GUIDED <u>Revision</u>

PHYSICS

Single Correct Answer Type

GR # ELECTROSTATICS

SECTION-I

14 Q. [3 M (-1)]

1. There are two point charges q_1 and q_2 lying on a circle of unit radius. Electric field intensity at the center of circle due to these charges is \vec{E} . Find the position vector of the center with respect to q_2 if the position vector of the center with respect to q_1 is $\vec{r_1}$.

(A)
$$\frac{\vec{E} + kq_1\vec{r_1}}{kq_2}$$
 (B) $-\left(\frac{\vec{E} + kq_1\vec{r_1}}{kq_2}\right)$ (C) $\frac{kq_1\vec{r_1} - \vec{E}}{kq_2}$ (D) $\frac{\vec{E} - kq_1\vec{r_1}}{kq_2}$

2. An infinite, uniformly charged sheet with surface charge density σ cuts through a spherical Gaussian surface of radius R at a distance x from its center, as shown in the figure. The electric flux Φ through the Gaussian surface is :-



3. The electric field intensity at all points in space is given by $\vec{E} = \sqrt{3}\hat{i} - \hat{j}$ volts/metre. The nature of equipotential lines in x-y plane is given by :-



4. A sphere of radius R carries charge such that its volume charge density is proportional to the square of the distance from the centre. What is the ratio of the magnitude of the electric field at a distance 2R from the centre to the magnitude of the electric field at a distance of R/2 from the centre (i.e. $E_{r=2R} / E_{r=R/2}$)?

(A) 1 (B) 2 (C) 4 (D) 8

5. A point positive charge is brought near an isolated conducting sphere. The electric field is best given by



 $(A) \operatorname{Fig}(i) \qquad (B) \operatorname{Fig}(iii) \qquad (C) \operatorname{Fig}(ii) \qquad (D) \operatorname{Fig}(iv)$

6. Figure shows two conducting thin concentric shells of radii r and 3r. The outer shell carries charge q and inner shell is neutral. The amount of charge which flows from inner shell to the earth after the key K is closed, is equal to :-



(A)
$$-q/3$$
 (B) $q/3$ (C) $3q$ (D) $-3q$

7. Consider a conductor with a spherical cavity in it. A point charge q_0 is placed at the centre of cavity and a point charge Q is placed outside conductor.

Statement-1: Total charge induced on cavity wall is equal and opposite to the charge inside.

Statement-2: If cavity is surrounded by a Gaussian surface, where all parts of Gaussian surface are

located inside the conductor, $\oint \vec{E} \cdot d\vec{A} = 0$; hence $q_{induced} = -q_0$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

- (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
- (C) Statement-1 is true, statement-2 is false.
- (D) Statement-1 is false, statement-2 is true.
- 8. A non conducting semicircular disc (as shown in figure) has a uniform surface charge density σ . The electric potential at the centre of the disc :-



(A)
$$\frac{\sigma}{2\pi \epsilon_0} \frac{\ell n(b/a)}{(b-a)}$$
 (B) $\frac{\sigma(b-a)}{2\epsilon_0}$ (C) $\frac{\sigma(b-a)}{4\epsilon_0}$ (D) $\frac{\sigma(b-a)}{4\pi \epsilon_0}$

9. The diagram shows a small bead of mass m carrying charge q. The bead can freely move on the smooth fixed ring placed on a smooth horizontal plane. In the same plane a charge +Q has also been fixed as shown. The potential at the point P due to +Q is V. The velocity with which the bead should projected from the point P so that it can complete a circle should be greater than



10. In a certain region of space, the potential field depends on x and y coordinates as $V = (x^2 - y^2)$. The corresponding electric field lines in x-y plane are correctly represented by :



11. Two short electric dipoles are placed as shown. The energy of electric interaction between these dipoles will be :-





12. A metal sphere A of radius r_1 charged to a potential ϕ_1 is enveloped by a thin walled conducting spherical shell B of radius r_2 . Then ϕ_2 of the sphere A after it is connected by a thin wire to the shell B will be :-



An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the center of 13. the cavity. The points A & B are on the cavity surface as shown in the figure. Then :



(A) electric field near A in the cavity = electric field near B in the cavity

(B) charge density at A = charge density at B

(C) potential at A = potential at B

- (D) total electric field flux through the surface of the cavity is q/ϵ_0 .
- Two charges, each equal to q, are kept at x = -a and x = a on the x-axis. A particle of mass m and charge 14.

