BINOMIAL THEOREM

$(x + y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_rx^{n-r}y^r + \ldots\ldots + {}^nC_ny^n$

$= \sum_{r=0}^{n} {}^nC_r x^{n-r} y^r$, where $n \in N$.

1. IMPORTANT TERMS IN THE BINOMIAL EXPANSION ARE :

(a) General term: The general term or the $(r+1)^{th}$ term in the expansion of $(x + y)^n$ is given by

$T_{r+1} = {}^nC_r\, x^{n-r} \cdot y^r$

(b) Middle term :

The middle term (s) is the expansion of $(x + y)^n$ is (are) :

(i) If n is even, there is only one middle term which is given by $T_{(n+2)/2} = {}^nC_{n/2} \cdot x^{n/2} \cdot y^{n/2}$

(ii) If n is odd, there are two middle terms which are $T_{(n+1)/2}$ & $T_{[(n+1)/2]+1}$

(c) Term independent of x :

Term independent of x contains no x ; Hence find the value of r for which the exponent of x is zero.

2. SOME RESULTS ON BINOMIAL COEFFICIENTS :

(a) ${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$

(b) ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

(c) $C_0 + C_1 + C_2 + \ldots\ldots = C_n = 2^n$, $C_r = {}^nC_r$

(d) $C_0 + C_2 + C_4 + \ldots\ldots = C_1 + C_3 + C_5 + \ldots\ldots = 2^{n-1}$, $C_r = {}^nC_r$

(e) $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = {}^{2n}C_n = \dfrac{(2n)!}{n!\,n!}$, $C_r = {}^nC_r$

3. Greatest coefficient & greatest term in expansion of $(x + a)^n$:

(a) If n is even, greatest binomial coefficient is ${}^nC_{n/2}$

If n is odd, greatest binomial coefficient is ${}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$

(b) For greatest term :

$$\text{Greatest term} = \begin{cases} T_p \ \& \ T_{p+1} & \text{if } \dfrac{n+1}{\left|\dfrac{x}{a}\right|+1} \text{ is an integer equal to p} \\ T_{q+1} & \text{if } \dfrac{n+1}{\left|\dfrac{x}{a}\right|+1} \text{ is non integer and} \in (q, q + 1), q \in I \end{cases}$$

4. BINOMIAL THEOREM FOR NEGATIVE OR FRACTIONAL INDICES :

If $n \in \mathbb{R}$, then $(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots \infty$ provided $|x| < 1$.

Note :

(i) $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots\ldots\ldots\ldots \infty$

(ii) $(1 + x)^{-1} = 1 - x + x^2 - x^3 + \ldots\ldots\ldots\ldots \infty$

(iii) $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots\ldots\ldots \infty$

(iv) $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots\ldots\ldots\ldots \infty$

5. EXPONENTIAL SERIES :

(a) $e^x = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots\ldots\infty$; where x may be any real or complex number & $e = \lim\limits_{n\to\infty}\left(1 + \dfrac{1}{n}\right)^n$

(b) $a^x = 1 + \frac{x}{1!}\ell na + \frac{x^2}{2!}\ell n^2 a + \frac{x^3}{3!}\ell n^3 a + ...\infty$, where $a > 0$

6. LOGARITHMIC SERIES :

(a) $\ell n(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\infty$, where $-1 < x \le 1$

(b) $\ell n(1 - x) = - x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - ....\infty$, where $-1 \le x < 1$

(c) $\ell n\frac{(1+x)}{(1-x)} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + ...\infty\right)$, $|x| < 1$