Topicwise Questions

Quadratic Equation & Nature of Roots

1. If $a + b + c = 0$, and $a, b, c \in R$, then the roots of the equation $4ax^2 + 3bx + 2c = 0$ are

(*a*) Equal (*b*) Imaginary

(*c*) Real (*d*) Both (*a*) and (*b*)

2. The roots of the given equation $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$ are
[Where $a \neq b$]

(*a*) Rational (*b*) Irrational

(*c*) Real (*d*) Imaginary

3. If $a, b, c \in Q$, then roots of the equation $(b + c - 2a)x^2 + (c + a - 2b)x + (a + b - 2c) = 0$ are

(*a*) Rational (*b*) Non-real

(*c*) Irrational (*d*) Equal

4. The value of k for which the quadratic equation, $kx^2 + 1 = kx + 3x - 11x^2$ has real and equal roots are

(*a*) $-11, -3$ (*b*) $5, 7$

(*c*) $5, -7$ (*d*) $-7, 25$

5. $x^2 + x + 1 + 2k(x^2 - x - 1) = 0$ is a perfect square for how many values of k

(*a*) 2 (*b*) 0

(*c*) 1 (*d*) 3

6. If $x^2 - 3x + 2$ be a factor of $x^4 - px^2 + q$, then (pq) =

(*a*) (3, 4) (*b*) (4, 5)

(*c*) (4, 3) (*d*) (5, 4)

7. The roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are

(*a*) $\frac{c-a}{b-c}, 1$ (*b*) $\frac{a-b}{b-c}, 1$

(*c*) $\frac{b-c}{a-b}, 1$ (*d*) $\frac{c-a}{a-b}, 1$

8. If a, b, c are integers and $b^2 = 4(ac + 5d^2)$, $d \in N$, then roots of the equation $ax^2 + bx + c = 0$ are

(*a*) Irrational (*b*) Rational & different

(*c*) Complex conjugate (*d*) Rational & equal

9. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then the equation $ax^2 + bx + c = 0$ has

(*a*) real roots (*b*) imaginary roots

(*c*) exactly one root (*d*) none of these

10. Consider the equation $x^2 + 2x - n = 0$, where $n \in N$ and $n \in [5, 100]$. Total number of different values of'n' so that the given equation has integral roots, is

(*a*) 4 (*b*) 6

(*c*) 8 (*d*) 3

11. The entire graph of the expression $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if

(*a*) $k < 7$ (*b*) $-5 < k < 7$

(*c*) $k > -5$ (*d*) None

12. If $a, b \in R$, $a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $a + b + 1$ is:

(*a*) positive (*b*) negative

(*c*) zero (*d*) depends on the sign of b

13. If a and b are the non-zero distinct roots of $x^2 + ax + b = 0$, then the least value of $x^2 + ax + b$ is

(*a*) $\frac{3}{2}$ (*b*) $\frac{9}{4}$

(*c*) $-\frac{9}{4}$ (*d*) 1

14. If both roots of the quadratic equation $(2 - x)(x + 1) = p$ are distinct & positive, then p must lie in the interval

(*a*) $(2, \infty)$ (*b*) $(2, 9/4)$

(*c*) $(-\infty, -2)$ (*d*) $(-\infty, \infty)$

15. If $(1 - p)$ is root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are

(*a*) 0, 1 (*b*) $-1, 1$

(*c*) $0, -1$ (*d*) $-1, 2$

Sum and Product of Roots

16. If one root of $5x^2 + 13x + k = 0$ is reciprocal of the other, then k =

(*a*) 0 (*b*) 5

(*c*) 1/6 (*d*) 6

17. If α and β are the roots of the equation $4x^2 + 3x + 7 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

(*a*) $-\frac{3}{7}$ (*b*) $\frac{3}{7}$

(*c*) $-\frac{3}{5}$ (*d*) $\frac{3}{5}$

18. If the sum of the roots of the equation $ax^2 + bx + c = 0$ be equal to the sum of their squares, then

(*a*) $a(a + b) = 2bc$ (*b*) $c(a + c) = 2ab$

(*c*) $b(a + b) = 2ac$ (*d*) $b(a + b) = ac$

19. If α, β be the roots of the equation $x^2 - 2x + 3 = 0$, then the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ is

(*a*) $x^2 + 2x + 1 = 0$ (*b*) $9x^2 + 2x + 1 = 0$

(*c*) $9x^2 - 2x + 1 = 0$ (*d*) $9x^2 + 2x - 1 = 0$

20. If the product of roots of the equation, $mx^2 + 6x + (2m - 1) = 0$ is -1, then the value of m will be

(*a*) 1 (*b*) -1

(*c*) $\frac{1}{3}$ (*d*) $-\frac{1}{3}$

21. If α and β are the roots of the equation $x^2 - 4x + 1 = 0$ the value of $\alpha^3 + \beta^3$ is

(*a*) 76
(*b*) 52
(*c*) –52
(*d*) –76

22. If α, β, γ are the roots of the equation $x^3 + x + 1 = 0$, then the value of $\alpha^3 \beta^3 \gamma^3$

(*a*) 0
(*b*) – 3
(*c*) 3
(*d*) – 1

23. If a, b are the roots of quadratic equation $x^2 + px + q = 0$ and g, d are the roots of $x^2 + px - r = 0$, then $(a - g) . (a - d)$ is equal to:

(*a*) $q + r$
(*b*) $q - r$
(*c*) $-(q + r)$
(*d*) $-(p + q + r)$

24. Two real numbers a & b are such that $a + b = 3$ & $|a - b| = 4$, then a & b are the roots of the quadratic equation:

(*a*) $4x^2 - 12x - 7 = 0$
(*b*) $4x^2 - 12x + 7 = 0$
(*c*) $4x^2 - 12x + 25 = 0$
(*d*) None of these

25. Let conditions C_1 and C_2 be defined as follows: $C_1 : b^2 - 4ac \geq 0$, C_2 : a, – b, c are of same sign. The roots of $ax^2 + bx + c = 0$ are real and positive, if

(*a*) both C_1 and C_2 are satisfied
(*b*) only C_2 is satisfied
(*c*) only C_1 is satisfied
(*d*) None of these

26. If α, β are roots of the equation $ax^2 + bx + c = 0$, then the value of $\alpha^3 + \beta^3$ is

(*a*) $\frac{3abc + b^3}{a}$
(*b*) $\frac{a^3 + b^3}{3abc}$
(*c*) $\frac{3abc - b^3}{a^3}$
(*d*) $\frac{-(3abc + b^3)}{a^3}$

Common Roots

27. If both the roots of $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then $2r - p$ is equal to

(*a*) –1
(*b*) 0
(*c*) 1
(*d*) 2

28. If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$, have a common root,(Where $p \neq q$) then $p + q + 1 =$

(*a*) 0
(*b*) 1
(*c*) 2
(*d*) – 1

29. If a, b, p, q are non-zero real numbers, then the equations $2a^2x^2 - 2abx + b^2 = 0$ and $p^2x^2 + 2pqx + q^2 = 0$ have:

(*a*) no common root
(*b*) one common root if $2a^2 + b^2 = p^2 + q^2$
(*c*) two common roots if $3pq = 2ab$
(*d*) two common roots if $3qb = 2ap$

Theory of Equation and Identity, Inequalities

30. If x is real and satisfies $x + 2 > \sqrt{x + 4}$, then

(*a*) $x < -2$
(*b*) $x > 0$
(*c*) $-3 < x < 0$
(*d*) $-3 < x < 4$

31. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$ is

(*a*) $(-\infty, -2) \cup (2, \infty)$
(*b*) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
(*c*) $(-\infty, -1) \cup (1, \infty)$
(*d*) $(\sqrt{2}, \infty)$

32. Number of values of 'p' for which the equation $(p^2 - 3p + 2)x^2 - (p^2 - 5p + 4)x + p - p^2 = 0$ possess more than two roots, is:

(*a*) 0
(*b*) 1
(*c*) 2
(*d*) None

33. The number of the integer solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is

(*a*) 1
(*b*) 2
(*c*) 3
(*d*) None of these

34. The complete set of values of 'x' which satisfy the inequations : $5x + 2 < 3x + 8$ and $\frac{x + 2}{x - 1} < 4$ is

(*a*) $(-\infty, 1)$
(*b*) $(2, 3)$
(*c*) $(-\infty, 3)$
(*d*) $(-\infty, 1) \cup (2, 3)$

35. The complete solution set of the inequality $\frac{x^4 - 3x^3 + 2x^2}{x^2 - x - 30} \geq 0$ is:

(*a*) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$
(*b*) $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$
(*c*) $(-\infty, -5] \cup [1, 2] \cup [6, \infty) \cup \{0\}$
(*d*) none of these

36. If the inequality $(m - 2)x^2 + 8x + m + 4 > 0$ is satisfied for all $x \in R$, then the least integral 'm' is:

(*a*) 4
(*b*) 5
(*c*) 6
(*d*) none

37. The complete set of real 'x' satisfying $||x - 1| - 1| \leq 1$ is:

(*a*) $[0, 2]$
(*b*) $[-1, 3]$
(*c*) $[-1, 1]$
(*d*) $[1, 3]$

38. If $\log_{1/3} \frac{3x - 1}{x + 2}$ is less than unity, then 'x' must lie in the interval:

(*a*) $(-\infty, -2) \cup (5/8, \infty)$
(*b*) $(-2, 5/8)$
(*c*) $(-\infty, -2) \cup (1/3, 5/8)$
(*d*) $(-2, 1/3)$

39. Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is:

(*a*) $[-4, 1]$
(*b*) $[-4, -3) \cup (0, 1]$
(*c*) $(-\infty, -3) \cup (1, \infty)$
(*d*) $(-\infty, -4) \cup [1, \infty)$

40. The set of all the solutions of the inequality $\log_{1-x}(x - 2) \geq 0$ is

(*a*) $(-\infty, 0)$
(*b*) $(2, \infty)$
(*c*) $(-\infty, 1)$
(*d*) ϕ

41. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

(*a*) $(2, \infty)$
(*b*) $(1, 2)$
(*c*) $(-2, -1)$
(*d*) None of these

42. If $\log_{0.5}\log_5(x^2-4) > \log_{0.5}1$, then 'x' lies in the interval

(*a*) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$
(*b*) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3\sqrt{5})$
(*c*) $(\sqrt{5}, 3\sqrt{5})$
(*d*) ϕ

43. The set of all solutions of the inequality $(1/2)^{x^2-2x} < 1/4$ contains the set

(*a*) $(-\infty, 0)$
(*b*) $(-\infty, 1)$
(*c*) $(1, \infty)$
(*d*) $(3, \infty)$

44. If $\dfrac{6x^2-5x-3}{x^2-2x+6} \le 4$, then least and the highest values of $4x^2$ are

(*a*) 0 & 81
(*b*) 9 & 81
(*c*) 36 & 81
(*d*) None of these

45. If two roots of the equation $x^3 - px^2 + qx - r = 0$ are equal in magnitude but opposite in sign, then:

(*a*) $pr = q$
(*b*) $qr = p$
(*c*) $pq = r$
(*d*) None

46. If α, β & γ are the roots of the equation $x^3 - x - 1 = 0$ then, $\dfrac{1+\alpha}{1-\alpha} + \dfrac{1+\beta}{1-\beta} + \dfrac{1+\gamma}{1-\gamma}$ has the value equal to:

(*a*) zero
(*b*) -1
(*c*) -7
(*d*) 1

47. For what value of a and b the equation $x^4 - 4x^3 + ax^2 + bx + 1 = 0$ has four real positive roots?

(*a*) $(-6, -4)$
(*b*) $(-6, 5)$
(*c*) $(-6, 4)$
(*d*) $(6, -4)$

48. If α, β are roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$ is

(*a*) $ab\,x^2 - (a+b)\,cx + (a+b)^2 = 0$
(*b*) $ac\,x^2 - (a+c)\,bx + (a+c)^2 = 0$
(*c*) $ac\,x^2\,(a+c)\,bx - (a+c)^2 = 0$
(*d*) None of these

49. If S is the set of all real x such that $\dfrac{2x-1}{2x^3+3x^2+x}$ is positive, then S contains

(*a*) $(-\infty, -3/2)$
(*b*) $(-3/2, 1/4)$
(*c*) $(-1/4, 1/2)$
(*d*) $(-1/2, 3)$

Max and Min Value, Factorization

50. If x is real, then the maximum and minimum values of the expression $\dfrac{x^2-3x+4}{x^2+3x+4}$ will be

(*a*) 2, 1
(*b*) $5, \dfrac{1}{5}$
(*c*) $7, \dfrac{1}{7}$
(*d*) 2, 7

51. The smallest value of $x^2 - 3x + 3$ in the interval $(-3, 3/2)$ is

(*a*) 3/4
(*b*) 5
(*c*) -15
(*d*) -20

52. If $y = -2x^2 - 6x + 9$, (for $x \in R$) then

(*a*) maximum value of y is -11 and it occurs at $x = 2$
(*b*) minimum value of y is -11 and it occurs at $x = 2$
(*c*) maximum value of y is 13.5 and it occurs at $x = -1.5$
(*d*) minimum value of y is 13.5 and it occurs at $x = -1.5$

