23. ALTERNATING CURRENT

1. INTRODUCTION

A majority of electrical power in the world is generated, distributed, and consumed in the form of 50-Hzor60-Hz sinusoidal alternating current (AC) and voltage. It is used for household andindustrial applications.

AC has several advantages over DC. The major advantage of AC is the fact that it can be transformedinto any form, whereas direct current (DC) cannot. A transformer permits voltage to be stepped up or down for the purpose of transmission. Transmission of high voltage (in terms of KV) implies that less current is required to produce the same amount of power. Less current permits thinner wires to be used for transmission.

In this chapter, we will introduce a sinusoidal signal and its basic mathematic equation. We will discuss and analyse circuits where currents i(t) and voltages v(t) vary with time. The phasor analysis techniques will be used to analyse electronic circuits under sinusoidal steady-state operating conditions. The chapter will conclude with single-phase power.

2. SINUSOIDAL WAVEFORMS

AC, unlike DC, flows first in one direction, then in the opposite direction. The most common AC waveform is a sine (or sinusoidal) waveform.

In discussing AC signal, it is necessary to express the current and voltage in termsof maximum or peak values, peak-to-peakvalues, effective values, average values, or instantaneous values. Each of these values has a different meaning and is used to describe a different amount of current or voltage. $V(t)=V_0 \sin \omega t$. Where V_o is the peak voltage, $\omega = 2\pi f$ is the angular frequency expressed in radian per second (rad/s), f is the frequencyexpressed in Hertz (Hz), t is time expressed in second (s).

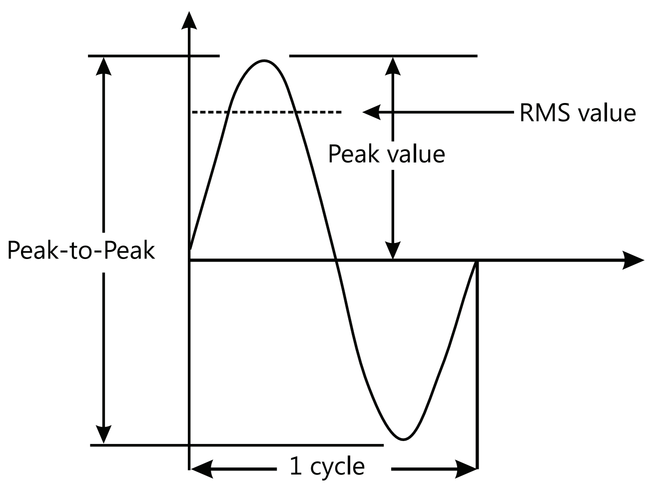


Figure 23.1: Sinusoidal Waveform.

2.1 Instantaneous Value

The instantaneous value of an AC signal is the value of voltage or current at one particular instant. The value may be zero, if the particular instant is the time in the cycle at which the polarity of the voltage is changing. It may also be the same as the peak value, if the selected instant is the time in the cycle at which the voltage or current stops increasing and starts decreasing. There are actually an infinitenumber of instantaneous values between zero and the peak value.

PLANCESS CONCEPTS

It is always advisable to find symmetries in functions while calculating rms and average value to reduce the period of integration.It helps a lot in avoiding unnecessary calculations when functions are defined part by part.

Yashwanth Sandupatla (JEE 2012, AIR 821)

2.2 Average Value

Average value of a function,from t_1 to t_2, is defined as $<f>=\frac{\int_{t_1}^{t_2} f dt}{t_2-t_1}$. We can find the value of $\int_{t_1}^{t_2} f dt$ graphically if the graph is simple. It is the area of f-t graph from t_2-t_1.

$I_{avg}=\frac{\int_0^t i dt}{\int_0^t dt}$,where i is the instantaneous value of the current.

