Session 5

Combinations from Identical Objects

Combinations from Identical Objects

(i) The number of combinations of n identical objects taking r objects $(r \le n)$ at a time $= 1$.

(ii) The number of combinations of zero or more objects from n identical objects $= n + 1$.

(iii) The total number of combinations of atleast one out of $a_1 + a_2 + a_3 + \ldots + a_n$ objects, where a_1 are alike of one kind, a_2 are alike of second kind, a_3 are alike of third kind, ..., a_n are alike of nth kind

$$= (a_1 + 1)(a_2 + 1)(a_3 + 1)\ldots(a_n + 1) - 1$$

Example 62. How many selections of atleast one red ball from a bag containing 4 red balls and 5 black balls, balls of the same colour being identical?

Sol. Number of selections of atleast one red ball from 4 identical red balls = 4

Number of selections of any number of black balls from 5 identical black balls

$$= 5 + 1 = 6$$

$\therefore$ Required number of selections of balls

$$= 4 \times 6 = 24$$

Example 63. There are p copies each of n different books. Find the number of ways in which a non-empty selection can be made from them.

Sol. Since, copies of the same book are identical.

$\therefore$ Number of selections of any number of copies of a book is $p + 1$. Similarly, in the case for each book.

Therefore, total number of selections is $(p + 1)^n$.

But this includes a selection, which is empty i.e., zero copy of each book. Excluding this, the required number of non-empty selections is $(p + 1)^n - 1$.

Example 64. There are 4 oranges, 5 apples and 6 mangoes in a fruit basket and all fruits of the same kind are identical. In how many ways can a person make a selection of fruits from among the fruits in the basket?

Sol. Zero or more oranges can be selected out of 4 identical oranges = 4 + 1 = 5 ways.

Similarly, for apples number of selection = 5 + 1 = 6 ways and mangoes can be selected in 6 + 1 = 7 ways.

$\therefore$ The total number of selections when all the three kinds of fruits are selected $= 5 \times 6 \times 7 = 210$

But, in one of these selection number of each kind of fruit is zero and hence this selection must be excluded.

$\therefore$ Required number $= 210 - 1 = 209$

Combinations when both Identical and Distinct Objects are Present

The number of combinations (selections) of one or more objects out of $a_1 + a_2 + a_3 + \ldots + a_n$ objects, where a_1 are alike of one kind, a_2 are alike of second kind, a_3 are alike of third kind, ..., a_n are alike of nth kind and k are distinct.

$$= \{(a_1 + 1)(a_2 + 1)(a_3 + 1)\cdots(a_n + 1)\}$$
$$({}^kC_0 + {}^kC_1 + {}^kC_2 + \ldots + {}^kC_k) - 1$$
$$= (a_1 + 1)(a_2 + 1)(a_3 + 1) + \ldots + (a_n + 1)2^k - 1$$

Example 65. Find the number of ways in which one or more letters can be selected from the letters

AAAAA BBBB CCC DD EFG.

Sol. Here, 5A's are alike, 4 B's are alike, 3 C's are alike, 2D's are alike and E, F, G are different.

$\therefore$ Total number of combinations

$$= (5 + 1)(4 + 1)(3 + 1)(2 + 1)2^3 - 1$$
$$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 8 - 1$$
$$= 2879$$

[excluding the case, when no letter is selected]

Explanation Selection from $(AAAAA)$ can be made by 6 ways such include no A, include one A, include two A, include three A, include four A, include five A. Similarly, selections from $(BBBB)$ can be made in 5 ways, selections from (CCC) can be made in 4 ways, selections from (DD) can be made in 3 ways and from E, F, G can be made in $2 \times 2 \times 2$ ways.

Number of Divisors of N

Every natural number N can always be put in the form $N = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \ldots p_k^{\alpha_k}$, where $p_1, p_2, p_3, \ldots, p_k$ are distinct primes and $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_k \in W$.