 $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement (y << a) along the [JEE-Main-2013]

y-axis, the net force acting on the particle is proportional to

(A) y (B) -y (C)
$$\frac{1}{y}$$
 (D) - $\frac{1}{y}$

Multiple Correct Answer Type

An insulating ring of charge $2\pi\lambda b$, radius b is concentric with a charged conducting solid sphere of 15. charge Q and radius a (b > a) as shown in the figure. Then



(A) Potential of the sphere is
$$\frac{Q}{4\pi \epsilon_0 a} + \frac{\lambda}{2\epsilon_0}$$

(B) If sphere is grounded then charge on the sphere will be $\frac{-a\lambda}{8\pi^2 \in \Omega}$

(C) Electric field inside the sphere is non-zero

- (D) The sphere is equipotential body.
- Two large thin conducting plates with small gap in between are placed in a uniform electric field E 16. (perpendicular to the plates). Area of each plate is A and charges +Q and -Q are given to these plates as shown in the figure. If points R,S and T as shown in the figure are three points in space, then the





(C)

(B) field at point S is E

field at point *T* is
$$\left(E + \frac{Q}{\varepsilon_o A}\right)$$
 (D) field at point *S* is $\left(E + \frac{Q}{A}\right)$

3 Q. [4 M (-1)]

17. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε₀. Which of the following statements is (are) true ? [JEE-Advance-2018]



- (A) The electric flux through the shell is $\sqrt{3} R\lambda / \epsilon_0$
- (B) The z-component of the electric field is zero at all the points on the surface of the shell
- (C) The electric flux through the shell is $\sqrt{2} R\lambda / \epsilon_0$
- (D) The electric field is normal to the surface of the shell at all points

Linked Comprehension Type(1 Para × 2Q.) [3 M (-1)](Single Correct Answer Type)

Paragraph for Question No. 18 and 19

A uniform ring of mass m and radius R can rotate freely about an axis passing through centre C and perpendicular to plane of paper. Half of ring is positively charge and other half is negatively charge. Uniform electric field E_0 is switched on along –ve x-axis (axis are shown in figure) [Magnitude of charge density λ]



- **18.** The dipole moment of ring is :-(A) $2 \lambda R^2$ (B) $4 \lambda R^2$ (C) $2 \pi \lambda R^2$ (D) $4 \pi \lambda R^2$
- 19. If ring is slightly disturb from given position, find the angular speed of ring when it rotate by $\pi/2$.

(A)
$$2\sqrt{\frac{\lambda E_0}{m}}$$
 (B) $\sqrt{\frac{\lambda E_0}{m}}$ (C) $\sqrt{\frac{8\lambda E_0}{m}}$ (D) None

SECTION-II

Numerical Answer Type Question (upto second decimal place)

1. The figure shows a conducting sphere 'A' of radius 'a' which is surrounded by a neutral conducting spherical shell B of radius 'b' (>a). Initially switches S_1 , S_2 and S_3 are open and sphere 'A' carries a charge Q. First the switch ' S_1 ' is closed to connect the shell B with the ground and then opened. Now the switch ' S_2 ' is closed so that the sphere 'A' is grounded and then S_2 is opened. Finally, the switch ' S_3 ' is closed to connect the shell (in joule) which is produced after closing the switch S_3 . [Consider b = 4 cm, a = 2 cm and Q = 80 μ C]



SECTION-III

7 Q. [4 M (0)]

Numerical Grid Type (Ranging from 0 to 9)

1. A sphere of radius R has charge density given by $\rho = \rho_0 \left(1 - \frac{nr}{3R} \right)$, where ρ_0 is a constant, r is distance

from centre of sphere. For a spherical gaussian surface of radius R centered at the centre of sphere, the flux is zero. Find 'n'.

2. A cube is made from six thin non-conducting square faces of size a each. Each face carries a uniformly

distributed charge Q. If electrostatic force acting on each face is $\frac{Q^2}{N \in_0 a^2}$, then find N.

3. Two fixed, equal positive charges, each +q are located at point A & B separated by a distance of 6 m. A particle of mass m having equal and opposite charge –q moves towards them along the perpendicular bisector line COD where O is the centre of line joining A and B. If –q charge is released from rest from

point C, then speed of charge –q at O is given by $v = q \sqrt{\frac{x}{15\pi \epsilon_0 m}}$ find value of x. (Neglect gravity)



1 Q. [3(0)]

4. An electric field given by $\vec{E} = 4\hat{i} - 3(y^2 + 2)\hat{j}$ pierces Gaussian's cube of side 1m placed at origin such that its three sides represents x, y and z axes. The net charge enclosed within the cube is given by $-n\varepsilon_0$. Find the value of n.



