CHAPTER 1

Electric Charges and Fields

Coulomb's law, electrostatic field and electric dipole

Electric Charge

- Electrostatic charge is a fundamental property of matter due to which it produces and experiences electrical and magnetic effects.
- Properties of atoms, molecules and bulk matter are determined by electric and magnetic forces.
- It can be inferred from simple experiments based on frictional electricity that there are two type of charges in nature: negative and positive; and like charges repel and unlike charges attract.
- By convention, the charge on electron is considered as negative and the charge on proton is considered as positive and the charge present is equal. The S.I. unit of electric charge is coulomb. Its C.G.S unit is stat coulomb.
- The nature and amount of electric charge present in a charged body is detected by Gold-leaf electroscope.
- Total charge on a body is expressed as $q = \pm ne$.

Conductors and Insulators

- Objects that allow charges to flow through them are called Conductors (metals) and objects that do not allow charges to flow through are called Insulators (rubber, wood, and plastic).
- Objects that behave as an intermediate between conductors and insulators are called semi-conductors, for example- silicon.
- The process of sharing charges with the earth, when we bring a charged body in contact with the earth is called grounding or earthing.

Charging by Induction

- Charging by induction means charging without contact.
- If a plastic comb is rubbed with wool, it becomes negatively charged.

Three basic properties of electric charge

- **Quantization:** When the total charge of a body is an integral multiple of a basic quantum of charge, this is known as quantization of electric charge. i.e., $q = ne$ where
  $n = \pm1, \pm2, \pm3, ...$
- **Additivity:** It means that the total charge of a system is the algebraic sum (adding taking into account negative and positive signs both) of all the charges in the system.
- **Conservation of charge:** Conservation of electric charges means that there will be no change in the total charge of the isolated system with time. There is transfer of the electric charge from one body to another, but no charge will be created or destroyed.

Coulomb's law

The force between two point charges q_1 and q_2 is directly proportional to the product of the two charges(q_1q_2) and inversely proportional to the square of the distance between them$(r^2)$and it acts along the straight line joining the two charges.

F_{21} = force on q_2 due to q_1 $= \dfrac{k(q_1q_2)}{r^2_{21}}\hat{r}_{21}$

where $k = \dfrac{1}{4\pi\varepsilon_0}$

The experimental value of the constant ε_0 is

$8.854 \times 10^{-12} C^2N^{-1}m^{-2}$

Therefore, the approximate value of k is

$9 \times 10^9 Nm^2C^{-2}$

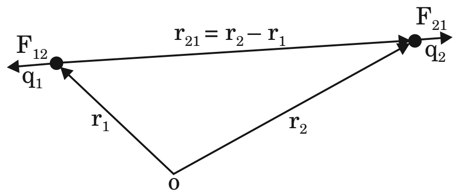


Fig. Depiction of Coulomb's law

Facts about Coulomb's law:

- Coulomb's law is not valid for charges in motion; it should only be used for point charges in vacuum at rest.
- The electrostatic force obeys Newton's third law of motion and acts along the line joining the two charges.
- Presence of other charges in the neighborhood does not affect Coulomb's force.
- The ratio of electric force and gravitational force between a proton and an electron is represented by $\frac{k e^2}{G m_e m_p} \cong 2.4 \times 10^{39}$

Superposition Principle

The presence of an (or more) additional charge does not affect the forces with which two charges attract or repel each other. Superposition principle states that the net force on any charge due to n number of charges at rest is the vector sum of all the forces on that charges, taken one at a time.

i.e. $\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + ..\vec{F}_{0n}$

- The force on a small positive test charge q placed at the point divided by the magnitude of the charge is the electric field E at a point due to charge configuration.

Electric Field

- The space around a charge up to which its force can be experienced is called electric field.
- Electric field due to a point charge q has a magnitude $E(r) = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$
  - ➢ It is radially outwards if q is positive.
  - ➢ It is radially inwards if q is negative.
- Electric field satisfies the superposition principle.
  - ➢ The unit of electric field is N/C.
  - ➢ Electric field inside the cavity of a charged conductor is zero.