53. If 'x' is real and $k = \dfrac{x^2-x+1}{x^2+x+1}$, then:

(*a*) $\dfrac{1}{3} \le k \le 3$
(*b*) $k \ge 5$
(*c*) $k \le 0$
(*d*) None of these

54. Consider $y = \dfrac{2x}{1+x^2}$, where x is real, then the range of expression $y^2 + y - 2$ is

(*a*) $[-1, 1]$
(*b*) $[0, 1]$
(*c*) $[-9/4, 0]$
(*d*) $[-9/4, 1]$

55. The values of x and y besides y can satisfy the equation (x, y $\in$ real numbers) $x^2 - xy + y^2 - 4x - 4y + 16 = 0$

(*a*) 2, 2
(*b*) 4, 4
(*c*) 3, 3
(*d*) None of these

56. If x is real, then $\dfrac{x^2-x+c}{x^2+x+2c}$ can take all real values if

(*a*) $c \in [0, 6]$
(*b*) $c \in [-6, 0]$
(*c*) $c \in (-\infty, -6) \cup (0, \infty)$
(*d*) $c \in (-6, 0)$

Location of Roots

57. The real values of 'a' for which the quadratic equation $2x^2 - (a^3 + 8a - 1)\,x + a^2 - 4a = 0$ possesses roots of opposite sign is given by:

(*a*) $a > 5$
(*b*) $0 < a < 4$
(*c*) $a > 0$
(*d*) $a > 7$

58. If a, b are the roots of the quadratic equation $x^2 - 2p\,(x-4) - 15 = 0$, then the set of values of 'p' for which one root is less than 1 & the other root is greater than 2 is:

(*a*) $(7/3, \infty)$
(*b*) $(-\infty, 7/3)$
(*c*) $x \in R$
(*d*) None of these

59. The values of k, for which the equation $x^2 + 2(k-1)\,x + k + 5 = 0$ possess atleast one positive root, are

(*a*) $[4, \infty)$
(*b*) $(-\infty, -1] \cup [4, \infty)$
(*c*) $[-1, 4]$
(*d*) $(-\infty, -1]$

Learning Plus

1. If $a > 0$, $b > 0$, $c > 0$ then both the roots of the equation $ax^2 + bx + c = 0$

(*a*) Are real and negative

(*b*) Have negative real parts

(*c*) Are rational numbers

(*d*) Both (*a*) and (*c*)

2. The value of k for which the equation $(k-2)x^2 + 8x + k + 4 = 0$ has both real, distinct and negative is -

(*a*) 0

(*b*) 2

(*c*) 3

(*d*) -4

3. If α, β are the roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is

(*a*) $acx^2 + (a + c)\, bx + (a + c)^2 = 0$

(*b*) $abx^2 + (a + c)\, bx + (a + c)^2 = 0$

(*c*) $acx^2 + (a + b)\, cx + (a + c)^2 = 0$

(*d*) $acx^3 + (a + c)\, bx + (a + c)^3$

4. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then

$$\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} =$$

(*a*) $\frac{2}{a}$

(*b*) $\frac{2}{b}$

(*c*) $\frac{2}{c}$

(*d*) $-\frac{2}{a}$

5. If the roots of the equation $12x^2 - mx + 5 = 0$ are in the ratio 2 : 3, then m =

(*a*) $5\sqrt{10}$

(*b*) $3\sqrt{10}$

(*c*) $2\sqrt{10}$

(*d*) $10\sqrt{5}$

6. $x^2 - 11x + a$ and $x^2 - 14x + 2a$ will have a common factor, if $a =$

(*a*) 24

(*b*) 0, 24

(*c*) 3, 24

(*d*) 0, 3

7. If α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$, then $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1}$

(*a*) 2

(*b*) 3

(*c*) 4

(*d*) 5

8. If the roots of $x^2 + x + a = 0$ exceed a , then

(*a*) $2 < a < 3$

(*b*) $a > 3$

(*c*) $-3 < a < 3$

(*d*) $a < -2$

9. The value of *p* for which both the roots of the equation $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2, lies in

(*a*) $(4/5, 2)$

(*b*) $(2, \infty)$

(*c*) $(-1, -4/5)$

(*d*) $(-\infty, -1)$

10. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has -

(*a*) No root

(*b*) One root

(*c*) Two equal root

(*d*) Infinitely many roots

11. The number of quardratic equations which are unchanged by squaring their roots

(*a*) 2

(*b*) 3

(*c*) 4

(*d*) None of these

12. For what value of the curve $y = x^2 + ax + 25$ touches the x-axis

(*a*) 0

(*b*) ± 5

(*c*) ± 10

(*d*) None of these

13. If $b^2 < 2ac$, then equation $ax^3 + bx^2 + cx + d = 0$ has

(*a*) exactly one real roots

(*b*) Has three real roots

(*c*) at least two roots

(*d*) None of these

14. Let α and β are the roots of the equation $x^2 + x + 1 = 0$ then

(*a*) $\alpha^2 + \beta^2 = 4$

(*b*) $(\alpha - \beta)^2 = 3$

(*c*) $\alpha^3 + \beta^3 = 2$

(*d*) $\alpha^4 + \beta^4 = 1$

15. If $\sec\alpha$, $\tan\alpha$ are roots of $ax^2 + bx + c = 0$, then

(*a*) $a^4 - b^4 + 4ab^2c = 0$

(*b*) $a^4 + b^4 - 4ab^2c = 0$

(*c*) $a^2 - b^2 = 4ac$

(*d*) $a^2 + b^2 = ac$

16. Let α, β be the roots of $ax^2 + bx + c = 0$, γ, δ be the roots of $px^2 + qx + r = 0$ and D_1 and D_2 be their respective discriminant. If $\alpha, \beta, \gamma, \delta$, are in A.P., then the ratio $D_1 : D_2$ is equal to

(*a*) $\frac{a^2}{b^2}$

(*b*) $\frac{a^2}{p^2}$

(*c*) $\frac{b^2}{q^2}$

(*d*) $\frac{c^2}{r^2}$

17. If one root of the equation $(l - m)x^2 + lx + 1 = 0$ is double the other and if *l* is real, then the greatest value of m is

(*a*) $\frac{9}{8}$

(*b*) $\frac{7}{8}$

(*c*) $\frac{8}{9}$

(*d*) $\frac{8}{9}$

18. If p, q, r are real numbers satisfying the condition $p + q + r = 0$, then the roots of the quadratic equation $3px^2 + 5qx + 7r = 0$ are

(a) Positive (b) Negative

(c) Real and distinct (d) Imaginary

19. If a, b, c are in G.P., then the equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

(a) A.P. (b) G.P.

(c) H.P. (d) $ab = cd$

20. If $a < b < c < d$ and $K > 0$, then the quadratic equation $(x - a)(x - c) + k(x - b)(x - d) = 0$ has

(a) All roots real and distinct

(b) All roots real but not necessarily distinct

(c) All root real and negative

(d) May be imaginary

21. The number of integers satisfying the inequality $\frac{x}{x+6} \le \frac{1}{x}$ is:

(a) 7 (b) 8

(c) 9 (d) 3

22. Sum of values of x and y satisfying the equation $3^x - 4^y = 77; 3^{x/2} - 2^y = 7$ is:

(a) 2 (b) 3

(c) 4 (d) 5

23. If the value of $m^4 + \frac{1}{m^4} = 119$, then the value of $\left|m^3 - \frac{1}{m^3}\right| =$

(a) 11 (b) 18

(c) 24 (d) 36

24. The number of integral roots of the equation $x^8 - 24x^7 - 18x^5 + 39x^2 + 1155 = 0$ is:

(a) 0 (b) 2

(c) 4 (d) 6

25. If the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the product of the roots is:

(a) $-2(p^2 + q^2)$ (b) $-(p^2 + q^2)$

(c) $-\frac{(p^2+q^2)}{2}$ (d) $-pq$

26. If $\alpha \ne \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$, then the equation with roots $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ is:

(a) $3x^2 - 25x + 3 = 0$

(b) $x^2 + 5x - 3 = 0$

(c) $x^2 - 5x + 3 = 0$

(d) $3x^2 - 19x + 3 = 0$

27. Minimum possible number of positive root of the quadratic equation $x^2 - (1 + \lambda)x + \lambda - 2 = 0, \lambda \in R$:

(a) 2 (b) 0

(c) 1 (d) Can not be determined

28. If $\alpha, \beta, \gamma, \delta \in R$ satisty

$$\frac{(\alpha+1)^2 + (\beta+1)^2 + (\gamma+1)^2 + (\delta+1)^2}{\alpha+\beta+\gamma+\delta} = 4$$

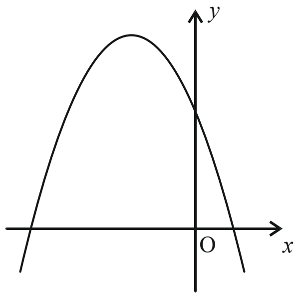
If biquadratic equation $a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$ has the roots $\left(\alpha + \frac{1}{\beta} - 1\right), \left(\beta + \frac{1}{\gamma} - 1\right), \left(\gamma + \frac{1}{\delta} - 1\right), \left(\delta + \frac{1}{\alpha} - 1\right)$.

Then the value of a_2/a_0 is:

(a) 4 (b) –4

(c) 6 (d) None the these

29. If graph of the quadratic $y = ax^2 + bx + c$ is given below:



(a) $a > 0, b > 0, c > 0$

(b) $a < 0, b > 0, c < 0$

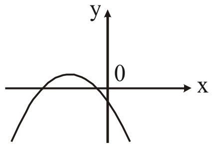
(c) $a < 0, b < 0, c > 0$

(d) $a < 0, b < 0, c < 0$

Advanced Level Multiconcept Questions

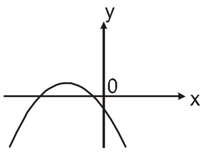
MCQ/COMPREHENSION/MATCHING/ NUMERICAL

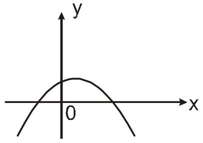
1. The graph of the quadratic polynomial $y = ax^2 + bx + c$ is as shown in the figure. Then:

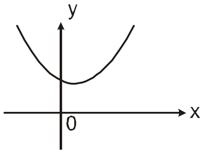


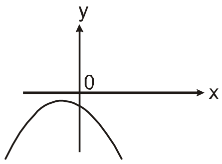
(*a*) $b^2 - 4ac > 0$ (*b*) $b < 0$
(*c*) $a > 0$ (*d*) $c < 0$

2. For which of the following graphs of the quadratic expression $y = ax^2 + bx + c$, the product a b c is negative?

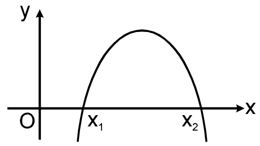
(*a*)


(*b*)


(*c*)


(*d*)


3. The adjoining figure shows the graph of $y = ax^2 + bx + c$. Then



(*a*) $a < 0$
(*b*) $b^2 < 4ac$
(*c*) $c > 0$
(*d*) a and b are of opposite sign

4. For the equation $|x|^2 + |x| - 6 = 0$, the correct statement (s) is (are):
(*a*) sum of roots is 0 (*b*) product of roots is – 4
(*c*) there are 4 roots (*d*) there are only 2 roots

5. If a, b are the roots of $ax^2 + bx + c = 0$, and a + h, b + h are the roots of $px^2 + qx + r = 0$, (where h ¹ 0), then

(*a*) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ (*b*) $h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

(*c*) $h = \frac{1}{2}\left(\frac{b}{a} + \frac{q}{p}\right)$ (*d*) $\frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

6. If a, b are non-zero real numbers and α, β the roots of $x^2 + ax + b = 0$, then
(*a*) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$
(*b*) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax + 1 = 0$
(*c*) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b - a^2)x + b = 0$
(*d*) $(\alpha - 1), (\beta - 1)$ are the roots of the equation $x^2 + x(a + 2) + 1 + a + b = 0$

7. $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$, then the real root of above equation is
(a, b, c, d ∈ R)
(*a*) – d/a (*b*) d/a
(*c*) (b – a)/a (*d*) (a – b)/a

8. If $\frac{1}{2} \le \log_{0.1} x \le 2$, then

(*a*) maximum value of x is $\frac{1}{\sqrt{10}}$
(*b*) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$
(*c*) minimum value of x is $\frac{1}{\sqrt{10}}$
(*d*) minimum value of x is $\frac{1}{100}$

9. If the quadratic equations $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root, then the equation containing their other roots is/are:
(*a*) $x^2 + a(b + c)x - a^2bc = 0$
(*b*) $x^2 - a(b + c)x + a^2bc = 0$
(*c*) $a(b + c)x^2 - (b + c)x + abc = 0$
(*d*) $a(b + c)x^2 + (b + c)x - abc = 0$

10. If the quadratic equations $ax^2 + bx + c = 0$ (a, b, c ∈ R, a ≠ 0) and $x^2 + 4x + 5 = 0$ have a common root, then a, b, c must satisfy the relations:
(*a*) $a > b > c$
(*b*) $a < b < c$
(*c*) a = k; b = 4k; c = 5k (k ∈ R, k ≠ 0)
(*d*) $b^2 - 4ac$ is negative.

