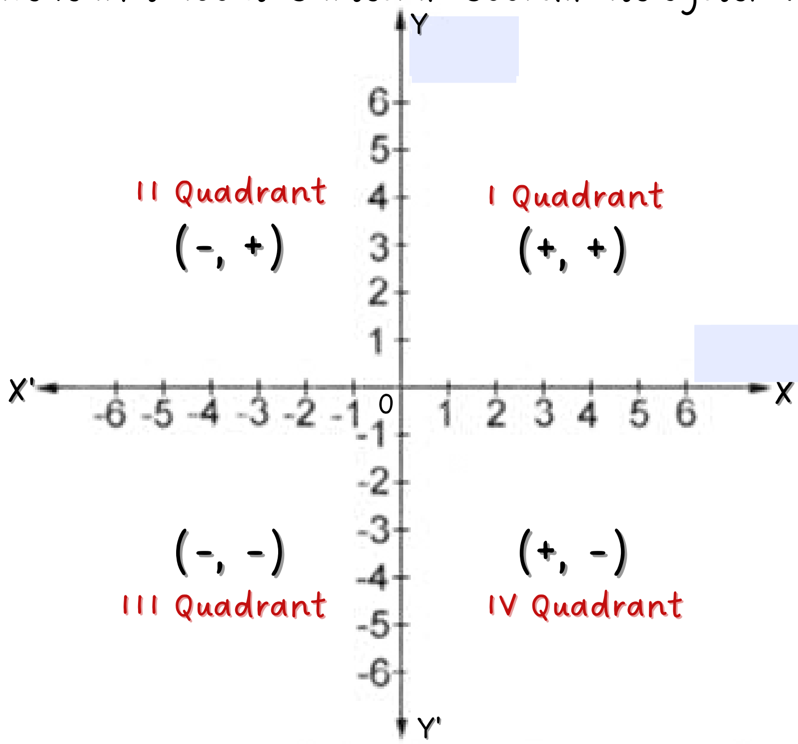
Coordinate Geometry

Introduction

In Class9, we have learnt about 'Cartesian Coordinate System'. Let us once recall it.



- In the cartesian co-ordinate system, there is a cartesian plane which is made up of two number lines i.e. X-axis (horizontal) ; Y-axis (Vertical)
- The intersection point of these two lines is known as centre or the "Origin" of the co-ordinate plane denoted by O.
- Any point on this co-ordinate plane is represented by the ordered pair of numbers. Let (a,b) is an ordered pair then a is the x-coordinate and b is the y-coordinate.
- The distance of any point from the y-axis is called its x-coordinate or abscissa and the distance of any point from x-axis is called its y-coordinate or ordinate . E.g. (7, 3) => here 7 is abscissa and 3 is ordinate.

Distance Formula

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$A(x_1, y_1)$ ——— d ——— $B(x_2, y_2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Remarks :

(i) The distance of a point $P(x, y)$ from the origin $O(0, 0)$ is given by

$$OP = \sqrt{x^2 + y^2}\ .$$

(ii) Ham agar chahe toh ye formula bhi use kar sakte hain, answer dono se same aata hai.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Let's Practice :

Example: Find the distance between the following pairs of points:

(i) (2, 3) , (4, 1) (ii) (a, b) , (-a, -b)

SOLUTION: Using above distance formula to find distance between given points.

(i) $d = \sqrt{(2-4)^2 + (3-1)^2}$

$= \sqrt{(-2)^2 + (2)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{8}$

$= 2\sqrt{2}$ units

(ii) $d = \sqrt{(a-(-a))^2 + (b-(-b))^2}$

$= \sqrt{(2a)^2 + (2b)^2}$

$= \sqrt{4a^2 + 4b^2}$

$= 2\sqrt{a^2 + b^2}$ units

Example: Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Three points A, B and C are collinear if AB + BC = AC.

Here points are A(1, 5), B(2, 3) and C(-2, -11).

$\therefore AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{1^2 + (-2^2)} = \sqrt{1 + 4} = \sqrt{5}$

$BC = \sqrt{((-2)-(2))^2 + ((-11)-(3))^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{212}$

$AC = \sqrt{((-2)-(1))^2 + ((-11)+(5))^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{265}$

Since, clearly $AB + BC \neq CA$.

Therefore, the points (1, 5), (2, 3) and (-2, -11) are not collinear.

