CHAPTER

4 Quadratic Equations

CASE STUDY / PASSAGE BASED QUESTIONS

Syllabus

Standard form of a quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

1

Nature of Roots

A quadratic equation can be defined as an equation of degree 2. This means that the highest exponent of the polynomial in it is 2. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$. Every quadratic equation has two roots depending on the nature of its discriminant, $D = b^2 - 4ac$.

Based on the above information, answer the following questions.

(i) Which of the following quadratic equation have no real roots?

(a) $-4x^2 + 7x - 4 = 0$ (b) $-4x^2 + 7x - 2 = 0$

(c) $-2x^2 + 5x - 2 = 0$ (d) $3x^2 + 6x + 2 = 0$

(ii) Which of the following quadratic equation have rational roots?

(a) $x^2 + x - 1 = 0$ (b) $x^2 - 5x + 6 = 0$

(c) $4x^2 - 3x - 2 = 0$ (d) $6x^2 - x + 11 = 0$

(iii) Which of the following quadratic equation have irrational roots?

(a) $3x^2 + 2x + 2 = 0$ (b) $4x^2 - 7x + 3 = 0$

(c) $6x^2 - 3x - 5 = 0$ (d) $2x^2 + 3x - 2 = 0$

(iv) Which of the following quadratic equations have equal roots?

(a) $x^2 - 3x + 4 = 0$ (b) $2x^2 - 2x + 1 = 0$

(c) $5x^2 - 10x + 1 = 0$ (d) $9x^2 + 6x + 1 = 0$

(v) Which of the following quadratic equations has two distinct real roots?

(a) $x^2 + 3x + 1 = 0$ (b) $-x^2 + 3x - 3 = 0$

(c) $4x^2 + 8x + 4 = 0$ (d) $3x^2 + 6x + 4 = 0$

2

Quadratic in Day to Day Life

In our daily life we use quadratic formula as for calculating areas, determining a product's profit or formulating the speed of an object and many more.

Based on the above information, answer the following questions.

(i) If the roots of the quadratic equation are 2, –3, then its equation is

(a) $x^2 - 2x + 3 = 0$ (b) $x^2 + x - 6 = 0$ (c) $2x^2 - 3x + 1 = 0$ (d) $x^2 - 6x - 1 = 0$

(ii) If one root of the quadratic equation $2x^2 + kx + 1 = 0$ is $-1/2$, then $k =$

(a) 3 (b) –5 (c) –3 (d) 5

(iii) Which of the following quadratic equations, has equal and opposite roots?

(a) $x^2 - 4 = 0$ (b) $16x^2 - 9 = 0$ (c) $3x^2 + 5x - 5 = 0$ (d) Both (a) and (b)

(iv) Which of the following quadratic equations can be represented as $(x - 2)^2 + 19 = 0$?

(a) $x^2 + 4x + 15 = 0$ (b) $x^2 - 4x + 15 = 0$ (c) $x^2 - 4x + 23 = 0$ (d) $x^2 + 4x + 23 = 0$

(iv) If one root of a quadratic equation is $\frac{1+\sqrt{5}}{7}$, then its other root is

(a) $\frac{1+\sqrt{5}}{7}$ (b) $\frac{1-\sqrt{5}}{7}$ (c) $\frac{-1+\sqrt{5}}{7}$ (d) $\frac{-1-\sqrt{5}}{7}$

3

Formation of Quadratic Equation

Quadratic equations started around 3000 B.C. with the Babylonians. They were one of the world's first civilisation, and came up with some great ideas like agriculture, irrigation and writing. There were many reasons why Babylonians needed to solve quadratic equations. For example to know what amount of crop you can grow on the square field.

Based on the above information, represent the following questions in the form of quadratic equation.

(i) The sum of squares of two consecutive integers is 650.

