CHAPTER 14

Oscillations

14.3 Simple Harmonic Motion

1. The displacement of a particle executing simple harmonic motion is given by $y = A_0 + A\sin\omega t + B\cos\omega t$. Then the amplitude of its oscillation is given by

(a) $A + B$ (b) $A_0 + \sqrt{A^2 + B^2}$

(c) $\sqrt{A^2 + B^2}$ (d) $\sqrt{A_0^2 + (A + B)^2}$

(NEET 2019)

2. The distance covered by a particle undergoing SHM in one time period is (amplitude = A)

(a) zero (b) A (c) $2A$ (d) $4A$

(Odisha NEET 2019)

3. Out of the following functions representing motion of a particle, which represents SHM ?

(1) $y = \sin\omega t - \cos\omega t$ (2) $y = \sin^3\omega t$

(3) $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$

(4) $y = 1 + \omega t + \omega^2 t^2$

(a) Only (1)

(b) Only (4) does not represent SHM

(c) Only (1) and (3) (d) Only (1) and (2) *(2011)*

4. Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite directions when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. The phase difference is

(a) $\frac{\pi}{6}$ (b) 0 (c) $\frac{2\pi}{3}$ (d) π

(Mains 2011)

5. The displacement of a particle along the x-axis is given by $x = a\sin^2\omega t$. The motion of the particle corresponds to

(a) simple harmonic motion of frequency ω/π

(b) simple harmonic motion of frequency $3\omega/2\pi$

(c) non simple harmonic motion

(d) simple harmonic motion of frequency $\omega/2\pi$ *(2010)*

6. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

(a) $T/8$ (b) $T/12$ (c) $T/2$ (d) $T/4$. *(2007)*

7. The circular motion of a particle with constant speed is

(a) periodic but not simple harmonic

(b) simple harmonic but not periodic

(c) period and simple harmonic

(d) neither periodic nor simple harmonic. *(2005)*

8. Two SHM's with same amplitude and time period, when acting together in perpendicular directions with a phase difference of $\pi/2$, give rise to

(a) straight motion (b) elliptical motion

(c) circular motion (d) none of these. *(1997)*

9. A simple harmonic oscillator has an amplitude A and time period T. The time required by it to travel from $x = A$ to $x = A/2$ is

(a) $T/6$ (b) $T/4$ (c) $T/3$ (d) $T/2$ *(1992)*

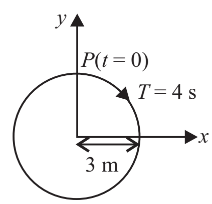
10. The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π results in the displacement of the particle along

(a) circle (b) figure of eight

(c) straight line (d) ellipse *(1990)*

14.4 Simple Harmonic Motion and Uniform Circular Motion

11. The radius of circle, the period of revolution, initial position and sense of revolution are indicated in the figure. y-projection of the radius vector of rotating particle P is



(a) $y(t) = 3\cos\left(\frac{\pi t}{2}\right)$, where y in m

(b) $y(t) = -3\cos 2\pi t$, where y in m

(c) $y(t) = 4\sin\left(\frac{\pi t}{2}\right)$, where y in m

(d) $y(t) = 3\cos\left(\frac{3\pi t}{2}\right)$, where y in m *(NEET 2019)*

14.5 Velocity and Acceleration in Simple Harmonic Motion

12. The phase difference between displacement and acceleration of a particle in a simple harmonic motion is
(a) π rad (b) $3\pi/2$ rad
(c) $\pi/2$ rad (d) zero *(NEET 2020)*

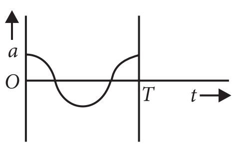
13. Average velocity of a particle executing SHM in one complete vibration is
(a) zero (b) $\frac{A\omega}{2}$
(c) $A\omega$ (d) $\frac{A\omega^2}{2}$ *(NEET 2019)*

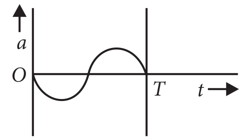
14. A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is
(a) $\frac{\sqrt{5}}{2\pi}$ (b) $\frac{4\pi}{\sqrt{5}}$ (c) $\frac{2\pi}{\sqrt{3}}$ (d) $\frac{\sqrt{5}}{\pi}$ *(NEET 2017)*

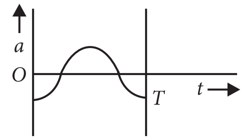
15. A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β. Then, its time period of vibration will be
(a) $\frac{\beta^2}{\alpha}$ (b) $\frac{2\pi\beta}{\alpha}$ (c) $\frac{\beta^2}{\alpha^2}$ (d) $\frac{\alpha}{\beta}$ *(2015)*

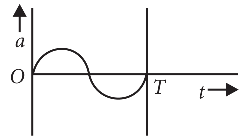
16. A particle is executing SHM along a straight line. Its velocities at distances x_1 and x_2 from the mean position are V_1 and V_2, respectively. Its time period is
(a) $2\pi\sqrt{\frac{V_1^2+V_2^2}{x_1^2+x_2^2}}$ (b) $2\pi\sqrt{\frac{V_1^2-V_2^2}{x_1^2-x_2^2}}$
(c) $2\pi\sqrt{\frac{x_1^2+x_2^2}{V_1^2+V_2^2}}$ (d) $2\pi\sqrt{\frac{x_2^2-x_1^2}{V_1^2-V_2^2}}$
(2015 Cancelled)

17. The oscillation of a body on a smooth horizontal surface is represented by the equation, $X = A\cos(\omega t)$
where X = displacement at time t
ω = frequency of oscillation
Which one of the following graphs shows correctly the variation of a with t ?
Here a = acceleration at time t, T = time period

(a) 

(b) 

(c) 

(d) 

(2014)