Example: Find the point on the X-axis which is equidistant from (2, -5) and (-2, 9).

SOLUTION: We know that y-coordinate of any point on x-axis is 0.

So, let the required point on X-axis be P(x, 0).

It is given that P is equidistant from A(2, -5) and B(-2, 9).

∴ PA = PB

=> $\sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x+2)^2 + (0-9)^2}$

=> $(x - 2)^2 + 25 = (x + 2)^2 + 81$

=> $x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

=> $x^2 - x^2 - 4x - 4x = 4 + 81 - 4 - 25$

=> $-8x = 85 - 29$

=> $x = \frac{56}{-8} = -7$

∴ Required point on X-axis = (-7, 0)

Example: Find a relation between X and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

Let the point P(x, y) is equidistant from the points A(3, 6) and B(-3, 4)

∴ PA = PB

$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$

Squarring both the sides, we get

=> $(x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$

=> $x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

=> $x^2 - x^2 - 6x - 6x + y^2 - y^2 - 12y + 8y = 9 + 16 - 9 - 36$

=> $-6x - 6x - 12y + 8y = 25 - 45$

=> $\frac{-12x}{4} - \frac{4y}{4} = \frac{-20}{4}$

=> $-3x - y = -5$

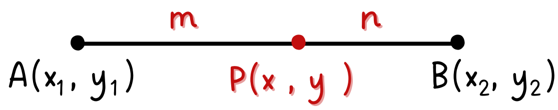
=> $3x + y = 5$

=> $3x + y - 5 = 0$

Hence, this is the required relation between X and y.

*Tip: Practice different types of questions from PYQ's , reference books, etc to be fully prepared from this topic, because NCERT me kam questions hain.

Section Formula

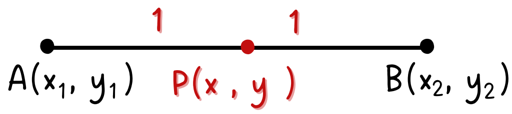


If P(x, y) is any point on the line segment AB, which divides AB in the ratio m : n then the coordinates of the point P(x, y) will be

$$\left(x = \frac{mx_2 + nx_1}{m + n} \;,\; y = \frac{my_2 + ny_1}{m + n} \right)$$

*Note: Point P(x, y) has divided the line segment internally here.

- ## Mid-Point Formula



If P(x, y) is the mid-point of the line segment AB, which divides AB in the ratio of 1 : 1 then the coordinates of the point P(x, y) will be

$$\left(x = \frac{x_1 + x_2}{2} \;,\; y = \frac{y_1 + y_2}{2} \right)$$

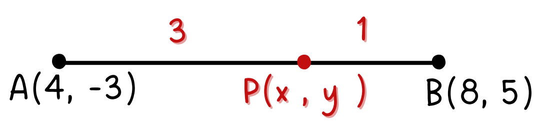
- ## IMP for Questions:

If the Point P(x, y) divides the line segment AB internally, but the ratio is not given in the question, then it is taken to be k : 1 and the coordinates of point P will be

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1} \right).$$

Example: Find coordinates of the point which divides the line segment joining (4, -3) and (8, 5) in 3 : 1 internally.

SOLUTION: Let P(x, y) the the required point which divides AB in the ratio of 3:1. By using the Section formula, we have



=> $x = \dfrac{3(8) + 1(4)}{3 + 1}$; $y = \dfrac{3(5) + 1(-3)}{3 + 1}$

=> $x = \dfrac{24 + 4}{4}$; $y = \dfrac{15 - 3}{4}$

=> $x = \dfrac{28}{4}$; $y = \dfrac{12}{4}$

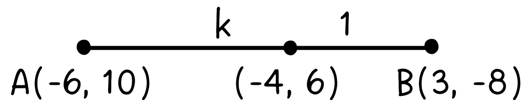
=> $x = 7$; $y = 3$

Therefore, (7, 3) is the required point.

Example: Find the ratio in which the line segment joining the points A(-6, 10) and B(3, -8) is divided by (-4, 6).

*suno ratio nahi diya hai, toh k : 1 lene ko kaha tha.....Remember!

