Chapter 4

Principle of Mathematical Induction

Solutions (Set-1)

Very Short Answer Type Questions :

1. Let $P(n)$ denotes the statement "$2^n \geq n!$. Show that $P(1)$, $P(2)$ and $P(3)$ are true but $P(4)$ is not true.

Sol. Here given that $P(n): 2^n > n!$

$\Rightarrow \quad P(1) = 2^1 > 1!$

$\Rightarrow \quad 2 > 1$

$\therefore \quad P(1)$ is true.

$P(2) = 2^2 > 2!$

$\Rightarrow 4 > 2$

$\therefore \quad P(2)$ is true.

Now $P(4) = 2^4 = 16$

$4! = 24$

$\therefore \quad 2^4 \not> 4!$

$\therefore \quad P(4)$ is false.

2. Give an example of a statement $P(n)$ which is true for all n.

Sol. $P(n) : 1 + 2 + 3 + ... + n = \dfrac{n(n+1)}{2}$ is true for all n.

3. Let $P(n)$ denote the statement "$2^{3n} - 1$ is a multiple of 7". Show that

   (i) $P(1)$, $P(2)$ and $P(3)$ are true.

   (ii) If $P(m)$ is true, then $P(m + 1)$ is also true.

Sol. (i) $P(1) = 2^{3\times1} - 1 = 7$ is a multiple of 7

$P(2) = 2^{3\times2} - 1 = 63$ is a multiple of 7

$P(3) = 2^9 - 1 = 511$ is a multiple of 7

Hence $P(1)$, $P(2)$ and $P(3)$ are true for the given statement.

(ii) $P(m)$ is true $\Rightarrow 2^{3m} - 1$ is a multiple of 7

$\Rightarrow 2^{3m} - 1 = 7\lambda$, for some integer λ.

$\Rightarrow 2^{3m} = 7\lambda + 1$ ...(i)

Now, $2^{3(m+1)} - 1 = 2^{3m+3} - 1 = 2^{3m} . 2^3 - 1$

$= (7\lambda + 1)8 - 1$

$= 56\lambda + 7 = 7(8\lambda + 1)$ is a multiple of 7

Hence if $P(m)$ is true, then $P(m + 1)$ is also true.

4. Let $P(n)$ be the statement : "$n(n + 1)(n + 2)$ is a multiple of 12". Show that $P(3)$ and $P(4)$ are true but $P(5)$ is not true.

Sol. $P(3)$ = "$3 \times 4 \times 5$ is a multiple of 12" *i.e* "60 is a multiple of 12", which is true.

$P(4) = 4 \times 5 \times 6$ is a multiple of 12

$= 120$ is a multiple of 12, which is true

$P(5) = 5 \times 6 \times 7$ is a multiple of 12

$= 210$ is a multiple of 12 which is false.

5. Let $P(n)$ denote the statement : "$\frac{n(n+1)}{6}$ is a natural number". Show that $P(2)$ and $P(3)$ are true but $P(4)$ is not true.

Sol. $P(2) = \frac{2(2+1)}{6} = \frac{6}{6} = 1$ is a natural number, which is true

$P(3) = \frac{3(3+1)}{6} = \frac{3\times 4}{6} = 2$ is a natural number, which is true

$P(4) = \frac{4\times(4+1)}{6} = \frac{20}{6}$ is a natural number, which is false.

6. Let $P(n)$ denote the statement : "$2^n \geq 3n$". Show that if $P(m)$ is true, then $P(m + 1)$ is also true.

Sol. $P(m)$ is true $\Rightarrow 2^m \geq 3m$, multiplying both sides by 2

$2^{m+1} \geq 6m$

$\Rightarrow 2^{m+1} \geq 3m + 3m$

$\Rightarrow 2^{m+1} \geq 3m + 3$ Since $m \geq 1$

$\Rightarrow 2^{m+1} \geq 3(m+1)$

$\Rightarrow P(m+1)$ is true

7. Let $P(n)$ denote the statement "$n^2 - n + 41$ is prime". Show that $P(1)$ and $P(2)$ are true but $P(41)$ is not true.