18. A particle of mass m oscillates along x-axis according to equation $x = a\sin\omega t$. The nature of the graph between momentum and displacement of the particle is
(a) Circle
(b) Hyperbola
(c) Ellipse
(d) Straight line passing through origin.
(Karnataka NEET 2013)

19. Two simple harmonic motions of angular frequency 100 and 1000 rad s^{-1} have the same displacement amplitude. The ratio of their maximum acceleration is
(a) $1 : 10^3$ (b) $1 : 10^4$ (c) $1 : 10$ (d) $1 : 10^2$ *(2008)*

20. A point performs simple harmonic oscillation of period T and the equation of motion is given by $x = a\sin(\omega t + \pi/6)$. After the elapse of what fraction of the time period, the velocity of the point will be equal to half of its maximum velocity?
(a) $T/3$ (b) $T/12$ (c) $T/8$ (d) $T/6$ *(2008)*

21. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is
(a) π (b) 0.707π
(c) zero (d) 0.5π *(2007)*

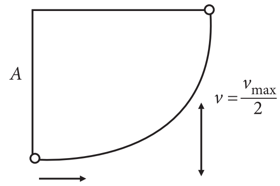
22. A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of its oscillation is
(a) 4 Hz (b) 3 Hz
(c) 2 Hz (d) 1 Hz. *(2004)*

23. Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion ?
(a) When v is maximum, a is maximum.
(b) Value of a is zero, whatever may be the value of v.
(c) When v is zero, a is zero.
(d) When v is maximum, a is zero. *(2003)*

24. A particle starts with S.H.M. from the mean position as shown in the figure. Its amplitude is A and its time period is T. At one time, its speed is half that of the maximum speed.
What is its displacement?

(a) $\frac{2A}{\sqrt{3}}$

(b) $\frac{3A}{\sqrt{2}}$



(c) $\frac{\sqrt{2}A}{3}$ (d) $\frac{\sqrt{3}A}{2}$ *(1996)*

25. If a simple harmonic oscillator has got a displacement of 0.02 m and acceleration equal to 2.0 m/s^2 at any time, the angular frequency of the oscillator is equal to

(a) 10 rad/s (b) 0.1 rad/s
(c) 100 rad/s (d) 1 rad/s *(1992)*

26. A body is executing simple harmonic motion. When the displacements from the mean position is 4 cm and 5 cm, the corresponding velocities of the body is 10 cm/sec and 8 cm/sec. Then the time period of the body is

(a) 2π sec (b) $\pi/2$ sec
(c) π sec (d) $3\pi/2$ sec *(1991)*

14.6 Force Law for Simple Harmonic Motion

27. Which one of the following equations of motion represents simple harmonic motion?

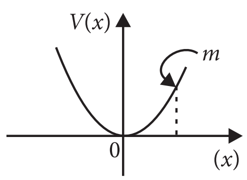
(a) Acceleration = $-k(x + a)$
(b) Acceleration = $k(x + a)$
(c) Acceleration = kx
(d) Acceleration = $-k_0x + k_1x^2$
where k, k_0, k_1 and a are all positive. *(2009)*

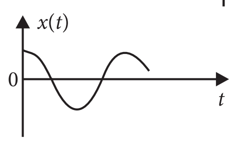
28. A particle executes S.H.M. along x-axis. The force acting on it is given by

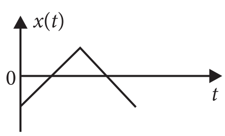
(a) $A\cos(kx)$ (b) Ae^{-kx}
(c) Akx (d) $-Akx$. *(1994, 1988)*

14.7 Energy in Simple Harmonic Motion

29. A particle of mass m is released from rest and follows a parabolic path as shown. Assuming that the displacement of the mass from the origin is small, which graph correctly depicts the position of the particle as a function of time?



(a)


(b)


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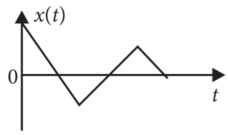
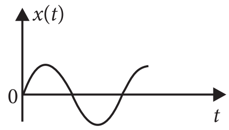


(d)


(2011)

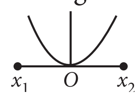
30. The particle executing simple harmonic motion has a kinetic energy $K_0\cos^2\omega t$. The maximum values of the potential energy and the total energy are respectively

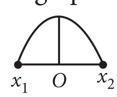
(a) $K_0/2$ and K_0 (b) K_0 and $2K_0$
(c) K_0 and K_0 (d) 0 and $2K_0$ *(2007)*

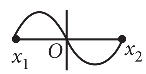
31. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is

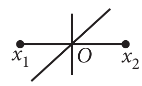
(a) $\frac{2}{3}E$ (b) $\frac{1}{8}E$ (c) $\frac{1}{4}E$ (d) $\frac{1}{2}E$ *(2003)*

32. A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position being O. Its potential energy is plotted. It will be as given below in the graph

(a)


(b)


(c)


(d)


(2003)

33. Displacement between maximum potential energy position and maximum kinetic energy position for a particle executing simple harmonic motion is

(a) $\pm a/2$ (b) $+a$
(c) $\pm a$ (d) -1 *(2002)*

34. The total energy of particle performing SHM depends on

(a) k, a, m (b) k, a
(c) k, a, x (d) k, x. *(2001)*

35. A linear harmonic oscillator of force constant 2×10^6 N/m and displacement 0.01 m has a total mechanical energy of 160 J. Its

(a) P.E. is 160 J (b) P.E. is zero
(c) P.E. is 100 J (d) P.E. is 120 J. *(1996)*

36. In a simple harmonic motion, when the displacement is one-half the amplitude, what fraction of the total energy is kinetic?

