1. UNITS, DIMENSIONS AND ERRORS

1. INTRODUCTION

Physics is an experimental science and experiments require measurement of physical quantities. Measuring a physical quantity involves comparing the quantity with a reference standard called the unit of the quantity. Some physical quantities are taken as base quantities and other quantities are expressed in terms of the base quantities called derived quantities. This forms a system of base quantities and their units. Without performing proper measurements we cannot describe the physical phenomena quantitatively.

2. UNITS

To measure a physical quantity we need some standard unit of that quantity. The measurement of the quantity is mentioned in two parts, the first part gives magnitude and the second part gives the name of the unit. Thus, suppose I say that length of this wire is 5 meters. The numeric part 5 says that it is 5 times of the unit of length and the second part meter says that unit chosen here is meter.

2.1 Fundamental and Derived Quantities

The basic physical quantities, which are independent of other quantities, are known as the fundamental quantities. For example, mass, length and time are considered to be the fundamental quantities. In the same manner, the units which can be derived from the fundamental units are known as derived units. In mechanics, virtually all quantities can be expressed in terms of mass, length and time. The main systems of units are given as follows:

(a) CGS or Centimetre, Gram, Second System

(b) FPS or Foot, Pound, Second System

(c) MKS or Metre, Kilogram, Second System

(d) SI system: Totally, there are seven basic or fundamental quantities in the international system of units called the SI system which can express all physical quantities including heat, optics and electricity and magnetism. We now provide these basic seven quantities with their units and symbols:

S. No.	Physical Quantity	SI Unit	Symbol
1	Mass	Kilogram	kg
2	Length	Metre	m
3	Time	Second	s
4	Temperature	Kelvin	K
5	Luminous intensity	Candela	cd

S. No.	Physical Quantity	SI Unit	Symbol
6	Electric current	Ampere	A
7	Amount of substance	Mole	mol.

There are also two supplementary units used as radian (rad) for plane angle and steradian (sr) for solid angle.

The above mentioned International System of Units (SI) is now extensively used in scientific measurements.

However, the following practical units of length are also conveniently used and are expressed in terms of SI system of units.

(a) **Micron** is a small unit for measurement of length. 1 micron $= 1\,\mu m = 10^{-6} m$

(b) **Angstrom** is a unit of length in which the size of an atom is measured and is used in atomic physics.

1 Angstrom$=1Å=10^{-10}$ m.

(c) **Light year** is a unit of distance travelled by light in 1 year free space and is used in astrophysics.

1 Light year = $3\times10^{8} m/s \times 365\times24\times60\times60 = 9.5\times10^{15}$ meters

(d) **Fermi** is a unit of distance in which the size of a nucleus is measured. 1 Fermi $= 10^{-15}$ m

(e) **Atomic mass unit:** It is a unit of mass equal to 1/12th of mass of carbon-12 atom.

1 atomic mass unit $\cong 1.67\times10^{-27}$ kg

Note: There are only seven fundamental units. Apart from these, there are two supplementary units—plane angle (radian) and solid angle (steradian). By using these units, all other units can be derived. However, we need to know the fact that both radian and steradian have no dimensions.

3. DIMENSIONS

All the physical quantities of interest can be derived from the base quantities. Thus, when a quantity is expressed in terms of the base quantities, it is written as a product of different powers of the base quantities. Further, the exponent of a base quantity that enters into the expression is called the dimension of the quantity in that base. To make it clear, consider the physical quantity "force." As we shall learn later, force is equal to mass times acceleration. We know that acceleration is change in velocity divided by time interval but velocity is length divided by time interval. Thus,

$$\text{Force} = \text{Mass}\times\text{Acceleration} = \text{Mass}\times\frac{\text{Velocity}}{\text{Time}} = \text{Mass}\times\frac{\text{Length / Time}}{\text{Time}} = \text{Mass}\times\text{Length}\times(\text{Time})^{-2}$$

Thus, the dimensions of force are 1 in mass, 1 in length and –2 in time. The dimensions in all other base quantities are zero. Note, however, that in this type of calculation, the magnitudes are not considered. This is because only equality of the type of quantity is what that matters. Thus, change in velocity, average velocity, or final velocity all are equivalent in this discussion, as each one is expressed in terms of length/time.

Illustration 1: Validate the relation $s = ut + \frac{1}{2}at^2$, where u is the initial velocity, a is the acceleration, t is the time and s is the displacement. **(JEE MAIN)**

Sol: The above relation is having units of displacement. To validate above relation dimensionally correct, we need to match the dimensions of each quantity to the right of equality with the dimensions of displacement.

By writing the dimensions of either side of the equation, we obtain

LHS = s = displacement = $[M^0LT^0]$; RHS = ut = velocity$\times$time $= [M^0LT^{-1}][T] = [M^0LT^0]$

Further, $\frac{1}{2}at^2 = (\text{acceleration})\times(\text{time})^2 = [M^0LT^{-2}][T]^2 = [M^0LT^0]$

As LHS = RHS, the formula is dimensionally correct.

Table 1.1: SI units and dimensions of commonly used quantities

S. No.	Quantity	SI Units	Dimensional Formula
1.	Area	m^2	$[L^2]$
2.	Density	kg m^{-3}	$[ML^{-3}]$
3.	Velocity	ms^{-1}	$[LT^{-1}]$
4.	Acceleration	ms^{-2}	$[LT^{-2}]$
5.	Angular velocity	rad s^{-1}	$[T^{-1}]$
6.	Frequency	s^{-1} or hertz(Hz)	$[T^{-1}]$
7.	Momentum	kg ms^{-1}	$[MLT^{-1}]$
8.	Force	kg ms^{-2} or newton (N)	$[MLT^{-2}]$
9.	Work, energy	kg m^2 s^{-2} or joule(J)	$[ML^2T^{-2}]$
10.	Power	kg m^2 s^{-3} or Js^{-1} orWatt	$[ML^2T^{-3}]$
11.	Pressure, stress	Nm^{-2} or pascal (Pa)	$[ML^{-1}T^{-2}]$
12.	Coefficient of elasticity	Nm^{-2}	$[ML^{-1}T^{-2}]$
13.	Moment of inertia	kg m^2	$[ML^2]$
14.	Torque	Nm	$[ML^2T^{-2}]$
15.	Angular momentum	kg m^2 s^{-1}	$[ML^2T^{-1}]$
16.	Impulse	Ns	$[MLT^{-1}]$
17.	Universal gravitational constant	Nm^2 kg^{-2}	$[M^{-1}L^3T^{-2}]$
18.	Latent heat	Jkg^{-1}	$[L^2T^{-2}]$
19.	Specific heat	Jkg^{-1} K^{-1}	$[L^2T^{-2}K^{-1}]$

S. No.	Quantity	SI Units	Dimensional Formula
20.	Thermal conductivity	$Jm^{-1} s^{-1} K^{-1}$	$[MLT^{-3}K^{-1}]$
21.	Electric charge	Coulomb(C)	$[AT]$
22.	Electric potential	JC^{-1} or volt (V)	$[ML^2T^{-3}A^{-1}]$
23.	Electric resistance	VA^{-1} or ohm (Ω)	$[ML^2T^{-3}A^{-2}]$
24.	Electric resistivity	(Ω) m	$[ML^3T^{-3}A^{-2}]$
25.	Capacitance	CV^{-1} or farad (F)	$[ML^{-1}T^{-2}\ T^4\ A^2]$
26.	Inductance	VsA^{-1} or henry (H)	$[ML^2T^{-2}\ A^{-2}]$
27.	Electric field	NC^{-1} or Vm^{-1}	$[ML^2T^{-3}\ A^{-1}]$
28.	Magnetic induction	$NA^{-1}\ m^{-1}$ or tesla(T)	$[MT^{-2}A^{-1}]$
29.	Magnetic flux	Tm^2 or weber (Wb)	$[ML^2T^{-2}A^{-1}]$
30.	Permittivity	$C^2\ N^{-1}\ m^{-2}$	$[M^{-1}L^{-3}T^4A^2]$
31.	Permeability	$Tm\ A^{-1}$ or $Wb\ A^{-1}\ m^{-1}$	$[MLT^{-2}A^{-2}]$
32.	Plank's constant	Js	$[ML^2T^{-1}]$
33.	Boltzmann constant	JK^{-1}	$[ML^2T^{-2}K^{-1}]$

Table 1.2: Table of physical quantity having same dimensional formula

S. No.	Dimensional Formula	Physical Quantities
1.	$[M^0\ L^0\ T^{-1}]$	(a) Frequency (b) Angular frequency (c) Angular velocity (d) Velocity gradient
2.	$[M^0\ L^2\ T^{-2}]$	(a) Square of velocity (b) Gravitational potential (c) Latent heat

S. No.	Dimensional Formula	Physical Quantities
3.	$\left[ML^2 T^{-2}\right]$	(a) Work (b) Energy (c) Torque (d) Heat
4.	$\left[MLT^{-2}\right]$	(a) Force (b) Weight (c) Thrust (d) Energy gradient
5.	$\left[ML^{-1}T^{-2}\right]$	(a) Pressure (b) Stress (c) Modulii of elasticity (d) Energy density

4. USES OF DIMENSIONS

The major uses of dimensions are listed hereunder:

(a) Conversion from one system of units to another.

(b) To test and validate the correctness of a physical equation or formula.

(c) To derive a relationship between different physical quantities in any physical phenomenon.

(d) **Conversion from one system of units to another:** If we consider n_1 as numerical value of a physical quantity with dimensions a, b and c for units of mass, length and time as M_1, L_1, and T_1, then the numerical value of the same quantity, n_2 can be calculated for different units of mass, length and time as M_2, L_2 and T_2, respectively.

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

(e) **To test and validate the correctness of a physical equation or formula:** The principle of homogeneity requires that the dimensions of all the terms on both sides of physical equation or formula should be equal if the physical equation of any derived formula is correct.

(f) **To derive a relationship between different physical quantities in any physical phenomenon:** Suppose that if a physical quantity depends upon a number of parameters whose dimensions are not known, then the principle of homogeneity of dimensions can be used. As we know that the dimensions of a correct dimensional equation are equal on both sides, it can be used to find the unknown dimensions of these parameters on which the physical quantity depends. Further, it can be used to derive the relationships between any physical quantity and its dependent parameters.

Derivation: $S_n = u + \frac{a}{2}(2n-1)$

$$S_n = S_n + S_{n-1} = \left(un + \frac{1}{2}an^2\right) - (n(n-1) + \frac{1}{2}a(n-1)^2)$$

$$S_n = u(1) + \frac{1}{2}a(1)(2n-1) = \left[u + \frac{a}{2}(2n-1)\right](1)$$ (We ignore ‘1’ in formula but it carries dimension of time.)

Where, n – dimension of time; u – dimension of velocity; s – dimension of displacement; and a – dimension of acceleration.

CONCEPTS

The formula for displacement in nth second by a moving body is wrong using dimensional analysis.

NO! Actually, if we go back deeper in derivation we would very easily find that although the equation looks dimensionally incorrect but it is precise and accurate.

Vaibhav Gupta (JEE 2009, AIR 54)

Illustration 2: A calorie is a unit of heat or energy and it equals about 4.2 J. Suppose that we employ a system of unit in which the unit of mass equals α kg, the unit of length equals β metre, and the unit of time is γ second. Then, show that a calorie has a magnitude $4.2\ \alpha^{-1}\beta^{-2}\gamma^{2}$ in terms of the new units. **(JEE MAIN)**

Sol: Here the system is expressed in one set of units. When we want to convert the units in order of magnitude only, the conversion factor is obtained by dividing the original units by new set of units.

1 cal = 4.2 kg m^2 s^{-2}

SI	New system
$N_1 = 4.2$	$N_2 = ?$
$M_1 = 1$ kg	$M_2 = \alpha$ kg
$L_1 = 1$ m	$L_2 = \beta$ m
$T_1 = 1$ s	$T_2 = \gamma$ s

Dimensional formula of energy is $\left[ML^2T^{-2}\right]$

Comparing with $\left[M^aL^bT^c\right]$, we find that a = 1, b = 2, and c = –2

$$\text{Now, } N_2 = N_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 4.2\left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1\left[\frac{1\text{ m}}{\beta\text{ m}}\right]^2\left[\frac{1\text{ s}}{\gamma\text{ s}}\right]^{-2} = 4.2\ \alpha^{-1}\beta^{-2}\gamma^{2}$$

Illustration 3: The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v), and radius (r) of the circle. Derive the formula for F using the method of dimensions. **(JEE MAIN)**

Sol: To obtain the relation between force F, mass M, velocity V and radius r, we use the dimensional analysis. The power of base units of each quantity on the right of the equality are matched with the power of same unit on left of the equality.

$$\text{Let } F = K(m)^x(v)^y(r)^z \qquad \ldots \text{(i)}$$

Here, k is a dimensionless constant of proportionality. By writing the dimensions of RHS and LHS in Eq. (i), we have

$$\left[MLT^{-2}\right] = \left[M\right]^x\left[LT^{-1}\right]^y\left[L\right]^z = \left[M^xL^{y+z}T^{-y}\right]$$

By equating the powers of M, L, and T of both sides, we have

$$x = 1, y = 2 \text{ and } y + z = 1 \text{ or } z = 1 - y = -1$$

By substituting the values in Eq. (i), we obtain $F = kmv^2r^{-1} = k\frac{mv^2}{r}$; $F = \frac{mv^2}{r}$

(where k = 1). [The value of K cannot be calculated wring dimensional analysis].

CONCEPTS

A dimensionally correct equation may or may not be an exact equation but an exact equation must be dimensionally correct.

Example: F = ma and F = 0.5ma, both are dimensionally correct but only one is correct w.r.t the physical relation.

Vaibhav Krishnan (JEE 2009, AIR 22)

5. LIMITATIONS OF DIMENSIONS

(a) From a dimensionless equation, the nature of physical quantities cannot be decided, i.e., whether a given quantity is scalar or vector.

(b) The value of proportionality constant also cannot be determined.

(c) The relationship among physical quantities having exponential, logarithmic, and trigonometric functions cannot be established.

6. ORDER OF MAGNITUDE

In physics, we often learn quantities which vary over a wide rage. For example, we discuss regarding the size of a mountain and the size of the tip of a pin. In the same way, we also discuss regarding the mass of our galaxy and the mass of a hydrogen atom. Sometimes, we also discuss regarding the age of universe and the time taken by an electron to complete a circle around the proton in a hydrogen atom. However, we observe that it is quite difficult to get a feel of largeness or smallness of such quantities. Therefore, to express such drastically varying numbers, we use the power of ten method.

In this method, each number is expressed as $a\times10^{b}$ where $1\le a\le10$ and b is an integer. Thus, we represent the diameter of the sun as 1.39×10^{9} m and diameter of a hydrogen atom as 1.06×10^{-10} m. However, to have an approximate idea of the number, we may round the number 'a' to 1 if it is less than or equal to 5 and 10 if it is greater than 5. Thereafter, the number can be expressed approximately as 10^b. Further, we then obtain the order of magnitude of that number. Thus, now we can more clearly state that the diameter of the sun is of the order of 10^9m and that of a hydrogen atom is of the order of 10^{-10}m. More precisely, we say that the exponent of 10 in such a representation is called the order of magnitude of that quantity. Thus, now we can say that the diameter of the sun is 19 orders of magnitude larger than the diameter of a hydrogen atom. This is due to the fact that the order of magnitude of 10^9 is 9 and of 10^{-10} is –10. The difference is 9 –(–10) = 19.

Table 1.3: Table of SI prefixes

Power of 10	Prefix	Symbol
18	exa	E
15	peta	P
12	tera	T
9	giga	g
6	mega	M
3	kilo	k
2	hecto	h
1	deka	da

–1	deci	d
–2	centi	c
–3	milli	m
–6	micro	m
–9	nano	n
–12	pico	p
–15	femto	f
–18	atto	a

Tip: The best way to remember is by memorizing from milli to atto, kilo to exa and thereafter to go the power of 3.

For example, if one asks for giga since we have already memorized from kilo to exa, then we need to go like kilo mega giga and since it is 3 in the order shown, giga would be assigned a value of 3 * 3 = 9, i.e., 10^9.

7. SIGNIFICANT FIGURES

Significant figures in the measured value of a physical quantity provide information regarding the number of digits in which we have confidence. Thus, the larger the number of significant figures obtained in a measurement, the greater is the precision of the measurement.

"All accurately known digits in a measurement plus the first uncertain digit together form significant figures."

7.1 Rules for Counting Significant Figures

For counting significant figures, we make use of the rules listed hereunder:

(a) All non-zero digits are significant. For example, x = 2567 has clearly four significant figures.

(b) The zeroes appearing between two non-zero digits are counted in significant figures. For example, 6.028 has 4 significant figures.

(c) The zeroes located to the left of the last non-zero digit are not significant. For example, 0.0042 has two significant figures.

(d) In a number without decimal, zeroes located to the right of the non-zero digit are not significant. However, when some value is assigned on the basis of actual measurement, then the zeroes to the right non-zero digit become significant. For example, L = 20 m has two significant figures but x = 200 has only one significant figure.

(e) In a number with decimal, zeroes located to the right of last non-zero digit are significant. For example, x = 1.400 has four significant figures.

(f) The power of ten is not counted as significant digit(s). For example, 1.4×10^{-7} has only two significant figures, i.e., 1 and 4.

(g) Change in the units of measurement of a quantity, however, does not change the number of significant figures. For example, suppose the distance between two stations is 4067 m. It has four significant figures. The same distance can be expressed as 4.067 km or 4.067×10^5 cm. In all these expressions, however, the number of significant figures continues to be four.

Table 1.4: Significant figures

Measured value	Number of significant figures	Rule
12376	5	1
6024.7	5	2

0.071	2	3
410 m	3	4
720	2	4
2.40	3	5
1.6×10^{14}	2	6

7.2 Rounding Off a Digit

The rules for rounding off a measurement are listed hereunder:

(a) If the number lying to the right of cut off digit is less than 5, then the cut off digit is retained as such. However, if it is more than 5, then the cut off digit is increased by 1.

For example, x = 6.24 is rounded off to 6.2 (two significant digits) and x = 5.328 is rounded off to 5.33 (three significant digits).

(b) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is increased by 1. For example, x = 14.252 is rounded off to x = 14.3 to three significant digits.

(c) If the digit to be dropped is simply 5 or 5 followed by zeroes, then the preceding digit is left unchanged if it is an even number. For example, x = 6.250 or x = 6.25 becomes x = 6.2 after rounding off to two significant digits.

(d) If the digit to be dropped is 5 or 5 followed by zeroes, then the preceding digit is raised by one if it is an odd number.

For example, x = 6.350 or 6.35 becomes x = 6.4 after rounding off to two significant digits.

Table 1.5: Significant digits

Measured value	After rounding off to three significant digits	Rules
7.364	7.36	1
7.367	7.37	1
8.3251	8.33	2
9.445	9.44	3
9.4450	9.44	3
15.75	15.8	4
15.7500	15.8	4

7.3 Algebraic Operations with Significant Figures

(a) **Addition and subtraction:** Suppose in the measured values to be added or subtracted, the least number of significant digits after the decimal is n. Then, in the sum or difference also, the number of significant digits after the decimal should be n.

Example: Suppose that we have to find the sum of number 420.42 m, 420.4m and 0.402m by arithmetic addition

$$\begin{array}{r} 420.42 \\ 420.4 \\ 0.402 \\ \hline 441.222 \end{array}$$

But the least precise measurement of 420.4 m is correct to only one decimal place. Therefore, the final answer will be 441.2 m.

(b) Multiplication or division: Suppose in the measured values to be multiplied or divided, the least number of significant digits is n; then, in the product or quotient, the number of significant digits should also be n.

Example: $1.2 \times 36.72 = 44.064 \approx 44$

In the example shown, the least number of significant digits in the measured values is two. Hence, the result when rounded off to two significant digits becomes 44. Therefore, the answer is 44.

Example: $\dfrac{1100\,ms^{-1}}{10.2\,ms^{-1}} = 107.8431373 \approx 108$

CONCEPTS

Tip: In algebraic operations with significant figures, the result shall have significant figures corresponding to their number in the least accurate variable involved.

Nivvedan (JEE 2009, AIR 113)

Illustration 4: Round off the following number to three significant digits: (a) 15462, (b) 14.745, (c) 14.750 and (d) 14.650×10^{12}. **(JEE MAIN)**

Sol: The values above when rounded off to the three significant figures, if the fourth digit of the number is greater than or equal to 5, we increase the third digit by 1 and discard the digits after third digit. If the fourth digit is not greater than or equal to 5, we discard the digits from fourth onwards and write the number up to third significant figure. The power of 10 is not considered as the significant number.

(a) The third significant digit is 4. Now, this digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore, be increased by 1. The digits to be dropped should be replaced by zeroes because they appear to the left of the decimal point. Thus, 15462 becomes 15500 on rounding to three significant digits.

(b) The third significant digits in 14.745 is 7. The number next to it is less than 5. Therefore, 14.745 becomes 14.7 on rounding to three significant digits.

(c) 14.750 will become 14.8 because the digit to be rounded is odd and the digit next to it is 5.

(d) 14.650×10^{12} will become 14.6×10^{12} because the digit to be rounded is even and the digit next to it is 5.

Illustration 5: Evaluate $\dfrac{25.2 \times 1374}{33.3}$. All the digits in this expression are significant. **(JEE MAIN)**

Sol: The result of the above fraction is rounded off to the same number of significant figure as is contained by the least precise term used in calculation, like 25.2 and 33.3.