Sol. $P(n)$: $n^2 - n + 41$ is prime

$P(1)$ is "$1^2 - 1 + 41$ is prime

i.e 41 is prime, which is true

$P(2)$ = "$2^2 - 2 + 41$ is prime"

i.e 43 is prime, which is true.

So, $P(1)$ and $P(2)$ is true.

However, $P(41)$ is "$41^2 - 41 + 41$ *i.e* 41^2 is prime, which is not true.

8. Let $P(n)$: "$4^n > n$". Show that if $P(m)$ is true, then $P(m + 1)$ is also true, $m \in \mathbb{N}$.

Sol. Let $P(m)$ be true $\Rightarrow 4^m > m$ ...(i)

To prove : $P(m + 1)$ *i.e* $4^{m+1} > m + 1$

Multiplying both sides of (i) by 4, we get

$4^m \times 4 > 4m \Rightarrow 4^{m+1} > m + 3m > m + 1$

$\Rightarrow \quad 4^{m+1} > m + 1 \Rightarrow P(m + 1)$ is true.

9. Using the principle of mathematical induction prove that for all $n \in \mathbb{N}$

$4 + 8 + 12 + ...$ to n terms $= 2n(n + 1)$.

Sol. n^{th} term of series $t_n = 4 + (n - 1)4 = 4n$

So we have to prove

$P(n)$: "$4 + 8 + 12 + ... + 4n = 2n(n + 1)$"

$P(m)$ is true $\Rightarrow 4 + 8 + 12 + ... + 4m = 2m(m + 1)$, $m \in \mathbb{N}$.

Adding $4(m+1)$ to both sides, we get

$4 + 8 + 12 + ... + 4m + 4(m + 1) = 2m(m + 1) + 4(m + 1)$

$= 2(m + 1)(m + 2)$

$\therefore \quad P(m + 1)$ is true.

Hence $P(x)$ is true for all $n \in \mathbb{N}$.

10. By the principle of mathematical induction prove that

$1 + 2 + 3 + ... n = \frac{n(n+1)}{2}$, $n \in \mathbb{N}$.

Sol. Given that

$P(n) = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

Let $P(k)$ is true for all $k \in \mathbb{N}$.

i.e $\quad 1 + 2 + 3 + ... + k = \frac{k(k+1)}{2}$

Now, $P(k + 1) = P(k) + (k + 1)$

$$= \frac{k(k+1)}{2} + (k+1) = (k+1)\left[\frac{k}{2} + 1\right]$$

$$= \frac{(k+1)(k+2)}{2}$$

$\Rightarrow \quad P(k + 1)$ is true.

Then, by the principle of mathematical induction $P(n)$ is true for all natural number *n.*

Short Answer Type Questions :

11. By using PMI, prove that for all $n \in \mathbb{N}$, $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+.........+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$.

Sol. $P(k + 1) = \frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}=\frac{1}{(3k+1)}\left[k+\frac{1}{3k+4}\right]$

$= \frac{1}{3k+1}\left[\frac{3k^2+4k+1}{3k+4}\right]$

$= \frac{1}{(3k+1)}\left[\frac{(k+1)(3k+1)}{3k+4}\right]$

$= \frac{k+1}{3k+4}$

Hence $P(n)$ is true for all natural number n.

12. By using PMI, prove that $10^{2n-1} + 1$ is divisible by 11.

Sol. Here

$P(n)$: $f(n) = 10^{(2n-1)} + 1$ is divisible by 11

$P(1)$: $f(1) = 10' + 1 = 11$ is divisible by 11

for $n = 1$

$\Rightarrow$ $P(1)$ is true ...(i)

Let $P(k)$: $f(k) = 10^{2k-1} + 1$ is divisible by 11

Hence $f(k) = 11\lambda$ ...(ii)

Now,

$P(k + 1) = f(k + 1) = 10^{2(k+1)-1} + 1$

$= 10^{2k+1} + 1 = 10^{2k-1}.\ 100 + 1$

$= (99.10^{2k-1}) + (10^{2k-1} + 1)$

$= 99.10^{(2k-1)} + 11\lambda$

$= 11\ [9.10^{2k-1} + \lambda]$ is divisible by 11

$\Rightarrow$ $P(k + 1)$ is also divisible by 11

$\therefore$ By the principle of mathematical induction $P(n)$ is true for all natural number n.