(a) 1/2 (b) 3/4 (c) zero (d) 1/4. *(1995)*

37. A loaded vertical spring executes S.H.M. with a time period of 4 sec. The difference between the kinetic energy and potential energy of this system varies with a period of

(a) 2 sec (b) 1 sec
(c) 8 sec (d) 4 sec *(1994)*

38. A body executes simple harmonic motion with an amplitude A. At what displacement from the mean position is the potential energy of the body is one fourth of its total energy ?
(a) $A/4$ (b) $A/2$
(c) $3A/4$
(d) Some other fraction of A *(1993)*

39. The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement x from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is
(a) $\frac{(a^2 - x^2\omega^2)}{x^2\omega^2}$ (b) $\frac{x^2\omega^2}{(a^2 - x^2\omega^2)}$
(c) $\frac{(a^2 - x^2)}{x^2}$ (d) $\frac{x^2}{(a^2 - x^2)}$ *(1991)*

14.8 Some Systems Executing Simple Harmonic Motion

40. A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20 m s^{-2} at a distance of 5 m from the mean position. The time period of oscillation is
(a) 2π s (b) π s (c) 2 s (d) 1 s
(NEET 2018)

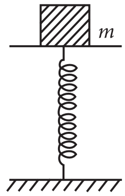
41. A spring of force constant k is cut into lengths of ratio 1 : 2 : 3. They are connected in series and the new force constant is k'. Then they are connected in parallel and force constant is k''. Then $k' : k''$ is
(a) 1 : 9 (b) 1 : 11 (c) 1 : 14 (d) 1 : 6
(NEET 2017)

42. A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. The value of m in kg is
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{16}{9}$ (d) $\frac{9}{16}$
(NEET-II 2016)

43. The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be
(a) T (b) $\frac{T}{\sqrt{2}}$ (c) $2T$ (d) $\sqrt{2}T$
(2010)

44. A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude a and time period T. The speed of the pendulum at $x = a/2$ will be
(a) $\frac{\pi a}{T}$ (b) $\frac{3\pi^2 a}{T}$ (c) $\frac{\pi a\sqrt{3}}{T}$ (d) $\frac{\pi a\sqrt{3}}{2T}$ *(2009)*

45. A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released, the mass executes a simple harmonic motion. The spring constant is 200 N/m. What should be the minimum amplitude of the motion so that the mass gets detached from the pan? (take g = 10 m/s^2)



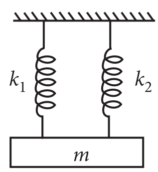
(a) 10.0 cm
(b) any value less than 12.0 cm
(c) 4.0 cm
(d) 8.0 cm *(2007)*

46. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T, then
(a) $T \propto \frac{1}{\sqrt{m}}$ (b) $T \propto \sqrt{\rho}$
(c) $T \propto \frac{1}{\sqrt{A}}$ (d) $T \propto \frac{1}{\rho}$ *(2006)*

47. Two springs of spring constant k_1 and k_2 are joined in series. The effective spring constant of the combination is given by
(a) $\sqrt{k_1 k_2}$ (b) $(k_1 + k_2)/2$
(c) $k_1 + k_2$ (d) $k_1k_2/(k_1 + k_2)$ *(2004)*

48. The time period of a mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be
(a) $T/4$ (b) T
(c) $T/2$ (d) $2T$ *(2003)*

49. A mass is suspendedseparately by two different springs in successive order then time periods is t_1 and t_2 respectively. If it is connected by both spring as shown in figure then time period is t_0, the correct relation is



(a) $t_0^2 = t_1^2 + t_2^2$
(b) $t_0^{-2} = t_1^{-2} + t_2^{-2}$
(c) $t_0^{-1} = t_1^{-1} + t_2^{-1}$
(d) $t_0 = t_1 + t_2$ *(2002)*

50. Two masses M_A and M_B are hung from two strings of length l_A and l_B respectively. They are executing SHM with frequency relation $f_A = 2f_B$, then relation

(a) $l_A = \frac{l_B}{4}$, does not depend on mass

(b) $l_A = 4l_B$, does not depend on mass

(c) $l_A = 2l_B$ and $M_A = 2M_B$

(d) $l_A = \frac{l_B}{2}$ and $M_A = \frac{M_B}{2}$ *(2000)*

51. The bob of simple pendulum having length l, is displaced from mean position to an angular position θ with respect to vertical. If it is released, then velocity of bob at equilibrium position

(a) $\sqrt{2gl(1-\cos\theta)}$ (b) $\sqrt{2gl(1+\cos\theta)}$

(c) $\sqrt{2gl\cos\theta}$ (d) $\sqrt{2gl}$ *(2000)*

52. Time period of a simple pendulum is 2 sec. If its length is increased by 4 times, then its time period becomes

(a) 8 sec (b) 12 sec

(c) 16 sec (d) 4 sec *(1999)*

53. Two simple pendulums of length 5 m and 20 m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed _______ oscillations.

(a) 2 (b) 1 (c) 5 (d) 3 *(1998)*

54. A mass m is vertically suspended from a spring of negligible mass; the system oscillates with a frequency n. What will be the frequency of the system, if a mass $4m$ is suspended from the same spring?

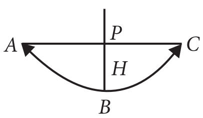
(a) $\frac{n}{2}$ (b) $4n$ (c) $\frac{n}{4}$ (d) $2n$ *(1998)*

55. If the length of a simple pendulum is increased by 2%, then the time period

(a) increases by 1% (b) decreases by 1%

(c) increases by 2% (d) decreases by 2%. *(1997)*

56. A simple pendulumwith a bob of mass m oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is g, then the velocity of the bob as it passes through B is



(a) mgH

(b) $\sqrt{2gH}$

(c) zero (d) $2gH$. *(1995)*

57. A body of mass 5 kg hangs from a spring and oscillates with a time period of 2π seconds. If the ball is removed, the length of the spring will decrease by

(a) g/k metres (b) k/g metres

(c) 2π metres (d) g metres. *(1994)*

58. A seconds pendulum is mounted in a rocket. Its period of oscillation will decrease when rocket is

(a) moving down with uniform acceleration

(b) moving around the earth in geostationary orbit

(c) moving up with uniform velocity

(d) moving up with uniform acceleration. *(1994)*

59. A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration a, then the time period is given by $T = 2\pi\sqrt{(l/a')}$, where a' is equal to

