Determinants

QUICK RECAP

DETERMINANT

- If $A = [a_{ij}]$ is a square matrix of order n, then a number (real or complex) associated to matrix A is called determinant of A.

  It is denoted by det A or $|A|$ or Δ.

- If $A = [a]$ be a matrix of order 1, then det $(A) = a$.
- If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2,

  then $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$

▶ If $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a matrix of order 3,

then

$$|A|=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}=(-1)^{1+1}a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$+(-1)^{1+2}a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}+(-1)^{1+3}a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}\,a_{33} - a_{32}\,a_{23}) - a_{12}\,(a_{21}\,a_{33} - a_{31}\,a_{23}) + a_{13}\,(a_{21}\,a_{32} - a_{31}\,a_{22})$$

Note : $|KA| = K^n|A|$, where A is of order n.

Properties of Determinants

▶ The value of the determinant remains unchanged if its rows and columns are interchanged.

▶ If any two rows (or columns) of a determinant are interchanged, then the value of the determinant is multiplied by –1.

▶ If any two rows (or columns) of a determinant are identical then the value of determinant is zero.

▶ If the elements of a row (or column) of a determinant are multiplied by a scalar, then the value of the new determinant is equal to same scalar times the value of the original determinant.

▶ If each element of any row (or column) of a determinant is the sum of two (or more) terms then the determinant is expressible as the sum of two (or more) determinants of the same order.

▶ The value of a determinant does not change when any row (or column) is multiplied by a number or an expression and then added to or subtracted from any other row (or column).

AREA OF A TRIANGLE

⏩ The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

Note :

(i) Since area is always a positive quantity, therefore we always take the absolute value of the determinant for the area.

(ii) Area of a triangle formed by three collinear points is always zero.

MINOR OF AN ELEMENT

⏩ Minor of an element a_{ij} of the determinant of matrix A is the determinant obtained by deleting i^{th} row and j^{th} column. Minor of a_{ij} is denoted by M_{ij}.

▶ Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order $n - 1$.

COFACTOR OF AN ELEMENT

⏩ Cofactor of an element a_{ij} of determinant of matrix A is, $A_{ij} = (-1)^{i+j} M_{ij}$.

▶ The determinant of a matrix A can also be obtained by using cofactors *i.e.*, sum of product of elements of a row (or column) with their corresponding cofactors.

$\therefore\ |A| = a_{11}\,A_{11} + a_{12}\,A_{12} + a_{13}\,A_{13}$

▶ If the elements of one row (or column) are multiplied with the cofactors of elements of any other row (or column), then their sum is zero *i.e.*, $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$

ADJOINT OF A MATRIX

⏩ The adjoint of a square matrix is the transpose of the matrix of cofactors.

▶ Adjoint of A is denoted by adj$A$.

Remark : For a matrix A of order n,

$A(\text{adj}A) = (\text{adj}A)A = |A|I_n$

SINGULAR AND NON-SINGULAR MATRIX

⏩ Let A be a square matrix, then A is called

▶ Singular matrix, iff $|A| = 0$

▶ Non-singular matrix, iff $|A| \neq 0$

Note : If A and B are non-singular matrices of same order, then AB and BA are also non-singular matrices of same order.

- A square matrix A is invertible iff A is non-singular matrix and

$$A^{-1} = \frac{1}{|A|}(\text{adj}A)$$

SOLUTION OF A SYSTEM OF LINEAR EQUATIONS

- For a square matrix A, a system of equations $AX = B$ is said to be
  - (i) Consistent, if it has one or more solutions.
  - (ii) Inconsistent, if its solution doesn't exist.

- For a system of equations, $AX = B$.
  - (i) If $|A| \neq 0$, then the given system of equations is consistent and has a unique solution.
  - (ii) If $|A| = 0$ and $(\text{adj}A)B \neq O$, then the solution does not exist and the given system is inconsistent.
  - (iii) If $|A| = 0$ and $(\text{adj}A)B = O$, then the given system may be either consistent or inconsistent, according as the system have either infinitely many solutions or no solution.

Previous Years' CBSE Board Questions

4.2 Determinant

VSA (1 mark)

1. Find the maximum value of
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}.$$ *(Delhi 2016)*

2. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x. *(AI 2016)*

3. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x. *(Foreign 2016)*

4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$, write the value of $|AB|$. *(Delhi 2015C)*

5. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x. *(Delhi 2014)*

6. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x. *(AI 2014)*

7. Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$. *(Delhi 2014C)*

8. Write the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$. *(AI 2014C)*

9. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x. *(Delhi 2013)*

10. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then write the value of x. *(Delhi 2013C)*

11. Evaluate : $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$. *(AI 2011)*

12. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, find the value of $3|A|$. *(AI 2011C)*

4.3 Properties of Determinants

VSA (1 mark)

13. If A is a square matrix of order 3 and $|A| = 5$, then the value of $|2A'|$ is
(a) -10 (b) 10
(c) -40 (d) 40 *(2020)*

14. If A is a skew-symmetric matrix of order 3, then the value of $|A|$ is
(a) 3 (b) 0
(c) 9 (d) 27 *(2020)*

15. If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals
(a) 8 (b) 24
(c) 72 (d) 216 *(2020)*

16. If A and B are square matrices each of order 3 and $|A| = 5$, $|B| = 3$, then the value of $|3AB|$ is ________. *(2020)*

17. If A and B are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$, write the value of $|B|$. *(Delhi 2019)*

18. Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$. *(AI 2015)*

19. If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k\,|A|$, then write the value of k. *(Foreign 2014)*

20. Let A be a square matrix of order 3×3. Write the value of $|2A|$, where $|A| = 4$. *(AI 2012, Delhi 2011C)*

21. The value of the determinant of a matrix A of order 3×3 is 4. Find the value of $|5A|$. *(Delhi 2012C)*

22. If the determinant of matrix A of order 3×3 is of value 4, write the value of $|3A|$.

(AI 2012C)

SA (2 marks)

23. If A is a skew-symmetric matrix of order 3, then prove that det $A = 0$. *(AI 2017)*

LA 1 (4 marks)

24. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3.$$

(Delhi 2019, AI 2017)

25. Using properties of determinants, find the value of x for which $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0.$

(AI 2019)

26. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz+xy+yz+zx)$$

(2018)

27. Using properties of determinants, prove that

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$$

(Delhi 2017, AI 2013)

28. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants find the value of $f(2x) - f(x)$.

(Delhi 2015)

29. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

(AI 2015, Foreign 2014)

30. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (1-a^3)^2$$ *(Foreign 2015)*

31. Using properties of determinants, prove that

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2.$$ *(Delhi 2015C)*

32. Using properties of determinants, solve for x :

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$$ *(AI 2015C, 2011)*

33. Using properties of determinants, prove that

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} = (x+y+z)^3$$

(Delhi 2014)

34. Prove the following using properties of determinants :

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$

(Delhi 2014, 2012C)

35. Using properties of determinants, prove the following:

$$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1+x^2+y^2+z^2$$

(Delhi 2014)

36. Using properties of determinants, prove the following :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab+bc+ca+abc$$

(AI 2014, Delhi 2012)

37. Using properties of determinants, prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$

(AI 2014)

38. Using properties of determinants, show that

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3.$$

(AI 2014)

39. Using properties of determinants, prove that :

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z).$$

(Foreign 2014)

40. Using properties of determinants, prove that :

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2.$$

(Foreign 2014)

41. Using properties of determinants, prove the following :

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$$

(Delhi 2014C, 2013)

42. Using properties of determinants, prove the following :

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(bc+ca+ab)$$

(Delhi 2014C)

43. Using properties of determinants, prove the following :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

(AI 2014C, 2012)

44. Show that $\Delta = \Delta_1$, where

$$\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}, \Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$$

(AI 2014C)

45. Using properties of determinants, prove the following :

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$$

(AI 2013)

46. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

(Delhi 2013C)

47. Using properties of determinants, prove that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

(AI 2013C, Delhi 2011C)

48. Using properties of determinants, prove that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}.$$

(Delhi 2012)

49. Using properties of determinants, prove the following :

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

(Delhi 2012, 2011C)

50. Using properties of determinants, prove the following :

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$

$= (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)(\alpha + \beta + \gamma)$

(Delhi 2012C)

51. Using properties of determinants, prove the following :

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc.$$

(Delhi 2012C)

52. Using properties of determinants, prove the following :

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

(AI 2012C)

53. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$$

$= (a - b)\ (b - c)\ (c - a)\ (a + b + c)\ (a^2 + b^2 + c^2)$

(AI 2012C)

54. Using properties of determinants, prove the following :

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2).$$

(AI 2012C)

55. Using properties of determinants, prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2.$$

(Delhi 2011, AI 2011C)

56. Using properties of determinants, prove that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x).$$

(Delhi 2011)

57. Using properties of determinants, prove that

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2.$$

(Delhi 2011)

58. Using properties of determinants, solve the following for x.

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$$

(AI 2011)

59. Using properties of determinants, solve the following for x.

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$$

(AI 2011)

60. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

(AI 2011C)

LA 2 (6 marks)

61. Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

(2020)

62. If a, b, c are p^{th}, q^{th} and r^{th} terms respectively of a G.P., then prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$$

(2020)

63. Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ is divisible by $(x + y + z)$, and hence find the quotient. *(Delhi 2016)*

64. Using properties of determinants, show that ΔABC is isosceles, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix} = 0$$

(AI 2016)

65. If a, b and c are all non-zero and

$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$, then prove that

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1=0$ *(Foreign 2016)*

4.4 Area of a Triangle

LA 1 (4 marks)

66. Find the equation of the line joining $A(1, 3)$ and $B(0, 0)$ using determinants and find the value of k if $D(k, 0)$ is a point such that area of ΔABD is 3 square units. *(AI 2013C)*

4.5 Minors and Cofactors

VSA (1 mark)

67. Find the cofactors of all the elements of $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$. *(2020)*

68. If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2^{nd} row. *(Foreign 2015)*

69. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32} \cdot A_{32}$. *(AI 2013)*

70. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}. *(Delhi 2012)*

71. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the cofactor of the element a_{32}. *(Delhi 2012)*

72. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 5 & 3 & 8 \end{vmatrix}$, write the minor of element a_{22}. *(Delhi 2012)*

4.6 Adjoint and Inverse of a Matrix

VSA (1 mark)

73. If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find A (adj A). *(2020)*

74. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is

(a) 64 (b) 16
(c) 0 (d) –8 *(2020)*

75. If A is a square matrix of order 3 with $|A| = 9$, then write the value of $|2 \cdot \text{adj } A|$. *(AI 2019)*

76. If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$? *(Delhi 2017)*

77. If for any 2×2 square matrix A,

$A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$. *(AI 2017)*

78. In the interval $\pi/2 < x < \pi$, find the value of x for which the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular. *(AI 2015C)*

79. Find (adj A), if $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$. *(Delhi 2014C)*

80. If A is a square matrix of order 3 such that $|\text{adj } A| = 64$, find $|A|$. *(Delhi 2013C)*

81. If A is an invertible square matrix of order 3 and $|A| = 5$, then find the value of $|\text{adj } A|$. *(AI 2013C, 2011C)*

82. For what value of x, is the given matrix $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ singular? *(AI 2013C)*

83. For what value of x, the matrix $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is a singular matrix? *(AI 2012C)*

84. Write A^{-1} for $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$. *(Delhi 2011)*

85. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular ? *(Delhi 2011)*

86. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then write A^{-1} in terms of A. *(AI 2011)*

87. For what value of x is the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ singular? *(Delhi 2011C)*

88. For what value of x is the matrix $\begin{bmatrix} 2x & 4 \\ x+2 & 3 \end{bmatrix}$ singular? *(Delhi 2011C)*

89. For what value of x is the matrix $\begin{bmatrix} 2x+4 & 4 \\ x+5 & 3 \end{bmatrix}$ a singular matrix? *(AI 2011C)*

90. For what value of x is $A = \begin{bmatrix} 2(x+1) & 2x \\ x & x-2 \end{bmatrix}$ a singular matrix? *(AI 2011C)*

SA (2 marks)

91. Find $(AB)^{-1}$ if $A = \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$. *(2020)*

92. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$. *(2018)*

LA 1 (4 marks)

93. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$. *(Delhi 2015)*

94. Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A\cdot(\text{adj } A) = |A|I_3$. *(AI 2015)*

95. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}. *(Foreign 2015)*

96. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. *(Delhi 2015C)*

97. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$. *(AI 2015C)*

LA 2 (6 marks)

98. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_3$. *(Foreign 2016)*

4.7 Applications of Determinants and Matrices

LA 1 (4 marks)

99. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value? *(Delhi 2016)*

100. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question? *(AI 2016)*

101. A coaching institute of English (Subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is ₹ 9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is ₹ 26,000. Using matrix method, find monthly fees paid by each child of two types. What values the coaching institute is inculcating in the society? *(Foreign 2016)*