SOLUTION: Let (-4, 6) divides AB internally in the ratio k : 1. Using section formula,



$$\Rightarrow \left(\frac{3k + 1(-6)}{k + 1}, \frac{-8k + 1(10)}{k + 1}\right) = (-4, 6)$$

$$\Rightarrow \left(\frac{3k - 6}{k + 1}, \frac{-8k + 10}{k + 1}\right) = (-4, 6)$$

Taking $\frac{3k - 6}{k + 1} = -4 \Rightarrow 3k - 6 = -4(k + 1) \Rightarrow 3k - 6 = -4k - 4$

$\Rightarrow 3k + 4k = -4 + 6 \Rightarrow 7k = 2$

$\Rightarrow k = \frac{2}{7}$

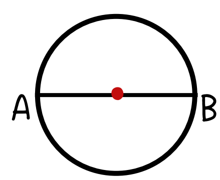
Ratio = $k : 1 \Rightarrow \frac{2}{7} : 1 \Rightarrow 2 : 7$

So, the point (-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8) in the ratio 2 : 7.

Example: Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).

SOLUTION: Let the coordinates of point A be (x, y).

AB is a diameter of circle, then mid-point of AB is centre of circle which is (2, -3).



Mid point of A and B = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$\therefore \quad (2, -3) = \left(\frac{x + 1}{2}, \frac{y + 4}{2}\right)$$

$\Rightarrow \quad \frac{x + 1}{2} = 2 \quad ; \quad \frac{y + 4}{2} = -3$

$\Rightarrow \quad x + 1 = 4 \quad ; \quad y + 4 = -6$

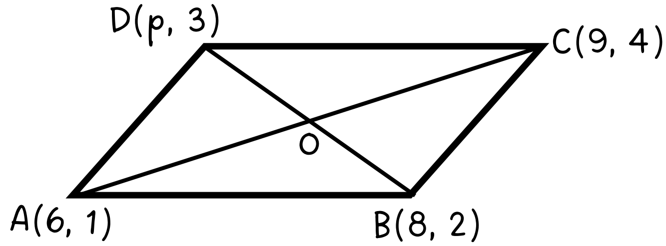
$\Rightarrow \quad x = 4 - 1 \quad ; \quad y = -6 - 4$

$\Rightarrow \quad x = 3 \quad ; \quad y = -10$

Therefore, coordinates of A are (3, -10)

Example: If the points (6, 1), (8, 2), (9, 4) and (p, 3) are the vertices of a parallelogram, find the value of p.

SOLUTION: Let the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram. Also we know that diagonals of a ||gm bisect each other.



If O is the mid point of AC, then coordinates of O are = $\left(\frac{6+9}{2}, \frac{1+4}{2}\right)$

If O is the mid point of BD, then coordinates of O are = $\left(\frac{8+p}{2}, \frac{2+3}{2}\right)$

Since both coordinates are of the same point O

$$\therefore \quad \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$$

$$\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

$$\frac{15}{2} = \frac{8+p}{2}$$

$$15 = 8 + p$$

$$p = 15 - 8$$

$$p = 7$$

Hence, the value of p is 7 and so the coordinates of D are (7, 3).

Ques: Find the area of a rhombus if its vertices are (3, 0), (4, 5), (- 1, 4) and (- 2, - 1) taken in order.

Solution:

A rhombus has all sides of equal length and opposite sides are parallel.
Let A(3, 0), B(4, 5), C(- 1, 4) and D(- 2, - 1) be the vertices of a rhombus ABCD.

Also, Area of a rhombus =1/2 × (product of its diagonals)

Hence we will calculate the values of the diagonals AC and BD.

We know that the distance between the two points is given by the distance formula,

Distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Therefore, distance between A (3, 0) and C (- 1, 4) is given by

Length of diagonal AC = $\sqrt{[3 - (-1)]^2 + [0 - 4]^2}$

= $\sqrt{(16 + 16)}$

= $4\sqrt{2}$

The distance between B (4, 5) and D (- 2, - 1) is given by

Length of diagonal BD = $\sqrt{[4 - (-2)]2 + [5 - (-1)]^2}$

= $\sqrt{(36 + 36)}$

= $6\sqrt{2}$

Area of the rhombus ABCD = 1/2 × (Product of lengths of diagonals) = 1/2 × AC × BD

Therefore, the area of the rhombus ABCD = 1/2 × $4\sqrt{2}$ × $6\sqrt{2}$ square units

= 24 square units