11. If α, β are the real and distinct roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always
(*a*) two real roots
(*b*) two negative roots
(*c*) two positive roots
(*d*) one positive root and one negative root

12. If $(x^2+x+1)+(x^2+2x+3)+(x^2+3x+5)+......+(x^2+20x+39)=4500$, then x is equal to
(*a*) 10 (*b*) – 10
(*c*) 20.5 (*d*) –20.5

13. If roots of equation, $x^3+bx^2+cx-1=0$ forms an increasing G.P., then
(*a*) $b+c=0$
(*b*) $b \in (-\infty, -3)$
(*c*) one of the roots = 1
(*d*) one root is smaller than 1 & other > 1

14. Let $f(x)=\frac{3}{x-2}+\frac{4}{x-3}+\frac{5}{x-4}$, then f(x) = 0 has
(*a*) exactly one real root in (2, 3)
(*b*) exactly one real root in (3, 4)
(*c*) at least one real root in (2, 3)
(*d*) None of these

Paragraph for question no. 15 & 16: A quadratic polynomial $f(x)=px^2+qx+r$ has two distinct roots x_1 & x_2. If its vertex (of parabola) is V and x_1, x_3, x_2 are in A.P., then answer the following

Comprehension – 1 (No. 15 to 17)

Consider the equation $x^4-\lambda x^2+9=0$. This can be solved by substituting $x^2=t$ such equations are called as pseudo quadratic equations.

15. If the equation has four real and distinct roots, then λ lies in the interval
(*a*) $(-\infty,-6)\cup(6,\infty)$ (*b*) $(0,\infty)$
(*c*) $(6,\infty)$ (*d*) $(-\infty,-6)$

16. If the equation has no real root, then λ lies in the interval
(*a*) $(-\infty, 0)$ (*b*) $(-\infty, 6)$
(*c*) $(6,\infty)$ (*d*) $(0,\infty)$

17. If the equation has only two real roots, then set of values of λ is
(*a*) $(-\infty,-6)$ (*b*) $(-6, 6)$
(*c*) $\{6\}$ (*d*) ϕ

18. **Match the column.**

Column – I	**Column – II**
(*a*) If α, α + 4 are two roots of $x^2-8x+k=0$, then possible value of k is	(P) 2
(*b*) Number of real roots of equation $x^2-5\|x\|+6=0$ are 'n', then value of $\frac{n}{2}$ is	(Q) 3
(*c*) If $3-i$ is a root of $x^2+ax+b=0$ (a, b ∈ R), then b is	(R) 12
(*d*) If both roots of $x^2-2kx+k^2+k-5=0$ are less than 5, then 'k' may be equal to	(S) 10

NUMERICAL BASED QUESTIONS

19. Find number of integer roots of equation $x(x+1)(x+2)(x+3)=120$.

20. Find product of all real values of x satisfying
$$(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$$

21. The least prime integral value of '2a' such that the roots α, β of the equation $2x^2+6x+a=0$ satisfy the inequality $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}<2$ is

22. If a, b are the roots of $x^2+px+1=0$ and c, d are the roots of $x^2+qx+1=0$. Then find the value of $(a-c)(b-c)(a+d)(b+d)/(q^2-p^2)$.

23. α, β are roots of the equation $\lambda(x^2-x)+x+5=0$. If λ_1 and λ_2 are the two values of λ for which the roots α, β are connected by the relation $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=4$, then the value of $\left(\frac{\frac{\lambda_1}{\lambda_2}+\frac{\lambda_2}{\lambda_1}}{14}\right)$ is

24. Let α, β be the roots of the equation $x^2+ax+b=0$ and γ, δ be the roots of $x^2-ax+b-2=0$. If $\alpha\beta\gamma\delta=24$ and $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}=\frac{5}{6}$, then find the value of a.

25. The least value of expression $x^2+2xy+2y^2+4y+7$ is:

26. If $a>b>0$ and $a^3+b^3+27ab=729$ then the quadratic equation $ax^2+bx-9=0$ has roots α, β (α < β). Find the value of $4\beta-a\alpha$.

27. Let α and β be roots of $x^2-6(t^2-2t+2)x-2=0$ with α > β. If $a_n=\alpha^n-\beta^n$ for n ≥ 1, then find the minimum value of $\frac{a_{100}-2a_{98}}{a_{99}}$ (where t ∈ R)

28. If roots of the equation $x^2-10ax-11b=0$ are c and d and those of $x^2-10cx-11d=0$ are a and b, then find the value of $\frac{a+b+c+d}{110}$. (where a, b, c, d are all distinct numbers)

Topicwise Questions

1. (*c*) We have $4ax^2 + 3bx + 2c = 0$ Let roots are α and β
Let $D = B^2 - 4AC = 9b^2 - 4(4a)(2c) = 9b^2 - 32\,ac$
Given that , $(a + b + c) = 0 \Rightarrow b = -(a + c)$
Putting this value, we get
$= 9(a + c)^2 - 32\,ac \; = 9(a - c)^2 + 4ac$
Hence roots are real.

2. (*d*) Given equation
$2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$
Let $A = 2(a^2 + b^2)$, $B = 2(a + b)$ and $C = 1$
$B^2 - 4AC = 4(a^2 + b^2 + 2ab) - 4.2\,(a^2 + b^2)1$
$\Rightarrow B^2 - 4AC = -4(a - b)^2 < 0$
Thus given equation has imaginary roots.

3. (*a*) Here $(b + c - 2a) + (c + a - 2b) + (a + b - 2c) = 0$
Therefore the roots are rational.

4. (*c*) The quadratic is $(k + 11)\,x^2 - (k + 3)x + 1 = 0$
Accordingly, $(k + 3)^2 - 4(\,k + 11)\,(1) = 0 \Rightarrow k = -7, 5$

5. (*a*) Given equation $(1 + 2k)x^2 + (1 - 2k)x + (1 - 2k) = 0$
If equation is a perfect square then root are equal
i.e., $(1 - 2k)^2 - 4(1 + 2k)(1 - 2k) = 0$
i.e., $k = \frac{1}{2}, \frac{-3}{10}$. Hence total number of values = 2.

6. (*d*) $x^2 - 3x + 2$ be factor of $x^4 - px^2 + q = 0$
Hence $(x^2 - 3x + 2) = 0 \Rightarrow (x - 2)\,(x - 1) = 0$
$\Rightarrow x = 2, 1$ putting these values in given equation
so $4p - q - 16 = 0$ .....(i)
and $p - q - 1 = 0$ .....(ii)
Solving (i) and (ii), we get $(p, q) = (5, 4)$

7. (*b*) check by options
$x = 1$ is root
Let other root = a
$\therefore$ Product of the roots $= (1)\,(1) = \frac{a - b}{b - c}$
$\Rightarrow$ roots are 1, $\frac{a - b}{b - c}$

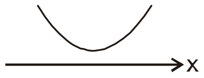
8. (*a*) $D = b^2 - 4ac = 20d^2$
$\sqrt{D} = 2\sqrt{5}d$ here $\sqrt{5}$ is irrational
So roots are irrational.

9. (*a*) $D = b^2 - 4ac = b^2 - 4a\,(-4a - 2b) = b^2 + 16a^2 + 8ab$
Since $ab > 0$
$\therefore D > 0$
So equation has real roots.

10. (*c*) For integral roots, D of equation should be perfect sq.
$\because D = 4(1 + n)$
By observation, for $n \in N$, D should be perfect sq. of even integer.
So $D = 4(1 + n) = 6^2, 8^2, 10^2, 12^2, 14^2, 16^2, 18^2, 20^2$
No. of values of n = 8.

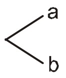
11. (*b*) Here for $D < 0$, entire graph will be above x-axis
$(\because\; a > 0)$
$\Rightarrow \quad (k - 1)^2 - 36 < 0$
$\Rightarrow \quad (k - 7)\,(k + 5) < 0$
$\Rightarrow \quad -5 < k < 7$

12. (*a*) Let $f(x) = ax^2 - bx + 1$
Given $D < 0$ & $f(0) = 1 > 0$
$\therefore$ possible graph is as shown



i.e. $f(x) > 0 \;\forall\; x \in R$
or $f(-1) > 0$
$f(-1) = a + b + 1 > 0$

13. (*c*) $x^2 + ax + b = 0$



$a + b = -a$
$\Rightarrow 2a + b = 0$
and $ab = b$
$ab - b = 0$
$b\,(a - 1) = 0$
$\Rightarrow$ Either $b = 0$ or $a = 1$
But $b \neq 0$(given)
$\therefore \quad a = 1$

$\therefore \quad b = -2$

$\therefore \quad f(x) = x^2 + x - 2$

Least value occurs at $x = -\frac{1}{2}$

Least value $= \frac{1}{4} - \frac{1}{2} - 2 = -\frac{9}{4}$

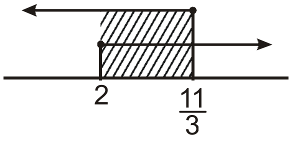
14. (*b*) $(2 - x)(x + 1) = p$

$(x - 2)(x + 1) + p = 0$

$\Rightarrow x^2 - x - 2 + p = 0$

$\frac{c}{a} > 0 \Rightarrow \; p - 2 > 0$

$\& \; D > 0 \Rightarrow \; 1 - 4(p - 2) > 0 \; \Rightarrow \; p < \frac{9}{4}$



$\frac{-b}{2a} > 0, \; \frac{-1}{2(2-p)} > 0, P \in (2, \infty)$

Taking intersection of all $p \in \left(2, \frac{9}{4}\right)$

15. (*c*) $x^2 + px + (1 - p) = 0$

$(1 - p)^2 + p(1 - p) + (1 - p) = 0$

$(1 - p)[1 - p + p + 1] = 0 \Rightarrow p = 1$

Q.E. will be $\Rightarrow \; x^2 + x = 0 \; \Rightarrow \; x(x + 1) = 0$

$\Rightarrow x = 0, -1$

Aliter

$\alpha + 1 - p = -p \Rightarrow \alpha = -1$

Satisfies

$1 - p + 1 - p = 0 \Rightarrow p = 1$

$\beta = 1 - p = 0 \Rightarrow \beta = 0$

16. (*b*) Let first root = α and second root = $\frac{1}{\alpha}$

Then $\alpha, \frac{1}{\alpha} = \frac{k}{5} \Rightarrow k = 5$

17. (*a*) Given equation $4x^2 + 3x + 7 = 0$, therefore

$\alpha + \beta = -\frac{3}{4}$ and $\alpha\beta = \frac{7}{4}$

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-3/7}{7/4} = \frac{-3}{4} \times \frac{4}{7} = -\frac{3}{7}$

18. (*c*) Let α and β be two roots of $ax^2 + bx + c = 0$

Then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{b^2}{a^2} - 2\frac{c}{a}$

So under condition $\alpha + \beta = \alpha^2 + \beta^2 \;\; \alpha + \beta = a^2 + \beta^2$

$\Rightarrow \; -\frac{b}{a} = \frac{b^2 - 2ac}{a^2} \; \Rightarrow b(a + b) = 2ac$

19. (*b*) α, β be the roots of $x^2 - 2x + 3 = 0$, then $\alpha + \beta = 2$ and $\alpha\beta = 3$.

Now required equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ is

$x^2 - \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)x + \frac{1}{\alpha^2\beta^2} = 0$

$\Rightarrow x^2 - \left(-\frac{2}{9}\right)x + \frac{1}{9} = 0 \Rightarrow 9x^2 + 2x + 1 = 0$

20. (*c*) According to condition

$\frac{2m - 1}{m} = -1 \; \Rightarrow \; 3m = 1 \Rightarrow m = \frac{1}{3}$

21. (*b*) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (4)^3 - 3 \times 1(4) = 52$

22. (*d*) We know that the roots of the equation

$ax^3 + bx^2 + cx + d = 0$ follows $\alpha\beta\gamma = -d/a$

Comparing above equation with given equation

we get $d = 1$, $a = 1$

So, $\quad \alpha\beta\gamma = -1$ or $\alpha^3\beta^3\gamma^3 = -1$.