5. The electric potential in a region is given by $V(x, y, z) = ax^2 + ay^2 + abz^2$. 'a' is a positive constant of appropriate dimensions and b, a positive constant such that V is volts when x, y, z are in m. Let b = 2. The work done by the electric field when a point charge +4µC moves from the point (0, 0, 0.1m) to the origin is 50 µJ. The radius of the circle of the equipotential curve corresponding to V = 6250 volts and

 $z = \sqrt{2}$ m is α m. Fill α^2 in OMR sheet.

6. The electric field strength depends only on the x, y and z coordinates according to the law

$$E = \frac{a(x\hat{i} + y\hat{j} + z\hat{k})}{(x^2 + y^2 + z^2)^{3/2}}, \text{ where a = 122.5 SI unit and is a constant. Find the potential difference (in volt)}$$

between (3, 2, 6) and (0, 3, 4).

7. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the

rectangular surface ABCD lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\epsilon_0}$ (ϵ_0 = perimittivity of free space) then the value of n is. [JEE-Advance-2015]



SECTION-IV

Matrix Match Type (4×5)

- 3 Q. [8 M (for each entry +2(0)]
- 1. As shown in column I their are graphs of electric field (E) and potential (V) along the line joining charges Q_1 and Q_2 are drawn against distance (r) on x-axis for charges Q_1 and Q_2 . Take potential at infinity equal to zero. [Take direction of E in righward direction as positive] Column-I Column-II



(T) $|Q_1| = |Q_2|$

2. Figure shows an uncharged conducting body having a spherical cavity. Charge Q is placed at the centre of the cavity.



Column-I (Point)

- (A) 1 (inside cavity near surface)
- (B) 2 (near but outside conductor surface)
- (C) 3 (inside bulk of conductor)
- (D) 4 (near but outside conductor surface)

- Column II (Components of electric field)
- (P) $E_x = 0$
- (Q) $E_v = 0$
- (R) $E_x^y \neq 0$
- (S) $E_{u}^{x} \neq 0$
- (T) None of these

3. Column-II shows some charge distributions and column-I has some statements about electric field at four points A, B, C, D. Match column-I with column-II.



Subjective Type

6 Q. [4 M (0)]

1. A simple pendulum of length ℓ and bob mass *m* is hanging in front of a large nonconducting sheet of surface charge density σ . If suddenly a charge +*q* is given to the bob in the position shown in figure. Find the maximum angle through which the string is deflected from vertical.



2. A dipole is placed at origin of coordinate system as shown in figure, find the electric field at point P(0, y).



- 3. Electric dipole of moment $P = P \hat{i}$ is kept at a point (x,y) in an electric field, $E = 4 xy^2 \hat{i} + 4 x^2 y \hat{j}$. Find the forces on the dipole.
- 4. Four point charges $+8\mu$ C, -1μ C, -1μ C and $+8\mu$ C, are fixed at the points, $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and of charge $+ 0.1 \mu$ C moves along the -x direction. Its speed at $x = +\infty$ is v_0 . Find the least value of v_0 for which the particle will cross the origin . Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. (Given : $1/(4\pi \epsilon_0) = 9 \times 10^9$ Nm²/C²)
- 5. Two concentric rings of radii r and 2r are placed with centre at origin. Two charges +q each are fixed at the diametrically opposite points of the rings as shown in figure. Smaller ring is now rotated by an angle 90° about Z-axis then it is again rotated by 90° about Y-axis. Find the work done by electrostatic forces in each step. If finally larger ring is rotated by 90° about X-axis, find the total work required to perform all three steps.



- 6. S_1 and S_2 are two conducting surfaces. Between S_1 and S_2 and inside S_1 is air. S_1 is spherical with A its centre. S_1 has total charge Q. S_2 is uncharged. Find (if possible) :
 - (i) Charges induced on inner and outer surface of S_2 .
 - (ii) Total electric field at A, B.
 - (iii) Electric field at B due to induced charges on S_2 .
 - (iv) Electric field at C due to induced charges on inner surface of S_2 .
 - (v) Electric field produced by induced charges on outer surface of S_2 inside the body of S_2 .
 - (vi)Can you find electric field at C easily ?
 - (take the required distances from A). Which charge will produce electric field here.