102. Two schools A and B decided to award prizes to their students for three values, team spirit, truthfulness and tolerance at the rate of ₹ x, ₹ y and ₹ z per student respectively. School A, decided to award a total of ₹ 1,100 for the three values to 3, 1 and 2 students respectively while school B decided to award ₹ 1,400 for the three values to 1, 2 and 3 students respectively. If one prize for all the three values together amount to ₹ 600 then
(i) Represent the above situation by a matrix equation after forming linear equations.
(ii) Is it possible to solve the system of equations so obtained using matrices?
(iii) Which value you prefer to be rewarded most and why? *(Delhi 2015C)*

LA 2 (6 marks)

103. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations :
$x + 2y - 3z = 6$,
$3x + 2y - 2z = 3$
$2x - y + z = 2$ *(2020)*

104. Solve the following system of equations by matrix method :
$x - y + 2z = 7$
$2x - y + 3z = 12$
$3x + 2y - z = 5$ *(2020)*

105. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}.
Using A^{-1}, solve the following system of equations :
$2x - 3y + 5z = 11$
$3x + 2y - 4z = -5$
$x + y - 2z = -3$ *(2020, 2018, AI 2012C)*

106. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations $x + y + z = 6$, $x + 2z = 7$, $3x + y + z = 12$. *(Delhi 2019)*

107. Using matrices, solve the following system of linear equations:
$x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$ *(AI 2019, 2011)*

108. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x + 3z = 9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$ *(Delhi 2017)*

109. Determine the product
$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$. *(AI 2017, Delhi 2012C)*

110. Using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and use it to solve the following system of linear equations:
$8x + 4y + 3z = 19$; $2x + y + z = 5$;
$x + 2y + 2z = 7$ *(Delhi 2016)*

111. A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for ₹ 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹ 70. Using matrix method, find cost of each variety of pen. *(AI 2016)*

112. Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students with a total award money of ₹ 1,000. School Q wants to spend ₹ 1,500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is ₹ 600, using matrices, find the award money for each value.

Apart from the above three values, suggest one more value for awards. *(Delhi 2014)*

113. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹ 1,600. School B wants to spend ₹ 2,300 to award its, 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award. *(AI 2014)*

114. Two schools P and Q want to award their selected students on the values of tolerance, kindness and leadership. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of ₹ 2200. School Q wants to spend ₹ 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as school P). If the total amount of award for one prize on each value is ₹ 1200, using matrices, find the award money for each value.

Apart from the above these three values, suggest one more value which should be considered for award. *(Foreign 2014)*

115. A total amount of ₹ 7,000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}\%$ respectively. The total annual interest from these three accounts is ₹ 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices. *(Delhi 2014C)*

116. Two schools, P and Q, want to award their selected students for the values of sincerity, truthfulness and hard work at the rate of ₹ x, ₹ y and ₹ z for each respective value per student. School P awards its 2, 3 and 4 students on the above respective values with a total prize money of ₹ 4,600. School Q wants to award its 3, 2 and 3 students on the respective values with a total award money of ₹ 4,100. If the total amount of award money for one prize on each value is ₹ 1,500, using matrices find the award money for each value. Suggest one other value which the school can consider for awarding the students. *(AI 2014C)*

117. A school wants to award its students for the value of honesty, regularity and hard work with a total cash award of ₹ 6,000. Three times the award money for hard work added to that given for honesty amounts to ₹ 11,000. The award money given for honesty and hard work together is double the one given for regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, honesty, regularity and hard work, suggest one more value which the school must include for awards.

(Delhi 2013)

118. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the numbers of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards. *(AI 2013)*

119. Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of ₹ 37,000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of ₹ 47,000. If all the three prizes per person together amount to ₹ 12,000 then using matrix method find the value of x, y and z.
What values are described in the question?
(Delhi 2013C)

120. Two factories decided to award their employees for three values of (a) adaptable to new techniques, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of ₹ x, ₹ y and ₹ z per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with a total prize money of ₹ 29,000. The second factory decided to honour respectively 5, 2 and 3 employees with the prize money of ₹ 30,500. It the three prizes per person together cost ₹ 9,500; then

(i) Represent the above situation by a matrix equation and form linear equations using matrix multiplication.
(ii) Solve these equations using matrices.
(iii) Which values are reflected in this question? *(AI 2013C)*

121. Using matrices, solve the following system of linear equations :
$x - y + 2z = 7$, $3x + 4y - 5z = -5$,
$2x - y + 3z = 12$ *(Delhi 2012)*

122. Using matrices, solve the following system of equations :
$2x + 3y + 3z = 5$, $x - 2y + z = -4$,
$3x - y - 2z = 3$ *(AI 2012)*

123. Using matrices, solve the following system of equations :
$3x + 4y + 7z = 4$; $2x - y + 3z = -3$;
$x + 2y - 3z = 8$ *(A I 2012)*

124. Using matrices, solve the following system of equations :
$x + y - z = 3$; $2x + 3y + z = 10$;
$3x - y - 7z = 1$ *(AI 2012)*

125. If $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations :
$x + 2y + z = 4$, $-x + y + z = 0$,
$x - 3y + z = 4$ *(Delhi 2012C)*

126. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$.
Hence solve the system of equations
$x + 2y - 3z = -4$; $2x + 3y + 2z = 2$;
$3x - 3y - 4z = 11$. *(Delhi 2012C)*

127. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of equations
$x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$.
(AI 2012C)

128. If $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$, find A^{-1}. Hence solve the following system of equations :
$x + 2y + 5z = 10$, $x - y - z = -2$,
$2x + 3y - z = -11$ *(AI 2012C)*

129. Using matrix method, solve the following system of equations :
$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2;$$
$x, y, z \neq 0$ *(Delhi 2011)*

130. Using matrix method, solve the following system of equations:
$x + 2y + z = 7$, $x + 3z = 11$, $2x - 3y = 1$.
(AI 2011)

131. Using matrices, solve the following system of equations :
$4x + 3y + 2z = 60$, $x + 2y + 3z = 45$,
$6x + 2y + 3z = 70$. *(AI 2011)*

132. If $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations :
$3x - 4y + 2z = -1$, $2x + 3y + 5z = 7$ and $x + z = 2$ *(Delhi 2011C)*

133. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB. Hence solve the system of equations :
$x - 2y = 10$, $2x + y + 3z = 8$ and $-2y + z = 7$. *(Delhi 2011C)*

134. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}$, find A^{-1} and hence solve the system of equations
$x - 2y + z = 0$, $-y + z = -2$, $2x - 3z = 10$. *(AI 2011C)*

135. If $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the following system of equations:
$2x - y + z = -3$, $3x - z = 0$, $2x + 6y - 2 = 0$ *(AI 2011C)*

Detailed Solutions

1. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

$\Rightarrow \Delta = 1[(1 + \sin\theta)(1 + \cos\theta) - 1] - 1(1 + \cos\theta - 1) + 1(1 - 1 - \sin\theta)$
$= 1 + \cos\theta + \sin\theta + \sin\theta\cos\theta - 1 - \cos\theta - \sin\theta$
$= \sin\theta\cos\theta$

$\therefore$ Maximum value of Δ is $\frac{1}{2}$.

2. Given, $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$

$\Rightarrow (x + 3)(2x) - (-2)(-3x) = 8$
$\Rightarrow 2x^2 + 6x - 6x = 8 \Rightarrow 2x^2 = 8$
$\Rightarrow x^2 = 4 \Rightarrow x = 2$ $[x \neq -2 \because x \in N]$

3. Given, $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$

$\Rightarrow x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta) = 8$
$\Rightarrow -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta = 8$
$\Rightarrow -x^3 - x + x(\sin^2\theta + \cos^2\theta) = 8$
$\Rightarrow -x^3 - x + x = 8 \Rightarrow x^3 + 8 = 0$
$\Rightarrow (x + 2)(x^2 - 2x + 4) = 0 \Rightarrow x + 2 = 0$ $[\because x^2 - 2x + 4 > 0 \ \forall x]$
$\Rightarrow x = -2$

4. Given that $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 4 & 8 \end{bmatrix}$

$\therefore |AB| = \begin{vmatrix} -1 & 5 \\ 4 & 8 \end{vmatrix} = (-1)\cdot 8 - 4\cdot 5 = -28.$

5. Given, $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow 2x^2 - 40 = 18 + 14$
$\Rightarrow 2x^2 = 72 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

6. Given, $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

$\Rightarrow 12x + 14 = 32 - 42$
$\Rightarrow 12x = -10 - 14 = -24 \Rightarrow x = -2.$

7. $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix} = p^2 - (p - 1)(p + 1)$
$= p^2 - (p^2 - 1) = 1.$

8. Let $\Delta = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

$\Rightarrow \Delta = 2[8(86) - 9(75)] - 7[3(86) - 5(75)] + 65[3(9) - 5(8)]$
$= 2(688 - 675) - 7(258 - 375) + 65(27 - 40)$
$= 2(13) - 7(-117) + 65(-13) = 26 + 819 - 845 = 0$

9. Given, $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow$ $(x+1)(x+2)-(x-3)(x-1) = 4\times 3-(1)\times(-1)$

$\Rightarrow$ $x^2 + x + 2x + 2 - (x^2 - 3x - x + 3) = 12 + 1$

$\Rightarrow$ $x^2 + 3x + 2 - x^2 + 4x - 3 = 13$

$\Rightarrow$ $7x = 13 + 1 \Rightarrow x = 2$

10. Given, $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$

$\Rightarrow$ $2x(x + 1) - 2(x + 1)(x + 3) = 3 - 15$

$\Rightarrow$ $2x^2 + 2x - 2(x^2 + 4x + 3) = -12$

$\Rightarrow$ $2x^2 + 2x - 2x^2 - 8x - 6 = -12$

$\Rightarrow$ $-6x = -12 + 6 \Rightarrow x = \frac{-6}{-6} = 1$

11. $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

$= \cos 15^\circ \cos 75^\circ - \sin 75^\circ \sin 15^\circ$

$= \cos(15^\circ + 75^\circ) = \cos 90^\circ = 0$

12. Here, $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow$ $|A| = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 3\times 2 - 1\times 4 = 2$

$\Rightarrow$ $3|A| = 3 \times 2 = 6.$

13. **(d) :** Given, A is a 3×3 matrix and $|A| = 5$

Now, $|2A'| = 2^3|A'|$

$= 2^3|A|$ $[\because |A'| = |A|]$

$= 8 \times 5 = 40$

14. **(b) :** We have, $A^T = -A$

$[\because A$ is skew-symmetric matrix$]$

$\therefore$ $|A^T| = |-A| \Rightarrow |A| = (-1)^3|A|$ $[\because A$ is of order 3$]$

$\Rightarrow$ $|A| = -|A| \Rightarrow 2|A| = 0 \Rightarrow |A| = 0$

15. **(d) :** We have, $|3A| = 3^3|A| = 3^3 \cdot 8$

[Given $|A| = 8$]

$= 27 \cdot 8 = 216$

16. We have, $|A| = 5$, $|B| = 3$

Now, $|3AB| = 3^3|AB|$ (As order of AB is 3)

$= 3^3|A||B| = 3^3 \times 5 \times 3 = 405$

17. We have, $AB = 2I \Rightarrow |AB| = |2I| \Rightarrow |A||B| = 2^3|I|$

$\Rightarrow$ $2|B| = 8$ $[\because |A| = 2$ (given)$]$

$\Rightarrow$ $|B| = 4$

18. Here, $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$

Applying $R_1 \to R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

Taking -3 common from R_3 and $x + y + z$ common from R_1, we get

$$\Delta = -3(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$= -3(x + y + z) \cdot 0$ $(\because R_1$ and R_3 are identical$)$

$= 0.$

19. We have, $|3A| = k|A|$

$\Rightarrow$ $3^3|A| = k|A|$

[Using $|mA| = m^n|A|$, where n is order of A]

$\Rightarrow$ $k = 27.$

20. *Refer to answer 15.*

21. *Refer to answer 15.*

22. *Refer to answer 15.*

23. *Refer to answer 14.*

24. Let $\Delta = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$, we get

$$\Delta = \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking out common $(a - 1)$ from R_1, we get

$$\Delta = (a-1)\begin{vmatrix} a+1 & 1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$, we get

$$\Delta = (a-1)\begin{vmatrix} a+1 & 1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking out common $(a - 1)$ from R_2, we get

$$\Delta = (a-1)^2\begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Now, expanding along C_3, we get
$\Delta = (a-1)^2 (a+1-2) = (a-1)^2 (a-1) = (a-1)^3$

25. Given, $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 12+x & 4+x & 4+x \\ 12+x & 4-x & 4+x \\ 12+x & 4+x & 4-x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} 12+x & 4+x & 4+x \\ 0 & -2x & 0 \\ 0 & 0 & -2x \end{vmatrix} = 0$$

$\Rightarrow \quad (12+x)(-2x)^2 = 0 \Rightarrow 4x^2(12+x) = 0$

$\Rightarrow \quad x = 0, -12$

26. L.H.S. = $\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we get

$$\begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix}$$

Taking out common 3 from both C_2 and C_3, we get

$$9\begin{vmatrix} 1 & 0 & x \\ 1+3y & -y & -y \\ 1 & z & 0 \end{vmatrix}$$

$= 9[1(0 + zy) - 0 + x(z + 3yz + y)]$
$= 9(zy + xz + 3xyz + xy) = 9(3xyz + xy + yz + zx)$
= R.H.S.

