6 Work, Energy and Power

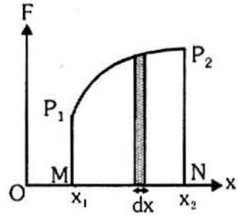
- **Work done** $W = \int dW = \int \vec{F}.d\vec{r} = \int Fdr\cos\theta$

  [where θ is the angle between $\vec{F}$ & $\vec{dr}$]

  - For constant force $W = \vec{F}.\vec{d} = Fd\cos\theta$
  - For Unidirectional force

    $W = \int dW = \int Fdx =$ Area between F–x curve and x-axis.

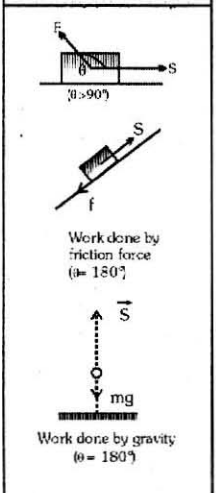
- **Calculation of work done from force–displacement graph :**



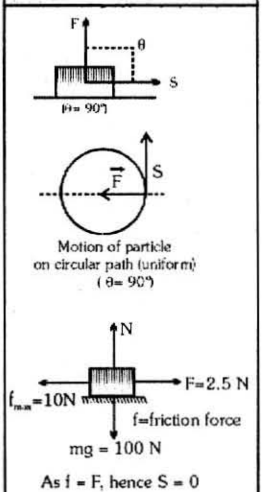
Total work done, $W = \sum_{x_1}^{x_2} dW = \sum_{x_1}^{x_2} Fdx =$ Area of $P_1P_2NM = \int_{x_1}^{x_2} Fdx$

- **Nature of work done** : Although work done is a scalar quantity, yet its value may be positive, negative or even zero

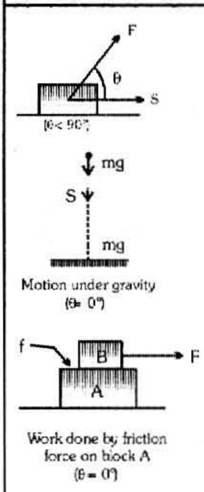
Negative work



Zero work



Positive work



Conservative Forces

- Work done does not depend upon path.
- Work done in a round trip is zero.
- Central forces, spring forces etc. are conservative forces
- When only a conservative force acts within a system, the kinetic energy and potential energy can change into each other. However, their sum, the mechanical energy of the system, doesn't change.
- Work done is completely recoverable.
- If $\vec{F}$ is a conservative force then $\vec{\nabla}\times\vec{F}=\vec{0}$ (i.e. curl of $\vec{F}$ is zero)

Non-conservative Forces

- Work done depends upon path.
- Work done in a round trip is not zero.
- Force are velocity-dependent & retarding in nature e.g. friction, viscous force etc.
- Work done against a non-conservative force may be dissipated as heat energy.
- Work done is not recoverable.

Kinetic energy

- The energy possessed by a body by virtue of its motion is called kinetic energy.

$$K=\frac{1}{2}mv^2=\frac{1}{2}m(\vec{v}.\vec{v})$$

- Kinetic energy is a frame dependent quantity because velocity is a frame depends.

Potential energy

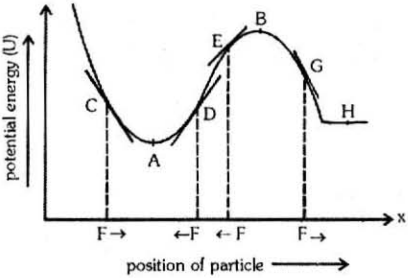
- The energy which a body has by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Relationship between conservative force field and potential energy :

$$\vec{F}=-\nabla U=-\text{grad}(U)=-\frac{\partial U}{\partial x}\hat{i}-\frac{\partial U}{\partial y}\hat{j}-\frac{\partial U}{\partial z}\hat{k}$$

- If force varies only with one dimension (along x-axis) then

$$F=-\frac{dU}{dx}\Rightarrow U=-\int_{x_1}^{x_2}Fdx$$

- **Potential energy curve and equilibrium**



It is a curve which shows change in potential energy with postion of a particle.

