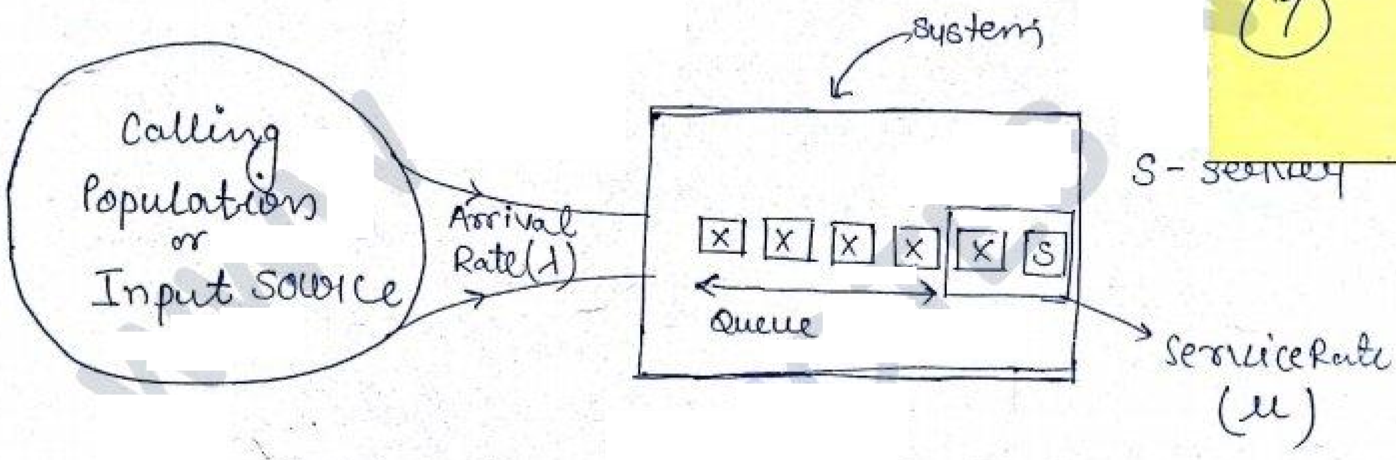
"Queing Theory" / Waiting line theory

The aim of queing theory is achievement of an economic balance between the cost of provideing service and cost associated with the wait required for that service. it is applied to service oriented organization like production shop, work shop, repair shop, bank, atm, etc.

Queuing Theory (7)

Characteristic of Queing Model:–

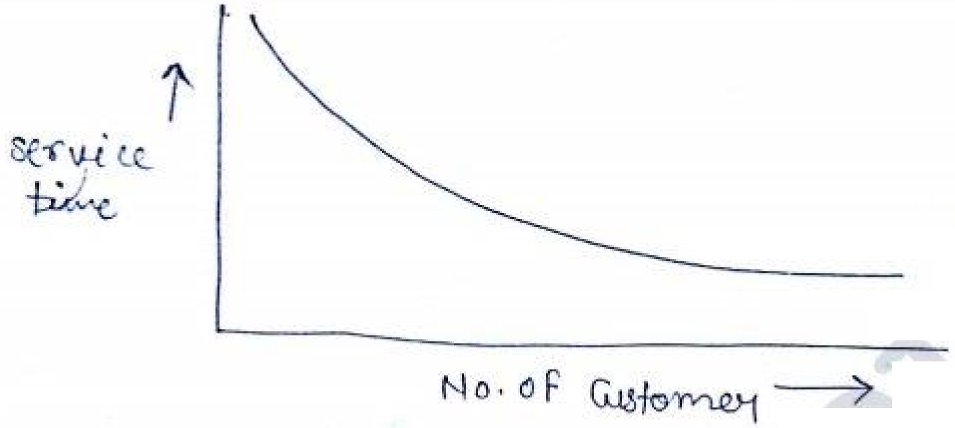


1. Arrival Rate/ Arrival Pattern:–(λ) →

The no. of customer arriving per unit time is termed as arrival rate. custome arrival is random and there for it is assume to follow poisson's distribution

2. Service Rate / Service Pattern:–(μ)

The number of customer serviced per unit time is termed as a service rate and it assumed to follow exponential distribution



(-ve exponential)

3. Service Rule / service order

Queuing Theory

It given information about the Q-discipline which mean the order by which the customers are picked up from the waiting line in order to provide service.