(a) g (b) $g - a$

(c) $g + a$ (d) $\sqrt{(g^2 + a^2)}$ *(1991)*

60. A mass m is suspended from the two coupled springs connected in series. The force constant for springs are k_1 and k_2. The time period of the suspended mass will be

(a) $T = 2\pi\sqrt{\frac{m}{k_1 - k_2}}$ (b) $T = 2\pi\sqrt{\frac{mk_1k_2}{k_1 + k_2}}$

(c) $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$ (d) $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$ *(1990)*

14.9 Damped Simple Harmonic Motion

61. When an oscillator completes 100 oscillations, its amplitude reduced to $\frac{1}{3}$ of initial value. What will be its amplitude, when it completes 200 oscillations?

(a) $\frac{1}{8}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$ *(2002)*

14.10 Forced Oscillations and Resonance

62. In case of a forced vibration, the resonance peak becomes very sharp when the

(a) damping force is small

(b) restoring force is small

(c) applied periodic force is small

(d) quality factor is small *(2003)*

63. A particle, with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force $F\sin\omega t$. If the amplitude of the particle is maximum for $\omega = \omega_1$ and the energy of the particle is maximum for $\omega = \omega_2$, then (ω_0 is natural frequency of oscillation of the particle)

(a) $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$

(b) $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$

(c) $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_0$

(d) $\omega_1 \neq \omega_0$ and $\omega_2 \neq \omega_0$ *(1998, 1989)*

ANSWER KEY

1. (c)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (c)
11. (a)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (b)	**16.** (d)	**17.** (c)	**18.** (c)	**19.** (d)	**20.** (b)
21. (d)	**22.** (d)	**23.** (d)	**24.** (d)	**25.** (a)	**26.** (c)	**27.** (*)	**28.** (d)	**29.** (a)	**30.** (c)
31. (c)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** (b)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (b)
41. (b)	**42.** (d)	**43.** (d)	**44.** (c)	**45.** (a)	**46.** (c)	**47.** (d)	**48.** (c)	**49.** (b)	**50.** (a)
51. (a)	**52.** (d)	**53.** (a)	**54.** (a)	**55.** (a)	**56.** (b)	**57.** (d)	**58.** (d)	**59.** (d)	**60.** (d)
61. (d)	**62.** (a)	**63.** (b)							

Hints & Explanations

1. **(c) :** $y = A_0 + A\sin\omega t + B\cos\omega t$.

or $(y - A_0) = A\sin\omega t + B\cos\omega t$

or $y' = A\sin\omega t + B\cos\omega t$

$$= A\cos\left(\frac{\pi}{2} - \omega t\right) + B\cos\omega t$$

$$\text{Amplitude} = \sqrt{A^2 + B^2 + 2AB\cos\frac{\pi}{2}} \quad \left[\because \phi = \frac{\pi}{2}\right]$$

$$= \sqrt{A^2 + B^2}$$

2. **(d)**

3. **(c) :** $y = \sin\omega t - \cos\omega t$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right] = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

It represents a SHM with time period, $T = \dfrac{2\pi}{\omega}$.

$$y = \sin^3\omega t = \frac{1}{4}[3\sin\omega t - \sin 3\omega t]$$

It represents a periodic motion with time period $T = \dfrac{2\pi}{\omega}$ but not SHM.

$$y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right) = 5\cos\left(3\omega t - \frac{3\pi}{4}\right)$$

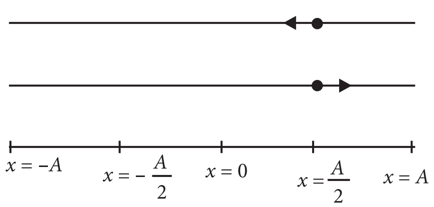
$$[\because \cos(-\theta) = \cos\theta]$$

It represents a SHM with time period, $T = \dfrac{2\pi}{3\omega}$.

$y = 1 + \omega t + \omega^2 t^2$

It represents a non-periodic motion. Also it is not physically acceptable as $y \to \infty$ as $t \to \infty$.

4. **(c) :**



The time taken by the particle to travel from

$$x = 0 \text{ to } x = \frac{A}{2} \text{ is } \frac{T}{12}$$

The time taken by the particle to travel from

$$x = A \text{ to } x = \frac{A}{2} \text{ is } \frac{T}{6}$$

$$\text{Time difference} = \frac{T}{6} + \frac{T}{6} = \frac{T}{3}$$

$$\text{Phase difference, } \phi = \frac{2\pi}{T} \times \text{Time difference}$$

$$= \frac{2\pi}{T} \times \frac{T}{3} = \frac{2\pi}{3}$$

5. **(c) :** $x = a\sin^2\omega t = a\left(\dfrac{1 - \cos 2\omega t}{2}\right) = \dfrac{a}{2} - \dfrac{a\cos 2\omega t}{2}$

$(\because \cos 2\theta = 1 - 2\sin^2\theta)$

$$\therefore \quad \text{Velocity, } v = \frac{dx}{dt} = \frac{2\omega a \sin 2\omega t}{2} = \omega a \sin 2\omega t$$

$$\text{Acceleration, } a = \frac{dv}{dt} = 2\omega^2 a\cos 2\omega t$$

For the given displacement $x = a\sin^2\omega t$,

$a \propto -x$ is not satisfied.

Hence, the motion of the particle is non simple harmonic motion.

The given motion is a periodic motion with a time period

$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

6. **(b) :** $x(t) = a\sin\omega t$ (from the equilibrium position)

At $x(t) = a/2$

$$\frac{a}{2} = a\sin(\omega t) \Rightarrow \sin\left(\frac{\pi}{6}\right) = \sin(\omega t) \text{ or, } \frac{\pi}{6} = \frac{2\pi t}{T}$$

or $t = T/12$

7. **(a)**

8. **(c) :** $x = a\sin\omega t$

$y = a\sin(\omega t + \pi/2) = a\cos\omega t$

$x^2 + y^2 = a^2$

It is an equation of a circle.