Electric Field lines

- The tangent at each point on the curve of electric field line, gives the direction of electric field at that point.
- The relative strength of electric field at different points is indicated by the relative closeness of field lines.
  - ➢ In regions of strong electric field, they crowd near each other.
  - ➢ In regions of weak electric field, they are far apart.
  - ➢ In regions of constant electric field, the field lines formed are uniformly spaced parallel straight lines.
- Field lines are continuous curves. There will be no breaks.

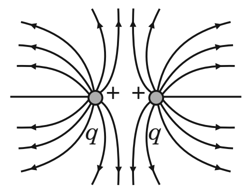


Fig. Electric field lines

- Field lines are not intersecting. They cannot cross each other.
- Electrostatic field lines begin at positive charges and terminate at negative charges.
- No closed loop can be formed by them.

Electric Dipole

- A pair of equal and opposite charges q and –q separated by small distance 2a is known as electric dipole. The magnitude of its dipole moment vector is 2qa and is in the direction of the dipole axis from –q to q.

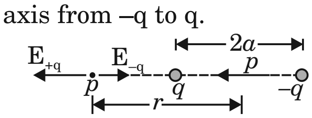


Fig. Electric dipole

- Field of an electric dipole in its equatorial plane at a distance r from the center:

$$E = \frac{-p}{4\pi\varepsilon_o}\frac{1}{\left(a^2 + r^2\right)^{3/2}}$$

$$\cong \frac{-p}{4\pi\varepsilon_o r^3} \qquad \text{for } r >> a$$

- Dipole electric field on the axis at a distance r from the center:

$$E = \frac{2pr}{4\pi\varepsilon_o\left(r^2 - a^2\right)^2}$$

$$\cong \frac{2p}{4\pi\varepsilon_o r^3} \qquad \text{for } r >> a$$

The $1/r^3$ dependence of dipole electric fields should be noted in contrast to the $1/r^2$ dependence of electric field due to a point charges.

- In a uniform electric field E, a dipole experiences a torque τ given by
  $\tau = p \times E$

But no net force will be experienced by it.

Electric Flux

- Electric flux is proportional to number of lines leaving a surface, outgoing lines with positive sign, incoming lines with negative sign.

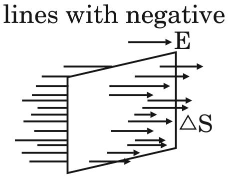


Fig. Electric flux

- Through a small area element ΔS, the flux Δϕ of electric field E is given by

  $$\Delta\phi = \vec{E}\,.\,\vec{\Delta S}$$

  And the vector area element ΔS is

  $$\vec{\Delta S} = \Delta S\hat{n}$$

Where ΔS is the magnitude of the area element and $\hat{n}$ is normal to the area element, which can be considered planar for the sufficiently small ΔS.

Gauss's Law and its application

- The flux of electric field through any closed surface S is $1/\varepsilon_0$ times the total charge enclosed by S.

  $$\phi = E\int dA = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

- The law is mainly useful in determining electric field E, when the source distribution has simple symmetry:
  - ➢ Thin infinitely long straight wire of uniform linear charge density λ

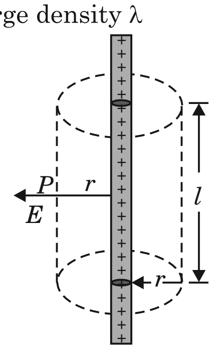


Fig. Thin infinitely long Straight wire

$$E = \frac{\lambda}{2\pi\varepsilon_o r}\hat{n}$$

Where, r is the radial (perpendicular) distance of the point from the wire and $\hat{n}$ is the radial unit vector in the plane normal to the wire passing through the point.

- Infinite plane sheet (thin) of uniform surface charge density σ

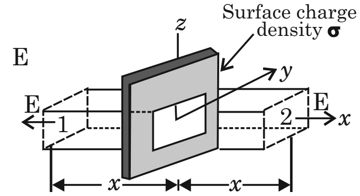


Fig. Infinite plane sheet (thin)

$$E = \frac{\sigma}{2\varepsilon_o}\hat{n}$$

Where $\hat{n}$ is a unit vector normal to the plane and going away from it.