1) FIFO or FCFS
2) LIFO or LCFS
3) SIRO (servic in random order)
4) Priority treatment

11. System and calling Population :-

System is the place or facility where Customer arrive in order to get service. Its Capacity may be finite or Infinite.

The entire sample of customer from which only few visit our system is termed as Calling population or input source. Its Capacity may be finite or infinite. It is infinite when the arrival of few customer does not make any effect on the arrival pattern of future Customer.

5. Customer Attitude

- Attitude
  - Patient
  - Impatient
    - Jockey
    - Balking
    - Reneging
    - Cheater

Jockey - Customer keep on changing queue in hope to get service faster.

Balking - Customer does not join the queue and leave the system as queue is very long.

Reneging - Customer join the queue for short duration and then leave the system. as queue is moving very slow.

Cheater - Customer take illegal means like fighting, bribing in hope to get faster

Representation of Queing Model:-

Queing Models are ~~to~~ represented by kendall and lee notations whos general form is $(a/b/c):(d/e/f)$

where a - probability dist. for arrival pattern
b - probability dist. for service pattern
c - No. of server within the system
d - service rule or service order
e - size of capacity of system
f - size or capacity of calling population

symbols

1. a & b

M - markovian (Poisson) for arrival pattern or exponential service pattern

E - Erlangian (Gamma) for arrival or service pattern

D - Deterministic Arrival or Service pattern

2. C 1, 2, 3, 4, 5 - - -
Numbers

3. d
1) FIFO or FCFS
2) LIFO or LCFS
3) SIRO
4) GD - General service discipline)

4. e & f
N - Finite
∞ - Infinite

e.g. $(E/M/2) : (SIRO/N/\infty)$

— Gamma arrival, exponential service, 2 server finite capacity of system and infinite capacity of calling population.

Single server model $(M/M/1 : FIFO/\infty/\infty)$

Arrival Rate $= \lambda$ (Poisson's) $= 15$ Cust/hr

Inter-arrival rate or Time $= \frac{1}{\lambda} = \frac{1}{15}$ hr/cust $=$ 4 min/Cust.
(Exponential)

Service Rate $= \mu$ (exponential) $=$ 20 Cust/hr.

Inter-Service Rate or Time $= \frac{1}{\mu} = \frac{1}{20}$ hr/cust. $=$ 3 min/Cust.
(Poisson's)

(1) <u>If $\lambda > \mu$</u>

Queue length will keep on increasing and after a certain period of time incoming population will not get service In these Condition System ultimately fail and there is no solution to such proplem.

(2) <u>If $\lambda \le \mu$</u> - system work

$\lambda < \mu$ - prefer (Service Rate more then arrival rate)

$$\boxed{\rho = \frac{\text{Arrival Rate}}{\text{Service Rate}} = \frac{\lambda}{\mu}}$$

ρ indicated the percentage time when server busy

e.g. $\mu = 20$ cust/hr

1) if $\lambda = 5$, $\rho = \frac{5}{20} = 0.25$

2) if $\lambda = 10$, $\rho = \frac{10}{20} = 0.50$

3) if $\lambda = 15$, $\rho = 15/20 = 0.75$

4) if $\lambda = 20$, $\rho = \frac{20}{20} = 1.00$.

The ratio of Arrival rate to service rate indicate the percentage time when server is busy and is know as utilization factor, Average utilization, system utilization channel efficiency and clearing ratio

☆ It aslo indicate the probability that a new customer have to wait.

