

## Mathematics

### Chapterwise Practise Problems (CPP) for JEE (Main & Advanced)

#### Chapter - Conic Sections

##### Level-1

#### SECTION - A

##### Straight Objective Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

- The locus of a point divides a chord of slope 2 of the parabola  $y^2 = 4x$  internally in the ratio 1 : 2 is  
 (A)  $\left(y + \frac{8}{9}\right)^2 = \frac{4}{9}\left(x + \frac{2}{9}\right)$   
 (B)  $\left(y - \frac{8}{9}\right)^2 = \frac{4}{9}\left(x - \frac{2}{9}\right)$   
 (C)  $\left(y - \frac{8}{9}\right)^2 = \frac{4}{9}\left(x + \frac{2}{9}\right)$   
 (D)  $\left(y + \frac{8}{9}\right)^2 = \frac{4}{9}\left(x + \frac{2}{9}\right)$
- If  $t_1$  and  $t_2$  are two points on the parabola  $y^2 = 4ax$ . If the focal chord joining them coincides with the normal chord, then  
 (A)  $t_1(t_1 + t_2) + 2 = 0$  (B)  $t_1 + t_2 = 0$   
 (C)  $t_1 t_2 = -1$  (D) none of these
- Tangent to the curve  $y = x^2 + 6$  at a point P (1, 7) touches the circle  $x^2 + y^2 + 16x + 12y + c = 0$  at a point Q. The coordinates of Q are  
 (A) (-6, -11) (B) (-9, -13)  
 (C) (-10, -15) (D) (-6, -7)
- The locus of mid point of chords of an ellipse  $\frac{x^2}{4} + y^2 = 1$  the tangents at the extremities of which intersect at right angle is  
 (A)  $16(x^2 + y^2)^2 = 5(x^2 + 4y^2)$   
 (B)  $16(x^2 + y^2)^2 = 5(x^2 + 4y^2)^2$   
 (C)  $5(x^2 + y^2)^2 = 16(x^2 + 4y^2)^2$   
 (D)  $16(x^2 + y^2) = 5(x^2 + 4y^2)^2$
- If  $\alpha, \beta$  are the eccentric angles of the extremities of a focal chord of an ellipse, then eccentricity of the ellipse is  
 (A)  $\frac{\sin \alpha + \sin \beta}{\sin(\alpha + \beta)}$  (B)  $\frac{\cos \alpha + \cos \beta}{\cos(\alpha + \beta)}$   
 (C)  $\frac{\sin(\alpha + \beta)}{\sin(\alpha + \beta)}$  (D) none of these
- The eccentricity of ellipse  $ax^2 + by^2 + 2fx + 2gy + c = 0$ , if major axis is parallel to x-axis, is  
 (A)  $\sqrt{\frac{b-a}{b}}$  (B)  $\sqrt{\frac{a-b}{b}}$   
 (C)  $\sqrt{\frac{a}{a+b}}$  (D)  $\sqrt{\frac{b}{a+b}}$
- If P is any point on ellipse with foci  $S_1$  &  $S_2$  and eccentricity is  $\frac{1}{2}$  such that  $\angle PS_1 S_2 = \alpha$ ,  $\angle PS_2 S_1 = \beta$ ,  $\angle S_1 P S_2 = \gamma$ , then  $\cot \frac{\alpha}{2}$ ,  $\cot \frac{\gamma}{2}$ ,  $\cot \frac{\beta}{2}$  are in  
 (A) A.P. (B) G.P.  
 (C) H.P. (D) NOT A.P., G.P. & H.P.
- Locus of the feet of the perpendiculars drawn from either foci on a variable tangent to the hyperbola  $16y^2 - 9x^2 = 1$  is  
 (A)  $x^2 + y^2 = 9$  (B)  $x^2 + y^2 = 1/9$   
 (C)  $x^2 + y^2 = 7/144$  (D)  $x^2 + y^2 = 1/16$
- If  $x, \{x\}$  and  $2[x]$  represent the segments of a focal chord and length of latus rectum of an ellipse respectively, then length of major axis of ellipse is always greater than (where  $x \notin \mathbb{Z}$ )  
 (A) 7 (B) 5  
 (C) 8 (D) 0

10. The equation of the line of latus rectum of the rectangular hyperbola  $xy = c^2$  is  
 (A)  $x - y = \sqrt{2}c$  (B)  $x + y = 2\sqrt{2}c$   
 (C)  $x + y = \sqrt{2}c$  (D)  $x + y = 0$
11. If the line  $y = \frac{x}{\sqrt{3}}$  cuts the curve  $x^4 - 2x^2y - pxy + qx + ry + 10 = 0$  at  $P, Q, R, S$ , then the value of  $(OP \cdot OQ \cdot OR \cdot OS)$ , where  $O$  is the origin, equals  
 (A) 36 (B)  $\frac{39}{2}$   
 (C)  $\frac{160}{9}$  (D) 90
12. A ray of light incident at the point  $(-2, -1)$  gets reflected from the tangent at  $(0, -1)$  to the circle  $x^2 + y^2 = 1$ . The reflected ray touches the circle. The equation of the line along which the incident ray moves is  
 (A)  $4x - 3y + 11 = 0$  (B)  $4x + 3y + 11 = 0$   
 (C)  $3x + 4y + 11 = 0$  (D) None of these
13. Consider a curve  $ax^2 + 2hxy + by^2 = 1$ . A line drawn from origin 'O' cuts the given curve at point  $A$  and  $B$  such that  $OA \cdot OB$  is independent of slope of the line then eccentricity of the conic obtained will be  
 (A)  $0 < e < 1$  (B)  $1 < e < \infty$   
 (C)  $e = 0$  (D)  $e = 1$
14. Locus of point of contact of the parabolas  $y^2 = 8x + c_1$  and  $x^2 = 3y + c_2$ , where  $c_1$  and  $c_2$  are parameters is  
 (A) A straight line (B) A circle  
 (C) A hyperbola (D) An ellipse
16. The range of  $\alpha$  such that tangents can be drawn to different branches of hyperbola  $\frac{x^2}{4} - \frac{y^2}{1} = 1$  from the point  $(\alpha, \alpha^2)$   
 (A)  $\left(-\infty, -\frac{1}{2}\right)$  (B)  $\left(\frac{1}{2}, \infty\right)$   
 (C)  $\left(-\frac{1}{2}, \frac{1}{2}\right)$  (D)  $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$
17. If the equation  $\left|\sqrt{(x-1)^2 + y^2} - \sqrt{(x+1)^2 + y^2}\right| = k$  represents a hyperbola, then  $k$  belongs to the set  
 (A)  $(1, \infty)$  (B)  $(0, 1)$   
 (C)  $(0, 2)$  (D)  $(0, \infty)$
18. If there exist a double ordinate  $PQ$  of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  with centre at  $O$  such that  $\Delta OPQ$  is an equilateral triangle, then the eccentricity  $e$  of the hyperbola satisfies  
 (A)  $1 < e < \frac{2}{\sqrt{3}}$  (B)  $e > \frac{2}{\sqrt{3}}$   
 (C)  $e > \sqrt{3}$  (D)  $e > 2$
19. The points on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  from where mutually perpendicular tangents can be drawn to circle  $x^2 + y^2 = \frac{a^2}{2}$  is /are.  
 (A)  $(a, 0)$  (B)  $(-a, 0)$   
 (C)  $(0, b)$  (D)  $(0, -b)$
20. A circle which touches the axes, and whose centre is at distance  $2\sqrt{2}$  from the origin, has the equation.  
 (A)  $x^2 + y^2 - 4x + 4y + 4 = 0$   
 (B)  $x^2 + y^2 + 4x - 4y + 4 = 0$   
 (C)  $x^2 + y^2 + 4x + 4y + 4 = 0$   
 (D)  $x^2 + y^2 + 8x + 8y + 8 = 0$