2.2.1 For Sinusoidal Variation of Current and Voltages

Case I: Average value over complete cycle $\frac{\int_0^t i_o \sin(\omega\tau+\theta) dt}{\int_0^t dt}$. Similarly $V_{avg}=0$

Case II: Average value over half cycle $I_{avg}=\frac{\int_0^{t/2} i_o \sin(\omega\tau+\theta) dt}{\int_0^{t/2} dt}=\frac{2i_o}{\pi}$; Similarly $V_{avg}=\frac{2\, i_o}{\pi}$

Illustration 1: An electric heater draws 2.5 A current from a 220-V, 60-Hz power supply. Find **(JEE MAIN)**

(a) The average current

(b) The average of the square of the current

(c) The current amplitude

(d) The supply voltage amplitude

Sol: In AC circuit, the average value of current over a long time interval is zero but I^2 is not zero. The r.m.s. value of current and voltage is given by $I_{rms}=\frac{I_{max}}{\sqrt{2}}$ and $V_{rms}=\frac{V_{max}}{\sqrt{2}}$.

(a) The average of sinusoidal AC values over any whole number of cycles is zero.

(b) RMS value of current = I_{rms} =2.5 A so, $\left(I^2\right)_{av} = \left(I_{rms}\right)^2 = 6.25\ A^2$

(c) $I_{rms} = \frac{I_m}{\sqrt{2}}$; So, current amplitude $I_m = \sqrt{2}I_{rms} = \sqrt{2}(2.5A) = 3.5\ A$

(d) $V_{rms} = 220V = \frac{V_m}{\sqrt{2}}$, So, supply voltage amplitude $V_m = \sqrt{2}\left(V_{rms}\right) = \sqrt{2}(220V) = 311\ V.$

2.3 Effective Value (RMS Value)

This is the value of AC signal that will have the same effect on a resistance as a comparable value of direct voltage or current will have on the same resistance. It is possible to compute the effective value of a sine wave of current to a good degree of accuracy by taking equally spacedinstantaneous values of current along the curve and extracting the square root of the average of the sum of the squared values. For this reason, effective value is sometimes called RMS value.

Root mean square value of a function, from t_1 to t_2 is defined as $f_{rms} = \frac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}$

The magnitude of I_{rms} is given by $I_{rms}^2 = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{\int_0^T I_0^2 \sin^2(\omega\tau)dt}{\int_0^T dt} = \frac{I_0^2}{2}$

$I_{eff} = I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\ I_0$ Where I_0 is the peak value of the current. Similarly V_{eff} or $V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\ E_0 A$

PLANCESS CONCEPTS

RMS value is actually more important because in the context of power transmission,the loss in energy due to a resistor plays an important role. And the power is given by i^2R, where R is the resistance.

Yashwanth Sandupatla (JEE 2012, AIR 821)

Illustration 2: Find the RMS value of current I = $I_m \sin\omega t$ from (i) t=0 to t=$\frac{\pi}{\omega}$ (ii)t=$\frac{\pi}{2\omega}$to t=$\frac{3\pi}{2\omega}$ **(JEE MAIN)**

Sol: In AC circuit over time interval $0 \le t \le T$ the RMS value of current is given by

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}} = \sqrt{\frac{\int_0^T I_0^2 \sin^2(\omega\tau)dt}{\int_0^T dt}} = \frac{I_0}{\sqrt{2}} \quad \text{where } T = \frac{2\pi}{\omega}$$

(i) $I_{rms} = \sqrt{\dfrac{\int_0^{\frac{\pi}{\omega}} I_m^2 \sin^2(\omega t)\,dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{I_m^2}{2}} = \dfrac{I_m}{\sqrt{2}}$ (ii) $I_{rms} = \sqrt{\dfrac{\int_{\frac{\pi}{2}}^{\frac{3\pi}{2\pi}} I_m^2 \sin^2(\omega t)\,dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{I_m^2}{2}} = \dfrac{I_m}{\sqrt{2}}$ A

PLANCESS CONCEPTS

The RMS value of one cycle or half cycle (either a positive or negative cycle) is same.

