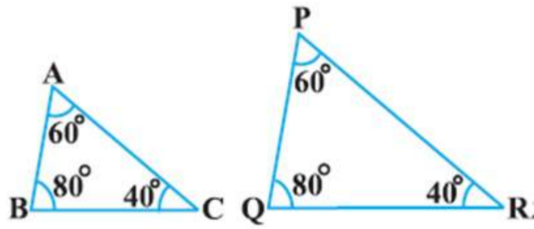
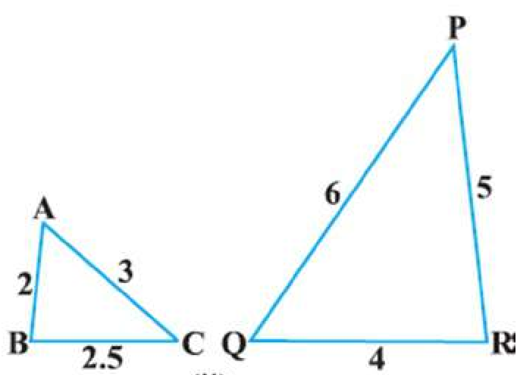
Chapter – 6
Triangle

Exercise 6.3

Q. 1 State which pairs of triangles in Fig. are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



(i)



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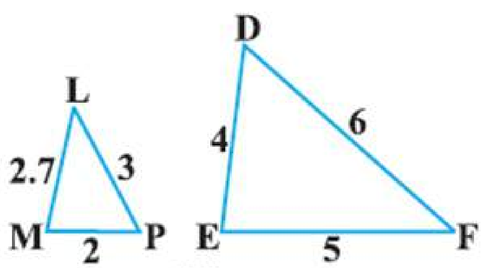

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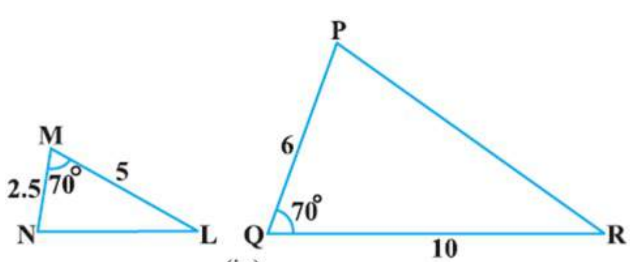
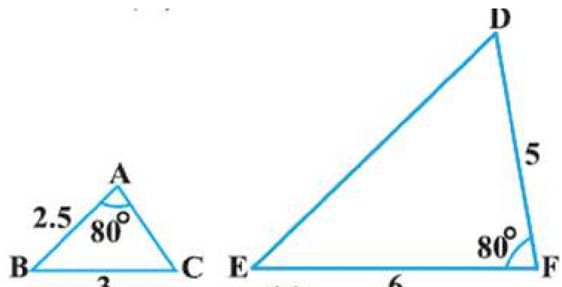
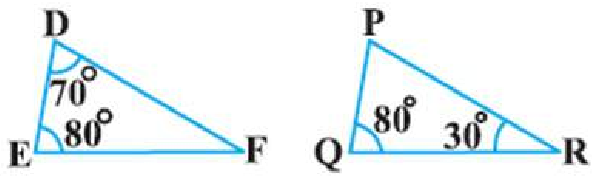


(iv)



(v)



(vi)

Answer: (i) From the figure:

∠A = ∠P = 60°

∠B = ∠Q = 80°

∠C = ∠R = 40°

Therefore, ΔABC ~ ΔPQR [By AAA similarity]

Now corresponding sides of triangles will be proportional,

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii) From the triangle, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ} = 0.5$

Hence the corresponding sides are propotional. Thus the corresponding angles will be equal. The triangles ABC and QRP are similar to each other by SSS similarity

(iii) The given triangles are not similar because the corresponding sides are not proportional

(iv) In triangle MNL and QPR, we have

∠M = ∠Q = 70°

But

$\frac{MN}{PQ} = \frac{2.5}{6} = \frac{5}{12}$

$\frac{ML}{PR} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \frac{MN}{PQ} \neq \frac{ML}{PR}$

Therefore, MNL and QPR are not similar.