ANSWER KEY			GR # ELECTROSTATICS
SECTION-I			
Single Correct Answer	r Type		14 Q. [3 M (-1)]
1. Ans. (D)	2. Ans. (D)	3. Ans. (C)	4. Ans. (B)
5. Ans. (A)	6. Ans. (B)	7. Ans. (A)	8. Ans. (C)
9. Ans. (A)	10. Ans. (D)	11. Ans. (B)	12. Ans. (A)
13. Ans. (C)	14. Ans. (A)		
Multiple Correct Answer Type			3 Q. [4 M (-1)]
15. Ans. (A,D)	16. Ans. (A,D)	17. Ans. (A,B)	
Linked Comprehension Type (1 Para × 2Q.) [.		M (-1)]	
(Single Correct Answer Type)			
18. Ans. (B)	19. Ans. (C)		
SECTION-II			
Numerical Answer Ty	pe Question		1 Q. [3(0)]
(upto second decimal)	place)		
1. Ans. 180	[)		
SECTION-III			
Numerical Grid Type (Ranging from 0 to 9)			7 Q. [4 M (0)]
1. Ans. 4	2. Ans. 2	3. Ans. 2	4. Ans. 3
5. Ans. 6	6. Ans. 7	7. Ans. 6	
SECTION-IV			
Matrix Match Type (4 × 5) 3 Q. [8 M (for each entry +2(0)]			
1. Ans. (A) \rightarrow (Q, R); (B) \rightarrow (Q,T); (C) \rightarrow (S,T) (D) \rightarrow (P)			
2. Ans. (A) \rightarrow (Q,R); (B) \rightarrow (R,S); (C) \rightarrow (P,Q); (D) \rightarrow (P,S)			
3. Ans. (A) P,R,S (B) P,Q,R,S,T (C) Q,T (D) R,S			
Subjective Type			6 Q. [4 M (0)]
1. Ans. $2 \tan^{-1} \left(\frac{\sigma q}{2 \varepsilon_0 mg} \right)$	2. Ans. $\frac{kP}{\sqrt{2}y^3}(-\hat{i}-2\hat{j})$)	3. Ans. 4 py $(y^2 + 4x^2)^{1/2}$
4. Ans. $v_0 = 3 \text{ m/s}$; K.E. at the origin = $(27 - 10\sqrt{6}) \times 10^{-4} \text{ J}$ approx.2.5 × 10 ⁻⁴ J			
5. Ans. $W_{\text{first step}} = \left(\frac{8}{3} - \frac{4}{\sqrt{5}}\right) \frac{Kq^2}{r}$, $W_{\text{second step}} = 0$, $W_{\text{total}} = 0$			
6. Ans. (i) $-Q$, $+Q$ (ii) $E_A = 0$, can't be found (iii) can't be found (iv) can't be found (v) 0 (vi) No, induced charge on outer surface]			

GUIDED REVISION

PHYSICS

GR # ELECTROSTATICS

14 Q. [3 M (-1)]

SOLUTIONS SECTION-I

Single Correct Answer Type

1. Ans. (D)



$$\begin{aligned} \mathbf{r}_{1} &= \mathbf{r}_{2} = 1 \\ \vec{\mathbf{E}} &= \vec{\mathbf{E}}_{1} + \vec{\mathbf{E}}_{2} \\ \vec{\mathbf{E}} &= \frac{\mathbf{k}\mathbf{q}_{1}\vec{\mathbf{r}}_{1}}{\mathbf{r}_{1}^{3}} + \frac{\mathbf{k}\mathbf{q}_{2}\vec{\mathbf{r}}_{2}}{\mathbf{r}_{2}^{3}} \\ \vec{\mathbf{E}} &= \mathbf{k}\mathbf{q}_{1}\vec{\mathbf{r}}_{1} + \mathbf{k}\mathbf{q}_{2}\vec{\mathbf{r}}_{2} \\ \vec{\mathbf{r}}_{2} &= \frac{\mathbf{E} - \mathbf{k}\mathbf{q}_{1}\vec{\mathbf{r}}}{\mathbf{k}\mathbf{q}_{2}} \end{aligned}$$

Sol. Using Gauss's law, $\phi = \frac{q}{\epsilon_0} = \frac{\sigma \pi (R^2 - x^2)}{\epsilon_0}$

3. Ans. (C)

Sol. $V = -\int \vec{E} \cdot d\vec{r}$

$$V = -\int \left(\sqrt{3}\hat{i} - \hat{j}\right) \cdot \left(dx\hat{i} + dy\hat{j}\right)$$
$$V = -\left[\sqrt{3}x - y\right]$$
$$V = -\sqrt{3}x + y$$