- **Stable Equilibrium :**

When a particle is slightly displaced from equilibrium position and it tends to come back towards equilibrium then it is said to be in stable equilibrium

At point **C** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **D** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **A** : It is the point of stable equilibrium.

$$U = U_{min}\ ,\ \frac{dU}{dx} = 0 \quad \text{and} \quad \frac{d^2U}{dx^2} = \text{positive}$$

- **Unstable equilibrium :**

When a particle is slightly displaced from equilibrium and it tends to move away from equilibrium position then it is said to be in unstable equilibrium

At point **E** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **G** : slope $\frac{dU}{dx}$ is negative so F is postive

At point **B** : It is the point of unstable equilibrium.

$$U=U_{max},\ \frac{dU}{dx} = 0 \text{ and } \frac{d^2U}{dx^2} = \text{negative}$$

- **Neutral equilibrium :**

  When a particle is slightly displaced from equilibrium position and no force acts on it then equilbirum is said to be neutral equilibrium. Point H is at neutral equilibrium $\Rightarrow U = \text{constant}$; $\frac{dU}{dx}=0,\ \frac{d^2U}{dx^2}=0$

- **Work energy theorem** : $W = \Delta KE$

  Change in kinetic energy = work done by all force

- **For conservative force** $F(x) = -\frac{dU}{dx}$

  change in potential energy $\Delta U = -\int F(x)dx$

- **Law of conservation of Mechanical energy**

  Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles or the work done by all other forces is zero. From work energy theorem $W = \Delta KE$

  Proof : For internal conservative forces $W_{int} = -\Delta U$

  So $W = W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = \Delta KE$

  $\Rightarrow \Delta(KE + U) = 0 \Rightarrow KE + U = \text{constant}$

- Spring force $F=-kx$, Elastic potential energy stored in spring $U(x) = \frac{1}{2}kx^2$

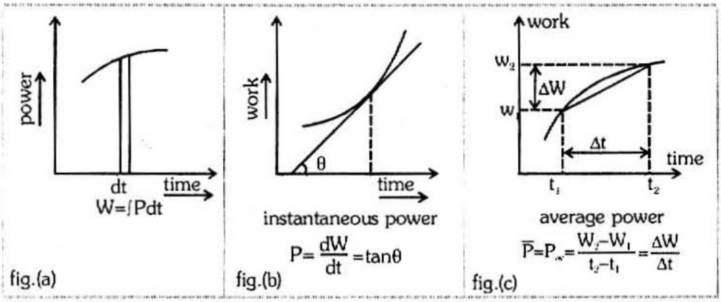
- Mass and energy are equivalent and are related by $E = mc^2$

- **Power**

  - Power is a scalar quantity with dimension $M^1L^2T^{-3}$
  - SI unit of power is J/s or watt
  - 1 horsepower = 746 watt = 550 ft–lb/sec.

- **Average power** $P_{av} = W/t$

- **Instantaneous power** $P = \dfrac{dW}{dt} = \dfrac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$



- For a system of varying mass $\vec{F} = \dfrac{d}{dt}(m\vec{v}) = m\dfrac{d\vec{v}}{dt} + \vec{v}\dfrac{dm}{dt}$
- If $\vec{v}$ = constant then $\vec{F} = \vec{v}\dfrac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\dfrac{dm}{dt}$
- In rotatory motion : $P = \tau\dfrac{d\theta}{dt} = \tau\omega$

KEY POINTS

- A body may gain kinetic energy and potential energy simultaneously because principle of conservation of mechanical energy may not be valid every time.
- Comets move around the sun in elliptical orbits. The gravitational force on the comet due to sun is not normal to the comet's velocity but the work done by the gravitational force is zero in complete round trip because gravitational force is a conservative force.
- Work done by static friction may be positive because static friction may acts along the direction of motion of an object.