13. By using PMI, prove that $5^{2n} - 1$ is divisible by 24.

Sol. Let $P(n)$: "$5^{2n} - 1$ is divisible by 24"

Now, $P(1)$ means $5^{2\times 1} - 1 = 24$ is divisible by 24,

Which is true.

So, $P(1)$ is true.

Let $P(k)$ be true for $k \in \mathbb{N}$. *i.e* $5^{2k} - 1$ is divisible by 24.

$\Rightarrow$ $5^{2k} - 1 = 24\ \lambda$

To prove : $P(k + 1)$ is true.

Now, $5^{2(k+1)} - 1 = 5^{2k} . 5^2 - 1$

$= 25.5^{2k} - 1.$

$= (24.5^{2k}) + (5^{2k} - 1).$

$= 24.5^{2k} + 24\lambda$

$= 24(5^{2k} + \lambda)$ is divisible by 24.

So, $P(k) \Rightarrow P(k + 1)$, Also $P(1)$ is true.

Therefore, by the principle of mathematical induction $P(n)$ is true for all $n \in \mathbb{N}$.

14. By using PMI, prove that $n(n + 1)(2n + 1)$ is divisible by 6.

Sol. $P(k) = k(k + 1)(2k + 1) = 6\lambda$

$\Rightarrow P(k + 1) = (k + 1)(k + 2)\{2(k + 1) + 1\}$

$= (k + 1)(k + 2)\{(2k + 1) + 2\}$

$= (k + 1)(k + 2)(2k + 1) + 2(k + 1)(k + 2)$

$= k(k + 1)(2k + 1) + 2(k + 1)(2k + 1) + 2(k + 1)(k + 2)$

$= k(k + 1)(2k + 1) + 2(k + 1)[2k + 1 + k + 2]$

$= k(k + 1)(2k + 1) + 2(k + 1)3(k + 1)$

$= k(k + 1)(2k + 1) + 6(k + 1)(k + 1)$

$= 6[\lambda + (k + 1)^2]$ is divisible by 6.

15. Show that by using PMI, $2^{3n} - 1$ is divisible by 7.

Sol. $P(k)$: $2^{3k} - 1$ is divisible by 7

$\Rightarrow 2^{3k} - 1 = 7\lambda$. for some integer λ

$P(k + 1) = 2^{3(k+1)} - 1 = 2^{3k}.2^3 - 1$

$= 2^{3k}(7 + 1) - 1 = 7.2^{3k} + 2^{3k} - 1 = 7.2^{3k} + 7\lambda$

$\Rightarrow P(k + 1) = 7(2^{3k} + \lambda)$

$\Rightarrow P(k + 1)$ is divisible by 7.

16. By the help of PMI, prove that $n(n + 1)$ is multiple of 2 or $n^2 + n$ is an even number.

Sol. $P(k)$: $k(k + 1)$ is divisible by 2

$\Rightarrow P(k) = 2\lambda$ for some integer λ

$P(k + 1) = (k + 1)(k + 2)$

$= k(k + 1) + 2(k + 1)$

$= 2\lambda + 2(k + 1)$

$= 2(\lambda + k + 1)$

$\Rightarrow P(k + 1)$ is divisible by 2.

17. By using the principle of mathematical induction prove that for all $n \in \mathbb{N}$

$1 + 4 + 7 + ... + 3n - 2 = \frac{1}{2}n(3n-1)$

Sol. Let $P(n) = 1 + 4 + 7 + ... + (3n - 2) = \frac{1}{2}n(3n-1)$

for $n = 1$

$P(1)$: 1 = $\frac{1}{2}\cdot 1(3\times 1-1) \Rightarrow 1=\frac{1}{2}\cdot 1\times 2$ $\Rightarrow$ 1 = 1 which is true.