## SECTION - B

### Multiple Correct Answer Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

15. If the orthocentre of the triangle formed by the points  $t_1, t_2, t_3$  on the parabola  $y^2 = 4ax$  is the focus, then  
 (A)  $t_1 + t_2 + t_3 = 0$   
 (B)  $t_1 t_2 + t_2 t_3 + t_3 t_1 = -5$   
 (C)  $t_1 + t_2 + t_3 + t_1 t_2 t_3 = 0$   
 (D)  $t_1 t_2 t_3 = t_1 t_2 + t_2 t_3 + t_3 t_1$

21. Let  $\Delta PQR$  be formed by the common tangents to the circle  $x^2 + y^2 + 6x = 0$  and  $x^2 + y^2 - 2x = 0$ , then which of the following is (are) true ?
- (A) Centroid of  $\Delta PQR$  is (1, 0)  
 (B) Radius of circle inscribed in  $\Delta PQR$  is 1.  
 (C) Area of  $\Delta PQR$  is  $3\sqrt{3}$ .  
 (D) The y-intercept of common tangent having negative slope is 3
22. PQ is a double ordinate of the parabola  $y^2 = 8x$ . If the normal at P intersect the line passing through Q and parallel to axis of x at G, then the locus of G is a parabola with
- (A) vertex at (8, 0)  
 (B) focus at (10, 0)  
 (C) length of latus rectum equals 8  
 (D) equation of directrix is  $x = 6$
23. The circle  $x^2 + y^2 + 2\lambda y = 0$  touches  $x^2 = -4ay$ ,  $a > 0$  externally, then  $\lambda$  may be
- (A) -1 (B) -2  
 (C) 3 (D) 10
24. Let  $L_1$  be  $x + y = 1$  and  $L_2$  be the line  $y = mx + 1$ . If the intercepts made by the circle  $x^2 + y^2 - x + 3y = 0$  on  $L_1$  and  $L_2$  are equal and  $m_1$  and  $m_2$  be two possible slopes of  $y = mx + 1$ , then
- (A)  $m_1 + m_2 = \frac{10}{7}$  (B)  $m_1 m_2 = -\frac{17}{7}$   
 (C)  $m_1 m_2 = -\frac{10}{7}$  (D)  $|(m_1 - m_2)| = \frac{24}{7}$
25. If the two chords of a circle each of length  $a^2 - 1$  and  $3(a + 1)$  respectively bisect each other, then the radius of the circle is
- (A) Greater than 12  
 (B) Greater than or equal to  $\frac{15}{2}$   
 (C) Greater than 9  
 (D) Less than or equal to 12
26. From (3, 5) secants are drawn to  $x^2 + y^2 = 16$ . Locus of midpoints of the secants intercepted by the given circle is

- (A)  $3x + 5y = 15$   
 (B)  $3x + 5y = x^2 + y^2$   
 (C)  $5x + 3y = x^2 + y^2$   
 (D)  $5x + 3y = 15$

27. Consider a branch of hyperbola  $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$  with vertex at the point A. Let P be one of the end points of its latus rectum. If S is the focus of the hyperbola nearest to the point A, then area of triangle APS is

- (A)  $\frac{\sqrt{3}}{2} + 1$  (B)  $\sqrt{\frac{3}{2}} + 1$   
 (C)  $\sqrt{\frac{3}{2}} - 1$  (D)  $1 - \sqrt{\frac{3}{2}}$

## SECTION - C

### Linked Comprehension Type

This section contains paragraph. Based upon this paragraph, 2 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **MORE THAN ONE CORRECT** is correct.

### Paragraph for Q Nos. 28 to 30

$x^2 + y^2 - 2x - 2ay - 8 = 0$ ,  $a$  is variable

28. The equation represents a family of circles passing through two fixed points whose co-ordinates are :
- (A) (-2, 0) (B) (2, 0)  
 (C) (4, 0) (D) (-4, 0)
29. Equation of a circle C of this family tangents to which at these fixed points intersects on the line  $x + 2y + 5 = 0$  is :
- (A)  $x^2 + y^2 - 2x - 8y - 8 = 0$   
 (B)  $x^2 + y^2 - 2x + 6y - 8 = 0$   
 (C)  $x^2 + y^2 - 2x + 8y - 8 = 0$   
 (D)  $x^2 + y^2 - 2x - 6y - 8 = 0$
30. If the chord joining the fixed points subtends an angle  $\theta$  at the center of the circle C, then  $\theta$  is :
- (A)  $\pi/6$  (B)  $\pi/4$   
 (C)  $\pi/3$  (D)  $\pi/2$

**Paragraph for question nos. 31 and 32**

Consider the parabola  $y^2 = 8x$

31. Area of the figure formed by the tangents and normals drawn at the extremities of its latus rectum is

(A) 8 (B) 16  
(C) 32 (D) 64

32. Distance between the tangent to the parabola and a parallel normal inclined at  $30^\circ$  with the x-axis, is

(A)  $\frac{16}{3}$  (B)  $\frac{16\sqrt{3}}{9}$   
(C)  $\frac{2}{3}$  (D)  $\frac{16}{\sqrt{3}}$

**Paragraph for Question Nos. 33 and 34**

Consider  $C_1 : x^2 + y^2 + 2ax = 0$  and  $S_1 : y^2 - 4ax = 0$ .  $L_1$  and  $L_2$  are tangents to  $S_1$  and pass through centre of  $C_1$

33. Locus of centre of circle touching  $C_1$  (externally) and the line  $L_1$  is

(A) Circle (B) Straight line  
(C) Pair of line (D) Parabola

34. If  $L_1$  and  $L_2$  touch  $S_1$  at  $P$  and  $Q$  respectively, then area of triangle  $CPQ$ , where  $C$  is the centre of  $C_1$  is

