Circular Motion

Definition

If a particle moves in a plane such that its distance from a fixed point remains constant then its motion is called as circular motion with respect to that fixed.

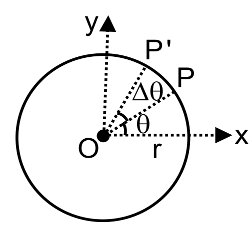
That fixed point is called the centre and the corresponding distance is called the radius of circular path. The vector joining the centre of the circle and the particle performing circular motion, directed towards the later is called the radius vector. It has constant magnitude but variable direction.

1. Terminology Related to Circular Motion

1.1 Angular Position

The angle made by the position vector w.r.t. origin, with the reference line is called angular position.

Clearly angular position depends on the choice of the origin as well as the reference line.



The angular position of particle P at a given instant may be described by the angle θ between OP and OX. This angle θ is called the angular position of the particle.

1.2 Angular Displacement

Angle traced by the position vector of particle moving w.r.t. some fixed point is called angular displacement.

$\Delta\theta$ = angular displacement

$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

$$\Delta\theta = \frac{\text{arc } PP'}{r}$$

Key Points

- Small angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is not a vector quantity.

  $d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$ But $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$

- Direction of angular displacement is perpendicular to the plane of rotation and is given by right hand thumb rule.

- Angular displacement is dimensionless and its S.I. unit is radian while other units are degree and revolution. 2π radian = 360° = 1 revolution

Example 1:

A particle moving on 7m radius circular path. If it covers 21 m distance on circular path then find angular displacement.

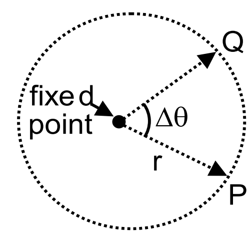
Solution:

$$\Delta\theta = \text{angular displacement} = \frac{\text{arc}}{\text{radius}} = \frac{21}{7} = 3 \text{ rad}$$

$$\text{in degree } \Delta\theta = 3 \times \frac{180}{\pi} = \frac{540^{\circ}}{\pi}$$

1.3 Angular Velocity (ω)

It is defined as the rate of change of angular displacement of a moving particle, w.r.t. to time.

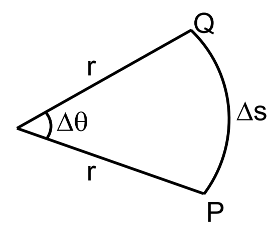


$$\omega = \frac{\text{angle traced}}{\text{time taken}} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$\Rightarrow$ Its unit is rad/s and dimensions is $[T^{-1}]$. It is a axial vector.

1.4 Relation Between Linear and Angular Velocity

$$\text{Angel } (\Delta\theta) = \frac{\text{arc}}{\text{radius}} = \frac{\Delta s}{r} \Rightarrow \Delta s = r\Delta\theta$$



$$\therefore \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} \text{ if } \Delta t \to 0 \text{ then } \frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow v = \omega r$$

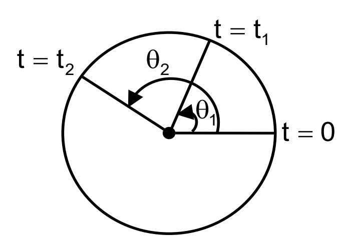
In vector form $\vec{v} = \vec{\omega} \times \vec{r}$ (direction of $\vec{v}$ is according to right hand thumb rule)

1.5 Instantaneous Angular Velocity

The angular velocity at some particular instant

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

1.6 Average Angular Velocity (ω_{av})



$$\omega_{av} = \frac{\text{total angle rotation}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are the angular positions of the particle at instants t_1 and t_2.

1.7 Frequency (n)

Number of revolutions described by particle per second is its frequency. Its unit is revolutions per second (rps) or revolutions per minute (rpm).

Note : 1 rps = 60 rpm

1.8 Time Period (T)

It is the time taken by particle to complete one revolution. i.e. T = 1/n

1.9 Relative Angular Velocity

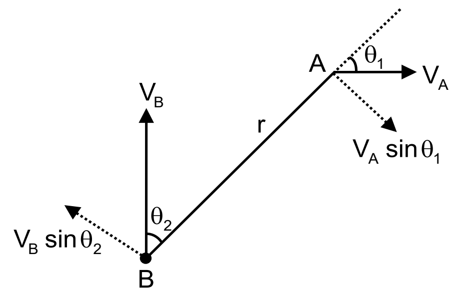
Relative angular velocity of a particle 'A' w.r.t. an other moving particle B is the angular velocity of the position vector of 'A' w.r.t. B. It means the rate at which the position vector of 'A' w.r.t. B rotates at the instant.

$$\omega_{AB} = \frac{(v_{AB})_{\perp}}{r_{AB}}$$

$$= \frac{\text{Relative velocity of A w.r.t B perpendicular to line AB}}{\text{separation between A and B}}$$

Here $(v_{AB})_{\perp} = v_A \sin\theta_1 + v_B \sin\theta_2$

$$\therefore \ \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$$

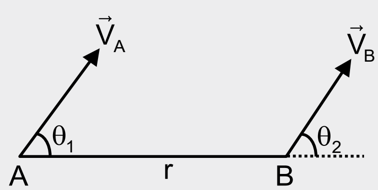


Key Points

- Instantaneous angular velocity is an axial vector quantity.
- Direction of angular velocity is same as that of angular displacement i.e., perpendicular to the plane of rotation and along the axis according to right hand screw rule or right hand thumb rule.
- If particles A and B are moving with a velocity $\vec{v}_A$ and $\vec{v}_B$ are separated by a distance r at a given instant then

(i) $\frac{dr}{dt} = v_B \cos\theta_2 - v_A \cos\theta_1$

(ii) $\frac{d\theta_{AB}}{dt} = \omega_{AB} = \frac{v_B \sin\theta_2 - v_A \sin\theta_1}{r}$



Example 2:

A particle moves on circular track of 7 cm radius its angular displacement is 225° in 50 sec find angular velocity of a particle.

Solution:

$$\omega = \frac{225^\circ}{50} \times \frac{\pi}{180} \qquad = \frac{\pi}{40} \text{rad/sec}$$

Example 3:

A particle revolving in a circular path completes first one third of the circumference in 2 s, while next one third in 1s. Calculate its average angular velocity.

Solution:

$$\theta_1 = \frac{2\pi}{3} \text{ and } \theta_2 = \frac{2\pi}{3}$$

total time T = 2 + 1 = 3 s

$$\therefore \ \omega_{av} = \frac{\theta_1 + \theta_2}{T} = \frac{\frac{2\pi}{3} + \frac{2\pi}{3}}{3} = \frac{\frac{4\pi}{3}}{3} = \frac{4\pi}{9} \text{rad / s}$$

Example 4:

In the above question linear speed of the point on the rim at t = 2s is :

Solution:

$\theta = 10 - 5t + 4t^2$

$\omega = \dfrac{d\theta}{dt}$

$= 0 - 5 + 8t = 8t - 5$

'ω' at t = 2

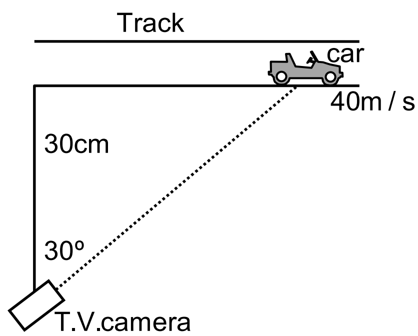
$\Rightarrow \quad 8 \times 2 - 5$

$= 16 - 5 = 11$ rad/sec

$\{v = \omega r = 11 \times 6 = 66$ cm/sec$\}$

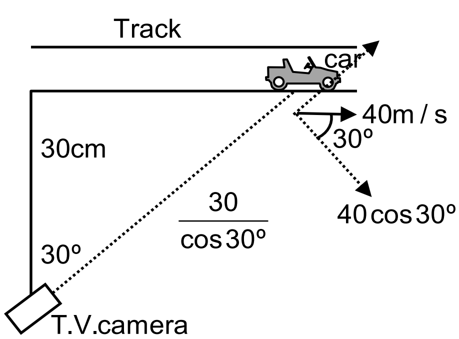
Example 5:

A racing car is travelling along a track at a constant speed of 40 m/s. A T.V. camera men is recording the ever from a distance of 30 m directly away from the track as shown in figure. In order to keep the car under view in the position shown, the angular speed with which the camera should be rotated, is :



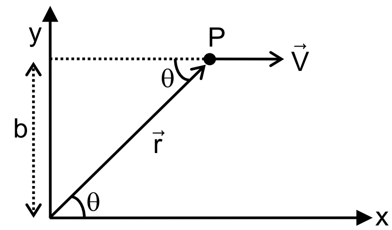
Solution:

$$\omega = \frac{v_\perp}{r} = \frac{40\cos 30^\circ}{\dfrac{30}{\cos 30^\circ}} = 1 \text{ rad/sec}$$

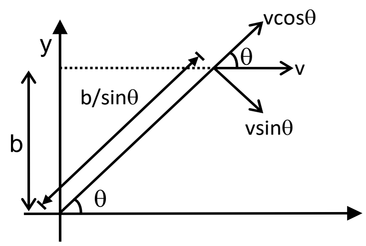


Example 6:

A particle is moving parallel to x-axis as shown in figure such that the y component of its position vector is constant at all instants and is equal to 'b'. Find the angular velocity of the particle about the origin when its radius vector makes an angle θ with the x-axis.



