DPP - Daily Practice Problems

Date : ____ Start Time : ____ End Time : ____

MATHEMATICS CM18

SYLLABUS : Matrices

Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. If $B^n - A = I$

and $A = \begin{bmatrix} 26 & 26 & 18 \\ 25 & 37 & 17 \\ 52 & 39 & 50 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 5 & 1 \\ 7 & 1 & 6 \end{bmatrix},$

then n =

(a) 2 (b) 3
(c) 4 (d) 5

2. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then A^{100} :

(a) $2^{100}A$ (b) $2^{99}A$
(c) $2^{101}A$ (d) None of these

3. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then $A^T + A = I_2$, if

(a) $\theta = n\pi, n \in Z$ (b) $\theta = (2n+1)\frac{\pi}{2}, n \in Z$
(c) $\theta = 2n\pi + \frac{\pi}{3}, n \in Z$ (d) None of these

4. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, I is the unit matrix of order 2 and a, b are arbitrary constants, then $(aI + bA)^2$ is equal to

(a) $a^2I + abA$ (b) $a^2I + 2abA$
(c) $a^2I + b^2A$ (d) None of these

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. If A is a square matrix, then AA^T is a
(a) skew-symmetric matrix (b) symmetric matrix
(c) diagonal matrix (d) None of these

6. If $f(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and if α, β, γ, are angle of a triangle, then $f(\alpha).f(\beta).f(\gamma)$ equals
(a) I_2 (b) $-I_2$
(c) 0 (d) None of these

7. Let A, B and C be $n \times n$ matrices. Which one of the following is a correct statement?
(a) If $AB = AC$, then $B = C$
(b) If $A^3 + 2A^2 + 3A + 5I = 0$; then A is invertible.
(c) If $A^2 = 0$, then $A = 0$
(d) None of these

8. If $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then
(a) $A_\alpha . A_{(-\alpha)} = I$ (b) $A_\alpha . A_{(-\alpha)} = O$
(c) $A_\alpha . A_\beta = A_{\alpha\beta}$ (d) $A_\alpha . A_\beta = A_{\alpha-\beta}$

9. If A is a square matrix such that $(A-2I)(A+I) = O$, then $A^{-1} =$
(a) $\frac{A-I}{2}$ (b) $\frac{A+I}{2}$
(c) $2(A-I)$ (d) $2A+I$

10. A square matrix P satisfies $P^2 = I - P$, where I is the identity matrix. If $P^n = 5I - 8P$, then n is equal to
(a) 4 (b) 5
(c) 6 (d) 7

11. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P(Q^{2005})P^T$ equal to
(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \sqrt{3}/2 & 2005 \\ 1 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 2005 \\ \sqrt{3}/2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & \sqrt{3}/2 \\ 0 & 2005 \end{bmatrix}$

12. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$ and I is the unit matrix of order 3, then $A^2 + 2A^4 + 4A^6$ is equal to
(a) $7A^8$ (b) $7A^7$
(c) 8I (d) 6I

13. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then
(a) there cannot exist any B such that AB = BA
(b) there exist more than one but finite number of B′s such that AB = BA
(c) there exists exactly one B such that AB = BA
(d) there exist infinitely many B′s such that AB = BA

14. Given that
$$\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} \begin{bmatrix} k & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
then k =
(a) 6 (b) 1 (c) 8 (d) 9

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, then $(I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ is equal to

(a) $I + A$ (b) $I - A$ (c) $A - I$ (d) A

16. If $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ is the sum of a symmetric matrix B and skew-symmetric matrix C, then B is

(a) $\begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 5 & -2 \\ 2 & 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 6 & 7 \\ -6 & 2 & -5 \\ -7 & 5 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 6 & -2 \\ 2 & 0 & -2 \\ -2 & -2 & 0 \end{bmatrix}$

17. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(a) A (b) $I - A$ (c) $I + A$ (d) 3A

18. If B is an idempotent matrix, and $A = I - B$, then

(a) $A^2 = A$ (b) $A^2 = I$

(c) $AB = I$ (d) $BA = I$

19. Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $F(\alpha).\,F(\beta)$ is equal to