(A)  $a^2$  (B)  $2a^2$   
(C)  $4a^2$  (D)  $8a^2$

**SECTION-D**

**Single-Match Type**

This section contains Single match questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled 1, 2, 3 and 4, while the statements in **Column II** are labelled p, q, r, s and t. Four options 1,2,3 and 4 are given below. Out of which, only one shows the right matching

35. Match the following

**Column I**

**Column II**

(A) The product of length of perpendicular from any point of hyperbola  $x^2 - y^2 = 10$  to its asymptotes is

(p) 0

- (B) The number of points on (q) 5

$$\frac{x^2}{4} + \frac{y^2}{3} = 1 \text{ from which}$$

mutually perpendicular tangents can be drawn to

$$\text{hyperbola } \frac{x^2}{4} - \frac{y^2}{1} = 1 \text{ is}$$

- (C) The distance between directrices of the ellipse (r) 16

$$(4x - 8)^2 + 16y^2 =$$

$$(x + \sqrt{3}y + 10)^2 \text{ is}$$

- (D) Tangents are drawn from any (s) 2

point on the line  $y - x + 2 = 0$

parabola  $y^2 = 4x$  such that

chords of contact pass

through the fixed point then

the sum of whose abscissa

and ordinates is

36. Match the following

**Column I**

**Column II**

- (A) The normal chord at a point 't' (p) 4

on the parabola  $y^2 = 4x$

subtends a right angle at the

vertex then  $t^2$  is

- (B) The area of the triangle (q) 2

inscribed in the curve  $y^2 = 4x$ ,

the parameter of coordinates

whose vertices are 1, 2 and

4 is

- (C) The number of distinct normals (r) 3

possible from  $\left(\frac{11}{4}, \frac{1}{4}\right)$  to the

parabola  $y^2 = 4x$  is

- (D) The normal at  $(a, 2a)$  on  $y^2 = 4ax$  meets the curve again (s) 6

at  $(at^2, 2at)$ , then the value of

$|t - 1|$  is

(t) 1

37. **Column-I** **Column-II**

- (A) The value of 'k' for which  $x^2 + y^2 = 4$  and  $x^2 + y^2 - 2x + 2ky = k$  cut orthogonally is (p)  $2\sqrt{5}$
- (B) Length of the chord cut by  $4y = 3x + 1$  on the circle  $x^2 + y^2 - 6x = 0$  is (q)  $-8$
- (C) The value of 'k' for which lines  $2x - ky + 3 = 0$ ,  $4x + y + 5 = 0$  cut axes in concyclic points is (r)  $4.5$
- (D) If any triangle, the maximum value of  $\frac{r_1 + r_2 + r_3}{R}$  must be (s)  $-4$
- (t)  $1$

38. Match the following :

**Column - I** **Column - II**

- (A) The point (2, 4) is one extremity of focal chord of parabola  $y^2 = 8x$ . The length of this focal chord is (p)  $1$
- (B) The equation  $(13x - 1)^2 + (13y - 1)^2 = k(5x - 12y + 1)^2$  will represent a parabola if k is (q)  $4/3$
- (C) The length of the common chord of the curves  $y^2 - 4x - 4 = 0$  and  $4x^2 + 9y^2 - 36 = 0$  is (r)  $8$
- (D) A focal chord of the parabola  $y^2 = 4ax$  is of length  $4a$ . The angle subtended by it at the vertex of the parabola is  $\theta$ , then  $|\tan \theta|$  is equal to (s)  $4$

39. Match the following :

**Column - I** **Column - II**

- (A) If P is point on the ellipse  $\frac{x^2}{16} + \frac{y^2}{20} = 1$  whose foci are S and S', then PS + PS' is (P)  $\frac{4}{5}$
- (B) The eccentricity of the ellipse  $2x^2 + 3y^2 - 4x - 12y + 13 = 0$  is (Q)  $4\sqrt{5}$
- (C) Tangents are drawn from the points on the line  $x - y - 5 = 0$  to  $x^2 + 4y^2 = 4$ . Then all the chords of contact pass through a fixed point, whose abscissa is (R)  $8$
- (D) The sum of the distance of any point on the ellipse  $3x^2 + 4y^2 = 12$  from is (S)  $\frac{1}{\sqrt{3}}$

40. Match the following :

**Column - I** **Column - II**

- (A) The focus of  $\frac{x^2}{16} - \frac{y^2}{9} = 1$  is (p)  $(\sqrt{2}, \sqrt{2})$
- (B) The focus of  $xy = 1$  is (q)  $(2, 0)$
- (C) The line  $5x + 2y = 9$  touches the hyperbola  $x^2 - 9y^2 = 9$  at the point (r)  $(5, 0)$
- (D) The focus of  $21x^2 - 4y^2 = 84$  is (s)  $\left(5, \frac{-2}{9}\right)$

## SECTION-E

### Integer Answer Type

This section contains Integer type questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y and Z(say) are 6, 0 and 9, respectively, then the correct darkening of bubbles will look like the following :

| X | Y | Z |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

41. If circles  $x^2 + y^2 = c$  with radius  $\sqrt{3}$  and  $x^2 + y^2 + ax + by + c = 0$  with radius  $\sqrt{6}$  intersect at two points A and B, If length of  $AB = \sqrt{\ell}$ . Find  $\ell$
42. Normals are drawn from the point P with slopes  $m_1, m_2, m_3$  to the parabola  $y^2 = 4x$ . If locus of P under

the condition that  $m_1 m_2 = \alpha$  is a part of the parabola itself then find the value of  $\alpha$ .

43. If the tangents drawn from the point (0, 2) to the parabola  $y^2 = 4ax$  are inclined at an angle  $\frac{3\pi}{4}$  then the number of values of  $a$  is / are.
44. Maximum length of perpendicular from centre of ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  on any normal to this ellipse is equal to  $a + 5$ , then value of  $|a|$  is \_\_\_\_\_
45. Equation of an ellipse is  $9x^2 + 16y^2 = 144$  and  $P$  is any point on it.  $PM$  is ordinate of the ellipse and meets the auxiliary circle in  $Q$ . If equation of normal at point  $P$  where  $x = 2$ , is  $8x - 2\sqrt{3}y = \lambda$ , then the value of  $\lambda$  is
46. The equation of mirror image of parabola  $y^2 = 4x$  in line  $x - y + 2 = 0$  is obtained. Let its vertex be  $(m, n)$  and its latus rectum be  $\ell$ . The value of  $5(m+n) \cdot \ell$  is
47. Normal at a point  $P(am^2, 2am)$  intersects the parabola  $y^2 = 4ax$  at point  $Q$ . If the tangent at  $P$  and  $Q$  meet at point  $R$ , then the area of triangle  $PQR$  is  $\frac{ka^2(1+m^2)^3}{m^3}$ . Find the numerical value of  $k$
48. The equation of the chord of the hyperbola  $25x^2 - 16y^2 = 400$  which is bisected at  $(5, 3)$  is  $125x - 48y = \lambda + 480$ , then the value of  $\lambda$  is \_\_\_\_\_