23. (*c*) $a + b = -p$

$ab = q$

$g + d = -p$

$gd = -r$

$(a - g)(a - d) = a^2 - a(g + d) + gd$

$= a^2 + pa - r = a(a + p) - r = -ab - r$

$= -q - r = -(q + r)$

24. (*a*) $|a - b| = 4 \Rightarrow (a - b)^2 = 16$

$\Rightarrow (a + b)^2 - 4ab = 16$

$\Rightarrow 9 - 4ab = 16 \Rightarrow ab = -\frac{7}{4}$

$\Rightarrow$ equation is $x^2 - 3x - \frac{7}{4} = 0$

25. (*a*) $C_1 : b^2 - 4ac \geqslant 0$,

$ax^2 + bx + c = 0$ real roots C_1 satisfied

C_2 : a, –b, c are same sign

$\alpha + \beta > 0 \Rightarrow \frac{-b}{a} > 0$

$\alpha\beta > 0 \Rightarrow \frac{c}{a} > 0$

C_2 satisfied C_1 & C_2 are satisfied

26. (*c*) $ax^2 + bx + c = 0, \alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

$\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$\alpha^3 + \beta^3 = \left(\frac{-b}{a}\right)\left[\left(\frac{-b}{a}\right)^2 - 3\frac{c}{a}\right]$

$= \frac{-b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right] = \frac{-b}{a}\frac{(b^2 - 3ac)}{a^2} = \frac{3abc - b^3}{a^3}$

27. (*b*) Given equation can be written as

$(6k+2)x^2+rx+3k-1=0$ .....(i)

and $2(6k+2)x^2+px+2(3k-1)=0$ .....(ii)

Condition for common roots is

$$\frac{12k+4}{6k+2}=\frac{p}{r}=\frac{6k-2}{3k-1}=2 \text{ or } 2r-p=0$$

28. (a) Let α is the common root,

so $\alpha^2+p\alpha+q=0$ .....(i)

and $\alpha^2+q\alpha+p=0$ ....(ii)

from (i) – (ii),

$\Rightarrow (p-q)\alpha+(q-p)=0 \Rightarrow \alpha=1$

Put the value of α in (i), $p+q+1=0$.

29. (*a*) $D_1=4a^2b^2-8a^2b^2=-4a^2b^2<0$ img. root

$D_2=4p^2q^2-4p^2q^2=0$ equal, real roots

So no common roots.

30. (*b*) Given , $x+2>\sqrt{x+4} \Rightarrow (x+2)^2>(x+4)$

$\Rightarrow x+4x+4>x+4 \Rightarrow x^2+3x>0$

$\Rightarrow x(x+3)>0 \Rightarrow x<-3 \text{ or } x>0 \Rightarrow x>0$

31. (*b*) **Case I:** When $x+2\geq 0$ i.e. $x\geq -2$

Then given inequality becomes

$x^2-(x+2)+x>0 \Rightarrow x^2-2>0 \Rightarrow |x|>\sqrt{2}$

$\Rightarrow x<-\sqrt{2}$ or $x>\sqrt{2}$

As $x\geq -2$, therefore, in this case the part of the solution set is $[-2,-\sqrt{2})\cup(\sqrt{2},\infty)$.

Case II: When $x+2\leq 0$ *i.e.* $x\leq -2$,

Then given inequality becomes $x^2+(x+2)+x>0$

$\Rightarrow x^2+2x+2>0 \Rightarrow (x+1)^2+1>0$, which is true for all real x

Hence, the part of the solution set in this case is $(-\infty,-2]$. Combining the two cases, the solution set is

$(-\infty,-2)\cup([-2,-\sqrt{2}]\cup(\sqrt{2},\infty)=(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$.

32. (*b*) For $(p^2-3p+2)x^2-(p^2-5p+4)x+p-p^2=0$ to be an identity

$p^2-3p+2=0 \Rightarrow p=1,2$ ...(1)

$p^2-5p+4=0 \Rightarrow p=1,4$ ...(2)

$p-p^2=0 \Rightarrow p=0,1$ ...(3)

For (1), (2) & (3) to hold simultaneously

$p=1$.

33. (*d*) $x^2+9<(x+3)^2<8x+25$

$x^2+9<x^2+6x+9 \Rightarrow x>0$

$\&\ (x+3)^2<8x+25$

$x^2+6x+9-8x-25<0$

$x^2-2x-16<0$

$1-\sqrt{17}<x<1+\sqrt{17}\ \&\ x>0$

$\Rightarrow x\in(0,1+\sqrt{17})$

Integer x = 1, 2, 3, 4, 5

No. of integer are = 5

34. (*d*) $5x+2<3x+8 \Rightarrow 2x<6 \Rightarrow x<3$ ...(i)

$\frac{x+2}{x-1}<4 \Rightarrow \frac{x+2}{x-1}-4<0 \Rightarrow \frac{-3x+6}{x-1}<0$

$\Rightarrow \frac{x-2}{x-1}>0 \Rightarrow x\in(-\infty,1)\cup(2,\infty)$ ...(ii)

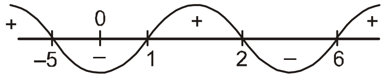
Taking intersection of (i) and (ii) $x\in(-\infty,1)\cup(2,3)$

35. (*b*) $\frac{x^2(x^2-3x+2)}{x^2-x-30}\geq 0$

$\Rightarrow \frac{x^2(x-1)(x-2)}{(x+5)(x-6)}\geq 0$

$x\neq -5, 6$

$x\in(-\infty,-5)\cup[1,2]\cup(6,\infty)\cup\{0\}$



36. (*b* $\because$ $(m-2)x^2+8x+m+4>0\ \forall x\in R$

$\Rightarrow m>2\ \&\ D<0$

$64-4(m-2)(m+4)<0$

$16-[m^2+2m-8]<0$

$\Rightarrow m^2+2m-24>0$

$\Rightarrow (m+6)(m-4)>0$

$m\in(-\infty,-6)\cup(4,\infty)$

But $m>2$

$\Rightarrow m\in(4,\infty)$

Then least integral m is m = 5.

37. (*b*) $-1\leq|x-1|-1\leq 1$

$\Rightarrow 0\leq|x-1|\leq 2$

$\Rightarrow 0\leq|x-1|$

$\Rightarrow x\in R$ ...(1)

and $|x-1|\leq 2$

$\Rightarrow -2\leq x-1\leq 2$

$\Rightarrow -1\leq x\leq 3$ ...(2)

$(1)\cap(2)$

$\Rightarrow x\in[-1,3]$.

38. (*a*) $\log_{1/3}\frac{3x-1}{x+2}<1$

$\Rightarrow \frac{3x-1}{x+2}>0 \Rightarrow x\in(-\infty,-2)\cup\left(\frac{1}{3},\infty\right)$ ....(i)

and $\frac{3x-1}{x+2}>\frac{1}{3}$

$\Rightarrow \frac{8x-5}{x+2}>0$

$\Rightarrow x\in(-\infty,-2)\cup\left(\frac{5}{8},\infty\right)$ .....(ii)

$(i)\cap(ii) \Rightarrow x\in(-\infty,-2)\cup\left(\frac{5}{8},\infty\right)$

39. (*b*) $2 - \log_2 (x^2 + 3x) \geq 0$

$\Rightarrow \quad \log_2 (x^2 + 3x) \leq 2$

$x^2 + 3x > 0$

$\Rightarrow \quad x \in (-\infty, -3) \cup (0, \infty)$ .....(i)

and $x^2 + 3x \leq 4$

$\Rightarrow \quad (x-1)(x+4) \leq 0$

$\Rightarrow \quad x \in [-4, 1]$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow x \in [-4, -3) \cup (0, 1]$

40. (*d*) $\log_{1-x} (x - 2) \geq 0$

$x > 2$ ....(1)

(i) When $0 < 1 - x < 1 \Rightarrow 0 < x < 1$

So no common range comes out.

(ii) When $1 - x > 1 \Rightarrow x < 0$ but $x > 2$

here, also no common range comes out. , hence no solution.

Finally, no solution

41. (*a*) $\log_{0.3} (x - 1) < \log_{0.09} (x - 1)$

$\log_{0.3} (x - 1) < \dfrac{\log_{0.3} (x - 1)}{2}$

$\Rightarrow \log_{0.3} (x - 1) < 0 \quad \Rightarrow \quad x - 1 > 1 \quad \Rightarrow \quad x > 2$

42. (*a*) $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$

$\log_{0.5} \log_5 (x^2 - 4) > 0$

$\Rightarrow x^2 - 4 > 0$

$\Rightarrow x \in (-\infty, -2) \cup (2, \infty)$ ....(i)

$\log_5 (x^2 - 4) > 0 \Rightarrow x^2 - 5 > 0$

$\Rightarrow x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$ ....(ii)

$\log_5 (x^2 - 4) < 1$

$\Rightarrow x^2 - 9 < 0 \Rightarrow x \in (-3, 3)$ ....(iii)

(i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow x \in (-3, \sqrt{5}) \cup (\sqrt{5}, 3)$

43. (*d*) $\left(\dfrac{1}{2}\right)^{x^2 - 2x} < \left(\dfrac{1}{2}\right)^2$ here base is less than zero so inequality change

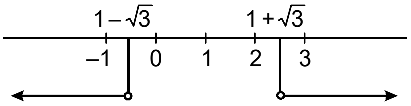
$\Rightarrow x^2 - 2x > 2 \Rightarrow x^2 - 2x - 2 > 0$

$\alpha, \beta = \dfrac{2 \pm \sqrt{4 + 8}}{2} = \dfrac{2 \pm 2\sqrt{3}}{2}$

$a = 1 - \sqrt{3}$, $b = 1 + \sqrt{3}$

$(x - a)(x - b) > 0$

$x \in \left(-\infty, \ 1 - \sqrt{3}\right) \cup \left(1 + \sqrt{3}, \ \infty\right)$,x can be in $(3, \infty)$



44. (*a*) $\dfrac{6x^2 - 5x - 3}{x^2 - 2x + 6} \leq 4$

D^r is always > 0

$6x^2 - 5x - 3 - 4x^2 + 8x - 24 \leq 0$

$\Rightarrow 2x^2 + 3x - 27 \leq 0$

$\Rightarrow (2x + 9)(x - 3) \leq 0 \Rightarrow x \in \left[\dfrac{-9}{2}, 3\right]$

least value of $4x^2 = 4.0^2 = 0$

Highest value of $4x^2$ is $= \max \left(4 \cdot \left(-\dfrac{9}{2}\right)^2, \ 4 \cdot 3^2\right)$

$= \max (81, 36) = 81$

45. (*c*) Let the roots be a, b, –b

then $\alpha + \beta - \beta = p$

$\Rightarrow \alpha = p$ ...(1)

and $\alpha\beta - \alpha\beta - \beta^2 = q$

$\Rightarrow \beta^2 = -q$ ...(2)

also $-\alpha\beta^2 = r$

$\Rightarrow pq = r$ [using (1)].

46. (*c*) $x^3 - x - 1 = 0$ α β γ

then $\alpha^3 - \alpha - 1 = 0$.............(1)

Let $\dfrac{1 + \alpha}{1 - \alpha} = y \Rightarrow \quad \alpha = \dfrac{y - 1}{y + 1}$

from equation (1) $\left(\dfrac{y - 1}{y + 1}\right)^3 - \left(\dfrac{y - 1}{y + 1}\right) - 1 = 0$

$\Rightarrow \quad y^3 + 7y^2 - y + 1 = 0$ $\dfrac{1 + \alpha}{1 - \alpha}$ $\dfrac{1 + \beta}{1 - \beta}$ $\dfrac{1 + \gamma}{1 - \gamma}$

then $\dfrac{1 + \alpha}{1 - \alpha} + \dfrac{1 + \beta}{1 - \beta} + \dfrac{1 + \gamma}{1 - \gamma} = -7$ Ans.

47. (*d*) $x^4 - 4x^3 + ax^2 + bx + 1 = 0$

real & positive roots

$\alpha + \beta + r + \delta = 4$ & $\alpha \beta r \delta = 1$

$\Rightarrow \alpha = \beta = r = \delta = 1$

$\Sigma\alpha\beta = a \Rightarrow a = 6$

$\Sigma\alpha\beta r = -b \Rightarrow b = -4$

or $(x - 1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$

48. (*d*) $ax^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

sum of roots $= (2\alpha + 3\beta) + (3\alpha + 2\beta)$

$= 5(\alpha + \beta) = 5\left(-\frac{b}{a}\right)$

Product of roots $= 6\alpha^2 + 6\beta^2 + 13\alpha\beta = 6(\alpha + \beta)^2 + \alpha\beta$

$= 6\left(\frac{-b}{a}\right)^2 + \frac{c}{a} \;\; = \frac{6b^2}{a^2} + \frac{c}{a}$

Q. E. $\;x^2 + \frac{5b}{a}x + \frac{6b^2}{a^2} + \frac{c}{a} = 0$

$a^2x^2 + 5abx + 6b^2 + ac = 0$

49. (*a*) $\Rightarrow \frac{(2x-1)}{x(2x^2+3x+1)} > 0$

$\Rightarrow \frac{(2x-1)}{x(x+1)(2x+1)} > 0$

$$\overset{+}{}\underset{-1}{|}\underset{-}{}\underset{-1/2}{|}\overset{+}{}\underset{0}{|}\underset{-}{}\underset{1/2}{|}\overset{+}{}$$

consontains $\left(-\infty, \frac{-3}{2}\right)$

50. (*c*) Let $y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$\Rightarrow (y-1)x^2 + 3(y+1)x + 4(y-1) = 0$

For x is real $D \geq 0$

$\Rightarrow 9(y+1)^2 - 16(y-1)^2 \geq 0 \; 9(y+1)^2 - 16(y-1)^2 \geq 0$

$\Rightarrow -7y^2 + 50y - 7 \geq 0 \Rightarrow 7y^2 - 50y + 7 \leq 0$

$\Rightarrow (y-7)(7y-1) \leq 0$

Now, the product of two factors is negative if one in –ve and one in +ve.