GV Abhinav (JEE 2012, AIR 329)

2.4 Difference between Sine and Cosine Representation of AC Signal

The sine and cosine are essentially the same function, but with a 90^0 phase difference. For example, $\sin\omega t = \cos\left(\omega t - 90^0\right)$. Multiples of 360^0 may be added to or subtracted from the argument of any sinusoidal function, without changing the value of the function. To realize this, let us consider

$V_1 = V_{P1}\cos\left(10t + 20^0\right) = V_{P1}\sin\left(10t + 90^0 + 20^0\right)$ ... (i)

$= V_{P1}\sin\left(10t + 110^0\right)$ Leads $V_2 = V_{P2}\sin\left(10t - 40^0\right)$ ... (ii)

by 150^0. It is also correct to say that v_1 lags v_2 by 210^0, since v_1 may be written as

$V_1 = V_{P1}\sin\left(10t - 250^0\right)$ V ... (iii)

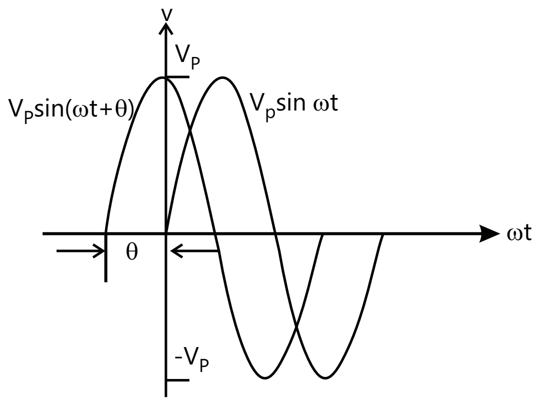


Figure 23.2: Representation of voltage as sine and cosine function

3. POWER IN AC CIRCUITS

Average power in alternating current circuit over time t is defined as $P_{avg} = \frac{\int_0^t vidt}{\int_0^t dt}$, where V andiare the instantaneous values of voltage and current respectively. Let V= $V_0 \sin \omega t$; $i = i_0 \sin(\omega t - \phi)$, Average power over a cycle

$$P_{avg} = \frac{\int_0^T v_o i_o \sin\omega t.\sin(\omega t - \phi)dr}{\int_0^T dt}; = \frac{v_o i_o \int_0^T \left(\sin^2 \omega t \cos\phi - \frac{1}{2}\sin 2\omega t \sin\phi\right)dt}{T} = \frac{1}{2}V_0 i_0 \cos\phi = V_{rms} i_{rms} \cos\phi$$

The term $\cos\phi$ is known as power factor.

If the current leads voltage, it is said to be leading, whereas, if it lags voltage, it is said to be lagging. Thus, a power factor of 0.5 lagging means the current lags voltage by 60° (as $\cos^{-1}0.5 = 60°$). The product of V_{rms} and i_{rms} gives the apparent power, while the true power is obtained by multiplying the apparentpower by the power factor $\cos\phi$

Thus, apparent power = $V_{rms} \times i_{rms}$ and true power=apparent power × power factor

For $\phi = 0°$, the current and voltage are in phase. The power is thus, maximum ($V_{rms} \times i_{rms}$). For $\phi = 90°$ the power is zero. The current is then stated wattless. Such a case will arise when resistance in the circuits is zero. The circuit is purely inductive or capacitive. The case is similar to that of a frictionless pendulum, where the total work done by gravity upon the pendulum cycle is zero.

We shall discuss more about the power and power factor later, shortly after we define impedance and its properties.

Illustration 3: When a voltage $V_s = 200\sqrt{2}$ sin (ωt+15°) is applied to an AC circuit, the current in thecircuit is found to be I=2 sin (ωt+π/4) then average power consumed in the circuit is **(JEE MAIN)**

(A) 200 W (B) 400 $\sqrt{2}$ W (C) 100 $\sqrt{2}$ W (D) 200 $\sqrt{2}$ W

Sol: Power in any AC circuit is calculated as $P_{av} = V_{rms} I_{rms} \cos\phi$ where ϕ is phase angle between V and I.

$$P_{av} = V_{rms}\ I_{rms} \cos\phi = \frac{200\sqrt{2}}{\sqrt{2}}.\frac{2}{\sqrt{2}}\cos\left(30^0\right) = 100\sqrt{6}\ \ W$$

4. SIMPLE AC CIRCUITS

4.1 Purely Resistive Load

Writing KVL along the circuit (see Fig. 23.3), V_S iR=0

Or $I = \frac{V_s}{R} = \frac{V_m \sin\omega t}{R} = I_m \sin\omega t$.