$\therefore$ $P(1)$ is true ...(i)

Let $P(k) = 1 + 4 + 7 + ... + (3k - 2) = \frac{1}{2}k(3k-1)$ is true for $k \in \mathbb{N}$.

$P(k + 1) = P(k) + (3k + 1)$

$$= \frac{1}{2}k(3k-1)+(3k+1)$$

$$= \frac{1}{2}\left[3k^2+5k+2\right] = \frac{1}{2}(k+1)(3k+2)$$

$$= \frac{1}{2}(k+1)\left[3(k+1)-1\right]$$

$\Rightarrow$ $P(k + 1)$ is also true

$\therefore$ $P(k)$ true $\Rightarrow P(k + 1)$ is true and $P(1)$ is also true, therefore by the principle of mathematical induction $P(n)$ is true for all $n \in \mathbb{N}$.

18. By using the principle of mathematical induction, prove that

$2 + 3 \cdot 2 + 4 \cdot 2^2 + ... + (n + 1)2^{n-1} = n \cdot 2^n$. for all $n \in \mathbb{N}$.

Sol. Let $P(n)$: $2 + 3\cdot 2 + 4\cdot 2^2 + ... + (n + 1)2^{n-1} = n\cdot 2^n$

$P(1)$ means $2 = 1\cdot 2$

$\Rightarrow$ 2 = 2, which is true

$\Rightarrow$ $P(1)$ is true ...(i)

Let us assume that $P(k)$ is true for all $k \in \mathbb{N}$

i.e $2 + 3\cdot 2 + 4\cdot 2^2 + ... + (k + 1)\, 2^{k-1} = k.2^k$ ...(ii)

Now to prove $P(k + 1)$ is true

$\therefore$ $P(k + 1) = P(k) + (k + 2)2^k$

$= k2^k + (k + 2)\cdot 2^k = 2^k[k + k + 2]$

$= (k + 1)\cdot 2^k \cdot 2$

$= (k + 1)2^{k+1}$

$\Rightarrow$ $P(k + 1)$ is also true

Hence, by induction, $P(n)$ is true for all $n \in \mathbb{N}$.

Long Answer Type Questions :

By using the principle of mathematical induction, prove the following for all $n \in \mathbb{N}$.

19. $12^n + 2 \cdot 5^{n-1}$ is divisible by 7.

Sol. Let $P(n)$ denote the statement: $12^n + 2\cdot 5^{n-1}$ is divisible by 7

Now, $P(1)$ means $12 + 2\cdot 5^{1-1}$

$= 12 + 2 = 14$ is divisible by 7

Therefore $P(1)$ is true.

Let us assume that $P(k)$ is true

i.e $12^k + 2\cdot5^{k-1} = 7\lambda$

we shall prove that $P(k + 1)$ is also true,

i.e $12^{k+1} + 2\cdot5k$ is also divisible by 7

Now, $12^{k+1} + 2\cdot5^k = 12\cdot12^k + 2\cdot5\cdot5^{k-1}$

$= (7 + 5)\cdot12^k + 2\cdot5\cdot5^{k-1}$

$= 7\cdot12^k + 5\cdot12^k + 2\cdot5\cdot5^{k-1}$

$= 7\cdot12^k + 5[12^k + 2\cdot5^k]$

$= 7\cdot12^k + 5 \times 7\lambda$

$= 7[12^k + 5\lambda]$

$\Rightarrow$ $P(k + 1)$ is divisible by 7.

Hence, by the principle of mathematical induction. $P(n)$ is true for all $n \in \mathbb{N}$.