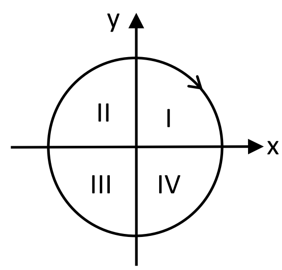
Solution:



$$\therefore \omega_{PQ} = \frac{v\sin\theta}{\frac{b}{\sin\theta}} = \frac{v}{b}\sin^2\theta$$

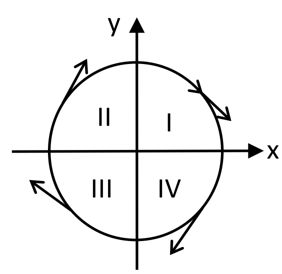
Example 7:

A particle is moving in clockwise direction in a circular path shown in figure. The instantaneous velocity of particle at a certain instant is $\vec{v} = (3\hat{i} + 3\hat{j})m/s$. Then in which quadrant does the particle lie at that instant? Explain your answer.



Solution:

II quadrant. According to following figure x & y components of velocity are positive when the particle is in II quadrant.



1.10 Angular Acceleration (α)

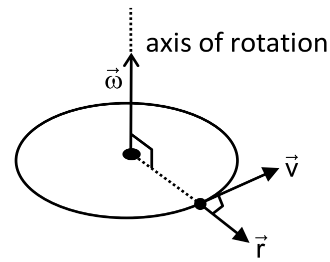
Rate of change of angular velocity is called angular acceleration i.e., $\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$

1.11 Average Angular Acceleration

$$\vec{\alpha}_{av} = \frac{\text{change in angular velocity}}{\text{time taken}}$$

$$= \frac{\Delta\vec{\omega}}{\Delta t} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1}$$

1.12 Relation Between Angular and Linear Acceleration



Velocity $\vec{v} = \vec{\omega} \times \vec{r}$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$

$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega}\frac{d\vec{r}}{dt}$

$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a} = \vec{a}_T + \vec{a}_C$

($\vec{a}_T = \vec{\alpha} \times \vec{r}$ is tangential acceleration & $\vec{a}_C = \vec{\omega} \times \vec{v}$ is centripetal acceleration)

$\vec{a} = \vec{a}_T + \vec{a}_C$ ($\vec{a}_T$ and $\vec{a}_C$ are the two component of net linear acceleration)

As $\vec{a}_T \perp \vec{a}_C$ so $|\vec{a}| = \sqrt{a_T^2 + a_C^2}$

1.13 Tangential Acceleration

$a = \vec{\alpha} \times \vec{r}$, its direction is parallel (or antiparallel) to velocity. $\vec{v} = \vec{\omega} \times \vec{r}$ as $\vec{\omega}$ and $\vec{\alpha}$ **both parallel (or antiparallel) and along the axis.**

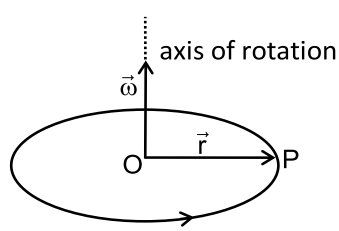
Magnitude of tangential acceleration in case of circular motion :

$a_T = \alpha r \sin 90° = \alpha r$ ($\vec{\alpha}$ is axial, $\vec{r}$ is radial so that $\vec{\alpha} \perp \vec{r}$)

As $\vec{a}_T$ is along the direction of motion (in the direction of $\vec{v}$) so $\vec{a}_T$ is responsible for change in speed of the particle. Its magnitude is the rate of change of speed of the particle.

1.14 Centripetal Acceleration

$\vec{a}_c = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r})(\because \vec{v} = \vec{\omega} \times \vec{r})$



Let $\vec{r}$ be in $\hat{i}$ direction and $\vec{\omega}$ be in $\hat{j}$ direction then the direction of $\vec{a}_c$ is along

$\hat{j} \times (\hat{j} \times \hat{i}) = \hat{j} \times (-\hat{k}) - \hat{i}$, opposite to the direction of $\vec{r}$ i.e., from P to O and it is centripetal in direction.

Magnitude of centripetal acceleration, $a_c = \omega v = \frac{v^2}{r}\omega^2 r$ therefore $\vec{a}_c = \frac{v^2}{r}(-\vec{r})$

Note : Centripetal acceleration is always perpendicular to the velocity at each point.

Key Points

- Angular acceleration is an axial vector quantity. It's direction is along the axis according to the right hand thumb rule or right hand screw rule.
- **Important difference between projectile motion and uniform circular motion:**
  In projectile motion, both the magnitude and the direction of acceleration (g) remains constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

Example 8:

A stone tied to the end of a 80 cm long string is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, the magnitude of its acceleration is :

(1) 20 ms^{-2} (2) 12 ms^{-2} (3) 9.9 ms^{-2} (4) 8 ms^{-2}

Solution:

Angular speed = $\frac{14 \times 2\pi}{25} = \frac{28\pi}{25}$

Acceleration = $\frac{v^2}{r} = \omega^2 r = \left(\frac{28\pi}{25}\right)^2 \times \frac{80}{100}$ = 9.9 m/sec^2.

Concept Builder-1

Q.1 An ant trapped in a circular groove of radius 13 cm moves along the groove steadily complete 14 revolution is 50 sec. What is angular speed.

Q.2 Two particles move in concentric circles of radii r_1 and r_2 such that they maintain a straight line through the centre. Find the ratio of their angular velocities.

Q.3 What is the ratio of angular speeds of minutes hand and hour hand of a watch is:

Q.4 A particle starts moving along a circle of radius (20/π) m with a constant tangential acceleration. If the velocity of the particle is 50 m/s at the end of the second revolution after the motion has began, the tangential acceleration is m/s^2 is :

(1) 1.6 (2) 4 (3) 15.6 (4) 13.2

Q.5 A particle rotates along a circle of radius $R = \sqrt{2}m$ with an angular acceleration $\alpha = \frac{\pi}{4} rad/s^2$ starting from rest. Calculate the magnitude of average velocity of the particle over the time it rotates a quarter circle.

Q.6 If the radii of circular paths of two particles are in the ratio of 1 : 2 then in order to have same centripetal acceleration, their speeds should be in the ratio of :

(1) 1 : 4 (2) 4 : 1 (3) $1 : \sqrt{2}$ (4) $\sqrt{2} : 1$

2. Uniform and Non-Uniform Circular Motion

2.1 Uniform Circular Motion

If a particle moves with a constant speed in a circle, the motion is called uniform circular motion. In uniform circular motion a resultant non-zero force acts on the particle. This force is provided by some external such as friction, magnetic force, coulomb force, gravitational force, tension. etc.

Key Points

- Speed = constant
- Velocity ≠ constant (because its direction continuously changes). But magnitude of velocity is constant.
- $\vec{\omega}$ = constant (because magnitude and direction, both are constants)
- Tangential acceleration

  $a_T = 0 \left[\because a_T = \frac{dv}{dt} = \frac{d(\text{constant})}{dt} = 0\right]$
- Angular acceleration $\alpha = 0$
- Direction of acceleration of particle is towards centre and its magnitude

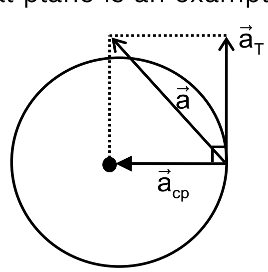
  $|\vec{a}| = |\vec{a}_{cp}| = \omega v = \omega^2 r = \frac{v^2}{r}$ = constant
- $= \vec{a} = \vec{a}_{cp} \neq$ constant

  (because the direction of a_{cp} is toward the centre of circle which changes as the particle revolves)
- The direction of the resultant force F is therefore, towards the centre and its magnitude is

  $F = \frac{mv^2}{r} = mr\omega^2 (\text{as } v = r\omega)$
- K.E. = $\frac{1}{2}mv^2$ = constant
- Uniform circular motion is usually executed in a horizontal plane.

2.2 Non-Uniform Circular Motion

If a particle moves with variable speed in a circle, then the motion is called non uniform circular motion. Circular motion in vertical plane is an example of non-uniform circular motion.



Key Points

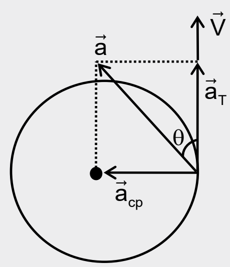
Acceleration (a) has two components :-

$\vec{a}_{CP}$ = responsible for change in direction only.