- Thin spherical shell of uniform surface charge density σ

  $$E = \frac{q}{4\pi\varepsilon_o r^2}\hat{r} \qquad (r \geq R)$$

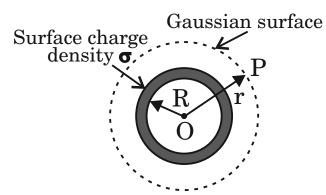


Fig.: Thin uniformly surface charged spherical shell (r > R)

(For r > R)

E = 0 (r < R)

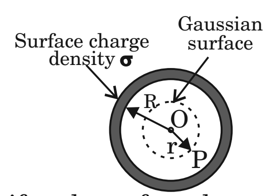


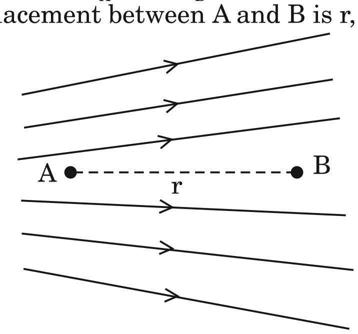
Fig.: Thin uniformly surface charged spherical shell (r < R)

(For r < R)

Where r is the distance of the point from the center of the shell whose radius is R with the total charge q. The electric field outside the shell is the same as the total charge is concentrated at the center. A solid sphere of uniform volume charge density shows the same result. Inside the shell at all the points, the field is zero.

Exercise

1. When the distance between the charged particles is halved, the force between them becomes.
   (*a*) One-fourth (*b*) Half
   (*c*) Double (*d*) Four times
2. A charge q_1 exerts some force on a second charge q_2. If third charge q_3 is brought near, the force of q_1 exerted on q_2.
   (*a*) Decreases
   (*b*) Increases
   (*c*) Remains unchanged
   (*d*) Increases if q_3 is of the same signs as q_1 and decreases if q_3 is of opposite sign
3. The minimum charge on an object is
   (*a*) 1 coulomb
   (*b*) 1 stat coulomb
   (*c*) 1.6×10^{-19} coulomb
   (*d*) 3.2×10^{-19} coulomb
4. Three charges 4q, Q and q are in a straight line in the position of 0, l/2 and l respectively. The resultant force on q will be zero, if Q =
   (*a*) –q (*b*) –2q
   (*c*) $-\frac{q}{2}$ (*d*) 4q
5. The number of electrons in 1.6 C charge will be
   (*a*) 10^{19} (*b*) 10^{20}
   (*c*) 1.1×10^{19} (*d*) 1.1×10^{2}
6. The electric charge in uniform motion produces
   (*a*) An electric field only
   (*b*) A magnetic field only
   (*c*) Both electric and magnetic field
   (*d*) Neither electric nor magnetic field
7. Figure shows the electric lines of force emerging from a charged body. If the electric field at A and B are E_A and E_B respectively and if the displacement between A and B is r, then



   (*a*) $E_A > E_B$ (*b*) $E_A < E_B$
   (*c*) $E_A = \frac{E_B}{r}$ (*d*) $E_A = \frac{E_B}{r^2}$
8. The electric field near a conducting surface having a uniform surface charge density σ is given by
   (*a*) $\frac{\sigma}{\varepsilon_0}$ and is parallel to the surface
   (*b*) $\frac{2\sigma}{\varepsilon_0}$ and is parallel to the surface
   (*c*) $\frac{\sigma}{\varepsilon_0}$ and is normal to the surface
   (*d*) $\frac{2\sigma}{\varepsilon_0}$ and is normal to the surface
9. Deutron and α-particle are put 1 Å apart in air. Magnitude of intensity of electric field due to deutron at α-particle is
   (*a*) zero (*b*) 2.88×10^{11} N/C
   (*c*) 1.44×10^{11} N/C (*d*) 5.76×10^{11} N/C
10. An electric dipole when placed in a uniform electric field E will have minimum potential energy, if the positive direction of dipole moment makes the following angle with E
    (*a*) π (*b*) $\frac{\pi}{2}$
    (*d*) zero (*d*) $\frac{3\pi}{2}$
11. The electric potential at a point on the axis of an electric dipole depends on the distance r of the point from the dipole as
    (*a*) $\propto \frac{1}{r}$ (*b*) $\propto \frac{1}{r^2}$
    (*c*) μr (*d*) $\propto \frac{1}{r^3}$
12. An electric dipole is kept in non-uniform electric field. It experiences
    (*a*) A force and a torque
    (*b*) A force but not a torque
    (*c*) A torque but not a force
    (*d*) Neither a force nor a torque
13. The distance between the two charges +q and –q of a dipole is r. On the axial line at a distance d from the centre of dipole, the intensity is proportional to
    (*a*) $\frac{q}{d^2}$ (*b*) $\frac{qr}{d^2}$
    (*c*) $\frac{q}{d^3}$ (*d*) $\frac{qr}{d^3}$