(i) The total number of divisors of N including 1 and N $=(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1)$

(ii) The total number of divisors of N excluding 1 and N $=(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1)-2$

(iii) The total number of divisors of N excluding either 1 or $N=(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1)-1$

(iv) Sum of all divisors $=(p_1^0+p_1^1+p_1^2+p_1^3+\ldots+p_1^{\infty_1})$
$(p_2^0+p_2^1+p_2^2+p_2^3+\ldots+p_2^{\alpha_2})\ldots$
$(p_k^0+p_k^1+p_k^2+p_k^3+\ldots+p_k^{\alpha_k})$

$$=\left(\frac{1-p_1^{\alpha_1+1}}{1-p_1}\right)\cdot\left(\frac{1-p_2^{\alpha_2+1}}{1-p_2}\right)\ldots\left(\frac{1-p_k^{\alpha_k+1}}{1-p_k}\right)$$

(v) Sum of proper divisors (excluding 1 and the expression itself)
$=$ Sum of all divisors $-(N+1)$

(vi) The number of even divisors of N are possible only if $p_1=2$, otherwise there is no even divisor.

$\therefore$ Required number of **even** divisors
$$=\alpha_1(\alpha_2+1)(\alpha_3+1)+\ldots+(\alpha_k+1)$$

(vii) The number of **odd** divisors of N

***Case* I** If $p_1=2$, the number of odd divisors
$$=(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1)$$

***Case* II** If $p_1\neq 2$, the number of odd divisors
$$=(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1)$$

(viii) The number of ways in which N can be resolved as a product of two factors

$$=\begin{cases}\frac{1}{2}(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1), & \text{if } N \text{ is not a perfect square}\\ \frac{1}{2}\{(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1)+1\}, & \text{if } N \text{ is a perfect square}\end{cases}$$

(ix) The number of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1}, where n is the number of different factors (or different primes) in N.

Example 66. Find the number of proper factors of the number 38808. Also, find sum of all these divisors.

Sol. The number $38808=2^3\cdot 3^2\cdot 7^2\cdot 11$

Hence, the total number of proper factors (excluding 1 and itself i.e., 38808)
$=(3+1)(2+1)(2+1)(1+1)-2$
$=72-2=70$

And sum of all these divisors (proper)
$=(2^0+2^1+2^2+2^3)(3^0+3^1+3^2)$
$(7^0+7^1+7^2)(11^0+11^1)-1-38808$
$=(15)(13)(57)(12)-38809$
$=133380-38809$
$=94571$

Example 67. Find the number of even proper divisors of the number 1008.

Sol. $\because 1008=2^4\times 3^2\times 7^1$

$\therefore$ Required number of even proper divisors
= Total number of selections of atleast one 2 and any number of 3's or 7's.
$=4\times(2+1)\times(1+1)-1=23$

Example 68. Find the number of odd proper divisors of the number 35700. Also, find sum of the odd proper divisors.

Sol. $\because 35700=2^2\times 3^1\times 5^2\times 7^1\times 17^1$

$\therefore$ Required number of odd proper divisors
= Total number of selections of zero 2 and any number of 3's or 5's or 7's or 17's
$=(1+1)(2+1)(1+1)(1+1)-1=23$

$\therefore$ The sum of odd proper divisors
$=(3^0+3^1)(5^0+5^1+5^2)(7^0+7^1)(17^0+7^1)-1$
$=4\times 31\times 8\times 18-1$
$=17856-1=17855$

Example 69. If $N=10800$, find the

(i) the number of divisors of the form $4m+2, \forall\, m\in W$.

(ii) the number of divisors which are multiple of 10.

(iii) the number of divisors which are multiple of 15.

Sol. We have, $N=10800=2^4\times 3^3\times 5^2$

(i) $\because (4m+2)=2(2m+1)$, in any divisor of the form $4m+2$, 2 should be exactly 1.
So, the number of divisors of the form
$(4m+2)=1\times(3+1)\times(2+1)=1\times 4\times 3=12$

(ii) $\therefore$ The required number of proper divisors
= Total number of selections of atleast one 2 and one 5 from 2, 2, 2, 2, 3, 3, 3, 5, 5
$=4\times(3+1)\times 2=32$

(iii) $\therefore$ The required number of proper divisors

= Total number of selections of atleast one 3 and one 5 from 2, 2, 2, 2, 3, 3, 3, 3, 5, 5

$= (4 + 1) \times 3 \times 2 = 30$

Example 70. Find the number of divisors of the number $N = 2^3 \cdot 3^5 \cdot 5^7 \cdot 7^9 \cdot 9^{11}$, which are perfect square.