$\vec{a}_T$ = responsible for change in speed only.

Hence due to a_T speed = | velocity | is variable,

- $|\vec{\omega}| = \frac{v}{r}$
- $a_T \neq 0$
- $\alpha \neq 0$
- $|\vec{a}_{cp}| = \omega v = \omega^2 r = \frac{v^2}{r}$
- $\vec{a} = \vec{a}_T + \vec{a}_{cp}$
- $|\vec{a}| = \sqrt{a_{T^2} + a_{cp^2}} = \sqrt{(\alpha r)^2 + \left(\frac{v^2}{r}\right)^2}$
- $\vec{F} = \vec{F}_T + \vec{F}_{cp}$
- $K.E. = \frac{1}{2}mv^2$
- Angle between velocity and acceleration is given by : $\tan\theta = \frac{a_{cp}}{a_T} = \frac{F_{cp}}{F_T}$



2.3 Equations of Circular Motion

Initial angular velocity (ω_0)

Final angular velocity (ω)

Angular displacement (θ)

Angular acceleration (α)

If α = constant, then

$$\vec{\omega} = \vec{\omega}_0 + \vec{\alpha}t$$

$$\vec{\theta} = \vec{\omega}_0 t + \frac{1}{2}\vec{\alpha}t^2$$

$$\vec{\omega}^2 = \vec{\omega}_0^{\,2} + 2\alpha\theta$$

$$\theta_{n^{th}} = \omega_0 + \frac{\alpha}{2}(2n - 1)$$

$$\theta = \left(\frac{\omega_0 + \omega}{2}\right)t$$

Example 9:

A particle is performing circular motion of radius 1m. Its speed is $v = (2t^2)$ m/s. What will be the magnitude of its acceleration at t = 1s?

Solution:

Tangential acceleration = $a_T = \frac{dv}{dt} = 4t$

at t = 1s, $a_T = 4\ m/s^2$

Centripetal acceleration $a_C = \frac{v^2}{r} = \frac{4t^4}{1} = 4t^4$

at t = 1s, $a_C = 4\ m/s^2$

Net acceleration

$(a) = \sqrt{a_T^2 + a_C^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2} m/s^2$

Example 10:

A disc starts from rest and gains an angular acceleration given by $\alpha = 3t - t^2$ (where t is in seconds) upon the application of a torque. Calculate its angular velocity after 2 s.

Solution:

$\alpha = \frac{d\omega}{dt} = 3t - t^2 \Rightarrow \int_0^\omega d\omega = \int_0^1 (3t - t^2)dt$

$\Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} \Rightarrow$ at t = 2 s, $\omega = \frac{10}{3} rad/s$

Example 11:

The tangential and angular acceleration of a particle are 10 m/sec^2 and 5 rad/sec^2 respectively it will be at a distance from the axis of rotation:

Solution:

$a_t = 10\ m/s^2$

$\alpha = 5\ rad/sec^2$

$a_t = \alpha R$

$10 = 5 \times R$

$R = 2$ m

Example 12:

A grinding wheel attained a angular velocity of 20 rad/sec in 5 sec starting from rest. Find the number of revolutions made by the wheel.

Solution:

$\omega_f = \omega_0 + \alpha t$

$20 = 0 + \alpha \times 5$

$\alpha = 4 rad/sec^2$

$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \Rightarrow \theta = 0 + \frac{1}{2} \times 4 \times 25$

$\{\theta = 2 \times 25 = 50\ rad\}$

$\{$No. of revolution $= \frac{\theta}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi}\}$

Concept Builder-2

Q.1 A car is moving in a circular path off radius 500 m with speed of 30 m/sec. If its speed is increasing at the rate of 2 m/sec^2, the resultant acceleration will be **:**

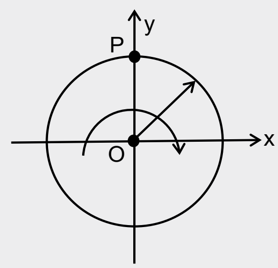
Q.2 If angular velocity of a particle depends on the angle rotated θ as $\omega = \theta^2 + 2\theta$, then its angular acceleration α at $\theta = 1$ rad is :

(1) 8 rad/s^2 (2) 10 rad/s^2 (3) 12 rad/s^2 (4) None of these

Q.3 A particle is moving on a circular path of radius 6 m. Its linear speed is v = 2t, here t is time in second and v is in m/s. Calculate its centripetal acceleration at t = 3 s.

Q.4 The angular velocity of a particle is given by $\omega = 1.5t - 3t^2 + 2$. (where t is in seconds). Find the instant when its angular acceleration becomes zero.

Q.5 A ring rotates about z axis as shown in figure. The plane of rotation is xy. At a certain instant the acceleration of a particle P (shown in figure) on the ring is $(6\hat{i} - 8\hat{j}) m/s^2$.



Find the angular acceleration of the ring and its angular velocity at that instant. Radius of the ring is 2 m.

Q.6 A wheel starts rotating at 10 rad/sec and attains the angular velocity of 100 rad/sec in 15 seconds. What is the angular acceleration in rad/sec^2 ?

3. Dynamics of Circular Motion

3.1 Centripetal Force

A force causing a centripetal acceleration towards the centre of the circular path and causes a change in the direction of velocity vector is called centripetal force

$$F = \frac{mv^2}{r} = m\omega^2 r$$

3.2 Centrifugal Force

- It gives a physical sensation of being centrifuged (pushed radially away).
- It’s magnitude is same as centripetal but direction is opposite to centripetal force.
- It never exists in inertial frame
- It is a pseudo force. It has no action-reaction pair.

Example 13:

The breaking tension of a string is 100 N. A particle of mass 0.1 kg tied to it, is rotated along horizontal circle of radius 0.5 meter. The maximum speed with which the particle can be rotated without breaking the string is :

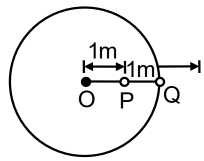
(1) $\sqrt{5}$ m / sec (2) $\sqrt{(50)}$ m / sec (3) $\sqrt{(500)}$ m / sec (4) $\sqrt{(1000)}$ m / sec

Solution:

$$T = \frac{mv^2}{r} \qquad \Rightarrow v = \sqrt{\frac{Tr}{m}} = 10\sqrt{5}\,m/s$$

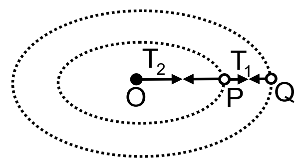
Example 14:

Two sphere of equal mass are attached to a string of length 2 m as shown in figure. The string and the spheres are then rotated in horizontal circle about 'O' at constant rate. The value of ratio of tension in string PQ & PO :



(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 2

Solution:



$T_1 = m\omega^2 R$

$T_1 = m\omega^2 (2)$

$T_2 - T_1 = m\omega^2$

$\Rightarrow T_2 = 3m\omega^2$

so $\frac{T_1}{T_2} = \frac{2}{3}$

Concept Builder-3

Q.1 A string of length 1 m is fixed at one end and carries a mass of 100 gm at the other end. The string makes $(2/\pi)$ revolutions per second around vertical axis through the fixed end. Calculate the tension in the string :

Q.2 A particle of mass m rotates in a circle of radius 'a' with a uniform angular speed ω. It is viewed from a frame rotating about the Z-axis with a uniform angular speed ω_0. The centrifugal force on the particle is :

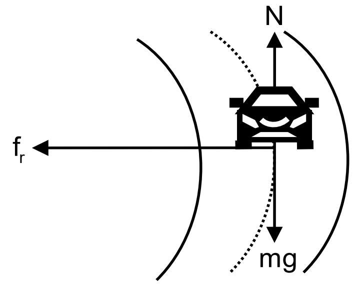
Q.3 A man of mass 60 kg standing on horizontal turn table of radius 2 m. If coefficient of friction between man and table is 0.3. Then find maximum value of angular velocity about table aries when man does not slip.

3.3 Circular Turning of Roads

When vehicles travel along a nearly circular arc. There must be some force which provides the required centripetal acceleration. In real life the necessary centripetal force is provided by friction and banking of roads both. The necessary centripetal force is being provided to the vehicles by the following :

- **By Friction Only**

  Suppose a car of mass m is moving with a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force will be provided to the car by the force of friction f acting towards centre of the circular path.



Thus, $f = \frac{mv^2}{r}$ $\therefore\ f_{max} = \mu N = \mu mg$

Therefore for a safe turn without skidding $\frac{mv^2}{r} \le f_{max} \Rightarrow \frac{mv^2}{r} \le \mu mg \Rightarrow \boxed{v \le \sqrt{\mu rg}}$

Example 15:

Find the maximum speed at which a car can turn round a curve of 30 m radius on a levelled road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity = 10 m/s^2]

Solution:

Here centripetal force is provided by friction so

$$\frac{mv^2}{r} \le \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 ms^{-1}$$

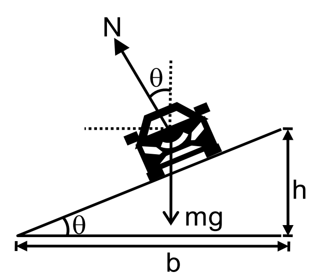
- **By Banking of Roads Only**

  Friction is not always reliable at turns particularly when high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn in the sense that the outer part of the road is some what lifted compared to the inner part.