Case I : $(y-7) \geq 0$ and $(7y-1) \leq 0$

$\Rightarrow y \geq 7$ and $y \geq \frac{1}{7}$. But it is impossible

Case II : $(y-7) \leq 0$ and $(7y-1) \geq 0$

$\Rightarrow y \leq 7$ and $y \geq \frac{1}{7} \Rightarrow \frac{1}{7} \leq y \leq 7$

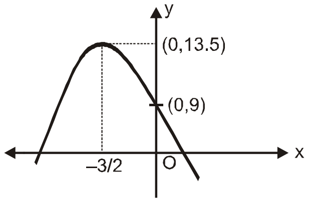
Hence maximum value is 7 and minimum value is $\frac{1}{7}$

51. (*a*) $x^2 - 3x + 3 = \left(x - \frac{3}{2}\right)^2 + \frac{3}{4}$

Therefore, smallest value is $\frac{3}{4}$, which lie in $\left(-3, \frac{3}{2}\right)$

52. (*c*) $y = -2x^2 - 6x + 9$

$\therefore \frac{-b}{2a} = \frac{6}{2(-2)} = -\frac{3}{2} = -1.5$



$\& D = 36 - 4(-2)(9) = 36 + 72 = 108$

$\therefore -\frac{D}{4a} = -\frac{108}{4(-2)} = +\frac{108}{8} = 13.5$

$\Rightarrow y \in (-\infty, 13.5]$

53. (*a*) $k = \frac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow (k-1)x^2 + (k+1)x + (k-1) = 0$

Q x is real

$\therefore D \geq 0$

$\Rightarrow (k+1)^2 - 4(k-1)^2 \geq 0$

$\Rightarrow (3k-1)(k-3) \leq 0$

$\Rightarrow k \in \left[\frac{1}{3}, 3\right]$

54. (*c*) $y = \frac{2x}{1+x^2}, x \in R$

$\Rightarrow yx^2 - 2x + y = 0$

$\Rightarrow D \geq 0 \Rightarrow 4 - 4y^2 \geq 0$

$\Rightarrow (y^2 - 1) \leq 0 \Rightarrow y \in [-1, 1]$

$\therefore$ Range of $f(y) = y^2 + y - 2$

Min value $= \frac{-D}{4a} = \frac{-9}{4}$ at $y = \frac{-b}{2a} = \frac{-1}{2}$

$y = \frac{-1}{2} \in [-1, 1]$

$f(-1) = 1 - 1 - 2 = -2$

$f(1) = 1 + 1 - 2 = 0$

max value is = 0

Range $\left[\frac{-9}{4}, \; 0\right]$

55. (*b*) $x^2 - xy + y^2 - 4x - 4y + 16 = 0$, $x, y \in R$

$x^2 - x(y+4) + (y^2 - 4y + 16) = 0 \qquad \ldots(1)$

$x \in R \Rightarrow D \geq 0$

$(y+4)^2 - 4(y^2 - 4y + 16) \geq 0$

$\Rightarrow y^2 + 8y + 16 - 4y^2 + 16y - 64 \geq 0$

$\Rightarrow\ y^2 - 8y + 16 \le 0$

$\Rightarrow\ (y-4)^2 \le 0 \Rightarrow y = 4$

Put is given equation (i)

$x^2 - 8x + 16 = 0$

$\Rightarrow (x-4)^2 = 0 \Rightarrow x = 4$

56. (*d*) $(y-1)x^2 + (y+1)x + (2cy - c) = 0$

$D \ge 0 \therefore\ x \in R$

$\Rightarrow (y+1)^2 - 4(y-1)(2cy - c) \ge 0$

$y^2 + 2y + 1 - 8cy^2 + 12cy - 4c \ge 0$

$(1-8c)y^2 + (2+12c)y + (1-4c) \ge 0$

$\forall\ y \in R, D \le 0$

$(2+12c)^2 - 4(1-8c)(1-4c) \le 0$

$(1+6c)^2 - (1-8c)(1-4c) \le 0$

$4c^2 + 24c \le 0 \Rightarrow c \in [-6, 0]$

& N^r & D^r have no any common root

(i) both common factor (root) (not possible)

$$\frac{1}{1} = \frac{-1}{+1} = \frac{c}{2c}$$

(ii) If one common root is α

$(\alpha^2 - \alpha + c = 0) \times 2$

& $\alpha^2 + \alpha + 2c = 0$

$\alpha^2 - 3\alpha = 0$

$\alpha = 0 \Rightarrow c = 0$

or $\quad \alpha = 3 \Rightarrow c = -6$

$\therefore \quad c \ne 0$ & $c \ne -6$

$\therefore \quad c \in (-6, 0)$

57. (*b*) $2x^2 - (a^3 + 8a - 1)x + (a^2 - 4a) = 0$

since the roots are of opposite sign, $f(0) < 0$

$\Rightarrow a^2 - 4a < 0$

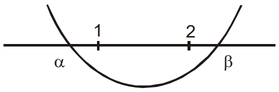
$\Rightarrow a(a-4) < 0$

$\Rightarrow a \in (0, 4)$

58. (*b*) $x^2 - 2px + (8p - 15) = 0$

$f(1) < 0$ and $f(2) < 0$

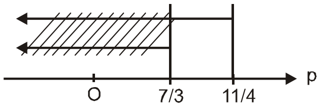
$\Rightarrow f(1) = 1 - 2p + 8p - 15 < 0$



$\Rightarrow p < 7/3$

and $f(2) = 4 - 4p + 8p - 15 < 0$

$\Rightarrow 4p - 11 < 0 \Rightarrow\ p < \dfrac{11}{4}$



Hence $p \in (-\infty, 7/3)$ Ans.

59. (*d*) $x^2 + 2(k-1)x + k + 5 = 0$

Case - I (i) D … 0

$\Rightarrow 4(k-1)^2 - 4(k+5) \ldots 0$

$\Rightarrow k^2 - 3k - 4 \ldots 0 \Rightarrow (k+1)(k-4) \ldots 0$

$\Rightarrow k \in (-\infty, -1] \cup [4, \infty)$

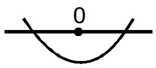
& (ii) $f(0) > 0 \Rightarrow k + 5 > 0 \Rightarrow k \in (-5, \infty)$

& (iii) $\dfrac{-b}{2a} > 0 \Rightarrow \dfrac{-2(k-1)}{2} > 0$

$\Rightarrow k \in (-\infty, 1) \therefore k \in [-5, -1]$

Case - II $f(0) \le 0 \Rightarrow k + 5 \le 0$

$\Rightarrow k \in (-\infty, -5]$



Finally $k \in$ (Case - I) $\cup$ (Case - II)

$k \in (-\infty, -1]$

Learning Plus

1. (*b*) The roots of the equations are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) Let $b^2 - 4ac > 0, b > 0$

Now if $a > 0$, $c > 0$, $b^2 - 4ac < b^2$

$\Rightarrow$ the roots are negative.

(ii) Let $b^2 - 4ac < 0$, then the roots are given by

$$x = \frac{-b \pm i\sqrt{(4ac - b^2)}}{2a}, \ (i = \sqrt{-1})$$

Which are imaginary and have negative real part

$(\because b > 0)$

$\therefore$ In each case, the roots have negative real part.

2. (*c*) From options put $k = 3 \Rightarrow x^2 + 8x + 7 = 0$

$\Rightarrow (x + 1)(x + 7) = 0 \Rightarrow x = -1, -7$

means for k = 3 roots are negative.

3. (*a*) Here $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

If roots are $\alpha + \frac{1}{\beta}$, $\beta + \frac{1}{\alpha}$ then sum of roots are

$$= \left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = (\alpha + \beta) + \frac{(\alpha + \beta)}{\alpha\beta} = \frac{b}{ac}(a + c)$$

and product $= \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

$$= \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta} = 2 + \frac{c}{a} + \frac{a}{c}$$

$$= \frac{2ac + c^2 + a^2}{ac} = \frac{(a + c)^2}{ac}$$

Hence required equation is given by

$$x^2 + \frac{b}{ac}(a + c)x + \frac{(a + c)^2}{ac} = 0$$

$\Rightarrow acx^2 + (a + c)bx + (a + c)^2 = 0$

Trick : Let $a = 1$, $b = -3$, $c = 2$, then $\alpha = 1$, $\beta = 2b$ $= -3$, $c = 2$, then $\alpha = 1$, $\beta = 2$

$\therefore \ \alpha + \frac{1}{\beta} = \frac{3}{2}$ and $\beta + \frac{1}{\alpha} = 3$

Therefore, required equation must be

$(x - 3)(2x - 3) = 0$ i.e. $2x^2 - 9x + 9 = 0$

Here (1) gives this equation on putting

$a = 1, b = -3, c = 2$

4. (*d*) $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

and $\alpha^2 + \beta^2 = \frac{(b^2 - 2ac)}{a^2}$

$$\text{Now } \frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} = \frac{\alpha(a\alpha + b) + \beta(a\beta + b)}{(a\beta + b)(a\alpha + b)}$$

$$= \frac{a(\alpha^2 + \beta^2) + b(\alpha + \beta)}{\alpha\beta a^2 + ab(\alpha + \beta) + b^2} = \frac{a\frac{(b^2 - 2ac)}{a^2} + b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2 + ab\left(-\frac{b}{a}\right) + b^2}$$

$$= \frac{b^2 - ac - b^2}{a^2c - ab^2 + ab^2} = \frac{-ac}{a^2c} = -\frac{2}{a}$$

5. (*a*) Let roots are α, β so, $\frac{\alpha}{\beta} = \frac{2}{3} \Rightarrow \alpha = \frac{2\beta}{3}$

$\therefore \ \alpha + \beta = \frac{m}{12}$

$$\Rightarrow \frac{2\beta}{3} + \beta = \frac{m}{12} \Rightarrow \frac{5\beta}{3} = \frac{m}{12} \qquad .....(i)$$

and $\alpha\beta = \frac{5}{12} \Rightarrow \frac{2\beta}{3}.\beta = \frac{5}{12} \Rightarrow \beta^2 = \frac{5}{8}$

$\Rightarrow \beta = \sqrt{5/8}$

Put the value of β in (i), $\frac{5}{3}.\sqrt{\frac{5}{8}} = \frac{m}{12} \Rightarrow m = 5\sqrt{10}$.

6. (*b*) Expressions are $x^2 - 11x + a$ and $x^2 - 14x + 2a$ will have a common factor, then

$$\Rightarrow \frac{x^2}{-22a + 14a} = \frac{x}{a - 2a} = \frac{1}{-14 + 11}$$

$$\Rightarrow \frac{x^2}{-8a} = \frac{x}{-a} = \frac{1}{-3} \Rightarrow x^2 = \frac{8a}{3} \text{ and } x = \frac{a}{3}$$

$$\Rightarrow \left(\frac{a}{3}\right)^2 = \frac{8a}{3} \Rightarrow \frac{a^2}{9} = \frac{8a}{3} \Rightarrow a = 0, 24.$$

Trick : We can check by putting the values of *a* from the options.

7. (*c*) If α, β, γ are the roots of the equation.

$x^3 - px^2 + qx - r = 0$

$$\therefore (\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1} = \frac{p^2 + q}{pq - r}$$

Given, $p = 0, q = 4, r = -1$

$$\Rightarrow \frac{p^2 + q}{pq - r} = \frac{0 + 4}{0 + 1} = 4$$

8. (*d*) If the roots of the quadratic equation $ax^2 + bx + c = 0$ exceed a number k , then $ak^2 + bk + c > 0$ if $a > 0$, $b^2 - 4ac \geq 0$ and sum of the roots > 2k. Therefore, if the roots of $x^2 + x + a = 0$ exceed a number a , then

$a^2 + a + a > 0$, $1 - 4a \geq 0$ and $-1 > 2a$

$$\Rightarrow a(a + 2) > 0, a \leq \frac{1}{4} \text{ and } a < -\frac{1}{2} \quad \Rightarrow a > 0 \text{ or}$$

$$a < -2, a < \frac{1}{4} \text{ and } a < -\frac{1}{2}$$

Hence $a < -2$.

9. (*d*) Let

$f(x) = 4x^2 - 20px + (25p^2 + 15p - 66) = 0$ .....(i)

The roots of (i) are real if $b^2 - 4ac = 400p^2 - 16(25p^2 + 15p - 66) = 16(66 - 15p) \geq 0$

$\Rightarrow p \leq 22/5$ .....(ii)

Both roots of (i) are less than 2. Therefore f(2) > 0 and sum of roots < 4.

$$\Rightarrow 4.2^2 - 20p.2 + (24p^2 + 15p - 66) > 0 \text{ and } \frac{20p}{4} < 4$$

$$\Rightarrow p^2 - p - 2 > 0 \text{ and } p < \frac{4}{5}$$

$$\Rightarrow (p + 1)(p - 2) > 0 \text{ and } p < \frac{4}{5}$$

$$\Rightarrow p < -1 \text{ or } p > 2 \text{ and } p < \frac{4}{5} \Rightarrow p < -1 \ .....(iii)$$

From (ii) and (iii), we get $p < -1$ i.e. $p \in (-\infty, -1)$.