(v) In triangle ABC and DEF, we have

AB = 2.5, BC = 3

$\angle A = 80°$

EF = 6

DF = 5

$\angle F = 80°$

$\frac{AB}{DF} = \frac{2.5}{5} = \frac{1}{2}$

And, $\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$

$\angle B \neq \angle F$

Hence, triangle ABC and DEF are not similar

(vi) In triangle DEF, we have

$\angle D + \angle E + \angle F = 180^o$ (Sum of angles of triangle)

$70^o + 80^o + \angle F = 180^o$

$\angle F = 30^o$

In PQR, we have

$\angle P + \angle Q + \angle R = 180^o$

$\angle P + 80^o + 30^o = 180^o$

$\angle P = 70^o$

In triangle DEF and PQR, we have

$\angle D = \angle P = 70^o$

$\angle F = \angle Q = 80^o$

∠F = ∠R = 30°
Hence, ΔDEF ~ ΔPQR (AAA similarity)

Q. 2 In Fig. 6.35, ΔODC ~ ΔOBA, ∠BOC = 125° and ∠CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB

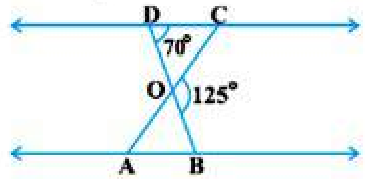


Fig. 6.35

Answer: From the figure,

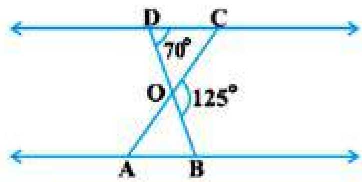


Fig. 6.35

We see, DOB is a straight line

∠ DOC + ∠ COB = 180° (angles on a straight line form a supplementary pair)

∠ DOC = 180° - 125°

∠ DOC = 55°

Now, In ΔDOC,

∠ DCO + ∠ CDO + ∠ DOC = 180°

(Sum of the measures of the angles of a triangle is 180°)

∠ DCO + 70° + 55° = 180°

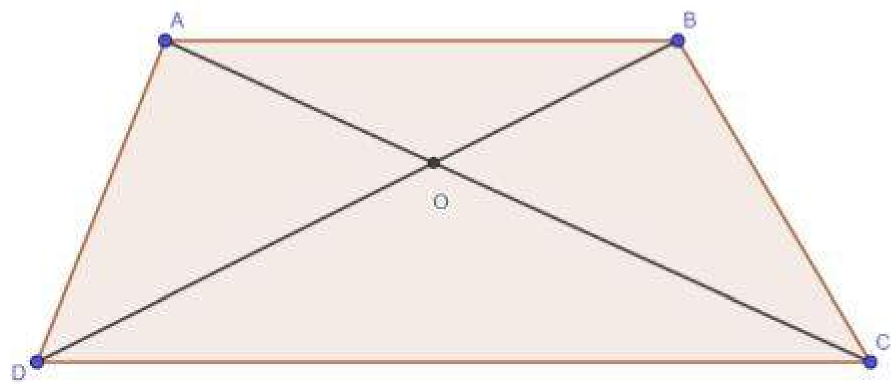
∠ DCO = 55°

It is given that ΔODC ~ΔOBA

$\angle$ OAB = $\angle$ OCD (Corresponding angles are equal in similar triangles)

Thus, $\angle$ OAB = 55°.

Q. 3 Diagonals AC and BD of a trapezium ABCD with AB ∥ DC intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.



In ΔDOC and ΔBOA,

∠CDO = ∠ABO (Alternate interior angles as AB ∥ CD)

∠DCO = ∠BAO (Alternate interior angles as AB ∥ CD)

∠DOC = ∠BOA (Vertically opposite angles)

Therefore,

ΔDOC ~ ΔBOA [BY AAA similarity]Now in similar triangles, the ratio of corresponding sides are proportional to each other. Therefore,

$\frac{OA}{OC} = \frac{OB}{OD}$ ….. (Corresponding sides are proportional)

or $\frac{AO}{CO} = \frac{BO}{DO}$

Hence proved.