## SECTION - A

## Straight Objective Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

- A rectangular hyperbola has one of the asymptotes  $x - y + 2 = 0$  and tangent at  $P(0, 1)$  is  $x = 0$ . Then the length of the latusrectum is  
(A)  $\sqrt{2}$  (B)  $\sqrt{3}$   
(C) 2 (D) None of these
- From any point R, two normals, which are at right angles to one another, are drawn to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , ( $a > b$ ). If the feet of the normals are P and Q then the locus of the circumcentre of the triangle PQR is  
(A)  $\frac{x^2 + y^2}{a^2 - b^2} = \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} \right)^2$   
(B)  $\frac{x^2 - y^2}{a^2 - b^2} = \left( \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^2$   
(C)  $\frac{x^2 + y^2}{a^2 - b^2} = \left( \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^2$   
(D)  $\frac{x^2 + y^2}{a^2 + b^2} = \left( \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^2$
- From any point P on the circle  $x^2 + y^2 = 9$  two tangents are drawn to circle  $x^2 + y^2 = 4$  which cut  $x^2 + y^2 = 9$  at A and B then locus of point of intersection of tangents drawn to  $x^2 + y^2 = 9$  at A and B is the curve  
(A)  $x^2 + y^2 = 27$  (B)  $x^2 + y^2 = 27^2$   
(C)  $x^2 + 2y^2 = 27^2$  (D)  $x^2 + y^2 = 8$
- Two tangents are drawn from point P (1,8) to the circle  $x^2 + y^2 - 6x - 4y - 11 = 0$  touching the circle at A and B. A circle passes through the point of intersection of circles  $x^2 + y^2 - 2x - 6y + 6 = 0$  and  $x^2 + y^2 + 2x - 6y$  + 6 = 0 and intersects the circumcircle of the  $\Delta PAB$  orthogonally. If r is the radius of the circle ; then  $[r]$  is (where  $[.]$  denotes the greatest integer function)  
(A) 1 (B) 2  
(C) 3 (D) 4
- If the normals drawn at the end points of a variable chord PQ of the parabola  $y^2 = 4ax$  intersect at parabola, then the locus of the point of intersection of the tangent drawn at the points P and Q is  
(A)  $x + a = 0$  (B)  $x - 2a = 0$   
(C)  $y^2 - 4x + 6 = 0$  (D)  $x = +a$
- The normal at the point P ( $ap^2, 2ap$ ) meets the parabola  $y^2 = 4ax$  again at Q ( $aq^2, 2aq$ ) such that the lines joining the origin to P and Q are at right angle. Then  
(A)  $p^2 = 2$  (B)  $q^2 = 2$   
(C)  $p = 2q$  (D)  $q = 4p$
- Locus of mid-point of the focal chord of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  with eccentricity e is  
(A)  $x^2 + y^2 = \left( \frac{a+b}{2} \right)^2$  (B)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{ex}{a}$   
(C)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{ey}{b}$  (D)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{ex}{a} + \frac{ey}{b}$
- From a point P chords are drawn to the ellipse  $\frac{x^2}{4} + \frac{y^2}{3} = 1$ , to cut it at R and S. If tangents at R and S always meets the line  $2x + 4y = 1$ , then coordinates of the point P are  
(A) (8, 6) (B) (6, 8)  
(C) (-6, 8) (D) (8, 12)

9. If normal at any point P to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ) meet the axes at M and N so that  $\frac{PM}{PN} = \frac{2}{3}$ , then the value of eccentricity is
- (A)  $\frac{1}{\sqrt{2}}$  (B)  $\frac{\sqrt{2}}{\sqrt{3}}$   
 (C)  $\frac{1}{\sqrt{3}}$  (D)  $\frac{\sqrt{3}}{\sqrt{5}}$
10. A ray emanating from the point  $(-4, 0)$  is incident on the ellipse  $9x^2 + 25y^2 = 225$  at the point P with ordinate 3. The equation of the reflected ray after first reflection is
- (A)  $3x + 4y = 10$  (B)  $3x + 4y = 12$   
 (C)  $4x + 3y = 12$  (D)  $x + 2y = 4$
11. If tangent at a point on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  cuts the axis at A and B and rectangle OAPB is completed, then locus of point P is given by (when O is the centre of the hyperbola)
- (A)  $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$   
 (B)  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$   
 (C)  $\frac{a^2}{y^2} - \frac{b^2}{x^2} = 1$   
 (D)  $\frac{x^2}{a\sqrt{1-b^2}} + \frac{y^2}{b\sqrt{1-a^2}} = 1$
12. PQ is a chord of parabola  $x^2 = 4y$  which subtends right angle at vertex. Then locus of centroid of triangle PSQ, where S is the focus of given parabola, is
- (A)  $x^2 = 4(y + 3)$  (B)  $x^2 = \frac{4}{3}(y - 3)$   
 (C)  $x^2 = \frac{-4}{3}(y + 3)$  (D)  $x^2 = \frac{4}{3}(y + 3)$
13. Normal to the parabola  $y^2 = 8x$  at the point P (2, 4) meets the parabola again at the point Q. If C is the centre of the circle described on PQ as diameter then the coordinates of the image of the point C in the line  $y = x$  are
- (A)  $(-4, 10)$  (B)  $(-3, 8)$   
 (C)  $(4, -10)$  (D)  $(-3, 10)$
14. Normals are concurrent drawn at points A, B and C on the parabola  $y^2 = 4x$  at P (h, k). The locus of the point P if the slope of the line joining the feet of two of them is 2, is
- (A)  $x + y = 1$  (B)  $x - y = 3$   
 (C)  $y^2 = 2(x - 1)$  (D)  $y^2 = 2\left(x - \frac{1}{2}\right)$
15. If the locus of the middle points of the chords of the parabola  $y^2 = 2x$  which touches the circle  $x^2 + y^2 - 2x - 4 = 0$  is given by  $(y^2 + 1 - x)^2 = \lambda(1 + y^2)$ , then the value of  $\lambda$  is equal to
- (A) 3 (B) 4  
 (C) 5 (D) 6
16. Locus of the point (h, k) if  $(\sqrt{3h}, \sqrt{3k+2})$  lies on the line  $x - y - 1 = 0$  is a
- (A) Straight line (B) Circle  
 (C) Parabola (D) Eclipse
17. A rectangular hyperbola passes through the points A(1, 1), B(1, 5) and C(3, 1). The equation of normal to the hyperbola at A(1, 1) is
- (A)  $2x + y = 3$  (B)  $2x - y = 1$   
 (C)  $x + y = 2$  (D) none of these
18. If two tangents can be drawn to the different branches of hyperbola  $\frac{x^2}{1} - \frac{y^2}{4} = 1$  from the point  $(\alpha, \alpha^2)$  then
- (A)  $\alpha \in (-2, 0)$  (B)  $\alpha \in (0, 2)$   
 (C)  $\alpha \in (-\infty, -2)$  (D)  $\alpha \in (2, \infty)$
19. Two chords are drawn from the point P(h, k) on the circle  $x^2 + y^2 = hx + ky$ . If the y-axis divides both chords in the ratio 2 : 3, then
- (A)  $k^2 > 15h^2$  (B)  $15k^2 > h^2$