10. (*a*) $x-\frac{2}{x-1}=1-\frac{2}{x-1}$

Since, $x-1$ is in denominator

$x-1\neq 0$

$x\neq 1$

$x-\frac{2}{x-1}= \ 1-\frac{2}{x-1}$

$\Rightarrow x=1$ But x can't be 1. So, No roots.

11. (*c*) If roots are α and β.

Then, $\alpha+\beta=\alpha^2+\beta^2$

$\alpha\beta=\alpha^2\beta^2$

$\Rightarrow \quad \alpha^2\beta^2-\alpha\beta=1$

$\alpha\beta(\alpha\beta-1)=0$

$\alpha\beta=0$ or $\alpha\beta=1$

$\Rightarrow \quad \alpha=0$ or $\beta=0$ or $\alpha\beta=1$

C1: if $\alpha=0$

$0+\beta=0+\beta^2\Rightarrow\beta^2-\beta=0$

$\beta(\beta-1)=0\Rightarrow\beta=0$ or 1.

Roots are 0,0 and 0,1

C2: Similarly, for $\beta=0\Rightarrow\alpha=0$ or 1

Roots will be 0,0 and 1,0

C3: $\alpha\beta=1$

$\alpha^2+\beta^2=\alpha+\beta$

$(\alpha+\beta)^2-2\alpha\beta=\alpha+\beta$

$(\alpha+\beta)^2-(\alpha+\beta)-2=0$

Let $\alpha+\beta=\mathrm{t}$

$\mathrm{t}^2-\mathrm{t}-2=0$

$(\mathrm{t}-2)(\mathrm{t}+1)=0$

$\mathrm{t}=2$ or $\mathrm{t}=-1$

$\alpha+\beta=2$ or $\alpha+\beta=-1$

for $\alpha+\beta=2, \alpha\beta=1$

Roots are 1, 1

for $\alpha+\beta=-1, \alpha\beta=1$

Roots are w, w^2.

Hences total roots are possible.

0, 0

0, 1

1, 1

ω, ω^2

12. (*c*) for $y=x^2+ax+25$ to touch the x - axis, it should have equal & real roots, i.e D = 0

$a^2-4.1.25=0$

$a^2-100=0$

$(a-10)(a+10)=0$

$a=10$ or -10

13. (*a*) $ax^3+bx^2+cx+d=0$

Let roots be α, β, γ.

$\alpha+\beta+\gamma=-\frac{b}{a}$

$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}$

$\alpha\beta\gamma=-\frac{d}{a}$

Now, $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$$=\left(\frac{-b}{a}\right)^2-\frac{2c}{a}=\frac{b^2}{a^2}-\frac{2c}{a}=\frac{b^2-2ac}{a^2}$$

As $b^2<2ac$

$b^2-2ac<0 \quad \Rightarrow \quad \alpha^2+\beta^2+\gamma^2<0$

↓

Not possible if all α,β,γ are real.

And complex root occurs in pair, so two out of α,β,γ are complex & one is real.

14. (*c*) $x^2+x+1=0$

$$x=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{-3}}{2}=\mathrm{w}, \mathrm{w}^2$$

$\alpha=\omega$, $\beta=\omega^2$, ω is cube toot of unity.

(*a*) $\alpha^2+\beta^2=\omega^2+\omega^4=\omega^2+\omega=-1$ $(1+\omega+\omega^2=0)$

(*b*) $(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta$

$=-1-2.\omega^3$

$=-1-2=-3 \qquad (\omega^3=1)$

(*c*) $\alpha^3+\beta^3=\omega^3+(\omega^2)^3=1+1=2$

15. (*a*) $ax^2+bx+c=0$

$\sec\alpha$ and $\tan\alpha$ are roots.

$\sec\alpha+\tan\alpha=-\frac{b}{a}$; $\sec\alpha. \tan\alpha=\frac{c}{a}$

$(\sec\alpha-\tan\alpha)^2=(\sec\alpha+\tan\alpha)^2-4\sec\alpha\tan\alpha$

$$=\frac{b^2}{a^2}-\frac{4c}{a}=\frac{b^2-4ac}{a^2}$$

$\sec\alpha-\tan\alpha=\sqrt{\frac{b^2-4ac}{a}}$

Now, we know that,

$\sec^2\alpha-\tan^2\alpha=1 \ \Rightarrow \ (\sec\alpha-\tan\alpha)(\sec\alpha+\tan\alpha)=1$

$\Rightarrow \left(\sqrt{\frac{b^2-4ac}{a}}\right)\left(\frac{-b}{a}\right)=1$

$\Rightarrow \frac{(b^2-4ac)b^2}{a^4}=1$ (Squaring both side)

$a^4-b^4+4ab^2c=0$

16. (*b*) $ax^2 + bx + c = 0$

$$\alpha + \beta = \frac{-b}{a};\ \alpha\beta = \frac{c}{a};\ \text{D1} = b^2 - 4ac$$

$px^2 + qx + r = 0$

$$\gamma + \delta = \frac{-q}{p};\ \gamma\delta = \frac{r}{p};\ \text{D}_2 = q^2 - 4pr$$

$\alpha, \beta, \gamma, \delta$ are in AP.

$\alpha = \beta - \alpha = \delta - \gamma$

Common difference

$$\Rightarrow \frac{\sqrt{\text{D}_1}}{a} = \frac{\sqrt{\text{D}_2}}{p} \qquad \text{(Difference between roots).}$$

$$\sqrt{\frac{\text{D}_1}{\text{D}_2}} = \frac{a}{p}$$

$$\frac{\text{D}_1}{\text{D}_2} = \frac{a^2}{p^2}$$

17. (*a*) $(l - m)x^2 + lx + 1 = 0$

Roots are α and 2α.

$$\alpha + 2\alpha = \frac{-l}{l - m} \Rightarrow \alpha = \frac{-l}{3(l - 3)} \qquad \text{...(i)}$$

$$\alpha.2\alpha = \frac{l}{l - m} \Rightarrow \alpha^2 = \frac{-l}{2(l - 3)} \qquad \text{...(ii)}$$

from (i) & (ii),

$$\frac{l^2}{9(l - m)^2} = \frac{1}{2(l - 3)}$$

$\Rightarrow 2(l - m)\, l^2 = 9(l - m)^2$

$\Rightarrow 2l^2 - 9l + 9m = 0$

for real l, $\text{D} \geq 0$

$81 - 4.2.9m \geq 0 \quad \Rightarrow 9 - 8m \geq 0$

$$m \leq \frac{9}{8}$$

18. (*c*) $3px^2 + 5qx + 7r = 0$

$\text{D} = (5q)^2 - 4.3p.7r$

$= 25q^2 - 84pr$

$= 25\,(p + r)2 - 84pr \qquad (p + q + r = 0)$

$= 25\,(p^2 + r^2 + 2pr) - 84pr$

$= 25p^2 + 25r^2 + 50pr - 84pr$

$= 25p^2 - 34pr + 25r^2$

$$= (5p)^2 - 2.(5p)\,\frac{(17r)}{5} + \frac{(17r)^2}{5} - \frac{(17r)^2}{5} + 25r^2$$

As $\text{D} > 0$

Roots are real & distinct.

19. (*a*) $ax^2 + 2bx + c = 0$

$\text{D} = (2b)^2 - 4ac = 4(b^2 - ac)$

As a,b,c are in G.P. So, $b^2 = ac$

$\text{D} = b^2 - ac = 0$

Hence, above eq^n has equal roots

$$x = \frac{-2b}{2a} = \frac{-b}{a}$$

This root will satisfy $dx^2 + 2ex + f = 0$

$$\text{So, } d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{a}\right) + f = 0$$

$db^2 - 2aeb + a^2f = 0$

$dac - 2aeb + a^2f = 0$

$dac + a^2f = 2aeb \Rightarrow dc + af = 2eb$

Dividing by ac, both side

$$\frac{d}{a} + \frac{f}{c} = \frac{2e}{b} \Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

20. (*a*) $a < b < c < d$ and $k > 0$

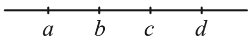
$f(x) = (x - a)\,(x - c) + k(x - b)\,(x - d)$

$f(a) = k(a - b)\,(a - b) \Rightarrow f(a) > 0$

$f(b) = (b - a)\,(b - c) \Rightarrow f(b) < 0$

$f(c) = (c - b)\,(c - d) \Rightarrow f(c) < 0$

$f(d) = (d - a)\,(d - c) \Rightarrow f(d) < 0$

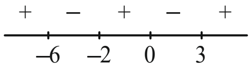


$f(x)$ has one root in (a,b) & another root in (c,d).

Hence, roots are real & distinct.

21. (*a*) $\dfrac{x}{x + 6} - \dfrac{1}{x} \leq 0 \Rightarrow \dfrac{x^2 - x - 6}{x(x + 6)} \leq 0$

$$\frac{(x + 2)(x - 3)}{x(x + 6)} \leq 0$$



No. of integral values of $x = 7$

22. (*d*) Given, $3x - 4y = 77$

$3^{x/2} - 2^y = 7$

Let $3^{x/2} = p$, $2^y = q$

$p - q = 7$

$$p^2 - q^2 = 77 \Rightarrow (p - q)\,(p + q) = 77 \Rightarrow p + q = \frac{77}{7} = 11$$

$$\left.\begin{aligned} p - q &= 7 \\ p + q &= 11 \end{aligned}\right\} \Rightarrow p = 9,\ q = 2$$

$$3^{x/2} = 9 \Rightarrow \frac{x}{2} = 2 \Rightarrow x = 4$$

$2^y = 2 \Rightarrow y = 1$

$x + y = 4 + 1 = 5$

23. (*d*) $\left(m^2+\frac{1}{m^2}\right)^2-2=119 \Rightarrow \left(m^2+\frac{1}{m^2}\right)^2=121$

$$m^2+\frac{1}{m^2}=11 \Rightarrow \left(m-\frac{1}{m}\right)^2+2=11$$

$$\left(m-\frac{1}{m}\right)^2=9 \Rightarrow m-\frac{1}{m}=3$$

$$\left|m^3-\frac{1}{m^3}\right|=\left|\left(m-\frac{1}{m}\right)\left(m^2+\frac{1}{m^2}+1\right)\right|$$

$$=|3.12|=36$$

24. (*a*) $x^8-24x^7-18x^5+39x^2+1155=0$

$x^2(x^6-24x^5-18x^3+39)=-1155$

$=-3\times5\times7\times11$

As $x \in Z$.

LHS contains x^2

But in RHS, there is no perfect square.

So, there is no integral value of x.

25. (*c*) $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$

$$\frac{(x+q)+(x+p)}{(x+p)(x+q)}=\frac{1}{r}$$

$(2x+p+q)r=x^2+(p+q)x+pq$

$x^2+(p+q-2r)x+pq-(p+q)r=0$

Roots are $\alpha_1-\alpha$.

$(\alpha)+(-d)=-(p+q-2\mathrm{r})$

$\Rightarrow 2r=p+q \Rightarrow r=\frac{p+q}{2}$

Product of roots $=\alpha.(-\alpha)=pq-(p+q)r$

$$=\mathrm{pq}-(\mathrm{p}+\mathrm{q})\ \frac{(p+q)}{2}\ =\ -\frac{(p^2+q^2)}{2}$$

26. (*d*) $\alpha^2=5\alpha-3;\ \beta^2=5\beta-3$

$$\left.\begin{matrix}\alpha^2-5\alpha+3=0\\ \beta^2-5\beta+3=0\end{matrix}\right\}\alpha\ \&\ \beta \text{ are roots of } x^2-5x+3=0$$

$\alpha+\beta=5$

$\alpha\beta=3$

Required Eq$^{\underline{n}}$:

$$x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+\left(\frac{\alpha}{\beta}.\frac{\beta}{\alpha}\right)=0$$

$$x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+1=0$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{25-2.3}{3}=\frac{19}{3}$$

$$x^2-\frac{19}{3}x+1=0$$

$3x^2-19x+1=0$

27. (*c*) $x^2-(1+\gamma)x+\gamma-2=0$

α, β are roots.

$\alpha+\beta=(1+\gamma)$

$\alpha\beta=\gamma-2$

$\alpha+\beta-\alpha\beta=3$

$\alpha\beta-\gamma-\beta+3=0$

$\alpha\beta-\alpha-\beta+1+2=0$

$\alpha(\beta-1)-(\beta-1)=-2$

$(\alpha-1)(\beta-1)=-2$

Since, product is (–*ve*)

At least one root is +*ve*.