$N \sin\theta = \frac{mv^2}{r}$ and $N \cos\theta = mg$

$\Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$ $\because \tan\theta = \frac{h}{b}$

Note : $\tan\theta = \frac{v^2}{rg} = \frac{h}{b}$



Example 16:

For traffic moving at 60 km/h, if the radius of the curve is 0.1 km, what is the correct banking angle of the road ? (g = 10 m/s^2)

Solution:

In case of banking $\tan\theta = \frac{v^2}{rg}$. Here v = 60 km/h $60 \times \frac{5}{18} ms^{-1} = \frac{50}{3} ms^{-1}$

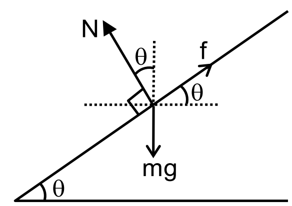
$r = 0.1 km = 100\ m$

So $\tan\theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$

- **Friction and Banking of Road Both**

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction. The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force, i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_{max} = \mu N$). So, direction and the magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the centre.

(a) If speed of the vehicle is small then friction acts outwards.



In this case,

$N\cos\theta + f\sin\theta = mg$ .....(i)

and $N\sin\theta - f\cos\theta = \frac{mv^2}{R}$ ....(ii)

For minimum speed $f = \mu N$

So, by dividing equation (i) by equation (ii)

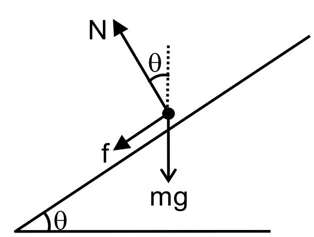
$$\frac{N\cos\theta + \mu N\sin\theta}{N\sin\theta - \mu N\cos\theta} = \frac{mg}{mv_{min}^2 / R};$$

Therefore $v_{min} = \sqrt{Rg\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$

If we assume $\mu = \tan\phi$, then

$$v_{min} = \sqrt{Rg\left(\frac{\tan\theta - \tan\phi}{1 + \tan\phi\tan\theta}\right)} = \sqrt{Rg\tan(\theta - \phi)}$$

(b) If speed of vehicle is high then friction force act inwards.



in this case for maximum speed

$N\cos\theta - \mu N\sin\theta = mg$

and $N\sin\theta + \mu N\cos\theta = \frac{mv_{max}^2}{R}$

which gives

$$v_{max} = \sqrt{Rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)} = \sqrt{Rg\tan(\theta + \phi)}$$

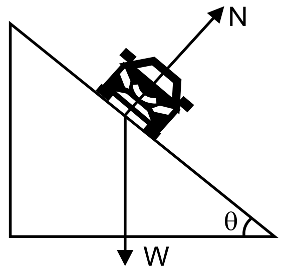
Hence for successful turning on a rough banked road, velocity of vehicle must satisfy following relation

$$\sqrt{Rg\tan(\theta - \phi)} \le v \le \sqrt{Rg\tan(\theta + \phi)}$$

where θ = banking angle and $\phi = \tan^{-1}(\mu)$.

Example 17:

A car is moving along a banked road laid out as a circle of radius r. (a) What should be the banking angle θ so that the car travelling at speed v needs no frictional force from the tyres to negotiate the turn ? (b) The coefficients of friction between tyres and road are μ_s = 0.9 and μ_k = 0.8. At what maximum speed can a car enter the curve without sliding toward the top edge of the banked turn ?



Solution:

(a) the banking angle θ so that car travelling at speed υ needs no frictional force from the tyres to negotiate the turn $\Rightarrow \tan\theta = v^2 / rg$

(b) The coefficients of friction between tyres and road are μ_s = 0.9 and μ_k = 0.8.

$$\Rightarrow V_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)} = \sqrt{rg\tan(\theta + \phi)}$$

3.4 Conical Pendulum

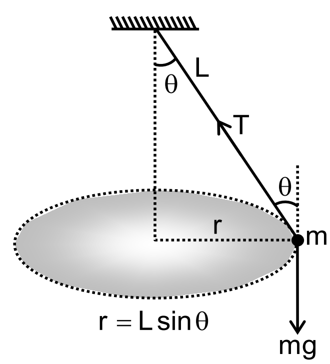
If a small particle of mass m tied to a string is whirled along a horizontal circle, as shown in figure then the arrangement is called a conical pendulum. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,

$$T\sin\theta = \frac{mv^2}{r} \text{ and } T\cos\theta = mg \Rightarrow v = \sqrt{rg\tan\theta}$$

$$\therefore \text{ Angular speed } \omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}}$$

So, the time period of pendulum is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$



Example 18:

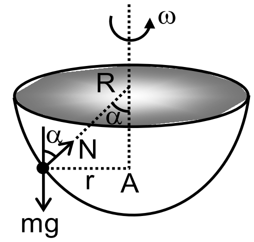
When the string of $5/\sqrt{3}\,m$ of conical pendulum makes an angle 30° with vertical. Find its time period?

Solution:

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}} = 2 \times \pi\sqrt{\frac{5/\sqrt{3} \times \sqrt{3}/2}{10}} = \pi$$

Example 19:

A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.



Solution:

$N \cos\alpha = mg$ .....(1)

$N \sin\alpha = mr\omega^2$ .....(2)

$r = R\sin\alpha$ .....(3)

Form equations (2) and (3)

$N\sin\alpha = m\omega^2 R\sin\alpha$

$N = mR\omega^2$ .....(4)

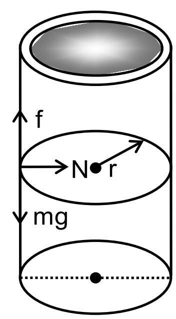
$\Rightarrow (mR\omega^2)\cos\alpha = mg \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$

3.5 Death Well or Rotor:

In case of ‘death well’ a person drives a motorcycle on the vertical surface of a large wooden well while in case of a rotor a person hangs resting against the wall without any support from the bottom at a certain angular speed of rotor.

In **death well** walls are at rest and person revolves while in case of **rotor** person is at rest and the walls rotate. In both cases friction balances the weight of person while reaction provides the centripetal force for circular motion, i.e.,

$f = mg$ and $N = \frac{mv^2}{r} = mr\omega^2$



Death well

Concept Builder-4

Q.1 A car has to move on a level turn of radius (R = 45 m). If the coefficient of static friction between tyre and road is $\mu = 0.2$. Find the maximum speed the car can take without skidding is given by:

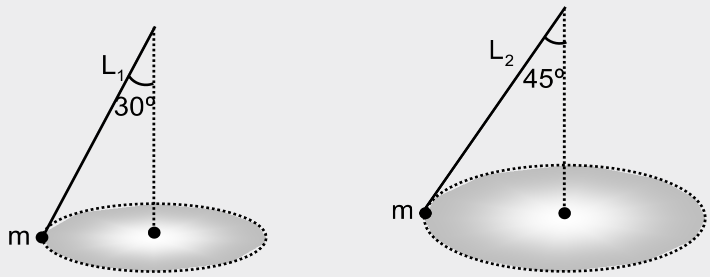
(1) 20 m/s (2) 10 m/s (3) 9.39 m/s (4) 25 m/s

Q.2 A car of mass 1000 kg moves on a circular track of radius 20 m. It the coefficient of friction is 0.64, what is the maximum velocity with which the car can be moved ?

Q.3 A road is 8 m wide. Its average radius of curvature is 40 m. The outer edge is above the lower edge by a distance of 1.28 m. Find the velocity of vehicle for which the road is most suited ? ($g = 10\ m/s^2$)

Q.4 A circular track has radius of 20 m. If banking angle is 45° then find optimum speed on circular track.

Q.5 Two particles tied to different strings are whirled in a horizontal circle as shown in figure. The ratio of lengths of the strings so that they complete their circular path with equal time period is :



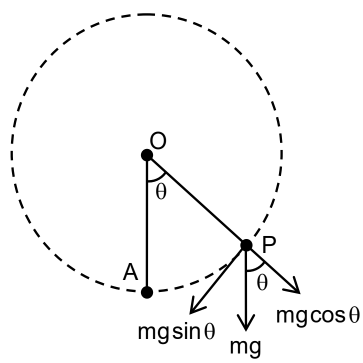
Q.6 A person wants to drive on the vertical surface of large cylindrical wooden 'well' commonly know as 'death well' in a circus. The radius of the well is 2 meter, and the coefficient of friction between the tyres of the motorcycle and the wall of the well is 0.2, the minimum speed the motorcyclist must have in order to prevent slipping should be:

(1) 10 m/s (2) 15 m/s (3) 20 m/s (4) 25 m/s

4. Motion in Vertical Circle

Suppose a particle of mass m is attached to a light inextensible string of length R. The particle is moving in a vertical circle of radius R about a fixed point O. It is imparted a velocity u in the horizontal direction at lowest point A. Let v be its velocity at point P of the circle as shown in the figure.