So equipotential line having slope 60°

4. Ans. (B)

Sol. $\rho = \alpha r^2$ [where α is a constant] Charge in the shell (element) $dq = \rho(4\pi r^2 dr) = \alpha(4\pi)r^4 dr$

 \therefore Charge enclosed in sphere of radius r, $q = 4\pi\alpha \int_{0}^{r} r^{4} dr = \frac{4\pi\alpha r^{5}}{5}$

By Gauss's theorem,

at
$$\mathbf{r} = \mathbf{R}/2$$
 $(\mathbf{E}_{\mathrm{r=R}/2}) \left[4\pi \left(\frac{\mathbf{R}}{2}\right)^2 \right] = \frac{4\pi\alpha(\mathbf{R}/2)^5}{5\varepsilon_0}$



$$E_{r=R/2} = \frac{\alpha R}{40\varepsilon_0}$$

Total charge enclosed, $Q = \frac{4\pi\alpha R^5}{5}$

- $\therefore \quad \mathbf{E}_{r=2R} = \frac{4\pi\alpha R^5}{5\epsilon_0} \times \frac{1}{4\pi(2R)^2} = \frac{\alpha R^3}{20\epsilon_0}$ $\frac{\mathbf{E}_{r=2R}}{\mathbf{E}_{r=R/2}} = 2$
- 5. Ans. (A)

6. Ans. (B)

Sol. For earthed conductor [the inner shell], V = 0 Here $V = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{3r} + \frac{q'}{r}\right]$

where q' is charge that would appear on inner shell as it is grounded \Rightarrow q' = -q/3 Hence, the charge flown to earth = 0 -(-q/3) = q/3

- 7. Ans. (A)
- **Sol.** Since E_{inside} conductor = 0

 $\oint \vec{E} \cdot \vec{ds} \text{ over guassian surface} = 0$

$$\Rightarrow \oint \vec{E} \cdot \vec{ds} = \frac{q_{in}}{\varepsilon_0} = \frac{q_0 + q_{in \text{ on surface}}}{\varepsilon_0} \Rightarrow q_{in} \text{ on surface} = -q_0$$

dr

8. Ans. (C)



Electric potential at "O" due to charge element

$$V_{0} = \int_{a}^{b} \frac{\sigma(2\pi r dr)}{(4\pi \in_{0})r} = \frac{\sigma}{4 \in_{0}} \int_{a}^{b} V_{0} = \frac{\sigma}{4 \in_{0}} (b-a)$$





After reaching at B, it will complete the circle Conservation of energy at A & B

$$\frac{KQ}{4a} = V_A$$

$$\frac{Kq(Q)}{4a} + \frac{1}{2}mv^2 = \frac{KqQ}{a}$$

$$\frac{1}{2}mv^2 = \frac{KqQ}{a}\left(\frac{3}{4}\right)$$

$$\frac{1}{2}mv^2 = 3Vq$$

$$V = \sqrt{\frac{6Vq}{m}}$$

10. Ans. (D)

Sol.
$$\vec{E} = -\frac{dv}{dx}\hat{i} - \frac{dv}{dy}\hat{j} = -2x\hat{i} + 2y\hat{j}$$

 $\tan \theta = -\frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$
 $\ell ny = -\ell nx + \ell nC$ $\ell n(xy) = \ell nC$
 $xy = C$
shape will be rectangular hyperbola

11. Ans. (B)

Sol. $U = -\vec{p}_1 \vec{E}$

$$U = -\frac{2kp_1p_2}{r^3}\cos\theta$$

E - electric field at p_1 due to p_2 .



12. Ans. (A)Sol. E' is the electric field due to conducting sheets.

- 13. Ans. (C) Sol. $V_A = V_B$ $r_A \ge r_B$ so, charge density at A > charge density at B $E_A > E_B$ flux = 0
- 14. Ans. (A)



net force =
$$2F\sin\theta = 2\left[\frac{kqq_0}{(a^2 + y^2)}\right] \cdot \frac{y}{(a^2 + y^2)^{1/2}}$$

$$=\frac{2kqq_0}{\left(a^2+y^2\right)^{3/2}}\approx\frac{2kqq_0y}{a^3}$$

$$\Rightarrow f_{net} \propto y$$

Multiple Correct Answer Type

15. Ans. (A,D)

Sol. (A) $V_{\text{centre}} = \frac{KQ}{a} + \frac{K2\pi\lambda b}{b} = \frac{KQ}{a} + \frac{\lambda}{2\epsilon_0}$ (B) $V_{\text{centre}} = 0$ for eathing $\frac{KQ'}{a} + \frac{K2\pi\lambda b}{b} = 0$ $Q' = -2\pi\lambda.a$ (C) $\vec{E}_{\text{inside sphere}} = \vec{0}$ always (property of conductor)

(D) Conductor without cavity is always equipotential.