We see that the phase difference between potential differences across resistance, V_R and i_R is 0.

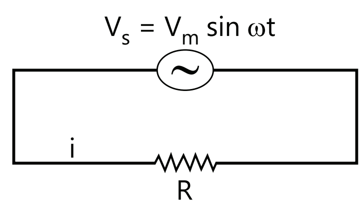


Figure 23.3: AC voltage applied to resistive load

$$I_m = \frac{V_m}{R} \Rightarrow I_{rms} = \frac{V_{rms}}{R} \quad <P> = V_{rms}\ I_{rms} \cos\phi = \frac{V_{rms}{}^2}{R}$$

4.2 Purely Capacitive

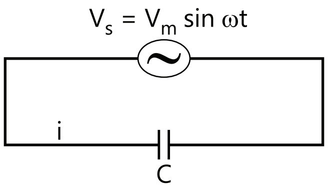


Figure 23.4: AC voltage applied to capacitive load

Writing KVL along the circuit shown in Fig. 23.4

$V_S - \frac{q}{c} = 0$ And current in the circuit is

$$I = \frac{dq}{dt} = \frac{d(cv)}{dt} = \frac{d(cv_m \sin\omega t)}{dt} = cv_m \omega \sin\omega t = \frac{V_m}{1/\omega C} \cos\omega t.$$

$$= \frac{V_m}{X_C} \cos\omega t = I_m \cos\omega t$$

where $X_C = \frac{1}{\omega C}$ and is called capacitive reactance. Its unit is Ohm (Ω).

From the graph of current versus time and voltage versus time,

it is clear that current attains its peak value at a time $\frac{T}{4}$ before the

time at which voltage attains its peak value. Corresponding to $\frac{T}{4}$ phase difference.

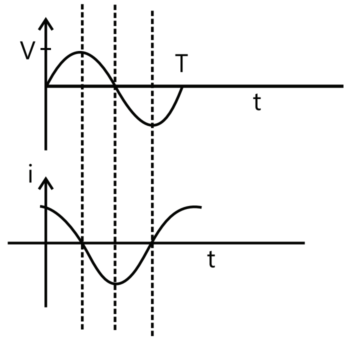
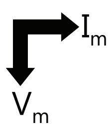


Figure 23.5

$= \omega \Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{\pi}{2}$ i_c leads v_c by $\pi/2$ diagrammatically (phasediagram) represented as



Since $\phi = 90^0$, $<P> = V_{rms}\ I_{rms} \cos\phi = 0$

The current leads the voltage by $\pi/2$ in a capacitive circuit

PLANCESS CONCEPTS

<P>=0 doesn't mean it is zero in any period less than the time period. In actuality, first the capacitor gets charged up, gaining energy during the first half cycle, and loses it for the next half cycle.So overall, power becomes zero.Same goes for the inductor in a different fashion (magnetic field plays a role there).

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4.3 Pure Inductive Circuit

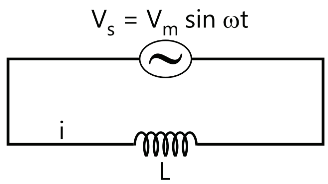


Figure 23.6: AC voltage applied to inductive load

Writing KVL along circuit, $V_s - L\frac{di}{dt} = 0$; $L\frac{di}{dt} = V_m \sin\omega t$; $\int L di = \int V_m \sin\omega t\, dt$;

$i = -\frac{V_m}{\omega L}\cos\omega t + C$; <i>=0 ; C=0;

$\therefore i = -\frac{V_m}{\omega L}\cos\omega t$ $I_m = \frac{Vm}{X_L}$ From the graph of current versus time and voltage versus time, it is clear that voltage attains its peak value at a time $\frac{T}{4}$ before the time at which current attains its peak value. Corresponding to $\frac{T}{4}$, the phase difference $= \omega \Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{T} = \frac{\pi}{2}$

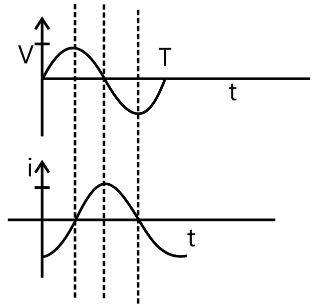
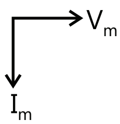


Figure 23.7: Variation of current and voltage with respect to time

Diagrammatically (See Fig. 23.7) it is represented  as

i_L lags behind V_L by $\pi / 2$ since $\phi = 90^0$, $<P> = V_{rms} I_{rms} \cos\phi = 0$. The current lags voltage by $\pi / 2$ in a purely inductive circuit.