When a particle is whirled in a vertical circle then three cases are possible-



Case I : Particle oscillates in lower half circle.

Case II : Particle moves to upper half circle but not able to complete loop.

Case III : Particle completes loop.

Case-I

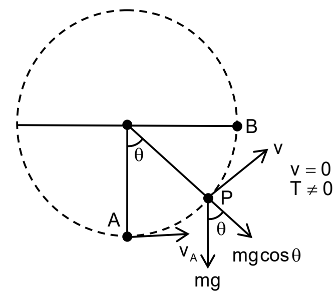
Condition of Oscillation

$(0 < u \leq \sqrt{2gR})$

The particle will oscillate if velocity of the particle becomes zero but tension in the string is not zero.

(In lower half circle (A to B))

Here, $T - mg\cos\theta = \frac{mv_A^2}{R}$; $T = \frac{mv_A^2}{R} + mg\cos\theta$



In the lower part of circle when velocity become zero and tension is non zero means when

$v = 0$, but $T \neq 0$

So, to make the particle oscillate in lower half cycle, maximum possible velocity at A can be given by

$\frac{1}{2}mv_A^2 + 0 = mgR + 0$ (by COME between A and B)

$v_A = \sqrt{2gR}$ ......(i)

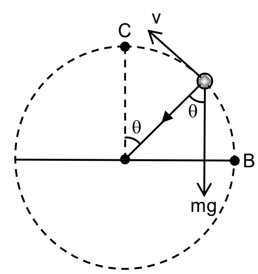
Thus, for $0 < u \leq \sqrt{2gR}$, particle oscillates in lower half of the circle ($0° < \theta \leq 90°$)

This situation is shown in the figure. $0 < u \leq \sqrt{2gR}$ or $0° < \theta \leq 90°$

Case-II

Condition of Leaving the Circle

($\sqrt{2gR} < u < \sqrt{5gR}$)



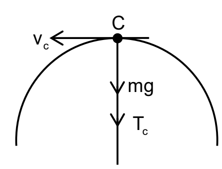
In upper half cycle (B to C)

Here, $T + mg\cos\theta = \frac{mv^2}{R}$

$T = \left(\frac{mv^2}{R} - mg\cos\theta\right)$ .....(ii)

In this part of circle tension force can be zero without having zero velocity mean when $T = 0$, $v \neq 0$ from equation (ii) it is clear that tension decreases if velocity decreases. So to complete the loop tension force should not be zero, in between B to C. Tension will be minimum at C i.e., $T_c \geq 0$ is the required condition.

At top $T_c + mg = \frac{mv_c^2}{R}$



If $T_c = 0$

Then $mg = \frac{mv_c^2}{R}$

$v_c^2 = gR \Rightarrow v_c = \sqrt{gR}$

By COME (Between A and C)

$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_c^2 + mg(2R)$

$v_A^2 = v_C^2 + 4gR \Rightarrow v_A^2 = 5gR \Rightarrow \sqrt{5gR}$

Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle leaves the circle.

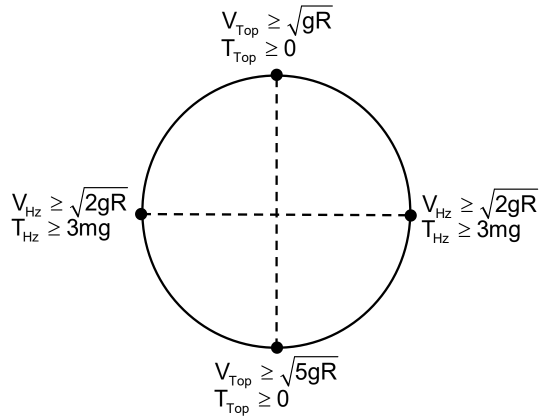
Note : After leaving the circle, the particle will follow a parabolic path.

Case-III

Condition of Looping the Loop

($u \geq \sqrt{5gR}$)

The particle will complete the circle if the string does not slack even at the highest point ($\theta = \pi$). Thus, tension in the string should be greater than or equal to zero ($T \geq 0$) at $\theta = \pi$. In critical case substituting T = 0



Thus, if $u \geq \sqrt{5gR}$, the particle will complete the circle.

Note : In case of rod tension at top most point can never be zero so velocity will become zero.

$\therefore$ For completing the loop $v_L \geq \sqrt{4gR}$

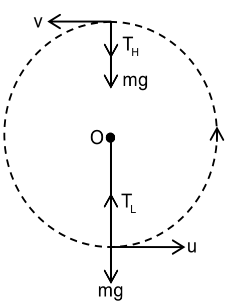
Example 20:

A particle of mass m tied to string of length ℓ and given a circular motion in the vertical plane.

If it performs the complete loop motion then prove that difference in tensions at the lowest and the highest point is 6 mg.

Solution:

Let the speeds at the lowest and highest points be u and v respectively.



At the lowest point, tension

$$= T_L = mg + \frac{mu^2}{\ell} \qquad \text{......(i)}$$

At the highest point, tension

$$= T_H = \frac{mv^2}{\ell} - mg \qquad \text{......(ii)}$$

By conservation of mechanical energy,

$$\frac{mu^2}{2} - \frac{mv^2}{2} = mg(2\ell) \Rightarrow u^2 = v^2 + 4g\ell$$

Substituting this in equation (i)

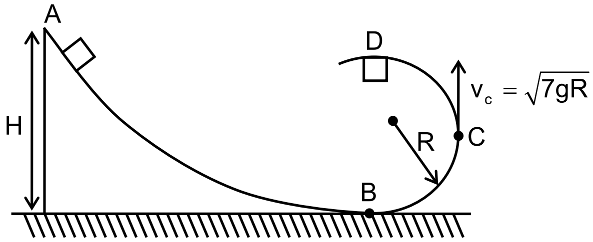
$$T_L = mg + \frac{m(v^2 + 4g\ell)}{\ell} \qquad \text{.....(iii)}$$

$\therefore$ From equation (ii) and (iii) $T_L - T_H = 6$ mg

Note : This implies that the Tension in string at lower point is always 6 mg more than Tension at upper point irrespective of velocity.

Example 21:

Calculate the following for the situation shown



(a) Speed at D (b) Normal reaction at D (c) Height H

Solution:

(a) Similarly At 'D'

$mg(R) + \frac{1}{2}m(7gR) = \frac{1}{2}mv_D^2 + mg(2R)$

$\Rightarrow v_D = \sqrt{5gR}$

(b) $N_D + mg = \frac{mv_D^2}{R}$

$N_D = 4\ mg$

(c) At C :

By energy conservation

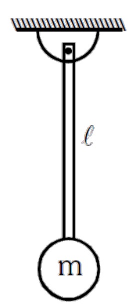
$mg(H - R) = 1/2mv^2$

$\Rightarrow v_C = \sqrt{2g(H-R)} = \sqrt{7gR}$

$\Rightarrow 2(H - R) = 7R \Rightarrow H = 4.5\ R$

Example 22:

A rigid rod of length ℓ and negligible mass has a ball of mass m attached to one end with its other end fixed, to form a pendulum as shown in figure. The pendulum is inverted, with the rod vertically up, and then released. Find the speed of the ball and the tension in the rod at the lowest point of the trajectory of ball.



Solution:

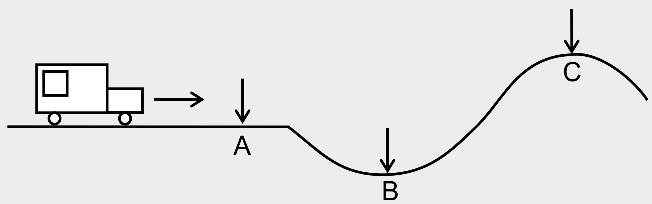
From COME: $2mg\ell = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{4g\ell} = 2\sqrt{g\ell}$

At the lowest point, laws of circular dynamics yield, $T - mg = \frac{mv^2}{\ell} \Rightarrow T = mg + \frac{m}{\ell}(4g\ell) = 5mg.$

Concept Builder-5

Q.1 A stone of mass 1 kg tied to a light string of length ℓ = 10 m is whirling in a circular path in the vertical plane. If the ratio of the maximum to minimum tensions in the string is 3, find the speeds of the stone at the lowest and highest points.

Q.2 A car is moving along a hilly road as shown (side view). The coefficient of static friction between the tyres and the pavement is constant and the car maintains a steady speed. If at one of the points shown the driver applies brakes as hard as possible without making the tyres slip, the magnitude of the frictional force immediately after the brakes are applied will be maximum if the car was at :



(1) point A
(2) point B
(3) point C
(4) friction force same for positions A, B and C

Q.3 A particle slides on the surface of a fixed smooth sphere starting from the topmost point. Find the angle rotated by the radius through the particle, when it leaves contact with the sphere.