We have $\dfrac{25.2 \times 1374}{33.3} = 1039.7838.$

Out of the three numbers given in the expression, both 25.0 and 33.3 have 3 significant digits, whereas 1374 has four. The answer, therefore, should have three significant digits. Rounding 1039.7838 to three significant digits, it hence becomes 1040. Thus, we write $\dfrac{25.2 \times 1374}{33.3} = 1040.$

8. ERROR ANALYSIS

We define the uncertainty in a measurement as an 'error'. By this we mean the difference between the measured and the true values of a physical quantity under investigation. There are three possible ways of calculating an error as listed hereunder:

(i) Absolute error (ii) Relative error (iii) Percentage error

Let us consider a physical quantity measured by taking repeated number of observations say $x_1, x_2, x_3, x_4, ...$ if <x> or $\bar{x}$ be the average value of the measurement, then the error in the respective measurement is $\Delta x_1 = x_1 - \bar{x}$; $\Delta x_2 = x_2 - \bar{x}$ ....; $\Delta x = \left|x_{\text{exp erimental value}} - x_{\text{true value}}\right|$

However, if we take the arithmetic mean of all absolute errors, then we obtain the final absolute error Δx_{mean} . When arithmetic mean alone is considered, then only the magnitudes of the absolute errors are taken into account.

$$\Delta x_{mean} = \frac{\left|\Delta x_1\right| + \left|\Delta x_2\right| + ... + \left|\Delta x_n\right|}{n} = \frac{1}{n}\sum_{i=1}^{n}\left|\Delta x_i\right|$$

It then follows clearly from the above discussion that any single measurement of x has to be such that

$$x_{mean} - \Delta x_{mean} \le x \le x_{mean} + \Delta x_{mean}$$

Relative error= $\frac{\Delta x_{mean}}{x_{mean}}$; percentage error= $\frac{\Delta x_{mean}}{x_{mean}} \times 100$

9. PROPAGATION OF ERRORS

9.1 Addition and Subtraction

If $x = A \pm B$; then $\Delta x = \Delta A + \Delta B$

i.e., for both addition and subtraction, the absolute errors are to be added up. The percentage error, then, in the value of x is

Percentage error in the value of $x = \left(\frac{\Delta A + \Delta B}{A \pm B}\right) \times 100\%$

9.2 Multiplication and Division

If y = AB or y = $\frac{A}{B}$ then, $\frac{\Delta y}{y} = \frac{\Delta A}{A} + \frac{\Delta B}{B} \Rightarrow \frac{\Delta y}{y} \times 100\% = \frac{\Delta A}{A} \times 100\% + \frac{\Delta B}{B} \times 100\%$

$\Rightarrow$ Percentage error in Value of y = percentage error in value of A + percentage error in value of B

Illustration 6: Two resistors $R_1 = 100 \pm 3\ \Omega$ and $R_2 = 200 \pm 4\ \Omega$ are connected in series. Find the equivalent resistance. **(JEE MAIN)**

Sol: When resistance are added in the series, the error in the resultant combination is given by $\Delta R_{eq} = \Delta R_1 + \Delta R_2$ where $\Delta R_1 = 3\ \Omega$ and $\Delta R_2 = 4\ \Omega$.

The equivalent resistance $R = R_1 + R_2 = (100 \pm 3)\ \Omega + (200 \pm 4)\Omega = 300 \pm 7\Omega$

Illustration 7: A capacitor of capacitance $C = 2.0 \pm 0.1\ \mu F$ is charged to a voltage $V = 20 \pm 0.2$ volt. What will be the charge Q on the capacitor? Use Q = CV. **(JEE MAIN)**

Sol: The relative error of result of the above product is given by $\frac{\Delta Q}{Q} = \pm\left(\frac{\Delta C}{C} + \frac{\Delta V}{V}\right)$ where $\frac{\Delta C}{C}$ and $\frac{\Delta V}{V}$ is the relative error in determination in C and V respectively.

If we omit all errors, then $Q = CV = 2.0 \times 10^{-6} \times 20\ C = 40 \times 10^{-6}\ C$

Error in C= 0.1 part in 2 = 1 part in 20 =5%

Error in V 0.2 part in 20 = 2 part in 200 = 1 part in 100 =1%; error $Q = 5\% + 1\% = 6\%$

$\therefore$ Charge, $Q = 40 \times 10^{-6} \pm 6\%\ C = 40 \pm 2.4 \times 10^{-6}\ C$

9.3 Power Functions

If $y = k\dfrac{A^{\ell}B^{m}}{C^{n}}$ then, $\dfrac{\Delta y}{y} = \ell\left(\dfrac{\Delta A}{A}\right) + m\left(\dfrac{\Delta B}{B}\right) + n\left(\dfrac{\Delta C}{C}\right)$

$$\begin{pmatrix}\text{Percentage error}\\ \text{in value of y}\end{pmatrix} = \ell\begin{pmatrix}\text{Percentage error}\\ \text{in value of A}\end{pmatrix} + m\begin{pmatrix}\text{Percentage error}\\ \text{in value of B}\end{pmatrix} + n\begin{pmatrix}\text{Percentage error}\\ \text{in value of C}\end{pmatrix}$$

CONCEPTS

- The error in a measurement is always equal to the least count of the measuring instrument.
- Errors never propagate particularly in case of constants.

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 8: A physical quantity P is related to four observables a, b, c and d as follows: $P = \dfrac{a^3b^2}{\sqrt{cd}}$. The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P? **(JEE MAIN)**

Sol: The relative error of ratio of $P = \dfrac{a^3b^2}{\sqrt{cd}}$ is calculated as $\dfrac{\Delta P}{P} = \pm\left(\dfrac{\Delta N}{N} + \dfrac{\Delta D}{D}\right)$ where $N = a^3b^2$ and $D = \sqrt{c}\,d$ and $\dfrac{\Delta N}{N}$ and $\dfrac{\Delta D}{D}$ are the relative error in the N and D.

$$P = \frac{a^3b^2}{\sqrt{cd}};\ \frac{\Delta P}{P} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}$$

$$\text{But } \frac{\Delta a}{a} = \frac{1}{100}, \frac{\Delta b}{b} = \frac{3}{100}, \frac{\Delta c}{c} = \frac{4}{100}, \frac{\Delta d}{d} = \frac{2}{100} \therefore \quad \frac{\Delta P}{P} = 3\times\frac{1}{100} + 2\times\frac{3}{100} + \frac{1}{2}\times\frac{4}{100} + \frac{2}{100}$$

% error in P = 3% + 6% + 2% + 2% = 13%.

10. LENGTH-MEASURING INSTRUMENTS

We know that length is an elementary physical quantity. The device generally used in everyday life for measurement of length is a metre scale. This scale can be used for measurement of length with accuracy to the extent of 1 mm. Therefore, the least count of a metre scale is 1 mm. Further, to measure length accurately up to $(1/10)$th or $\left(\dfrac{1}{100}\right)$th of a millimetre, we use the following instruments.

(1) Vernier calipers (2) Micrometer (3) Screw gauge

10.1 Vernier Calipers

This instrument has three parts.

(i) Main scale: It consists of a strip M, graduated in cm and mm at one of its edge. Also, it carries two fixed jaws A and C as shown in the Fig. 1.1.

(ii) Vernier scale: Vernier scale V slides on metallic strip M. This scale can be fixed in any position

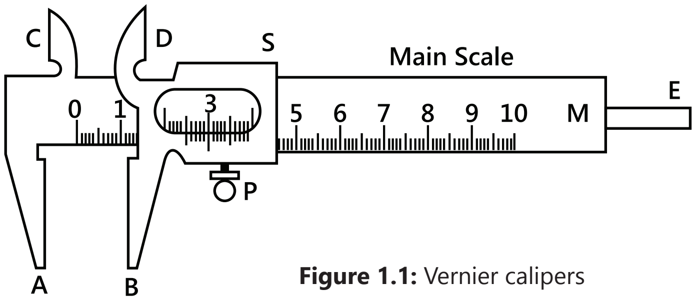


Figure 1.1: Vernier calipers

using the screw S. The side of the Vernier scale which slides over the mm sides has 10 divisions over a length of 9 mm. Further, B and D are two movable jaws that are fixed with it. When the Vernier scale is pushed toward A and C, then B touches A and straight side of C will touch straight side of D. In this position, however, if the instrument is free from error, zeroes of Vernier scale will coincide with zeroes of the main scales. Further, to measure the external diameter of an object, it is held between the jaws A and B, while the straight edges of C and D are used for measuring the internal diameter of a hollow object.

(iii) Metallic strip: There is a thin metallic strip E attached to the back side of M and connected with Vernier scale. When jaws A and B touch each other, the edge of E touches the edge of M. When the jaws A and B are separated, the E moves outward. This strip E is used for measuring the depth of a vessel.

10.1.1 Principle (Theory)

In the common form, the divisions on the Vernier scale V are smaller in size than the smallest division on the main scale M; however, in some special cases the size of the Vernier division may be larger than the main scale division.

Let n Vernier scale divisions (VSD) coincide with (n–1) main scale divisions (MSD). Then,

$$\text{nV.S.D.} = (n-1)\ \text{M.S.D.};\ \ 1\text{V.S.D.} = \left(\frac{n-1}{n}\right)\ \text{M.S.D.};\ 1\ \text{M.S.D.} - 1\ \text{V.S.D.}$$

$$= 1\ \text{M.S.D.} - \left(\frac{n-1}{n}\right)\text{M.S.D.} = \frac{1}{n}\text{M.S.D.}$$

The difference between the values of one main scale division and one Vernier scale division is known as Vernier constant (VC) or the **least count** (LC). This is precisely the smallest distance that can be accurately measured with the Vernier scale. Thus,

$$\text{V.C} = \text{L.C} = 1\text{M.S.D.-}1\text{V.S.D.} = \left(\frac{1}{n}\right)\text{M.S.D.} = \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

In the ordinary Vernier caliper, one main scale division is 1 mm and 10 Vernier scale divisions coincide with 9 main scale divisions.

$$1\text{V.S.D.} = \frac{9}{10}\text{M.S.D.} = 0.9\text{mm};\ \text{V.C.} = 1\text{M.S.D.} - 1\text{V.S.D.} = 1\text{mm-}0.9\text{mm} = 0.1\text{mm} = 0.01\text{cm}$$

10.1.2 Reading a Vernier Caliper

If we have to measure a length AB, then the end of A is coincided with the zero of the main scale. Now, suppose that the end B lies between 1.0 cm and 1.1 cm on the main scale. Then, let the 5th division of Vernier scale coincides with 1.5 cm of the main scale.

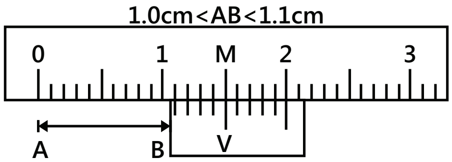


Figure 1.2: Reading main scale and vernier scale

Then, AB=1.0+5 × V.C = (1.0 +5 × 0.001) cm = 1.05 cm

Thus, we can make use of the following formula, i.e.,

Total reading = N+n × V.C

Here, N = main scale reading just before on the left of the zero of the Vernier scale.

n = number of Vernier division which just coincides with any of the main scale divisions.

CONCEPTS

The main scale reading with which the Vernier scale division coincides has no connection with reading.

Chinmay S Purandare (JEE 2012, AIR 698)

10.1.3 Zero Error and Zero Correction

If the zero of the Vernier scale does not coincide with the zero of the main scale when jaw B touches jaw A and the straight edge of D touches the straight edge of C, then the instrument has an error called zero error. This zero error is always algebraically subtracted from the measured length.

Zero correction has a magnitude that is equal to zero error but its sign is opposite to that of the zero error. Zero correction is, as a rule of the thumb, always algebraically added to the measured length.

Zero error $\rightarrow$ algebraically subtracted

Zero correction $\rightarrow$ algebraically added

10.1.4 Positive and Negative Zero Error

If the zero of Vernier scale lies to the right of the main scale, then the zero error is positive and if it lies to the left of the main scale then the zero error is negative (when jaws A and B are in contact)

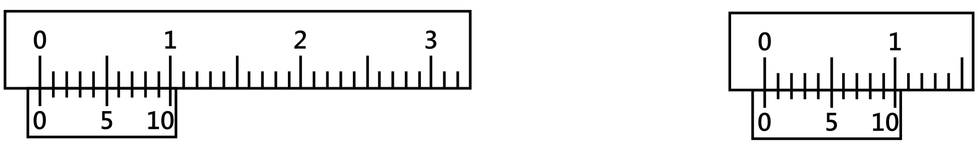
Positive zero error$=\left(N+x\times V.C.\right)$

Here, N = main scale reading on the left of zero of Vernier scale.

X = Vernier scale division which coincides with any main scale division.

When the Vernier zero lies before the main scale zero, then the error is said to be negative zero error. If the 5th Vernier scale division coincides with the main scale division, then

$$\text{Negative zero error}=-\left[0.00\text{cm}+5\times V.C.\right]=-\left[0.00\text{cm}+5\times 0.01\text{cm}\right]=-0.05\text{cm}$$



(A) Positive zero error (B) Negative zero error

Figure 1.3: Types of error of vernier scale

Illustration 9: N-divisions on the main scale of a Vernier caliper coincides with N + 1 divisions on the Vernier scale. If each division on the main scale is of 'a' units, then determine the least count of the instrument. **(IIT JEE 2003)**

Sol: Least count of Vernier caliper is given by L. C. =1 M.S.D – 1 V.S.D.

(N+1) divisions on the Vernier scale = N divisions on the main scale

$\therefore$ 1 division on vernier scale$=\frac{N}{N+1}$divisions on main scale

If each division on the main scale is of 'a' units

$\therefore$ 1 division on vernier scale$=\left(\frac{N}{N+1}\right)\times$a unit=a'(say)

Least count = 1 main scale division – 1 Vernier scale division = $a-a'=a-\left(\frac{N}{N+1}\right)a=\frac{a}{N+1}$

Illustration 10: In the diagram provided, find the magnitude and the nature of zero error. **(JEE MAIN)**

Sol: Here as the zero division of the main scale is to the left of the zero division of the Vernier scale, thus Vernier caliper is said to have positive error. Thus the measured length of any object will be greater than actual length. This error is to be subtracted from measured length to get actual length.

Here, zero of Vernier scale lies to the right of zero of the main scale; hence, it has positive zero error.

Further, N = 0, x = 5,

L.C. of, V.C. = 0.01 cm.

Hence, zero error $= N + x \times 0.01 = 0.05\text{cm}$ Zero correction = –0.05cm

$\therefore$The actual length will be 0.05 cm less than the measured length.

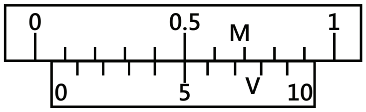


Figure 1.4

10.2 Micrometer Screw

10.2.1 Principle of a Micrometer Screw

The least count of Vernier calipers ordinary available in the laboratory is 0.01 cm. However, when lengths are to be measured with greater accuracy, say up to 0.001 cm, then screw gauge and spherometre are used which are based on the principle of micrometer screw as discussed here under.

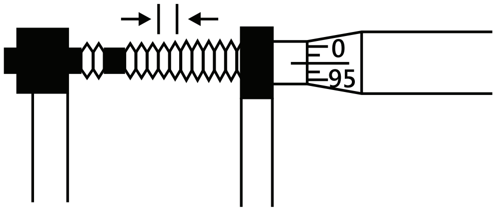


Figure 1.5: Micrometer screw gauge

If an accurately cut single threaded screw is rotated in a closely fitted nut, then in addition to the circular motion of the screw there is also a linear motion of the screw head in the forward or backward direction, along the axis of the screw. The linear distance moved by the screw, when it is given one complete rotation, is called the pitch (p) of the screw. The pitch is basically equal to the distance between two consecutive threads as measured along the axis of the screw. In most of the cases, it is either 1 mm or 0.5 mm. The screw moves forward or backward by $\frac{1}{100}\left(\text{or}\frac{1}{50}\right)$ of the pitch, if the circular scale (we will discuss later about the circular scale) is rotated through one circular division. This is exactly the minimum distance which can be accurately measured and hence called the least count (LC) of the screw.

Thus, $\text{Least count} = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$. If pitch is 1 mm and there are 100 divisions on the circular scale, then

$$\text{L.C.} = \frac{1\text{mm}}{100} = 0.01\text{mm} = 0.001\text{cm} = 10\mu\text{m}$$

Since, LC is of the order of $10\mu\text{m}$, the screw is called a micrometer screw.

10.3 Screw Gauge

Screw gauge functions based on the principle of a micrometer screw. It consists of a U-shaped metal frame M. At one end of it is fixed a small metal piece A. It is called stud and has a plane face. The other end N of M carries a cylindrical hum, H. It is graduated in millimeters or half millimeters based on the pitch of the screw. This scale is called a linear scale or a pitch scale.

A nut is threaded through both the hub and the frame N. Through the nut moves a screw S. The front face B of the screw, however, facing the plane face A, is also plane in nature. There is a hollow cylindrical cap L that is capable of rotating over the hub when screw is rotated. As the cap is rotated, the screw either moves in or out. The surface E of the cap K is divided into 50 or 100 equal parts. It is called the circular scale or the head scale. In an accurately adjusted instrument when the faces A and B are just touching each other, then the zero of the circular scale should coincide with the zero of the linear scale.

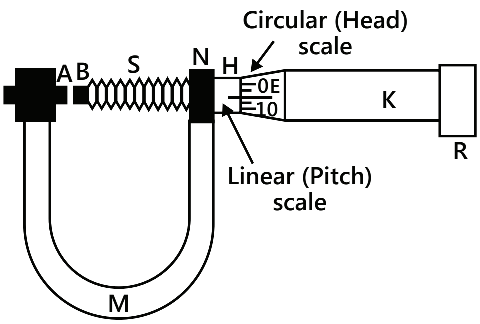


Figure 1.6: Main scale and circular scale of screw gauge

10.3.1 To measure diameter of a given wire using a screw gauge

If with the wire between plane faces A and B, the edge of the cap lies ahead of Nth division of the linear scale, and nth division of circular scale lies over reference line then, Total reading = N + n × L.C

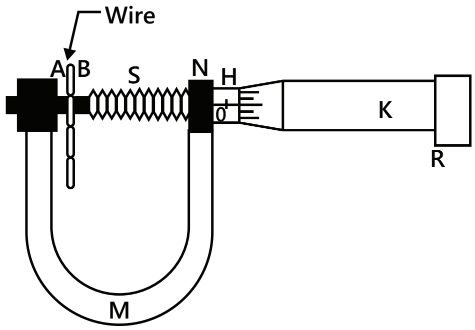


Figure 1.7: Determination of thickness of wire

10.3.2 Zero error and zero correction

If the zero mark of the circular scale does not coincide with the zero of the pitch scale when the faces A and B are just touching each other, then the instrument is said to possess zero error. However, if the zero of the circular scale advances beyond the reference line then the zero error is negative and zero correction is positive. Further, if it is left behind the reference line the zero error is positive and zero correction is negative. For example, if zero of the circular scale advances beyond the reference line by 5 divisions, then zero correction $=+5\times(\text{L.C.})$ and if the zero of the circular scale is left behind the reference line by 4 divisions, then zero correction $=-4\times(\text{L.C.})$.

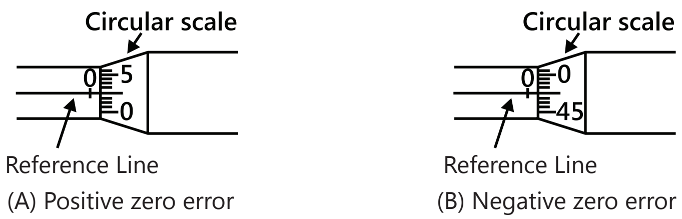


Figure 1.8: Types of errors in reading of screw gauge

10.3.3 Backlash error

When the direction of rotation of the screw is suddenly changed, then the screw head may rotate, but the screw itself may not move forward or backward. Consequently, the scale reading may not change even by the actual movement of the screw. This is what we meant by backlash error. This error is basically due to loose fitting of the screw. This arises mainly due to wear and tear of the threading due to prolonged use of the screw. To reduce this error, we recommend that the screw must always be rotated in the same direction for a particular set of observations.

PROBLEM-SOLVING TACTICS

1. While changing units, one can visualize them as some constants multiplied to the numbers. This trick is very helpful particularly in understanding units. Like you can cut out units from both sides of equations very easily just like constants.

2. Application of dimensional analysis can possibly rule out inappropriate answers in multiple choice questions (MCQs).

Further, dimensional analysis can also be used to eliminate invalid choices in MCQ even in mathematics. It definitely saves lot of time.

3. Be careful in handling error approximation. Binomial theorem can only be applied in very low percentage of errors (generally less than 5% error), else not.

4. One must accurately know the rules for determining significant digits and be precise while dealing with 0s and 5.

FORMULAE SHEET

(a) $\text{V.C.} = \text{L.C.} = \dfrac{1\text{M.S.D.}}{n} = \dfrac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}} = 1\text{M.S.D.} - 1\text{V.S.D.}$

(b) In ordinary Vernier calipers, $= 1\text{M.S.D.} = 1\text{mm}$ and $n=10$; V.C. or L.C.$=\dfrac{1}{10}\text{mm} = 0.01\text{cm}$

(c) Total reading = $(N + n \times \text{V.C.})$

(d) Zero correction=–zero error. Zero error is algebraically subtracted, whereas the zero correction is algebraically added. If zero of Vernier scale lies to the right of zero of the main scale, then the error is positive. The actual length in this case is less than the observed length.

(e) If zero of Vernier scale lies to the left of zero of the main scale, then the error is negative and the actual length is more than the observed length.

(f) Positive zero error $= (N + x \times \text{V.C.})$

(g) **Least count:** The minimum measurement that can be measured accurately by an instrument is called the least count.

Least count of Vernier caliper

$= \{\text{value of 1 part of main scale(s)}\} - \{\text{value of one part of vernier scale (V)}\}$

or least count of Vernier calipers = 1MSD – 1VSD

where, MSD = Main scale division; VSD = Vernier scale division

$$\text{Least count} = \frac{\text{Value of 1 part of main scale (s)}}{\text{Number of parts on vernier scale (n)}}$$

$$\text{Least count of screw gauge} = \frac{\text{Pitch (p)}}{\text{Number of parts on circular scale (n)}}$$

(h) The maximum absolute error in x is $\Delta x = +(\Delta a + \Delta b)$

(i) Percentage error $= +- \dfrac{\Delta y}{y_m} \times 100$

(j) Errors never propagate in case of constants.

