.** (4)	**30.** (4)	**31.** (3)	**32.** (1)
33. (9)	**34.** (1)	**35.** (2)	**36.** (1)	
37. (5.00)	**38.** (1.00)	**39.** (a)	**40.** (a)	
41. (b)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (d)