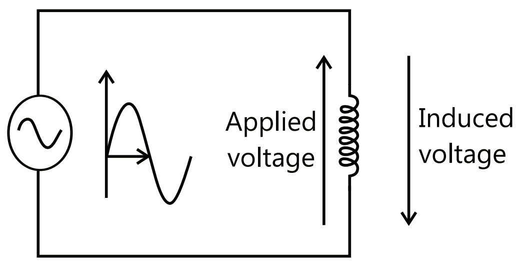


Figure 23.8: AC voltage applied to purely inductive circuit

5. IMPEDANCE

We have already seen that the inductive reactance $X_L = \omega L$ and capacitance reactance $X_C = 1 / \omega L$ play the role of an effective resistance in apurely inductive and capacitive circuit respectively. In the series RLC circuit, the effective resistance is the impedance,defined as $Z = \sqrt{R + \left(X_L - X_C\right)^2}$ ...(iv)

The relationship between Z, X_L, and X_C can be represented by the diagram shown in Fig. 23.9.

Following is a diagrammatic representation of the relationship between Z, X_L and X_C.

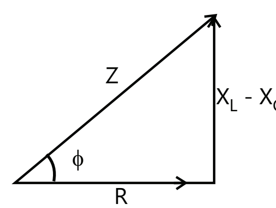


Figure 23.9: Impedance Triangle

The impedance has SI unit of Ω. In terms of Z the current may be rewritten as I(t)

$= \frac{V_0}{Z} \sin\left(\omega t - \phi\right)$ ...(v)

Notice that the impedance Z also depends on the angular frequency ω ,as do X_L and X_C.

Using the above equations for phase ϕ and Z, we may readily recover the limit for simple circuit (with only one element).

PLANCESS CONCEPTS

By now, students should get a clear idea of individual behaviour of inductor, capacitor and resistor and be able to visualize phasors. They should never get confused whetherinductor, capacitor is leading, etc.

Chinmay S Purandare (JEE 2012, AIR 698)

The upcoming series of circuits would be easy to understand because they are just a superposition of individual phasor diagrams.

6. MIXED AC CIRCUITS

6.1 LR Circuit

If V_R, V_L and $V_s$are the RMS voltage across are R, L and the AC source respectively. Then,

$V_S = \sqrt{V_R^2 + V_L^2} = I_2\sqrt{R^2 + X_L^2}$ Where I_s is r.m.s value of source current.

The total opposition to the current is called impedance and it is denoted by Z.

$$Z = \frac{V_S}{I_S} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

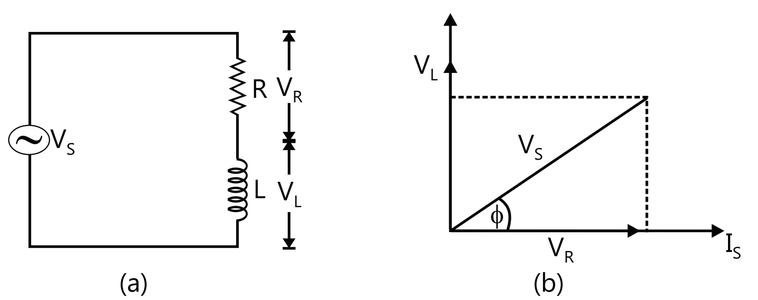


Figure 23.10: (a) AC voltage applied to LR circuit (b) Phasor diagram of voltage drops across R and L

The phase angel ϕ by which the applied voltage leads the current is $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Illustration 4: An alternating voltage of 220V RMS at a frequency of 40 cycles/second is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6Ω in series. Calculate (a) The current, (b) Potential difference across the resistance, (c) Potential different across inductance, (d) The time lag. **(JEE MAIN)**

Sol: Theimpedance of LR circuit is $Z = \sqrt{R^2 + (\omega L)^2}$. The RMS value of the current is $I_{rms} = \frac{V_{rms}}{Z}$. In LR circuit, the current lags the applied voltage by phase angleϕ obtained as $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$.