Q. 4 In Fig. 6.36, $\frac{QR}{QS} = \frac{QT}{PR}$ and < 1 = <2. Show that $\Delta PQS \sim \Delta TQR$.

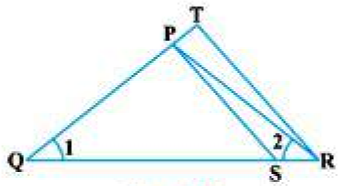


Fig. 6.36

Answer:

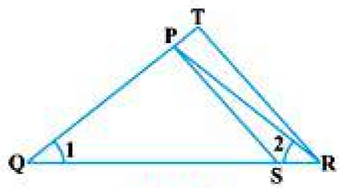


Fig. 6.36

To Prove: Δ PQS ~ Δ TQR
Given: In ΔPQR,

∠PQR = ∠PRQ

Proof: As ∠PQR = ∠PRQ

PQ = PR [sides opposite to equal angles are equal] (i)
Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$$\frac{QR}{QS} = \frac{QT}{QP} \quad \text{by (1)}$$

In Δ PQS and Δ TQR, we get

$$\frac{QR}{QS} = \frac{QT}{QP}$$

∠Q = ∠Q

Therefore,

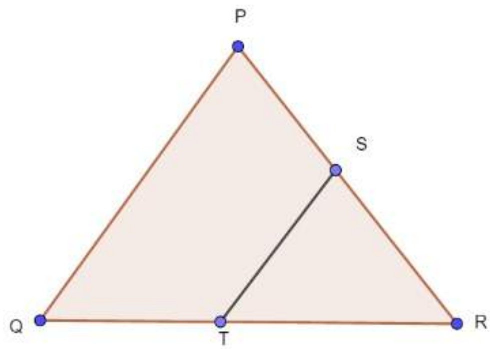
By SAS similarity Rule which states that Triangles are similar if two sides in one triangle are in the same proportion to the corresponding sides in the other, and the included angle are equal.

Δ PQS ~ Δ TQR

Hence, Proved.

Q. 5 S and T are points on sides PR and QR of Δ PQR such that ∠P = ∠RTS. Show that Δ RPQ ~ Δ RTS

Answer:



In ΔRPQ and ΔRST,

∠ RTS = ∠ QPS (Given)

∠ R = ∠ R (Common to both the triangles)

If two angles of two triangles are equal, third angle will also be equal. As the sum of interior angles of triangle is constant and is 180°

∴ ΔRPQ ~ ΔRTS (By AAA similarity).

Q. 6 In Fig. 6.37, if Δ ABE ≅Δ ACD, show that Δ ADE ~ Δ ABC.
Answer:
To Prove: Δ ADE ~ Δ ABC
Given: ΔABE ≅ ΔACD
Proof: ΔABE ≅ ΔACD
∴ AB = AC (By CPCT) (i)
And,
AD = AE (By CPCT) (ii)

In ΔADE and ΔABC,
Dividing equation (ii) by (i)

$$\frac{AB}{AD} = \frac{AC}{AE}$$

∠A = ∠A (Common)
SAS Similarity: **Triangles** are **similar** if two sides in one **triangle** are in the same proportion to the corresponding sides in the other, and the included angle are equal.

Therefore,

ΔADE ~ ΔABC (By SAS similarity)
Hence, Proved.

Q. 7 In Fig. 6.38, altitudes AD and CE of Δ ABC intersect each other at the point P.
Show that:

(i) Δ AEP ~ Δ CDP
(ii) Δ ABD ~ Δ CBE
(iii) Δ AEP ~ Δ ADB
(iv) Δ PDC ~ Δ BEC

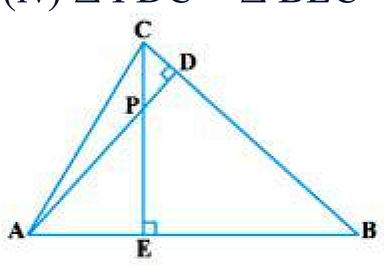


Fig. 6.38

(i) In ΔAEP and ΔCDP,

∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity,

ΔAEP ~ ΔCDP

(ii) In ΔABD and ΔCBE,

∠ADB = ∠CEB (Each 90°)
∠ABD = ∠CBE (Common)
Hence, by using AA similarity,
ΔABD ~ ΔCBE

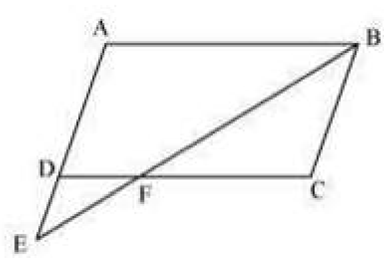
(iii) In ΔAEP and ΔADB,

∠AEP = ∠ADB (Each 90°)
∠PAE = ∠DAB (Common)
Hence, by using AA similarity,
ΔAEP ~ ΔADB

(iv) In ΔPDC and ΔBEC,

∠PDC = ∠BEC (Each 90°)
∠PCD = ∠BCE (Common angle)
Hence, by using AA similarity,
ΔPDC ~ ΔBEC

Q. 8 E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.
Answer:



To Prove: Δ ABE ~ Δ CFB
Given: E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. As shown in the figure.