ANSWER KEY FOR CONCEPT BUILDERS

CONCEPT BUILDER-1

1. $\frac{14\pi}{25}$ rad/sec **2.** 1 : 1

3. $\frac{12}{1}$ **4.** (3)

5. 1 m/s **6.** (3)

CONCEPT BUILDER-2

1. $\frac{\sqrt{181}}{5}$ m/s^2 **2.** (3)

3. 6 m/s^2 **4.** $\frac{1}{4}$ second

5. $-3\hat{k}$ rad/s^2, $-2\hat{k}$ rad/s

6. α = 6 rad/sec^2

CONCEPT BUILDER-3

1. 1.6 N **2.** $m\omega_0^2 a$

3. 1.225 rad/sec

CONCEPT BUILDER-4

1. (3) **2.** 11.2 m/s

3. 8 m/s **4.** $10\sqrt{2}$ m/s

5. $\sqrt{\frac{2}{3}}$ **6.** (1)

CONCEPT BUILDER-5

1. V_{lowest} = 20 ms^{-1}; $V_{highest}$ = 20 ms^{-1}

2. (2) **3.** $\cos^{-1}(2/3)$

Exercise - I

Kinematics of Circular Motion

1. A particle of mass 'm' describes a circle of radius (r). The centripetal acceleration of the particle is $\frac{4}{r^2}$. The momentum of the particle :-

(1) $\frac{2m}{r}$ (2) $\frac{2m}{\sqrt{r}}$

(3) $\frac{4m}{r}$ (4) $\frac{4m}{\sqrt{r}}$

2. A particle is moving around a circular path with uniform angular speed (ω). The radius of the circular path is r. The acceleration of the particle is :-

(1) $\frac{\omega^2}{r}$ (2) $\frac{\omega}{r}$

(3) $v\,\omega$ (4) $v\,r$

3. A car moves on a circular road, describing equal angles about the centre in equal intervals of times. Which of the statements about the velocity of car it true :-

(1) velocity is constant

(2) magnitude of velocity is constant but the direction of velocity change

(3) both magnitude and direction of velocity change

(4) velocity is directed towards the centre of circle

4. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. What is the linear speed of the motion :-

(1) 2.3 cm/s (2) 5.3 cm/s

(3) 0.44 cm/s (4) None of these

5. A particle moves in a circle of the radius 25 cm at two revolutions per second. The acceleration of the particle in m/sec^2 is :-

(1) π^2 (2) $8\pi^2$

(3) $4\pi^2$ (4) $2\pi^2$

6. A particle moves in a circle describing equal angle in equal times, its velocity vector :-

(1) remains constant

(2) change in magnitude

(3) change in direction

(4) changes in magnitude and direction

7. A mass of 2 kg is whirled in a horizontal circle by means of a string at an initial speed of 5 r.p.m. Keeping the radius constant the tension in the string is doubled. The new speed is nearly :-

(1) 7 r.p.m. (2) 14 r.p.m.

(3) 10 r.p.m. (4) 20 r.p.m.

8. A particle moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega},\vec{v},\vec{\alpha},\vec{a}_c$. Which of the following relation is/are correct :-

(a) $\vec{\omega}\perp\vec{v}$ (b) $\vec{\omega}\perp\vec{\alpha}$

(c) $\vec{v}\perp\vec{a}_c$ (d) $\vec{\omega}\perp\vec{a}_c$

(1) a,b,c (2) b,c,d

(3) a,b,c (4) a,c,d

9. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows, that :-

(1) its velocity is constant

(2) its K.E. is constant

(3) its acceleration is constant

(4) it moves in a straight line

10. If the equation for the displacement of a particle moving on a circular path is given by (θ) = $2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2 s from its start is ;-

(1) 8 rad/s (2) 12 rad/s

(3) 24 rad/s (4) 36 rad/s

11. A body moves with constant angular velocity on a circle. Magnitude of angular acceleration :-

(1) $r\omega^2$

(2) Constant

(3) Zero

(4) None of the above

12. A particle of mass m revolving in horizontal circle of radius R with uniform speed v. When particle goes from one end to other end of diameter, then :-

(1) K.E. changes by $\frac{1}{2}mv^2$

(2) K.E. change by mv^2

(3) no change in momentum

(4) change in momentum is 2 mv

13. A stone is tied to one end of string 50 cm long and is whirled in a horizontal circle with constant speed. If the stone makes 10 revolutions in 20 s, then what is the magnitude of acceleration of the stone :-

(1) 493 cm/s^2 (2) 720 cm/s^2

(3) 860 cm/s^2 (4) 990 cm/s^2

14. For a particle in a non-uniform accelerated circular motion :-

(1) velocity is radial and acceleration is transverse only

(2) velocity is transverse and acceleration is radial only

(3) velocity is radial and acceleration has both radial and transverse components

(4) velocity is transverse and acceleration has both radial and transverse components

15. Two particles having mass 'M' and 'm' are moving in a circular path having radius R and r. If their time period are same then the ratio of angular velocity will be :-

(1) $\frac{r}{R}$ (2) $\frac{R}{r}$

(3) 1 (4) $\sqrt{\frac{R}{r}}$

16. Angular velocity of minute hand of a clock is :-

(1) $\frac{\pi}{30}$ rad/s (2) 8 π rad/s

(3) $\frac{2\pi}{1800}$ rad/s (4) $\frac{\pi}{1800}$ rad/s

17. A car moving with speed 30 m/s on a circular path of radius 500 m. Its speed is increasing at the rate of $2 m/s^2$. The acceleration of the car is:-

(1) 9.8 m/s^2 (2) 1.8 m/s^2

(3) 2 m/s^2 (4) 2.7 m/s^2

18. A body of mass 1 kg tied to one end of string is revolved in a horizontal circle of radius 0.1 m with a speed of 3 revolution /sec, assuming the effect of gravity is negligible, then linear velocity, acceleration and tension in the string will be :-

(1) 1.88 m/s, 35.5 m/s^2, 35.5 N

(2) 1.88 m/s, 45.5 m/s^2, 45.5 N

(3) 3.88 m/s, 55.5 m/s^2, 55.5 N

(4) None of these

19. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is :-

(1) 40 m/s^2 (2) 640 π m/s^2

(3) 160 π m/s^2 (4) 40 π m/s^2

20. The angular velocity of a wheel is 70 rad/s. If the radius of the wheel is 0.5 m, then linear velocity of the wheel is :-

(1) 70 m/s (2) 35 m/s

(3) 30 m/s (4) 20 m/s

21. A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolution in 44 seconds, what is the magnitude and direction of acceleration of the stone :-

(1) π^2 ms^{-2} and direction along the tangent to the circle.

(2) π^2 ms^{-2} and direction along the radius towards the centre.

(3) $\frac{\pi^2}{4} ms^{-2}$ and direction along the radius towards the centre.

(4) π^2 ms^{-2} and direction along the radius away from the centre.

22. A fly wheel rotating at 600 rev/min is brought under uniform deceleration and stopped after 2 minutes, then what is angular deceleration in rad/sec^2?

(1) $\frac{\pi}{6}$ (2) 10π

(3) $\frac{1}{12}$ (4) 300

23. The linear and angular acceleration of a particle are 10 m/sec^2 and 5 rad/sec^2 respectively. It will be at a distance from the axis of rotation.

(1) 50 m (2) $\frac{1}{2}$

(3) 1 m (4) 2 m

24. The angular acceleration of particle moving along a circular path with uniform speed :-

(1) uniform but non zero

(2) zero

(3) variable

(4) as can not be predicted from given information

25. A body is revolving with a constant speed along a circle. If its direction of motion is reversed but the speed remains the same then :-

(a) the centripetal force will not suffer any change in magnitude

(b) the centripetal force will have its direction reversed

(c) the centripetal force will not suffer any change in direction

(d) the centripetal force is doubled

(1) a, b (2) b, c

(3) c, d (4) a, c

26. a_r and a_t represent radial and tangential acceleration. The motion of a particle will be uniform circular motion if :-

(1) $a_r = 0$ and $a_t = 0$

(2) $a_r = 0$ but $a_t \neq 0$

(3) $a_r \neq 0$ but $a_t = 0$

(4) $a_r \neq 0$ and $a_t \neq 0$

27. In uniform circular motion the velocity vector and acceleration vector are

(1) Perpendicular to each other

(2) Same direction

(3) Opposite direction

(4) Not related to each other

28. A string of length 10 cm breaks if its tension exceeds 10 newton. A stone of mass 250 g tied to this string, is rotated in a horizontal circle. The maximum angular velocity of rotation can be:

(1) 20 rad/s (2) 40 rad/s

(3) 100 rad/s (4) 200 rad/s

Dynamics of Circular Motion

29. If the speed and radius both are tripled for a body moving on a circular path, then the new centripetal force will be :

(1) Doubled of previous value

(2) Equal to previous value

(3) Triple of previous value

(4) One third of previous value

30. A string of length 0.1 m cannot bear a tension more than 100N. It is tied to a body of mass 100 g and rotated in a horizontal circle. The maximum angular velocity can be -