Sol. $\because N = 2^3 \cdot 3^5 \cdot 5^7 \cdot 7^9 \cdot 9^{11}$

$= 2^3 \cdot 3^5 \cdot 5^7 \cdot 7^9 \cdot 3^{22}$

$= 2^3 \cdot 3^{27} \cdot 5^7 \cdot 7^9$

For perfect square of N, each prime factor must occur even number of times.

2 can be taken in 2 ways (i.e., 2^0 or 2^2)

3 can be taken in 14 ways (i.e., 3^0 or 3^2 or 3^4 or 3^6 or 3^8 or 3^{10} or 3^{12} or 3^{14} or 3^{16} or 3^{18} or 3^{20} or 3^{22} or 3^{24} or 3^{26})

5 can taken in 4 ways (i.e., 5^0 or 5^2 or 5^4 or 5^6)

and 7 can taken in 5 ways (i.e., 7^0 or 7^2 or 7^4 or 7^4 or 7^6 or 7^8)

Hence, total divisors which are perfect squares

$= 2 \times 14 \times 4 \times 5 = 560$

Example 71. In how many ways the number 10800 can be resolved as a product of two factors?

Sol. Let $N = 10800 = 2^4 \times 3^3 \times 5^2$

Here, N is not a perfect square [$\because$ power of 3 is odd]

Hence, the number of ways $= \frac{1}{2}(4 + 1)(3 + 1)(2 + 1) = 30$

Example 72. In how many ways the number 18900 can be split in two factors which are relatively prime (or coprime)?

Sol. Let $N = 18900 = 2^2 \cdot 3^3 \cdot 5^2 \cdot 7^1$

Relatively prime or coprime Two numbers not necessarily prime are said to be relatively prime or coprime, if their HCF (highest common factor) is one as 2, 3, 5, 7 are relatively prime numbers.

$\therefore$ $n = 4$ [number of different primes in N]

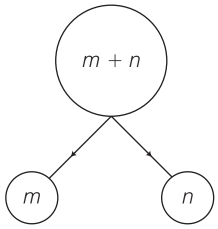
Hence, number of ways in which a composite number N can be resolved into two factors which are relatively prime or coprime $= 2^{4-1} = 2^3 = 8$

Division of Objects Into Groups

(a) Division of Objects Into Groups of Unequal Size

Theorem Number of ways in which $(m+n)$ distinct objects can be divided into two unequal groups containing m and n objects is $\frac{(m+n)!}{m!\,n!}$.

Proof The number of ways in which $(m+n)$ distinct objects are divided into two groups of the size m and n
= The number of ways m objects are selected out of $(m+n)$ objects to form one of the groups, which can be done in ${}^{m+n}C_m$ ways. The other group of n objects is formed by the remaining n objects.



(Tree diagram)

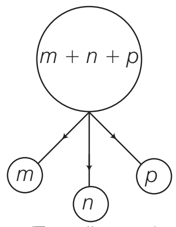
$$= {}^{m+n}C_m \cdot {}^nC_n = \frac{(m+n)}{m!\,n!}$$

***Corollary* I** The number of ways to distribute $(m+n)$ distinct objects among 2 persons in the groups containing m and n objects

= (Number of ways to divide) × (Number of groups)

$= \frac{(m+n)}{m!\,n!} \times 2!$

***Corollary* II** The number of ways in which $(m+n+p)$ distinct objects can be divided into three unequal groups containing m, n and p objects, is



(Tree diagram)