9. **(a) :** For S.H.M., $x = A\sin\left(\dfrac{2\pi}{T}t\right)$

when $x = A$, $A = A\sin\left(\dfrac{2\pi}{T}t\right)$

$$\therefore \quad \sin\left(\frac{2\pi}{T}\cdot t\right) = 1 \Rightarrow \sin\left(\frac{2\pi}{T}\cdot t\right) = \sin\left(\frac{\pi}{2}\right)$$

$\Rightarrow \quad t = (T/4)$

When $x=\frac{A}{2},\ \frac{A}{2}=A\sin\left(\frac{2\pi}{T}\cdot t\right)$

or $\sin\frac{\pi}{6}=\sin\left(\frac{2\pi}{T}t\right)$ or $t=(T/12)$

Now, time taken to travel from $x = A$ to $x = A/2$

$= T/4 - T/12 = T/6$

10. (c) : $x = a\sin\omega t$

and $y = b\sin(\omega t + \pi) = -b\sin\omega t$.

or $\frac{x}{a}=-\frac{y}{b}$ or $y=-\frac{b}{a}x$

It is an equation of straight line.

11. (a) : Here $T = 4$ s, $A = 3$ m

Time period $T=\frac{2\pi}{\omega} \Rightarrow 4=\frac{2\pi}{\omega} \Rightarrow \omega=\frac{\pi}{2}$

As the time is noted from the extreme position,

so, $y = A\cos(\omega t) \Rightarrow y=3\cos\left(\frac{\pi}{2}t\right)$

12 (a) : Displacement of the particle, $y = a\sin\omega t$,

$v=\frac{dy}{dt}=a\omega\cos\omega t$

Acceleration, $a=\frac{dv}{dt}=-a\omega^2\sin\omega t$

So, phase difference between displacement and acceleration is π.

13. (a) : Since the displacement for a complete vibration is zero, therefore the average velocity will be zero.

14. (b) : Given, $A = 3$ cm, $x = 2$ cm

The velocity of a particle in simple harmonic motion is given as $v=\omega\sqrt{A^2-x^2}$

and magnitude of its acceleration is $a = \omega^2 x$

Given $|v| = |a|$ $\therefore$ $\omega\sqrt{A^2-x^2}=\omega^2 x$

$$\omega x=\sqrt{A^2-x^2} \text{ or } \omega^2x^2=A^2-x^2$$

$$\omega^2=\frac{A^2-x^2}{x^2}=\frac{9-4}{4}=\frac{5}{4} \text{ or } \omega=\frac{\sqrt{5}}{2}$$

Time period, $T=\frac{2\pi}{\omega}=2\pi.\frac{2}{\sqrt{5}}=\frac{4\pi}{\sqrt{5}}$ s

15. (b) : If A and ω be the amplitude and angular frequency of vibration, then

$\alpha = \omega^2 A$ ...(i)

and $\beta = \omega A$ ...(ii)

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{\alpha}{\beta}=\frac{\omega^2 A}{\omega A}=\omega$$

$\therefore$ Time period of vibration is $T=\frac{2\pi}{\omega}=\frac{2\pi}{(\alpha/\beta)}=\frac{2\pi\beta}{\alpha}$

16. (d) : In SHM, velocities of a particle at distances x_1 and x_2 from mean position are given by

$V_1^2=\omega^2(a^2-x_1^2)$ ...(i)

$V_2^2=\omega^2(a^2-x_2^2)$ ...(ii)

From equations (i) and (ii), we get

$$V_1^2-V_2^2=\omega^2(x_2^2-x_1^2)$$

$$\omega=\sqrt{\frac{V_1^2-V_2^2}{x_2^2-x_1^2}} \Rightarrow T=2\pi\sqrt{\frac{x_2^2-x_1^2}{V_1^2-V_2^2}}$$

17. (c) : Here, $X = A\cos\omega t$

$\therefore$ Velocity, $v=\frac{dX}{dt}=\frac{d}{dt}(A\cos\omega t)=-A\omega\sin\omega t$

Acceleration, $a=\frac{dv}{dt}=\frac{d}{dt}(-A\omega\sin\omega t)=-A\omega^2\cos\omega t$

Hence the variation of a with t is correctly shown by graph (c).

18. (c) : $x=a\sin\omega t$ or $\frac{x}{a}=\sin\omega t$ ...(i)

Velocity, $v=\frac{dx}{dt}=a\omega\cos\omega t$

$\frac{v}{a\omega}=\cos\omega t$ or, $\frac{p}{ma\omega}=\cos\omega t$ ...(ii)

Squaring and adding (i) and (ii), we get

$$\frac{x^2}{a^2}+\frac{p^2}{m^2a^2\omega^2}=\sin^2\omega t+\cos^2\omega t;\ \frac{x^2}{a^2}+\frac{p^2}{m^2a^2\omega^2}=1$$

It is an equation of ellipse.

19. (d) : $\omega_1 = 100$ rad s^{-1}; $\omega_2 = 1000$ rad s^{-1}.

Maximum acceleration of (1) $= -\omega_1^2 A$

Maximum acceleration of (2) $= -\omega_2^2 A$

$$\therefore \frac{\text{accln (1)}}{\text{accln (2)}}=\frac{\omega_1^2}{\omega_2^2}=\frac{(100)^2}{(1000)^2}=\frac{1}{100}$$

$a(1) : a(2) = 1 : 100$.

