MATHEMATICS PAPER IIB - MARCH 2009

COORDINATE GEOMETRY & CALCULUS

TIME : 3hrs **Max. Marks.75**

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. **10 × 2 = 20**

Noe : Attempt all questions. Each question carries 2 marks.

1. If the equation $x^2 + y^2 - 4x + 6y + c = 0$ represents a circle with radius 6, find c.

2. Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z = 11$.

3. Find the coodinates of the points on the parabola $y^2=2x$ whose focal distance is 5/2.

4. Find the equation of the Hyperbola whose foci are (4,2);(8,2) and eccentricity is 2.

5. If $y = ae^{nx} + be^{-nx}$ then show that $y_2=n^2y$

6. Evaluate $\int\sqrt{1-\cos 2x}dx$

7. Evaluate $\int\frac{x^8}{1+x^{18}}dx$ on R

8. Find the value of $\int_0^2|1-x|dx$

9. Find the area bounded by the parabola $y = x^2$, the x-axis and the lines x = -1, x =2.

10. Find the order and degree of $\left[\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3\right]^{\frac{6}{5}} = 6y$

SECTION B

SHORT ANSWER TYPE QUESTIONS. **5X4 =20**

Note: Answer any FIVE questions. Each question carries 4 marks.

11. If a point P is moving such that the Lengths of tangents drawn from P to

$X^2 + y^2 + 6x + 18y + 26 = 0$ are in the Ratio 2:3, then find the equation of the Locus of P.

12. Show that the equations of the commmon tangents to the circle $x^2+y^2=2a^2$ and the parabola $y^2=8ax$ are y=(x+2a).

13. Find eccentricity, coordinates of foci and equations of directrices of the ellispse $9x^2+16y^2-36x+32y-92 = 0$

14. Show that the points with polar coordinates (0,0), $(3,\pi/2)$ and $(3,\pi/6)$ form an equilateral triangle .

15. Evaluate $\int x\cos^{-1}x\,dx, x\in(-1,1)$

16. Solve the differential equations $\sqrt{1+x^2}\,dx+\sqrt{1+y^2}\,dy=0$

17. Solve $\frac{dy}{dx}-y\tan x=e^x\sec x$

SECTION C

LONG ANSWER TYPE QUESTIONS. 5X7 =35

Note: Answer any Five of the following. Each question carries 7 marks.

18. Show that the circles $x^2+y^2-6x-2y+1=0; x^2+y^2+2x-8y+13=0$ touch each other. Find the point of contact and the equation of common tangent at the point of contact.

19. Find the limiting points of the coaxial system determined by the circles $x^2+y^2+10x-4y-1=0$, $x^2+y^2+5x+y+4=0$.

20. If the polar of P with respect to the Parabola $y^2=4ax$ touches the circle $x^2+y^2=4a^2$, then show that P lies on the curve $x^2-y^2 = 4a^2$.

21. If $y=\frac{\sinh^{-1}x}{\sqrt{1+x^2}}$, then show that $(1+x^2)y_2+3xy_1+y=0$ and hence deduce that $(1+x^2)y_{n+2}+(2n+3)xy_{n+1}+(n+1)^2y_n=0$

22. Evaluate $\int(6x+5)\sqrt{6-2x^2+x}\,dx$

23. Show that $\int_0^{\pi/2}\frac{x}{\sin x+\cos x}dx=\frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right)$

24. Find the approximate value of π from $\int_0^1\frac{1}{1+x^2}dx$ by using Simpson's rule by dividing [0, 1] into 4 equal parts.

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