20. The sum of cubes of three consecutive natural numbers is divisible by 9.

Sol. $P(k + 1) = (k + 1)^3 + (k + 2)^3 + (k + 3)^3$

$= (k + 1)^3 + (k + 2)^3 + k^3 + 9k^2 + 27k + 27$

$= k^3 + (k + 1)^3 + (k + 2)^3 + 9(k^2 + 3k + 3)$

$= P(k) + 9(k^2 + 3k + 3)$

Since $P(k)$ is divisible by 9

Therefore $P(k) + 9(k^2 + 3k + 3)$ is also divisible by 9

$\Rightarrow$ $P(k + 1)$ is divisible by 9

21. $15^{2n-1} + 1$ is divisible by 16.

Sol. $P(k + 1) = 15^{2k+1} + 1 = 15^2 \times 15^{2k-1} + 1$

$= (15^2 - 1)15^{2k-1} + 15^{2k-1} + 1$

$= 224\cdot15^{2k-1} + (15^{2k-1} + 1)$

$= 224\cdot15^{2k-1} + P(k)$

Since $P(k)$ is divisible by 16

Hence $P(k + 1) = 224\cdot15^{2k-1} + P(k)$ is also divisible by 16

22. $n(n^2 - 1)$ is divisible by 24 where n is an odd number greater than 2.

Sol. $P(k + 2) = (k + 2)\{(k + 2)^2 - 1\} = (k + 2)[k^2 + 4k + 3]$

$= k^3 + 6k^2 + 11k + 6$

$= k(k^2 - 1) + 6(k + 1)^2$

as k is odd, $(k + 1)$ would be even and $6(k + 1)^2$ would be divisible by 24.

23. $(7^{2n} + 2^{3n-1.}\,.\,3^{n-1})$ is divisible by 25.

Sol. $P(k + 1) = 7^{2k+2} + 2^{3k}\cdot3^k = 7^{2k}.49 + 2^{3k-3}\cdot8\cdot3^{k-1}\cdot3$

$= 25\,[2\cdot7^{2k} + 2^{3k-3}\cdot3^{k-1}] - [7^{2k} + 2^{3k-3}\cdot3^{k-1}]$

Since $7^{2k} + 2^{3k-3}\,3^{k-1}$ is divisible by 25

$\therefore$ $P(k + 1)$ is divisible by 25

24. $3 \cdot 2^2 + 3^2 2^3 + 3^3 2^4 + ... + 3^n 2^{n+1} = \frac{12}{5}(6^n - 1)$.

Sol. $P(n)$: $3{\cdot}2^2 + 3^2{\cdot}2^3 + 3^2{\cdot}2^4 + ... + 3n{\cdot}2^{n+1} = \frac{12}{5}(6^n - 1)$

$P(1)$: $3{\cdot}2^2 = \frac{12}{5}(6-1) \Rightarrow 12 = 12$, which is true

Therefore $P(1)$ is true.

Let $P(k)$ be true, $k \in \mathbb{N}$

$$3{\cdot}2^2 + 3^2{\cdot}2^3 + ... + 3^k\,2^{k+1} = \frac{12}{5}(6^k - 1)$$

Consider $P(k + 1)$

$$3{\cdot}2^2 + 3^2{\cdot}2^3 + ... + 3^k{\cdot}2^{k-11} + 3^{k+1}{\cdot}2^{k+2} = \frac{12}{5}\left(6^{k+11} - 1\right)$$

$$\text{L.H.S} = \frac{12}{5}\left(6^k - 1\right) + 3^k \cdot 3 \cdot 2^k \cdot 2^2$$

$$= \frac{12}{5}\left(6^k - 1\right) + 6^k \cdot 12$$

$$= \frac{12}{5}\left(6^k - 1 + 6^k \cdot 5\right)$$

$$= \frac{12}{5}\left[(5+1)6^k - 1\right]$$

$$= \frac{12}{5}\left[6 \cdot 6^k - 1\right] = \frac{12}{5}\left[6^{k+1} - 1\right]$$

$\therefore$ $P(k)$ is true $\Rightarrow P(k + 1)$ is true

Hence, by the principle of mathematical induction $P(n)$ is true for $n \in \mathbb{N}$.

25. $1 + 5 + 9 + 13 + ... + (4n - 3) = n(2n - 1)$.