20. (b) : $x = a\sin(\omega t + \pi/6)$

$$\frac{dx}{dt}=a\omega\cos(\omega t+\pi/6)$$

Max. velocity $= a\omega$

$$\therefore \frac{a\omega}{2}=a\omega\cos(\omega t+\pi/6) \text{ or, } \cos(\omega t+\pi/6)=\frac{1}{2}$$

$$\Rightarrow \frac{2\pi}{6}=\frac{2\pi}{T}\cdot t+\frac{\pi}{6} \Rightarrow \frac{2\pi}{T}\cdot t=\frac{2\pi}{6}-\frac{\pi}{6}=+\frac{\pi}{6}$$

$$\therefore t=+\frac{\pi}{6}\times\frac{T}{2\pi}=\left|+\frac{T}{12}\right|$$

21. (d) : Let $y = A\sin\omega t$

$$\frac{dy}{dt}=A\omega\cos\omega t=A\omega\sin\left(\omega t+\frac{\pi}{2}\right)$$

Acceleration $= -A\omega^2\sin\omega t$

The phase difference between acceleration and velocity is $\pi/2$.

22. (d) : $a = 5$ cm, $v_{max} = 31.4$ cm/s

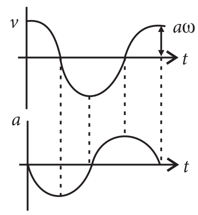
$v_{max} = \omega a \Rightarrow 31.4 = 2\pi\upsilon \times 5$

$\Rightarrow 31.4 = 10 \times 3.14 \times \upsilon \Rightarrow \upsilon = 1$ Hz

23. (d) : In SHM

$v = A\omega\sin(\omega t + \pi/2)$,

$a = A\omega^2\sin(\omega t + \pi)$. From this we can easily find out that when v is maximum, then a is zero.



24. (d) : Maximum velocity,

$v_{max} = A\omega$

According to question, $\frac{v_{max}}{2} = \frac{A\omega}{2} = \omega\sqrt{A^2 - y^2}$

$\frac{A^2}{4} = A^2 - y^2 \Rightarrow y^2 = A^2 - \frac{A^2}{4} \Rightarrow y = \frac{\sqrt{3}A}{2}$

25. (a) : Acceleration = $-\omega^2$ displacement

$\omega^2 = \frac{\text{acceleration}}{\text{displacement}} = \frac{2.0}{0.02}$

$\omega^2 = 100$ or $\omega = 10$ rad/s

26. (c) : For simple harmonic motion velocity at displacement x, $v = \omega\sqrt{a^2 - x^2}$

$10 = \omega\sqrt{a^2 - 16}$ ...(i) $\quad 8 = \omega\sqrt{a^2 - 25}$ ...(ii)

$\frac{100}{\omega^2} = a^2 - 16$ ...(iii) $\quad \frac{64}{\omega^2} = a^2 - 25$ ...(iv)

$\therefore$ Equation(iii) – (iv) gives $\frac{36}{\omega^2} = 9$

$\Rightarrow \omega = 2$ rad/s or $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ sec

27. (*) : Simple harmonic motion is defined as follows

Acceleration $\frac{d^2x}{dt^2} = -\omega^2 x$

The negative sign is very important in simple harmonic motion. Acceleration is independent of any initial displacement of equilibrium position.

Then acceleration = $-\omega^2 x$.

*Option is not given.

28 (d) : For simple harmonic motion

$\frac{d^2x}{dt^2} \propto -x$

Therefore force acting on the particle = $-Akx$.

29. (a)

30. (c) : Kinetic energy + potential energy = total energy

When kinetic energy is maximum, potential energy is zero and vice versa.

$\therefore$ Maximum potential energy = total energy

$0 + K_0 = K_0$

31. (c) : Potential energy of simple harmonic oscillator

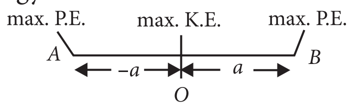
$= \frac{1}{2}m\omega^2 y^2$

For $y = \frac{a}{2}$, P.E. $= \frac{1}{2}m\omega^2\frac{a^2}{4}$

$\Rightarrow$ P.E. $= \frac{1}{4}\left(\frac{1}{2}m\omega^2 a^2\right) = \frac{E}{4}$

32. (a) : Potential energy of particle performing SHM varies parabolically in such a way that at mean position it becomes zero and maximum at extreme position.

33. (c) : For a simple harmonic motion between A and B, with O as the mean position, maximum kinetic energy of the particle executing SHM will be at O and maximum potential energy will be at A and B.



$\therefore$ Displacement between maximum potential energy and maximum kinetic energy is $\pm a$.

34. (b) : Total energy $= \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}ka^2$

35. (c) : Force constant (k) = 2×10^6 N/m; displacement (x) = 0.01 m and total mechanical energy = 160 J.

Potential energy $= \frac{1}{2}k\,x^2 = \frac{1}{2}\times(2\times10^6)(0.01)^2 = 100$ J

36. (b) : Displacement $(x) = \frac{a}{2}$

Total energy $= \frac{1}{2}m\omega^2 a^2$ and

kinetic energy when displacement is (x)

$= \frac{1}{2}m\omega^2(a^2 - x^2)$

$= \frac{1}{2}m\omega^2\left(a^2 - \frac{a^2}{4}\right) = \frac{3}{4}\left(\frac{1}{2}m\omega^2 a^2\right)$

Therefore fraction of the total energy at x,

$= \frac{\frac{3}{4}\left(\frac{1}{2}m\omega^2 a^2\right)}{\frac{1}{2}m\omega^2 a^2} = \frac{3}{4}$

37. (a) : Time period = 4 sec. In one complete oscillation, the same kinetic and potential energies are repeated two times. So the difference will vary with a period of 2 seconds.

38. (b) : P.E $= \frac{1}{2}M\omega^2 x^2 \Rightarrow \frac{1}{4}\left(\frac{1}{2}M\omega^2 A^2\right)$

$\therefore x = \frac{A}{2}$

39. (c) : P.E., $V = \frac{1}{2}m\omega^2 x^2$ and K.E., $T = \frac{1}{2}m\omega^2(a^2 - x^2)$

$\therefore \frac{T}{V} = \frac{a^2 - x^2}{x^2}$

40. (b) : Magnitude of acceleration of a particle moving in a SHM is, $|a| = \omega^2 y$; where y is amplitude.