Proof:
In ΔABE and ΔCFB,

∠A = ∠C (Opposite angles of a parallelogram are equal)

∠AEB = ∠CBF (Alternate interior angles are equal because AE ∥ BC)

Therefore,

ΔABE ~ ΔCFB (By AA similarity)

Hence, Proved.

Q. 9 In Fig. 6.39, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:
(i) Δ ABC ~ Δ AMP
(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

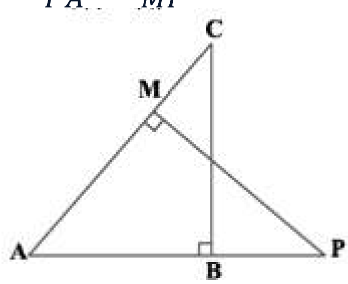


Fig. 6.39

Answer: (i) To Prove: Δ ABC ~ Δ AMP
Given: In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

Proof:
∠ABC = ∠AMP (Each 90°)
∠A = ∠A (Common)
∴ΔABC ~ ΔAMP (By AA similarity)
Hence, Proved.

(ii) **Δ ABC ~ Δ AMP**

Now we get that, Similarity Theorem - If the lengths of the corresponding sides of two triangles are proportional, then the triangles must be similar. And the converse is also true, so we have $\frac{CA}{PA} = \frac{BC}{MP}$

Hence, Proved.

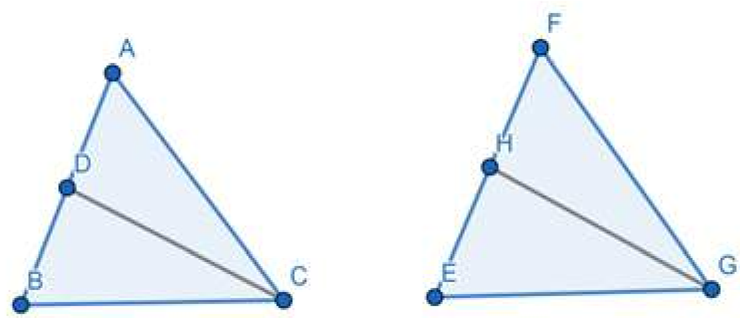
Q. 10 CD and GH are respectively the bisectors of ∠ ACB and ∠ EGF in such a way that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If Δ ABC ~ Δ FEG, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) Δ DCB ~ Δ HGE

(iii) Δ DCA ~ Δ HGF

Answer:



Given, Δ ABC ~ Δ FEG …..eq(1)

⇒ corresponding angles of similar triangles

⇒ ∠ BAC = ∠ EFG ….eq(2)

And ∠ ABC = ∠ FEG …….eq(3)

⇒ ∠ ACB = ∠ FGE

$\Rightarrow \frac{1}{2} < ACB = \frac{1}{2} < FGE$

⇒ ∠ ACD = ∠ FGH and ∠ BCD = ∠ EGH ……eq(4)

Consider Δ ACD and Δ FGH

⇒ From eq(2) we have

⇒ ∠ DAC = ∠ HFG

⇒ From eq(4) we have

⇒ ∠ ACD = ∠ EGH

Also, ∠ ADC = ∠ FGH

⇒ If the 2 angle of triangle are equal to the 2 angle of another triangle, then by angle sum property of triangle 3rd angle will also be equal.

⇒ by AAA similarity we have in two triangles if the angles are equal, then sides opposite to the equal angles are in the same ratio (or proportional) and hence the triangles are similar.

∴ Δ ADC ~ Δ FHG

⇒ By Converse proportionality theorem

$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

Consider Δ DCB and Δ HGE

From eq(3) we have

⇒ ∠ DBC = ∠ HEG

⇒ From eq(4) we have

⇒ ∠ BCD = ∠ FGH

Also, ∠ BDC = ∠ EHG

∴ Δ DCB ~ ΔHGE

Hence proved.