27. L.H.S. = $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 3x+3y & x+y & x+2y \\ 3x+3y & x & x+y \\ 3x+3y & x+2y & x \end{vmatrix}$$

Taking $3(x + y)$ common from C_1, we get

$$3(x+y)\begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_2$, we get

$$3(x+y)\begin{vmatrix} 1 & x+y & x+2y \\ 0 & -y & -y \\ 0 & 2y & -y \end{vmatrix}$$

Taking y common from R_2 and R_3 both, we get

$$3(x+y)\cdot y \cdot y\begin{vmatrix} 1 & x+y & x+2y \\ 0 & -1 & -1 \\ 0 & 2 & -1 \end{vmatrix}$$

$= 3y^2 (x+y) \cdot 1(1+2) = 9y^2 (x+y)$ = R.H.S.

28. $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$

$\Rightarrow \quad f(x) = a\begin{vmatrix} 1 & -1 & 0 \\ x & a & -1 \\ x^2 & ax & a \end{vmatrix}$

Applying $C_2 \to C_2 + C_1$, we get

$$f(x) = a\begin{vmatrix} 1 & 0 & 0 \\ x & x+a & -1 \\ x^2 & x^2+ax & a \end{vmatrix}$$

$\Rightarrow \quad f(x) = a[a(x+a) + (x^2 + ax)]$
$\Rightarrow \quad f(x) = a(a^2 + ax + ax + x^2) = a(a^2 + 2ax + x^2)$
Now, $f(2x) = a\{a^2 + 2a(2x) + (2x)^2\} = a(a^2 + 4ax + 4x^2)$
$\therefore \quad f(2x) - f(x) = a(a^2 + 4ax + 4x^2 - a^2 - 2ax - x^2)$
$= ax(3x + 2a)$

29. L.H.S.= $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

Taking a, b, c common from C_1, C_2, C_3 respectively, we get

$$abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $C_3 \to C_3 - C_1 - C_2$, we get

$$abc\begin{vmatrix} a & c & 0 \\ a+b & b & -2b \\ b & b+c & -2b \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$, we get

$$abc\begin{vmatrix} a & c & 0 \\ a & -c & 0 \\ b & b+c & -2b \end{vmatrix}$$

$$= abc(-2b)(-ac-ac) = -2ab^2c(-2ac)$$
$$= 4a^2b^2c^2 = \text{R.H.S.}$$

30. L.H.S. $= \begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 1+a+a^2 & a & a^2 \\ 1+a+a^2 & 1 & a \\ 1+a+a^2 & a^2 & 1 \end{vmatrix}$$

Taking $(1 + a + a^2)$ common from C_1, we get

$$(1+a+a^2)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$, we get

$$(1+a+a^2)\begin{vmatrix} 1-a & a & a^2 \\ 0 & 1 & a \\ 1-a^2 & a^2 & 1 \end{vmatrix}$$

Taking $(1 - a)$ common from C_1, we get

$$(1+a+a^2)(1-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 1+a & a^2 & 1 \end{vmatrix}$$

$$= (1-a^3)\,[(1-a^3) + (1+a)(a^2-a^2)]$$
$$= (1-a^3)^2 = \text{R.H.S.}$$

31. L.H.S. $= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$

$$= \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ a^2+5a+6 & a+3 & 1 \\ a^2+7a+12 & a+4 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_2$, we get

$$\begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2a+4 & 1 & 0 \\ 2a+6 & 1 & 0 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get

$$\begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2a+4 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix} = 0 - 2 = -2 = \text{R.H.S.}$$

32. Given, $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} 3a-x & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

$\Rightarrow\ (3a - x)\cdot(2x)^2 = 0 \ \Rightarrow\ x = 0, 3a.$

33. L.H.S. $= \begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\begin{vmatrix} x+y+z & x+y+z & x+y+z \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

Taking $(x + y + z)$ common from R_1, we get

$$(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$, we get

$$(x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ 2z & 0 & -(x+y+z) \\ x-y-z & x+y+z & 0 \end{vmatrix}$$

$= (x + y + z) \cdot 1 \cdot (x + y + z)^2 = (x + y + z)^3 =$ R.H.S.

34. L.H.S. $= \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$ and taking $2(a + b + c)$ common from C_1, we get

$$2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1; R_3 \to R_3 - R_1$, we get

$$2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & b+c+a & 0 \\ 0 & 0 & c+a+b \end{vmatrix}$$

$= 2(a + b + c) \cdot 1 \cdot (b + c + a) \cdot (c + a + b)$
$= 2(a + b + c)^3 =$ R.H.S.

35. L.H.S. $= \begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix}$

Applying $R_1 \to \frac{1}{x}R_1, R_2 \to \frac{1}{y}R_2, R_3 \to \frac{1}{z}R_3$, we get

$$xyz\begin{vmatrix} x+\frac{1}{x} & y & z \\ x & y+\frac{1}{y} & z \\ x & y & z+\frac{1}{z} \end{vmatrix}$$

Applying $C_1 \to xC_1, C_2 \to yC_2, C_3 \to zC_3$, we get

$$\frac{xyz}{xyz}\begin{vmatrix} x^2+1 & y^2 & z^2 \\ x^2 & y^2+1 & z^2 \\ x^2 & y^2 & z^2+1 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} x^2+y^2+z^2+1 & y^2 & z^2 \\ x^2+y^2+z^2+1 & y^2+1 & z^2 \\ x^2+y^2+z^2+1 & y^2 & z^2+1 \end{vmatrix}$$

Taking $(x^2 + y^2 + z^2 + 1)$ common from C_1, we get

$$(x^2+y^2+z^2+1)\begin{vmatrix} 1 & y^2 & z^2 \\ 1 & y^2+1 & z^2 \\ 1 & y^2 & z^2+1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, we get

$$(1+x^2+y^2+z^2)\begin{vmatrix} 1 & y^2 & z^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= (1 + x^2 + y^2 + z^2)\ (1)\ (1 - 0)$
$= (1 + x^2 + y^2 + z^2) =$ R.H.S.

36. L.H.S. $= \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} 1+a & 1 & 1 \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix}$$

$= -a[0 - (b)] + c[b(1 + a) + a]$
$= ab + bc + abc + ac$
$= ab + bc + ca + abc =$ R.H.S.

37. L.H.S. $= \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$ and taking 2 common from C_1, we get

$$2\begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1; C_3 \to C_3 - C_1$, we get

$$2\begin{vmatrix} a+b+c & -b & -c \\ p+q+r & -q & -r \\ x+y+z & -y & -z \end{vmatrix} = 2(-1)(-1)\begin{vmatrix} a+b+c & b & c \\ p+q+r & q & r \\ x+y+z & y & z \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2 - C_3$, we get

$$2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = \text{R.H.S.}$$

38. L.H.S. $= \begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$.

Since each element in the first column of determinant is the sum of two elements, therefore, determinant can be expressed as the sum of two determinants given by

$$\begin{vmatrix} x & x & x \\ 5x & 4x & 2x \\ 10x & 8x & 3x \end{vmatrix} + \begin{vmatrix} y & x & x \\ 4y & 4x & 2x \\ 8y & 8x & 3x \end{vmatrix}$$

Taking x common from R_1, R_2, R_3 in first determinant and x common from C_2, C_3, y common from C_1 in second determinant, we get

$$x^3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + yx^2 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 2 \\ 8 & 8 & 3 \end{vmatrix}$$

$$= x^3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + yx^2 \cdot 0$$

($\because C_1$ and C_2 are identical in the second determinant)

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$, we get

$$x^3 \begin{vmatrix} 0 & 0 & 1 \\ 3 & 2 & 2 \\ 7 & 5 & 3 \end{vmatrix} = x^3 \cdot 1 \cdot (15 - 14) = x^3 = \text{R.H.S.}$$

39. L.H.S. $= \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix}$$

Taking $(a + x + y + z)$ common from C_1, we get

$$(a+x+y+z) \begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$, we get

$$(a+x+y+z) \begin{vmatrix} 0 & -a & 0 \\ 0 & a & -a \\ 1 & y & a+z \end{vmatrix}$$

$= (a+x+y+z)1(a^2 - 0)$

$= a^2 (a + x + y + z) =$ R.H.S.

40. L.H.S. $= \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 5x+\lambda & 2x & 2x \\ 5x+\lambda & x+\lambda & 2x \\ 5x+\lambda & 2x & x+\lambda \end{vmatrix}$$

Taking $(5x + \lambda)$ common from C_1, we get

$$(5x+\lambda) \begin{vmatrix} 1 & 2x & 2x \\ 1 & x+\lambda & 2x \\ 1 & 2x & x+\lambda \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$(5x+\lambda) \begin{vmatrix} 1 & 2x & 2x \\ 0 & -x+\lambda & 0 \\ 0 & 0 & -x+\lambda \end{vmatrix}$$

Taking $(\lambda - x)$ common from R_2 and R_3 both, we get

$$(5x+\lambda)(\lambda - x)^2 \begin{vmatrix} 1 & 2x & 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= (5x + \lambda)(\lambda - x)^2\, 1 \cdot (1 - 0)$

$= (5x + \lambda)(\lambda - x)^2 =$ R.H.S.

41. *Refer to answer 30.*

42. L.H.S. $= \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$

Applying $R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$, we get

$$\frac{1}{abc} \begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} = \frac{1}{abc} \cdot abc \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} a^2-b^2 & a^3-b^3 & 0 \\ b^2-c^2 & b^3-c^3 & 0 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a-b)(a+b) & (a-b)(a^2+ab+b^2) & 0 \\ (b-c)(b+c) & (b-c)(b^2+bc+c^2) & 0 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} a+b & a^2+ab+b^2 & 0 \\ b+c & b^2+bc+c^2 & 0 \\ c^2 & c^3 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, we get

$$(a-b)(b-c)\begin{vmatrix} a+b & a^2+ab+b^2 & 0 \\ c-a & (bc-ab)+c^2-a^2 & 0 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)\begin{vmatrix} a+b & a^2+ab+b^2 & 0 \\ 1 & b+c+a & 0 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$= (a - b)\ (b - c)\ (c - a)$
$\quad [(a + b)\ (a + b + c) - (a^2 + ab + b^2)]$
$= (a - b)\ (b - c)\ (c - a)$
$\quad [a^2 + ab + ac + ba + b^2 + bc - a^2 - ab - b^2]$
$= (a - b)\ (b - c)\ (c - a)\ (bc + ca + ab) =$ R.H.S.

43. L.H.S. $= \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Applying $R_1 \to R_1 - R_2 - R_3$, we get

$$\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking –2 common from R_1, we get

$$(-2)\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $C_2 \to \frac{1}{c}C_2, C_3 \to \frac{1}{b}C_3$, we get

$$(-2)\,bc\begin{vmatrix} 0 & 1 & 1 \\ b & 1+a/c & 1 \\ c & 1 & a/b+1 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_3$, we get

$$-2bc\begin{vmatrix} 0 & 0 & 1 \\ b & a/c & 1 \\ c & -a/b & a/b+1 \end{vmatrix}$$

$= -\ 2bc\ (-\ a - a) = 4abc =$ R.H.S.

44. Here, $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$

Multiplying C_1, C_2 and C_3 by x, y and z respectively, we get

$$\Delta_1 = \frac{1}{xyz}\begin{vmatrix} Ax & By & Cz \\ x^2 & y^2 & z^2 \\ xzy & xyz & xyz \end{vmatrix}$$

Taking xyz common from R_3, we get

$$\Delta_1 = \frac{1}{xyz}\cdot xyz\begin{vmatrix} Ax & By & Cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

Interchanging corresponding rows and columns, we get

$$\Delta_1 = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix} = \Delta.$$

45. L.H.S. $= \begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} x+y+z & -x+y & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$$

Taking $(x + y + z)$ common from C_1, we get

$$(x+y+z)\begin{vmatrix} 1 & y-x & z-x \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$, we get

$$(x+y+z)\begin{vmatrix} 1 & y-x & z-x \\ 0 & 2y+x & x-y \\ 0 & x-z & 2z+x \end{vmatrix}$$

$= (x + y + z)\ [(2y + x)\cdot(2z + x) - (x - z)\cdot(x - y)]$

$= (x + y + z) \cdot [(4yz + 2xy + 2zx + x^2)$
$- (x^2 - xy - zx + yz)]$

$= 3(x + y + z)\ (xy + yz + zx) =$ R.H.S.