16. Ans. (A,D)



3 Q. [4 M (-1)]

17. Ans. (A,B)



Field due to straight wire is perpendicular to the wire & radially outward. Hence $E_z = 0$ Length, PQ = 2R sin 60 = $\sqrt{3}R$ According to Gauss's law

total flux =
$$\oint \vec{E} \cdot \vec{ds} = \frac{q_{in}}{\epsilon_0} = \frac{\lambda \sqrt{3R}}{\epsilon_0}$$

Linked Comprehension Type (Single Correct Answer Type) 18. Ans. (B) Sol. dP = (dq)2R

$$P_{Net} = \int (dp)\cos\theta = \int (dq)2R\cos\theta$$
$$= \int (\lambda R d\theta)2R\cos\theta = 2R^2 \lambda \int_{(-\pi/2)}^{(+\pi/2)} \cos\theta d\theta = 4R^2 \lambda$$

19. Ans. (C)

Sol. Conservation of energy $\Delta U_{loss} = \Delta K_{gain}$

$$PE_0(\cos 90^\circ - \cos 180^\circ) = \frac{1}{2}I\omega^2$$
$$4R^2\lambda E_0 = \frac{1}{2}(mR^2)\omega^2 \Longrightarrow \omega = \sqrt{\frac{8\lambda E_0}{m}}$$

SECTION-II

Numerical Answer Type Question (upto second decimal place)

1. Ans. 180

Sol. When outer surface is grounded charge '–Q' resides on the inner surface of sphere 'B' Now sphere A is connected to earth potential on its surface becomes zero. Let the charge on the surface A becomes q

$$\frac{\mathrm{kq}}{\mathrm{a}} - \frac{\mathrm{kQ}}{\mathrm{b}} = 0 \Longrightarrow \mathrm{q} = \frac{\mathrm{a}}{\mathrm{b}}\mathrm{Q}$$

Consider the figure. In this position energy stored

$$E_1 = \frac{1}{8\pi\varepsilon_0 a} \left[\frac{a}{b}Q\right]^2 + \frac{Q^2}{8\pi\varepsilon_0 b} + \frac{1}{4\pi\varepsilon_0 b} \left[\frac{a}{b}Q\right](-Q)$$





(1 Para × 2Q.) [3 M (-1)]



when S_3' is closed, total charge will appear on the outer surface of shell 'B'. In this position energy

dq

dr

R

stored
$$E_2 = \frac{1}{8\pi\epsilon_0 b} \left(\frac{a}{b} - 1\right)^2 Q^2$$
 Heat produced $= E_1 - E_2 = \frac{Q^2 a(b-a)}{8\pi\epsilon_0 b^3} = 180$
SECTION-III

Numerical Grid Type (Ranging from 0 to 9)

7 Q. [4 M (0)]

1. Ans. 4
Sol. For flux to be zero

$$q_{in} = 0$$

 $dq = \rho 4\pi^2 dr$
 $dq = \rho_0 \left(1 - \frac{nr}{3R}\right) 4\pi^2 dr$
 $d_{inc} = 4\pi\rho_0 \int_y^R \left(r^2 - \frac{nr^3}{3R}\right) dr$
 $= 4\pi\rho_0 \left(\frac{R^2}{3} - \frac{nR^4}{4 \times 3R}\right) = \frac{4\pi\rho_0 R^3}{3} \left[1 - \frac{n}{4}\right]$
 $q_{inc} = 0$ when $1 - \frac{n}{4} = 0$
 $n = 4$
2. Ans. 2
Sol. Force $= \left[\frac{\sigma^2}{2\epsilon_0}\right] \times A$
 $= \left[\frac{Q}{A}\right]^2 \times \frac{A}{2\epsilon_0} = \frac{Q^2}{2A\epsilon_0} = \frac{Q^2}{2a^2\epsilon_0}$
 $N = 2$
3. Ans. 2
Sol. By energy conservation
 $k_1 + pE_i = kE_f + PE_f$
 $0 + \left(-\frac{kq^2}{5}\right) \times 2 = \frac{1}{2}mv^2 + \left(-\frac{kqL}{3}\right)q$
 $\frac{1}{2}mv^2 = 2\frac{kq^2}{5}\left(\frac{1}{3} - \frac{1}{5}\right)$
 $\frac{1}{2}mv^2 = \frac{4kq^2}{15\pi}$
 $v = \sqrt{\frac{8kq^2}{15\pi}} = q\sqrt{\frac{8}{4\pi\epsilon_0 \times 15m}}$
 $v = q\sqrt{\frac{2}{15\pi\epsilon_0 m}}$
 $x = 2$