Formulae :-

1. Probability that the system is idle or zero customer in the system ⇒ $\boxed{P_0 = 1 - \rho}$

2. Probability of having exactly n customer in the system

$$\boxed{P_n = \rho^n . P_0} \qquad \boxed{P_n = \rho^n (1-\rho)}$$

eg. For $n = 4$

$$\Rightarrow P_4 = \rho^4 (1-\rho)$$

eg.

Probability of having 3 or more than 3 customer in system

$$P_0 + P_1 + P_2 + P_3 + \cdots = 1$$

$$P_3 + P_4 + P_5 + \cdots = 1 - (P_0 + P_1 + P_2)$$

$$= 1 - (P_0 + \rho^1 . P_0 + \rho^2 . P_0)$$

$$P_3 + P_4 + P_5 + \cdots = 1 - P_0 (1 + \rho^1 + \rho^2)$$

☆ Probability of at least n customer in system $= \underline{(\rho)^n}$

3. Average number of Customer in the system →

In this we include both the customers waiting in the queue along with those getting service

$$\boxed{L_s = \sum_{n=0}^{\infty} n \cdot P_n}$$

$$\boxed{L_s = \frac{\rho}{1-\rho}}$$

4. Average number of Customer in the queue →

In this we do not include the Customer getting service

$$\boxed{L_q = \sum_{n=2}^{\infty} (n-1) \cdot P_n}$$

$$\boxed{L_q = \frac{\rho^2}{1-\rho} = L_s - \rho = L_s \cdot \rho}$$

Little's law :- for a stable system average number of customer in the system or queue is given by average Customer arrival Rate multiplied by average waiting time of Customer in the system or queue.

$$L_s = \lambda \cdot W_s$$

$$L_q = \lambda W_q$$

For Remember
चाल x समय = दूरी

So here we can find

waiting in system $\boxed{W_s = \frac{L_s}{\lambda}}$

waiting time in queue $\boxed{W_q = \frac{L_q}{\lambda} = W_s - \frac{1}{\mu}}$

*

Average $\xrightarrow{\text{for}}$ System
Mean $\xrightarrow{\text{for}}$ Queue

if Nothing mention in question

Problem: 35 The number of person arriving at a service center is 8 cust/hr and the service provider take 5 min/cust on an average then determine

1. L_s and L_q
2. W_s and W_q

Sol^n $\lambda = 8$ cust/hr $\quad 1/\mu = 5$ min/cust.

$\mu = \frac{1}{5} \times 60$ ~~cust~~/hr $= 12$ cust/hr

$\rho = \frac{8}{12} = 2/3$

$L_s = \frac{\rho}{1-\rho} = \frac{2/3}{1-2/3} = 2 \qquad W_s = \frac{L_s}{\lambda} = \frac{2}{8}$ hr ~~/cust~~ $= 15$ min

$L_q = L_s \cdot \rho = 2 \times 2/3 = 4/3 \qquad W_q = \frac{L_q}{\lambda} = \frac{4}{3 \times 8} = 10$ mins

Problem 36 : A shopkeeper service 10 cust/hr and customer arrival is 7 cust/hr find the probability that at least two (2) customer waiting in the queue.

Soln

$$\mu = 10 \text{ cust/hr}$$

$$\lambda = 7 \text{ cust/hr}$$

$$P_0 + P_1 + P_2 + P_3 - - \quad = 1$$

$$P_3 + P_4 - - \quad = 1 - P_0 - P_1 - P_2$$

At least 2 customer waiting in queue it means atleast three in the system

So Probability $= 1 - (P_0 + \rho^1 P_0 + \rho^2 P_0)$

$$= 1 - P_0 (1 + \rho^1 + \rho^2)$$

$$= 1 - (1-\rho)(1+\rho^1+\rho^2)$$

$$= \rho^3$$

$$= (0.7)^3 = 0.343$$

Problem (workbook 13) Page No. (78) ESE 2004

$$\lambda = 6/hr$$

$A \rightarrow \frac{1}{\mu_A} = 6 \text{ min} \Rightarrow \mu_A = 10/hr$, wages = Rs 8/hr

$B \rightarrow \frac{1}{\mu_B} = 5 \text{ min} \Rightarrow \mu_B = 12/hr$, wages = Rs 10/hr.

estimate cost of idle machine = Rs 15/hr.