46. L.H.S. $= \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} 0 & a-b & a^3-b^3 \\ 0 & b-c & b^3-c^3 \\ 1 & c & c^3 \end{vmatrix}$$

Taking $(a - b)$ and $(b - c)$ common from R_1 and R_2 respectively, we get

$$(a-b)(b-c)\begin{vmatrix} 0 & 1 & a^2+ab+b^2 \\ 0 & 1 & b^2+bc+c^2 \\ 1 & c & c^3 \end{vmatrix}$$

$= (a - b)\ (b - c)\ [(b^2 + bc + c^2) - (a^2 + ab + b^2)]$

$= (a - b)\ (b - c)\ [(c^2 - a^2) + b(c - a)]$

$= (a - b)\ (b - c)\ (c - a)\ (c + a + b) =$ R.H.S.

47. *Refer to answer 42.*

48. *Refer to answer 37.*

49. *Refer to answer 46.*

50. L.H.S. $= \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$

Applying $R_3 \to R_3 + R_1$, we get

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \alpha+\beta+\gamma & \alpha+\beta+\gamma & \alpha+\beta+\gamma \end{vmatrix}$$

Taking $(\alpha + \beta + \gamma)$ common from R_3, we get

$$(\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$, we get

$$(\alpha+\beta+\gamma)\begin{vmatrix} \alpha-\beta & \beta-\gamma & \gamma \\ \alpha^2-\beta^2 & \beta^2-\gamma^2 & \gamma^2 \\ 0 & 0 & 1 \end{vmatrix}$$

Taking $(\alpha - \beta)$ and $(\beta - \gamma)$ common from C_1 and C_2 respectively, we get

$$(\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)\begin{vmatrix} 1 & 1 & \gamma \\ \alpha+\beta & \beta+\gamma & \gamma^2 \\ 0 & 0 & 1 \end{vmatrix}$$

$= (\alpha + \beta + \gamma)\ (\alpha - \beta)\ (\beta - \gamma)\ [(\beta + \gamma) - (\alpha + \beta)]$

$= (\alpha - \beta)\ (\beta - \gamma)\ (\gamma - \alpha)\ (\alpha + \beta + \gamma) =$ R.H.S.

51. L.H.S. $= \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$$

Taking $(a + b + c)$ common from C_1, we get

$$(a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

Applying $R_3 \to R_3 - 2R_1$, we get

$$(a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

$= (a + b + c)\ [(b - c)(a + b - 2c) - (c - a)(c + a - 2b)]$

$= (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3\ abc =$ R.H.S.

52. *Refer to answer 51.*

53. L.H.S. $= \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$, we get

$$\begin{vmatrix} a^2 & -(b-c)^2 & bc \\ b^2 & -(c-a)^2 & ca \\ c^2 & -(a-b)^2 & ab \end{vmatrix}$$

Taking (–1) common from C_2, we get

$$-\begin{vmatrix} a^2 & b^2+c^2-2bc & bc \\ b^2 & c^2+a^2-2ca & ca \\ c^2 & a^2+b^2-2ab & ab \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1 + 2C_3$, we get

$$-\begin{vmatrix} a^2 & a^2+b^2+c^2 & bc \\ b^2 & a^2+b^2+c^2 & ca \\ c^2 & a^2+b^2+c^2 & ab \end{vmatrix}$$

Taking $(a^2 + b^2 + c^2)$ common from C_2, we get

$$-(a^2+b^2+c^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$, we get

$$-(a^2+b^2+c^2)\begin{vmatrix} a^2-b^2 & 0 & c(b-a) \\ b^2-c^2 & 0 & a(c-b) \\ c^2 & 1 & ab \end{vmatrix}$$

Taking $(a - b)$ and $(b - c)$ common from R_1 and R_2 respectively, we get

$$-(a-b)(b-c)(a^2+b^2+c^2)\begin{vmatrix} a+b & 0 & -c \\ b+c & 0 & -a \\ c^2 & 1 & ab \end{vmatrix}$$

$$=-(a-b)(b-c)(a^2+b^2+c^2)[(a+b)a-c(b+c)]$$

$= -(a - b)(b - c)(a^2 + b^2 + c^2)[a^2 + ab - bc - c^2 + ac - ac]$

$= -(a - b)(b - c)(a^2 + b^2 + c^2)[a(a + b + c) - c(a + b + c)]$

$= -(a - b)(b - c)(a^2 + b^2 + c^2)(a + b + c)(a - c)$

$= (a - b)(b - c)(c - a)(a + b + c)(a^2 + b^2 + c^2)$

= R.H.S.

54. $\text{L.H.S.} = \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}$

Applying $C_1 \to aC_1$, we get

$$\frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & b+a & c \end{vmatrix}$$

Applying $C_1 \to C_1 + bC_2 + cC_3$, we get

$$\frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & b+a & c \end{vmatrix}$$

Taking $(a^2 + b^2 + c^2)$ common from C_1, we get

$$\frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_2$, we get

$$\frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & a & a \end{vmatrix}$$

$$=\frac{a^2+b^2+c^2}{a}\cdot 1(ac+a^2+ab)$$

$$=\frac{a^2+b^2+c^2}{a}\cdot a(a+b+c)$$

$=(a + b + c)(a^2 + b^2 + c^2) = \text{R.H.S.}$

55. $\text{L.H.S.} = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

Taking a, b, c common from R_1, R_2 and R_3, respectively, we get

$$abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

Taking a, b, c common from C_1, C_2 and C_3, respectively, we get

$$a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$, we get

$$a^2b^2c^2\begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$=a^2b^2c^2 \cdot 2(1 + 1) = 4a^2b^2c^2 = \text{R.H.S.}$

56. $\text{L.H.S.} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$

Taking x, y and z common from C_1, C_2 and C_3 respectively, we get

$$xyz\begin{vmatrix}1 & 1 & 1\\ x & y & z\\ x^2 & y^2 & z^2\end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$; $C_3 \to C_3 - C_1$, we get

$$xyz\begin{vmatrix}1 & 0 & 0\\ x & y-x & z-x\\ x^2 & y^2-x^2 & z^2-x^2\end{vmatrix}$$

Taking $(y - x)$ and $(z - x)$ common from C_2 and C_3 respectively, we get

$$xyz(y-x)(z-x)\begin{vmatrix}1 & 0 & 0\\ x & 1 & 1\\ x^2 & y+x & z+x\end{vmatrix}$$

$= xyz\,(y - x)\,(z - x) \cdot 1[(z + x) - (y + x)]$
$= xyz\,(y - x)\,(z - x)\,[z + x - y - x]$
$= xyz\,(y - x)\,(z - x)\,(z - y)$
$= xyz\,(x - y)\,(y - z)\,(z - x) =$ R.H.S.

57. *Refer to answer 40.*

58. Given, $\begin{vmatrix}x-2 & 2x-3 & 3x-4\\ x-4 & 2x-9 & 3x-16\\ x-8 & 2x-27 & 3x-64\end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix}2 & 6 & 12\\ 4 & 18 & 48\\ x-8 & 2x-27 & 3x-64\end{vmatrix} = 0$$

Taking 2 common from R_1 and R_2 both, we get

$$2\times 2\begin{vmatrix}1 & 3 & 6\\ 2 & 9 & 24\\ x-8 & 2x-27 & 3x-64\end{vmatrix} = 0$$

Applying $C_2 \to C_2 - 2C_1$ and $C_3 \to C_3 - 3C_1$, we get

$$4\begin{vmatrix}1 & 1 & 3\\ 2 & 5 & 18\\ x-8 & -11 & -40\end{vmatrix} = 0$$

Applying $R_2 \to R_2 - 2R_1$, we get

$$4\begin{vmatrix}1 & 1 & 3\\ 0 & 3 & 12\\ x-8 & -11 & -40\end{vmatrix} = 0$$

$\Rightarrow$ $4[(-120 + 132) + (x - 8)\,(12 - 9)] = 0$
$\Rightarrow$ $4(12 + 3x - 24) = 0$
$\Rightarrow$ $3x - 12 = 0$
$\Rightarrow$ $x = 4.$

59. Given, $\begin{vmatrix}x+a & x & x\\ x & x+a & x\\ x & x & x+a\end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix}3x+a & x & x\\ 3x+a & x+a & x\\ 3x+a & x & x+a\end{vmatrix} = 0$$

Taking $(3x + a)$ common from C_1, we get

$$(3x+a)\begin{vmatrix}1 & x & x\\ 1 & x+a & x\\ 1 & x & x+a\end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$, we get

$$(3x+a)\begin{vmatrix}1 & x & x\\ 0 & a & 0\\ 0 & 0 & a\end{vmatrix} = 0$$

$\Rightarrow$ $(3x + a)\,(1 \cdot a \cdot a) = 0$
$\Rightarrow$ $a^2\,(3x + a) = 0$
$\Rightarrow$ $x = -\dfrac{a}{3}.$ $[\because a \neq 0]$

60. *Refer to answer 35.*

61. We have, $\begin{vmatrix}(b+c)^2 & a^2 & bc\\ (c+a)^2 & b^2 & ca\\ (a+b)^2 & c^2 & ab\end{vmatrix}$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix}(b+c)^2-(a+b)^2 & a^2-c^2 & bc-ab\\ (c+a)^2-(a+b)^2 & b^2-c^2 & ca-ab\\ (a+b)^2 & c^2 & ab\end{vmatrix}$$

$$= \begin{vmatrix}(c-a)(a+2b+c) & -(c+a)(c-a) & b(c-a)\\ -(b-c)(2a+b+c) & (b+c)(b-c) & -a(b-c)\\ (a+b)^2 & c^2 & ab\end{vmatrix}$$

$$= (b-c)(c-a)\begin{vmatrix}a+2b+c & -c-a & b\\ -2a-b-c & b+c & -a\\ (a+b)^2 & c^2 & ab\end{vmatrix}$$

Applying $R_1 \to R_1 + R_2$, we get

$$(b-c)(c-a)\begin{vmatrix} -(a-b) & -(a-b) & -(a-b) \\ -2a-b-c & b+c & -a \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$= (b-c)(c-a)(a-b)\begin{vmatrix} -1 & -1 & -1 \\ -2a-b-c & b+c & -a \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$, we get

$$(a-b)(b-c)(c-a)\begin{vmatrix} 0 & 0 & -1 \\ -(a+b+c) & (a+b+c) & -a \\ (a+b)^2-ab & c^2-ab & ab \end{vmatrix}$$

$= (a-b)(b-c)(c-a)(-1)[-(a+b+c)(c^2-ab) - (a+b+c)((a+b)^2-ab)]$

$= (a-b)(b-c)(c-a)(a+b+c)[c^2 - ab + a^2 + b^2 + 2ab - ab]$

$= (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

62. We have, a, b, c are p^{th}, q^{th} and r^{th} terms of G.P.

Then $A_p = AR^{p-1} = a$, $A_q = AR^{q-1} = b$, $A_r = AR^{r-1} = c$, where A and R be the first term and common ratio of the G.P. respectively.

Now, L.H.S. = $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = \begin{vmatrix} \log(AR^{p-1}) & p & 1 \\ \log(AR^{q-1}) & q & 1 \\ \log(AR^{r-1}) & r & 1 \end{vmatrix}$

L.H.S. = $\begin{vmatrix} \log A+(p-1)\log R & p & 1 \\ \log A+(q-1)\log R & q & 1 \\ \log A+(r-1)\log R & r & 1 \end{vmatrix}$

Applying $C_1 \to C_1 - (\log A)C_3$, we get

L.H.S. = $\begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix} = \log R\begin{vmatrix} (p-1) & p & 1 \\ (q-1) & q & 1 \\ (r-1) & r & 1 \end{vmatrix}$

Applying $C_1 \to C_1 + C_3$, we get

L.H.S. = $\begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix} = 0$

(Since, C_1 and C_2 are identical)

= R.H.S.