(a) $F(\alpha\beta)$ (b) $F\left(\frac{\alpha}{\beta}\right)$

(c) $F(\alpha + \beta)$ (d) $F(\alpha - \beta)$

20. For each real number x such that $-1 < x < 1$, let $A(x)$ be the matrix $(1 - x)^{-1}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \frac{x + y}{1 + xy}$ Then

(a) $A(z) = A(x) + A(y)$ (b) $A(z) = A(x)[A(y)]^{-1}$

(c) $A(z) = A(x)A(y)$ (d) $A(z) = A(x) - A(y)$

21. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then A^{16} is equal to :

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

22. If $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is square root of identity matrix of order 2 then –

(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 + \alpha^2 - \beta\gamma = 0$

(c) $1 - \alpha^2 + \beta\gamma = 0$ (d) $\alpha^2 + \beta\gamma = 1$

23. If A and B are matrices of same order, then $(AB' - BA')$ is a

(a) skew symmetric matrix (b) null matrix

(c) symmetric matrix (d) unit matrix

24. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the matrix A equals

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

RESPONSE GRID					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ

25. If A is symmetric as well as skew-symmetric matrix, then A is
(a) Diagonal (b) Null
(c) Triangular (d) None of these

26. If AB = A and BA = B, then B^2 is equal to
(a) B (b) A
(c) 1 (d) O

27. If A and B are two square matrices such that $B = -A^{-1}BA$, then $(A+B)^2 =$
(a) O (b) $A^2 + B^2$
(c) $A^2 + 2AB + B^2$ (d) A + B

28. Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then A^n is equal to
(a) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{bmatrix}$ (b) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$
(c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$ (d) $\begin{bmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & na \end{bmatrix}$

29. If $A = \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 \\ 7 & 2 \end{pmatrix}$ then which statement is true ?
(a) $AA^T = I$ (b) $BB^T = I$
(c) $AB \neq BA$ (d) $(AB)^T = I$

30. If A is any square matrix, then which of the following is skew-symmetric?
(a) $A + A^T$ (b) $A - A^T$ (c) AA^T (d) $A^TA - A$

RESPONSE GRID	
	25. ⓐⓑⓒⓓ 26. ⓐⓑⓒⓓ 27. ⓐⓑⓒⓓ 28. ⓐⓑⓒⓓ 29. ⓐⓑⓒⓓ 30. ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM DPP CHAPTERWISE 18 - MATHEMATICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	40	Qualifying Score	55
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

1. **(a)** $\because$ $B^n - A = I$

$\therefore$ $B^n = I + A$

$$B^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 26 & 26 & 18 \\ 25 & 37 & 17 \\ 52 & 39 & 50 \end{bmatrix}$$

$$B^n = \begin{bmatrix} 27 & 26 & 18 \\ 25 & 38 & 17 \\ 52 & 39 & 51 \end{bmatrix}$$

or $$\begin{bmatrix} 1 & 4 & 2 \\ 3 & 5 & 1 \\ 7 & 1 & 6 \end{bmatrix}^n = \begin{bmatrix} 27 & 26 & 18 \\ 25 & 38 & 17 \\ 52 & 39 & 51 \end{bmatrix} \quad \text{....... (i)}$$

$\therefore$ $n \neq 1$

Now put n = 2, then

$$B^2 = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 5 & 1 \\ 7 & 1 & 6 \end{bmatrix}^2 = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 5 & 1 \\ 7 & 1 & 6 \end{bmatrix}\begin{bmatrix} 1 & 4 & 2 \\ 3 & 5 & 1 \\ 7 & 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+12+14 & 4+20+2 & 2+4+12 \\ 3+15+7 & 12+25+1 & 6+5+6 \\ 7+3+42 & 28+5+6 & 14+1+36 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 26 & 18 \\ 25 & 38 & 17 \\ 52 & 39 & 51 \end{bmatrix}$$

Which is equal to R.H.S. of eq. (i).