Q. 11 In Fig. 6.40, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that Δ ABD ~ Δ ECF

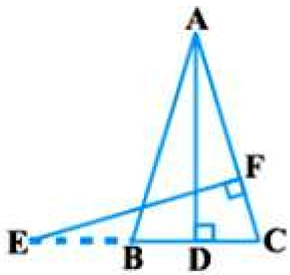
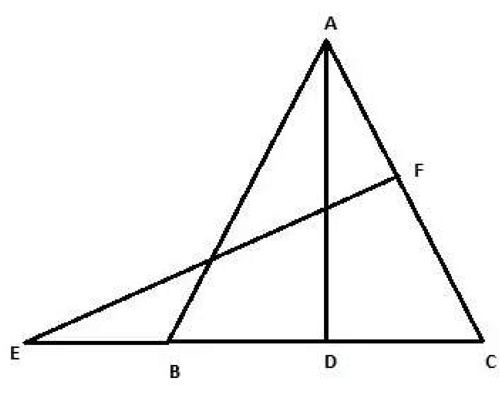


Fig. 6.40

Answer:



To Prove: Δ ABD ~ Δ ECF

Given: ABC is an isosceles triangle, AD is perpendicular to BC

BC is produced to E and EF is perpendicular to AC

Proof:

Given that ABC is an isosceles triangle

AB = AC

⇒ ∠ABD = ∠ECF

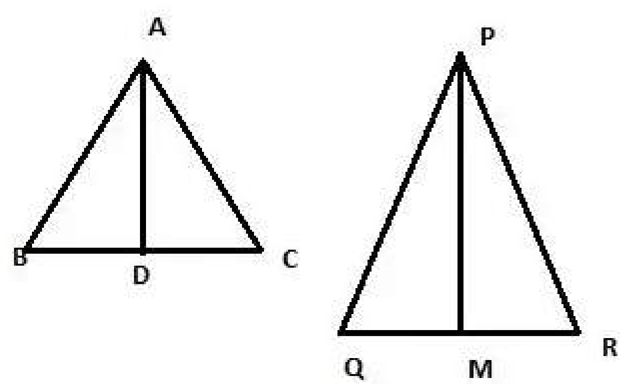
In ΔABD and ΔECF,

∠ADB = ∠EFC (Each 90°)

∠ABD = ∠ECF (Proved above)

Therefore,
ΔABD ~ ΔECF (By using AA similarity criterion)
AA Criterion: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
Hence, Proved.

Q. 12 Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR (see Fig. 6.41). Show that Δ ABC ~ Δ PQR.
Answer:



To Prove: Δ ABC ~ Δ PQR

Given:

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

Proof: Median divides the opposite side

$BD = \frac{BC}{2}$ and,

$QM = \frac{QR}{2}$

Now,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

Multiplying and dividing by 2, we get

$\frac{AB}{PQ} = \frac{\frac{2}{2BC}}{\frac{1}{2QR}} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

In Δ ABD and Δ PQM,

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Side-Side-Side (SSS) Similarity Theorem - If the lengths of the corresponding sides of two triangles are proportional, then the triangles must be similar.

ΔABD ~ ΔPQM (By SSS similarity)

∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

$\frac{AB}{PQ} = \frac{BC}{QR}$

The SAS Similarity Theorem states that if two sides in one triangle are proportional to two sides in another triangle and the included angle in both are **congruent**, then the two triangles are similar.

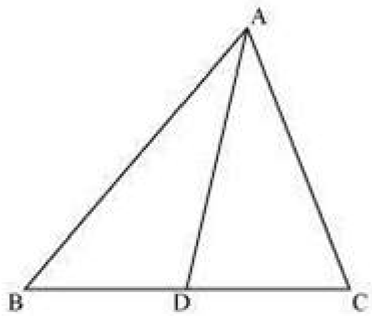
ΔABC ~ ΔPQR (By SAS similarity)

Hence, Proved.