63. Let $\Delta = \begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} -(x^2+y^2+z^2-xy-yz-zx) & zx-y^2 & xy-z^2 \\ -(x^2+y^2+z^2-xy-yz-zx) & xy-z^2 & yz-x^2 \\ -(x^2+y^2+z^2-xy-yz-zx) & yz-x^2 & zx-y^2 \end{vmatrix}$$

Taking $-(x^2 + y^2 + z^2 - xy - yz - zx)$ common from C_1, we get

$$\Delta = -(x^2+y^2+z^2-xy-yz-zx)\begin{vmatrix} 1 & zx-y^2 & xy-z^2 \\ 1 & xy-z^2 & yz-x^2 \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$, we get

$$\Delta = -(x^2+y^2+z^2-xy-yz-zx)\begin{vmatrix} 0 & (x-y)(x+y+z) & (y-z)(x+y+z) \\ 0 & (x-z)(x+y+z) & (y-x)(x+y+z) \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

Taking $(x + y + z)$ common from R_1 and R_2 both, we get

$$\Delta = -(x^2+y^2+z^2-xy-yz-zx)(x+y+z)^2\begin{vmatrix} 0 & x-y & y-z \\ 0 & x-z & y-x \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

$$\Rightarrow \Delta = -(x+y+z)(x^3+y^3+z^3-3xyz)\begin{vmatrix} 0 & x-y & y-z \\ 0 & x-z & y-x \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

$\Rightarrow \Delta = -(x+y+z)(x^3+y^3+z^3-3xyz)[(x-y)(y-x)-(x-z)(y-z)]$

$\Rightarrow \Delta = -(x+y+z)(x^3+y^3+z^3-3xyz)(xy+yz+zx-x^2-y^2-z^2)$

$\Rightarrow \Delta = (x+y+z)(x^3+y^3+z^3-3xyz)(x^2+y^2+z^2-xy-yz-zx)$

Hence, Δ is divisible by $(x + y + z)$ and quotient is $(x^3+y^3+z^3-3xyz)(x^2+y^2+z^2-xy-yz-zx)$

64. We have,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix}=0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & \cos B-\cos A & \cos C-\cos A \\ \cos^2 A+\cos A & (\cos B-\cos A)\times(\cos A+\cos B+1) & (\cos C-\cos A)\times(\cos C+\cos A+1) \end{vmatrix}=0$$

Taking common $(\cos B - \cos A)$ from C_2 and $(\cos C - \cos A)$ from C_3, we get

$(\cos B - \cos A)(\cos C - \cos A) \times$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & 1 & 1 \\ \cos^2 A+\cos A & (\cos A+\cos B+1) & (\cos C+\cos A+1) \end{vmatrix}=0$$

$\Rightarrow$ $(\cos B - \cos A)(\cos C - \cos A)(\cos C - \cos B) = 0$

$\Rightarrow$ $\cos B = \cos A \Rightarrow B = A$

or $\cos C = \cos A \Rightarrow C = A$ or $\cos C = \cos B \Rightarrow C = B$

$\therefore$ ΔABC is an isosceles triangle.

65. It is given that, $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}=0$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} a & -b & 0 \\ 0 & b & -c \\ 1 & 1 & 1+c \end{vmatrix}=0$$

$\Rightarrow$ $a(b + bc + c) + b(0 + c) = 0$

$\Rightarrow$ $ab + bc + ac + abc = 0$

Dividing both sides by abc, we get

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1=0$$

66. Using determinants, the line joining $A(1, 3)$ and $B(0,0)$ is given by $\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix}=0$

$\Rightarrow$ $1(3x - y) = 0 \Rightarrow y = 3x$

Now, $D(k, 0)$ is a point s.t. area of $\Delta ABD = 3$ sq. units

$$\therefore \quad \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix}=3$$

$\Rightarrow$ $(0 + 3k) = \pm 6 \Rightarrow k = \pm 2$

67. Let $A = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$

Cofactor of 1 = 3, Cofactor of –2 = –4.
Cofactor of 4 = 2, Cofactor of 3 = 1.

68. We have, $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$

$\therefore$ Cofactor of $a_{21} = (-1)^{2+1}\begin{vmatrix} 6 & -3 \\ -7 & 3 \end{vmatrix}$

$= -1(18 - 21) = 3$

69. Let $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Now, $a_{32} = 5$

A_{32} = cofactor of a_{32} in $\Delta = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$

$= -(8 - 30) = 22$

$\therefore$ $a_{32} \cdot A_{32} = 5 \cdot 22 = 110.$

70. Here, $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

$\therefore$ Minor of $a_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$

71. Here, $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

$\therefore$ Cofactor of $a_{32} = (-1)^{3+2}\begin{vmatrix} 5 & 8 \\ 2 & 1 \end{vmatrix} = -(5-16) = 11$

72. We have, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 5 & 3 & 8 \end{vmatrix}$

$\therefore$ Minor of $a_{22} = \begin{vmatrix} 1 & 3 \\ 5 & 8 \end{vmatrix} = 8 - 15 = -7$

73. Given, $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$

Now, $A(\text{adj } A) = |A| = \begin{vmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{vmatrix}$

$= 2(10 - 9) - 0 + 0 = 2(1) = 2$

74. **(a) :** We have, $|A| = \begin{vmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{vmatrix}$

$= -2(4 - 0) - 0 + 0 = -8$

Now, we know that $|\text{adj } A| = |A|^{n-1}$, where n is the order of A.

$\therefore \quad |\text{adj } A| = (-8)^2 = 64$

75. Given, $|A| = 9$

We know that, $|k \text{ adj } A| = k^n |A|^{n-1}$

where n is the order of the matrix A.

$\therefore \quad |2 \cdot \text{adj } A| = 2^3 (9)^2 = 8 \times 81 = 648.$

76. Given that, $\det(A^{-1}) = (\det A)^k$

i.e., $|A^{-1}| = |A|^k$

We know that $|A^{-1}| = \dfrac{1}{|A|} = |A|^{-1}$

$\therefore \quad k = -1$

77. We have, $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$|A| \cdot I = 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad [\because A(\text{adj } A) = |A| \cdot I]$

$\Rightarrow |A| = 8$

78. For the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ to be singular, its determinant = 0

$\therefore \quad \begin{vmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{vmatrix} = 0$

$\Rightarrow 4\sin^2 x - 3 = 0 \Rightarrow \sin^2 x = \dfrac{3}{4} \Rightarrow \sin x = \pm \dfrac{\sqrt{3}}{2}$

$\therefore \quad x = \dfrac{2\pi}{3} \quad \left(\because \dfrac{\pi}{2} < x < \pi\right)$

79. Here, $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

Cofactor of matrix $A = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$

$\therefore \quad \text{adj } A = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}^t = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

80. A is a 3×3 matrix s.t. $|\text{adj } A| = 64$.

We know that

$|\text{adj } A| = |A|^{n-1}$

$\Rightarrow \quad 64 = |A|^{3-1} \Rightarrow |A|^2 = 64 \Rightarrow |A| = \pm 8.$

81. Given, $|A| = 5$ and A is invertible matrix of order 3.

We know that $|\text{adj } A| = |A|^{n-1}$

$\Rightarrow \quad |\text{adj } A| = (5)^{3-1} \Rightarrow |\text{adj } A| = 25.$

82. Given, $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$.

Matrix A is singular, iff $|A| = 0$

$\Rightarrow \quad \begin{vmatrix} 3-2x & x+1 \\ 2 & 4 \end{vmatrix} = 0$

$\Rightarrow \quad 4(3 - 2x) - 2(x + 1) = 0 \Rightarrow 12 - 8x - 2x - 2 = 0$

$\Rightarrow \quad -10x + 10 = 0 \Rightarrow x = 1$

83. Let $A = \begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$

For A to be singular, $|A| = 0$

$\Rightarrow \quad \begin{vmatrix} 1+x & 7 \\ 3-x & 8 \end{vmatrix} = 0$

$\Rightarrow \quad 8 + 8x - 21 + 7x = 0 \Rightarrow 15x = 13 \Rightarrow x = \dfrac{13}{15}.$

84. We have, $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1 \neq 0$, so A^{-1} exists.

$\Rightarrow \quad \text{adj } A = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Now, $A^{-1} = \dfrac{\text{adj } A}{|A|} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

85. Let $A = \begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$

Matrix A is singular, iff $|A| = 0$

$\Rightarrow \quad \begin{vmatrix} 5-x & x+1 \\ 2 & 4 \end{vmatrix} = 0$

$\Rightarrow \quad 20 - 4x - 2x - 2 = 0$

$\Rightarrow \quad -6x + 18 = 0 \Rightarrow x = 3$

86. We have, $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19 \neq 0$, so A^{-1} exists.

$\text{adj } A = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

Now, $A^{-1}=\frac{\text{adj}A}{|A|}=-\frac{1}{19}\begin{bmatrix}-2 & -3\\-5 & 2\end{bmatrix}=\frac{1}{19}\begin{bmatrix}2 & 3\\5 & -2\end{bmatrix}$

$\Rightarrow A^{-1}=\frac{1}{19}A.$

87. *Refer to answer 83.*

88. Let $A=\begin{bmatrix}2x & 4\\x+2 & 3\end{bmatrix}$

For the matrix A to be singular, $|A| = 0$

$\Rightarrow \begin{vmatrix}2x & 4\\x+2 & 3\end{vmatrix}=0$

$\Rightarrow 6x - 4x - 8 = 0 \Rightarrow 2x = 8 \Rightarrow x = 4.$

89. *Refer to answer 88.*

90. For the given matrix, $A=\begin{bmatrix}2(x+1) & 2x\\x & x-2\end{bmatrix}$ to be singular, $|A| = 0$

$\Rightarrow \begin{vmatrix}2(x+1) & 2x\\x & x-2\end{vmatrix}=0$

$\Rightarrow 2(x + 1)(x - 2) - 2x^2 = 0$

$\Rightarrow 2(x^2 - x - 2) - 2x^2 = 0 \Rightarrow 2(-x - 2) = 0$

$\Rightarrow x = -2$

91. Given, $A=\begin{bmatrix}1 & 0\\-4 & 2\end{bmatrix}$ and $B^{-1}=\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}$

Now, $|A|=\begin{vmatrix}1 & 0\\-4 & 2\end{vmatrix}=2$ and adj $A=\begin{bmatrix}2 & 0\\4 & 1\end{bmatrix}$

$\therefore A^{-1}=\frac{1}{|A|}(\text{adj }A)=\frac{1}{2}\begin{bmatrix}2 & 0\\4 & 1\end{bmatrix}$

Now, $(AB)^{-1}=B^{-1}A^{-1}$

$=\frac{1}{2}\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}\begin{bmatrix}2 & 0\\4 & 1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}10 & 1\\18 & 2\end{bmatrix}$

92. We have, $A=\begin{bmatrix}2 & -3\\-4 & 7\end{bmatrix}$

$\therefore |A|=\begin{vmatrix}2 & -3\\-4 & 7\end{vmatrix}=14-12=2\neq 0$

So, A is a non-singular matrix and therefore it is invertible.

$\therefore \text{adj } A=\begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}$

Hence, $A^{-1}=\frac{1}{|A|}\text{adj }A=\frac{1}{2}\begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}$

$\Rightarrow 2A^{-1}=\begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}$ ...(i)

Now, $9I-A=9\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}2 & -3\\-4 & 7\end{bmatrix}$

$=\begin{bmatrix}9 & 0\\0 & 9\end{bmatrix}-\begin{bmatrix}2 & -3\\-4 & 7\end{bmatrix}=\begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}=2A^{-1}$ [From (i)]

93. $A=\begin{bmatrix}1 & -2 & 3\\0 & -1 & 4\\-2 & 2 & 1\end{bmatrix}\Rightarrow A'=\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 2\\3 & 4 & 1\end{bmatrix}$

$|A'|=\begin{vmatrix}1 & 0 & -2\\-2 & -1 & 2\\3 & 4 & 1\end{vmatrix}=1(-1-8)-2(-8+3)$

$=-9+10=1\neq 0$. So, $(A')^{-1}$ exists.