(k) If $x = a$ then $\dfrac{\Delta x}{x} = \dfrac{\Delta a}{a}$

(l) If $x = a - b$, then $\dfrac{\Delta x}{x} = \dfrac{\Delta a + \Delta b}{a - b}$

(m) If x = a b or $x = \dfrac{a}{b}$, then $\dfrac{\Delta x}{x} = \left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

(n) If $b = \dfrac{a^n b^m}{z^p y^k}$, then $\dfrac{\Delta x}{x} = \left(\dfrac{n\Delta a}{a} + \dfrac{m\Delta b}{b} + \dfrac{p\Delta z}{z} + \dfrac{k\Delta y}{y}\right)$

(o) Absolute error $\Delta x = x\left(\dfrac{n\Delta a}{a} + \dfrac{m\Delta b}{b} + \dfrac{p\Delta z}{z} + \dfrac{k\Delta y}{y}\right)$ or in general ; $\Delta x = x$ (relative error)

Solved Examples

JEE Main/Boards

Example 1: The value of acceleration due to gravity is 980 cm/s^2. What will be its value if the unit of length is kilometer and that of time is minute?

Sol: Here we have to express value of g in units of km/min^2. Thus the conversion factor from cm/s^2, to km/min^2 we need to divide the old units to the new units.

Dimension of acceleration due to gravity is $\left[LT^{-2}\right]$. In the CGS system, let L_1, T_1 represent length and time measured in cm and second. The numerical value n_1 = 980 cm/sec^2. Let n_2 be the value of acceleration due to gravity in the new system. The length L_2 and time T_2 are measured in kilometer and minute respectively. Now

$$n_1\left[L_1T_1^{-2}\right]=n_2\left[L_2T_2^{-2}\right] \text{ or } n_2=n_1\left[\frac{L_1}{L_2}\right]\left[\frac{T_1^{-2}}{T_2^{-2}}\right]^2$$

$$\therefore n_2=980\left[\frac{1}{10^5}\right]\left[\frac{1}{60}\right]^{-2}=\frac{980\times 60\times 60}{10^5}\Rightarrow n_2=35.3$$

Example 2: A body of mass m hung at one end of the spring executes a simple harmonic motion. The force constant of a spring is k while its period of vibration is T. Prove by dimensional method that the equation $T=\frac{2\pi m}{k}$ is incorrect. Derive the correct equation, assuming that they are related by a lower power.

Sol: To prove the dimensional inequality of the above relation we use the dimensional analysis. In dimensional analysis, the power of each basic unit to the left of the equality sign is matched with the dimensions of each basic units of quantity on the right of the equality.

The given equation is $T=\frac{2\pi m}{k}$

Taking the dimensions of both sides, we have

$$\left[T\right]=\frac{\left[M\right]}{\left[ML^0T^{-2}\right]}=T^2$$

As the dimensions of two sides are not equal,

the equation is incorrect. Let the correct relation be $T = Cm^a k^b$ where C is a constant.

Equating the dimensions of both sides, we get $\left[T\right]=\left[M\right]^a\left[MT^{-2}\right]^b$;

Or $\left[M^0L^0T\right]=\left[M^{a+b}L^0T^{-2b}\right]$ Comparing the power of M, L and T on both sides a + b = 0 and –2b = 1

$$\therefore b=-\frac{1}{2} \text{ and } a=\frac{1}{2};\quad \therefore T=Cm^{1/2}k^{-1/2}=C\sqrt{\left(\frac{m}{k}\right)}$$

This is the correct equation.

Example 3: The radius of the earth is 6.37×10^6 m and its mass is 5.975×10^{24} kg. Find the earth's average density to appropriate significant figures.

Sol: The average density of earth is given as $D_E=\frac{M_E}{V_E}$.

The result is rounded off to the same number of significant figures as is contained by the least precise term used in the calculation.

Given, mass of the earth (M) = 5.975×10^{24} kg. Further, radius of the earth (R) = 6.37×10^6 m and volume of the earth (V)

$$=\frac{4}{3}\times \pi R^3 m=\frac{4}{3}\times\left(3.142\right)\times\left(6.37\times 10^6\right)^3 m^3$$

Average density (D)

$$=\frac{\text{Mass}}{\text{Volume}}=\frac{m}{v}=\frac{5.975\times 10^{24}}{\frac{4}{3}\times\left(3.142\right)\times\left(6.37\times 10^6\right)^3}$$

$$=0.005517\times 10^6\,kgm^{-3}=5.52\times 10^3\,kgm^{-3}$$

(to three significant figures)

The density is accurate only up to three significant figures which is the accuracy of the least accurate term, namely, the radius of the earth.

Example 4: A man runs 100.5 m in 10.3 sec. Find his average speed up to appropriate significant figures.

Sol: The average speed of man is given by $V_{avg}=\frac{\text{distance travelled}}{\text{Time taken}}$. The result is rounded off to the same number of significant figures as is contained by the least precise term used in the calculation.

Using average speed $=\frac{\text{distance travelled}}{\text{Time taken}}$

$$=\frac{100.5m}{10.3s}=9.757\ ms^{-1}$$

Note that, the distance 100.5 m has four significant figures but the time of 10.3 sec has only three. Thus, we round off the final result to three significant figures.

The average speed must be correctly expressed as $9.76\ ms^{-1}$.

Example 5: The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. L is about 10 cm and is known to 1 mm accuracy. The period of oscillations is about 0.5 second. The time of 100 oscillations is measured with a wrist watch of 1 s resolution. What is the accuracy in the determination of g?

Sol: The accuracy in determination of g is found in terms of minimum percentage error in calculation. The percentage error in g $= \frac{\Delta g}{g} \times 100\%$. Where $\frac{\Delta g}{g}$ is the relative error in determination of g.

$T = 2\pi\sqrt{\frac{L}{g}}$ or $T^2 = 4\pi^2\sqrt{\frac{L}{g}}$ or $g = \frac{4\pi^2 L}{T^2}$;

Now, $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\times\frac{\Delta T}{T}$

In terms of percentage,

$\frac{\Delta g}{g}\times 100 = \frac{\Delta L}{L}\times 100 + 2\times\frac{\Delta T}{T}\times 100$;

Percentage error in L

$100\times\frac{\Delta L}{L} = 100\times\frac{0.1}{10} = 1\%$

Percentage error in T $= 100\times\frac{\Delta T}{T} = 100\times\frac{1}{50} = 2\%$

Thus percentage error in g

$= 100\times\frac{\Delta g}{g} = 1\% + 2\times 2\% = 5\%$

Example 6: The error in the measurement of the radius of a sphere is 0.5%. What is the permissible percentage error in the measurement of its (a) surface area and (b) volume?

Sol: Percentage error in determination of any quantity = Relative error in determination of quantity $\times 100\%$. The relative error in area and volume of sphere are $\frac{\Delta A}{A} = \frac{2\Delta r}{r}$ and $\frac{\Delta V}{V} = \frac{3\Delta r}{r}$ respectively.

Given $\frac{\Delta r}{r} = 0.5\%$

(a) The surface area of a sphere of radius r is $A = 4\pi r^2$

$\therefore$ Percentage error in A $= \frac{\Delta A}{A} = \frac{2\Delta r}{r} = 2\times 0.5\% = 1\%$

(b) The volume of a sphere of radius r is $V = \frac{4\pi}{3}r^3$

$\therefore$ Percentage error in $V = \frac{\Delta V}{V} = \frac{3\Delta r}{r} = 3\times 0.5\% = 1.5\%$

Example 7: In an experiment on the determination of Young's modulus of a wire by Searle's method, the following data is available:

Normal length of the wire = 110 cm Diameter of the wire d = 0.01 cm

Elongation in the wire ℓ = 0.125 cm

This elongation is for a tension of 50 N. The least counts for corresponding quantities are 0.01 cm, 0.00005 cm and 0.001 cm, respectively. Calculate error in calculating the value of Young's modulus (Y).

Sol: The relative error in determination of Young's modulus is $\frac{\Delta Y}{Y} = \pm\left(\frac{\Delta N}{N} + \frac{\Delta D}{D}\right)$ where N = 4TL and D = $\pi d^2\ell$ and $\frac{\Delta N}{N}$ and $\frac{\Delta D}{D}$ are the relative error in the N and D.

$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{\ell/L}; \Rightarrow Y = \frac{TL}{A\ell} = \frac{4TL}{\pi d^2\ell}$

$\frac{\Delta Y}{Y}\ \frac{\Delta L}{L} = \frac{\Delta\ell}{\ell} + 2\frac{\Delta d}{d}$

Since $\frac{4T}{\pi}$ is a constant, so it does not contribute anything to the net error:

$\Rightarrow \frac{\Delta Y}{Y} = \frac{0.01}{110} + \frac{0.001}{110} + 2\left(\frac{0.00005}{0.01}\right) = 0.01809$

$\Rightarrow \frac{\Delta Y}{Y}\times 100\% = 0.009 + 0.8 + 1 = 1.809\%$

Example 8: It has been observed that velocity of ripple waves produced in water depends upon their wavelength(λ), density of water (ρ) and surface tension (T).

Prove that $V^2 \propto \frac{T}{\lambda\rho}$.

Sol: To prove above inequality we need to use the dimensional analysis. The dimensions of λ, ρ and T are need to be matched with dimensions of V^2. The power of same base units on either side of equality sign are equated to get the correct relation.

According to the problem we have

$V^2 \propto \lambda^a \rho^b T^c \Rightarrow V^2 = K\,\lambda^a \rho^b T^c$ ... (i)

Dimensions of V, ρ, λ and T are $[L T^{-1}]$, $[M L^{-3}]$, $[L]$ and $[M T^{-2}]$ respectively.

Thus according to the equation (i),

$$\left[V^2\right] = \left[\lambda\right]^a \left[\rho\right]^b \left[T\right]^c \Rightarrow L^2 T^{-2}$$

$$= \left[L\right]^a \left[ML^{-3}\right]^b \left[MT^{-2}\right]^c$$

Matching the powers of the same units we get a−3b =2, b + c = 0 and 2c = 2

⇒c = 1, b = −1 and a = −1.

Thus we get $V^2 = K \times \frac{T}{\lambda\rho} \Rightarrow V^2 \propto \frac{T}{\lambda\rho}$.

Hence proved.

Example 9: In an experiment for determining the value of acceleration due to gravity (g) using a simple pendulum, the following observations were recorded.

Length of the string (ℓ) = 98.0 cm

Diameter of the bob (d) = 2.56 cm

Time for 10 oscillations (T) = 20.0 sec

Calculate the value of g with maximum permissible absolute error and the percentage relative error.

Sol: The absolute error in g is $\Delta g = g_m - g_i$ where I = 1,2,3……, etc. and the percentage relative error in g = Relative error in g $\times 100\% = \frac{\Delta g}{g} \times 100\%$.

Time period for a simple pendulum is $T = 2\pi\sqrt{\frac{\ell_{eff}}{g}}$……..(i)

where ℓ_{eff} is the effective length of the pendulum equal to $\left(\ell + \frac{d}{2}\right)$ and time period equals $T = \frac{20.0}{10} = 2.00s$

from (i), we get

$$g = \frac{4\pi^2\left(\ell_{eff}\right)}{T^2}$$

To calculate actual value of g

$$\text{Since } g = \frac{4\pi^2\left(\ell_{eff}\right)}{T^2} = \frac{4\pi^2\left(\ell + \frac{d}{2}\right)}{T^2} = \frac{4\pi^2\left(\ell + r\right)}{T^2}$$

$$g = \frac{4\pi^2\left(98 + 1.28\right)}{\left(2.00\right)^2} = 980\ cms^{-2} = 9.80 ms^{-2}$$

Error in the value of g

$$\frac{\Delta g}{g} = \frac{\Delta \ell_{eff}}{\Delta \ell_{eff}} + 2\left(\frac{\Delta T}{T}\right);\Rightarrow \frac{\Delta g}{g} = \frac{\Delta\ell + \Delta r}{\ell + r} + 2\left(\frac{\Delta T}{T}\right)$$

Further, since errors can never exceed the least count of the measuring instrument. Therefore,

$\Delta\ell = 0.1cm;\ \Delta r = 0.01cm;$

$$\Delta T = 0.1s \Rightarrow \frac{\Delta g}{g} = \left(\frac{0.1 + 0.01}{98.0 + 1.28}\right) + 2\left(\frac{0.1}{20.0}\right)$$

$$\Rightarrow \frac{\Delta g}{g} = 0.0011 + 0.01;\Rightarrow \frac{\Delta g}{g} = 0.0111$$

$$\Rightarrow \text{Percentage error} \Rightarrow \frac{\Delta g}{g} \times 100 = 1.1\%$$

and absolute error $= \Delta g = g\left(0.011\right) = 0.11 ms^{-2}$;

Thus, $g = \left(9.80 ms^{-2} \pm 1.1\%\right)$

or, $g = \left(9.80 \pm 1.1\%\right) ms^{-2}$

Example 10: A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty ± 0.01 mm. Take g = 9.8 m/s² (exact). The Young's modulus obtained from the reading is close to

(A) $\left(2.0\ \pm 0.3\right)10^{11}\ n/m^2$

(B) $\left(2.0\ \pm 0.2\right)\times 10^{11}\ n/m^2$

(C) $\left(2.0\ \pm 0.1\right)\times 10^{11}\ n/m^2$

(D) $\left(2.0\ \pm 0.05\right)\times 10^{11}\ n/m^2$

Sol: As relative error in the length and diameter are given, the relative error in determination of Young's modulus is calculated as $\frac{\Delta Y}{Y} = \pm\left(\frac{\Delta N}{N} + \frac{\Delta D}{D}\right)$ where N = 4TL and D = $\pi d^2 \ell$ and $\frac{\Delta N}{N}$ and $\frac{\Delta D}{D}$ are the relative error in the N and D.

$$Y = \frac{FL}{Al} = \frac{4FL}{\pi d^2 \ell} = \frac{\left(4\right)\left(1.0 \times 9.8\right)\left(2\right)}{\pi\left(0.4 \times 10^{-3}\right)^2\left(0.8 \times 10^{-3}\right)}$$

$$= 2.0 \times 10^{-3}\ N/m^2$$

Further $\frac{\Delta Y}{Y} = 2\left(\frac{\Delta d}{d}\right) + \left(\frac{\Delta l}{l}\right)$; $\Delta Y = \left\{2\left(\frac{\Delta d}{d}\right) + \left(\frac{\Delta l}{l}\right)\right\}y$

$= \left\{2 \times \frac{0.01}{0.4} + \frac{0.05}{0.8}\right\} \times 2.0 \times 10^{11}$

$= 0.225 \times 10^{11}\ N/m^2 = 0.2 \times 10^{11}\ N/m^2$

(by rounding off)

Or $(Y + \Delta Y) = (2 + 0.2) \times 10^{11} N/m^2$

$\therefore$ Correct option is (B).

JEE Advanced/Boards

Example 1: The pitch of a screw gauge is 1 mm and there are 100 divisions on circular scale. When faces A and B are just touching each other without putting anything between the studs, 32nd division of the circular scale coincides with the reference line. When a glass plate is placed between the studs, the linear scale reads 4 divisions and the circular scale reads 16 divisions. Find the thickness of the glass plate. The zero of linear scale is not hidden from circular scale when A and B touch each other.

Sol: The gauge is found to have positive error. This has to be subtracted from measured value to get actual value. The error is e= number of division coinciding with main scale (n)$\times$ leas count.

Least count L.C.=$\frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

$= \frac{1}{100}\text{mm} = 0.01\text{mm}$

As zero is not hidden from circular scale when A and B touch each other, the screw gauge has positive error.

$e = +n(L.C.) = 32 \times 0.01 = 0.32\text{mm}$.

Therefore, Linear scale reading=$4 \times (1\text{mm}) = 4\text{mm}$

Circular scale reading=$16 \times (0.01\text{mm}) = 0.16\text{mm}$

$\therefore$Measured reading=$(4 + 0.16)\text{mm} = 4.16\text{mm}$

$\therefore$Absolute reading=Measures reading-e

=$(4.16 - 0.32)\text{mm} = 3.84\text{mm}$

Thickness of the glass plate is 3.84 mm.

Example 2: The smallest division of the main scale of a Vernier calipers is 1 mm and 10 Vernier divisions coincide with 9 main scale divisions. While measuring the length of a line, the zero mark Vernier scale lies between 10.2 cm and 10.3 cm and the third division of Vernier scale coincides with the main scale division.

(a) Determine the least count of the caliper.

(b) Find the length of the line.

Sol: The smallest count of caliper is L. C.

$= \frac{\text{Samllest division on main scale}}{\text{Number of divisions on vernier scale}}$.

(a) Least count (L.C)

$=\frac{\text{Samllest division on main scale}}{\text{Number of divisions on vernier scale}}$

$= \frac{1}{10}\text{mm} = 0.1\text{mm} = 0.01\text{cm}$

(b) $L = N + n(L.C.) = (10.2 + 3 \times 0.01)\text{cm} = 10.23\text{cm}$

Example 3: The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. In measuring the diameter of a sphere, 6 divisions on the linear scale and 40 divisions on circular scale coincide with the reference line. Find the diameter of the sphere.

Sol: The 6th division of main scale coincides with the 40th division of circular scale, the diameter of sphere is obtained as L.S.D + C.S.D.

$L.C. = \frac{1}{100} = 0.01\text{mm}$

Linear scale reading=6 (pitch)=6mm

Circular scale reading=n $(L.C.) = 40 \times 0.01 = 0.4\text{mm}$

$\therefore$ Total reading =$(6 + 0.4) = 6.4\text{mm}$

Example 4: Least count of Vernier calipers is 0.01 cm. When the two jaws of the instrument touch each other, the 5th division of the Vernier scale coincides with a main scale division and the zero of the scale lies to the left of the zero of the main scale. Furthermore while measuring the diameter of a sphere, the zero mark of the Vernier scale lies between 2.4 cm and 2.5 cm and the 6th Vernier division coincides with a main scale division. Calculate the diameter of the sphere.

Sol: As the instrument is noted to have negative error, the measured diameter will be less then original. Thus it has to be added to measured length to get otiginal length of diameter.

The instrument has a negative error,

$e = (-5 \times 0.01)\text{cm} = -0.05\text{cm}$

Measured reading $= (2.4 + 6 \times 0.01) = 2.46\text{cm}$ True reading = Measured reading

$= 2.46 - (-0.05) = 2.51\text{cm}$

Therefore, diameter of the sphere is 2.51 cm.

Example 5: The pitch of a screw gauge is 1 mm and there are 100 divisions on its circular scale. When nothing is put in between its jaws, the zero of the circular scale lies 6 divisions below the reference line. When a wire is placed between the jaws, 2 linear scale divisions are clearly visible while 62 divisions on circular scale coincide with the reference line. Determine the diameter of the wire.

Sol: As zero mark of circular gauge lies 6 division below the main reference line, the gauge is noted to have positive error. Positive error e = n × Least count. This error has to be subtracted from the measured reading.

$$\text{L.C.} = \frac{P}{N} = \frac{1\text{mm}}{100} = 0.01\text{mm}$$

The instrument has a positive zero error,

$e = +n(\text{L.C.}) = +(6 \times 0.01) = +0.06\text{mm}$

Linear scale reading $= 2 \times (1\text{mm}) = 2\text{mm}$

Circular scale reading $= 62 \times (0.01\text{mm}) = 0.62\text{mm}$

Measured reading $= 2 + 0.62 = 2.62\text{mm}$

True reading $= 2.62 - 0.06 = 2.56\text{mm}$

JEE Main/Boards

Exercise 1

Q.1 The force acting on an object of mass m, travelling at velocity V in a circle of radius r is given by $F = \frac{mv^2}{r}$

The measurements are recorded as

$m = (3.5 \pm 0.1)$ kg; $v = (20 \pm 1)\ \text{ms}^{-1}$ and

$r = (12.5 \pm 0.5)$ m.

Find the maximum possible (i) relative error and (ii) percentage error in the measurement of force.

Q.2 The side of a cube is measured as

(75 ± 0.1) cm. Find the volume of the cube.

Q.3 In the formula $g = \frac{4\pi^2 L}{T^2}$, (ℓ) has 2% uncertainty and (T) has 5% uncertainty. What is the maximum uncertainty in the value of g?

Q.4 The length and breadth of a rectangular field are measured as length, $\ell = (250 \pm 5)$ m; breadth $b = (150 \pm 4)$ m. What is the area of the field?

Q.5 The initial temperature of a body is (15 ± 0.5)°C and the final temperature is (17 ± 0.3)°C. What is the rise in temperature of the body?

Q.6 The error in measurement of radius of a sphere is 0.4%. What is the permissible error in the measurement of its surface area?

Q.7 5.74 gm of a substance occupies 1.2cc. Find the density of the substance to correct significant figures.

Q.8 The diameter of a circle is 1.06 m. Calculate its area with regard to significant figures.

Q.9 A substance of mass 5.74 g, occupies a volume of 1.2 cm^3. Find its density with due regard to significant figures.

Q.10 If $m_1 = 1.2\text{kg}$ and $m_2 = 5.42\text{gm}$. Find $(m_1 + m_2)$ with due regard to significant figures.

Q.11 Assuming force (F), length (L) and time (T) as fundamental units, what should be the dimensions of mass?

Q.12 The velocity (v) of a particle depends on time (t) according to the relation: $v = At^2 + Bt + C$ where V is in m/s and t is in s. Write the units and dimensions of constants A, B and C.

Q.13 A calorie is a unit of heat or energy and it equals about 4.2 J, where J = 1 kg $\text{m}^2\ \text{s}^{-2}$. Suppose we employ a system of units in which the unit of mass equals (α)

kilogram, the unit of length equals (β) meter and unit of time (γ) seconds. Show that a calorie has a magnitude $4.2\,\alpha^{-1}\beta^{-2}\gamma^{2}$ in terms of new units.

Q.14 The centripetal force (F) acting on a particle moving in the circumference of a circle depends upon its mass (m), linear velocity (v) and radius (r) of the circle. Use method of dimensions to find the expression for centripetal force.

Q.15 Show by method of dimensions:

(i) Joule = 10^7 Erg (ii) 10^5 dyne/cm^2 = 10^4 N/m^2

Q.16 The latent head of ice is 80 cal/ gm. Express it in J/kg.