4. Ans. 3

Sol. Net flux in x-direction = 0;

Net flux in y-direction =
$$A[3(1^2+2)] - A[3(0^2+2)] \Rightarrow q = -3 \in_0^{-1} (as A = 1m^2)$$

5. Ans. 6

Sol.
$$V_{(0,0,0)} = 0$$
 and $V_{(0,0,0,1)} = \frac{a(2)}{100} = \frac{a}{50}$

$$W_{elect} = q(V_1 - V_2) \Longrightarrow 50 \times 10^{-6} = 4 \times 10^{-6} \times \frac{a}{50}$$

$$\implies a = 625$$

$$6250 \text{ volts} = 625 (x^2) + 625 (y^2) + 625 (2) (2)^2$$

$$\implies 6 = x^2 + y^2 \text{ Thus } R = \sqrt{6}$$

6. Ans. 7

Sol.
$$\vec{E} = \frac{a\hat{r}}{r^2}$$
; $V = \frac{-a}{r}$; $\Delta V = \frac{a}{r_2} - \frac{a}{r_1} = \frac{a}{5} - \frac{a}{7} = \frac{2a}{35} = 2 \times \frac{122.5}{35} = 7 V$
 $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
 $|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$

7. Ans. 6



$$d\phi = \frac{\lambda}{2\pi\epsilon_0} \frac{\lambda}{\sqrt{d^2 + x^2}} \cdot \frac{d}{\sqrt{d^2 + x^2}} Ldx$$

$$\phi = \frac{\lambda dL}{2\pi\epsilon_0} \int_{-a/2}^{a/2} \frac{dx}{d^2 + x^2}$$

$$= \frac{\lambda(d)(L)}{2\pi\epsilon_0} \left(\frac{1}{d}\right) 2 \left[\tan^{-1}\frac{x}{d}\right]_0^{a/2} = \frac{\lambda L}{\pi\epsilon_0} \left(\tan^{-1}\frac{a}{2\cdot\frac{a\sqrt{3}}{2}}\right) = \frac{\lambda L}{6\pi\epsilon_0}$$

SECTION-IV

Matrix Match Type (4×5)

3 Q. [8 M (for each entry +2(0)]

- 1. Ans. (A) \rightarrow (Q, R); (B) \rightarrow (Q,T); (C) \rightarrow (S,T) (D) \rightarrow (P)
- 2. Ans. (A) \rightarrow (Q,R); (B) \rightarrow (R,S); (C) \rightarrow (P,Q); (D) \rightarrow (P,S)
- Sol. Since cavity is spherical charge induces uniformly on its inner surface but an outer surface charge distribution is non-uniform.Due to point charge electric field is radially outwards.Near conducting surface net electric field is always perpendicular to the conductor.
- 3. Ans. (A) P,R,S (B) P,Q,R,S,T (C) Q,T (D) R,S



E at C is zero, field at D may not to zero as sizes and location of cavities are unknown.



$$\theta_{\text{max}} = 2\theta = 2 \tan^{-1} \left(\frac{\sigma q}{2 \text{mg} \epsilon_0} \right)$$
2. Ans. $\frac{kP}{\sqrt{2} y^3} (-\hat{i} - 2\hat{j})$



$$\begin{split} \vec{E}_{\rm P} &= \frac{kp\cos 45}{y^3} \Big(-\hat{i}\Big) + \Bigg(\frac{2kp}{y^3}\cos 45\Bigg) \Big(-\hat{j}\Big) \\ E_{\rm P} &= \frac{kp}{\sqrt{2}y^3} \Big(-\hat{i} - 2\hat{j}\Big) \end{split}$$