(a) $x^2 + 2x - 650 = 0$ (b) $2x^2 + 2x - 649 = 0$ (c) $x^2 - 2x - 650 = 0$ (d) $2x^2 + 6x - 550 = 0$

(ii) The sum of two numbers is 15 and the sum of their reciprocals is 3/10.

(a) $x^2 + 10x - 150 = 0$ (b) $15x^2 - x + 150 = 0$ (c) $x^2 - 15x + 50 = 0$ (d) $3x^2 - 10x + 15 = 0$

(iii) Two numbers differ by 3 and their product is 504.

(a) $3x^2 - 504 = 0$ (b) $x^2 - 504x + 3 = 0$ (c) $504x^2 + 3 = x$ (d) $x^2 + 3x - 504 = 0$

(iv) A natural number whose square diminished by 84 is thrice of 8 more of given number.

(a) $x^2 + 8x - 84 = 0$ (b) $3x^2 - 84x + 3 = 0$ (c) $x^2 - 3x - 108 = 0$ (d) $x^2 - 11x + 60 = 0$

(v) A natural number when increased by 12, equals 160 times its reciprocal.

(a) $x^2 - 12x + 160 = 0$ (b) $x^2 - 160x + 12 = 0$ (c) $12x^2 - x - 160 = 0$ (d) $x^2 + 12x - 160 = 0$

4

Factorization Method

Amit is preparing for his upcoming semester exam. For this, he has to practice the chapter of Quadratic Equations. So he started with factorization method. Let two linear factors of $ax^2 + bx + c$ be $(px + q)$ and $(rx + s)$.

$\therefore \quad ax^2 + bx + c = (px + q)(rx + s) = prx^2 + (ps + qr)x + qs.$

Now, factorize each of the following quadratic equations and find the roots.

(i) $6x^2 + x - 2 = 0$

(a) 1, 6 (b) $\frac{1}{2}, \frac{-2}{3}$ (c) $\frac{1}{3}, \frac{-1}{2}$ (d) $\frac{3}{2}, -2$

(ii) $2x^2 + x - 300 = 0$

(a) $30, \frac{2}{15}$ (b) $60, \frac{-2}{5}$ (c) $12, \frac{-25}{2}$ (d) None of these

(iii) $x^2 - 8x + 16 = 0$

(a) 3, 3 (b) 3, –3 (c) 4, –4 (d) 4, 4

(iv) $6x^2 - 13x + 5 = 0$

(a) $2, \frac{3}{5}$ (b) $-2, \frac{-5}{3}$ (c) $\frac{1}{2}, \frac{-3}{5}$ (d) $\frac{1}{2}, \frac{5}{3}$

(v) $100x^2 - 20x + 1 = 0$

(a) $\frac{1}{10}, \frac{1}{10}$ (b) –10, –10 (c) $-10, \frac{1}{10}$ (d) $\frac{-1}{10}, \frac{-1}{10}$

5

Concept of Quadratic Equation

If $p(x)$ is a quadratic polynomial *i.e.*, $p(x) = ax^2 + bx + c$, $a \neq 0$, then $p(x) = 0$ is called a quadratic equation. Now, answer the following questions.

(i) Which of the following is correct about the quadratic equation $ax^2 + bx + c = 0$?

(a) a, b and c are real numbers, $c \neq 0$ (b) a, b and c are rational numbers, $a \neq 0$

(c) a, b and c are integers, a, b and $c \neq 0$ (d) a, b and c are real numbers, $a \neq 0$

(ii) The degree of a quadratic equation is

(a) 1 (b) 2 (c) 3 (d) other than 1

(iii) Which of the following is a quadratic equation?

(a) $x(x + 3) + 7 = 5x - 11$ (b) $(x - 1)^2 - 9 = (x - 4)(x + 3)$

(c) $x^2(2x + 1) - 4 = 5x^2 - 10$ (d) $x(x - 1)(x + 7) = x(6x - 9)$

(iv) Which of the following is incorrect about the quadratic equation $ax^2 + bx + c = 0$?