Q.17 A satellite is revolving around the earth in a circular orbit. The period of revolution (T) depends on

(i) Mass of earth (M)

(ii) Radius of orbit (r) and

(iii) Gravitational constant (G)

Use the method of dimensions to prove that $T \propto \sqrt{\left(\frac{r^3}{GM}\right)}$

Q.18 The pressure (P), volume (V) and temperature (T) of a real gas are related through Van der Waals equation:

$$\left(P+\frac{q}{V^2}\right)(V-b)=RT$$

Find the dimensions of constants a and b and also write the units of a and b in the SI system.

Q.19 If the dimensions of length are expressed as $\left[G^xC^yh^z\right]$ where G, C and h are universal gravitational constant, speed of light in vacuum and Plank's constant respectively, then what are the values x, y and z?

Q.20 Laplace corrected Newton's calculation for the velocity of sound. Laplace said that speed of sound in a solid medium depends upon the coefficient of elasticity of the medium under adiabatic conditions (E) and the density of the medium (ρ).

Prove that $v=k\sqrt{\frac{E}{\rho}}$

Q.21 The coefficient of viscosity (η) of a liquid by the method of flow through a capillary tube is given by the formula

$$\eta=\frac{\pi R^4 P}{8\,\ell\,Q}$$

Where R= radius of the capillary tube,

ℓ =length of the tube, P= pressure difference between its ends, and Q=volume of liquid flowing per second.

Which measurement needs to be made most accurately and why?

Q.22 Consider a planet of mass (m), revolving round the sun. The time period (T) of revolution of the planet depends upon the radius of the orbit (r), mass of the sun (M) and the gravitational constant (G). Using dimensional analysis, verify Kepler's third law of planetary motion.

Exercise 2

Single Correct Choice Type

Q.1 The dimensional formula for Planck's constant is

(A) $\left[ML^2T^{-1}\right]$ (B) $\left[ML^2T^3\right]$ (C) $\left[ML^{-1}T^{-2}\right]$ (D) $\left[MLT^{-2}\right]$

Q.2 Turpentine oil is flowing through a tube of length ℓ and radius r. The pressure difference between the two ends of the tube is P; the viscosity of the oil is given by $\eta=\frac{\rho\left(r^2-x^2\right)}{4v\ell}$ where v is the velocity of oil at a distance x from the axis of the tube. From this relation, the dimensions of viscosity η are

(A) $\left[M^0L^0T^0\right]$ (B) $\left[MLT^{-1}\right]$ (C) $\left[ML^2T^{-2}\right]$ (D) $\left[ML^{-1}T^{-1}\right]$

Q.3 The time dependence of a physical quantity is given by $P=P_0\exp\left(-\alpha t^2\right)$ [Where α is a constant and t is time]. The constant α

(A) Is dimensionless (B) Has dimensions [T^{-2}]

(C) Has dimensions [T^2] (D) Has dimensions of P

Q.4 which of the following quantities can be written in SI units in kg m^2 A^{-2} s^{-3}

(A) Resistance (B) Inductance

(C) Capacitance (D) Magnetic flux

Q.5 If L and R denote inductance and resistance respectively, then the dimensions of L/R is

(A) $\left[M^0L^0T^0\right]$ (B) $\left[M^0L^0T\right]$ (C) $\left[M^2L^0T^2\right]$ (D) $\left[MLT^2\right]$

Q.6 The dimensions of $\left(\frac{1}{2}\right)\in E^2$

($\in_0$: permittivity of free space; E: electric field) is

(A) MLT^{-1} (B) ML^2T^{-2} (C) $ML^{-1}T^{-2}$ (D) ML^2T^{-1}

Q.7 Which of the following measurements is most accurate?

(A) 0.005 mm (B) 5.00 mm

(C) 50.00 mm (D) 5.0 mm

Q.8 When 97.52 is divided by 2.54, the correct result is

(A) 38.3937 (B) 38.394

(C) 38.39 (D) 38.4

Q.9 The density of a cube is measured by measuring its mass and the length of its sides. If the maximum error in the measurement of mass and length are 3% and 2% respectively, then the maximum error in the measurement of density is

(A) 9% (B) 7% (C) 5% (D) 1%

Q.10 A physical quantity is represented by $X=M^a L^bT^{-c}$. If percentage error in the measurement of M, L and T are α%, β% and γ% respectively, then total percentage error is

(A) $(\alpha a-\beta b+\gamma c)\%$ (B) $(\alpha a+\beta b+\gamma c)\%$

(C) $(\alpha a-\beta b-\gamma c)\%$ (D) none of the above

Q.11 The volume of a sphere is 1.76 cm^3. The volume of 25 such spheres taking into account the significant figures is

(A) $0.44\times10^2\ cm^3$ (B) $44.0\ cm^3$

(C) $44\,cm^3$ (D) $44.00 cm^3$

Q.12 The measurement of radius of a sphere is $(4.22\pm2\%)$ cm. The percentage error in volume of the sphere is

(A) $(315\pm6\%)$ (B) $(315\pm2\%)$

C) $(315\pm4\%)$ (D) $(315\pm5\%)$

Q.13 In the measurement of n from the formula $n=\frac{2Wg\ell}{\pi r^4\theta}$, the quantity which should be measured with the best care is

(A) W (B) ℓ (C) r (D) θ

Q.14 When the number 6.03587 is rounded off to the second place of decimals, it becomes

(A) 6.035 (B) 6.04 (C) 6.03 (D) None

Q.15 If the velocity (V) acceleration (A) and force (F) are taken as fundamental quantities instead of mass (M), length (L) and time (T), the dimension of Young's modulus would be

(A) $FA^2\ V^{-2}$ (B) $FA^2\ V^{-3}$ (C) $FA^2\ V^{-4}$ (D) $FA^2\ V^{-5}$

Q.16 The number of particles crossing per unit area perpendicular to x- axis in unit time is $N=-D\frac{n_2-n_1}{x_2-x_1}$ where n_1 and n_2 are number of particles per unit volume for x_1 and x_2 respectively. The dimensions of diffusion constant D are

(A) $\left[ML^0T^2\right]$ (B) $\left[M^0L^2T^{-4}\right]$ (C) $\left[M^0LT^{-3}\right]$ (D) $\left[M^0L^2T^{-1}\right]$

Q17 If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be

(A) FT^2 (B) $F^{-1}\ A^2\ T^{-1}$ (C) $FA^2\ T$ (D) AT^2

Q.18 In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and unit of length is 10m. In this system, one unit of power will correspond to

(A) 16 watts (B) 1/16 watts

(C) 25 watts (D) None of these

Q.19 While measuring acceleration due to gravity by a simple pendulum, a student makes a positive error of 1% in the length of a pendulum and a negative error of 3% in the time period. His actual percentage error in the measurement of the value of g will be:

(A) 2% (B) 4% (C) 7% (D) 10%

Q.20 A body is moving from height x=0.1 m to x=1.2 in 1 sec under constant acceleration of $0.5m/s^2$. What was the initial velocity with which it started? (Correct to significant digits)

(A) 0.85m/s (B) 0.9 m/s (C) 1.0 m/s (D) 0.8 m/s

Q.21 A quantity y is related to another quantity x by the equation $y=kx^a$ where k and a are constant. If percentage error in the measurement of x is p, then that in y depends upon

(A) K and a (B) x and a

(C) p and a (D) p,k and a all

Q.22 Which of the following quantities has smallest number of significant digits?

(A) 0.00145 cm (B) 14.50 cm

(C) 145.00 cm (D) 145.0×10^{-6} cm

Q.23 $\frac{3.06}{1.2}+1.15$ and express the answer in correct significant digits

(A) 3.70 (B) 3.7 (C) 3.75 (D) 3.8

Q.24 Which of the following pairs don't have same dimensions?

(A) Solid angle and vector

(B) Potential energy and torque

(C) (Area $\times$ velocity) and rate of change of volume with time

(D) None of these

Q.25 Which of the following quantities are dimensionless? (Symbols have their usual meaning)

(A) $\frac{I\omega^2}{m\,v\,r}$ (B) $\frac{Gp}{T}$ (C) $\frac{\rho v r}{\eta}$ (D) $\frac{\tau\theta}{I\omega}$

[Useful relation $I=\frac{2}{5}mr^2$, $F= =6\pi\eta rv$]

Q.26 Suppose $A=B^n C^m$, where A has dimensions LT, B has dimensions $L^2\,T^{-1}$, and C has dimensions LT^2. Then the exponents n and m have values:

(A) 2/3; 1/3 (B) 2;3 (C) 4/5; -1/5 (D) 1/5; 3/5

Q.27 A uniform wire of length L and mass M is stretched between two fixed points, keeping a tension F. A sound of frequency μ is aimed on it. Then the maximum vibrational energy is existing in the wire when μ =

(A) $\frac{1}{2}\sqrt{\frac{ML}{F}}$ (B) $\sqrt{\frac{FL}{M}}$ (C) $2\times\sqrt{\frac{FM}{L}}$ (D) $\frac{1}{2}\sqrt{\frac{F}{ML}}$

Q.28 The dimension $ML^{-1}\,T^{-1}$ can correspond to

(A) Moment of a force

(B) Surface tension

(C) Modulus of elasticity

(D) Coefficient of viscosity

Q.29 Which of the following physical quantities represents the dimensional formula $\left[M^1 L^{-2} T^{-2}\right]$

(A) Energy/ area (B) Pressure

(C) Force $\times$ length (D) pressure per unit length

Q.30 In a particular system of unit, if the unit of mass become twice & that of time becomes half, then 8 joules will be written as______ units of work

(A) 16 (B) 1 (C) 4 (D) 64

Q.31 Which of the following is not one of the seven fundamental SI units?

(A) Henry (B) Ampere (C) Candela (D) Mole

Q.32 The dimensional formula for which of the following pairs is not the same

(A) Impulse and momentum

(B) Torque and work

(C) Stress and pressure

(D) Momentum and angular momentum

Q.33 Dimensional formula for coefficient of viscosity (η)[use $F=6\pi\eta r\,v$ (r = radius; v = velocity; F = viscous force]

(A) $ML^{-2}T^{-1}$ (B) $M^{-1}L^1T^{-1}$ (C) $M^1L^1T^{-2}$ (D) $ML^{-1}T^{-1}$

Q34 The time dependence of a physical quantity P is given by $p=p_0\,e^{\left(-\alpha t^2\right)}$ where α is constant and t is time. The constant α

(A) Is dimensionless (B) Has dimensions T^{-2}

(C) Has dimensions T^2 (D) Has dimensions of p

Q.35 From the following pairs of physical quantities, in which group dimensions are not same:

(A) Momentum and impulse

(B) Torque and energy

(C) Energy and work

(D) Light year and minute

Previous Years' Questions

Q.1 In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in MKS system? ***(1995)***

(A) $\left[M^{-3}L^{-1}T^{3}Q^{4}\right]$ (B) $\left[M^{-3}L^{-2}T^{4}Q^{4}\right]$

(C) $\left[M^{-2}L^{-2}T^{4}Q^{4}\right]$ (D) $\left[M^{-3}L^{-2}T^{4}Q\right]$

Q.2 The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0: permittivity of free space; E: electric field) is ***(1996)***

(A) $\left[MLT^{-1}\right]$ (B) $\left[ML^{2}T^{-1}\right]$

(C) $\left[MLT^{-2}\right]$ (D) None of these

Q.3 A quantity X is given by $\varepsilon_0 L\frac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of ***(1994)***

(A) Resistance (B) Charge

(C) Voltage (D) Current

Q.4 A cube has a side of length $1.2 x 10^{-2} m$. Calculate its volume ***(1999)***

(A) $1.7x10^{-6}m^3$ (B) $1.73x10^{-6}m^3$

(C) $1.70x10^{-6}m^3$ (D) $1.732x10^{-6}m^3$

Q.5 In the relation $\rho = \frac{\alpha}{\beta}e^{-\frac{a}{10}}$ ρ is pressure, Z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be ***(2007)***

(A) $\left[M^{0}L^{2}T^{0}\right]$ (B) $\left[ML^{2}T\right]$

(C) $\left[ML^{0}T^{-1}\right]$ (D) $\left[M^{0}L^{-2}T^{-1}\right]$

Q.6 A wire has mass ΔT radius (0.5 ± 0.005)mm and length (6 ± 0.06)cm. The maximum percentage error in the measurement of its density is ***(2005)***

(A) 1 (B) 2 (C) 3 (D) 4

Q.7 Which of the following sets have different dimensions? ***(2005)***

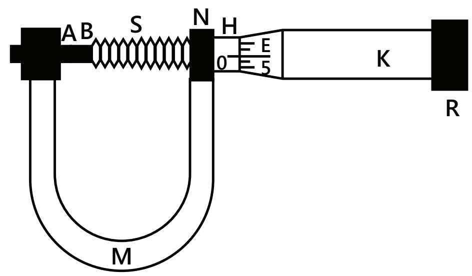
(A) Pressure, Young's modulus, Stress

(B) Emf, potential difference, Electric potential

(C) Heat, Work done, Energy

(D) Dipole moment, Electric flux, Electric field

Q.8 The circular scale of a screw gauge has 50 divisions and pitch of 0.5mm. Find the diameter of sphere. Main scale reading is 2. ***(2006)***



(A) 1.2 mm (B) 1.25 mm

(C) 2.20 mm (D) 2.25 mm

Q.9 A student performs an experiment to determine the Young's modulus of a wire, exactly 2m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8mm with an uncertainly of ± 0.05mm at a load of exactly 1.0kg. The student also measures the diameter of the wire to be 0.4mm with an uncertainly of ± 0.01mm. Take $g = 9.8 m/s^2$ (exact). The young's modulus obtained from the reading is close to ***(2007)***

(A) $(2.0 \pm 0.3)10^{11} N/m^2$ (B) $(2.0 \pm 0.2)10^{11} N/m^2$

(C) $(2.0 \pm 0.1)10^{11} N/m^2$ (D) $(2.0 \pm 0.5)10^{11} N/m^2$

Q.10 In the experiment to determine the speed of sound using a resonance column ***(2007)***

(A) Prongs of the tuning fork are kept in a vertical plane

(B) Prongs of the tuning fork are kept in a horizontal plane

(C) In one of the two resonances observed, the length of the resonating air column in close to the wavelength of sound in air.

(D) In one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air.

Q.11 Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum.

They use different lengths of the pendulum and / or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1cm, Least count for time = 0.1s

Student	Length of pendulum (cm)	Number of oscillations (n)	Total time for (n) Oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors is g.i.e., $\left(\frac{\Delta g}{g} x100\right)$ for students I, II and III respectively ***(2008)***

(A) $E_I = 0$ (B) E_I is minimum

(C) $E_I = E_{II}$ (D) E_{II} is maximum

Q.12 A Vernier callipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For this vernier callipers, the least count is ***(2010)***

(A) 0.02 mm (B) 0.05 mm

(C) 0.1 mm (D) 0.2 mm

Q.13 The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2% the relative percentage error in the density is ***(2011)***

(A) 0.9% (B) 2.4% (C) 3.1% (D) 4.2%

Q.14 Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is ***(2012)***

(A) 6% (B) zero (C) 1% (D) 3%

Q.15 A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading: 58.5 degree

Vernier scale reading: 09 divisions

Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data ***(2012)***

(A) 58.59° (B) 58.77° (C) 58.65° (D) 59°

Q.16 Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then: ***(2013)***

(A) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

(B) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^{-2}]$

(B) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A]$

(D) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$

Q.17 A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it? ***(2014)***

(1) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.

(2) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

(3) A meter scale.

(4) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.

Q.18 A student measures the time period of 100 oscillations of a simple pendulum four times. That data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be: ***(2016)***

(A) 92 ± 5.0 s (B) 92 ± 1.8 s

(C) 92 ± 3 s (D) 92 ± 2 s

Q.19 A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45^{th} division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5mm and the 25^{th} division coincides with the main scale line? ***(2016)***

(A) 0.80 mm (B) 0.70 mm

(C) 0.50 mm (D) 075 mm

JEE Advanced/Boards

Exercise 1

Q.1 A research worker takes 100 careful readings in an experiment. If he repeats the same experiment by taking 400 readings, then by what factor will be the probable error be decreased?

Q.2 The length, breadth and thickness of a rectangular sheet of metal are 4.234m, 1.005m and 2.01 cm respectively. Find the area and volume of the sheet to correct significant figures.

Q.3 The intensity of X- rays decreases exponentially according to the law $I = i\,e^{-\mu x}$, where i is the initial intensity of X-rays and I is the intensity after it penetrates a distance X through lead. If μ be the absorption coefficient, then find the dimensional formula for μ.

Q.4 Two resistors have resistance $R_1 = (24 \pm 0.5)\Omega$ and $R_2 = (8 \pm 0.3)\Omega$. Calculate the absolute error and the percentage relative error in calculating the combination of two resistors when they are in (a) Series (b) Parallel

Q.5 In an electrical set up, the following readings are obtained.

Voltmeter reading (V)=6.4V

Ammeter reading (I) = 2.0A

The respective least counts of the instruments used in these measurements are 0.2V and 0.1A. Calculate the value of resistance of the wire with maximum permissible absolute error and relative percentage error.

Q.6 The radius of a proton is 10^{-9} micron and that of universe is 10^{27}m. Identify an object whose size lies approximately midway between these two extremes on the logarithmic scale.

Q.7 If the velocity of light (c), gravitational constant (G) and the plank's constant (h) are selected as the fundamental units, find the dimensional formulae for mass, length and time in this new system of units.

Q.8 The critical velocity (V_c) of flow of a liquid through a pipe depends upon the diameter (d) of the pipe, density (ρ), and the coefficient of viscosity (η) of the liquid. Obtain an expression for the critical velocity.

Q.9 The mass m of the heaviest stone that can be moved by the water flowing in a river varies with the speed of water (V), density of water (d) and the acceleration due to gravity. Prove that the heaviest mass moved is proportional to the sixth power of speed. Also find the complete dependence.

Q.10 The frequency (f) of a stretched string of linear mass density (m), length (ℓ) depends (in addition to quantities specified before) on the force of stretching (F). Prove that $f = \frac{k}{\ell}\sqrt{\frac{F}{m}}$ where k is a dimensionless constant.

Q.11 Find out the maximum percentage error while the following observations were taken in the determination of the value of acceleration of the value of acceleration due to gravity. Length of thread=100.2cm; radius of bob=2.34cm; Time of one oscillation=2.3s. Calculate the value of maximum percentage error up to the required significant figures. Which quantity will be measured more be measured more accurately?

Q.12 Determine the focal length of the lens from the following readings:

Object distance, $u = 20.1 \pm 0.2\text{cm}$;

Image distance, $v = 50.1 \pm 0.5\text{cm}$.

Q.13 The specific gravity of the material of a body is determined by weighing the body first in air and then in water. If the weight in air is 10.0 ± 0.1 gw and weight in water is $50. \pm 0.1$ gw, then what is the maximum possible percentage error in the specific gravity?

Q.14 The following observation were actually made during an experiment to find the radius of curvature of a concave mirror using a spherometre: ℓ = 4.4 cm; h=0.085 cm. The distance ℓ between the legs of the spherometre was measured with a metre rod and the least count of the spherometre was 0.001 cm. Calculate the maximum possible error in the radius of curvature.

Q.15 It has been observed that the rate of flow (V) of a liquid of viscosity η through a capillary tube of radius (r) depends upon η, r and the pressure gradient P maintained across the length (ℓ) of the tube. Assuming a power law dependence, prove that the rate of flow of liquid is proportional to r^4. Also find the exact expression up to a constant.

Q.16 The height h to which a liquid rises in a tube of radius (r) depends upon the density of the liquid (d), surface tension (T), and acceleration due to gravity (g). Show that it would not be possible to derive the relation without the additional information that h is inversely proportional to r. Also find the relation.

Q.17 The viscosity η of gas depends upon its mass m, the effective diameter D and the mean speed v of the molecules present in the gas. Assuming a power law, find dependence of η on all these quantities.

Q.18 The distance moved by a particle in time from the center of a ring under the influence of its gravity is given by x=a sin ωt where a and ω are constant. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitation constant (G), find using dimensional analysis an expression for ω in terms of r, m and G.

Q.19 The centripetal force is given by $F=\frac{mv}{r}$. The mass, velocity and radius of the circular path of an object are 0.5kg, 10m/s and 0.4 m respectively. Find the percentage error in the force. Given: m,v and are measured to accuracies of 0.005 kg, 0.01m/s and 0.01 m respectively.

Q.20 An experiment to determine the specific resistance ρ of a metal wire provided the following observations.

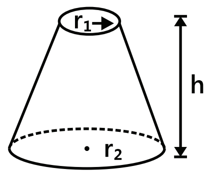
Resistance of $R=(64\pm 2)$ ohm; Length $\ell=(156\pm 0.1)$ cm; Radius $r=(0.26\pm 0.02)$ cm

If s is expressed as: $\rho=\frac{\pi r^2 R}{\ell}$ Find the percentage error in ρ.

Q.21 The consumption of natural gas by a company satisfied the empirical equation $V=1.50\ t+0.008t^2$, where 'V' is the volume in millions of cubic metre and 't' is the time in months. Expressed this equation in units of cubic metre and seconds, put the proper units on the coefficients. Assume a month is of 30 days.

Q.22 As part of their introduction of the metric system the national convention made an attempt to introduce decimal time. In this plan, which was not successful, the day-starts at midnight into 10 decimal hours consisting of 100 decimal minutes each. The hands of a surviving decimal pocket watch are stopped at 8 decimal hours, 22.8 decimal minutes. What time is it representing in the usual system?

Q.23 Figure shows a frustum of a cone



Match the following dimensionally:

(a) Total circumference of the flat circular faces	(i) $\pi(r_1+r_2)\left[h^2+(r_1-r_2)^2\right]^{1/2}$
(b) Volume	(ii) $2\pi(r_1+r_2)$
(c) Area of the curved surface	(iii) $\pi h\left(r_1^2+r_1r_2+r_2^2\right)$

Q.24 Suppose that a man defines a unit of force as that which acts due to gravitation between two point masses each of 1 kg and 1 m apart. What would be the value of 'G' in this new system? What would be the value of one newton in this new system?