(C)  $h^2 = 15k^2$  (D)  $k^2 < 5h^2$

20. The locus of the centre of a circle which cuts the parabola  $y^2 = 4x$  orthogonally at  $(1, 2)$  will pass through the point

(A)  $(3, 4)$  (B)  $(4, 3)$   
(C)  $(5, 3)$  (D)  $(2, 4)$

21. The conic represented of the equation

$$\sqrt{px} + \sqrt{qy} = 1 \text{ where } p > 0, q > 0, \text{ is}$$

(A) An ellipse (B) A circle  
(C) A parabola (D) A hyperbola

22. Two circles of radii  $R$  and  $r$  ( $R > r$ ) touch each other externally. Then the radius of circle which touches both of them externally and also their direct common tangent is

(A)  $\sqrt{Rr}$  (B)  $\frac{Rr}{R+r}$

(C)  $\frac{Rr}{(\sqrt{R} + \sqrt{r})^2}$  (D)  $\frac{R+r}{2}$

23. Let  $P$  be a point on parabola  $y^2 = 4ax$ . Tangent at  $P$  meets the axis of parabola at  $T$  and normal at  $P$  meets the axis at  $G$ , then

(A) Circumcentre of triangle  $PGT$  is vertex of parabola  
(B) Circumcentre of triangle  $PGT$  lies on directrix  
(C) Circumcentre of triangle  $PGT$  lies on focus  
(D) Circumcentre lies on the tangent at vertex

24. A triangle  $ABC$  of area  $\Delta$  is inscribed in the parabola  $y^2 = 4ax$  such that the vertex  $A$  lies at the vertex of the parabola and  $BC$  is a focal chord. The difference of the distances of  $B$  and  $C$  from the axis of the parabola is

(A)  $\frac{2\Delta}{a^2}$  (B)  $\frac{a}{2\Delta}$

(C)  $\frac{2\Delta}{a}$  (D)  $\frac{\Delta}{a^2}$

## SECTION - B

### Multiple Correct Answer Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer,

out of which **ONE OR MORE** is/are correct.

25. Let  $A$ ,  $B$  and  $C$  are three distinct points on  $y^2 = 8x$  such that normals at these points are concurrent at  $P$ . The slope of  $AB$  is 2 and abscissa of centroid of  $\Delta ABC$  is  $\frac{4}{3}$ . Which of the following is (are) correct

(A) area of  $\Delta ABC = 8$   
(B) coordinate of  $P$  is  $(6, 0)$   
(C) angle between normals are  $45^\circ, 45^\circ, 90^\circ$   
(D) angle between normals are  $30^\circ, 30^\circ, 60^\circ$

26. Tangent drawn at point  $P(1, 3)$  of a parabola intersects its tangent at vertex at  $V(-1, 5)$ . If  $R(-5, 5)$  is a point on  $SP$ ; where  $S$  is focus of the parabola then

(A) Slope of directrix is  $\frac{1}{3}$   
(B) Radius of circumcircle of  $\Delta SVO$  is  $\sqrt{10}$  units ('O' being origin)  
(C) Tangent cuts the axis of parabola at  $(-3, 7)$   
(D) Focus is  $(-2, 4)$

27. If a parabola touches the lines  $y = x$  and  $y = -x$  at  $A(3, 3)$  and  $B(1, -1)$  respectively, then

(A) Equation of directrix is  $2x - y = 0$   
(B) Equation of line through origin and focus is  $x + 2y = 0$   
(C) Equation of line through origin and focus is  $x + 2y = 4$

(D) Focus is  $\left(-\frac{3}{5}, \frac{6}{5}\right)$

28. If  $T(3, 2)$  is the foot of perpendicular drawn from focus  $S(2, -1)$  on a tangent to a parabola and directrix passes through  $P(0, 9)$ , then

(A) Length of latus rectum of parabola is  $8\sqrt{2}$   
(B) Equation of tangent at the vertex is  $x + y - 5 = 0$   
(C) Equation of axis of the parabola is  $x - y - 3 = 0$   
(D) Directrix is at a distance  $2\sqrt{2}$  from focus

29. Let  $P$  be any point on the curve  $S = 0$  such that tangents from  $P$  to  $x^2 + y^2 - 2x - 4y - 4 = 0$  make  $60^\circ$  with each other and from point  $Q$  perpendicular tangents are drawn to  $S$  then

- (A) Locus of P is circle of radius  $2\sqrt{3}$   
 (B) Locus of P is circle of radius 6  
 (C) Locus of Q is a circle of radius  $3\sqrt{2}$   
 (D) Locus of Q is a circle of radius  $6\sqrt{2}$
30. A(1, 2) and B(7, 10) are two fixed points. If P(x, y) is such a point where  $\angle APB = 60^\circ$  and the area of triangle APB is maximum, then the point P  
 (A) is on the line  $3x + 4y = 36$   
 (B) is on a line perpendicular to AB  
 (C) is on the perpendicular bisector of AB  
 (D) is on the circle passing through the points (1,2), (7,10) and having radius 10
31. P is a point on the parabola  $y^2 = 4x$  and Q is a point on the line  $2x + y + 4 = 0$ . If the line  $x - y + 1 = 0$  is the perpendicular bisector of PQ, then the coordinates of P can be  
 (A) (1, -2) (B) (4, 4)  
 (C) (9, -6) (D) (16, 8)
32. A point P moves in the x-y plane such that the point P remains at equidistant from the centre of a square and its sides. If the coordinates of four vertices of the square are  $(\pm a, \pm a)$ , then the point P lies on the curve whose equation is given by  
 (A)  $y^2 = a^2 + 2ax$  (B)  $x^2 = a^2 + 2ay$   
 (C)  $y^2 + 2ax = a^2$  (D)  $x^2 + 2ay = a^2$
33. A variable chord PQ of the parabola  $y^2 = 4ax$  is drawn parallel to the line  $y = x$ . If the parameter of the points P and Q on the parabola be  $t_1$  and  $t_2$  respectively, then  
 (A)  $t_1 + t_2 = 2$   
 (B)  $t_1 t_2 = \frac{2}{a}$   
 (C) locus of point of intersection of tangents at P and Q is  $y = 2a$   
 (D) locus of point of intersection of normals at P and Q is  $2x - y = 12a$
34. A chord AB of circle  $x^2 + y^2 = a^2$  touches the circle  $x^2 + y^2 - 2ax = 0$  locus of the point of intersections of tangents at A and B is  
 (A)  $x^2 + y^2 = (x - a)^2$  (B)  $x^2 + y^2 = (y - a)^2$   
 (C)  $x^2 = a(a - 2y)$  (D)  $y^2 = a(a - 2x)$
35. The focus of the parabola is (1, 1) and the tangent at the vertex has the equation  $x + y = 1$ . Then  
 (A) equation of the parabola is  $(x - y)^2 = 2(x + y - 1)$   
 (B) equation of the parabola is  $(x - y)^2 = 4(x + y - 1)$   
 (C) the co-ordinates of the vertex are  $\left(\frac{1}{2}, \frac{1}{2}\right)$   
 (D) length of the latus rectum is  $2\sqrt{2}$
36. Tangents PA and PB are drawn to the circle  $x^2 + y^2 - 2y - 3 = 0$  from the point P(3, 4). Then which of the following is/are correct ?  
 (A) The length of tangent from P to the circle is  $\sqrt{14}$   
 (B) The equation of circumcircle of  $\triangle PAB$  is  $x^2 + y^2 - 3x - 5y + 4 = 0$   
 (C) The equation of circumcircle of  $\triangle PAB$  is  $x^2 + y^2 - 3x - 5y + 1 = 0$   
 (D) The angle between tangents from P(3, 4) to the circle is  $\frac{\pi}{3}$
37. Points on the curve  $5x^2 + 5y^2 = 4 + 6xy$  which are nearest to the origin are  
 (A)  $\left(\frac{1}{2}, \frac{-1}{2}\right)$  (B)  $\left(0, \frac{2}{\sqrt{5}}\right)$   
 (C)  $\left(\frac{2}{\sqrt{5}}, 0\right)$  (D)  $\left(\frac{-1}{2}, \frac{1}{2}\right)$