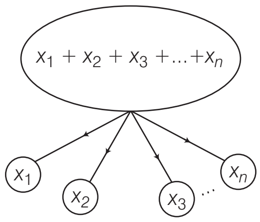
$${}^{m+n+p}C_m \cdot {}^{n+p}C_n \cdot {}^pC_p = \frac{(m+n+p)!}{m!\,n!\,p!}$$

***Corollary* III** The number of ways to distribute $(m+n+p)$ distinct objects among 3 persons in the groups containing m, n and p objects

= (Number of ways to divide) × (Number of groups)

$= \frac{(m+n+p)!}{m!\,n!\,p!} \times 3!$

***Corollary* IV** The number of ways in which $(x_1 + x_2 + x_3 + \cdots + x_n)$ distinct objects can be divided into n unequal groups containing x_1, $x_2, x_3, \ldots, x_n$ objects, is



(Tree diagram)

$$\frac{(x_1 + x_2 + x_3 + \cdots + x_n)!}{x_1!\, x_2!\, x_3! \cdots x_n!}.$$

***Corollary* V** The number of ways to distribute $(x_1 + x_2 + x_3 + \ldots + x_n)$ distinct objects among n persons in the groups containing $x_1, x_2, \ldots, x_n$ objects

= (Number of ways to divide) × (Number of groups)

$$= \frac{(x_1 + x_2 + x_3 + \ldots + x_n)!}{x_1!\, x_2!\, x_3! \ldots x_n!} \times n!$$

(b) Division of Objects Into Groups of Equal Size

The number of ways in which mn distinct objects can be divided equally into m groups, each containing n objects and

(i) If order of groups is not important is.

$$= \left(\frac{(mn)!}{(n!)^m}\right) \times \frac{1}{m!}.$$

(ii) If order of groups is important is.

$$\left(\frac{(mn)!}{(n!)^m} \times \frac{1}{m!}\right) \times m! = \frac{(mn)!}{(n!)^m}.$$

Note Division of $14n$ objects into 6 groups of $2n, 2n, 2n, 2n, 3n, 3n$, size is $\dfrac{\left(\dfrac{(14n)!}{(2n)!\,(2n)!\,(2n)!\,(2n)!\,(3n)!\,(3n)!}\right)}{4!\,2!} = \dfrac{(14n)}{((2n)!)^4\ ((3n)!)^2} \times \dfrac{1}{4!\,2!}$

Now, the distribution ways of these 6 groups among 6 persons is

$$\frac{(14n)!}{[(2n)!)]^4\ [(3n)!]^2} \times \frac{1}{4!\,2!} \times 6! = \frac{(14n)!}{[(2n)!]^4\ [(3n)!]^2} \times 15$$

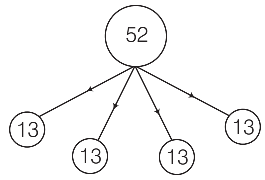
Example 73. *In how many ways can a pack of 52 cards be*

(i) distributed equally among four players in order?

(ii) divided into four groups of 13 cards each?

(iii) divided into four sets, three of them having 17 cards each and fourth just one card?

Sol. (i) Here, order of group is important, then the numbers of ways in which 52 different cards can be divided equally into 4 players is



(Tree diagram)

$$\frac{52!}{4!\,(13!)^4} \times 4! = \frac{52!}{(13!)^4}$$

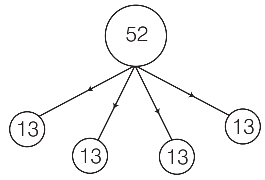
Aliter Each player will get 13 cards. Now, first player can be given 13 cards out of 52 cards in $^{52}C_{13}$ ways. Second player can be given 13 cards out of remaining 39 cards (i.e., $52 - 13 = 39$) in $^{39}C_{13}$ ways. Third player can be given 13 cards out of remaining 26 cards (i.e., $39 - 13 = 26$) in $^{26}C_{13}$ ways and fourth player can be given 13 cards out of remaining 13 cards (i.e., $26 - 13 = 13$) in $^{13}C_{13}$ ways.