Given: G (in SI unit system) $=6.6\times 10^{-11}$.

$\left[\text{Use}\left(F=\frac{Gm_1m_2}{r^2}\right)\right]$

Q.25 The distance between neighbouring atoms or molecules, in a solid substance can be estimated by calculating twice the radius of a sphere with volume equal to the volume per atoms of the material. Calculate the distance between neighboring atoms in the following: (a) iron (b) sodium

Given: The densities of iron and sodium are 7870 kg/m^3 and 1013 kg/m^3 respectively, the mass of an iron atom is 9.27×10^{-26} kg and the mass of sodium atom is 3.82 $\times 10^{-26}$ kg.

Q.26 If force 'F' and density'd' are related as $F=\frac{\alpha}{\beta+\sqrt{d}}$, then find out the dimensions of α & β.

Q.27 If the velocity of light 'c' Gravitational constant 'G' & Plank's constant 'h' be chosen as fundamental units, find the dimensions of mass, length & time in this new system.

Q.28 In the formula; $\rho = \frac{nRT}{v-b}e^{-\frac{a}{RTV}}$, find the dimensions of 'a' and 'b' where P=pressure, n=no. of moles, T=temperature, V=volume and R=universal gas constant.

Q.29 A ball thrown horizontally from a height 'H' with speed 'v' travels a total horizontal distance 'R'. From dimensional analysis, find a possible dependence of 'R' on H, v and g. It is known that 'R' is directly proportional to 'v'.

Exercise 2

Single Correct Choice Type

Q.1 The percentage error in measurement of a physical quantity m given by $m = \pi \tan\theta$ is minimum when

(A) $\theta = 45^0$ (B) $\theta = 90^0$

(C) $\theta = 60^0$ (D) $\theta = 30^0$

(Assumed that error in θ remain constant)

Q.2 A vernier calliper having 1 main scale division =0.1 cm is designed to have a least count of 0.02cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then

(A) n=10, m=0.5cm (B) n=9, m=0.4 cm

(C) n=10, m=0.8 cm (D) n=10, m=0.2cm

Q.3 In a vernier caliper, N divisions of vernier scale coincides with N-1 divisions of main scale (in which length of one division is 1 mm). The least count of the instrument should be

(A) N mm (B) N-1 mm

(C) $\frac{1}{10\ N}$ cm (D) $\frac{1}{N-1}$ mm

Q.4 Choose the option whose pair doesn't have same dimensions.

(A) (Pressure $\times$ volume) & Work done

(B) (Force $\times$ Time) & Change in momentum

(C) Kilocalorie & joule

(D) Angle & no. of moles

Multiple Correct Choice Type

Q.5 If dimension of length are expressed as G^x, c^y, h^z where G, c and h are the universal gravitational constant, speed of light and plank's constant respectively, then:

(A) x=(1/2),y=(1/2) (B) x=(1/2),z=(1/2)

(C) y=(-3/2), z=(1/2) (D) y=(1/2), z=(3/2)

Q.6 Which of the following groups have the same dimensions?

(A) Velocity, speed (B) Pressure, stress

(C) Force, impulse (D) Work, energy

Comprehension Type

Paragraph 1: The van-der Waals equation is $\left(P+\frac{a}{V^2}\right)(V-b)=RT$, where P is pressure, V is molar volume and T is the temperature of the given sample of gas. R is called molar gas constant, a and b are called Vander walls constants.

Q.7 The dimensional formula for b is same as that for

(A) P (B) V (C) PV^2 (D) RT

Q.8 The dimensional formula for a is same as that for

(A) V^2 (B) P (C) PV^2 (D) RT

Q.9 Which of the following does not possess the same dimensional formula as that for RT

(A) PV (B) Pb (C) a/V^2 (D) ab/V^2

Q.10 The dimensional formula for ab/RT is

(A) ML^5T^{-2} (B) $M^0L^3T^0$ (C) $ML^{-1}T^{-2}$ (D) $M^0L^6T^0$

Q.11 The dimensional formula of RT is same as that of

(A) Energy (B) Force

(C) Specific heat (D) Latent heat

Paragraph 2: The power of a hovering helicopter depends on its linear size, the density of air and (g$\times$ density of the helicopter) as power $\propto$ (linear size)^x (density of air)^y (g$\times$ density of helicopter)^z where g is acceleration due to gravity. Given: [power] $=ML^2T^{-3}$

[Linear Size]=L; [Density]= ML^{-3}

[g$\times$ density]= $ML^{-2}T^{-2}$

Q.12 The value of y in above expression is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{7}{2}$

Q.13 The value of x in the above expression is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{7}{2}$

Q.14 The ratio of power output of engine of two helicopters when linear size of one helicopter is one fourth of linear size of other and all other parameters remaining same is

(A) 132 (B) 16 (C) 128 (D) 4

Match the Columns

Q.15 Match the physical quantities in column I with their dimensional formulae expressed in column II.

Column I		Column II	
(A)	Angular momentum	(p)	ML^2T^{-2}
(B)	Latent heat	(q)	$ML^2T^{-2}A^{-2}$
(C)	Torque	(r)	ML^2T^{-1}
(D)	Capacitance	(s)	$ML^3T^{-3}A^{-2}$
(E)	Inductance	(t)	$M^{-1}L^{-2}T^4A^2$
(F)	Resistivity	(u)	$ML^2T^{-2}A^{-1}$
(G)	Magnetic Flux	(v)	$ML^{-1}T^{-2}$
(H)	Magnetic energy density	(w)	L^2T^{-2}

Q.16 Entries in column I are representing physical quantities whereas entries in column II are representing dimensions. Match the columns.

Column I		Column II	
(A)	Angle	(p)	$M^1L^2T^{-3}$
(B)	Power	(q)	$M^0L^0T^0$
(C)	Work	(r)	$M^1L^2T^{-2}$
(D)	Unit vector	(s)	$M^1L^1T^{-2}$

Q.17 Considering force (F), velocity (V) and Energy € as fundamental quantities, match the correct dimensions of following quantities.

Column I		Column II	
(A)	Mass	(p)	$[F^1V^0E^1]$
(B)	Light year	(q)	$[F^1V^1E^{-1}]$
(C)	Frequency $\left(\frac{1}{T}\right)$	(r)	$[F^3V^0E^{-2}]$
(D)	Pressure	(s)	$[F^0V^{-2}E^1]$

Previous Years' Questions

Paragraph 1: A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency $\omega_{p'}$, which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together, and all the energy is reflected. This is the explanation of high reflectivity of metals ***(2011)***

Q.1 Taking the electronic charge as ℓ and the permittivity as ε_0, use dimensional analysis to determine the correct expression for ω_p.

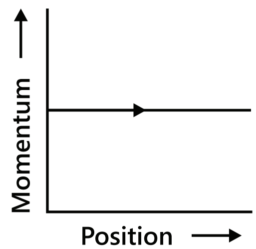
(A) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\frac{m\varepsilon_0}{Ne}}$ (C) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$

Q.2 Estimate the wavelength at which plasma reflection will occur for a metal having the density of electronics $N = 4 \times 10^{27}\ m^{-3}$. Take $\varepsilon_0 = 10^{-11}$ and $m = 10^{-30}$, where these quantities are in proper SI units.

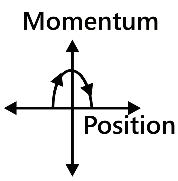
(A) 800 mm (B) 600 mm

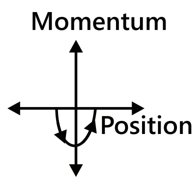
(C) 300 mm (D) 200 mm

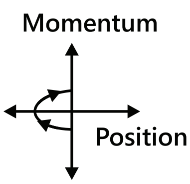
Paragraph 2: Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative. ***(2011)***

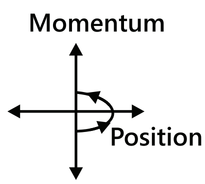


Q.3 The phase space diagram for a ball thrown vertically up from ground is

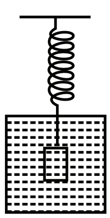
(A)


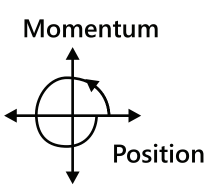
(B)


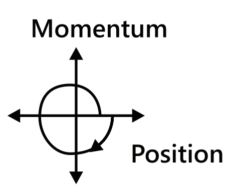
(C)


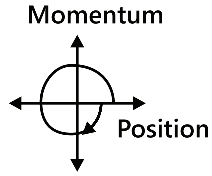
(D)


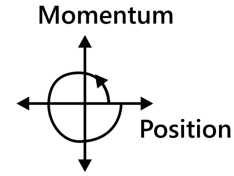
Q.4 Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(A)


(B)


(C)


(D)


Match the Columns

Q.5 Column I gives three physical quantities. Select the appropriate units for the choices given in column II. Some of the physical quantities may have more than one choice. ***(1990)***

Column I	Column II
(A) Capacitance	(p) Ohm-second
(B) Inductance	(q) Coulomb^2-Joule $^{-1}$
(C) Magnetic Induction	(r) Coulomb (volt)$^{-1}$
	(s) Newton (ampere metre)$^{-1}$
	(t) Volt-second (ampere)$^{-1}$

Q.6 Match the physical quantities given in column I with dimensions expressed in terms of mass (M), length (L), time (T), and charge (Q) given in column II and write the correct answer against the match quantity in a tabular form in your answer book ***(1993)***

Column I	Column II
(A) Angular momentum	(p) $\left[ML^2T^{-2}\right]$
(B) Latent heat	(q) $\left[ML^2Q^{-2}\right]$
(C) Torque	(r) $\left[ML^2T^{-1}\right]$
(D) Capacitance	(s) $\left[ML^3T^{-1}Q^2\right]$
(E) Inductance	(t) $\left[M^{-1}L^{-2}T^2Q^2\right]$
(F) Resistivity	(u) $\left[L^2T^{-2}\right]$

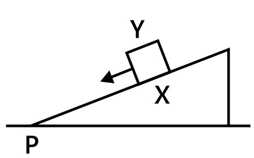
Q.7 Some physical quantities are given in column I and some possible SI units in which these quantities may be expressed are given in column II. Match the physical quantities in column I with the units in column II. ***(2007)***

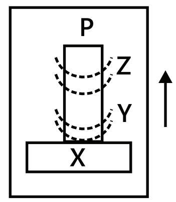
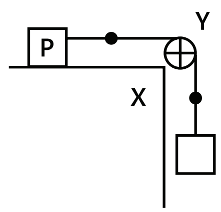
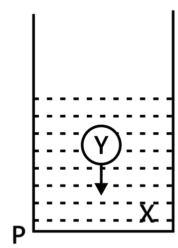
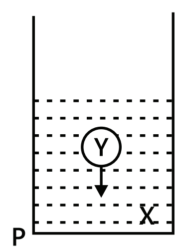
Column I		Column II	
(A)	GM_eM_s G - Universal gravitational constant, M_e - Mass of the earth, M_s - Mass of the sun.	(p)	(volt) (coulomb) (metre)
(B)	$\frac{3RT}{M}$ R – Universal gas constant, T – Absolute temperature, M- Molar mass.	(q)	(kilogram) $(metre)^3$ $(second)^{-2}$
(C)	$\frac{F^2}{q^2B^2}$ F – Force, q- Charge, B – Magnetic field.	(r)	$(metre)^2$ $(second)^{-2}$
(D)	$\frac{GM_e}{R_e}$ G – Universal gravitational constant, M_e - Mass of the earth, R_e - Radius of the earth.	(s)	(farad) $(volt)^2$ $(kg)^{-1}$

Q.8 Column II gives certain systems undergoing a process. Column II suggests changes in some of the parameters related to the system. Match the statements in column I to the appropriate process (es) from column II. ***(2009)***

Column I		Column II	
(A)	The energy of the system is increased.	(p)	System: A capacitor, initially uncharged. Process: It is connected to a battery.
(B)	Mechanical energy is provided to the system, which is converted into energy of random motion of its parts.	(q)	System: A gas in an adiabatic container fitted with an adiabatic piston.
(C)	Internal energy of the system is converted into mechanical energy	(r)	System: A gas in rigid container. Process: The gas gets cooled due to colder atmosphere surrounding it.
(D)	Mass of the system is decreased	(s)	System: A heavy nucleus fission into two fragments of nearly equal masses and some neutrons are emitted.
		(t)	System: A resistive wire loop. Process: The loop is placed in a time varying magnetic field perpendicular to its plane.

Q.9 Column II shows five systems in which two objects are labelled as x and Y. Also in each case a point P is shown. column I gives some statements about X and / or Y. Match these statements to the appropriate systems (s) from column II. ***(2009)***

Column I		Column II		
(A)	The force exerted by X on Y has a magnitude Mg.	(p)		Block Y of mass M left on a fixed inclined plane X slides on it with a constant velocity.

Column I		Column II		
(B)	The gravitational potential energy of X is continuously increasing.	(q)		Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in lift that is going up with a constant velocity.
(C)	Mechanical energy of the system X+Y is continuously decreasing.	(r)		A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.
(D)	The torque of the weight of Y about point P is zero	(s)		A sphere Y of mass M is put in a non-viscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid
		(t)		A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

Q.10 L, C and R represent the physical quantities inductance, capacitance and resistance respectively. The combinations of which have the dimensions of frequency ***(1984)***

(A) 1/RC (B) R/L (C) $1/\sqrt{LC}$ (D) C/L

Q.11 The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair are the same. ***(1986)***

(A) Torque and work

(B) Angular momentum and work

(C) Energy and Young's modulus

(D) Light year and wavelength

Q.12 The pairs of physical quantities that have the same dimensions is (are) ***(1995)***

(A) Reynolds number and coefficient of friction

(B) Curie and frequency of a light wave

(C) Latent heat and gravitational potential

(D) Plank's constant and torque.

Q.13 Let $\left[\varepsilon_0\right]$ denote the dimensional formula of the permittivity of the vacuum and $\left[\mu_0\right]$ that of the permeability of the vacuum. If M = mass, L=length, T=time and I=electric current. ***(1998)***

(A) $\left[\varepsilon_0\right]=\left[M^{-1}L^{-3}T^{-2}I\right]$ (B) $\left[\varepsilon_0\right]=\left[M^{-1}L^{-3}T^{4}I^{2}\right]$

(C) $\left[\mu_0\right]=\left[MLT^{-2}I^{-2}\right]$ (D) $\left[\mu_0\right]=\left[ML^2T^{-1}I\right]$

Q.14 The SI unit of inductance, Henry can by written as ***(1998)***

(A) Weber / ampere (B) Volt-second / ampere

(C) Joule / (ampere)^2 (D) Ohm-second

Q.15 A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He used a stop watch with the least count of 1s for this and recorded 40s for 20 oscillations. For this observation, which of the following statement (s) is / are true? ***(2004)***

(A) Error ΔT in measuring T, the time period, is 0.05s.

(B) Error ΔT in measuring T, the time period, is 1s.

(C) Percentage error in the determination of g is 5%.

(D) Percentage error in the determination of g is 2.5%.

Q.16 In the determination of Young's modulus $\left(Y=\frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length L = 2m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension ℓ = 0.25 mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement ***(2012)***

(A) Due to the errors in the measurements of d and ℓ are the same.

(B) Due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.

(C) Due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.

(D) Due to the error in the measurement of d is four times that due to the error in the measurement of ℓ.

Q.17 The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is ***(2013)***

(A) 5.112 cm (B) 5.124 cm

(C) 5.136 cm (D) 5.148 cm

Q.18 During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions.

When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is ***(2014)***

Q.19 The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2\ s^{-1}$. The measurement of A has an error of 1.25 %. If the error in the measurement of time is 1.50 %, the percentage error in the value of E(t) at t = 5 s is ***(2015)***

Q.20 Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then: ***(2015)***

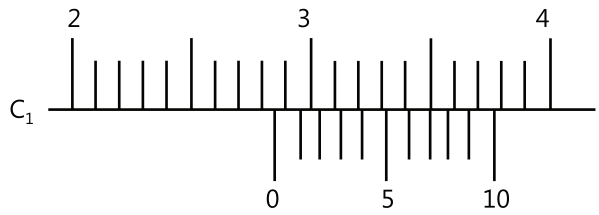
(A) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

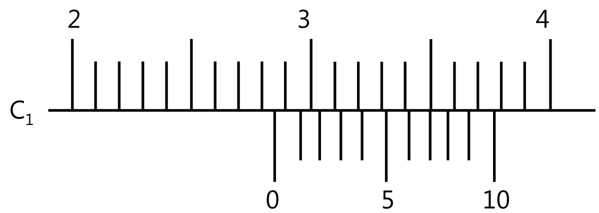
(B) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

(C) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(D) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

Q.21 There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2 respectively, are ***(2016)***





(A) 2.87 and 2.86 (B) 2.87 and 2.87 (C) 2.87 and 2.83 (D) 2.85 and 2.82

Q.22 In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true? ***(2016)***

(A) The error in the measurement of r is 10%

(B) The error in the measurement of T is 3.57%

(C) The error in the measurement of T is 2%

(D) The error in the determined value of g is 11%

Questions

JEE Main/Boards

Exercise 1

Q. 1 Q. 3 Q.13

Exercise 2

Q. 9 Q. 10 Q. 21
Q. 36 Q. 54 Q. 56

JEE Advanced/Boards

Exercise 1

Q. 3 Q. 5 Q. 14
Q. 22

Exercise 2

Q. 1 Q. 3 Q. 21

Answer Key

JEE Main/Boards

Exercise 1

Q.1 0.17,17%

Q.2 $(422+1.7) \times 10^3 cm^3$

Q.3 12%

Q.4 $(375 \pm 0.17) \times 10^4$ m^2

Q.5 $(2 \pm 0.8)^0$ C

Q.6 0.8%

Q.7 4.8gm/cc

Q.8 0.882 m^2

Q.9 4.8g/cm^3

Q.10 1.2 kg

Q.11 $\left[FL^{-1}T^2\right]$

Q.12 m/s^3, ms^2, ms and [LT^{-3}], [LT^{-2}], [LT^{-1}]

Q.13 $\left[4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2\right]$

Q.14 $\left[F = \frac{mv^2}{r}\right]$

Q.16 $\left[3.3\times10^5 J/kg\right]$

Q.18 $\left[ML^5\ T^{-2}\right],\left[L^3\right]$and kg $m^5\ s^{-2}, m^3$

Q.19 $\left[x = \frac{1}{2}, y = -\frac{3}{2}, z = \frac{1}{2}\right]$

Q.21 R

Q.22 $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** D	**Q.3** B	**Q.4** A	**Q.5** B	**Q.6** C
Q.7 C	**Q.8** D	**Q.9** A	**Q.10** B	**Q.11** B	**Q.12** A
Q.13 C	**Q.14** B	**Q.15** C	**Q.16** D	**Q.17** D	**Q.18** A
Q.19 C	**Q.20** B	**Q.21** C	**Q.22** A	**Q.23** D	**Q.24** D
Q.25 C	**Q.26** D	**Q27** D	**Q.28** D	**Q.29** D	**Q.30** B
Q.31 A	**Q.32** D	**Q.33** D	**Q34** B	**Q.35** D	

Previous Years' Questions

Q.1 B	**Q.2** D	**Q.3** D	**Q.4** A	**Q.5** A	**Q.6** D
Q.7 D	**Q.8** A	**Q.9** B	**Q.10** A	**Q.11** B	**Q.12** D
Q.13 C	**Q.14** A	**Q.15** C	**Q.16** A	**Q.17** D	**Q.18** D
Q.19 A					

JEE Advanced/Boards

Exercise 1

Q.1 By a factor of 4

Q.3 L^{-1}

Q.4 $R_S = (32 \pm 0.8)\Omega, R_S = (32\Omega \pm 2.5\%)$;

$R_P = (6 \pm 0.2)\Omega, R_S = (6\Omega \pm 3.33\%)$

Q.5 $R = (3.2 \pm 0.26)\Omega, R = (3.2 \pm 8\%)\Omega$

Q.6 10^6 m, Earth

Q.7 $M = c^{\frac{1}{2}} G^{-\frac{1}{2}} h^{\frac{1}{2}}, L = c^{\frac{-3}{2}} G^{\frac{1}{2}} h^{\frac{1}{2}}, T = c^{\frac{-5}{2}} G^{\frac{1}{2}} h^{\frac{1}{2}}$

Q.8 $V_c = \frac{K\eta}{\rho d}$

Q.9 $m\frac{KV^6 d}{g^3}$

Q.11 9.2%, Time period

Q.12 14.3 ± 0.4 cm

Q.13 3%

Q.14 0.068

Q.15 $V = K\frac{r^4 p}{\eta}$

Q.16 $h = k\left(\frac{T}{rdg}\right)$

Q.17 $\eta = \frac{kmv}{D^2}$

Q.19 3.7%

Q.20 18.6%

Q.21 R

Q.22 7 hour 44 minute 50 second PM

Q.23 (a) (ii); (c) (i)

Q.24 1 newton $= 1.5 \times 10^{10}$ G unit

Q.25 (a) 282 pm (b) 416 pm

Q.26 $M^{3/2} L^{-1/2} T^{-2}, M^{1/2} L^{-3/2} T^0$

Q.27 $\left[h^{1/2}.c^{1/2}.G^{-1/2}\right]$;

$[L] = \left[h^{1/2}.c^{-3/2}.G^{1/2}\right]$;

$[T] = \left[h^{1/2}.c^{-5/2}.G^{1/2}\right]$

Q.28 $[a] = ML^5T^{-2}$, $[b] = L^3$

Q.29 $R = kv\sqrt{\frac{H}{g}}$

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** C
Q.3 C	**Q.4** D

Multiple Correct Choice Type

Q.5 B, C	**Q.6** A, B, D

Comprehension Type

Q.7 B	**Q.8** C	**Q.9** C
Q.10 D	**Q.11** A	**Q.12** B
Q.13 D	**Q.14** C	

Matric Match Type

Q.15 A→r, B→ w, C→p, D→t, E→q, F→s, G→u, H→v

Q.16 A→q, B→p, C→r, D→q

Q.17 A→s, B→p, C→q, D→r

Previous Years' Questions

Q.1 C **Q.2** B **Q.3** D **Q.4** B **Q.7** A→p,q; B→r,s; C→r,s; D→r,s

Q.8 A→p,q,s,t; B→q; C→s; D→s **Q.9** A→p,t; B→q,s,t; C→p,r,t; D→q **Q.10** A, B, C

Q.11 A, D **Q.12** A, D **Q.13** C **Q.14** A, B, D **Q.15** A, C

Q.16 A **Q.17** B **Q.18** 4 **Q.19** 4 **Q.20** B, C

Q.21 C **Q.22** A, B, D

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $F = \frac{mv^2}{r}$

$$\frac{\Delta F}{F} = \left(\frac{\Delta m}{m}\right) + 2\left(\frac{\Delta v}{v}\right) + \left(\frac{\Delta r}{r}\right) \quad \text{... (i)}$$

Generally any measured physical quantity is noted in the form of $(S \pm \Delta S)$. So, here ΔS is the absolute error.