14. The electric field due to an electric dipole at a distance r from its centre in axial position is E. If the dipole is rotated through an angle of 90° about its perpendicular axis, the electric field at the same point will be

(a) E (b) $\frac{E}{4}$

(c) $\frac{E}{2}$ (d) 2E

15. An electric dipole of moment $\vec{\rho}$ placed in a uniform electric field $\vec{E}$ has minimum potential energy when the angle between $\vec{\rho}$ and $\vec{E}$ is

(a) Zero (b) $\frac{\pi}{2}$

(c) π (d) $\frac{3\pi}{2}$

16. A Cylinder of radius R and length L is placed in a uniform electric field E parallel to the cylinder axis. The total flux for the surface of the cylinder is given by

(a) $2\pi R^2 E$ (b) $\frac{\pi R^2}{E}$

(c) $\frac{\left(\frac{\pi R^2}{\pi R}\right)}{E}$ (d) zero

17. An electric charge q is placed at the centre of a cube of side a. The electric flux on one of its faces will be

(a) $\frac{q}{6\varepsilon_0}$ (b) $\frac{q}{\varepsilon_0 a^2}$

(c) $\frac{q}{4\pi\varepsilon_0 a^2}$ (d) $\frac{q}{\varepsilon_0}$

18. Total electric flux coming out of a unit positive charge put in air is

(a) ε_o (b) ε_o^{-1}

(c) $(4\rho\varepsilon_o)^{-1}$ (d) $4\pi\varepsilon_o$

19. For a given surface the Gauss's law is stated as $\oint E \cdot ds = 0$. From this we can conclude that

(a) E is necessarily zero on the surface

(b) E is perpendicular to the surface at every point

(c) The total flux through the surface is zero.

(d) The flux is only going out of the surface

20. A cube of side ℓ is placed in a uniform field E, where $E = E\hat{i}$. The net electric flux through the cube is

(a) zero

(b) $\ell^2 E$

(c) $4\ \ell^2 E$

(d) $6\ \ell^2 E$

21. A charge q is placed at the centre of the open end of cylindrical vessel. The flux of the electric field through the surface of the vessel is

(a) zero (b) $\frac{q}{\varepsilon_o}$

(c) $\frac{q}{2\varepsilon_o}$ (d) $\frac{2q}{\varepsilon_o}$

22. According to Gauss's Theorem, electric field of an infinitely long straight wire is proportional to

(a) r (b) $\frac{1}{r^2}$

(c) $\frac{1}{r^3}$ (d) $\frac{1}{r}$

23. The S.I. unit of electric flux is

(a) Weber (b) Newton per coulomb

(c) Volt × meter (d) Joule per coulomb

24. Gauss's law is true only if force due to a charge varies as

(a) r^{-1} (b) r^{-2}

(c) r^{-3} (d) r^{-4}

25. An electric dipole is put in north-south direction in a sphere filled with water. Which statement is correct

(a) Electric flux is coming towards sphere

(b) Electric flux is coming out of sphere

(c) Electric flux entering into sphere and leaving the sphere are same

(d) Water does not permit electric flux to enter into sphere.

Answer Keys

1. (*d*)	**2.** (*c*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)	**6.** (*c*)	**7.** (*a*)	**8.** (*c*)	**9.** (*c*)	**10.** (*c*)
11. (*d*)	**12.** (*a*)	**13.** (*d*)	**14.** (*c*)	**15.** (*a*)	**16.** (*d*)	**17.** (*a*)	**18.** (*b*)	**19.** (*c*)	**20.** (*a*)
21. (*c*)	**22.** (*d*)	**23.** (*c*)	**24.** (*b*)	**25.** (*c*)					

Solutions

1. $\because f \propto \frac{1}{r^2}$

$\therefore$ when r is halved the force becomes four times.