Hence, required number of ways

$$= {}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13}$$

$$= \frac{52!}{13!\,39!} \times \frac{39!}{13!\,26!} \times \frac{26!}{13!\,13!} \times 1 = \frac{52!}{(13!)^4}$$

(ii) Here, order of group is not important, then the number of ways in which 52 different cards can be divided equally into 4 groups is



(Tree diagram)

$$\frac{52!}{4!\,(13!)^4}$$

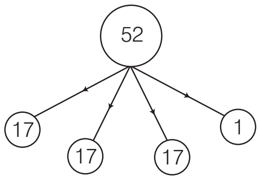
Aliter Each group will get 13 cards. Now, first group can be given 13 cards out of 52 cards in $^{52}C_{13}$ ways. Second group can be given 13 cards out of remaining 39 cards (i.e., $52 - 13 = 39$) in $^{39}C_{13}$ ways. Third group can be given 13 cards out of remaining 26 cards (i.e., $39 - 13 = 26$) in $^{26}C_{13}$ ways and fourth group can be given 13 cards out of remaining 13 cards (i.e., $26 - 13 = 13$) in $^{13}C_{13}$ ways. But the all (four) groups can be interchanged in 4! ways. Hence, the required number of ways

$$= {}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13} \times \frac{1}{4!}$$

$$= \frac{52!}{13!\,39!} \times \frac{39!}{13!\,26!} \times \frac{26!}{13!\,13!} \times 1 \times \frac{1}{4!} = \frac{52!}{(13!)^4\,4!}$$

(iii) First, we divide 52 cards into two sets which contains 1 and 51 cards respectively, is

$$\frac{52!}{1!\,51!}$$



Now, 51 cards can be divided equally in three sets each contains 17 cards (here order of sets is not important) in $\dfrac{51!}{3!\,(17!)^3}$ ways.

Hence, the required number of ways

$$= \frac{52!}{1!\,51!} \times \frac{51!}{3!(17!)^3}$$

$$= \frac{52!}{1!\,3!(17)^3} = \frac{52!}{(17!)^3\,3!}$$

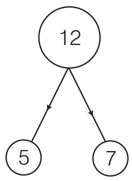
Aliter First set can be given 17 cards out of 52 cards in $^{52}C_{17}$. Second set can be given 17 cards out of remaining 35 cards (i.e., 52 – 17 = 35) in $^{35}C_{17}$. Third set can be given 17 cards out of remaining 18 cards (i.e., 35 – 17 = 18) in $^{18}C_{17}$ and fourth set can be given 1 card out of 1 card in $^{1}C_{1}$. But the first three sets can interchanged in 3! ways. Hence, the total number of ways for the required distribution

$$= {}^{52}C_{17} \times {}^{35}C_{17} \times {}^{18}C_{17} \times {}^{1}C_{1} \times \frac{1}{3}!$$

$$= \frac{52!}{17!\,35!} \times \frac{35!}{17!\,1!} \times \frac{18!}{17!\,18!} \times 1 \times \frac{1}{3!} = \frac{(52)!}{(17!)^3\,3!}$$

Example 74. In how many ways can 12 different balls be divided between 2 boys, one receiving 5 and the other 7 balls? Also, in how many ways can these 12 balls be divided into groups of 5, 4 and 3 balls, respectively?

Sol. **I Part** Here, order is important, then the number of ways in which 12 different balls can be divided between two boys which contains
5 and 7 balls respectively, is



(Tree diagram)

$$= \frac{12!}{5!\,7!} \times 2! = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7!}{(5\cdot 4\cdot 3\cdot 2\cdot 1)7!}\cdot 2 = 1584$$

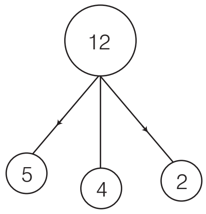
Aliter First boy can be given 5 balls out of 12 balls in $^{12}C_5$. Second boy can be given 7 balls out of 7 balls (i.e., 12 – 5 = 7) but there order is important boys interchange by (2 types), then required number of ways

$$= {}^{12}C_5 \times {}^{7}C_7 \times 2! = \frac{12!}{5!\,7!} \times 1 \times 2!$$

$$= \frac{12! \times 2}{5! \times 7!} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7!\cdot 2}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 7!} = 1584.$$