38. An ellipse intersects the hyperbola  $2x^2 - 2y^2 = 1$  orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

- (A) Equation of the ellipse is  $x^2 + 2y^2 = 2$   
 (B) The foci of ellipse are  $(\pm 1, 0)$   
 (C) Equation of ellipse is  $x^2 + 2y^2 = 4$   
 (D) The foci of ellipse are  $(\pm\sqrt{2}, 0)$

39. If the normal at any point  $P(\theta)$  on ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  meets its auxiliary circle at Q and R such that  $\angle QOR = 90^\circ$  where O is centre of ellipse then

- (A)  $2(a^2 - b^2)^2 = a^4 \sec^2\theta + a^2b^2 \operatorname{cosec}^2\theta$   
 (B)  $a^4 + 5a^2b^2 + 2b^4 = a^4 \tan^2\theta + a^2b^2 \cot^2\theta$   
 (C)  $a^4 + 5a^2b^2 + 2b^4 \geq 2a^3b$   
 (D)  $a^4 + 2b^4 \geq 5a^2b^2 + 2a^3b$

## SECTION - C

### Linked Comprehension Type

This section contains paragraphs. Based upon each paragraph, some multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

#### Paragraph for Q. Nos. 40 to 42

Locus of centre of circle touching the circle  $x^2 + y^2 - 4y - 2x = \frac{7}{4}$  internally and tangents on which from  $(1, 2)$  is making an angle  $60^\circ$  with each other is director circle (C) of a variable ellipse E.

40. Equation of director circle (C) is

- (A)  $(x-2)^2 + (y-1)^2 = \sqrt{3}$  (B)  $(x-1)^2 + (y-2)^2 = 3$   
 (C)  $(x+1)^2 + (y-2)^2 = \sqrt{3}$  (D)  $(x-1)^2 + (y+2)^2 = 3$

41. Locus of focus of variable ellipse (E) when major axis

is parallel to x-axis is

- (A)  $\left\{ (x, y) / \frac{\sqrt{3}}{2} < |x-1| < \sqrt{3} \text{ and } y = 2 \right\}$   
 (B)  $\left\{ (x, y) / 0 < |x| < \sqrt{3} \text{ and } y = 2 \right\}$   
 (C)  $\left\{ (x, y) / 0 < |x+1| < \sqrt{3} \text{ and } y = 2 \right\}$   
 (D)  $\left\{ (x, y) / 0 < |x-1| < \sqrt{3} \text{ and } y = 2 \right\}$

42. Locus of focus of variable ellipse (E)

- (A)  $1 < (x-1)^2 + (y-2)^2 < 3$   
 (B)  $\frac{2}{3} < (x-1)^2 + (y-2)^2 < 3$   
 (C)  $\frac{3}{2} < x^2 + y^2 < 3$   
 (D)  $0 < (x-1)^2 + (y-2)^2 < 3$

#### Paragraph for question nos. 43 to 45

From a point P (h, k), in general, three normals can be drawn to the parabola  $y^2 = 4ax$ . If  $t_1, t_2, t_3$  are the parameters associated with the feet of these normals, then  $t_1, t_2, t_3$  are the roots of the equation  $at^3 + (2a-h)t - k = 0$ . Moreover, from the line  $x = -a$ , two perpendicular tangents can be drawn to the parabola.

43. If the tangents at the feet Q  $(at_1^2, 2at_1)$  and R  $(at_2^2, 2at_2)$  to the parabola meet on the line  $x = a$ , then  $t_1, t_2$  are the roots of the equation

- (A)  $t^2 - t_3t + 1 = 0$  (B)  $t^2 + t_3t + 1 = 0$   
 (C)  $t^2 - t_3t - 1 = 0$  (D)  $t^2 + t_3t - 1 = 0$

44. If the feet Q  $(at_1^2, 2at_1)$  and R  $(at_2^2, 2at_2)$  are the ends of a focal chord of the parabola, then the locus of P (h, k) is

- (A)  $y^2 = a(x-2a)$  (B)  $y^2 = a(x-a)$   
 (C)  $y^2 = a(x-3a)$  (D)  $y^2 = 3a(x-a)$

45. If P (h, k) is a vertex of the square comprising normals to the parabola from P and tangents from the directrix, then (h, k) is the same as

- (A) (a, 0) (B) (2a, 0)  
(C) (3a, 0) (D) (4a, 0)

**Paragraph for question nos. 46 to 48**

An ellipse whose distance between foci S and S' is 4 units is inscribed in the triangle ABC touching the sides AB, AC and BC at P, Q and R. If centre of ellipse is at origin and major axis along x-axis,  $SP + S'P = 6$ , then answer the following questions.