2. The force will still remain unchanged.

$\therefore F = \frac{q_1 q_2}{4\pi\,\varepsilon_0 r^2}$

3. All other charges are its integral multiple.

$\therefore$ Minimum charge on an object = 1.6×10^{-19} coulomb

4. The force between 4q and q.

$F_1 = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{4q \times q}{l^2}$

The force between Q and q

$F_2 = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{l}{2}\right)^2}$

$\therefore F_1 + F_2 = 0$ or $\frac{4q^2}{l^2} = -\frac{4Qq}{l^2} \Rightarrow Q = -q$

5. $n = \frac{q}{e} = \frac{1.6}{1.6 \times 10^{-19}} = 10^{19}$

6. A movable charge produces electric field and magnetic field both.

7. In non-uniform electric field. Intensity is more, where the lines are more denser.

8. Electric field near the conductor surface is given by $\frac{\sigma}{\varepsilon_0}$ and it is perpendicular to surface.

9. Due to deutron, intensity of electric field at 1 Å distance

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e}{r^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{10^{-20}}$

$= 1.44 \times 10^{11}$ N/C

10. Potential energy = $-pE\cos\theta$.

when $\theta = 0$,

Potential energy = $-pE$ (minimum)

11. Electric potential due to dipole in it's general position is given by $v = \frac{k.p\cos\theta}{r^2} \Rightarrow v \alpha \frac{1}{r^2}$

12. As the dipole will feel two forces which are although opposite but not equal.

$\therefore$ A net force will be there and as these forces act at different points of a body. A torque is also parent.

13. Field along the axis of the dipole

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{d^3} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2(q \times r)}{d^3}$

$\therefore\; E \propto \frac{qr}{d^3}$

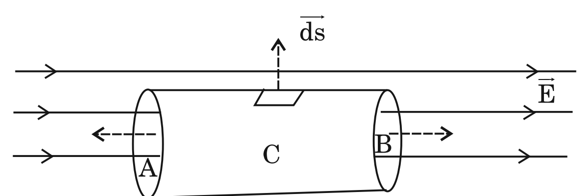
14. When the dipole is rotated through at an angle of 90° about it's perpendicular axis then given point comes out to be on equator. So field will become $\frac{E}{2}$ at the given point.

15. $U = -PE\cos\theta$.

It has minimum value when $\theta = 0°$.

i.e. $U_{min} = -PE\cos 0° = -PE$

16. Flux through surface A $\phi_A = E \times \pi R^2$ and $\phi_B = E \times \pi R^2$.



Flux through curved surface

$C = \int \vec{E} \cdot \vec{ds} = \int E\, ds \cos 90° = 0$

$\therefore$ Total flux through cylinder = $\phi_A + \phi_B + \phi_C = 0$

17. By Gauss's theorem,

Electric flux $(\phi) = \frac{q}{6\varepsilon_o}$

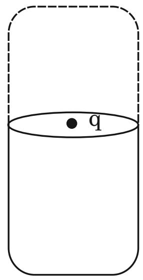
18. Total flux coming out from unit charge

$\phi = \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_o} \times 1 = \varepsilon_o^{-1}$

19. The total flux through the surface is zero.

20. As there is no charge residing inside the cube, hence net flux is zero.

21. To apply Gauss's theorem it is essential that charge should be placed inside a closed surface. So, imagine another similar cylindrical vessel above it as shown in figure (dotted).



$\therefore$ Required flux $\phi = \dfrac{q}{2\varepsilon_o}$

22. $E = \dfrac{\lambda}{2\pi\varepsilon_o r} \Rightarrow E \propto \dfrac{1}{r}$

23. S.I. unit of electric flux is

$$\frac{N \times m^2}{C} = \frac{J \times m}{C} = \text{volt} \times \text{metre}$$

24. Gauss's law is true only if force due to a charge varies as r^{-2}.

25. In electric dipole the flux coming out from positive charge is equal to the flux coming in at negative charge i.e. total charge on sphere = 0. From Gauss law, total flux passing through the sphere = 0.