II Part Here, order is not important, then the number of ways in which 12 different balls can be divided into three groups of 5,4 and 3 balls respectively, is

$$= \frac{12!}{5!\,4!\,3!} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5!}{5!\cdot 4\cdot 3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1} = 27720$$



(Tree diagram)

Aliter First group can be given 5 balls out of 12 balls in $^{12}C_5$ ways. Second group can be given 4 balls out of remaining 7 balls $(12 - 5 = 7)$ in $^{7}C_4$ and 3 balls can be given out of remaining 3 balls in $^{3}C_3$.

Hence, the required number of ways (here order of groups are not important)

$$= {}^{12}C_5 \times {}^{7}C_4 \times {}^{3}C_3$$

$$= \frac{12!}{5!\,7!} \times \frac{7!}{4!\,3!} \times 1$$

$$= \frac{12!}{5!\,4!\,3!}$$

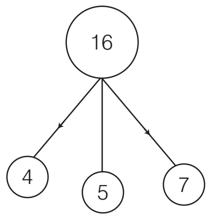
Example 75. In how many ways can 16 different books be distributed among three students A, B, C so that B gets 1 more than A and C gets 2 more than B ?

Sol. Let A gets n books, then B gets $n + 1$ and C gets $n + 3$.

Now, $n + (n + 1) + (n + 3) = 16$

$\Rightarrow \quad 3n = 12$

$\therefore \quad n = 4$



(Tree diagram)

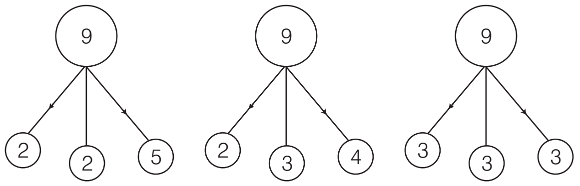
$\Rightarrow$ A, B, C gets 4, 5 and 7 books, respectively.

Hence, the total number of ways for the required distribution

$$= \frac{16!}{4!\,5!\,7!}$$

Example 76. In how many ways can 9 different books be distributed among three students if each receives atleast 2 books?

Sol. If each receives atleast 2 books, then the division as shown by tree diagrams



(i) (ii) (iii)

The number of division ways for tree diagrams (i), (ii) and (iii) are

$$\frac{9!}{(2!)^2\,(5!)}\times\frac{1}{2!},\frac{9!}{2!\,3!\,4!}\text{ and }\frac{9!}{(3!)^3}\times\frac{1}{3!},\text{ respectively.}$$

Hence, the total number of ways of distribution of these groups among 3 students is

$$\left[\frac{9!}{(2!)^2\,(5!)}\times\frac{1}{2!}+\frac{9!}{2!\,3!\,4!}+\frac{9!}{(3!)^3}\times\frac{1}{3!}\right]\times 3!$$

$$=[378+1260+280]\times 6$$

$$=11508$$

Exercise for Session 5

1. There are 3 oranges, 5 apples and 6 mangoes in a fruit basket (all fruits of same kind are identical). Number of ways in which fruits can be selected from the basket, is

(a) 124 (b) 125 (c) 167 (d) 168

2. In a city no two persons have identical set of teeth and there is no person without a tooth. Also, no person has more than 32 teeth. If we disguard the shape and size of tooth and consider only the positioning of the teeth, the maximum population of the city is

(a) 2^{32} (b) $(32)^2-1$ (c) $2^{32}-1$ (d) 2^{31}

3. If $a_1, a_2, a_3, \ldots, a_{n+1}$ be $(n+1)$ different prime numbers, then the number of different factors (other than 1) of $a_1^m\cdot a_2\cdot a_3, \ldots, a_{n+1}$, is