28. (*c*) $\frac{(\alpha+1)^2+(\beta+1)^2+(\gamma+1)^2(\delta+1)^2}{\alpha+\beta+\gamma+\delta}=4$

$(\alpha+1)^2+(\beta+1)^2+(\gamma+1)^2+(\delta+1)^2$

$=4\alpha+4\beta+4\gamma+4\delta$

$(\alpha+1)^2-4\alpha+(\beta+1)^2-4\beta+(\gamma+1)^2$

$-4\gamma+(\delta+1)^2-4\delta=0$

$(\alpha-1)^2+(\beta-1)^2+(\gamma-1)^2+(\delta-1)^2=0$

$\alpha=\beta=\gamma=\delta=1$

$a_0x^4+a_1x^3+a_2x^2+a_3x+a_4=0$

Roots are 1, 1, 1, 1

$\frac{a_2}{a_0}=\mathrm{S}_2=$ Sum of product of roots taken two at a time.

$\frac{a_2}{a_0}=6$

29. (*c*) $ax^2+bx+c=0$

(i) Since downward parabola, $a<0$

(ii) As graph cuts +*ve* y-axis, $c>0$

(iii) Vertex lies in 2nd Quardrant,

$$\frac{-b}{2a}<0 \Rightarrow \frac{b}{2a}>0$$

$\Rightarrow b\ \&\ a$ must have same sign.

$b<0$

Hence, $a<0, b<0, c>0$

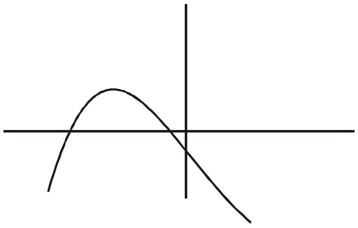
Advanced Level Multiconcept Questions

1. (*a*, *b*, *d*)

$y = ax^2 + bx + c$

Clearly $a < 0$

and $\frac{-b}{2a} < 0$



$\Rightarrow b < 0$

also $f(0) < 0 \Rightarrow c < 0$

and $D > 0$

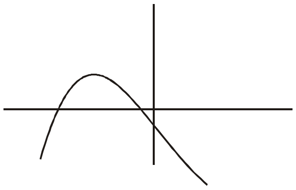
$\therefore$ (A), (B) and (D).

2. (*a*,*b*,*c*,*d*)

(*a*) $a < 0$,

$-\frac{-b}{2a} < 0 \Rightarrow b < 0$

& $f(0) < 0 \Rightarrow c < 0$



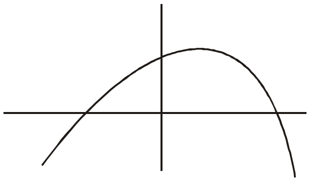
$\therefore$ $abc < 0$

(*b*) $a < 0$,

$\frac{-b}{2a} > 0 \Rightarrow b > 0$

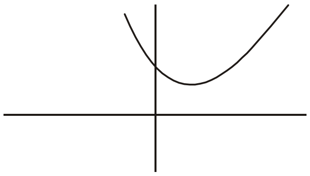
$f(0) > 0 \Rightarrow c > 0$

$\Rightarrow abc < 0$



(*c*) $a > 0$

$\frac{-b}{2a} > 0 \Rightarrow b < 0$



$f(0) > 0$ $c > 0$

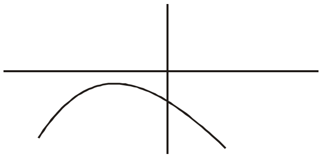
$\Rightarrow abc < 0$

(*d*) $a < 0$

$\frac{-b}{2a} < 0 \Rightarrow b < 0$

$f(0) < 0 \Rightarrow c < 0$

$\therefore$ (*a*), (*b*), (*c*), (*d*)



3. (*a*,*d*)

Clearly $a < 0$

$\frac{-b}{2a} > 0 \Rightarrow b > 0$

$\therefore$ (*a*), (*d*)

4. (*a*, *b*, *d*)

$|x|^2 + |x| - 6 = 0 \Rightarrow |x| = -3, 2 \Rightarrow |x| = 2$

$\Rightarrow x = \pm 2$

5. (*b*, *d*)

$ax^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

$a + b = -b/a$, $ab = c/a$

$px^2 + qx + r = 0 \begin{matrix} \nearrow \alpha + h \\ \searrow \beta + h \end{matrix}$

$$(a + b) + 2h = \frac{-q}{p}$$

$$h = \frac{\frac{-q}{p} + \frac{b}{a}}{2} = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right) \text{ Ans.}$$

$$|\alpha - \beta| = |(\alpha + h) - (\beta + h)|$$

$$= \sqrt{[(\alpha + h) + (\beta + h)]^2 - 4(\alpha + h)(\beta + h)}$$

$$= \frac{b^2}{a^2} - \frac{4c}{a} = \frac{q^2}{p^2} - \frac{4r}{p} \Rightarrow \frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2} \text{ Ans.}$$

6. (*b*,*c*,*d*)

(*a*) $S = a^2 + b^2 = a^2 - 2b$

$P = a^2 b^2 = b^2$

$\therefore$ equation is $x^2 - (a^2 - 2b) x + b^2 = 0$

(*b*) $S = \frac{1}{\alpha} + \frac{1}{\beta} = -\frac{a}{b}$, $P = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{b}$

$\therefore x^2 + \frac{a}{b} x + \frac{1}{b} = 0$

$\Rightarrow bx^2 + ax + 1 = 0$

(*c*) $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{a^2 - 2b}{b}$

$P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

$x^2 - \frac{a^2 - 2b}{b} x + 1 = 0 \Rightarrow bx^2 - (a^2 - 2b) x + b = 0$

(*d*) $S = a + b - 2 = - a - 2$

$P = (a - 1)(b - 1)$

$= ab - (a + b) + 1$

$= b + a + 1$

$\therefore$ equation is

$x^2 + (a + 2)x + (a + b + 1) = 0.$

7. (*a*,*d*)

$$ax^3 + bx^2 + cx + d = 0 \begin{array}{l} \diagup\ \alpha \\ \text{—}\ \beta \\ \diagdown\ \gamma \end{array}$$

Let $ax^3 + bx^2 + cx + d \equiv (x^2 + x + 1)(Ax + B)$

Roots of $x^2 + x + 1 = 0$ are imaginary, Let these are α, β

So the third root 'γ' will be real.

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$-1 + \gamma = \frac{-b}{a} \quad \Rightarrow \quad \gamma = \frac{a - b}{a}$$

Also $\alpha\beta\gamma = \dfrac{-d}{a}$

But $\alpha\beta = 1$

$$\therefore \ \gamma = \frac{-d}{a}$$

$\therefore$ Ans are (*a*) & (*d*).

8. (*a*, *b*, *d*)

$$\frac{1}{2} \le \log_{1/10} x \le 2$$

$$\Rightarrow \frac{1}{100} \le x \le \frac{1}{\sqrt{10}}$$

9. (*b*, *d*)

$$x^2 + abx + c = 0 \begin{array}{l} \diagup\ \alpha \\ \diagdown\ \beta \end{array} \qquad \text{...(1)}$$

$\alpha + \beta = - ab, \ \alpha\,\beta = c$

$$x^2 + acx + b = 0 \begin{array}{l} \diagup\ \alpha \\ \diagdown\ \delta \end{array} \qquad \text{...(2)}$$

$\alpha + \delta = - ac, \ \alpha\,\delta = b$

$\alpha^2 + ab\,\alpha + c = 0$

$\alpha^2 + ac\,\alpha + b = 0$

$$\frac{\alpha^2}{ab^2 - ac^2} = \frac{\alpha}{c - b} = \frac{1}{a\,c - ab}$$

$$\Rightarrow \alpha^2 = \frac{a\,(b^2 - c^2)}{a(c - b)} = -(b + c)$$

$$\& \ \alpha = \frac{c - b}{a(c - b)} = \frac{1}{a} \ \therefore \text{ common root, } \alpha = \frac{1}{a}$$

$$\therefore \ -(b + c) = \frac{1}{a^2} \Rightarrow a^2(b + c) = - 1$$

Product of the roots of equation (1) & (2) gives

$$\beta \times \frac{1}{a} = c \Rightarrow \beta = ac$$

$$\& \ \delta \times \frac{1}{a} = b \Rightarrow \delta = ab.$$

$\therefore$ equation having roots β, δ is

$x^2 - a\,(b + c)\,x + a^2bc = 0$

$a\,(b + c)\,x^2 - a^2(\,b + c)^2\,x + a.(b + c)\,a^2bc = 0$

$a\,(b + c)\,x^2 + (b + c)\,x - abc = 0.$

10. (*c*, *d*)

$\because$ D of $x^2 + 4x + 5 = 0$ is less than zero

$\Rightarrow$ both the roots are imaginary

$\Rightarrow$ both the roots of quadratic are same

$$\Rightarrow b^2 - 4ac < 0 \ \& \ \frac{a}{1} = \frac{b}{4} = \frac{c}{5} = k$$

$\Rightarrow a = k, b = 4k, c = 5k.$

11. (*a*, *d*)

$$x^2 + px + q = 0 \begin{array}{l} \diagup\ \alpha \\ \diagdown\ \beta \end{array}$$

$\alpha + \beta = - p, \ \alpha\beta = q$ and $p^2 - 4q > 0$

$$x^2 - rx + s = 0 \begin{array}{l} \diagup\ \alpha^4 \\ \diagdown\ \beta^4 \end{array} \qquad \text{.....(1)}$$

Now $\alpha^4 + \beta^4 = r$

$\Rightarrow \quad \alpha^4 + \beta^4 = r\,, (\alpha\beta)^4 = s = q^4$

$\therefore \ (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = r$

$\Rightarrow [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2 = r$

$\Rightarrow (p^2 - 2q)^2 - 2q^2 = r$

$\Rightarrow (p^2 - 2q)^2 = 2q^2 + r > 0$ .....(2)

Now, for $x^2 - 4qx + 2q^2 - r = 0$

$D = 16q^2 - 4(2q^2 - r)$ by equation (2)

$= 8q^2 + 4r = 4(2q^2 + r) > 0$

$\Rightarrow$ D > 0 two real and distinct roots

Product of roots $= 2q^2 - r$

$= 2q^2 - [(p^2 - 2q)^2 - 2q^2]$

$= 4q^2 - (p^2 - 2q)^2$

$= - p^2 (p^2 - 4q) < 0$ from (1)

So product of roots is – ve

hence roots are opposite in sign

12. (*a*, *d*)

$20x^2 + 210x + 400 = 4500 \Rightarrow 2x^2 + 21x - 410 = 0$

$\Rightarrow (2x + 41)(x - 10) = 0$

$$\Rightarrow x = \frac{-41}{2},\ x = 10 \ \Rightarrow x = -20.5,\ x = 10$$

13. (*a*,*b*,*c*,*d*)

$$x^3 + bx^2 + cx - 1 = 0 \begin{cases} \alpha = \frac{a}{r} \\ \beta = a \\ r = ar \end{cases}$$

$$\frac{a}{r} + a + ar = -b \Rightarrow a\left(\frac{1}{r} + 1 + r\right) = -b$$

$$\& \quad \frac{a}{r} \times a \times ar = 1$$

$$a^3 = 1 \Rightarrow a = 1$$

$$\& \quad \frac{a}{r} a + a \cdot ar + \frac{a}{r} \cdot ar = c$$

$$a^2\left(\frac{1}{r} + r + 1\right) = c$$

$$\frac{1}{r} + r + 1 = -b \;\&\; \frac{1}{r} + r + 1 = c \Rightarrow b + c = 0$$

we know $\frac{1}{r} + r > 2 \Rightarrow \left(\frac{1}{r} + r + 1\right) > 3$

$-b > 3 \Rightarrow b < -3 \Rightarrow b \in (-\infty, -3)$

& other two roots are $\frac{1}{r}$ & r

if $\frac{1}{r} > 1 \Rightarrow r < 1$ if $r > 1 \Rightarrow r < 1$

14. (*a*,*b*)

$$f(x) = \frac{3}{(x-2)} + \frac{4}{(x-3)} + \frac{5}{(x-4)} = 0$$

$$6x^2 - 14x - 21x + 49 = 0$$

$$(3x - 7)(2x - 7) = 0$$

$$x = \frac{7}{2}, x = \frac{7}{2}$$

$$2 < \frac{7}{2} < 33 < \frac{7}{2} < 4$$

2nd Method

$$g(x) = 3(x-3)(x-4) + 4(x-2)(x-4) + 5(x-2)(x-3) = 0$$

$$g(2) > 0; g(3) < 0, g(4) > 0$$

one root lie b/w (2, 3) & other root lie b/w (3, 4)

15. (*c*)

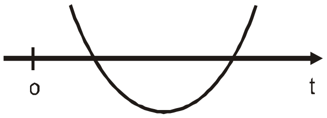
16. (*b*)

17. (*d*)

Sol. (15 to 16)

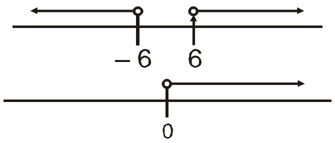
$$x^4 - \lambda x^2 + 9 = 0 \Rightarrow \quad x^2 = t \geq 0 \Rightarrow f(t) = t^2 - \lambda t + 9 = 0$$