(a) If $a\alpha^2 + b\alpha + c = 0$, then $x = -\alpha$ is the solution of the given quadratic equation.

(b) The additive inverse of zeroes of the polynomial $ax^2 + bx + c$ is the roots of the given equation.

(c) If α is a root of the given quadratic equation, then its other root is $-\alpha$.

(d) All of these

(v) Which of the following is not a method of finding solutions of the given quadratic equation?

(a) Factorisation method (b) Completing the square method

(c) Formula method (d) None of these

HINTS & EXPLANATIONS

1. (i) (a): To have no real roots, discriminant ($D = b^2 - 4ac$) should be < 0.

(a) $D = 7^2 - 4(-4)(-4) = 49 - 64 = -15 < 0$

(b) $D = 7^2 - 4(-4)(-2) = 49 - 32 = 17 > 0$

(c) $D = 5^2 - 4(-2)(-2) = 25 - 16 = 9 > 0$

(d) $D = 6^2 - 4(3)(2) = 36 - 24 = 12 > 0$

(ii) (b): To have rational roots, discriminant ($D = b^2 - 4ac$) should be > 0 and also a perfect square.

(a) $D = 1^2 - 4(1)(-1) = 1 + 4 = 5$, which is not a perfect square.

(b) $D = (-5)^2 - 4(1)(6) = 25 - 24 = 1$, which is a perfect square.

(c) $D = (-3)^2 - 4(4)(-2) = 9 + 32 = 41$, which is not a perfect square.

(d) $D = (-1)^2 - 4(6)(11) = 1 - 264 = -263$, which is not a perfect square.

(iii) (c): To have irrational roots, discriminant ($D = b^2 - 4ac$) should be > 0 but not a perfect square.
(a) $D = 2^2 - 4(3)(2) = 4 - 24 = -20 < 0$
(b) $D = (-7)^2 - 4(4)(3) = 49 - 48 = 1 > 0$ and also a perfect square.
(c) $D = (-3)^2 - 4(6)(-5) = 9 + 120 = 129 > 0$ and not a perfect square.
(d) $D = 3^2 - 4(2)(-2) = 9 + 16 = 25 > 0$ and also a perfect square.

(iv) (d): To have equal roots, discriminant ($D = b^2 - 4ac$) should be = 0.
(a) $D = (-3)^2 - 4(1)(4) = 9 - 16 = -7 < 0$
(b) $D = (-2)^2 - 4(2)(1) = 4 - 8 = -4 < 0$
(c) $D = (-10)^2 - 4(5)(1) = 100 - 20 = 80 > 0$
(d) $D = 6^2 - 4(9)(1) = 36 - 36 = 0$

(v) (a): To have two distinct real roots, discriminant ($D = b^2 - 4ac$) should be > 0.
(a) $D = 3^2 - 4(1)(1) = 9 - 4 = 5 > 0$
(b) $D = 3^2 - 4(-1)(-3) = 9 - 12 = -3 < 0$
(c) $D = 8^2 - 4(4)(4) = 64 - 64 = 0$
(d) $D = 6^2 - 4(3)(4) = 36 - 48 = -12 < 0$

2. (i) (b): Roots of the quadratic equation are 2 and –3.
$\therefore$ The required quadratic equation is
$(x - 2)(x + 3) = 0 \Rightarrow x^2 + x - 6 = 0$

(ii) (a): We have, $2x^2 + kx + 1 = 0$
Since, –1/2 is the root of the equation, so it will satisfy the given equation.

$$\therefore \quad 2\left(-\frac{1}{2}\right)^2 + k\left(-\frac{1}{2}\right) + 1 = 0 \Rightarrow 1 - k + 2 = 0 \Rightarrow k = 3$$

(iii) (d): If the roots of the quadratic equations are opposites to each other, then coefficient of x (sum of roots) is 0.
So, both (a) and (b) have the coefficient of $x = 0$.