$\rho_A = 0.6$, $\rho_B = 0.5$

Total cost = Cost of idle mc + salary

A-11

$$(L_s)_A = \frac{\rho_A}{1-\rho_A} = \frac{0.6}{1-0.4} = 1.5 \text{ mc}$$

$$\begin{pmatrix}\text{idle Cost}\\ \text{of mc}\end{pmatrix} = 1.5 \text{ mc} \times \frac{\text{Rs } 15}{\text{hr.mc}} \times 8 \text{ hr} = \text{Rs } 180$$

$$(\text{salary})_A = \text{Rs } 8/_{hr} \times 8 \text{hr} = \text{Rs } 64$$

$$\text{Total Cost} = (T.C.)_A = \text{Rs } 180 + \text{Rs } 64 = \text{Rs } 244$$

B-11

$$(L_s)_B = \frac{\rho_B}{1-\rho_B} = \frac{0.5}{1-0.5} = 1 \text{ mc}$$

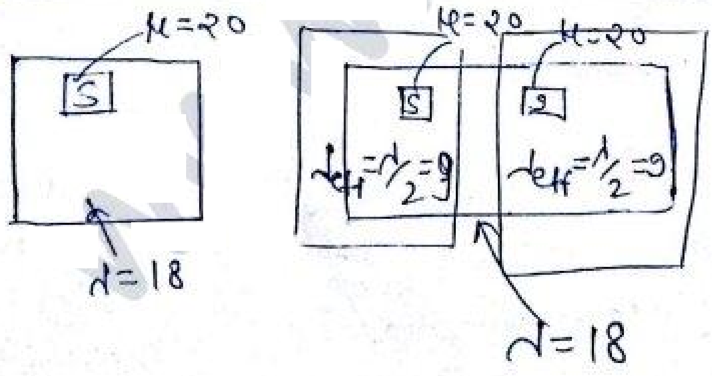
$$\begin{pmatrix}\text{idle Cost}\\ \text{of mc}\end{pmatrix} = 1 \text{ mc} \times \frac{\text{Rs } 15}{\text{hr.mc}} \times 8 \text{ hr} = \text{Rs } 120$$

$$(\text{salary})_B = \text{Rs } 10/_{hr} \times 8 \text{hr} = \text{Rs } 80$$

$$(T.C.)_B = \text{Rs } 200$$

So Repairman B is used because the total cost of repairman B is less than A

⇒



When a additional server put with same the incoming population Get decided.

* Some more Formulae

① Average length of non empty queue / Average length of queue containing at least one customer

$$\boxed{L_q' = \frac{1}{1-\rho}}$$

S.No.	System	Queue
1	4	3
2	2	1
3	1	~~0~~ ×
4	0	~~0~~ ×
5	2	1
6	6	5
7	4	3

②

② Probability of n arrival in the system during period T.

$$\boxed{P(n,T) = \frac{(\exp)^{-\lambda T}(\lambda T)^n}{n!}}$$

λ – cust/hr

T – hr.

③ Probability that more then T time period is needed to service a customer

$$\boxed{P = (\exp)^{-\mu . T}}$$

④ Probability that the waiting in the queue is greater then T.

$$\boxed{P(W_q > T) = \rho . (\exp)^{-T/W_s}}$$

⑤ Probability that the waiting time in the system is greater then T

$$\boxed{P(W_s > T) = (\exp)^{-T/W_s}}$$