$\Rightarrow 20 = \omega^2(5) \Rightarrow \omega = 2$ rad s^{-1}

$\therefore$ Time period of oscillation, $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ s

41. (b) : Let us assume, the length of spring be l.

When we cut the spring into ratio of length 1 : 2 : 3, we get three springs of lengths $\frac{l}{6}, \frac{2l}{6}$ and $\frac{3l}{6}$ with force constant,

$k_1 = \frac{kl}{l_1} = \frac{kl}{l/6} = 6k$, $k_2 = \frac{kl}{l_2} = \frac{kl}{2l/6} = 3k$

and $k_3 = \frac{kl}{l_3} = \frac{kl}{3l/6} = 2k$

When connected in series,

$$\frac{1}{k'}=\frac{1}{6k}+\frac{1}{3k}+\frac{1}{2k}=\frac{1+2+3}{6k}=\frac{1}{k}$$

$\therefore \quad k' = k$

When connected in parallel,

$k'' = 6k + 3k + 2k = 11k$

$$\therefore \quad \frac{k'}{k''}=\frac{k}{11k}=\frac{1}{11}$$

42. (d) : Time period of spring - block system,

$$T=2\pi\sqrt{\frac{m}{k}}$$

For given spring, $T \propto \sqrt{m}$

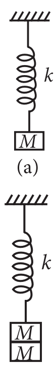
$$\frac{T_1}{T_2}=\sqrt{\frac{m_1}{m_2}}$$

Here, $T_1 = 3$ s, $m_1 = m$, $T_2 = 5$ s, $m_2 = m + 1$, $m = ?$

$$\frac{3}{5}=\sqrt{\frac{m}{m+1}} \text{ or } \frac{9}{25}=\frac{m}{m+1}$$

$$25m = 9m + 9 \Rightarrow 16m = 9; \therefore \ m=\frac{9}{16}\text{ kg}$$

43. (d) : A mass M is suspended from a massless spring of spring constant k as shown in figure (a). Then, time period of oscillation is



(a) (b)

$$T=2\pi\sqrt{\frac{M}{k}} \qquad ...(i)$$

When a another mass M is also suspended with it as shown in figure (b). Then, time period of oscillation is,

$$T'=2\pi\sqrt{\frac{M+M}{k}}=2\pi\sqrt{\frac{2M}{k}}$$

$$=\sqrt{2}\left(2\pi\sqrt{\frac{M}{k}}\right)=\sqrt{2}\,T \quad \text{(Using (i))}$$

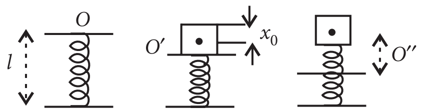
44. (c) : For simple harmonic motion,

$$v=\omega\sqrt{a^2-x^2}.$$

$$\text{When } x=\frac{a}{2}, v=\omega\sqrt{a^2-\frac{a^2}{4}}=\omega\sqrt{\frac{3}{4}a^2}$$

$$\text{As } \omega=\frac{2\pi}{T}, \ \therefore \ v=\frac{2\pi}{T}\cdot\frac{\sqrt{3}}{2}a \Rightarrow v=\frac{\pi\sqrt{3}a}{T}.$$

45. (a) :



(a) (b) (c)

The spring has a length l. Whem m is placed over it, the equilibrium position becomes O'.

If it is pressed from O' (the equilibrium position) to O'', $O'O''$ is the amplitude.

$$\therefore \quad OO'=\frac{mg}{k}=\frac{2\times 10}{200}=0.10\text{ m}$$

$mg = kx_0$.

If the restoring force $mA\omega^2 > mg$, then the mass will move up with acceleration, detached from the pan.

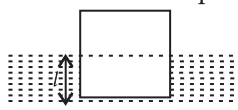
$$i.e., \ A>\frac{g}{k/m} \Rightarrow A > \frac{20}{200} > 0.10\text{ m}$$

The amplitude > 10 cm.

i.e. the minimum is just greater than 10 cm.

(The actual compression will include x_0 also. But when talking of amplitude, it is always from the equilibrium position with respect to which the mass is oscillating.

46. (c) : Let l be the length of block immersed in liquid as shown in the figure.



When the block is floating,

$\therefore \quad mg = Al\rho g$

If the block is given vertical displacement y then the effective restoring force is

$F = -[A(l + y)\rho g - mg] = -[A(l + y)\rho g - Al\rho g]$

$= -A\rho gy$

Restoring force $= -[A\rho g]y$. As this F is directed towards equilibrium position of block, so it will execute simple harmonic motion.

Here inertia factor = mass of block = m

Spring factor = $A\rho g$

$$\therefore \text{ Time period, } T=2\pi\sqrt{\frac{m}{A\rho g}} \ i.e., \ T\propto\frac{1}{\sqrt{A}}.$$

47. (d) : When the spring joined in series, the total extension in spring is

$$\Rightarrow y = y_1 + y_2 = \frac{-F}{k_1}-\frac{F}{k_2} \Rightarrow y=-F\left[\frac{1}{k_1}+\frac{1}{k_2}\right]$$

Thus spring constant in this case becomes

$$k=\frac{k_1k_2}{k_1+k_2}$$

48. (c) : Let k be the force constant of spring. If k' is the force constant of each part, then

$$\frac{1}{k}=\frac{4}{k'} \Rightarrow k'=4k$$

$$\therefore \text{ Time period} = 2\pi\sqrt{\frac{m}{4k}}=\frac{1}{2}\times 2\pi\sqrt{\frac{m}{k}}=\frac{T}{2}$$

49. (b) : The time period of a spring mass system as shown in figure 1 is given by $T=2\pi\sqrt{m/k}$, where k is the spring constant.

