Arithmetic Progressions

Assertion & Reason Type Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 1.

Assertion (A): $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$ is in Arithmetic Progression.

Reason (R): The terms of an Arithmetic Progression cannot have both positive and negative rational numbers.

Answer :

(c) **Assertion (A):** Given sequence: $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$

Here, $a_1 = -5, a_2 = -\frac{5}{2}, a_3 = 0, a_4 = \frac{5}{2}$

Difference of two consecutive terms

$$a_2 - a_1 = \frac{-5}{2} - (-5) = -\frac{5}{2} + 5 = \frac{5}{2}$$

$$a_3 - a_2 = 0 - \left(-\frac{5}{2}\right) = \frac{5}{2}$$

$$a_4 - a_3 = \frac{5}{2} - 0 = \frac{5}{2}$$

Since, the difference of two consecutive terms is constant *i.e.,* $\frac{5}{2}$.

Therefore, given sequence is an AP.

So, Assertion (A) is true.
Reason (R): The terms of an AP. can have both positive and negative rational numbers.
So, Reason (R) is false.
Hence, Assertion (A) is true but Reason (R) is false.

Q 2. Assertion (A): The nth term of the sequence -8,- 4,0,4,... is 4n- 12.
Reason (R): The nth term of an AP is determined by T_1 = a + (n-1)d.

Answer : (a) **Assertion (A):** Given sequence is -8,-4, 0, 4,...
a_2-a--4-(-8)=4.
a_3-a_2-0-(-4)=4,
a_4-a_3-4-0-4
Here, we see that difference of two consecutive terms
is same constant. So, given sequence is an AP.
First term, a=-8
and common difference, d=4
T_n=-8+ (n-1) (4)
=-8+4n-4-4n-12
So, Assertion (A) is true.
Reason (R): It is also true that nth term of an AP is
determined by Tn = a+ (n-1)d.
Hence, both Assertion (A) and Reason (R) are true and
Reason (R) is the correct explanation of Assertion (A).

Q 3. Assertion (A): The common difference of an AP in
which 4_{20}-a_{16}=20 is 5
Reason (R): The nth term of the sequence √2, √4, √18,... is √2 n.

Answer : (b) **Assertion (A):** Let a and d be the first term and
common difference of an AP. Then, nth term of an AP is
an=a+ (n-1)d
Given, a20-a16=20
:- [a+(20-1)d) - (a + (16 - 1)d) = 20
19d-15d=20
= 4d=20
= d=5

So, Assertion (A) is true.
Reason (R): Given sequence is
√2.√4.√18...
or √2, 2√2.3√2........
Here a=√2,d=2√2-√2=3√2-2√2=√2
T_n=a+ (n-1)d
T_n = √2+ (n-1)√2 = √2n
So, Reason (R) is also true.
Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Q4. Assertion (A): a, b, c are in AP if and only if 2b= a + c.
Reason (R): The sum of first n odd natural numbers is n^2.

Answer : (b) **Assertion (A):**
If part: Given a, b, c are in AP.
Then b-a-c-b
= b+b=a+c = 2b=a+c
Only part: Given, 2b=a+c
= b+b=a+c b-a-c-b
= a_2-a_1-a_3-a_2 (let a1 a, a_2-band a3 = c)
Since, each term differs from its preceding term are equal.
:- The sequence a,, a_2, a_3 or a, b, c are in AP.
Therefore, a, b, c are in AP if and only if 2b=a+c.
So, Assertion (A) is true.
Reason (R): First n odd natural numbers are:
1, 3, 5, 7...
Here, first term (a)=1
and common difference (d)-3-1-5-3-2
Since, the difference between each consecutive terms
is constant i.e., 2.
So, the sequence forms an AP.
:- Sum of first n terms of an AP,

$$S_n = \frac{n}{2}[2a+(n-1)d] \quad = \frac{n}{2}[2\times 1+(n-1)\times 2]$$

$$= \frac{n}{2}\times 2(1+n-1) = n\cdot n = n^2$$

So, Reason (R) is true.
Hence, both Assertion (A) and Reason (R) are true
but Reason (R) is not the correct explanation of
Assertion (A).

Q 5. Assertion (A): If sum of first n terms of an AP is $S_n = 6n^2-2n$, then nth term of an AP is 12n-8.
Reason (R): Suppose S_n be the sum of n terms of an AP, then nth term of an AP is $T_n = S_n - 1 - S_n$

Answer : (c**) Assertion (A):** Given,$S_n = 6n^2 - 2n$.
Using formula,
T_n-5_n-5_{n-1}-$(6n^2-2n)-(6(n-1)^2-2(n-1))$
$=6n^2-2n-[6(n^2+1-2n) - 2n+2)$
$-6n^2-2n-(6n^2-14n+8)$
$=-2n+14n-8=12n-8$
So, Assertion (A) is true.
Reason (R): It is not true that
$T_n=5_{n-1}-5$
Thus, the correct relation is
$Tn=5_n-5_{n-1}$
Hence, Assertion (A) is true but Reason (R) is false.

Q.6. Assertion (A) : Let the positive numbers a,b,c be in A.P., then $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are also in A.P.