46. Equation of ellipse is

- (A)  $\frac{x^2}{5} + \frac{y^2}{9} = 1$  (B)  $\frac{x^2}{9} + \frac{y^2}{4} = 1$   
(C)  $\frac{x^2}{9} + \frac{y^2}{5} = 1$  (D)  $\frac{x^2}{4} + \frac{y^2}{9} = 1$

47. If  $\angle BAC = 90^\circ$ , then locus of points A is

- (A)  $x^2 + y^2 = 12$  (B)  $x^2 + y^2 = 4$   
(C)  $x^2 + y^2 = 14$  (D)  $x^2 + y^2 = 5$

48. If chord PQ subtends  $90^\circ$  angle at centre of ellipse, then locus of A is

- (A)  $25x^2 + 81y^2 = 620$  (B)  $25x^2 + 81y^2 = 630$   
(C)  $9x^2 + 16y^2 = 25$  (D)  $9x^2 + 16y^2 = 125$

**Paragraph for question nos. 49 to 51**

Let two circles  $S_1 = 0$  and  $S_2 = 0$  intersect at point A and B.  $L_1 = 0$  is the line joining A and B where

$$S_1 = x^2 + y^2 + 2ax + 2by - 5 = 0 \text{ and}$$

$$S_2 = x^2 + y^2 + 2x + 4y - 4 = 0.$$

Let AB subtend an angle  $\theta$  at (0, 0) and angle subtended by  $S_1 = 0$  and  $S_2 = 0$  at P (3, 4) is  $\alpha$  and  $\beta$  respectively.

49. If equation  $L_1 \equiv 3x + 4y = 7$  of , then the value of  $4a + b$  is equal to

- (A)  $\frac{48}{7}$  (B)  $\frac{50}{7}$   
(C)  $\frac{60}{7}$  (D)  $\frac{55}{7}$

50. If AB passes through (1, 1) and parallel to a line which touches  $S_2 = 0$  at (2, -2), then  $4a + 2b$  is equal to

- (A) 4 (B) 6  
(C) 8 (D) 10

51. If  $\alpha = 60^\circ$ , then (a, b) lies on a circle whose radius is

- (A)  $\frac{\sqrt{10}}{3}$  (B)  $\frac{4\sqrt{10}}{3}$   
(C)  $\frac{2\sqrt{10}}{3}$  (D)  $\sqrt{\frac{10}{3}}$

**Paragraph for question nos. 52 and 53**

The line  $y = x - 2$  cuts the parabola  $y^2 = 8x$  in the points A and B. The normals drawn to the parabola at A and B intersect at G. A line passing through G intersects the parabola at right angles at the point C, and the tangents at A and B intersect at the point T.

52. The sum of the co-ordinates of possible point C is

- (A) 0 (B) -1  
(C) 1 (D) -2

53. Minimum radius of the circle passing through A and B is

- (A) 8 (B)  $4\sqrt{5}$   
(C)  $3\sqrt{6}$  (D) 10

**Paragraph for question nos. 54 to 57**

Let the two foci of an ellipse be  $(-1, 0)$  and  $(3, 4)$  and the foot of perpendicular from the focus  $(3, 4)$  upon a tangent to the ellipse be  $(4, 6)$

54. The foot of perpendicular from the focus  $(-1, 0)$  upon the same tangent to the ellipse is

- (A)  $\left(\frac{12}{5}, \frac{34}{5}\right)$  (B)  $\left(\frac{7}{3}, \frac{11}{3}\right)$   
(C)  $\left(2, \frac{17}{4}\right)$  (D)  $(-1, 2)$

55. The equation of auxiliary circle of the ellipse is

- (A)  $x^2 + y^2 - 2x - 4y - 5 = 0$   
(B)  $x^2 + y^2 - 2x - 4y - 20 = 0$   
(C)  $x^2 + y^2 + 2x + 4y - 20 = 0$   
(D)  $x^2 + y^2 + 2x + 4y - 5 = 0$

56. The length of semi-minor axis of the ellipse is

- (A) 1 (B)  $2\sqrt{2}$   
(C)  $\sqrt{17}$  (D)  $\sqrt{19}$

57. The equation of directrices of the ellipse are

- (A)  $x - y + 2 = 0, x - y - 5 = 0$   
(B)  $x + y - \frac{21}{2} = 0, x + y + \frac{17}{2} = 0$   
(C)  $x - y + \frac{3}{2} = 0, x - y - \frac{5}{2} = 0$   
(D)  $x + y - \frac{31}{2} = 0, x + y + \frac{19}{2} = 0$

**Paragraph for question nos. 58 to 60**

Tangents are drawn to the parabola  $y^2 = 4x$  from the point  $P(5, 6)$  to touch the parabola at  $Q$  and  $R$ .  $C_1$  is a circle which touches the parabola at  $Q$  and  $C_2$  is a circle which touches the parabola at  $R$ . Both the circles  $C_1$  and  $C_2$  pass through the focus the parabola.

58. Area of the  $\Delta PQR$  equals

- (A)  $\frac{1}{2}$  (B) 1  
(C) 32 (D)  $\frac{1}{4}$

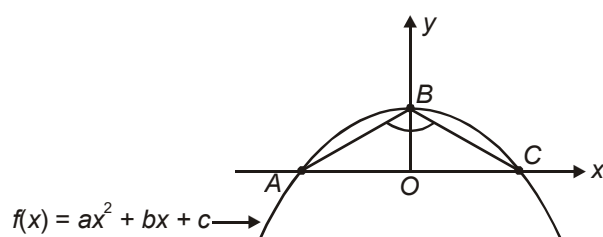
59. Radius of the circle  $C_2$  is

- (A)  $5\sqrt{5}$  (B)  $\sqrt{4394}$   
(C)  $10\sqrt{2}$  (D)  $\sqrt{210}$

60. The common chord of the circles  $C_1$  and  $C_2$  passes through the

- (A) incentre of the  $\Delta PQR$   
(B) circumcentre of the  $\Delta PQR$   
(C) centroid of the  $\Delta PQR$   
(D) orthocentre of the  $\Delta PQR$

**Paragraph for Question Nos. 61 and 62**



In the given figure vertices of  $\Delta ABC$  lie on  $y = f(x) = ax^2 + bx + c$ . The triangle  $ABC$  is equilateral and  $AC = 4\sqrt{3}$  units, then

61.  $y = f(x)$  is given by

- (A)  $12 - x^2$  (B)  $12 + x^2$   
(C)  $\frac{1}{2}(12 - x^2)$  (D)  $\frac{1}{2\sqrt{3}}(12 - x^2)$

62. Maximum value of  $y = f(x)$  is

- (A) 6 (B) 12  
(C)  $8\sqrt{3}$  (D)  $12\sqrt{3}$

**SECTION-D**

**Matrix-Match Type**

This section contains Single match questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s. Four options A, B, C and D are given below. Out of which, only one shows the right matching

63. Consider the parabola  $y^2 = 12x$

**Column - I**

**Column - II**

- (A) Tangent and normal at the extremities of the latus rectum intersect the x axis at T and G respectively. The coordinates of the middle point of T and G are.
- (B) Variable chords of the parabola passing through a fixed point K on the axis, such that sum of the squares of the reciprocals of the two parts of the chords through K, is a constant. The coordinate of the point K are.
- (C) All variable chords of the parabola subtending a right angle at the origin are concurrent at the point
- (D) AB and CD are the chords of the parabola which intersect at a point E on the axis. The radical axis of the two circles described on AB and CD as diameter always passes through

(P) (0, 0)