$\therefore$ n = 2

2. **(b)** Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$$

$$A^3 = 2^2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, A^4 = 2^3\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$A^3 = 2^2 A,\quad A^4 = 2^3 A$

$\therefore$ $A^n = 2^{n-1}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow A^{100} = 2^{100-1} A$

$\therefore$ $A^{100} = 2^{99} A$

3. **(c)** We have, $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\therefore$ $A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Now, $A^T + A = I_2$ (given)

$$\Rightarrow \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow 2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{3}, n \in Z$

4. **(b)** $(aI + bA)^2 = a^2I^2 + b^2A^2 + 2ab\,AI$

$= a^2I^2 + b^2A^2 + 2abA$

But $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\therefore (aI + bA)^2 = a^2I + 2abA$.

5. **(b)** Let $A=\begin{bmatrix}1&-1&1\\2&1&0\\1&-1&2\end{bmatrix}$, then $A'=\begin{bmatrix}1&2&1\\-1&1&-1\\1&0&2\end{bmatrix}$

$$\therefore \quad AA'=\begin{bmatrix}1&-1&1\\2&1&0\\1&-1&2\end{bmatrix}\begin{bmatrix}1&2&1\\-1&1&-1\\1&0&2\end{bmatrix}=\begin{bmatrix}3&1&4\\1&5&1\\4&1&4\end{bmatrix}$$

6. **(b)** Hence $f(\alpha)f(\beta)=\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix}\begin{bmatrix}\cos\beta&\sin\beta\\-\sin\beta&\cos\beta\end{bmatrix}$

$$=\begin{bmatrix}\cos\alpha\cos\beta-\sin\alpha\sin\beta&\cos\alpha\sin\beta+\sin\alpha\cos\beta\\-\sin\alpha\cos\beta-\cos\alpha\sin\beta&-\sin\alpha\sin\beta+\cos\alpha\cos\beta\end{bmatrix}$$

$$=\begin{bmatrix}\cos(\alpha+\beta)&\sin(\alpha+\beta)\\-\sin(\alpha+\beta)&\cos(\alpha+\beta)\end{bmatrix}$$

similarly $f(\alpha)f(\beta)f(\gamma)=\begin{bmatrix}\cos(\alpha+\beta+\gamma)&\sin(\alpha+\beta+\gamma)\\-\sin(\alpha+\beta+\gamma)&\cos(\alpha+\beta+\gamma)\end{bmatrix}$

$$=\begin{bmatrix}\cos\pi&\sin\pi\\-\sin\pi&\cos\pi\end{bmatrix} \text{ as } \alpha+\beta+\gamma=\pi$$

$$=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}=-\begin{bmatrix}1&0\\0&1\end{bmatrix}=-I_2$$

7. **(b)** We have a theorem that if a square matrix A satisfies the equation

$$a_0+a_1x+a_2x^2+.......+a_nx^n=0,$$

where $a_0\neq 0$ then A is invertible.

Since A, B and C are $n\times n$ matrices and A satisfies the equation $x^3+2x^2+3x+5=0$ as $A^3+2A^2+3A+5I=0$, therefore, A is invertible.

8. **(a)** $A_\alpha\cdot A_{(-\alpha)}=\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix}$

$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha&\sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\\sin\alpha\cos\alpha-\sin\alpha\cos\alpha&\cos^2\alpha+\sin^2\alpha\end{bmatrix}$$

$$=\begin{bmatrix}1&0\\0&1\end{bmatrix}=I$$

9. **(a)** $(A-2I)(A+I)=0$

$\Rightarrow AA-A-2I=0 \qquad (\because AI=A)$

$\Rightarrow A\left(\dfrac{A-I}{2}\right)=I \qquad \therefore \dfrac{A-I}{2}=A^{-1}$