$\therefore$ When we look at $m = 3.5 \pm 0.1$;

$\Delta m = 0.1$ and $m = 3.5$

Now using formula (i)

$$\frac{\Delta F}{F} = \left(\frac{0.1}{3.5}\right) + 2\left(\frac{1}{20}\right) + \left(\frac{0.5}{12.5}\right) = 0.168$$

$$\frac{\Delta F}{F} = 0.17$$

Hence the relative error is 0.17 and the percentage would be $(0.17) \times 100 = 17\%$

Sol 2: Volume of the cube for side of length 'L' is L^3 Cu. Units.

$\therefore\ V = L^3$

$$\left(\frac{\Delta V}{V}\right) = 3 \cdot \left(\frac{\Delta L}{L}\right) \quad \text{... (i)}$$

Here we have to write the volume in standard form i.e. $V + \Delta V$

$V = (75)^2\ cm^3$

$V = 421875\,cm^3$

$\Rightarrow\ V = 422000\,cm^3$ ... (ii)

Now $\frac{\Delta V}{V} = 3.\left(\frac{\Delta L}{L}\right).V$

$$\Delta V = 3\left(\frac{0.1}{75} \times 421875\right)$$

$\Delta V = 1687.5$

$\Rightarrow\ \Delta V = 1700$ ... (iii)

$\therefore$ Volume of the cube = $(422+1.7) \times 10^3 cm^3$

Sol 3: $g = \frac{4\pi^2\ell}{T^2}$

$$\left(\frac{\Delta g}{g}\right) = \left(\frac{\Delta L}{L}\right) + 2\left(\frac{\Delta T}{T}\right)$$

Now what we mean by 2% uncertainty in ℓ is $\left(\frac{\Delta L}{L}\right) \times 100$.

Accordingly;

$$\left(\frac{\Delta g}{g}\right) \times 100 = \left(\frac{\Delta L}{L}\right) \times 100 + 2\left(\frac{\Delta T}{T}\right) \times 100$$

$= 2\% + 2 \times 5\% = 12\%$

Sol 4: $\ell = (250 \pm 5)m$ and $b = (150 \pm 4)m$

Area of the rectangle = $\ell \times b$ sq.units

$\therefore\ A = Lb$ ... (i)

$$\frac{\Delta A}{A} = \left(\frac{\Delta L}{L}\right) + \left(\frac{\Delta b}{b}\right) \quad \text{... (ii)}$$

Now let us first find the area,

$A = 250 \times 150 m^2$

$A = 375 \times 10^2\ m^2$

Now $\frac{\Delta A}{A} = \left(\frac{5}{250}\right) + \left(\frac{4}{150}\right)$

$\frac{\Delta A}{A} = 0.046$

$\Delta A = 1750\ m^2$

$\therefore$ Area $= (375 \times 10^2 + 1750) m^2$

Area $= (375 + 0.17) \times 10^4 m^2$

Sol 5: $T_{initial} = (15 \pm 0.5)°C$

$T_{final} = (17 \pm 0.3)°C$

Rise in temperature = = (2 ± 0.8).

Wondering why it's not 0.2.....99

There we not the error value such that in the given reading will not fluctuate above the error value. So let us say $T_1 = (15 - 0.5)°C$ and $T_f = (17 + 0.3)°C$ then in this case we get $\Delta T = (2 \pm 0.8)°C$!

Now by no means we can get ΔT more then $2.8^0 c$ this is the inner meaning of error.

Sol 6: Surface area of the sphere = $4\pi r^2$ Sq. units

$$\frac{\Delta S}{S} = 2.\left(\frac{\Delta r}{r}\right) \Rightarrow \left(\frac{\Delta S}{S}\right) \times 100 = 2\left(\frac{\Delta r}{r}\right) \times 100$$

$$\Rightarrow \left(\frac{\Delta S}{S}\right) \times 100 = 2(0.4\%) = 0.8\ \%$$

Sol 7: Density $= \dfrac{\text{mass}}{\text{volume}}$

Mass = 5.74 gm $\rightarrow$ 3 significant digits

Volume = 1.2 cc $\rightarrow$ 2 significant digits

$\therefore d = \dfrac{5.74}{1.2}$ = 4.78 = 4.8 gm/cc

$\because$ Final result should be in 2 significant digits.

Sol 8: Diameter given

= 1.06 m $\rightarrow$ 3 significant digits.

And now Area $= \dfrac{\pi.d^2}{4}$

$A = 0.88206 \Rightarrow A = 0.882 m^2$

Sol 9: Solution similar to Q.7

Sol 10: $m_1 = 1.2 kg \rightarrow 2$ Significant digits

$m_2 = 5.42 g \rightarrow 3$ Significant digits

$\therefore (m_1 + m_2)$ would be of 2 significant digit ... (i)

$m_1 + m_2 = (1.2 \times 10^3 + 5.42) gms = 1205.42 gm$

$\Rightarrow 1200 gm$ [using(i)]

$\therefore m_1 + m_2 = 1.2 kg$

[This is what happens when we add 10 to a million]

Sol 11: All the problems of this kind; can be solved by the following method.

$M = F^a\ L^b\ T^c$

But we know that;

$F = [M\ L\ T^{-2}]$

$\therefore\ M = [M L T^{-2}]^a [L]^b [T]^c$

$M = [M^a\ L^{a+b}\ T^{-2a+c}]$

Comparing the corresponding coefficients

$a = 1;\ a + b = 0;\ \ -2a + c = 0$

$\Rightarrow\ a = 1;\ b = -1;\ c = 2.$

$\therefore\ M = [F\ L^{-1}\ T^2]$

Sol 12: $V = At^2 + Bt + C$

Now using the concept of Dimensions; all the individual terms i.e. At^2, Bt, C should have the dimension of velocity v.

$\therefore C = [L\ T^{-1}] = m / s.$

And $Bt = [L\ T^{-1}]$

$\Rightarrow B[T] = [L T^{-1}]\ \Rightarrow\ B = [L\ T^{-2}] = m / s^2$

And $At^2 = [L T^{-1}]$

$A[t^2] = [L\ T^{-1}]$

$A = [L\ T^{-3}] \Rightarrow A = [L\ T^{-3}] = m / s^3.$

Calorie = $4.2 [M^2\ L^2\ T^{-2}]$

Now we change the system of units to m', L', T'

$m' = \alpha m$

$L' = \beta \ell$

$t' = r t.$

Hence $C = 4.2\left[\dfrac{m'}{\alpha} \cdot \dfrac{L'^2}{\beta^2} \cdot \left(\dfrac{T'}{\gamma}\right)^{-2}\right]$

$$C = \frac{4.2}{\alpha\beta^2\gamma^{-2}} [m'\ L'^2\ T'^{-2}]$$

$$\Rightarrow C = 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2\ [m'\ L'^2\ T'^{-2}]$$

Hence the magnitude in new units is 4.2 $\alpha^{-1}\ \beta^{-2}\ \gamma^2$.

(*) Do the same procedure for any question on change in the units.

Sol 14: This is again a very standard problem.

$F = m^a\ v^b\ r^c$

And $F = [M L T^{-2}]$

$\Rightarrow [M\ L\ T^{-2}] = [M]^a \cdot [L T^{-1}]^b\ [L]^c$

$\Rightarrow [M\ L\ T^{-2}] = [M^a\ L^{b+c}\ T^{-b}]$

Comparing the powers;

$a = 1,\ b = 2,\ c = -1$

$\therefore F = \dfrac{mv^2}{r}$

Sol 15: (i) Dimensions of energy = $[M L^2 T^{-2}]$

Let M_1, L_1, T_1 represent mass in gram, length in cm and time in second.

And M_2, L_2, T_2 represents mass in kilogram, length in meters and time in second.

Now $n_1[M_1 L_1^2 T_1^{-2}] = n_2[M_2 L_2^2 T_2^{-2}]$

$$n_1 = n_2\left[\frac{M_2}{M_1}\left(\frac{L_2}{L_1}\right)^2\left(\frac{T_2}{T_1}\right)^{-2}\right]$$

$$\Rightarrow n_1 = n_2\left(10^3(10^2)^2 1\right) \Rightarrow n_1 = n_2 \times \left[10^{3+4}\right]$$

$$\Rightarrow n_1 = n_2 \times 10^7$$

$\Rightarrow$ 1 Joule = 10^7 erg. $[\because n_2 = 1]$

(ii) Similar method for (ii).

Sol 16: L = 80 cal/gm. Now we have to express it in J/kg.

We know that 1 cal = 4.2 J and 1 gm = 10^{-3} Kg.

$$\Rightarrow L = 80 \times \left[\frac{4.2}{10^{-3}} J/kg\right] = 80 \times 4.2 \times 10^3 \ J/kg$$

$$L = 336 \times 10^3 \ J/kg$$

$$\Rightarrow L = 3.3 \times 10^5 \ J/kg$$

Sol 17: Method is explained in detail in the solution of 11. Try this yourself.

[Hint: - $G = [M^{-1}L^3 T^{-2}]$]

Sol 18: $\left(p + \frac{q}{V^2}\right)(V - b) = RT$

Using the concept of dimensional analysis;

$\frac{q}{V^2}$ must have dimension of P (pressure)

$\Rightarrow q[L^{-6}] = [m L^{-1} T^{-2}] \Rightarrow q = [ML^5 T^{-2}] = kgm^5 s^{-2}$

And for b; it will have dimension of V,

$b = [L^3] = m^3$

Sol 19: L = $G^x C^y h^z$

$G = [M^{-1}L^3T^{-2}]$ $C = [LT^{-1}]$

$h = [ML^2 T^{-1}]$

$L = [M^{-1} L^3 T^{-2}]^a [L T^{-1}]^b [ML^2 T - 1]^c$

$L = [M^{-a+c} L^{3a+b+2c} T^{-2a-b-c}]$

Comparing the corresponding component;

$c - a = 0$

$3a + b + 2c = 1\,c$

$-2a - b - c = 0$

Solve for a,b,c.

∗This is a typical question from this chapter. So keep practicing problems of this type.

Sol 20: $V \propto (k)^a (E)^b (\rho)^c$

$k = [ML^{-1} T^{-2}]$, $E = [ML^2 T^{-2}]$, $\rho = [ML^{-3}]$, $V = [L T^{-1}]$

Now follow the same procedure as above to find a=1, $b = 1/2,\ \ c = -1/2$

Sol 21: $n = \frac{\Pi}{8} . \frac{R^4}{\ell} . \ \frac{P}{Q}$

$$\left(\frac{\Delta n}{n}\right) = 4.\left(\frac{\Delta R}{R}\right) + \left(\frac{\Delta L}{L}\right) + \left(\frac{\Delta P}{P}\right) + \left(\frac{\Delta Q}{Q}\right)$$

From this it is evident that an error in R gets magnified by four times. So we have to be careful in measuring R.

Sol 22: $T \propto r^a M^b G^c$

$G = [M^{-1} L^3 T^{-2}]$

Use the standard method followed above to derive

$$T \propto \left(\frac{r^3}{GM}\right)^{1/2}$$

Infact Keplers third law is $\frac{4\pi^2}{T^2} = \frac{GM}{r^3}$

This is the real application of dimensional Analysis. One can derive the body of any formula. Constant are then found or performing a couple of experiments.

Exercise 2

Single Correct Choice Type

Sol 1: (A) For the dimension formula for Plank's Constant, we need to know the relation $E = \frac{hc}{\lambda}$

Where E is the Energy

C is speed of light

λ is the wave length

$\therefore \frac{\lambda.E}{c} = h$

$\Rightarrow h = \frac{[ML^2 \ T^{-2}][L]}{[L T^{-1}]} = [ML^2 \ T^{-1}]$

Sol 2: (D) $\eta = \frac{p.(r^2 - x^2)}{4\mu\ell}$

For the dimension of viscosity $\eta = \frac{[\text{pressure}] \ [L^2]}{[L T^{-1}] \ [L]}$

We have to know the dimensions of pressure; which in turn is force per unit area.

$\therefore$ [pressure] $= [M L^{-1} T^{-2}]$

$\eta = \frac{[ML^{-1} \ T^{-2}] \ [L^2]}{[L T^{-1}] \ [L]} = [M L^{-1} T^{-1}]$

Sol 3: (B) $p = p_0 \ \exp(-\propto t^2)$

To find the constant $\propto$;

Hint :- All the question inside the exp ( ) will finally end up with dimension $[M^0 \ L^0 \ T^0]$.

So,

$\therefore [\propto] \ [T^2] = [M^0 \ L^0 \ T^0]$

$[\propto] = [M^0 \ L^0 \ T^{-2}]$

$\therefore$ The dimension are $[M^0 \ L^0 \ T^{-2}]$ or $[T^{-2}]$ which is option B.

Sol 4: (A) In this, we verity each option to match the given dimension.

Capacitance: $[M^{-1} \ L^{-2} \ T^4 \ A^2]$

Resistance: $[ML^2 \ T^{-3} \ A^{-2}]$

Inductance: $[ML^2 \ T^{-2} \ A^{-2}]$

Mag. Flux: $[ML^2 \ T^{-2} \ A^{-1}]$

So, the answer is resistance. i.e option (A).

Tip: - Don't get tensed up if you don't know these terms. You will learn them later. For time being, do remember them.

Sol 5: (B) Using the dimension mentioned in the above question,

We get $[L / R] = [M^0 \ L^0 \ T]$

Tip: - Once check the dimension of (R*C)!!

Sol 6: (C) Physically the term $\frac{1}{2} \in E^2$ equals the electric energy per unit volume. i.e Energy/Volume.

$\therefore \frac{[M L^2 \ T^{-2}]}{[L^3]} = [M L^{-1} \ T^{-2}]$

Sol 7: (C) Writing down the significant figures of all the options.

0.005 – 1

5.00 – 3

50.00 – 4

5.0 – 2

Sol 8: (D) $\underset{4}{\underset{\downarrow}{(97.52)}} \times \underset{3}{\underset{\downarrow}{(2.54)}}$

$\therefore$ The final answer should be with 3 Significant figure.

By observing the options, we can see that option (D) satisfies this condition.

Sol 9: (A) density $= \frac{\text{mass}}{\text{volume}}$ $\quad \therefore \rho = \frac{m}{a^3}$

$\frac{\Delta\rho}{\rho} = \left(\frac{\Delta m}{m}\right) + 3\left(\frac{\Delta a}{a}\right) = 3 + 3 \ (2) \ \therefore \left(\frac{\Delta\rho}{\rho}\right) = 9\%$

Sol 10: (B) This is just a generalization of the previous question.

$X = M^a \ L^b \ T^{-c}$

$\frac{\Delta x}{x} = a.\left(\frac{\Delta m}{m}\right) + b\left(\frac{\Delta L}{L}\right) + c\left(\frac{\Delta T}{T}\right)$

$\left(\frac{\Delta x}{x}\right) = (a \propto + \beta b + \gamma c)\%$

P.S:- don't get confused with $\pm c$.

Sol 11: (B) Here volume is an intrinsic property of each sphere. So, it will have the same number of significant digit even they are measured in bulk.

$\therefore$ The final result should be having 3 significant digit. Just multiply 1.75×25 and then scale the result to 3 significant digits.

Sol 12: (A) $V = \frac{4}{3}\pi r^3$

$\frac{\Delta v}{v} = 3.\left(\frac{\Delta r}{r}\right)$

$\left(\frac{\Delta v}{v}\right) = 3(2\%) \Rightarrow \frac{\Delta v}{v} = 6\%$

Sol 13: (C) $\eta = \frac{2WgL}{\pi r^4 \theta}$

$$\frac{\Delta n}{n} = \left(\frac{\Delta w}{w}\right) + \left(\frac{\Delta g}{g}\right) + \left(\frac{\Delta L}{L}\right) + 4\left(\frac{\Delta r}{r}\right) + \left(\frac{\Delta \theta}{\theta}\right)$$

Here a small error in r gets magnified by four times in the final result. So, it has to be measured with care.

Sol 14: (B) Refer theory.

Sol 15: (C) Let us first write dimension of Young's

Module in fundamental units $Y = [M L^{-1} T^{-1}]$

And now let $Y = v^a A^b F^c$

$y = [L T^{-1}]^a [L T^{-2}]^b [M L T^{-2}]^c$

$y = [M^c L^{a+b+c} T^{-a-2b-2c}]$

But actual $Y = [M L^{-1} T^{-2}]$

$\therefore$ Comparing respectively;

$c = 1;\ a + b + c = -1;\ -a - 2b - 2c = -2$

Solving them gives the result.

Sol 16: (D) $N = -D\frac{n_2 - n_1}{x_2 - x_1}$

$D = -N\frac{(x_2 - x_1)}{(n_2 - n_1)}$

$D = [M^0 L^0 T^0]\ \frac{[M^0 L T^0]}{[M^0 L^{-3} T^0]}$

$D = [M^0 L^2 T^{-1}]$

Sol 17: (D) This is very same as Q.15

Try this yourself !

Sol 18 : (A) Power $= [M L^2 T^{-3}]$

$n_1 [M_1 L_1^2 T_1^{-3}] = n_2 [M_2 L_2^2 T_2^{-3}]$

$m_1 = 20\text{kg} = 20M_2 \rightarrow (1)$

$L_1 = 10\text{m} = 10L_2 \rightarrow (2)$

$T_1 = 5\text{ s} = 5\ T_2 \rightarrow (3)$

Now $n_1 = n_2\left[\frac{m_2}{m_1}.\left(\frac{L_2}{L_1}\right)^2.\left(\frac{T_2}{T_1}\right)^{-3}\right]$

$n_1 = n_2\left[\frac{1}{20}.\left(\frac{1}{10}\right)^2.(5)^3\right]$

$n_1 = n_2\left[\frac{1}{16}\right]$

Now $n_1 = 1\ \ n_2 = 16$ watts.

Sol 19: (C) $T = 2\pi\sqrt{\ell / g}$

$g = 4\pi^2\frac{L}{T^2}$

$\therefore \frac{\Delta g}{g} = \left(\frac{\Delta L}{L}\right) + 2\left(\frac{\Delta T}{T}\right) \Rightarrow \left(\frac{\Delta g}{g}\right) = 1 + 2(3)\ = 7\%$

(P.S:- Error is an error either it is +ve or –ve. It effect the end result)

Sol 20: (B) $S_1 - S_2 = ut + \frac{1}{2}at^2$

$\Delta s = 1.1\text{m}$ and $t = 1\text{s},\ a = 0.5\text{m}/\text{s}^2$

Solving the above question;

$u = 0.85\text{m}/\text{s}$

But we have to around it to 1 significant digit .(why....??)

$\therefore$ u = 0.9 m/s

Sol 21: (C) $y = k x^a$

Now don't get confused with k. It's just a constant !

$\frac{\Delta y}{y} = a.\frac{\Delta x}{x}$

$\frac{\Delta y}{y} = a.p$

$\therefore$ Depends on both a and p.

Sol 22: (A) Refer theory.

Sol 23: (D)

$\frac{3.06}{1.2} + 1.15$

↓

2.55 + 1.15

↓

2.6 + 1.15

↓

3.75

↓

3.8

Sol 24: (D) (A) Solid angel and unit vector.

Both are dimension. Unit vector is just unit magnitude with a direction.

(B)Potential energy and torque

In a crude way, energy is similar to work

Which is $\vec{F}.\vec{s}$ and torque is $\vec{s}\times\vec{F}$. Hence the dimension would of course be the same.

We can also check by comparing down the dimension of them.

(C) Area × Velocity = $[L^2][LT^{-1}] = [L^3T^{-1}]$

$\frac{\Delta v}{\Delta t} = \frac{[L^3]}{[T]} = [L^3\ T^{-1}]$. Hence same.

Sol 25: (C) (A) $\frac{I\omega^2}{mvr} = \frac{[M\,L^2][T^{-2}]}{[M\,LT^{-1}\,L]}$ $\therefore$ Using hint.

$= [T^{-1}]$

(B) $\frac{G\rho}{T} = \frac{[M^{-1}\,L^3\,T^{-2}]\,[M\,L^{-3}]}{T} = [T^{-1}]$

(C) $\frac{\rho vr}{\eta} = \frac{[M\,L^{-3}]\,[L\,T^{-1}]\,[L]}{[M\,L^{-2}]\,[L^{-1}]\,[L^{-1}T]} = [M^0\ L^0\ T^0]$

Write dimension of η using F = $6\pi\eta rv$

(D) In an easy method,

We know that $T = I\propto$

$\Rightarrow \frac{I\alpha\theta}{I\omega} = \frac{\alpha\theta}{\omega} = \frac{[T^{-2}]}{[T^{-1}]} = [T^{-1}]$

Sol 26: (D) $A = B^n\ C^m$

$[L\ T] = [L^2\ T^{-1}]^n\,[L\ T^2]^m = [L^{2n+m}\ T^{-n+2m}]$

Comparing respective exponents;

$2n + m = 1$ ...(i)

$2m - n = 1$ ...(ii)

Give the value of n and m.

Sol 27: (D) Aim of the question is to use dimensional analysis.