3. Ans. 4 py
$$(y^2 + 4x^2)^{1/2}$$

Sol. In non uniform electric field force on dipole

$$\begin{split} F &= \vec{p}. \frac{\vec{d} \in}{dr} \\ \vec{E} &= 4xy^2 \,\hat{i} + 4x^2y \,\hat{j} \\ \frac{d \in}{dx} &= 4y^2 \,\hat{i} + 8xy \,\hat{j} \\ \frac{d \in}{dx} &= 4y^2 \,\hat{i} + 8xy \,\hat{j} \\ \frac{d \in}{dy} &= 8y \,\hat{i} + 4x^2 \,\hat{j} \\ F_x &= \vec{p} \cdot \frac{\vec{d} \in}{dx} = 4py^2 \\ F_y &= \vec{p} \cdot \frac{\vec{d} \in}{dy} = 8xyp \\ F_{Net} &= \sqrt{F_x^2 + F_y^2} = \sqrt{16p^2y^4 + 64p^2x^2y^2} \\ &= 4py \left(y^2 + 4x^2\right)^{\frac{1}{2}} \end{split}$$

4. Ans. $v_0 = 3 \text{ m/s}$; K.E. at the origin = $(27 - 10\sqrt{6}) \times 10^{-4} \text{ J}$ approx.2.5 × 10⁻⁴ J



P is origin. We can see from diagram electric field at P is zero but we have to check if it is zero at any other point x-axis we have to give particle just as much velocity to reach to point where electric field is zero. (Called neutral point) Let field is zero at θ .

$$(E_{\theta})_{Net} = 0$$

$$2E\cos\theta = 2E_{0}\cos\theta_{1}$$

$$2 \times \frac{kq_{0}}{r^{2}} \times \frac{x}{r} = 2 \times \frac{k8q_{0}}{r_{1}^{2}} \times \frac{x}{r_{1}}$$

$$\frac{1}{r^{3}} = \frac{8}{r_{1}^{3}} \Longrightarrow \frac{r_{1}}{r} = 2$$
For Q $\frac{r_{1}}{r} = +2$

$$r_{1} = 2r$$

$$quad x^{2} + \frac{27}{2} = 2\sqrt{x^{2} + \frac{3}{2}}$$

$$r = 2$$

$$x^{2} + \frac{27}{2} = 4x^{2} + \frac{12}{2}$$

$$3x^{2} = \frac{15}{2} \Longrightarrow x = \sqrt{\frac{5}{2}}$$

To reach at origin particle has to reach at Q the it will be attracted towards origin by electric fields of $-1\mu C$

Apply work energy theorem from ∞ to Q.

$$-q\Delta V = k_{f} - k_{i}$$

$$-q(v_{Q} - v_{\infty}) = 0 - \frac{1}{2}mv_{0}^{2}$$

$$-1 \times 10^{-6} \left[\left(\frac{2 \times 8 \times 10^{-6} \times 9 \times 10^{9}}{r_{1}} - \frac{2 \times 10^{-6} \times 9 \times 10^{9}}{r} \right) - 0 \right] = -\frac{1}{2}mv_{0}^{2}$$

$$-10^{6} \times 2 \times 10^{-6} \times 9 \times 109 \left(\frac{8}{4} - \frac{1}{2} \right) = -\frac{1}{2}mv_{0}^{2}$$

Solving we get $v_0 = 3$ m/s

To find kinetic energy at P apply work energy theorem from $Q \rightarrow P$ -q ($v_p - v_q$) = KE_p - KE_q.

$$v_{\rm p} = \frac{k\left(-1 \times 10^{-6}\right)}{\frac{\sqrt{3}}{2}} \times 2 + \frac{k\left(8 \times 10^{-6}\right)}{\frac{\sqrt{27}}{2}} \times 2$$

$$v_{\rm Q} = \frac{k\left(-1 \times 10^{-6}\right) \times 2}{2} + \frac{k\left(8 \times 10^{-6}\right)}{4} \times 2$$

 $\begin{aligned} -q & (v_{p} - v_{Q}) = k_{p} - E_{Q} \\ -1 \times 10^{-6} & (v_{p} - V_{Q}) = k_{p} - 0 \\ \text{Put value of } v_{p} & v_{Q} \text{ and find KE at P.} \end{aligned}$

5. Ans.
$$W_{\text{first step}} = \left(\frac{8}{3} - \frac{4}{\sqrt{5}}\right) \frac{\text{Kq}^2}{\text{r}}$$
, $W_{\text{second step}} = 0$, $W_{\text{total}} = 0$

6. Ans. (i) -Q, +Q (ii) $E_A = 0$, can't be found (iii) can't be found (iv) can't be found (v) 0 (vi) No, induced charge on outer surface]



- (i) on inner surface of S_1 charge induce = -Qon outer surface = +Q
- (ii) $E_A = 0$ inside spherical shells at B can not be found
- (iii) S₂ is assymetric, its electric field can not be calculated
- (iv)at C electric field can not be calculated as outer surface S_2 is not symmetric