10. **(c)** $\because \quad P^3=P(I-P)$

$=PI-P^2=PI-(I-P)$

$=P-I+P=2P-I$

Now, $P^4=P.P^3$

$\Rightarrow \quad P^4=P(2P-I)$

$\Rightarrow \quad P^4=2P^2-P$

$\Rightarrow \quad P^4=2I-2P-P$

$\Rightarrow \quad P^4=2I-3P$

and $P^5=P(2I-3P)$

$\Rightarrow \quad P^5=2P-3(I-P)$

$\Rightarrow \quad P^5=5P-3I$

Also, $P^6=P(5P-3I)$

$\Rightarrow \quad P^6=5P^2-3P$

$\Rightarrow \quad P^6=5(I-P)-3P$

$\Rightarrow \quad P^6=5I-8P$

So, $n=6$

11. **(a)** Given $Q=PAP^T$

$\Rightarrow \quad P^TQ=AP^T, (\because PP^T=I)$

$\Rightarrow \quad P^TQ^{2005}P=AP^TQ^{2004}P=AP^TQ^{2003}PA$

$(\because Q=PAP^T\Rightarrow QP=PA)$

$=AP^TQ^{2002}PA^2=AP^TPA^{2004}$

$$=AIA^{2004}=A^{2005}=\begin{bmatrix}1&1\\0&1\end{bmatrix}^{2005}=\begin{bmatrix}1&2005\\0&1\end{bmatrix}$$

12. **(a)** $A^2=\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$A^2=A^4=A^6=I_3 \Rightarrow A^2+2A^4+4A^6$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}+\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$$

$$=\begin{bmatrix}7&0&0\\0&7&0\\0&0&7\end{bmatrix}=7I_3=7A^8$$

13. **(d)** $A=\begin{bmatrix}1&2\\3&4\end{bmatrix} \qquad B=\begin{bmatrix}a&0\\0&b\end{bmatrix}$

$$AB=\begin{bmatrix}a&2b\\3a&4b\end{bmatrix}$$

$$BA=\begin{bmatrix}a&0\\0&b\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}=\begin{bmatrix}a&2a\\3b&4b\end{bmatrix}$$

Hence, $AB=BA$ only when $a=b$

$\therefore$ There can be infinitely many B's for which $AB=BA$

14. **(b)** $\begin{bmatrix}1&\omega&\omega^2\\\omega&\omega^2&1\\\omega^2&1&\omega\end{bmatrix}\begin{bmatrix}k&1&1\\1&1&1\\1&1&1\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}k+\omega+\omega^2&1+\omega+\omega^2&1+\omega+\omega^2\\k\omega+\omega^2+1&\omega+\omega^2+1&\omega+\omega^2+1\\k\omega^2+1+\omega&\omega^2+1+\omega&\omega^2+1+\omega\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1+\omega+\omega^2+k-1&0&0\\1+\omega+\omega^2+k\omega-\omega&0&0\\1+\omega+\omega^2+k\omega^2-\omega^2&0&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}k-1&0&0\\(k-1)\omega&0&0\\(k-1)\omega^2&0&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

Which gives $k-1=0$ or $k=1$

15. (a) Here, $A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$, where $t = \tan\left(\frac{\alpha}{2}\right)$

Now, $\cos\alpha = \dfrac{1-\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \dfrac{1-t^2}{1+t^2}$

and $\sin\alpha = \dfrac{2\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \dfrac{2t}{1+t^2}$

$$= (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ +t & 0 \end{bmatrix}\right)\begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t(1-t^2)}{1+t^2} \\ \frac{-t(1-t^2)+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{1+t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

Also, $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0+1 & -t+0 \\ t+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

16. (a) If $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ is the sum of a symmetric matrix B and skew symmetric matrix C,

Transpose of $A = \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}$

So that $B = \frac{1}{2}\left[\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}\right]$

$$B = \frac{1}{2}\begin{bmatrix} 12 & 12 & 14 \\ 12 & 4 & 10 \\ 14 & 10 & 2 \end{bmatrix}$$

$$\therefore \quad B = \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$$

17. (a) $A^2 = I$

Now, $(A-I)^3 + (A+I)^3 - 7A$

$= A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3A^2I + 3AI^2 - 7A$

$= 2A^3 + 6AI^2 - 7A = 2A^2A + 6AI - 7A$

$= 2IA + 6A - 7A = 2A + 6A - 7A = A \quad [\because A^2 = I]$

18. (a) Since B is an idempotent matrix, $\therefore B^2 = B$.