$\mu = F^a\ M^b\ L^c$

A,b,c are constants

$[T^{-1}] = [M\ L\ T^{-2}]^a\,[M]^b\,[L]^c$

$[T^{-1}] = [M^{a+b}\ L^{a+c}\ T^{-2a}]$

$\therefore$ We get three equations;

$a + b = 0$ ...(i)

$a + c = 0$ ...(ii)

$-2a = -1$ ...(iii)

$\therefore$ We get $a = 1/2,\ b = c = -1/2$

$\therefore$ We have $\mu = \lambda.\sqrt{\frac{F}{ML}}$ [λ is any constant]

Sol 28: (D)

(A) Moment of a force = Force × Perpendicular distance

$= M\ L\ T^{-2} \times L = M\ L^2\ T^{-2}$

(B) Surface tension $= \frac{Force}{Length} = \frac{M\,L\,T^{-1}}{L} = MT^{-2}$

(C) Modulus of elasticity = $\frac{Stress}{Strain}$ = Unitless

(D) Coefficient of viscosity $= \frac{Fr}{A\,v} = \frac{M\,L\,T^{-2}.rL}{L^2.LT^{-1}} = M\,L^{-1}T^{-1}$

Sol 29: (D)

(A) $\frac{Energy}{Area} = \frac{M\,L^{-1}\,T^{-1}}{L^2} = M\,L^{-3}T^{-2}$

(B) Pressure = $M\,L^{-1}T^{-2}$ user

(C) Force × length

(D) pressure per unit length

Sol 30: (B) Let M', T', L' be the value of mass, time and length respectively in the new system.

We know that $M' = 2M$ and $T' = \frac{1}{2}T$

$\therefore\ n'[M^1\,L'^2\,T^{-2}] = n\,[M\ L^2\ T^{-2}]$

$n'[M^1\,L'^2T'^{-2}] = 8\,[\frac{m'}{2}\ L'^2\,2^{-2}\,T'^{-2}]$

$n^1 = 8 \times \frac{1}{8} = 1$

Sol 31: (A) Refer theory.

Sol 32: (D) Momentum and angular momentum, Both have different dimension and for momentum $\to$ mv

and angular momentum $\to$ mvr

Hence it's very obvious from the above.

Sol 33: (D) $F = 6\pi\eta rv$.

$$\eta = \left(\frac{F}{6\pi rv}\right)$$

$$\eta = \frac{[M\,L\,T^{-2}]}{[L]\,[L\,T^{-1}]}$$

$$\eta = [M\,L^{-1}\;T^{-1}]$$

Sol 34: (B) $p = p_0 \exp[-\alpha t^2]$

As describe in Q3;

$\alpha = [T^{-2}]$

Sol 35: (D) Light year is the distance travelled by the light in one year.

Previous Years' Questions

Sol 1: (B) $[Y] = \left[\frac{X}{Z^2}\right] = \left[\frac{\text{Capacitance}}{(\text{Magnetic induction})^2}\right]$

$$= \left[\frac{M^{-1}L^{-2}Q^2T^2}{M^2Q^{-2}T^{-2}}\right] = [M^{-3}\,L^{-2}\,T^4\,Q^4]$$

Sol 2: (D) $\frac{1}{2}\varepsilon_0 E^2$ is the expression of energy density (Energy per unit volume)

$$\left[\frac{1}{2}\varepsilon_0 E^2\right] = \left[\frac{ML^2T^{-2}}{L^3}\right] = [ML^{-1}T^{-2}]$$

Sol 3: (D) $C = \frac{\Delta q}{\Delta V}$

or $\varepsilon_0 \frac{A}{L} = \frac{\Delta q}{\Delta V}$ or $\varepsilon_0 = \frac{(\Delta q)L}{A.(\Delta V)}$

$$X = \varepsilon_0 L \frac{\Delta V}{\Delta t} = \frac{(\Delta q)L}{A(\Delta V)} L \frac{\Delta V}{\Delta t}$$

But $[A] = [L^2]$

$\therefore X = \frac{\Delta q}{\Delta t} = \text{current}$

Sol 4: (A) $V = l^3 = (1.2 \times 10^{-2}\ \text{m})^3 = 1.728 \times 10^{-6}\,\text{m}^3$

$\because$ Length (l) has two significant figures, the volume (V) will also have two significant figures. Therefore, the correct answer is $V = 1.7 \times 10^{-6}\,\text{m}^3$

Sol 5: (A) $\left[\frac{\alpha Z}{k\theta}\right] = [M^0L^0T^0]\ \therefore\ [\alpha] = \left[\frac{k\theta}{Z}\right]$

Further $[p] = \left[\frac{\alpha}{\beta}\right]\ \therefore\ [\beta] = \left[\frac{\alpha}{p}\right] = \left[\frac{k\theta}{Zp}\right]$

Dimension of $k\theta$ are that of energy. Hence,

$$[\beta] = \left[\frac{ML^2T^{-2}}{LML^{-1}T^{-2}}\right] = [M^0L^2T^0]$$

Sol 6: (D) Density $\rho = \frac{m}{\pi r^2 L}$

$$\therefore\ \frac{\Delta\rho}{\rho} \times 100 = \left(\frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}\right) \times 100$$

After substituting the values, we get the maximum percentage error in density = 4 %

Sol 7: (D)

Dipole moment = (charge) × (distance)

Electric flux = (electric field) × (area)

Sol 8: (A) Least count (LC)

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}} = \frac{0.5}{50} = 0.01$$

Now, diameter of ball

$= (2 \times 0.5\ \text{mm}) + (25 - 5)\ (0.001) = 1.2\ \text{mm}$

Sol 9: (B) $Y = \frac{FL}{Al} = \frac{4FL}{\pi d^2 l} = \frac{(4)(1.0 \times 9.8)(2)}{\pi(0.4 \times 10^{-3})^2(0.8 \times 10^{-3})}$

$= 2.0 \times 10^{11}\ \text{N/m}^2$

Further $\frac{\Delta Y}{Y} = 2\left(\frac{\Delta d}{d}\right) + \left(\frac{\Delta l}{l}\right)$

$$\therefore \Delta Y = \left\{2\left(\frac{\Delta d}{d}\right) + \left(\frac{\Delta l}{l}\right)\right\} y = \left\{2 \times \frac{0.01}{0.4} + \frac{0.05}{0.8}\right\} \times 2.0 \times 10^{11}$$

$= 0.225 \times 10^{11}\ \text{N/m}^2 = 0.2 \times 10^{11}\ \text{N/m}^2$

(By rounding off)

or $(Y + \Delta Y) = (2 + 0.2) \times 10^{11}\ \text{N/m}^2$

Sol 10: (A) Length of air column in resonance is odd integer multiple of $\frac{\lambda}{4}$.

Sol 11: (B) $T = 2\pi\sqrt{\frac{l}{g}}$ or

$\frac{t}{n} = 2\pi\sqrt{\frac{l}{g}} \therefore g = \frac{(4\pi^2)(n^2)l}{t^2}$

% error in $g = \frac{\Delta g}{g} \times 100 = \left(\frac{\Delta l}{l} + \frac{2\Delta t}{t}\right) \times 100$

$E_I = \left(\frac{0.1}{64} + \frac{2\times 0.1}{128}\right) \times 100 = 0.3125\%$

$E_{II} = \left(\frac{0.1}{64} + \frac{2\times 0.1}{128}\right) \times 100 = 0.46875\%$

$E_{III} = \left(\frac{0.1}{20} + \frac{2\times 0.1}{36}\right) \times 100 = 1.055\ \%$

Hence E_I is minimum.

Sol 12: (D) Least count of vernier calipers

$LC = 1MSD - 1VSD$

$= \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$

20 divisions of vernier scale = 16 divisions of main scale

$\therefore 1\ VSD = \frac{16}{20}\ mm = 0.8\ mm$

$\therefore LC = 1MSD - 1VSD = 1mm - 0.8\ mm$

$\therefore\ \ = 0.2\ mm$

Sol 13: (C) Least count screw gauge

$= \frac{0.5}{50} = 0.01\ mm = \Delta r$

Diameter $r = 2.5\ mm + 20 \times \frac{0.5}{50} = 2.70\ mm$

$\frac{\Delta r}{r} = \frac{0.01}{2.70}$ or $\frac{\Delta r}{r} \times 100 = \frac{1}{2.7}$

Now density $d = \frac{m}{v} = \frac{m}{\frac{4}{3}\pi\left(\frac{r}{2}\right)^3}$

Here, r is the diameter.

$\therefore \frac{\Delta d}{d} \times 100 = \left\{\frac{\Delta m}{m} + 3\left(\frac{\Delta r}{r}\right)\right\} \times 100$

$= \frac{\Delta m}{m} \times 100\ + 3 \times \left(\frac{\Delta r}{r}\right) \times 100 = 2\% + 3 \times \frac{1}{2.7} = 3.11\ \%$

Sol 14: (A) $R = \frac{V}{i}$

$\Rightarrow \quad \left|\frac{\Delta R}{R}\right| = \left|\frac{\Delta V}{V}\right| + \left|\frac{\Delta i}{i}\right|$

$\frac{\Delta V}{V} \times 100 = 3$

$\Rightarrow \quad \frac{\Delta V}{V} = 0.03$

Similarly, $\frac{\Delta i}{i} = 0.03$

Hence $\frac{\Delta R}{R} = 0.06$

So percentage error is $\frac{\Delta R}{R} \times 100 = 6\%$

Sol 15: (C)

$L.C = \frac{1}{60}$

Total Reading = $585 + \frac{9}{60} = 58.65$

Sol 16: (A)

$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = F$

$\varepsilon_0 = \frac{[A^2T^2]}{[MLT^{-2}L^2]} = [M^{-1}L^{-3}A^2T^4]$

Sol 17: (D) Least count of vernier calliper is $\frac{1}{10}$ mm =0.1 mm = 0.01 cm

Sol 18: (D) $t_{mean} = \frac{90+91+95+92}{4} = 92$ sec

Absolute error in each reading = 2, 1, 3, 0

mean error $= \frac{2+1+3+0}{2} = 1.5$ sec

Put the least count of the measuring clock is 1 sec.

So it cannot measure upto 0.5 second so we have to round it off.

So mean error will be 2 second

So t = 92 ± 2 sec.

Sol 19: (A) Reading $= 0.5 + 25\left(\frac{0.5}{50}\right) + 5\left(\frac{0.5}{50}\right) = 0.8$ mm

JEE Advanced/Boards

Exercise 1

Sol 1: Increasing the number of readings reduces the errors. This is because, we have more chances to get the mean closer to the actual value.

So increasing reading from 100 to 400; reduce the problem error by a factor of four.

Sol 2: Length $(L) = 4.234\,m \rightarrow 4$ significant digits

Breadth $(B) = 1.005\,m \rightarrow 4$ significant digits

Thickness $(H) = 2.01\times10^{-2}\,m \rightarrow 3$ significant digits

$\therefore$ The volume = Lbh will have 3 significant digits

$\Rightarrow\ V = (4.234\times1.005\times2.01)\times10^{-2}\,m^3$

$\Rightarrow\ V = 8.5528\times10^{-2}\ m^3$

$\Rightarrow\ V = 8.55\times10^{-2}\ m^3$

Sol 3: $I = I_0\ e^{-\mu x}$

Now μx should have the dimension $[M^0 L^0 T^0]$

$\Rightarrow\ \mu.[L] = [M^0 L^0 T^0]$

$\mu = [M^0 L^{-1} T^0]$

Sol 4: $R_1 = (24\pm0.5)\Omega$

$R_2 = (8\pm0.3)\Omega$

(a) Series :-

$R_{eff.} = R_1 + R_2$

$R_{eff.} = 32$

$\Delta R_{eff} = \Delta R_1 + \Delta R_2 = 0.5 + 0.3\ = 0.8$

$\therefore R_{eff} = (32\pm0.8)\Omega$

Now absolute error = 0.8 and

Relative error $= \dfrac{0.8}{32}\times100 = 2.5\%$

(b) Parallel:-

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{...(i)}$$

$$R_{eff} = \frac{24\times8}{24+8} = \frac{24\times8}{32} = 6\Omega$$

And for R_{eff}; using (i)

$$\frac{\Delta R_{eff}}{R^2_{eff}} = \frac{\Delta R_1}{R_2^1} + \frac{\Delta R_2}{R_2^2}$$

Remember this formula !

$$\left[\text{Hint}: \frac{d}{dx}\left(\frac{1}{\theta}\right) = -\frac{1}{\theta^2}.\frac{d\theta}{dx} = \frac{\Delta\theta}{\theta^2}!\right]$$

$$\Delta R_e = \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right).R_{eff}^{\ 2}$$

$$\Delta R_e = \left(\frac{0.5}{24\times24} + \frac{0.3}{8\times8}\right)\times(6)^2$$

$\Delta R_e = 0.2$

$\Delta R_e = (6\pm0.2)\Omega$

And for relative error;

$$\left(\frac{\Delta R_e}{R_e}\right) = \frac{0.2}{6} = \left(\frac{1}{30}\right)$$

$$\%\text{ relative error} = \frac{1}{30}\times100 = \frac{10}{3} = 3.33\%$$

Sol 5: Ohm's Law: V=IR $\Rightarrow\ R = \dfrac{V}{I}$

$$R = \frac{6.4}{2} = 3.2\Omega\ \text{ and }\ \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$$\Delta R = \left[\frac{0.2}{6.4} + \frac{0.1}{2}\right]3.2$$

$\Delta R = 0.26$

Resistance $R = (3.2\pm0.26)\ \Omega$

And relative error $= \dfrac{\Delta R}{R} = \left(\dfrac{0.26}{3.2}\right)$

$$\%\text{ Relative error } = \frac{0.26}{3.2}\times100\ = 8.1\%$$

Sol 6: Radius of proton

$= 10^{-9}\mu = 10^{-9}\times10^{-6}m\ = 10^{-15}m$

Size of universe $= 10^{27}m$

Now let us use $\log_{10}(r)$ as an operator.

$\log(r_p) = \log_{10}(10^{-15}) = -15$

and

$$\log(r_4) = \log_{10}(10^{27}) = 27\ \log(r) = \frac{27+(-15)}{2} = \frac{12}{2} = 6$$

$\Rightarrow r = 10^6\,m$

Sol 7: Refer to the solution of Q_{11} (Ex – 1) and Q_{19} (Ex – 2) and try it yourself.

Sol 8: $V_c \propto [d]^a (\rho)^b (\eta)^c$

$V_c = [L\ T^{-1}]$

$\rho = [M\,L^{-3}]$

$d = [L]$

$\eta = [ML^{-1}\,T^{-1}]$

$[L\ T^{-1}] = [L]^a\,[ML^{-3}]^b\,[ML^{-1}T^{-1}]^c$

$[L\ T^{-1}] = [M^{b+c}\,L^{a-3b-c}T^{-c}]$

$b + c = 0$

$a - 3b - c = 1$

$-c = -1$

We get $c = 1,\ b = -1,$ and $a = -1$

$\therefore\ V_c \propto \frac{\eta}{d\rho}$

Sol 9: $m \propto V^a\,(d)^b\,(g)^c$

$m = [M]$

$V = [L\ T^{-1}]$

$d = [M\,L^{-3}]$

$g = [L\ T^{-2}]$

$[M] = [LT^{-1}]^a\,[ML^{-3}]^b [L\,T^{-2}]^c$

$[M] = [M^6\,L^{a-3b+c}\ T^{-a-2c}]$

$b = 1;\ a - 3b + c = 0;\ -a - 2c = 0$

$\Rightarrow\ c = -3$ and $a = 6$

$\therefore\ m \propto V^6.\ d\ g^{-3}$

$\Rightarrow\ m = \frac{kV^6 d}{g^3}$

Sol 10: $f \propto m^a\ \ell^6\ F^c$

$[f] = [T^{-1}]$

$m \equiv [M]$

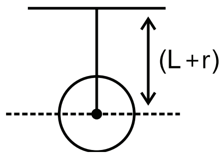
$\ell \equiv [L]$

$F \equiv [M\,L\ T^{-1}]$

Now proceeding the same way as we did in Q11 – (Ex – 1)

Sol 11: Now if there is an error, the next possible value of L would be 100.3 or 100.4 cm.

i.e least count for r=2.34 cm, L.C=0.01 cm



[∴ 2.35 or 2.36]

and for $t = 2.3s$, L.C$= 0.1s$

$\therefore\ \left(\frac{\Delta g}{g}\right) = \left(\frac{\Delta L}{L}\right) + \left(\frac{\Delta r}{r}\right) + 2\left(\frac{\Delta T}{T}\right)$

$\therefore\ \left(\frac{\Delta g}{g}\right) = \left(\frac{0.1}{100.2}\right) + \left(\frac{0.01}{2.34}\right) + 2.\left(\frac{0.1}{2.3}\right)$

$\left(\frac{\Delta g}{g}\right) = 0.092$

$\left(\frac{\Delta g}{g}\right) \times 100 = 9.2\%$

'T' has to be measured more accurately because each error gets double magnified in calculating g.

Sol 12: $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow f = 14.3$

$\frac{1}{f} = \frac{1}{50.1} + \frac{1}{20.1}$

And then $\frac{\Delta f}{f^2} = \frac{\Delta V}{v^2} + \frac{\Delta 4}{u^2}$

$\Delta f = \left(\frac{\Delta V}{v^2} + \frac{\Delta u}{u^2}\right) f^2$

$\Delta f = \left(\frac{0.5}{(50.1)^2} + \frac{(0.2)}{(20.1)^2}\right)(14.3)^2$

$\Delta f = 0.4\text{cm}$

Focal length $= (14.3 \pm 0.4)\text{cm}$

Sol 13: Specific gravity (s) $= \frac{w_{air}}{w_{water}}$

$\left(\frac{\Delta s}{s}\right) = \left(\frac{\Delta w_{air}}{w_{air}}\right) + \left(\frac{\Delta w_{water}}{w_{water}}\right)$

$\left(\frac{\Delta s}{s}\right) = \left(\frac{0.1}{10}\right) + \left(\frac{0.1}{5}\right)$

$\left(\frac{\Delta s}{s} \times 100\right) = 3\%$

Sol 14: $R = \left(\frac{h}{2}\right) + \left(\frac{L^2}{6h}\right)$

$R = \frac{h}{2} + \frac{L^2}{6h}$

$h = (0.085 \pm 0.001)\text{cm}$

$L = (4.4 \pm 0.1)\text{cm}$

$\frac{\Delta R}{R} = \frac{\Delta h}{h} + 2\frac{\Delta L}{L} + \frac{\Delta h}{h}$

$\frac{\Delta R}{R} = 2\left(\frac{\Delta h}{h} + 2\frac{\Delta L}{L}\right)$

$\left(\frac{\Delta R}{R}\right) = 2\left(\frac{0.001}{0.085} + \frac{0.1}{4.4}\right)$

$\left(\frac{\Delta R}{R}\right) = 0.068$

Sol 15: $V \propto n^a r^b \left(\frac{p}{\ell}\right)^c$

V-Rate of flow $\Rightarrow$ Volume/time $\equiv [L^3\ T^{-1}]$

$\eta \equiv [m\,L^{-1}\ T^{-1}]$

$r \equiv [L]$

$p \equiv [m\,L^{-1}\ T^{-2}]\ *\frac{p}{L} \equiv [m\,L^{-2}\,T^{-2}]$

Now solving for the constants a, b, c gives us the result.

$[L^3\,T^{-1}] = [M\,L^{-1}\ T^{-1}]^a\,[L]^b\,[M\,L^{-2}\ T^{-2}]^c$

$[L^3\ T^{-1}] = [M^{a+c}\,L^{-a+b-2c}T^{-a-2c}]$

$a + c = 0$

$-a + b - 2c = 3$

$-a - 2c = -1$

$a = -1 \quad c = 1 \text{ and } b = 4$

Sol 16: $h \propto (\rho)^a\ (T)^b\,(g)^c\,(r)^d$

$h \equiv [L]$

$\delta \equiv [ML^{-3}]$

$T \equiv [M\,L\,T^{-2}]$

$g = [L\ T^{-2}]$

$r = [L]$

Now proceeding;

$[L] \equiv [ML^{-3}]^a[M\,L\ T^{-2}]^6[L\,T^{-2}]^c[L]^d$

$[L] \equiv [M^{a+b}\,L^{-3a+b+c+d}\ T^{-2b-2c}]$

$\Rightarrow\ a + b = 0;\ -2(b + c) = 0;\ -3a + b + c + d = 1$

Here we have 3 equations and four variables to solve
$\therefore$ Not possible.

And now;

Given that h is inversely proportional to 'r'

$\therefore d = -1$

Now we can solve for a, b, c

Sol 17: $\eta \propto m^a\ D^b\ v^c$

$\eta \equiv [M\,L^{-1}\ T^{-1}]$

$m \equiv [M]$

$D \equiv [L]$

$V \equiv [L\,T^{-1}]$

Solve in a similarly way to Q.16

Sol 18: $\omega \propto r^a\,m^b\,G^c$

$[\omega] \equiv [m^0\ L^0\ T^0]$

$[r] \equiv [L]$

$m \equiv [M]$

$G = [M^{-1}\ L^3\ T^{-2}]$

And solve for a, b, c

Sol 19: $F = \frac{mv^2}{r}$

$m = (0.5 \pm 0.005)\text{kg}$

$v = (10 \pm 0.01)\text{m / s}$

$r = (0.4 \pm 0.01)\text{m}$

$\frac{\Delta F}{F} = \left(\frac{\Delta m}{m}\right) + 2\left(\frac{\Delta v}{v}\right) + \left(\frac{\Delta r}{r}\right)$

$= \left(\frac{0.005}{0.5}\right) + 2\left(\frac{0.01}{10}\right) + \left(\frac{0.01}{0.4}\right)$

$\left(\frac{\Delta F}{F}\right) = 0.037$

$\left(\frac{\Delta F}{F} \times 100\right) = 3.7\%$

Sol 20: $\rho = \frac{\pi r^2 R}{\ell}$

$$\left(\frac{\Delta\rho}{\rho}\right) = 2\cdot\left(\frac{\Delta r}{r}\right) + \left(\frac{\Delta R}{R}\right) + \left(\frac{\Delta L}{L}\right)$$

$$\left(\frac{\Delta\rho}{\rho}\times 100\right) = 2\cdot\left(\frac{0.02}{0.26}\times 100\right)$$

$$+\left(\frac{2}{64}\times 100\right) + \left(\frac{0.1}{156}\times 100\right) = 18.6\%$$

Sol 21: $V = \underset{(2)}{\frac{+.50t}{\downarrow}} + \underset{(2)}{\frac{0.008}{\downarrow}}t^2$

Now let us examine the units of (1) and (2) for (1); unit is m^2/s and dimension is $[L^3\ T^{-1}]$. And for 2; unit is m^3/s^2 and dimension is $[L^3\ T^{-2}]$.

$$\therefore\ V = (1.50\ m^3/s_1)t + \left(0.008\cdot\frac{m_1^3}{s_1^2}\right)t^2$$

By changing the unit system;

The value of coefficients of 't' and 't^2' change.

$$n_1\left[L_1^3\ T_1^{-1}\right] = n_2\left[L_2^3\ T_2^{-1}\right]$$

Using this we can find the values of new coefficients.