Q. 13 D is a point on the side BC of a triangle ABC such that ∠ ADC = ∠ BAC. Show that $CA^2 = CB.CD$

Answer:

In ΔADC and ΔBAC,

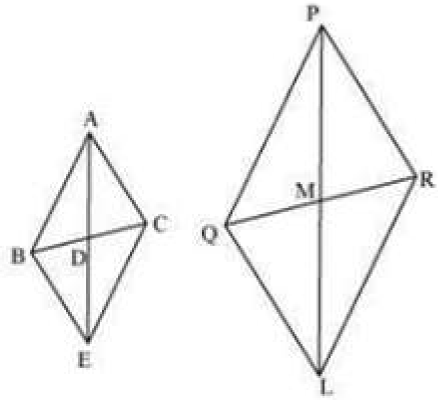


To Prove: CA^2 = CB . CD
Given: ∠ADC = ∠BAC
Proof: Now In ΔADC and ΔBAC,
∠ADC = ∠BAC

∠ACD = ∠BCA (Common angle)
According to AA similarity, if two corresponding angles of two triangles are equal then the triangles are similar
ΔADC ~ ΔBAC (By AA similarity)

We know that corresponding sides of similar triangles are in proportion

Hence in ΔADC and ΔBAC,
$$\frac{CA}{CB} = \frac{CD}{CA}$$
$$CA^2 = CB \times CD$$
Hence Proved.
Q. 14 Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR.
Show that Δ ABC ~ Δ PQR
Answer: To Prove: Δ ABC ~ Δ PQR
Given:
$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$
Proof



Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML.
Then, join B to E, C to

E, Q to L, and R to L

We know that medians divide opposite sides.

Hence, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC,

Diagonals AE and BC bisect each other at point D.

Therefore,

Quadrilateral ABEC is a parallelogram.

AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given in the question that,

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AR}{PL}$

ΔABE ~ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∠BAE = ∠QPL ..... (i)

Similarly, it can be proved that

ΔAEC ~ ΔPLR and

∠CAE = ∠RPL ..... (ii)

Adding equation (i) and (ii), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒∠CAB = ∠RPQ .... (iii)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

∠CAB = ∠RPQ [Using equation (iii)]

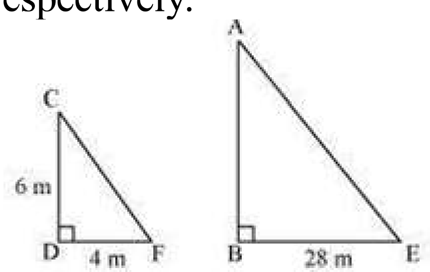
ΔABC ~ ΔPQR (By SAS similarity criterion)

Hence, Proved.

Q. 15 A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower

Answer: Let AB and CD be a tower and a pole respectively

And, the shadow of BE and DF be the shadow of AB and CD respectively.



To find: AB

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle

Therefore,

∠DCF = ∠BAE

And,

∠DFC = ∠BEA

∠CDF = ∠ABE (Tower and pole are vertical to the ground)

ΔABE ~ ΔCDF (AAA similarity)

Hence, By the properties of similar triangles that if two triangles are similar, their corresponding sides will be proportional.

$\frac{AB}{CD} = \frac{BF}{DF}$

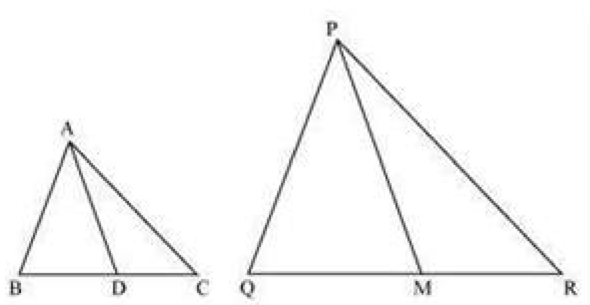
$\frac{AB}{6} = \frac{28}{4}$

AB = 42 m

Height of the Tower = 42 m

Q. 16 If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer: It is given that ΔABC is similar to ΔPQR



To Prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

Given: Δ ABC ~ Δ PQR

AD and PM are medians

We know that the corresponding sides of similar triangles are in proportion

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ … eq(i)

And also the corresponding angles are equal

∠A = ∠P

∠B = ∠Q

∠C = ∠R ....eq(ii)

Since AD and PM are medians, they divide their opposite sides in two equal parts

$BD = \frac{BC}{2}$ *and*

$QM = \frac{QR}{2}$ … eq. (iii)

From (i) and (iii), we get

$\frac{AB}{PQ} = \frac{BD}{QM}$ (iv)

In ΔABD and ΔPQM,

∠B = ∠Q [Using (ii)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Using (iv)]

ΔABD ~ ΔPQM (Since two sides are proportional and one angle is equal then by SAS similarity)

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Hence, Proved.