(1) 100 rad/s (2) 1000 rad/s

(3) 10000 rad/s (4) 0.1 rad/s

31. The radius of the circular path of a particle is doubled but its frequency of rotation is kept constant. If the initial centripetal force be F, then the final value of centripetal force will be :-

(1) F (2) F/2
(3) 4F (4) 2F

32. A 0.5 kg ball moves in a circle of radius 0.4 m at a speed of 4 ms^{-1}. The centripetal force on the ball is :-

(1) 10 N (2) 20 N
(3) 40 N (4) 80 N

33. The earth ($M_e = 6 \times 10^{24}$ kg) is revolving round the sun in an orbit of radius (1.5×10^8) km with angular velocity of (2×10^{-7}) rad/s. The force (in newton) exerted on the earth by the sun will be :-

(1) 36×10^{21} (2) 16×10^{24}
(3) 25×10^{16} (4) Zero

34. A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km/hr. The centripetal force is :-

(1) 250 N (2) 1000 N
(3) 750 N (4) 1200 N

35. A motor cycle driver doubles its velocity when he is taking a turn. The force exerted towards the centre will become :-

(1) double (2) half
(3) 4 times (4) $\frac{1}{4}$

36. The force required to keep a body in uniform circular motion is :-

(1) Centripetal force
(2) Centrifugal force
(3) Resistance
(4) None of the above

37. A car moving on a horizontal road may be thrown out of the road in taking a turn :-

(1) by the gravitational force
(2) due to lack of proper centripetal force
(3) due to rolling friction between the tyres and the road
(4) due to reaction of the road

Motion in Vertical Circle

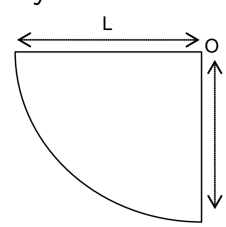
38. Let 'θ' denote the angular displacement of a simple pendulum oscillating in a vertical plane. If the mass of the bob is (m), then the tension in string is mg cos θ :-

(1) always
(2) never
(3) at the extreme positions
(4) at the mean position

39. A pendulum bob has a speed 3 m/s while passing through its lowest position, length of the pendulum is 0.5 m then its speed when it makes an angle of 60° with the vertical is :-

(1) 2 m/s (2) 1 m/s
(3) 4 m/s (4) 3 m/s

40. The mass of the bob of a simple pendulum of length L is m. If the bob is left from its horizontal position then the speed of the bob and the tension in the thread at the lowest position of the bob will be respectively :-



(1) $\sqrt{2gL}$ and 3 mg (2) 3 mg and $\sqrt{2gL}$
(3) 2 mg and $\sqrt{2gL}$ (4) 2 gL and 3 mg

41. A stone of mass 1 kg is tied to the end of a string of 1 m length. It is whirled in a vertical circle. If the velocity of the stone at the top be 4 m/s. What is the tension in the string at that instant ?

(1) 6 N (2) 16 N
(3) 5 N (4) 10 N

42. In a vertical circle of radius (r), at what point in its path a particle may have tension equal to zero :-

(1) highest point
(2) lowest point
(3) at any point
(4) at a point horizontal from the centre of radius

43. A stone attached to one end of a string is whirled in a vertical circle. The tension in the string is maximum when :-

(1) the string is horizontal

(2) the string is vertical with the stone at highest position

(3) the string is vertical with the stone at the lowest position

(4) the string makes an angle of 45° with the vertical

44. A particle is moving in a vertical circle the tension in the string when passing through two position at angle 30° and 60° from vertical from lowest position are T_1 and T_2 respectively then :-

(1) $T_1 = T_2$ (2) $T_1 > T_2$

(3) $T_1 < T_2$ (4) $T_1 \geq T_2$

45. A body crosses the topmost point of a vertical circle with critical speed. What will be its centripetal acceleration when the string is horizontal :-

(1) g (2) 2g

(3) 3g (4) 6g

46. Stone tied at one end of light string is whirled round a vertical circle. If the difference between the maximum and minimum tension experienced by the string wire is 2 kg wt, then the mass of the stone must be :-

(1) 1 kg (2) 6 kg

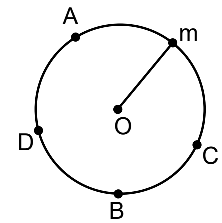
(3) 1/3 kg (4) 2 kg

47. If the over bridge is concave instead of being convex, then the thrust on the road at the lowest position will be :-

(1) $mg + \frac{mv^2}{r}$ (2) $mg - \frac{mv^2}{r}$

(3) $\frac{mv^2g}{r}$ (4) $\frac{v^2g}{r}$

48. A particle of mass m is performing vertical circular motion (see figure). If the average velocity of the particle is increased, then at which point maximum breaking possibility of the string :-



(1) A (2) B

(3) C (4) D

49. A fighter plane is moving in a vertical circle of radius 'r'. Its minimum velocity at the highest point of the circle will be :-

(1) $\sqrt{3gr}$ (2) $\sqrt{2gr}$

(3) $\sqrt{gr}$ (4) $\sqrt{\frac{gr}{2}}$

50. A stone of mass 0.2 kg is tied to one end of a thread of length 0.1 m whirled in a vertical circle. When the stone is at the lowest point of circle, tension in thread is 52 N then velocity of the stone will be :-

(1) 4 m/s (2) 5 m/s

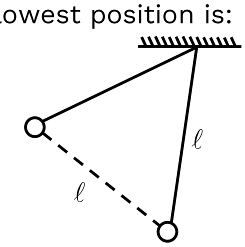
(3) 6 m/s (4) 7 m/s

51. A suspended simple pendulum of length ℓ is making an angle θ with the vertical. On releasing, its velocity at lowest point will be :-

(1) $\sqrt{2g\ell(1+\cos\theta)}$ (2) $\sqrt{2g\ell\sin\theta}$

(3) $\sqrt{2g\ell(1-\cos\theta)}$ (4) $\sqrt{2g\ell}$

52. A bob hangs from a rigid support by an inextensible string of length ℓ. If it is displaced through a distance ℓ (from the lowest position) keeping the string straight & released, the speed of the bob at the lowest position is:



(1) $\sqrt{g\ell}$ (2) $\sqrt{3g\ell}$

(3) $\sqrt{2g\ell}$ (4) $\sqrt{5g\ell}$

ANSWER KEY

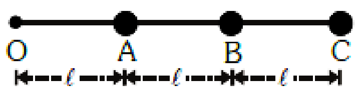
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	2	3	2	2	3	3	1	4	2	3	3	4	1	4	3	4	4	1	1	2	2	1	4	2	4
Que.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Ans.	3	1	1	3	1	4	2	1	2	3	1	2	3	1	1	1	1	3	2	3	3	1	2	3	2
Que.	51	52																							
Ans.	3	1																							

Exercise - II

1. Keeping the banking angle of the road constant, the maximum safe speed of passing vehicles is to be increased by 10%. The radius of curvature of the road will have to be changed from 20 m to :-

(1) 16 m (2) 18 m
(3) 24.20 m (4) 30.5 m

2. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is :-



(1) 3 : 5 : 7 (2) 3 : 4 : 5
(3) 7 : 11 : 6 (4) 3 : 5 : 6

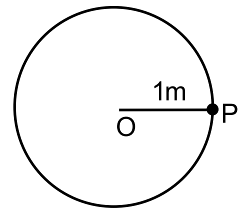
3. A stone is tied to a string of length ‘ℓ’ and is whirled in a vertical circle with the other end of the string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed ‘u’. The magnitude of the change in velocity as it reaches a position where the string is horizontal (g being acceleration due to gravity) is :–

(1) $\sqrt{u^2 - g\ell}$ (2) $u - \sqrt{u^2 - 2g\ell}$
(3) $\sqrt{2g\ell}$ (4) $\sqrt{2(u^2 - g\ell)}$

4. A weightless thread can withstand tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2 m in a vertical plane. If g = 10 m/s^2, then the maximum angular velocity of the stone can be :-

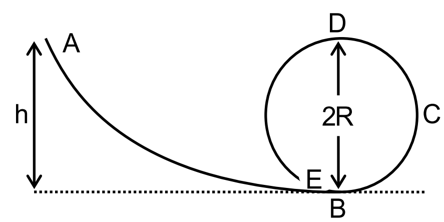
(1) 5 rad/s (2) $\sqrt{30}$ rad/s
(3) $\sqrt{60}$ rad/s (4) 10 rad/s

5. A mass tied to a string moves in a vertical circle with a uniform speed of 5 m/s as shown. At the point P the string breaks. The mass will reach a height above P of nearly (g = 10 m/s^2)