The impedence of the L-R series circuit is given by:

$$Z^2 = \left[R^2 + (\omega L)^2\right]^{1/2} = \left[(R)^2 + (2\pi f L)^2\right]^{1/2}$$

$= \left[62 + (2\times 3.14\times 40\times 0.01)^2\right]^{1/2}$ = 6.504 Ω

(a) RMS value of the current: $I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{6.504} = 33.83$ A

(b) The potential difference across the resistance is given by: $V_R = I_{rms} \times R = 33.83 \times 6 = 202.83$ V

(c) Potential difference across the inductance is given by:

$V_L = I_{rms} \times (\omega L) = 33.83 \times (2\times 3.14\times 0.01) = 96.83$ V

(d) Phase angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$; so, $\phi = \tan^{-1}(0.4189 = 22.46)$

Now time lag= $\frac{\phi}{360} = T = \frac{22.46}{360} = 0.0623$ s.

Illustration 5: A $\frac{9}{100\pi}$ H inductor and a 12 Ω resistance are connected in a series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage. **(JEE MAIN)**

Sol: Theimpedance of LR circuit is $Z = \sqrt{R^2 + (\omega L)^2}$. The RMS value of the current is $I_{rms} = \frac{V_{rms}}{Z}$. In LR circuit, the current lags the applied voltage by phase angleϕ obtained as $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$.

Here $X_L = \omega L = 2\pi fL = 2\pi\times 50\times\frac{9}{100\pi} = 9\Omega$

So, $Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 9^2} = 15\Omega$

(a) $I = \frac{V}{Z} = \frac{225}{15} = 15$A and (b) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{9}{12}\right) = \tan^{-1} 3/4 = 37^\circ$

i.e., the current will lag the applied voltage by 37^0 in phase.

Illustration 6: A chokecoil is needed to operate an arc lamp at 160 V (RMS) and 50 Hz. The arc lamp has an effective resistance of 5Ω when running of 10 A (RMS). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160V (dc), what additional resistance is required? Compare the power losses in both cases.

(JEE ADVANCED)

Sol: The choke coil is a LR circuit having large inductanceand small resistance. The potential difference across the resistor and inductor is added vectorially:$V^2 = V_R^2 + V_L^2$.

As for the lamp, $V_R = IR = 10\times 5 = 50V$, so when it is connected to 160 V ac source though a choke in series,$V^2 = V_R^2 + V_L^2$, $V_L = \sqrt{160^2 - 50^2} = 152V$

And as, $V_L = IX_L = I\omega L = 2\pi fLI$ $L = \frac{V_L}{2\pi fI} = \frac{152}{2\times\pi\times 50\times 10} = 4.84\times 10^{-2}H$

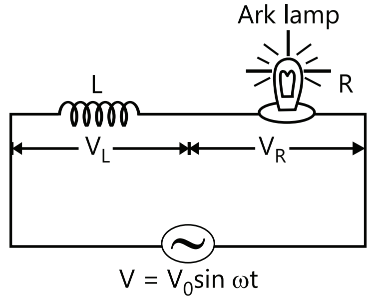
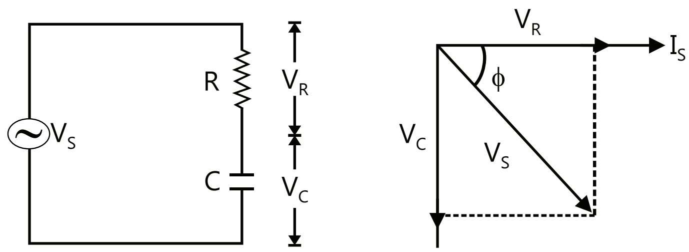


Figure 23.11

Now the lamp is to be operated at 160 V dc; instead of choke, if additional resistance r is I put in a series with it, V = I(R+r), i.e. 160 = 10(5+r) i.e. r = 11Ω In case of AC, as choke has no resistance, power loss in the choke will be zero, while the bulb will consume $P=I^2 R=10^2 \times 5=500$ W. However, in case of DC,as resistance r is to be used instead of choke, the power loss in the resistance r will be $PL=10^2 \times 11=1100$ W

While the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke, more than double the power consumed by the lamp is wasted by the resistance r.