(a) $m+1$ (b) $(m+1)2^n$ (c) $m\cdot 2^n+1$ (d) None of these

4. Number of proper factors of 2400 is equal to

(a) 34 (b) 35

(c) 36 (d) 37

5. The sum of the divisors of $2^5\cdot 3^4\cdot 5^2$ is

(a) $3^2\cdot 7^1\cdot 11^2$ (b) $3^2\cdot 7^1\cdot 11^2\cdot 31$

(c) $3\cdot 7\cdot 11\cdot 31$ (d) None of these

6. The number of proper divisors of $2^p\cdot 6^q\cdot 21^r$, $\forall\, p, q, r\in N$, is

(a) $(p+q+1)(q+r+1)(r+1)$ (b) $(p+q+1)(q+r+1)(r+1)-2$

(c) $(p+q)(q+r)r-2$ (d) $(p+q)(q+r)r$

7. The number of odd proper divisors of $3^p\cdot 6^q\cdot 15^r$, $\forall\, p, q, r\in N$, is

(a) $(p+1)(q+1)(r+1)-2$ (b) $(p+1)(q+1)(r+1)-1$

(c) $(p+q+r+1)(r+1)-2$ (d) $(p+q+r+1)(r+1)-1$

8. The number of proper divisors of 1800, which are also divisible by 10, is

(a) 18 (b) 27 (c) 34 (d) 43

9. Total number of divisors of 480 that are of the form $4n+2$, $n\geq 0$, is equal to

(a) 2 (b) 3 (c) 4 (d) 5

10. Total number of divisors of $N=2^5\cdot 3^4\cdot 5^{10}\cdot 7^6$ that are of the form $4n+2$, $n\geq 1$, is equal to

(a) 54 (b) 55 (c) 384 (d) 385

11. Total number of divisors of $N = 3^5 \cdot 5^7 \cdot 7^9$ that are of the form $4n + 1$, $n \geq 0$ is equal to

(a) 15 (b) 30 (c) 120 (d) 240

12. Number of ways in which 12 different books can be distributed equally among 3 persons, is

(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(3!)^4}$ (c) $\frac{12!}{(4!)^4}$ (d) $\frac{12!}{(3!)^3}$

13. Number of ways in which 12 different things can be distributed in 3 groups, is

(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{3!\,(4!)^3}$ (c) $\frac{12!}{4!\,(3!)^3}$ (d) $\frac{12!}{(3!)^4}$

14. Number of ways in which 12 different things can be distributed in 5 sets of 2, 2, 2, 3, 3, things is

(a) $\frac{12!}{(3!)^2\,(2!)^3}$ (b) $\frac{12!\,5!}{(3!)^2\,(2!)^3}$ (c) $\frac{12!}{(3!)^3\,(2!)^4}$ (d) $\frac{12!\,5!}{(3!)^2\,(2!)^4}$

15. Number of ways in which 12 different things can be divided among five persons so that they can get 2, 2, 2, 3, 3 things respectively, is

(a) $\frac{12!}{(3!)^2\,(2!)^3}$ (b) $\frac{12!\,5!}{(3!)^2\,(2!)^3}$ (c) $\frac{12!}{(3!)^2\,(2!)^4}$ (d) $\frac{12!\,5!}{(3!)^2\,(2!)^4}$

16. The total number of ways in which $2n$ persons can be divided into n couples, is

(a) $\frac{2n!}{(n!)^2}$ (b) $\frac{2n!}{(2n!)^n}$ (c) $\frac{2n!}{n!\,(2n!)^2}$ (d) None of these

17. n different toys have to be distributed among n children. Total number of ways in which these toys can be distributed so that exactly one child gets no toy, is equal to

(a) $n!$ (b) $n!\,{}^nC_2$ (c) $(n-1)!\,{}^nC_2$ (d) $n!\,{}^{n-1}C_2$

18. In how many ways can 8 different books be distributed among 3 students if each receives atleast 2 books?

(a) 490 (b) 980 (c) 2940 (d) 5880

Answers

Exercise for Session 5

1. (c)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (b)
7. (d)	**8.** (a)	**9.** (c)	**10.** (c)	**11.** (d)	**12.** (a)
13. (b)	**14.** (c)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (c)