(Q) (3, 0)

(R) (6, 0)

(S) (12, 0)

64. There are 2 circles  $C_1$  and  $C_2$  such that  $C_1 : x^2 + y^2 = 4$  and  $C_2 : x^2 + y^2 = 8$ . Two tangents PQ and PR are drawn from point  $P(\sqrt{6}, \sqrt{2})$  on circle  $C_2$  to the circle  $C_1$  and a. New circle  $C_3$  is drawn assuming QR as a diameter

**Column - I**

**Column - II**

- (A) The length of the largest chord of  $C_3$
- (B) The length of tangent PQ
- (C) The perimeter of  $\Delta PQR$
- (p) 2
- (q)  $2\sqrt{2}$
- (r) 1

- (D) The area of  $\Delta PQR$

(s)

$$4+2\sqrt{2}$$

**SECTION-E**

**Integer Answer Type**

This section contains Integer type questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y and Z(say) are 6, 0 and 9, respectively, then the correct darkening of bubbles will look like the following :

| X | Y | Z |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

65. If the locus of the focus of the variable parabola  $C_1$ , which always touch the curve  $C_2 \equiv x^2 = -y$  at the origin, is  $x^2 + y^2 = 2ay^4$ , (given that  $C_1$  and  $C_2$  have the same latus rectum and they have no other point of intersection). Then the value of a is \_\_\_\_\_
66. A parabola which have directrix  $x + y + 2 = 0$  and touches a line  $2x + y - 5 = 0$  at (2, 1). If m is the latus rectum of the parabola, then the value of  $\frac{m}{\sqrt{2}}$  is \_\_\_\_\_
67. Let circle  $C_1 : x^2 + (y - 4)^2 = 12$  intersects circle  $C_2 : (x - 3)^2 + y^2 = 13$  at A and B. A quadrilateral ACBD is formed by tangents at A and B to both circles. The diameter of circum circle of quadrilateral ACBD is \_\_\_\_\_

68. If the minimum value of

$$(x_1 - x_2)^2 + \left( \frac{x_1^2}{20} - \sqrt{(17 - x_2)(x_2 - 13)} \right)^2,$$

where  $x_1 \in \mathbb{R}^+, x_2 \in (13, 17)$  is  $(a\sqrt{b} - b)^2$  where  $a$  and  $b$  are coprime number then  $a + b =$

69. Let  $A(0, 1), B(1, 1), C(1, -1), D(-1, 0)$  be four points. If  $P$  be any other point, then  $PA + PB + PC + PD \geq d$ , then  $[d] =$  (where  $[.]$  denotes greatest integer function).

70. If the normals at the four points  $(x_r, y_r)$  (where  $r = 1, 2, 3, 4$ ) on the ellipse  $4x^2 + 9y^2 = 36$  are concurrent,

then the value of  $\sum_{r=1}^4 x_r \cdot \sum_{r=1}^4 \frac{1}{x_r}$  is

71. Number of possible values of  $\alpha^2$  such that there exists a tangent of slope 1 to the ellipse  $x^2 + \alpha^2 y^2 = \alpha^2$  such that the portion of the tangent intercepted by the hyperbola  $\alpha^2 x^2 - y^2 = 1$  subtends a right angle at the centre of the curve is \_\_\_\_\_.

72. A parabola with latus rectum  $4a$  ( $a > 0$ ) slides such that it touches the positive co-ordinate axes. The locus of its focus is  $x^{-2} + y^{-2} = \lambda a^{-2}$  where  $x > 0, y > 0$ . Then the value of  $\lambda$  is \_\_\_\_\_



## ANSWERS

### LEVEL-1

- |                       |               |                       |                  |                          |               |
|-----------------------|---------------|-----------------------|------------------|--------------------------|---------------|
| 1. (B)                | 2. (D)        | 3. (D)                | 4. (D)           | 5. (A)                   | 6. (A)        |
| 7. (A)                | 8. (D)        | 9. (D)                | 10. (B)          | 11. (C)                  | 12. (B)       |
| 13. (C)               | 14. (C)       | 15. (B, C)            | 16. (A, B)       | 17. (B, C)               | 18. (B, C, D) |
| 19. (A, B)            | 20. (A, B, C) | 21. (A, B, C)         | 22. (A, B, C, D) | 23. (A, B)               | 24. (A, B, D) |
| 25. (B, D)            | 26. (B)       | 27. (C)               | 28. (A, C)       | 29. (D)                  | 30. (D)       |
| 31. (C)               | 32. (A)       | 33. (D)               | 34. (C)          | 35. (A-q B-s C-r D-p)    |               |
| 36. (A-q B-s C-q D-p) |               | 37. (A-s B-p C-q D-r) |                  | 38. (A-r, B-p, C-s, D-q) |               |
| 39. (A-Q B-S C-P D-R) |               | 40. (A-r B-p C-s D-r) |                  | 41. (8)                  | 42. (2)       |
| 43. (2)               | 44. (4)       | 45. (7)               | 46. (0)          | 47. (4)                  |               |
| 48. (1)               |               |                       |                  |                          |               |

### LEVEL-2

- |               |                  |                       |                       |               |               |
|---------------|------------------|-----------------------|-----------------------|---------------|---------------|
| 1. (C)        | 2. (C)           | 3. (B)                | 4. (B)                | 5. (B)        | 6. (A)        |
| 7. (B)        | 8. (D)           | 9. (C)                | 10. (B)               | 11. (A)       | 12. (B)       |
| 13. (A)       | 14. (B)          | 15. (C)               | 16. (C)               | 17. (A)       | 18. (D)       |
| 19. (A)       | 20. (A)          | 21. (C)               | 22. (C)               | 23. (C)       | 24. (C)       |
| 25. (A, B, C) | 26. (A, C, D)    | 27. (A, B, D)         | 28. (A, B, C)         | 29. (B, D)    | 30. (A, B, C) |
| 31. (A, C)    | 32. (A, B, C, D) | 33. (A, C, D)         | 34. (A, D)            | 35. (B, C, D) | 36. (A, B)    |
| 37. (A, D)    | 38. (A, B)       | 39. (A, D)            | 40. (B)               | 41. (D)       | 42. (D)       |
| 43. (B)       | 44. (C)          | 45. (C)               | 46. (C)               | 47. (C)       | 48. (B)       |
| 49. (B)       | 50. (D)          | 51. (C)               | 52. (A)               | 53. (A)       | 54. (A)       |
| 55. (B)       | 56. (C)          | 57. (D)               | 58. (C)               | 59. (B)       | 60. (C)       |
| 61. (C)       | 62. (A)          | 63. (A-Q B-R C-S D-P) | 64. (A-q B-p C-s D-p) | 65. (8)       |               |
| 66. (9)       | 67. (5)          | 68. (7)               | 69. (4)               | 70. (4)       | 71. (1)       |
| 72. (1)       |                  |                       |                       |               |               |