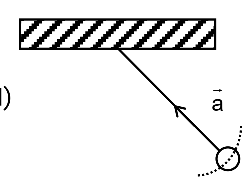
(1) 1 m (2) 0.5 m
(3) 1.75 m (4) 1.25 m

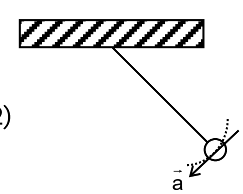
6. A frictionless track ABCDE ends in a circular loop of radius R. A body slides down the track from point A which is at a height h = 5 cm. Maximum value of R for the body to successfully complete the loop is :- (The velocity at point B is $\sqrt{5Rg}$)

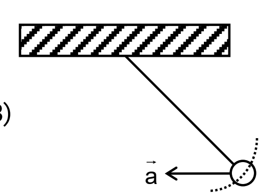


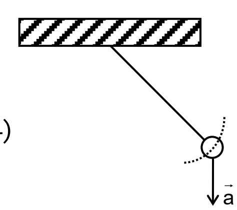
(1) 5 cm (2) $\frac{15}{4}$ cm
(3) $\frac{10}{3}$ cm (4) 2 cm

7. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector is correctly shown in:

(1)


(2)


(3)


(4)


8. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is :

(1) $\frac{ML\omega^2}{2}$ (2) $\frac{ML^2\omega}{2}$

(3) $ML\omega^2$ (4) $\frac{ML^2\omega^2}{2}$

9. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 seconds for every circular lap. The average velocity and average speed for each circular lap respectively is :

(1) 0,0 (2) 0, 10 m/s

(3) 10 m/s, 10 m/s (4) 10 m/s, 0

10. A roller coaster is designed such that riders experience "weightlessness" as they go round the top of a hill whose radius of curvature is 20 m. The speed of the car at the top of the hill is between. ($g = 10\ m/s^2$)

(1) 16 m/s and 17 m/s

(2) 13 m/s and 14 m/s

(3) 14 m/s and 15 m/s

(4) 15 m/s and 16 m/s

11. A particle moves in x-y plane according to rule x = a sin ωt and y = a cos ωt. The particle follows:

(1) An elliptical path

(2) A circular path

(3) A parabolic path

(4) A straight line path inclined equaled to x and y-axes

12. A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance r from the centre of the record. The static coefficient of friction is μ. The coin will revolve with the record if :

(1) $r \geq \frac{\mu g}{\omega^2}$ (2) $r = \mu g\omega^2$

(3) $r < \frac{\omega^2}{\mu g}$ (4) $r \leq \frac{\mu g}{\omega^2}$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	3	4	4	1	4	4	3	1	2	3	2	4

Exercise – III (Previous Year Question)

1. A particle moves in a circle of radius 5 cm with constant speed and time period 0.2 π s. The acceleration of the particle is : **[AIPMT 2011]**

(1) 15 m/s^2 (2) 25 m/s^2

(3) 36 m/s^2 (4) 5 m/s^2

2. A car of mass 1000 kg negotiates a banked curve of radius 90 m on a frictionless road. If the banking angle is 45°, the speed of the car is : **[AIPMT -2012]**

(1) 5 ms^{-1} (2) 10ms^{-1}

(3) 20 ms^{-1} (4) 30 ms^{-1}

3. A car mass m is moving on a level circular track of radius R. If μ_s represents the static friction between the road and tyres of the car, the maximum speed of the car in circular motion is given by : **[AIPMT 2012]**

(1) $\sqrt{mRg/\mu_s}$ (2) $\sqrt{\mu_s Rg}$

(3) $\sqrt{\mu_s mRg}$ (4) $\sqrt{Rg/\mu_s}$

4. Two stones of masses m and 2 m are whirled in horizontal circles, the heavier one in a radius $\frac{r}{2}$ and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they experience same centripetal forces. The value of n is : **[AIPMT 2012]**

(1) 1 (2) 2

(3) 3 (4) 4

5. The position vector of a particle $\vec{R}$ as a function of time is given by :

$\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$

Where R is in meters, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x and y-directions, respectively. Which one of the following statements is wrong for the motion of particle ? **[AIPMT 2015]**

(1) Path of the particle is a circle of radius 4 meter

(2) Acceleration vectors is along $-\vec{R}$

(3) Magnitude of acceleration vectors is $\frac{v^2}{R}$ where v is the velocity of particle.

(4) Magnitude of the velocity of particle is 8 meter /second

6. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion? **[AIPMT 2015]**

(1) 0.1 m/s^2 (2) 0.15 m/s^2

(3) 0.18 m/s^2 (4) 0.2 m/s^2

7. What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R at lowest point so that it can complete the loop ? **[AIPMT 2015]**

(1) $\sqrt{gR}$

(2) $\sqrt{2gR}$

(3) $\sqrt{3gR}$

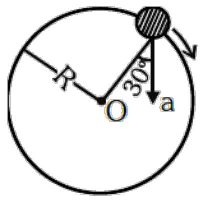
(4) $\sqrt{5gR}$

8. A car is negotiating a curved road of radius R. The road is banked at an angle θ. The coefficient of friction between the tyres of the car and the road is μ_s. The maximum safe velocity on this road is : **[NEET-2016]**

(1) $\sqrt{gR^2\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$ (2) $\sqrt{gR\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$

(3) $\sqrt{\frac{g}{R}\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$ (4) $\sqrt{\frac{g}{R^2}\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$

9. In the given figure, $a = 15\ m/s^2$ represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. The speed of the particle is : **[NEET-2016]**



(1) 5.7 m/s (2) 6.2 m/s
(3) 4.5 m/s (4) 5.0 m/s

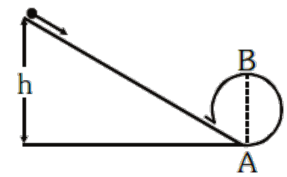
10. A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\hat{x} + \sin\omega t\hat{y}$. Where ω is a constant. Which of the following is true ? **[NEET-I 2016]**

(1) Velocity and acceleration both are perpendicular to $\vec{r}$
(2) Velocity and acceleration both are parallel to $\vec{r}$
(3) Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin
(4) Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin

11. One end of string of length ℓ is connected to a particle of mass 'm' and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed 'v' the net force on the particle (directed towards centre) will be (T represents the tension in the string) : **[NEET-2017]**

(1) $T + \frac{mv^2}{\ell}$ (2) $T - \frac{mv^2}{\ell}$
(3) Zero (4) T

12. A body initially at rest and sliding along a frictionless track from a height h (as shown in the figure) just completes a vertical circle of diameter AB = D. The height h is equal to : **[NEET-2018]**



(1) $\frac{3}{2}D$ (2) D
(3) $\frac{7}{5}D$ (4) $\frac{5}{4}D$

13. A block of mass 10 kg is in contact against the inner wall of a hollow cylindrical drum of radius 1 m. The coefficient of friction between the block and the inner wall of the cylinder is 0.1. The minimum angular velocity needed for the cylinder to keep the block stationary when the cylinder is vertical and rotating about its axis, will be: ($g = 10\ m/s^2$) **[NEET-2019]**

(1) $\sqrt{10}$ rad/s (2) $\frac{10}{2\pi}$ rad/s
(3) 10 rad/s (4) 10π rad/s

14. A mass m is attached to a thin wire and whirled in a vertical circle. The wire is most likely to break when : **[NEET-2019]**

(1) the mass is at the highest point
(2) the wire is horizontal
(3) the mass is at the lowest point
(4) inclined at an angle of 60° from vertical

15. Two particles A and B are moving in uniform circular motion in concentric circles of radius r_A and r_B with speed υ_A and υ_B respectively. The time period of rotation is the same. The ratio of angular speed of A to that of B will be : **[NEET-2019]**

(1) $r_A : r_B$ (2) $\upsilon_A : \upsilon_B$
(3) $r_B : r_A$ (4) 1 : 1

16. A particle starting from rest, moves in a circle of radius 'r'. It attains a velocity of V_0 m/s in the nth round. Its angular acceleration will be :**[NEET-2019 (Odisha)]**

(1) $\frac{V_0}{n} rad/s^2$ (2) $\frac{V_0^2}{2\pi nr^2} rad/s^2$
(3) $\frac{V_0^2}{4\pi nr^2} rad/s^2$ (4) $\frac{V_0^2}{4\pi nr} rad/s^2$

17. The angular speed of the wheel of a vehicle is increased from 360 rpm to 1200 rpm in 14 second. Its angular acceleration is : **[NEET-Covid-2020]**

(1) 2π rad/s^2 (2) 28π rad/s^2

(3) 120π rad/s^2 (4) 1 rad/s^2

18. A point mass 'm' is moved in a vertical circle of radius 'r' with the help of a string. The velocity of the mass is $\sqrt{7gr}$ at the lowest point. The tension in the string at the lowest point is : **[NEET-Covid-2020]**

(1) 6 mg (2) 7 mg

(3) 8 mg (4) 1 mg

19. The angular speed of a fly wheel moving with uniform angular acceleration changes from 1200 rpm to 3120 rpm in 16 seconds. The angular acceleration in rad/s^2 is: **[NEET-2022]**

(1) 2π (2) 4π

(3) 12π (4) 104π

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ans.	4	4	2	2	4	1	4	2	1	3	4	4	3	3	4	3	1	3	2