Sol 22: 24 hours ≡ 10 Decimal hours

$\Rightarrow$ 1 D h = 2.4 hrs. $\rightarrow$ (1)

and 1 Dh = 100 D mins

$\Rightarrow$ 1 D min $= \frac{2.4}{100} \rightarrow (2)$

Given time = 8 Dh and 22.8 Dmin

$$= 8(2.4) + 22.8\left(\frac{2.4}{100}\right)\text{hours} = 19.747\text{ hours}$$

$\Rightarrow$ 19 Hr, 10 minutes, 50 seconds

Sol 23: Aim of the question is to do dimensional analysis. Circumference will have dimension [L]

Volume $[L^3]$

Area $[L^2]$

Verify the options for correct choices

Sol 24: For $m_1 = m_2 = 1\text{kg}$ and $r = 1$ m,

The force $F = \frac{G.(1)^2}{1} = GN$

$\therefore\ G = 6.6\times 10^{-11}$ N.

But according to the problem, the force is 1 unit.

$\Rightarrow F = 6.6\times 10^{-11}\ N \equiv 1$ unit

$\therefore\ 1\ N \equiv \frac{1}{6.6\times 10^{-11}}$ unit

We call this unit as Gunit.

$\therefore\ 1\ N = 1.5\times 10^{10}$ Gunit

Sol 25: For each atom, we have,

$$v = \frac{m}{\rho} = \frac{9.27\times 10^{-26}}{7870} = 1.178\times 10^{-29} m^3/\text{atom}$$

Now, $\frac{4\pi r^3}{3} = 1.178\times 10^{-29} m^3 \Rightarrow r = 1.41\times 10^{-10} m$

Hence, the distance between atoms is d = 2r= 2.82 × 10^{-10}m

Sol 26: $F = \frac{\alpha}{\beta + \sqrt{d}}$

Here the dimension of β and $\sqrt{d}$ should be same.

Hence $[m^{1/2} L^{-3/2}] = \beta$

And now the dimension of $\frac{\alpha}{\sqrt{d}}$ should be

Same as F.

$$\frac{\alpha}{[m^{-1/2}L^{-3/2}]} = [MLT^{-2}]$$

$$\Rightarrow \alpha = [m^{3/2}L^{-1/2}T^{-2}]$$

Sol 27: $c = [L\ T^{-1}]$ $G = [M^{-1}\ L^3\ T^{-2}]$

$h = [M\ L^2\ T^{-1}]$

Now for mass $M = c^x G^y h^z$

Finding the value x, y, by following the method described in Q11 (Ex1)

Sol 28: $p = \left(\frac{nRT}{v-b}\right)e^{\frac{-a}{RTV}}$

There we can use the ideal gas equation;

$pv = nRT$

In solving question involving RT; we can replace RT by PV and then proceed.

So, now $\frac{a}{RTV}$ will have a dimensionally $[M^0\ L^0\ T^0]$

$$\frac{a}{pv^2} = [M^0 L^0 T^0]$$

$a \equiv [M L^{-1} T^{-2}][L^6]$

$a = [M L^5 T^{-2}]$

And now for b;

$\frac{nRT}{v-b}$ should be dimension equal to P.

$\therefore \frac{pv}{v-b} \equiv p$

$\Rightarrow v - b \equiv v$

$\Rightarrow$ b should be dimensionally equal to V

$\therefore b = [L^3]$

Sol 29: $R \propto H^a v^b g^c$

$r \equiv [L]$

$H \equiv [L]$

$v \equiv [L\ T^{-1}]$

$g \equiv [L\ T^{-2}]$

$[L] \equiv [L]^a [L\ T^{-1}]^b [L\ T^{-2}]^c$

$[L] \equiv [L^{a+b+c}\ T^{-b-2c}]$

$a + b + c = 1$ ... (i)

$b + 2c = 0$ ... (ii)

And also given that, $R \propto v$, $a = 1$, $b = 1$

So $a + c = 0$

$1 + 2c = 0$

$\Rightarrow c = -1/2$ and $a = 1/2$

$\therefore R \propto \sqrt{\frac{H}{g}}\, v$

$\therefore R = k\sqrt{\frac{H}{g}}\, v$

Exercise 2

Single Correct Choice Type

Sol 1: (A) $m = \pi \tan\theta$

In solving this problem; we shall use basic calculus. Let us say we find to find the Minimum value of y=f(x) then that would be at a point, (x_0) where

$\frac{dy}{dx} = f^1(x) = 0$

And that minimum/maximum value would be $f(x_0)$ now here;

$m = \pi \tan\theta$

$\frac{dm}{d\theta} = \pi \frac{d}{d\theta}(\tan\theta) \equiv \pi.\sec^2\theta = 0$

$\sec^2\theta = 0$

This would give us no value of θ .!

And now we don't have any choice rather than to go for an analytical method.

First let us find $\left(\frac{\Delta m}{m}\right)$

$\Delta m = \pi \sec^2\theta$

$\frac{\Delta m}{m} = \frac{1}{\sin\theta\cos\theta} = \left(\frac{2}{\sin 2\theta}\right) \rightarrow (1)$

Now for $\frac{\Delta m}{m}$ to be minimum;

$\sin 2\theta$ has to be maximum

$\sin 2\theta = n\,\pi / 2$ (n is odd)

$\theta = n.\pi / 4$

Hence $\theta = 45^0$ is the answer.

Sol 2: (C) The least count of the vernier can be measured by using the formula;

L.C = 1 M.S.D – 1 V.S.D

M.S.D $\rightarrow$ Main scale division.

VSD $\rightarrow$ Vernier scale division.

Now in most cases MSD is given.

Our task would be to find VSD.

$\therefore$ Let us say N division of vernier scale coincides with N-1 division of main scale, then

$1\ \text{VSD} = \left(\frac{N-1}{N}\right)$

And least count $= 1 - \left(\frac{N-1}{N}\right)$

$\text{L.C} = \left(\frac{1}{N}\right)$

$\therefore$ Here given; L.C = 0.02 cm

$\Rightarrow$ 1 USD = 0.1 – 0.02

1 USD = 0.08 cm

And the number of division of vernier scale =10.

$\therefore$ Length of vernier scale = 0.8

$[\therefore 0.8 \times 10]$

Sol 3: (C) Explained briefly in the above question.

(a) $0.00145 \rightarrow 3$

(b) $14.50 \rightarrow 4$

(c) $145.00 \rightarrow 5$

(d) $145.0 \times 10^{-6} \rightarrow 4$

Sol 4: (D) Option A, B, C are obvious.

Now in option D.

Angle doesn't have any dimension. And no. of moles being representing the number of particles, but still we define a dimension μ for moles.

Multiple Correct Choice Type

Sol 5: (B, C) $L = G^x\ c^y\ h^z$

$$L = [M^{-1} L^3 T^{-2}]^x [L\ T^{-1}]^y [M L^2 T^{-1}]^z$$

$$L = [M^{z-x} L^{3x+y+2z} T^{-2x-y-z}]$$

Comparing the respective powers;

$$z - x = 0 \qquad \text{...(i)}$$

$$3x + y + 2z = 1 \qquad \text{...(ii)}$$

$$-2x - y - z = 0 \qquad \text{...(iii)}$$

$$x = \frac{1}{2},\ y = \frac{-3}{2},\ z = \frac{1}{2}$$

Sol 6: (A, B, D) (A) Velocity and speed – yes $[L\ T^{-1}]$

(B) Pressure and stress – yes $[M L^{-1}\ T^{-2}]$

(C) $\underset{[M L T^{-2}]}{\underset{\downarrow}{\text{force}}}$ and $\underset{[M L T^{-1}]}{\underset{\downarrow}{\text{impulse}}}$ – No

(D) Work, energy – yes $[M L^2\ T^{-2}]$

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

Now we need to understand that;

(i) Only quantities with same physical dimension can be added or subtracted.

$\therefore$ b must be having a dimension of volume.

Comprehension Type

Sol 7: (B) b is dimension same as V.

Sol 8: (C) Now $\frac{a}{V^2}$ must be having dimensioned same as P.

$\Rightarrow\ a \equiv PV^2$

Sol 9: RT = [RT] = ML^2T^{-2}

(A) PV = [PV] = $ML^{-1}T^{-2}L^3 = ML^2T^{-2}$

(B) Pb = [pb] = $ML^{-1}T^{-2}L^3 = ML^2T^{-2}$

(C) $\frac{a}{v^2} = \left[\frac{a}{v^2}\right] = ML^5T^{-2}L^6 = ML^1T^{-2}$

(D) $\frac{ab}{v^2} = \left[\frac{ab}{v^2}\right] = ML^5T^{-2}L^3L^{-6} = ML^1T^{-2}$

$\Rightarrow$ (C) does not match

Sol 10: (D) $\frac{ab}{RT}$

Now we know ideal gas equation PV=RT

for one mole of ideal gas.

$$\Rightarrow \frac{ab}{RT} \equiv \frac{ab}{PV} \equiv \frac{PV^2.V}{PV} \equiv V^2 \equiv L^6$$

Sol 11: (A) $RT \equiv PV$

$\Rightarrow [M L^{-1}\ T^{-2}].[L^3]$

$\Rightarrow [M L^2\ T^{-2}]$

It is equal to energy.

Sol 12 – Sol 13: (B, D)

Power $\propto$ (size)^x (Density of air)^y

(Density of Helicopter $\times$ g]^z

$$[M L^2\ T^{-3}] \propto [L]^x [M L^{-3}]^y\ [M L^{-2}\ T^{-2}]^z$$

$$[M L^2\ T^{-3}] \propto [M^{y+2}\ L^{x-3y-2z}\ T^{-2z}]$$

Comparing the corresponding exponents;

$$y + z = 1$$

$x - 3y - 2z = 2$

$-2z = -3$

Solving them gives;

$\lambda = \frac{3}{2},\ y = -\frac{1}{2},\ x = \frac{7}{2}$

Sol 14: (C) $\frac{P_1}{P_2} = \left(\frac{L_1}{L_2}\right)^x$

$\frac{P_1}{P_2} = (4)^{7/2} = 128$

Match the Columns

Sol 15: A→r, B→ w, C→p, D→t, E→q, F→s, G→u, H→v

Angular momentum = MVR = $MLT^{-1}(L) = MLT^{-1}$

Latent heat = L^2T^{-2}

Torque = F×R = $MLT^{-2} \times L = ML^2T^{-2}$

Capacitance

$$= \frac{\text{Charge}}{\text{Potential Difference}} = \frac{AT}{ML^2T^{-3}A^{-1}} = M^{-1}L^{-2}T^4A^2$$

Inductance = $\frac{\text{Magnetic Flux}}{\text{Current}}$ = $ML^2T^{-2}A^{-2}$

Resistivity = $ML^3T^{-3}A^{-2}$

Magnetic Flux = $ML^3T^{-3}A^{-2}$ (B =F/qv)

Magnetic Energy Density = $ML^{-1}T^{-2}$

Sol 16: A→q, B→p, C→r, D→q

Angle and unit vector = $M^oL^oT^o$

Power = W/T=$M^1L^2T^{-3}$

Work = F,D=$M^1L^2T^{-2}$

Therefore, A→q, B→p, C→r, D→q

Sol 17 A→s, B→p, C→q, D→r

Force → F, Velocity → V, Energy → E

(p) $[F^1V^oE^1] \Rightarrow MLT^{-2}.\ ML^2T^{-2} \Rightarrow ML^3T^{-4}$ ((B) Light year)

(q) $[F^1V^oE^1] \Rightarrow MLT^{-2}.\ LT^{-1}.M^{-1}L^{-2}T^2 \Rightarrow T^{-1}$ ((C) Frequency)

(r) $[F^1V^oE^1] \Rightarrow M^3L^3T^{-2}.\ M^{-2}L^{-4}T^{-4} \Rightarrow ML^{-1}T^{-2}$ ((D) Pressure)

(s) $[F^1V^oE^1] \Rightarrow L^{-2}T^2.\ ML^2T^{-2} \Rightarrow M$ ((A) Mass)

Previous Years' Questions

Sol 1: (C) N= Number of electrons per unit volume

$\therefore [N] = [L^{-3}],\ [e] = [q] = [It] = [AT]$

$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

Substituting the dimension we can see that,

$$\left[\sqrt{\frac{Ne^2}{m\varepsilon_0}}\right] = [T^{-1}]$$

Angular frequency has also the dimension $[T^{-1}]$

Sol 2: (B) $\omega = 2\pi f = \frac{2\pi c}{\lambda} \therefore\ \lambda = \frac{2\pi c}{\omega} = \frac{2\pi c}{\sqrt{Ne^2/m\varepsilon_0}}$

Substituting the values, we get $\lambda \cong 600$ mm

Sol 3: (D) Momentum is first positive but decreasing. Displacement (or say position) is initially zero. It will first increase. At highest point, momentum is zero and displacement is maximum. After that momentum is downwards (negative) and increasing but displacement is decreasing. Only (D) option satisfies these conditions.

In all the given four figures, at mean position the position coordinate is zero.

At the same time mass is starting from the extreme position in all four case. In figures (C) and (D), extreme position is more than the initial extreme position. But due to viscosity opposite should be the case.

Correct answer is (B), because mass starts from positive extreme position (from uppermost position). Then, it will move downwards or, momentum should be negative.

Sol 4: (B) $t \equiv \frac{L}{R}$ $\quad \therefore L \equiv tR \equiv$ ohm-second

$U \equiv \frac{q^2}{2C} \quad \therefore\ C \equiv \frac{q^2}{U} \equiv \text{coulomb}^2/\text{joule}$

$q \equiv CV \quad \therefore C \equiv \frac{q}{V} \equiv \text{coulomb}/\text{volt}$

$L \equiv \frac{-e}{di/dt}$

$\therefore L \equiv \frac{e(dt)}{(di)} \equiv$ volt-second/ampere

$F = ilB$

$\therefore B \equiv \frac{F}{il} \equiv$ newton/ampere-metre

Sol 5:

Column I	Column II
Capacitance	Coulomb-volt coulomb^2 joule^{-1}
Inductance	Ohm-sec, volt second ampere^{-1}
Magnetic induction	Newton (ampere-metre)$^{-1}$

Sol 6: The correct table is as under

Column I	Column II
Angular momentum	$[ML^2T^{-1}]$
Latent heat	$[L^2T^{-2}]$
Torque	$[ML^2T^{-2}]$
Capacitance	$[M^{-1}L^{-2}T^2Q^2]$
Inductance	$[ML^2Q^{-2}]$
Resistivity	$[ML^3T^{-1}Q^{-2}]$

Sol 7: A $\rightarrow$ p, q; B $\rightarrow$ r, s; C $\rightarrow$ r, s; D $\rightarrow$ r, s

Sol 8: A $\rightarrow$ p, q, t; B $\rightarrow$ q; C $\rightarrow$ s, D $\rightarrow$ s

Sol 9: A $\rightarrow$ p, t; B $\rightarrow$ q, s, t; C $\rightarrow$ p, r, t, D $\rightarrow$ q

Sol 10: (A, C) Resistance = $ML^2T^{-3}A^{-2}$

Inductance = L = $ML^2T^{-2}A^{-2}$

Capacitance = C = $ML^{-1}T^{-2}A^2$

$$\therefore \frac{1}{\sqrt{LC}} = \frac{1}{T} = \text{Frequency} \quad \text{... (i)}$$

$$\text{Again } \frac{R}{L} = \frac{1}{T} = \text{Frequency} \quad \text{... (ii)}$$

Sol 11: (A, D) (A) Torque and work both have the dimensions $[ML^2T^{-2}]$.

(D) Light year and wavelength both have the dimension of length ie., $[L]$.

Sol 12: Reynold's number and coefficient of friction are dimensionless quantities.

Curie is the number of atoms decaying per unit time and frequency is the number of oscillations per unit time.

Latent heat and gravitational potential both have the same dimension corresponding to energy per unit mass.

Sol 13: (C) $F = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r^2}$

$$[\varepsilon_0] = \frac{[q_1][q_2]}{[F][r^2]} = \frac{[IT^2]}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4I^2]$$

Speed of light, $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$

$$\therefore [\mu_0] = \frac{1}{[\varepsilon_0][c]^2} = \frac{1}{[M^{-1}L^{-3}T^4I^2][LT^{-1}]^2} = [MLT^{-2}I^{-2}]$$

Sol 14: (A, B, D) (A) $L = \frac{\phi}{i}$ or henry$=\frac{\text{weber}}{\text{ampere}}$

(B) $e = -L\left(\frac{di}{dt}\right) \therefore L = -\frac{e}{(di/dt)}$

or henry$=\frac{\text{volt-second}}{\text{ampere}}$

(D) $U = \frac{1}{2}Li^2 = i^2 Rt$

$\therefore L = Rt$ or henry = ohm –second

Sol 15: (A, C) $T = \frac{40\,s}{20} = 2s.$

Further, $t = nT = 20T$ or $\Delta t = 20\,\Delta T$

$\therefore \frac{\Delta t}{t} = \frac{\Delta T}{T}$ or

$$\Delta T = \frac{T}{t}.\Delta t = \left(\frac{2}{40}\right)(1) = 0.05\text{ s}$$

Further, $T = 2\pi\sqrt{\frac{l}{g}}$ or $T \propto g^{-1/2}$

$$\therefore \frac{\Delta T}{T} \times 100 = -\frac{1}{2} \times \frac{\Delta g}{g} \times 100$$

or % error in determination of g is

$$\frac{\Delta g}{g}\times 100 = -200\times\frac{\Delta T}{T} = -\frac{200\times 0.05}{2} = -5\%$$

Sol 16: (A) $Y = \frac{4MLg}{\pi l d^2}$ & % y_{max} = %M + %L + %l + 2%d

Least count of both instrument, $\Delta\ell = \Delta d = \frac{0.5}{100} = 5\times 10^{-3}$

$$\%\ell\,\frac{\Delta\ell}{\ell}\times 100 = \frac{5\times 10^{-3}}{0.25} = 2\%$$

$$\%d\,\frac{\Delta d}{d}\times 100 = \frac{5\times 10^{-3}}{0.5}\times 100 = 1\%$$

Here we see that, contribution of ℓ, = 2%

Contribution of d = 2% d = 2 × 1 = 2%

Hence both terms ℓ and d contribute equally.

Sol 17: (B)

Main scale division (s) = .05 cm

Vernier scale division (v) = $\frac{49}{100}$ = .049

Least count = .05 – .049 = .001 cm

Diameter: 5.10+24 × .001 =5.124 cm

Sol 18: (4) $Y = \frac{FL}{\ell A}$ since the experiment measures only change in the length of wire

$$\therefore \frac{\Delta Y}{Y}\times 100 = \frac{\Delta\ell}{\ell}\times 100$$

From the observation ℓ_1 =MSR+20(LC)

ℓ_2 =MSR+40(LC)

⇒ Change in lengths = 25(LC)

and the maximum permissible error in elongation is one LC

$$\therefore \frac{\Delta Y}{Y}\times 100 = \frac{(LC)}{25(LC)}\times 100 = 4\ \%$$

Sol 19: (4) $E(t) = A^2 e^{-\alpha t}$

$$\Rightarrow dE = -\alpha A 2 e^{-\alpha t}dt + 2AdAe^{-\alpha t}$$

Putting the values for maximum error,

$$\Rightarrow \frac{dE}{E} = \frac{4}{100} \Rightarrow \%\ \text{error} = 4\ \%$$

Sol 20: (B, C) For vernier callipers,

1 main scale division = $\frac{1}{8}$ cm

1 vernier scale division = $\frac{1}{10}$ cm

So least count = $\frac{1}{40}$ cm

For screw gauge,

pitch (p) = 2 main scale division

So least count p = $\frac{P}{100}$

Sol 21: (C)

In first; main scale reading = 2.8 cm.

Vernier scale reading = $7\times\frac{1}{10} = 0.07$ cm

So reading = 2.87 cm;

In second; main scale reading = 2.8 cm

Vernier scale reading =

$$7\times\frac{-0.1}{10} = \frac{-0.7}{10} = -0.07\ \text{cm}$$

so reading = (2.80 + 0.10 – 0.07) cm = 2.83 cm

Sol 22: (A, B, D) Error in T

$$T_{mean} = \frac{0.52+0.56+0.57+0.54+0.59}{5} = 0.556 \approx 0.56s$$

$$\Delta T_{mean} = 0.02$$

$$\therefore \text{Error in T is given by } \frac{0.02}{0.56}\times 100 = 3.57\%$$

$$\text{Error in r} = \frac{1}{10}\times 100 = 10\%$$

Error in g

$$\because T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

$$T^2 = 4\pi^2\frac{7}{5}\left(\frac{R-r}{g}\right)$$

$$g = \frac{28\pi^2}{5}\left(\frac{R-r}{T^2}\right)$$

$$\frac{\Delta g}{g} = \left(\frac{\Delta R+\Delta r}{R-r}\right) + 2\frac{\Delta T}{T} = \frac{2}{50} + 2\times 0.0357$$

$$\therefore \frac{\Delta g}{g}\times 100 \approx 11\%$$