6.2 RC Circuits



(a) (b)

Figure 23.12: (a) AC voltage applied to RC circuit (b) Phasor diagram of voltage drops across R and C

If V_s, V_R and V_C are RMS voltages across a source, resistance and capacitor respectively

$$V_S = \sqrt{V_R^2 + V_C^2} = I_S = \sqrt{R^2 + X_C^2}$$

Impedance of circuit, $$Z=\frac{V_S}{I_S} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

V_S leads I_S by $$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

The current leads the applied voltage by angel ϕ.

Illustration 7: An ac source of angular frequency ω is fed across a resister R and a capacitor C in series. The current registered is I. If now, the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency ω.

(JEE MAIN)

Sol: The impedance of RC circuit is:

$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$. The RMS current is $I_{rms} = \frac{V_{rms}}{Z}$

According to the given problem, $I=\frac{V}{Z} = \frac{V}{\left[R^2 + (1/C\omega)^2\right]^{1/2}}$ ... (i)

And for frequency of $\frac{\omega}{3}$, $\frac{I}{2} = \frac{V}{\left[R^2 + (3/C\omega)^2\right]^{1/2}}$ ... (ii)

Substituting the value of I from equation (i) in (ii),

$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2} \text{ i.e.. } \frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that, $\frac{X}{R}=\frac{(1/c\omega)}{R}=\frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R}=\sqrt{\frac{3}{5}}$

Illustration 8: In an RC series circuit, the RMSvoltage of source is 200V, and its frequency is 50 Hz. If R = 100 Ω and $C=\frac{100}{\pi}\mu F$, find

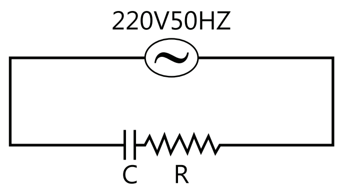


Figure 23.13

(a) Impedance of the circuit (b) Power factor angle

(c) Power factor (d) Current

(e) Maximum current (f) Voltage across R

(g) Voltage across C (h) Max voltage across R

(i) Maxvoltage cross C (j)<P>

(k) $<P_R>$ (l) $<P_C>$

(JEE ADVANCED)

Sol: The impedance of RC circuit is

$$Z=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$$

The RMS current is $I_{rms}=\frac{V_{rms}}{Z}$. The phase angle between current and voltage is given by $\tan\phi=\frac{X_C}{R}$. The RMS value of current and voltage is $I_{rms}=\frac{I_0}{\sqrt{2}}$ and $V_{rms}=\frac{V_0}{\sqrt{2}}$. Power developed in circuit is $P=V_{rms}I_{rms}\cos\phi$.

$$X_C=\frac{10^6}{\frac{100}{\pi}(2\pi 50)}=100\ \Omega$$

(a) $Z=\sqrt{R^2+XC^2}=\sqrt{100^2+(100)^2}=100\sqrt{2}\Omega$

(b) $\tan\phi=\frac{X_C}{R}=1 \qquad \therefore \phi=45^0$

(c) Power factor= $\cos\phi=\frac{1}{\sqrt{2}}$

(d) Current $I_{rms}=\frac{V_{rms}}{Z}=\frac{200}{100\sqrt{2}}=\sqrt{2}\ A$

(e) Maximum current $=I_{rms}\sqrt{2}=\sqrt{2}\ A$

(f) Voltage across R=$V_{R,rms}$=I_{rms} R= $\sqrt{2}\times 100\ V$

(g) Voltage across C= $V_{Crms}=I_{rms}X_C=\sqrt{2}\times 100\ V$

(h) Max voltage across R= $\sqrt{2}\ V_{R,rms}$=200 V

(i) Max voltage across C= $\sqrt{2}\ V_{Crms}$=200 V

(j) $\langle P \rangle = V_{rms} I_{rms} \cos\phi = 200 \times \sqrt{2} \times \frac{1}{\sqrt{2}}$ W

(k) $\langle P_R \rangle I_{rms}{}^2 R = 200$ W

(l) $\langle P_c \rangle = 0$

PLANCESS CONCEPTS

We observed here that inductor's reactance is directly proportional to the frequency used in the circuit and vice-versa for capacitor. So a combined circuit of them can be used as a frequency filter. High frequencies can be received by noting the voltage across capacitor and low frequencies can be noted using the inductor.