15. given equation has four real & distinct roots



$D > 0$

$\Rightarrow \lambda^2 - 36 > 0$



$\frac{-b}{2a} > 0$

$\Rightarrow \frac{\lambda}{2} > 0$

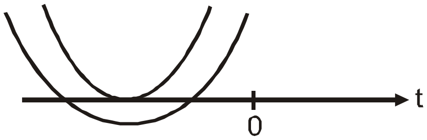
$\Rightarrow \quad \lambda > 0$

$f(0) > 0$

$\Rightarrow 9 > 0$

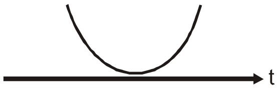
$\therefore \quad \lambda \in (6, \infty)$

16. Equation has no real roots.



case-I $\qquad D \geq 0 \Rightarrow \lambda^2 - 36 \geq 0$

$\frac{-b}{2a} < 0 \qquad \Rightarrow \lambda < 0$



$f(0) > 0 \qquad \Rightarrow \quad 9 > 0.$

$\therefore \; \lambda \in (-\infty, -6]$

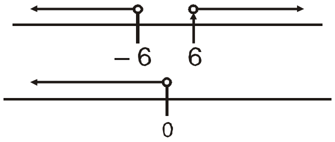
case-II $\qquad D < 0$

$\Rightarrow \quad \lambda^2 - 36 < 0$

$\Rightarrow \quad \lambda \in (-6, 6)$

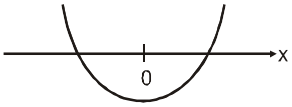
union of both cases gives

$\lambda \in (-\infty, 6)$



17. Equation has only two real roots

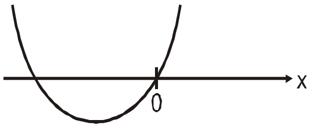
case-I $\qquad f(0) < 0 \qquad 9 < 0$



which is false

case-II $f(0) = 0$

and $\frac{-b}{2a} < 0$



$\therefore$ No solution

$\therefore$ Final answer is ϕ

18. (*a*) → (r), (*b*) → (p), (*c*) → (s), (*d*) → (p, q)

(*a*) $x^2 - 8x + k = 0 \begin{matrix}\diagup \alpha \\ \diagdown \alpha + 4 = \beta\end{matrix}$

$\because (\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow 16 = 64 - 4k \Rightarrow 4k = 48 \Rightarrow k = 12$

(*b*) $\because (|x| - 2)(|x| - 3) = 0$

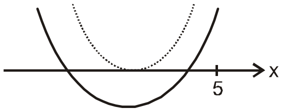
$\Rightarrow x = \pm 2 ; x = \pm 3$

$\therefore n = 4 \therefore \frac{n}{2} = 2$

(*c*) $\because b = (3 - i)(3 + i)$

$b = 10$

(*d*) $x^2 - 2kx + (k^2 + k - 5) = 0$



(*i*) $D \geq 0$

$\Rightarrow 4k^2 - 4(k^2 + k - 5) \geq 0$

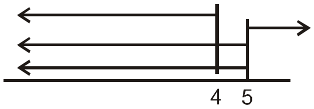
$\Rightarrow k - 5 \leq 0$

(*ii*) $f(5) > 0$

$\Rightarrow 25 - 10k + k^2 + k - 5 > 0$

$\Rightarrow k^2 - 9k + 20 > 0 \Rightarrow (k - 5)(k - 4) > 0$

(*iii*) $-\frac{b}{2a} < 5 \Rightarrow k < 5$



$\Rightarrow k \in (-\infty, 4)$

So k may be 2, 3.

NUMERICAL VALUE BASED

19. [2] $\left(x^2 + 3x + 2\right)\left(x^2 + 3x\right) = 120$

Let $x^2 + 3x = y$

$\Rightarrow y^2 + 2y - 120 = 0$

$\Rightarrow (y + 12)(y - 10) = 0$

$\Rightarrow y = -12 \Rightarrow x^2 + 3x + 12 = 0$

$\Rightarrow x \in \phi$

$y = 10 \Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow (x + 5)(x - 2) = 0 \quad \Rightarrow x = \{-5, 2\}$

x = 2, – 5 are only two integer roots.

20. [8] $\left(5 + 2\sqrt{6}\right)^{x^2-3} + \frac{1}{\left(5 + 2\sqrt{6}\right)^{x^2-3}} = 10$

$\Rightarrow t + \frac{1}{t} = 10$

$\Rightarrow t^2 - 10t + 1 = 0 \quad t = \frac{10 \pm \sqrt{96}}{2} = 5 \pm 2\sqrt{6}$

$\Rightarrow (5 + 2\sqrt{6})^{x^2-3} = (5 + 2\sqrt{6}) \quad$ or $\quad \frac{1}{5 + 2\sqrt{6}}$

$\Rightarrow x^2 - 3 = 1 \qquad$ or $x^2 - 3 = -1$

$\Rightarrow x = 2$ or $-2 \qquad$ or $-\sqrt{2}$ or $\sqrt{2}$

Product 8

21. [11] $2x^2 + 6x + a = 0$

Its roots are α, β

$\Rightarrow \alpha + \beta = -3 \qquad \& \ \alpha\beta = \frac{a}{2} \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$

$\Rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} < 2$

$\Rightarrow \frac{9 - a}{a} < 1$

$\Rightarrow \frac{2a - 9}{a} > 0$

$\Rightarrow a \in (-\infty, 0) \cup \left(\frac{9}{2}, \infty\right)$

$\Rightarrow$ 2a = 11 is least prime.

22. [1]

$x^2 + px + 1 = 0 \begin{matrix}\diagup a \\ \diagdown b\end{matrix} \ a + b = -p, ab = 1 ; x^2 + qx +$

$1 = 0 \begin{matrix}\diagup c \\ \diagdown d\end{matrix} \ c + d = -q, cd = 1$

$a + b = -p, \ ab = 1 \Rightarrow c + d = -q, cd = 1$

RHS = $(a - c)(b - c)(a + d)(b + d)$

$= (ab - ac - bc + c^2)(ab + ad + bd + d^2)$

$= (1 - ac - bc + c^2)(1 + ad + bd + d^2)$

$= 1 + ad + bd + d^2 - ac - a^2cd - abcd - acd^2 - bc$
$\quad - abcd - b^2cd - bcd^2 + c^2 + adc^2 + bdc^2 + c^2d^2$

$= 1 + ad + bd + d^2 - ac - a^2 - 1 - ad - bc - 1 - b^2$
$\quad - bd + c^2 + ac + bc + 1$

$[\because ab = cd = 1]$

$= c^2 + d^2 - a^2 - b^2 = (c + d)^2 - 2cd - (a + b)^2 + 2ab$

$= q^2 - 2 - p^2 + 2 = q^2 - p^2$ = LHS. **Proved.**

2nd Method :

RHS = $(ab - c(a + b) + c^2)(ab + d(ab + d(a + b) + d^2)$

$= (c^2 + pc + 1)(1 - pd + d^2) \qquad$ ...(1)

Since c & d are the roots of the equation $x^2 + qx + 1 = 0$

$\therefore c^2 + qc + 1 = 0 \Rightarrow c^2 + 1 = -qc \ \& \ d^2 + qd + 1 = 0$

$\Rightarrow d^2 + 1 = -qd.$

$\therefore$ (i) Becomes = $(pc - qc)(-pd - qd) = c(p - q)(-d)$
$(p + q) = -cd(p^2 - q^2)$

$= cd(q^2 - p^2) = q^2 - p^2$ = LHS. **Proved.**

23. [73] $\because$ α, β are roots of $\lambda x^2 - (\lambda - 1)x + 5 = 0$

$\therefore \alpha + \beta = \frac{\lambda - 1}{\lambda}$ and $\alpha\beta = \frac{5}{\lambda}$

$\because \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 4 \qquad \Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} = 4$

$\Rightarrow (\alpha+\beta)^2 = 6\,\alpha\,\beta \Rightarrow \frac{(\lambda-1)^2}{\lambda^2} = \frac{30}{\lambda}$

$\Rightarrow \lambda^2 - 32\lambda + 1 = 0$ ...(1)

$\because \lambda_1, \lambda_2$ are roots of (1)

$\therefore \lambda_1 + \lambda_2 = 32$ and $\lambda_1\lambda_2 = 1$

$$\therefore \frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} = \frac{(\lambda_1+\lambda_2)^2 - 2\lambda_1\lambda_2}{\lambda_1\lambda_2} = \frac{(32)^2-2}{1}$$

$$= 1022 \Rightarrow \left(\frac{\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}}{14}\right) = 73$$

24. [10] $\alpha.\beta = b\,;\ \gamma\delta = b-2$

$\Rightarrow \alpha\beta\gamma\delta = b(b-2) = 24$

$\therefore bx^2 + ax + 1 = 0$ has roots $\frac{1}{\alpha}, \frac{1}{\beta}$

$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{-a}{b}$

$(b-2)x^2 - ax + 1 = 0$ has root $\frac{1}{\gamma}, \frac{1}{\delta} \Rightarrow \frac{1}{\gamma} + \frac{1}{\delta} = \frac{a}{b-2}$

$\frac{1}{\gamma} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{-a}{b} + \frac{a}{b-2} = \frac{5}{6}\ ;\ \frac{+2a}{b(b-2)} = \frac{5}{6}\ ;$

$\frac{+2a}{24} = \frac{5}{6}\,;\ a = 10.$

25. [3] $x^2 + px + 1 = 0$

Roots = a,b

$a + b = -p;\ ab = 1$ ...(i)

$x^2 + qx + 1 = 0 \Rightarrow c + d = -q;\ cd = 1$ ...(ii)

Roots = c,d

Also, $c^2 + qc + 1 = 0;\ d^2 + qd + 1 = 0$ ...(iii)

$(a-c)(b-c)(a+b)(b+d) = (ab - (a+b)c + c^2)(ab + (a+b)d + d^2)$

$= (1 + p.c + c^2)(1 - pd + d^2)$ from ...(i)

$= (c^2 + 1 + pc)(d^2 + 1 - pd)$

$= (-qc + pc)(-qd - pd)$ from ...(iii)

$= -c\,(p-q)(p+q)d$

$= -cd\,(p^2 - q^2)$

$= -1(p^2 - q^2) = q^2 - p^2$

Required value $= \frac{(a-c)(b-c)(a+d)(b+d)}{q^2 - p^2}$

$= \frac{q^2 - p^2}{q^2 - p^2} = 1$

26. [13] $a^3 + b^3 + (-9)^3 = 3 \cdot a \cdot b\,(-9)$

$\Rightarrow a + b - 9 = 0$ or

$a = b = -9$. Which is rejected.

As $a > b > -9$

$\Rightarrow a + b - 9 = 0 \Rightarrow x = 1$ is a root

other root $= \frac{-9}{a}$. $\therefore\ \alpha = \frac{-9}{a}, \beta = 1$

$\Rightarrow 4\beta - a\alpha = 4 - a\left(\frac{-9}{a}\right) = 4 + 9 = 13.$

27. [6] Let $t^2 - 2t + 2 = k$

$\Rightarrow \alpha^2 - 6k\alpha - 2 = 0$

$\Rightarrow \alpha^2 - 2 = 6\,k\alpha$

$a_{100} - 2a_{98} = \alpha^{100} - 2.\alpha^{98} - \beta^{100} + 2.\beta^{98}$

$= \alpha^{98}(\alpha^2 - 2) - \beta^{98}(\beta^2 - 2) = 6k(\alpha^{99} - \beta^{99})$

$a_{100} - 2a_{98} = 6k.a_{99}$

$\frac{a_{100} - 2a_{98}}{a_{99}} = 6k = 6(t^2 - 2t + 2) = 6[(t-1)^2 + 1]$

$\therefore$ min. value of $\frac{a_{100} - 2a_{98}}{a_{99}}$ is 6.

28. [11] Given that, roots of equation $x^2 - 10ax - 11b = 0$ are c, d

So $c + d = 10a$ and $cd = -11b$ and a, b are the roots of equation $x^2 - 10cx - 11d = 0$

$\therefore a + b = 10c,\ ab = -11d$

So $a + b + c + d = 10(a + c)$ and $(c + d) - (a + b) = 10(a - c)$

$(c - a) - (b - d) + 10(c - a) = 0$

$\Rightarrow b + d = 9(a + c)$ ....(i)

$abcd = 121\,bd$

$\Rightarrow ac = 121$ ....(ii)

$b - d = 11(c - a)$ ....(iii)

c & a satisfies the equation $x^2 - 10ax - 11b = 0$ and $x^2 - 10cx - 11d = 0$ respectively

$\therefore\ c^2 - 10ac - 11b = 0$

$a^2 - 10ca - 11\,d = 0$

$(c^2 - a^2) - 11(b - d) = 0$

$(c - a)(c + a) = 11(b - d) = 11.\,11\,(c - a)$

(by equation (iii))

$c + a = 121$

$\Rightarrow a + b + c + d = 10(c + a)$

$\Rightarrow 10.121 \Rightarrow \frac{a + b + c + d}{110} = 11.$