(iv) (c): The given equation is $(x - 2)^2 + 19 = 0$
$\Rightarrow x^2 - 4x + 4 + 19 = 0 \Rightarrow x^2 - 4x + 23 = 0$

(v) (b): If one root of a quadratic equation is irrational, then its other root is also irrational and also its conjugate *i.e.*, if one root is $p + \sqrt{q}$, then its other root is $p - \sqrt{q}$.

3. (i) (b): Let two consecutive integers be x, $x + 1$.
Given, $x^2 + (x + 1)^2 = 650$
$\Rightarrow 2x^2 + 2x + 1 - 650 = 0$
$\Rightarrow 2x^2 + 2x - 649 = 0$

(ii) (c): Let the two numbers be x and $15 - x$.
Given, $\frac{1}{x} + \frac{1}{15 - x} = \frac{3}{10}$
$\Rightarrow 10(15 - x + x) = 3x(15 - x)$
$\Rightarrow 50 = 15x - x^2 \Rightarrow x^2 - 15x + 50 = 0$

(iii) (d): Let the numbers be x and $x + 3$.
Given, $x(x + 3) = 504$
$\Rightarrow x^2 + 3x - 504 = 0$

(iv) (c): Let the number be x.
According to question, $x^2 - 84 = 3(x + 8)$
$\Rightarrow x^2 - 84 = 3x + 24 \Rightarrow x^2 - 3x - 108 = 0$

(v) (d): Let the number be x.
According to question, $x + 12 = \frac{160}{x}$
$\Rightarrow x^2 + 12x - 160 = 0$

4. (i) (b): We have, $6x^2 + x - 2 = 0$
$\Rightarrow 6x^2 - 3x + 4x - 2 = 0$
$\Rightarrow (3x + 2)(2x - 1) = 0$
$\Rightarrow x = \frac{1}{2}, \frac{-2}{3}$

(ii) (c): $2x^2 + x - 300 = 0$
$\Rightarrow 2x^2 - 24x + 25x - 300 = 0$
$\Rightarrow (x - 12)(2x + 25) = 0$
$\Rightarrow x = 12, \frac{-25}{2}$

(iii) (d): $x^2 - 8x + 16 = 0$
$\Rightarrow (x - 4)^2 = 0 \Rightarrow (x - 4)(x - 4) = 0 \Rightarrow x = 4, 4$

(iv) (d): $6x^2 - 13x + 5 = 0$
$\Rightarrow 6x^2 - 3x - 10x + 5 = 0$
$\Rightarrow (2x - 1)(3x - 5) = 0$
$\Rightarrow x = \frac{1}{2}, \frac{5}{3}$

(v) (a): $100x^2 - 20x + 1 = 0$
$\Rightarrow (10x - 1)^2 = 0 \Rightarrow x = \frac{1}{10}, \frac{1}{10}$

5. (i) (d) **(ii) (b)**

(iii) (a): $x(x + 3) + 7 = 5x - 11$
$\Rightarrow x^2 + 3x + 7 = 5x - 11$
$\Rightarrow x^2 - 2x + 18 = 0$ is a quadratic equation.
(b) $(x - 1)^2 - 9 = (x - 4)(x + 3)$
$\Rightarrow x^2 - 2x - 8 = x^2 - x - 12$
$\Rightarrow x - 4 = 0$ is not a quadratic equation.
(c) $x^2(2x + 1) - 4 = 5x^2 - 10$
$\Rightarrow 2x^3 + x^2 - 4 = 5x^2 - 10$
$\Rightarrow 2x^3 - 4x^2 + 6 = 0$ is not a quadratic equation.
(d) $x(x - 1)(x + 7) = x(6x - 9)$
$\Rightarrow x^3 + 6x^2 - 7x = 6x^2 - 9x$
$\Rightarrow x^3 + 2x = 0$ is not a quadratic equation.

(iv) (d) **(v) (d)**