Let the cofactors of a_{ij}'s are A_{ij} in A'

Now, $A_{11}=-9, A_{12}=8, A_{13}=-5,$

$A_{21}=-8, A_{22}=7, A_{23}=-4,$

$A_{31}=-2, A_{32}=2, A_{33}=-1$

$\therefore \text{adj}(A')=\begin{bmatrix}-9 & -8 & -2\\8 & 7 & 2\\-5 & -4 & -1\end{bmatrix}$

$\therefore (A')^{-1}=\frac{\text{adj}(A')}{|A'|}=\begin{bmatrix}-9 & -8 & -2\\8 & 7 & 2\\-5 & -4 & -1\end{bmatrix}$

94. Here, $A=\begin{bmatrix}-1 & -2 & -2\\2 & 1 & -2\\2 & -2 & 1\end{bmatrix}$

$\Rightarrow |A|=\begin{vmatrix}-1 & -2 & -2\\2 & 1 & -2\\2 & -2 & 1\end{vmatrix}$

$=-1(1-4)-(-2)(2+4)-2(-4-2)$

$=3+12+12=27$

Now, $A_{11}=-3, A_{12}=-6, A_{13}=-6,$

$A_{21}=6, A_{22}=3, A_{23}=-6,$

$A_{31}=6, A_{32}=-6, A_{33}=3$

$\therefore \text{adj }A=\begin{bmatrix}-3 & 6 & 6\\-6 & 3 & -6\\-6 & -6 & 3\end{bmatrix}$

$$\therefore \quad A\cdot(\text{adj } A)=\begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}\cdot\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix}$$

$$=\begin{bmatrix}27 & 0 & 0\\ 0 & 27 & 0\\ 0 & 0 & 27\end{bmatrix}=27\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=|A|I_3.$$

95. $A=\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix} \Rightarrow A^2=\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}$

$$\Rightarrow \quad A^2=\begin{bmatrix}5 & -4\\ -4 & 5\end{bmatrix} \quad \text{...(i)}$$

Now, $4A-3I=4\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}-3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$\Rightarrow \quad 4A-3I=\begin{bmatrix}5 & -4\\ -4 & 5\end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we get

$A^2 = 4A - 3I$

Pre-multiplying by A^{-1} on both sides, we get

$A^{-1}(A^2) = 4A^{-1}A - 3A^{-1}I$

$\Rightarrow \quad A = 4I - 3A^{-1} \quad [\because AA^{-1} = I]$

$\Rightarrow \quad 3A^{-1} = 4I - A$

$\Rightarrow \quad A^{-1}=\frac{4}{3}I-\frac{1}{3}A$

$$=\frac{4}{3}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\frac{1}{3}\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}=\begin{bmatrix}2/3 & 1/3\\ 1/3 & 2/3\end{bmatrix}$$

96. Given $A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$

Now, $A_{11} = 7, A_{12} = -1, A_{13} = -1,$

$A_{21} = -3, A_{22} = 1, A_{23} = 0,$

$A_{31} = -3, A_{32} = 0, A_{33} = 1$

$$\therefore \quad \text{adj } A=\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix} \text{ and } |A|=\begin{vmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{vmatrix}$$

$= 1(16 - 9) - 3(4 - 3) + 3(3 - 4)$

$= 7 - 3 - 3 = 1 \neq 0$

So, A^{-1} exists and it is given by

$$A^{-1}=\frac{1}{|A|}\cdot\text{adj } A=\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}.$$

97. Here, $A=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}$

$\Rightarrow \quad |A| = -11, |B| = 1$

$$\therefore \quad A^{-1}=\frac{1}{|A|}\text{adj } A=-\frac{1}{11}\begin{bmatrix}-4 & -3\\ -1 & 2\end{bmatrix} \text{ and } B^{-1}=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$$

$\therefore \quad$ R.H.S. $= B^{-1}A^{-1}$

$$=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}\cdot\left(-\frac{1}{11}\right)\begin{bmatrix}-4 & -3\\ -1 & 2\end{bmatrix}=\left(-\frac{1}{11}\right)\begin{bmatrix}-14 & -5\\ -5 & -1\end{bmatrix} \quad \text{...(i)}$$

Now, $A\cdot B=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}\cdot\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 5 & -14\end{bmatrix}$

$\Rightarrow \quad |AB| = 14 - 25 = -11$

$$\therefore \quad \text{L.H.S.} =(AB)^{-1}=\left(-\frac{1}{11}\right)\begin{bmatrix}-14 & -5\\ -5 & -1\end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), $(AB)^{-1} = B^{-1}A^{-1}$.

98. Given, $A=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$

Now, $A_{11} = \cos\alpha, A_{12} = -\sin\alpha, A_{13} = 0,$

$A_{21} = \sin\alpha, A_{22} = \cos\alpha, A_{23} = 0,$

$A_{31} = 0, A_{32} = 0, A_{33} = 1$

$$\therefore \quad \text{adj}(A)=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$A.\text{adj}(A)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \quad \text{...(i)}$$

$$\text{adj}(A)\cdot A=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \quad \text{...(ii)}$$

$$|A|=\begin{vmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{vmatrix}$$

$= \cos\alpha(\cos\alpha - 0) + \sin\alpha(\sin\alpha - 0) + 0 = 1$ ...(iii)

From (i), (ii) and (iii), we get

$A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I_3$.

99. Let the monthly income of Aryan be ₹ $3x$ and that of Babban be ₹ $4x$

Also, let monthly expenditure of Aryan be ₹ $5y$ and that of Babban be ₹ $7y$

According to question,

$3x - 5y = 15000$

$4x - 7y = 15000$

These equations can be written as

$AX = B$

where, $A = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -5 \\ 4 & -7 \end{vmatrix} = (-21 + 20) = -1 \neq 0$

Thus, A^{-1} exists. So, system of equations has a unique solution and given by $X = A^{-1}B$

Now, $\text{adj}(A) = \begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix}$

$\therefore \quad A^{-1} = \dfrac{\text{adj}(A)}{|A|} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$

Now, $X = A^{-1}B = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30000 \\ 15000 \end{bmatrix} \Rightarrow x = 30000$ and $y = 15000$

So, monthly income of Aryan $= 3 \times 30000$

$= ₹90000$

Monthly income of Babban $= 4 \times 30000 = ₹120000$

From this question we are encouraged to save a part of money every month.

100. Let ₹ x be invested in the first bond and ₹ y be invested in the second bond.

According to question,

$\dfrac{10x}{100} + \dfrac{12y}{100} = 2800 \Rightarrow 10x + 12y = 280000$ ...(i)

If the rate of interest had been interchanged, then the total interest earned is ₹ 100 less than the previous interest *i.e.*, ₹ 2700.

$\therefore \dfrac{12x}{100} + \dfrac{10y}{100} = 2700 \Rightarrow 12x + 10y = 270000$...(ii)

The system of equations (i) and (ii) can be represented as

$AX = B$ where $A = \begin{bmatrix} 10 & 12 \\ 12 & 10 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 280000 \\ 270000 \end{bmatrix}$

$|A| = \begin{vmatrix} 10 & 12 \\ 12 & 10 \end{vmatrix} = 100 - 144 = -44 \neq 0$

Thus A^{-1} exists. So, system of equations has a unique solution and given by $X = A^{-1}B$

$\text{adj } A = \begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}$

Now, $X = A^{-1}B \Rightarrow X = \dfrac{\text{adj } A}{|A|} B$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{(-44)}\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}\begin{bmatrix} 280000 \\ 270000 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{(-44)}\begin{bmatrix} -440000 \\ -660000 \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix}$

$\Rightarrow x = 10000$ and $y = 15000$

Therefore, ₹ 10,000 be invested in the first bond and ₹ 15,000 be invested in the second bond. Thus, the total amount invested by the trust

$= 10,000 + 15,000 = ₹ 25,000.$

The interest received will be given to Helpage India as donation reflects the helping and caring nature of the trust.

101. Let the monthly fees paid by poor and rich children be ₹ x and ₹ y respectively.

For batch I : $20x + 5y = 9000$ ...(i)

For batch II : $5x + 25y = 26000$ ...(ii)

The system of equations (i) and (ii) can be written as

$AX = B$

where, $A = \begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$

$|A| = \begin{vmatrix} 20 & 5 \\ 5 & 25 \end{vmatrix} = 500 - 25 = 475 \neq 0$

Thus, A^{-1} exists. So, the given system has a unique solution and it is given by $X = A^{-1}B$.

$\text{adj } A = \begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$

$A^{-1} = \dfrac{\text{adj } A}{|A|} = \dfrac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$

Now, $X = A^{-1}B$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 95000 \\ 475000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$\Rightarrow \; x = 200, y = 1000$

Hence, the monthly fees paid by each poor child is ₹ 200 and the monthly fees paid by each rich child is ₹ 1000.

By offering discount to the poor children, the coaching institute offers an unbiased chance for the development and enhancement of the weaker section of our society.

102. (i) Given, value of prize for team spirit = ₹ x

Value of prize for truthfulness = ₹ y

Value of prize for tolerance = ₹ z

Linear equation for School A is $3x + y + 2z = 1100$

Linear equation for School B is $x + 2y + 3z = 1400$

Linear equation for Prize is $x + y + z = 600$

The corresponding matrix equation is $PX = Q$

$$\text{where, } P = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } Q = \begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$$

$$\text{(ii) Now } |P| = \begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 3\cdot(2 - 3) - 1\cdot(1 - 3) + 2\cdot(1 - 2)$

$= -3 + 2 - 2 = -3 \neq 0$

Thus, P^{-1} exists. So, system of equations has unique solution and it is given by $X = P^{-1}Q$

Now, cofactors of elements of P are

$A_{11} = -1, A_{12} = 2, A_{13} = -1,$

$A_{21} = 1, A_{22} = 1, A_{23} = -2,$

$A_{31} = -1, A_{32} = -7, A_{33} = 5$

$$\therefore \; \text{adj } P = \begin{bmatrix} -1 & 1 & -1 \\ 2 & 1 & -7 \\ -1 & -2 & 5 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|}\cdot \text{adj } P = -\frac{1}{3}\begin{bmatrix} -1 & 1 & -1 \\ 2 & 1 & -7 \\ -1 & -2 & 5 \end{bmatrix}$$

Now, $X = P^{-1}Q$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -1 & 1 & -1 \\ 2 & 1 & -7 \\ -1 & -2 & 5 \end{bmatrix}\begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -300 \\ -600 \\ -900 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix} \Rightarrow x = 100, y = 200, z = 300.$$

Thus, the above system of equations is solvable.

(iii) The value truthfulness should be rewarded the most because a student who is truthful will be also tolerant and will work with a team spirit in the school.

$$\textbf{103. } \text{Given, } A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix} = 1(2 - 2) - 2(3 + 4) - 3(-3 - 4)$$

$= -14 + 21 = 7 \neq 0$

$\therefore \; A^{-1}$ exists

Now, $A_{11} = 0, A_{12} = -7, A_{13} = -7, A_{21} = 1, A_{22} = 7,$ $A_{23} = 5, A_{31} = 2, A_{32} = -7, A_{33} = -4$

$$\therefore \; \text{adj } A = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$$\Rightarrow \; A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

The given system of equations is

$x + 2y - 3z = 6$

$3x + 2y - 2z = 3$

$2x - y + z = 2$

The system of equations can be written as $AX = B$

$$\text{where } A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$\because \; A^{-1}$ exists, so system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\Rightarrow \; x = 1, y = -5, z = -5$

104. We have, $x - y + 2z = 7$

$2x - y + 3z = 12$

$3x + 2y - z = 5$

The given system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix}$$

Here, $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{vmatrix}$

$= 1(1-6) + 1(-2-9) + 2(4+3)$
$= -5 - 11 + 14 = -2 \neq 0$

$\therefore$ A^{-1} exists. So system of equations has a unique solution given by $X = A^{-1}B$

$$\therefore \text{ adj } A = \begin{bmatrix} -5 & 11 & 7 \\ 3 & -7 & -5 \\ -1 & 1 & 1 \end{bmatrix}' = \begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{-1}{2}\begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -4 \\ -2 \\ -6 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 2, y = 1, z = 3$

105. We have, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

$= 2(-4+4) + 3(-6+4) + 5(3-2)$
$= -6 + 5 = -1 \neq 0$

$\therefore$ A^{-1} exists.

Now, $A_{11} = 0, A_{12} = 2, A_{13} = 1, A_{21} = -1, A_{22} = -9,$ $A_{23} = -5, A_{31} = 2, A_{32} = 23, A_{33} = 13$

$$\therefore \text{ adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|}\text{adj } A = (-1)\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations is

$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3$

The system of equations can be written as $AX = B$,

$$\text{where } A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

Since A^{-1} exists, therefore, system of equations has a unique solution given by

$$X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow$ $x = 1, y = 2$ and $z = 3$.

106. We have $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

Now, $|A| = 1(0-2) - 1(1-6) + 1(1)$
$= -2 + 5 + 1 = 4 \neq 0$

$\therefore$ A^{-1} exists.

Now, $A_{11} = -2, A_{12} = 5, A_{13} = 1, A_{21} = 0, A_{22} = -2,$ $A_{23} = 2, A_{31} = 2, A_{32} = -1, A_{33} = -1$

$$\therefore \text{ adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

Now the given equations are

$x + y + z = 6$
$x + 0y + 2z = 7$
$3x + y + z = 12$

The given system of equations can be written as $AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$\because A^{-1}$ exists, so system has a unique solution given by $X = A^{-1}B$.