Sol. $P(n)$: $1 + 5 + 9 + ... + (4n - 3) = n(2n - 1)$

$P(1)$: $1 = 1{\cdot}(2 - 1) \Rightarrow 1 = 1$, which is true.

Let $P(k)$ be true, *i.e*,

$1 + 5 + 9 + ... + (4k - 3) = k(2k - 1)$

To show $P(k + 1)$ is true

i.e. $1 + 5 + ... + (4k - 3) + (4k + 1) = (k + 1)(2k + 1)$

consider LHS $= k(2k - 1) + (4k + 1)$

$= 2k^2 + 3k + 1$

$= 2k^2 + 2k + k + 1$

$= 2k(k + 1) + 1(k + 1)$

$= (k + 1)(2k + 1) =$ R.H.S

$\therefore$ $P(k)$ is true $\Rightarrow P(k + 1)$ is true, therefore, by the principle of mathematical induction $P(n)$ is true for $n \in \mathbb{N}$.

26. $4^n + 15n - 1$ is divisible by 9.

Sol. $P(k + 1) \Rightarrow 4^{k+1} + 15(k + 1) - 1$

$= 4^k \cdot 4 + 15k + 15 - 1$

$= 4^k \cdot 4 + 15k + 14$

$= 4^k(9 - 5) + (90k - 75k) + (9 + 5)$

$= 9 \cdot 4^k + 90k + 9 - 5(4^k + 15k - 1)$

$= 9(4^k + 10k + 1) - 5P(k)$

[Since $P(k) = 4^k + 15k - 1$]

$\Rightarrow$ $P(k + 1)$ is divisible by 9 since $P(k)$ is divisible by 9.

27. $7^{2n} - 1$ is divisible by 48.

Sol. $P(k + 1) = 7^{2k+2} - 1 = 7^{2k} \cdot 49 - 1$

$= 49(7^{2k} - 1) + 48$

$\Rightarrow$ $P(k + 1)$ is divisible by 48 since $P(k) = 7^{2k} - 1$ is divisible 48.

28. $5^{2n+2} - 24n - 25$ is divisible by 576.

Sol. $P(k + 1) = 5^{2k+4} - 24k - 49$

$= 25[5^{2k+2} - 24k - 25] + 576(k + 1)$

$\Rightarrow$ $P(k + 1)$ is divisible by 576 since $P(k) = 5^{2k+2} - 24k - 25$ is divisible by 5·76

29. $6^{n+2} + 7^{2n+1}$ is divisible by 43.

Sol. $P(k + 1) = 6^{k+1+2} + 7^{2(k+1)+1}$

$= [6^{k+2} + 7^{2k+1}]\, 6 + 7^{2k+1} \cdot 43$

$\Rightarrow P(k + 1)$ is divisible by 43 since $P(k) = 6^{k+2} + 7^{2k+1}$ is divisible by 43.

30. $12^n + 25^{n-1}$ is divisible by 13.

Sol. $P(k + 1) = 12^{k+1} + 25^k = 12^k(13 - 1) + 25^{k-1}(26 - 1)$

$= 13(12^k + 2 \cdot 25^{k-1}) - [12^k + 25^{k-1}]$

$\Rightarrow$ $P(k + 1)$ is divisible by 13 since $P(k) = 12^k + 25^{k-1}$ is divisible by 13.

31. Prove by the method of induction that every even power of every odd integer greater than 1, When divided by 8 leaves the remainder 1.

Sol. First odd integer greater than one is 3, general odd integer is $(2r + 1)$.

To show $(2r + 1)^{2n} = 8m + 1$, $m \in \mathbb{N}$.

i.e $(2r + 1)^{2n} - 1$ is divisible by 8

Let $P(n) : (2r + 1)^{2n} - 1$

$P(1) = (2r + 1)^2 - 1 = 4r^2 + 4r = 4r(r + 1)$

As, $r(r + 1)$ is always even, therefore, $P(1)$ is true.