Now, $A^2 = (I-B)^2 = (I-B)(I-B)$

$= I - IB - BI + B^2 = I - B - B + B^2 = I - 2B + B^2$

$= I - 2B + B = I - B = A$

$\therefore$ A is idempotent.

19. (c) $F(\alpha)\,.\,F(\beta) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$F(\alpha)\,.\,F(\beta) = \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 0 \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(\alpha+\beta)$$

20. (c) $A(z) = A\left(\frac{x+y}{1+xy}\right) = \left[\frac{1+xy}{(1-x)(1-y)}\right]$

$$\begin{bmatrix} 1 & -\left(\frac{x+y}{1+xy}\right) \\ -\left(\frac{x+y}{1+xy}\right) & 1 \end{bmatrix}$$

$\therefore$ $A(x).\,A(y) = A(z)$.

21. (d) We have $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Now, $A^2 = A\,.\,A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is identity matrix

$(A^2)^8 = (-I)^8 = I$

Hence, $A^{16} = I$

22. **(d)** $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \sqrt{I_2}$; $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 + \beta\gamma = 1$

23. **(a)** $(AB' - BA')' = (AB')' - (BA')'$

$= (B')' A' - (A')' B' = BA' - AB' = -(AB' - BA')$

Hence, $(AB' - BA')$ is a skew-symmetric matrix.

24. **(a)** Let $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$

Given $BAC = I \Rightarrow B^{-1}(BAC) = B^{-1}I$

$\Rightarrow I(AC) = B^{-1} \Rightarrow AC = B^{-1}$

$\Rightarrow ACC^{-1} = B^{-1}C^{-1} \Rightarrow AI = B^{-1}C^{-1} \therefore A = (B^{-1})(C^{-1})$

Now $B^{-1} = \frac{1}{4-3}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$C^{-1} = \frac{1}{9-10}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$\therefore (B^{-1})(C^{-1}) = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

25. **(b)** Let $A = [a_{ij}]_{n \times m}$. Since A is skew–symmetric $a_{ii} = 0$ $(i = 1, 2, ..., n)$ and $a_{ji} = -a_{ji} (i \neq j)$

Also, A is symmetric so $a_{ji} = a_{ji}$ $\forall$ i and j

$\therefore$ $a_{ji} = 0$ $\forall$ $i \neq j$

Hence $a_{ij} = 0$ $\forall$ i and j $\Rightarrow$ A is a null zero matrix

26. **(a)** Since $BA = B$, $\therefore (BA)B = BB = B^2$

$\Rightarrow$ $B(AB) = B^2$ $\Rightarrow$ $BA = B^2$ $(\because AB = A)$

$\Rightarrow$ $B = B^2$ $(\because BA = B)$

27. **(b)** $B = -A^{-1}BA \Rightarrow AB = -BA \Rightarrow AB + BA = 0$

$\therefore (A + B)^2 = A^2 + AB + BA + B^2 = A^2 + B^2$

28. **(c)** $A^2 = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$

$A^3 = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} = \begin{bmatrix} a^3 & 0 & 0 \\ 0 & a^3 & 0 \\ 0 & 0 & a^3 \end{bmatrix}$

29. **(d)** Here $AA^T = \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}\begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$(BB^T)_{11} = (d)^2 + (a)^2 \neq 1$

$(AB)_{11} = 8 - 7 = 1, (BA)_{11} = 8 - 7 = 1$

$\therefore AB \neq BA$ may be not true

Now $AB = \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 7 & 2 \end{pmatrix}$

$= \begin{pmatrix} 8-7 & 2-2 \\ -28+28 & -7+8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $(AB)^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

30. **(b)** $(A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T)$

Hence, $(A - A^T)$ is skew-symmetric.