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -12+0+24 \\ 30-14-12 \\ 6+14-12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\Rightarrow$ $x = 3, y = 1, z = 2$

107. Here, $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$$\Rightarrow \quad |A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}$$

$= 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9)$
$= -6 + 28 + 45 = 67 \neq 0$
$\therefore$ A^{-1} exists.
Now, $A_{11} = -6, A_{12} = 14, A_{13} = -15, A_{21} = 17, A_{22} = 5,$
$A_{23} = 9, A_{31} = 13, A_{32} = -8, A_{33} = -1$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\cdot \text{adj } A = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

The given system of equations is
$x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$
The system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

$\therefore$ A^{-1} exists, so system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \Rightarrow x = 3, y = -2, z = 1.$$

108. We have, $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$\text{or} \quad \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}$$

Now the given system of equations is
$x + 3z = 9$
$-x + 2y - 2z = 4$
$2x - 3y + 4z = -3$
The system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$

Since, A^{-1} exists, so system of equations has a unique solution, given by
$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}\begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -18+36-18 \\ 8-3 \\ 9-12+6 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

$\Rightarrow$ $x = 0, y = 5, z = 3$

109. We have, $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$$\Rightarrow \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = I$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

The given system of equations is
$x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$ and it can be written as
$AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

Here, $|A| = 1(-6 + 2) + 1(3 + 4) + 1(1 + 4)$
$= -4 + 7 + 5 = 8 \neq 0$

So, the given system of equations has a unique solution given by $X = A^{-1}B$.

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \Rightarrow x = 3, y = -2, z = -1$$

110. $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Since, $AA^{-1} = I$

$$\Rightarrow \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_3$, we get

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 8R_1$, we get

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & -3 & -3 \\ 0 & -12 & -13 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & -8 \end{bmatrix}$$

Applying $R_3 \to (-1)\, R_3$, we get

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & -3 & -3 \\ 0 & 12 & 13 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 0 & 8 \end{bmatrix}$$

Applying $R_3 \to R_3 + 4R_2$, we get

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & -3 & -3 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 4 & 0 \end{bmatrix}$$

Applying $R_2 \to \left(-\frac{1}{3}\right) R_2$, we get

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}$$

Applying $R_2 \to R_2 - R_3$ and $R_1 \to R_1 - 2R_3$, we get

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & -8 & 1 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}$$

Applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}$$

$$\text{So, } A^{-1} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix} \quad \text{...(i)}$$

The given system of linear equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 19 \\ 5 \\ 7 \end{bmatrix}$$

$\therefore$ The solution of above equation is $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}\begin{bmatrix} 19 \\ 5 \\ 7 \end{bmatrix} \quad \text{[From (i)]}$$

$$=\begin{bmatrix} 0+10/3-7/3 \\ 19-65/3+14/3 \\ -19+20+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \Rightarrow x=1, y=2, z=1$$

111. Let one pen of variety 'A' costs ₹ x, one pen of variety 'B' costs ₹ y and one pen of variety 'C' costs ₹ z.

According to question,

$x + y + z = 21$ (For Meenu)

$4x + 3y + 2z = 60$ (For Jeevan)

$6x + 2y + 3z = 70$ (For Shikha)

The given system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix} = 1(9-4)-1(12-12)+1(8-18) = -5 \neq 0$$

$\therefore$ A^{-1} exists and system of equations has a unique solution given by $X = A^{-1}B$.

Now, $A_{11} = 5, A_{12} = 0, A_{13} = -10,$

$A_{21} = -1, A_{22} = -3, A_{23} = 4,$

$A_{31} = -1, A_{32} = 2, A_{33} = -1$

$$\therefore \text{ adj } A = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{(-5)} \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-5)} \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix} \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-5)} \begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \Rightarrow x = 5, y = 8, z = 8$$

$\therefore$ Cost of 1 pen of variety 'A' = ₹ 5

Cost of 1 pen of variety 'B' = ₹ 8

Cost of 1 pen of variety 'C' = ₹ 8

112. According to question, we have,

$3x + 2y + z = 1000$ ...(i)

$4x + y + 3z = 1500$ ...(ii)

$x + y + z = 600$ ...(iii)

The given system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -5 \neq 0$$

$\therefore$ A is invertible and system of equations has a unique solution given by $X = A^{-1} B$

Now, $A_{11} = -2, A_{12} = -1, A_{13} = 3,$

$A_{21} = -1, A_{22} = 2, A_{23} = -1,$

$A_{31} = 5, A_{32} = -5, A_{33} = -5$

$$\therefore \text{ adj } A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{-1}{5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix} = \frac{-1}{5} \begin{bmatrix} -500 \\ -1000 \\ -1500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix} \Rightarrow x = 100, y = 200, z = 300$$

Hence the money awarded for discipline, politeness and punctuality are ₹ 100, ₹ 200 and ₹ 300 respectively.

Apart from the above three values schools can award children for sincerity.

113. *Refer to answer 112.*

114. *Refer to answer 112.*

115. Let ₹ x, ₹ y and ₹ z be deposited at the rates of interest 5%, 8% and $8\frac{1}{2}\%$ respectively.

According to question,

$x + y + z = 7000$

$x - y = 0$

$$x \cdot \frac{5}{100} + y \cdot \frac{8}{100} + z \cdot \frac{17}{2} \times \frac{1}{100} = 550$$

$\Rightarrow$ $10x + 16y + 17z = 110000$

The system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 10 & 16 & 17 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 7000 \\ 0 \\ 110000 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 10 & 16 & 17 \end{vmatrix} = 1(-17) - (17) + 1(16 + 10)$$

$= -8 \neq 0$

$\therefore$ A^{-1} exists. So, system of equations has a unique solution and it is given by $X = A^{-1}B$

Now, $A_{11} = -17, A_{12} = -17, A_{13} = 26,$
$A_{21} = -1, A_{22} = 7, A_{23} = -6,$
$A_{31} = 1, A_{32} = 1, A_{33} = -2$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -17 & -1 & 1 \\ -17 & 7 & 1 \\ 26 & -6 & -2 \end{bmatrix}$$

$$\text{and } A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-8} \begin{bmatrix} -17 & -1 & 1 \\ -17 & 7 & 1 \\ 26 & -6 & -2 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} -17 & -1 & 1 \\ -17 & 7 & 1 \\ 26 & -6 & -2 \end{bmatrix} \cdot \begin{bmatrix} 7000 \\ 0 \\ 110000 \end{bmatrix}$$

$$= -\frac{1}{8} \begin{bmatrix} -9000 \\ -9000 \\ -38000 \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \\ 4750 \end{bmatrix}$$

$\Rightarrow$ $x = 1125 = y, z = 4750$

116. According to question, we have

$x + y + z = 1500$
$2x + 3y + 4z = 4600$
$3x + 2y + 3z = 4100$

The system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 2 & 3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1500 \\ 4600 \\ 4100 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 2 & 3 \end{vmatrix}$$

$= 1(9 - 8) - 1(6 - 12) + 1(4 - 9)$
$= 1 + 6 - 5 = 2 \neq 0$

$\therefore$ A^{-1} exists and so, system of equations has a unique solution given by $X = A^{-1}B$

Now, $A_{11} = 1, A_{12} = 6, A_{13} = -5,$
$A_{21} = -1, A_{22} = 0, A_{23} = 1,$
$A_{31} = 1, A_{32} = -2, A_{33} = 1$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 1 & -1 & 1 \\ 6 & 0 & -2 \\ -5 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ 6 & 0 & -2 \\ -5 & 1 & 1 \end{bmatrix}$$

Now, $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ 6 & 0 & -2 \\ -5 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1500 \\ 4600 \\ 4100 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1000 \\ 800 \\ 1200 \end{bmatrix} = \begin{bmatrix} 500 \\ 400 \\ 600 \end{bmatrix} \Rightarrow x = 500; y = 400; z = 600.$$

Apart from sincerity, truthfulness and hard work, the schools can include an award for regularity.

117. Let the award money for honesty = ₹ x
Let the award money for regularity = ₹ y
Let the award money for hard work = ₹ z
According to question, we have
$x + y + z = 6000; 3z + x = 11000; x + z - 2y = 0$
The system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{vmatrix}$$

$= 1(0 + 6) - 1(1 - 3) + 1(-2 - 0)$
$= 6 - (-2) - 2 = 6 \neq 0$

$\therefore$ A is invertible. So, the given system has a unique solution given by $X = A^{-1}B$

Now, $A_{11} = 6, A_{12} = 2, A_{13} = -2$
$A_{21} = -3, A_{22} = 0, A_{23} = 3$
$A_{31} = 3, A_{32} = -2, A_{33} = -1$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix} \Rightarrow x = 500, y = 2000, z = 3500$$

One more value which the school can include for awards is discipline.

118. According to question, we have $x + y + z = 12$

$2x + 3(y + z) = 33 \Rightarrow 2x + 3y + 3z = 33$

$x + z = 2y \Rightarrow x - 2y + z = 0$

The system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{vmatrix} = 1(3 + 6) - 1(2 - 3) + 1(-4 - 3)$$

$= 9 + 1 - 7 = 3 \neq 0$

$\therefore$ A^{-1} exists. So, system of equations has a unique solution and it is given by $X = A^{-1}B$

Now, $A_{11} = 9, A_{12} = 1, A_{13} = -7,$

$A_{21} = -3, A_{22} = 0, A_{23} = 3, A_{31} = 0, A_{32} = -1, A_{33} = 1$

$$\therefore \quad \text{adj}\, A = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A) = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$\Rightarrow \quad x = 3, y = 4, z = 5.$

The management of the colony can include the awards for those members of the colony who help for keeping the environment of the colony free from pollution.

119. According to question, we have

$x + y + z = 12000$

$4x + 3y + 2z = 37000$

$5x + 3y + 4z = 47000$

The system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 5 & 3 & 4 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 12000 \\ 37000 \\ 47000 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 5 & 3 & 4 \end{vmatrix}$$

$= 1(12 - 6) - 1(16 - 10) + 1(12 - 15)$

$= 6 - 6 - 3 = -3 \neq 0$

$\therefore$ A^{-1} exists. So, system of equations has a unique solution and it is given by $X = A^{-1}B$

Now, $A_{11} = 6, A_{12} = -6, A_{13} = -3,$

$A_{21} = -1, A_{22} = -1, A_{23} = 2,$

$A_{31} = -1, A_{32} = 2, A_{33} = -1$

$$\therefore \quad \text{adj}\, A = \begin{bmatrix} 6 & -1 & -1 \\ -6 & -1 & 2 \\ -3 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\cdot \text{adj}\, A = -\frac{1}{3}\begin{bmatrix} 6 & -1 & -1 \\ -6 & -1 & 2 \\ -3 & 2 & -1 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 6 & -1 & -1 \\ -6 & -1 & 2 \\ -3 & 2 & -1 \end{bmatrix}\begin{bmatrix} 12000 \\ 37000 \\ 47000 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -12000 \\ -15000 \\ -9000 \end{bmatrix} = \begin{bmatrix} 4000 \\ 5000 \\ 3000 \end{bmatrix}$$

$\Rightarrow \quad x = 4000, y = 5000; z = 3000$

The values described in this question are resourcefulness, competence and determination.

120. (i) According to question, we have

$x + y + z = 9500$

$2x + 4y + 3z = 29000$

$5x + 2y + 3z = 30500$

The system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 5 & 2 & 3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 9500 \\ 29000 \\ 30500 \end{bmatrix}$

(ii) $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 5 & 2 & 3 \end{vmatrix}$

$= 1(12 - 6) - 1(6 - 15) + 1(4 - 20)$

$= 6 + 9 - 16 = -1 \neq 0$

$\therefore$ A^{-1} exists. So, system of equations has a unique solution and it is given by $X = A^{-1}B$

Now, $A_{11} = 6, A_{12} = 9, A_{13} = -16$,
$A_{21} = -1, A_{22} = -2, A_{23} = 3$,
$A_{31} = -1, A_{32} = -1, A_{33} = 2$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 6 & -1 & -1 \\ 9 & -2 & -1 \\ -16 & 3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = (-1)\begin{bmatrix} 6 & -1 & -1 \\ 9 & -2 & -1 \\ -16 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 1 & 1 \\ -9 & 2 & 1 \\ 16 & -3 & -2 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B = \begin{bmatrix} -6 & 1 & 1 \\ -9 & 2 & 1 \\ 16 & -3 & -2 \end{bmatrix}\begin{bmatrix} 9500 \\ 29000 \\ 30500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2500 \\ 3000 \\ 4000 \end{bmatrix}$$

$\Rightarrow$ $x = 2500, y = 3000, z = 4000$.

(iii) The factories honours the most, those employees who are keeping calm in tense situations.

121. We have, $x - y + 2z = 7$
$3x + 4y - 5z = -5$
$2x - y + 3z = 12$

The given system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

Here $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$

$= 1(12 - 5) - 3(-3 + 2) + 2(5 - 8)$

$= 7 + 3 - 6 = 4 \neq 0$.