Let $P(k)$ be true *i.e* $(2r + 1)^{2k} - 1$ is divisible by 8

$\Rightarrow (2r + 1)^{2k} = 8m + 1$ for $m \in \mathbb{N}$.

Consider $P(k + 1)$: $(2r + 1)^{2(k+1)} - 1$ is divisible by 8

consider

$(2r + 1)^{2k} . (2r + 1)^2 - 1$

$= (8m + 1) (8p + 1) - 1$

$= 64mp + 8m + 8p + 1 - 1$

$= 8[8mp + m + p]$, which is divisible by 8.

$\therefore$ $P(k)$ is true $\Rightarrow$ $P(k + 1)$ is true.

Hence, by the principle of mathematical Induction $P(n)$ is true for all $n \in \mathbb{N}$.

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Chapter 4

Principle of Mathematical Induction

Solutions (Set-2)

1. Let $P(n)$ be a statement and let $P(n) \Rightarrow P(n + 1)$ for all natural numbers n, then $P(n)$ is true

   (1) For all $n \in \mathbb{N}$.
   (2) For all $n > m$, m being a fixed positive integer
   (3) For all $n > 1$
   (4) Nothing can be said.

Sol. Answer (4)

2. The statement $(n + 3)^2 > 2^{n+3}$ is true.

   (1) For all n
   (2) For all $n \geq 3$
   (3) For all $n \geq 2$
   (4) No $n \in \mathbb{N}$

Sol. Answer (4)

3. A student was asked to prove a statement $P(n)$ by using the principal of mathematical induction. He proved that $P(n) \Rightarrow P(n + 1)$ for all $n \in \mathbb{N}$ and also that $P(4)$ is true:

   On the basis of the above he can conclude that $P(n)$ is true

   (1) For all $n \in \mathbb{N}$
   (2) For all $n > 4$
   (3) For all $n \geq 4$
   (4) For all $n < 4$

Sol. Answer (3)

4. Let $P(n)$: $n^2 - n + 41$ is a prime number, then

   (1) $P(1)$ is not true
   (2) $P(5)$ is not true
   (3) $P(9)$ is not true
   (4) $P(41)$ is not true

Sol. Answer (4)

5. Let $P(n)$ be a statement such that $P(n) \Rightarrow P(n + 1)$ for all $n \in \mathbb{N}$. Also, if $P(k)$ is true, $k \in \mathbb{N}$, then we can conclude that

   (1) $P(n)$ is true for all n
   (2) $P(n)$ is true for all $n \geq k$
   (3) $P(n)$ true for any $n < k$
   (4) None of these

Sol. Answer (2)

6. Let $P(n)$: $n^2 + n$ is odd, then $P(n) \Rightarrow P(n + 1)$ for all n. and $P(1)$ is not true.

From here, we can conclude that

(1) $P(n)$ is true for all $n \in \mathbb{N}$ (2) $P(n)$ is true for all $n \geq 2$

(3) $P(n)$ is false for all $n \in \mathbb{N}$ (4) $P(n)$ is true for all $n \geq 3$

Sol. Answer (3)

7. Consider the statement $P(n)$: $n^2 \geq 100$. Here.

$P(n) \Rightarrow P(n + 1)$ for all n. Does it mean that

(1) $P(n)$ is true for all n (2) $P(n)$ is true for all $n \geq 2$

(3) $P(n)$ is true for all $n \geq 3$ (4) None of these

Sol. Answer (4)

8. The statement $x^n - y^n$ is divisible by $(x - y)$ where n is a positive integer is

(1) Always true (2) Only true for $n < 10$

(3) Always false (4) Only true for $n > 10$

Sol. Answer (1)

9. $9^n - 8n - 1$ is divisible by 64 is

(1) Always true (2) Always false

(3) Always true for rational values of n (4) Always true for irrational values of n

Sol. Answer (1)

10. For all natural number n, $3^{2n} - 1$ is divisible by

(1) 3 (2) 5 (3) 6 (4) 8

Sol. Answer (4)