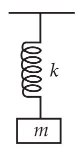
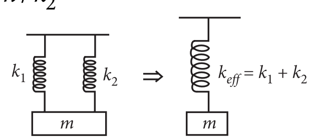


Figure 1

$\therefore \quad t_1=2\pi\sqrt{m/k_1} \qquad ...(i)$

and $t_2=2\pi\sqrt{m/k_2} \qquad ...(ii)$



(a) (b)

Figure 2

Now, when they are connected in parallel as shown in figure 2(a), the system can be replaced by a single spring of spring constant, $k_{eff} = k_1 + k_2$, as shown in figure 2(b).

Since $mg = k_1x + k_2x = k_{eff}\,x$

$\therefore \quad t_0 = 2\pi\sqrt{m/k_{eff}} = 2\pi\sqrt{m/(k_1+k_2)}$ ...(iii)

From (i), $\frac{1}{t_1^2} = \frac{1}{4\pi^2} \times \frac{k_1}{m}$ ...(iv)

From (ii), $\frac{1}{t_2^2} = \frac{1}{4\pi^2} \times \frac{k_2}{m}$ ...(v)

From (iii), $\frac{1}{t_0^2} = \frac{1}{4\pi^2} \times \frac{k_1+k_2}{m}$ ...(vi)

From eqns (iv), (v) and (vi)

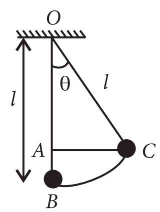
$$= \frac{1}{t_1^2} + \frac{1}{t_2^2} = \frac{1}{t_0^2}; \quad \therefore \quad t_0^{-2} = t_1^{-2} + t_2^{-2}$$

50. (a) : $f_A = 2f_B$

$$\Rightarrow \quad \frac{1}{2\pi}\sqrt{\frac{g}{l_A}} = 2 \times \frac{1}{2\pi}\sqrt{\frac{g}{l_B}} \quad \text{or,} \quad \frac{1}{l_A} = 4 \times \frac{1}{l_B}$$

or, $l_A = \frac{l_B}{4}$, which does not depend on mass.

51. (a) : In ΔOAC, $\cos\theta = OA/l$



or, $OA = l\cos\theta$

$\therefore \quad AB = l(1 - \cos\theta) = h$

At point, C, the velocity of bob = 0.

The vertical acceleration = g

$\therefore \quad v^2 = 2gh$

or, $v = \sqrt{2gl(1-\cos\theta)}$

52. (d) : Time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \Rightarrow \quad T \propto \sqrt{l}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2} \quad \text{or,} \quad T_2 = 2T_1 = 4 \text{ sec}$$

53. (a) : Frequency of the pendulum $\upsilon_{l=5} = \frac{1}{2\pi}\sqrt{\frac{g}{5}}$;

$$\upsilon_{l=20} = \frac{1}{2\pi}\sqrt{\frac{g}{20}}$$

$$\therefore \quad \frac{\upsilon_{l=5}}{\upsilon_{l=20}} = \sqrt{\frac{20}{5}} = 2 \Rightarrow \upsilon_{l=5} = 2\upsilon_{l=20}$$

As shorter length pendulum has frequency double the larger length pendulum. Therefore shorter pendulum should complete 2 oscillations before they will be again in phase.

54. (a) : $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$; $n' = \frac{1}{2\pi}\sqrt{\frac{k}{4m}}$

$\therefore \quad n' = n/2$

55. (a) : $l_2 = 1.02 l_1$.

Time period $(T) = 2\pi \times \sqrt{\frac{l}{g}} \propto \sqrt{l}$

Therefore $\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{1.02 l_1}{l_1}} = 1.01$

Thus time period increased by 1%.

56. (b) : Potential energy at A or C = Kinetic energy at B.

Thus, $\frac{1}{2}mv_B^2 = mgH$ or $v_B = \sqrt{2gH}$

57. (d) : Mass (m) = 5 kg and time period $(T) = 2\pi$ sec.

Therefore time period $T = 2\pi \times \sqrt{\frac{m}{k}} \Rightarrow \sqrt{\frac{5}{k}} = 1$

or k = 5 N/m.

According to Hooke's Law, $F = -kl$.

Therefore decrease in length (l)

$$= -\frac{F}{k} = -\frac{5g}{5} = -g \text{ metres}$$

58. (d) : Period of oscillation $T = 2\pi\sqrt{\frac{l}{g}}$. Therefore T will decrease when acceleration (g) increases. And g will increase when the rocket moves up with a uniform acceleration.

59. (d) : The effective value of acceleration due to gravity is $\sqrt{(a^2 + g^2)}$.

60. (d) : The effective spring constant of two springs in series is $k = \frac{k_1 k_2}{k_1 + k_2}$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}}$

61. (d) : This is a case of damped oscillation as the amplitude of oscillation is decreasing with time.

Amplitude of oscillations at any instant t is given by $a = a_0 e^{-bt}$, where a_0 is the initial amplitude of oscillations and b is the damping constant.

Now, when $t = 100T$, $a = a_0/3$ [T is time period]

Let the amplitude be a' at $t = 200T$ *i.e.* after completing 200 oscillations.

$\therefore \quad a = a_0/3 = a_0 e^{-100Tb}$ ...(i)

and $a' = a_0 e^{-200Tb}$ ...(ii)

From (i), $\frac{1}{3} = e^{-100Tb} \quad \therefore \quad e^{-200Tb} = \frac{1}{9}$

From (ii), $a' = a_0 \times \frac{1}{9} = \frac{a_0}{9}$

$\therefore$ The amplitude will be reduced to 1/9 of initial value.

62. (a) : Smaller damping gives a taller and narrower resonance peak.

63. (b) : The amplitude and velocity resonance occurs at the same frequency.

At resonance, *i.e.*, $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$, the amplitude and energy of the particle would be maximum.

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