$\therefore$ A^{-1} exists. So, system of equations has a unique solution given by $X = A^{-1}B$

Now, $A_{11} = 7, A_{12} = -19, A_{13} = -11, A_{21} = 1, A_{22} = -1$,
$A_{23} = -1, A_{31} = -3, A_{32} = 11, A_{33} = 7$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\Rightarrow$ $x = 2, y = 1, z = 3$

122. We have,

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

The given system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$

$= 2(4 + 1) - 3(-2 - 3) + 3(-1 + 6)$

$= 10 + 15 + 15 = 40 \neq 0$

$\therefore$ A^{-1} exists. So, system of equations has a unique solution and it is given by $X = A^{-1}B$

Now, $A_{11} = 5, A_{12} = 5, A_{13} = 5, A_{21} = 3, A_{22} = -13$,
$A_{23} = 11, A_{31} = 9, A_{32} = 1, A_{33} = -7$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\cdot \text{adj } A=\frac{1}{40}\begin{bmatrix}5 & 3 & 9\\ 5 & -13 & 1\\ 5 & 11 & -7\end{bmatrix}$$

$$\text{Now, } X=A^{-1}B=\frac{1}{40}\begin{bmatrix}5 & 3 & 9\\ 5 & -13 & 1\\ 5 & 11 & -7\end{bmatrix}\begin{bmatrix}5\\ -4\\ 3\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{40}\begin{bmatrix}40\\ 80\\ -40\end{bmatrix}=\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}$$

$\Rightarrow$ $x = 1, y = 2, z = -1$.

123. We have,

$3x+4y+7z=4$

$2x-y+3z=-3$

$x+2y-3z=8$

The system of equations can be written as $AX = B$

$$\text{where, } A=\begin{bmatrix}3 & 4 & 7\\ 2 & -1 & 3\\ 1 & 2 & -3\end{bmatrix};X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\text{ and } B=\begin{bmatrix}4\\ -3\\ 8\end{bmatrix}$$

$$\text{Now, } |A|=\begin{vmatrix}3 & 4 & 7\\ 2 & -1 & 3\\ 1 & 2 & -3\end{vmatrix}$$

$= 3(3 - 6) - 4(- 6 - 3) + 7(4 + 1)$

$= - 9 + 36 + 35 = 62 \neq 0$

$\therefore$ A^{-1} exists. So, system of equations has a unique solution given by $X = A^{-1}B$

Now, $A_{11} = -3, A_{12} = 9, A_{13} = 5, A_{21} = 26, A_{22} = -16,$ $A_{23} = -2, A_{31} = 19, A_{32} = 5, A_{33} = -11$

$$\therefore \quad \text{adj } A=\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\cdot \text{adj } A=\frac{1}{62}\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}$$

$$\text{Now, } X=A^{-1}B=\frac{1}{62}\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}\begin{bmatrix}4\\ -3\\ 8\end{bmatrix}=\frac{1}{62}\begin{bmatrix}62\\ 124\\ -62\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}\Rightarrow x=1, y=2, z=-1.$$

124. We have,

$x+y-z=3$

$2x+3y+z=10$

$3x-y-7z=1$

The system of equations can be written as $AX = B$

$$\text{where, } A=\begin{bmatrix}1 & 1 & -1\\ 2 & 3 & 1\\ 3 & -1 & -7\end{bmatrix};X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\text{ and } B=\begin{bmatrix}3\\ 10\\ 1\end{bmatrix}$$

$$\text{Now, } |A|=\begin{vmatrix}1 & 1 & -1\\ 2 & 3 & 1\\ 3 & -1 & -7\end{vmatrix}$$

$= 1(-21 + 1) - 1 (-14 - 3) - 1(- 2 - 9)$

$= - 20 + 17 + 11 = 8 \neq 0$

$\therefore$ A^{-1} exists. So, system of equation has a unique solution given by $X = A^{-1}B$

Now, $A_{11} = -20, A_{12} = 17, A_{13} = -11, A_{21} = 8,$ $A_{22} = -4, A_{23} = 4, A_{31} = 4, A_{32} = -3, A_{33} = 1$

$$\therefore \quad \text{adj}A=\begin{bmatrix}-20 & 8 & 4\\ 17 & -4 & -3\\ -11 & 4 & 1\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\cdot \text{adj } A=\frac{1}{8}\begin{bmatrix}-20 & 8 & 4\\ 17 & -4 & -3\\ -11 & 4 & 1\end{bmatrix}$$

$$\text{Now, } X=A^{-1}B=\frac{1}{8}\begin{bmatrix}-20 & 8 & 4\\ 17 & -4 & -3\\ -11 & 4 & 1\end{bmatrix}\begin{bmatrix}3\\ 10\\ 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{8}\begin{bmatrix}24\\ 8\\ 8\end{bmatrix}=\begin{bmatrix}3\\ 1\\ 1\end{bmatrix}\Rightarrow x=3, y=1, z=1$$

$$\textbf{125.}\text{ Here, } A=\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 1\\ 1 & -3 & 1\end{bmatrix}$$

$$\Rightarrow |A|=\begin{vmatrix}1 & 2 & 1\\ -1 & 1 & 1\\ 1 & -3 & 1\end{vmatrix}$$

$= 1(1 + 3) - 2(- 1 - 1) + 1(3 - 1) = 10 \neq 0$

$\therefore$ A^{-1} exists.

Now, $A_{11} = 4, A_{12} = 2, A_{13} = 2, A_{21} = -5, A_{22} = 0,$ $A_{23} = 5, A_{31} = 1, A_{32} = -2, A_{33} = 3$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$

The given system of equations is

$x + 2y + z = 4; - x + y + z = 0; x - 3y + z = 4$

The system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

Since A^{-1} exists, therefore, system of equations has a unique solution given by

$$X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \Rightarrow x = 2, y = 0, z = 2.$$

126. *Refer to answer 107.*

127. Here, $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}; \ B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$$\therefore \quad AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

$$\Rightarrow \quad A \cdot \left(\frac{1}{6}B\right) = I \Rightarrow A \text{ is invertible and } A^{-1} = \frac{1}{6}B$$

Now the given system of equations is

$x - y = 3$

$2x + 3y + 4z = 17$

$y + 2z = 7$

The system of equations can be written as

$AX = P$

$$\text{where, } A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; P = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

Since A^{-1} exists, so system of equations has a unique solution given by $X = A^{-1} P$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}BP = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\Rightarrow \quad x = 2, y = - 1, z = 4.$

128. Here, $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$

$$\Rightarrow \quad |A| = \begin{vmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{vmatrix}$$

$= 1(1 + 3) - 2(- 1 + 2) + 5(3 + 2) = 4 - 2 + 25 = 27 \neq 0$

$\therefore \quad A^{-1}$ exists.

Now, $A_{11} = 4, A_{12} = -1, A_{13} = 5, A_{21} = 17, A_{22} = -11,$ $A_{23} = 1, A_{31} = 3, A_{32} = 6, A_{33} = -3$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

The given system of equations is

$x + 2y + 5z = 10$

$x - y - z = - 2$

$2x + 3y - z = - 11$

The given system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

$\therefore \quad A^{-1}$ exists. So, system of equations has a unique solution given by $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27}\begin{bmatrix} -27 \\ -54 \\ 81 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$$

$\Rightarrow$ $x = -1, y = -2, z = 3$

129. The given equations are

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z} = 1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z} = 2$$

Given equations can be written as $AX = B$

where, $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$= [2(120 - 45) - 3(-80 - 30) + 10(36 + 36)]$$

$$= [150 + 330 + 720] = 1200 \neq 0$$

Since A^{-1} exists, therefore system of equations has a unique solution given by $X = A^{-1} B$

$A_{11} = 75, A_{12} = 110, A_{13} = 72, A_{21} = 150, A_{22} = -100, A_{23} = 0, A_{31} = 75, A_{32} = 30, A_{33} = -24$

$$\therefore \text{ adj}A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A) = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now, $X = A^{-1} B$.

$$\Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{2} \Rightarrow x = 2, \frac{1}{y} = \frac{1}{3} \Rightarrow y = 3, \frac{1}{z} = \frac{1}{5} \Rightarrow z = 5$$

130. The given equations are

$x + 2y + z = 7$

$x + 3z = 11$

$2x - 3y = 1$

The given system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix}$

$=2(6 - 0) + 3(3 - 1) = 12 + 6 = 18 \neq 0$

$\therefore$ A^{-1} exists. So, system of equations has a unique solution given by $X = A^{-1}B$

Now, $A_{11} = 9, A_{12} = 6, A_{13} = -3, A_{21} = -3, A_{22} = -2, A_{23} = 7, A_{31} = 6, A_{32} = -2, A_{33} = -2$

$$\therefore \text{ adj}A = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\Rightarrow$ $x = 2, y = 1, z = 3$

131. The given system of equations are

$4x + 3y + 2z = 60,$

$x + 2y + 3z = 45,$

$6x + 2y + 3z = 70$

The given system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{vmatrix}$

$= 4(6-6) - 3(3-18) + 2(2-12)$

$= 0 + 45 - 20 = 25 \neq 0$

$\therefore$ A^{-1} exists. So system of equations has a unique solution $X = A^{-1}B$

Now,

$A_{11} = 0, A_{12} = 15, A_{13} = -10, A_{21} = -5, A_{22} = 0,$
$A_{23} = 10, A_{31} = 5, A_{32} = -10, A_{33} = 5,$

$$\therefore \quad \text{adj}A = \begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\therefore \quad X = \frac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}\begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 0-225+350 \\ 900+0-700 \\ -600+450+350 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 125 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore \quad x = 5, y = 8, z = 8.$

132. Here, $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{vmatrix}$$

$= 3(3-0) + 4(2-5) + 2(0-3)$

$= 9 - 12 - 6 = -9 \neq 0$

$\therefore$ A^{-1} exists.

Now, $A_{11} = 3, A_{12} = 3, A_{13} = -3, A_{21} = 4, A_{22} = 1,$
$A_{23} = -4, A_{31} = -26, A_{32} = -11, A_{33} = 17$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\cdot\text{adj } A = -\frac{1}{9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

The given system of equations is

$3x - 4y + 2z = -1,$

$2x + 3y + 5z = 7$

$x + z = 2$

The given system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

Since A^{-1} exists, so, system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}\begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$= -\frac{1}{9}\begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$\Rightarrow \quad x = 3, y = 2, z = -1.$

133. Here, $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}; B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

$$\therefore \quad AB = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\cdot\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 11I$$

$$\Rightarrow \quad A \times \left(\frac{1}{11}B\right) = I \Rightarrow A^{-1} = \frac{1}{11}B$$

Now the given system of equations is

$x - 2y = 10$

$2x + y + 3z = 8$

$-2y + z = 7$

The system of equations can be written as $AX = P$

$$\text{where, } A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, P = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$\therefore$ A^{-1} exists, so given system of equations has a unique solution given by $X = A^{-1}P$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} BP = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

$\Rightarrow \ x = 4, y = -3, z = 1.$

134. Here, $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{vmatrix} = 1(3-0) + 2(-2+1) = 1 \neq 0$$

$\therefore \ A^{-1}$ exists.

Now, $A_{11} = 3, A_{12} = 2, A_{13} = 2, A_{21} = -6, A_{22} = -5,$
$A_{23} = -4, A_{31} = -1, A_{32} = -1, A_{33} = -1$

$$\therefore \ \text{adj } A = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix}$$

Now the given linear equations are

$x - 2y + z = 0$

$-y + z = -2$

$2x - 3z = 10$

The system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}$$

Since A^{-1} exists, so given system of equations has a unique solution given by

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

$\Rightarrow \ x = 2, y = 0, z = -2.$

135. Here, $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{vmatrix}$

$= 2(0+6) + 1(0+2) + 1(18-0) = 12 + 2 + 18 = 32 \neq 0$

$\therefore \ A^{-1}$ exists.

Now, $A_{11} = 6, A_{12} = -2, A_{13} = 18, A_{21} = 6, A_{22} = -2,$
$A_{23} = -14, A_{31} = 1, A_{32} = 5, A_{33} = 3$

$$\therefore \ \text{adj } A = \begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{32} \begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix}$$

Now the given equations are

$2x - y + z = -3,$

$3x - z = 0,$

$2x + 6y - 2 = 0$

The given system of equations can be written as

$AX = B$

$$\text{where, } A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

$\therefore \ A^{-1}$ exists, so, the system has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{32} \begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} = \frac{1}{32} \begin{bmatrix} -16 \\ 16 \\ -48 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ -3/2 \end{bmatrix}$$

$$\Rightarrow \ x = -\frac{1}{2}, y = \frac{1}{2}, z = -\frac{3}{2}$$