11. Let $P(n) = 10n + 3$ is a prime number. $P(n)$ is true $\forall n \in N$, such that

(1) $n > 2$ (2) $n > 3$ (3) $n < 4$ (4) $n \leq 2$

Sol. Answer (4)

12. "$x^n - y^n$ is divisible by $(x + y)$" is true when $n(\in N)$ is of the form $(k \in N)$

(1) $4k + 1$ (2) $4k + 3$ (3) $4k + 7$ (4) $2k$

Sol. Answer (4)

13. The statement which is correct for all $n \in N$, is

(1) $2^n > 2n + 1$ (2) $x^n - y^n$ is divisible by $(x + y)$

(3) $x^n - y^n$ is divisible by $(x - y)$ (4) $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots+\frac{1}{n^2}<2-\frac{1}{n}$

Sol. Answer (3)

14. $n(n + 1)(n + 2)$ is divisible by k for $\forall\ n \in N$. The largest k is

(1) 2 (2) 3 (3) 6 (4) 12

Sol. Answer (3)

15. Mathematical induction is a tool or technique which is used to prove a proposition about all

(1) Number (2) Integers (3) Whole number (4) None of these

Sol. Answer (4)

16. For each $n \in N$, $3^{2n} - 1$ is divisible by

(1) 8 (2) 16 (3) 32 (4) 10

Sol. Answer (1)

17. The statement $n! > 2^{n-1}$, $n \in N$ is true for

(1) $n > 1$ (2) $n > 2$ (3) All n (4) No n

Sol. Answer (2)

18. For each $n \in N$, $3\cdot(5^{2n+1}) + 2^{3n+1}$ is divisible by

(1) 17 (2) 19 (3) 21 (4) 23

Sol. Answer (1)

19. If b and $c \in N$ and $b < c$ then values of b and c in terms of $n \in N$ such that the statement "$3.6 + 6.9 + 9.12 + \ldots + 3n.(3n + 3) = 3nbc$" is true for all n

(1) $b = n$, $c = n$ (2) $b = n + 1$, $c = n + 2$

(3) $b = n$, $c = n + 1$ (4) $b = n + 1$, $c = n + 1$

Sol. Answer (2)

20. The proposition among the following that is not true for all $n \in N$, is

(1) $2^n > n$ (2) $2^n > n^2$

(3) $1 + 2 + 3 + \ldots + n < (2n + 1)^2$ (4) $(2n + 7) < (n + 3)^2$

Sol. Answer (2)

21. If $7 + 77 + 777 + \ldots$ upto n^{th} term

$= \frac{7}{81}[a.10^{n+1} - bn - c]$ is true $\forall\ n \in N$, then

(1) $a = 1$, $b = 9$, $c = 9$ (2) $a = 1$, $b = 9$, $c = 10$

(3) $a = 1$, $b = -9$, $c = -9$ (4) $a = 1$, $b = -9$, $c = -10$

Sol. Answer (2)

22. The proposition that is not true for $n > 1$ ($n \in N$), is

(1) $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n}$ (2) $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$

(3) $\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} > \frac{13}{24}$ (4) $\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6} \ldots\ldots \frac{2n-1}{2n} > \frac{1}{\sqrt{3n+1}}$

Sol. Answer (4)

23. Which of the following is true for $n \in N$?

(1) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} < 1$ (2) $\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{3n+1} < 1$

(3) $n^4 > 10^n$ (4) ${}^{2n}C_n > \frac{4^n}{n+1}$

Sol. Answer (1)

24. For all $n(> 1) \in N$, by using mathematical induction or otherwise $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}$ in its lowest form is

(1) Odd integer

(2) Even integer

(3) $\frac{\text{Odd integer}}{\text{Even integer}}$

(4) $\frac{\text{Even integer}}{\text{Odd integer}}$

Sol. Answer (3)

25. The sum of the squares of three consecutive odd numbers increased by 1 is divisible by (use mathematical induction)

(1) 12 as well as 24

(2) 12 but not 24

(3) Neither by 12 nor by 24

(4) By all multiples of 12

Sol. Answer (2)

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