Reason (R) : If each term of an A.P. is divided by abc , then the resulting sequence is also in A.P.

Answer : (a)

Q.7. Assertion (A) : Common difference of the AP -5, -1, 3, 7, ... is 4.

Reason (R) : Common difference of the AP a, a + d, a + 2d,..........is given by d = 2nd term-1st term.

Answer : (a) Common difference, d = - 1 - 1 (-5) = 4 So, both A and R are correct and R explains A.

Q.8. Assertion (A) : Sum of first 10 terms of the arithmetic progression -0.5, -1.0, -1.5, ... is 27.5

Reason (R) : Sum of n terms of an A.P. is given as $S_n = \frac{n}{2}[2a + (n-1)d]$ where a = first term, d = common difference.

Answer : (a) Both are correct. Reason is the correct reasoning for Assertion.

Assertion,

$$S_{10} = \frac{10}{2}[2(-0.5) + (10-1)(-0.5)]$$

$$= 5[-1 - 4.5]$$

$$= 5(-5.5) = 27.5$$

Q.9. Assertion (A) : $a_n - a_{n-1}$ is not independent of n then the given sequence is an AP.

Reason (R) : Common difference $d = a_n - a_{n-1}$ is constant or independent of n.

Answer : (d) Assertion is incorrect.

We have, common difference of an AP $d = a_n - a_{n-1}$ is independent of n or constant. So, A is correct but R is incorrect.

Q.10. Assertion (A) : The sum of the series with the nth term. $t_n = (9 - 5n)$ is (465), when no. of terms n = 15.

Reason (R) : Given series is in A.P. and sum of n terms of an A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Answer : (d)

Q.11. Assertion (A) : Three consecutive terms 2k + 1, 3k + 3 and 5k – 1 form an AP than k is equal to 6.

Reason (R) : In an AP a, a + d, a + 2d,........, the sum to n terms of the AP be

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

Answer : (b)

$$\text{For } 2k+1, 3k+3 \text{ and } 5k-1 \text{ to form an AP}$$

$$(3k+3)-(2k+1) = (5k-1)-(3k+3)$$

$$k+2 = 2k-4$$

$$2+4 = 2k-k = k$$

$$k = 6$$

So, both A and R are correct but R does not explain A

Q.12. Assertion (A) : If n^{th} term of an A.P. is 7 – 4n, then its common differences is -4.

Reason (R) : Common difference of an A.P. is given by $d = a_{n+1} - a_n$.

Answer : (a) Both are correct. Reason is the correct explanation.

Assertion,

$$a_n = 7 - 4n$$

$$d = a_{n+1} - a_n$$

$$= 7 - 4(n+1) - (7-4n)$$

$$= 7 - 4n - 4 - 7 + 4n = -4$$

Q.13. Assertion (A) : The sum of the first n terms of an AP is given by

$S_n = 3n^2 - 4n$. Then its n th term $a_n = 6n - 7$.

Reason (R) : n th term of an AP, whose sum to n terms is S_n, is given by

$a_n = S_n - S_{n-1}$.

Answer : (a) n th term of an AP be

$$a_n = S_n - S_{n-1}$$

$$a_n = 3n^2 - 4n - 3(n-1)^2 + 4(n-1)$$

$$a_n = 6n - 7$$

So, both A and R are correct and R explains A.

Q.14. Assertion (A) : If S_n is the sum of the first n terms of an A.P., then its n^{th} term a_n is given by $a_n = S_n - S_{n-1}$.

Reason (R) : The 10th term of the A.P. 5, 8, 11, 14, ... is 35.

Answer : (c) $a_{10} = a + 9d$

$$= 5 + 9(3) = 5 + 27 = 32$$

Q.15. Assertion (A) : Common difference of an AP in which $a_{21} - a_7 = 84$ is 14.

Reason (R) : n th term of AP is given by $a_n = a + (n-1)d$

Answer : (d) Assertion is incorrect.

We have, $a_n = a + (n-1)d$

$$a_{21} - a_7 = \{a + (21-1)d\} - \{a + (7-1)d\} = 84$$

$$a + 20d - a - 6d = 84$$

$$14d = 84$$

$$d = \frac{18}{14} = 6$$

$$d = 6$$

So, A is incorrect but R is correct.

Q.16. Assertion (A) : Sum of first hundred even natural numbers divisible by 5 is 500.

Reason (R) : Sum of first n-terms of an A.P. is given by $S_n = \frac{n}{2}[a + \ell]$ where l = last term.

Answer : (d) Assertion is incorrect.

Assertion : Even natural numbers divisible by 5 are 10, 20, 30, 40, ...

They form an A.P. with,

$$a = 10, \; d = 10$$

$$S_{100} = \frac{100}{2}[2(10) + 99(10)] = 50500$$

Reason is correct.

Q.17. Assertion (A) : Arithmetic between 8 and 12 is 10.

Reason (R) : Arithmetic between two numbers 'a ' and 'b' is given as $\frac{a+b}{2}$.

Answer : (a) Both are correct and Reason is the correct explanation for the Assertion.