Nitin Chandrol (JEE 2012, AIR 134)

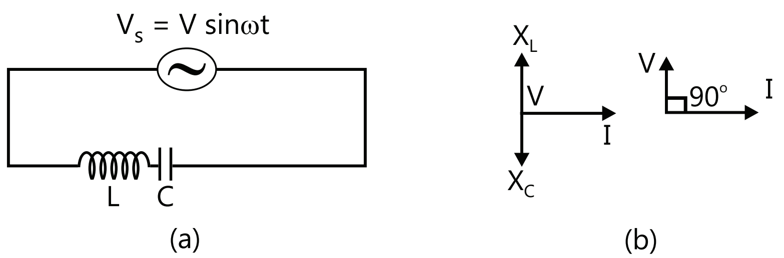


Figure 23.14: (a) AC voltage applied to LC circuit (b) Phasor diagram for voltage drops across L and R

6.3 LC Circuits

From the phasor diagram $V = I\left|\left(X_L - X_C\right)\right| = IZ;\ \phi = 90^0$

6.4 RLC Circuits

For LCR series circuits $V_s = \sqrt{V_R^2 + \left(V_L - V_C\right)^2}$

Impedance of circuits $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}$

V_S leads I_S by $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega c}}{R}\right)$

Power in LCR circuit= $V_{rms} I_{rms} \cos f = V_{rms} I_{rms} \frac{R}{Z} = V_R I_{rms}$

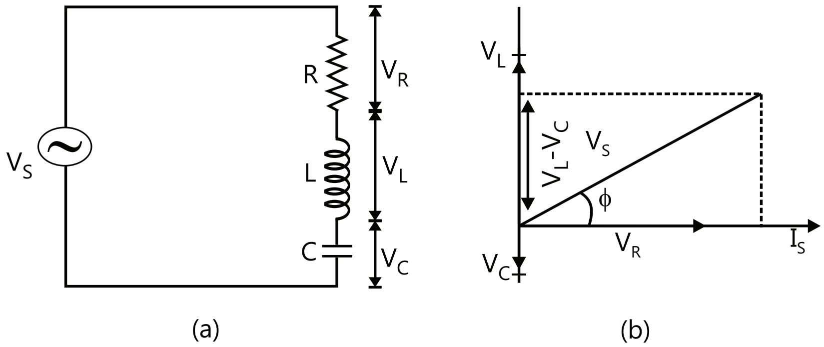


Figure 23.15: (a) AC voltage applied to LCR circuit. (b) Phasor diagram of voltage drops across L, C and R

Where $\cos\phi$ is called the power factor of the LCR circuit.

6.4.1 Resonance in RLC Circuits

At a particular angular frequency ω_o of the source, when $X_L = X_C$ or $\omega_o L = \frac{1}{\omega_0 C}$, the impedance of the circuit becomes minimum and equal to R and therefore, the current will be maximum. The circuit is then said to be in resonance. The resonance angular frequency ω_o andfrequency V_0 given by

$$\omega_0 = \frac{1}{\sqrt{2}} . \nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

The variation of RMS current with the frequency of the applied voltage is shown in the Fig. 23.16. If the applied voltage consists of a number of frequency components, the current will be large for the components having frequency V_0. The Q factor of an LCR series circuit is given by Q= $\frac{\omega_0 L}{R}$. A direct current of a flows uniformly throughout the cross-section of theconductor. An alternating current on the other hand, flows mainly along the surface of the conductor. This effect is known as the skin effect. The reason is that when ac flows through aconductor, the flux change in the inner part of the conductor is higher.

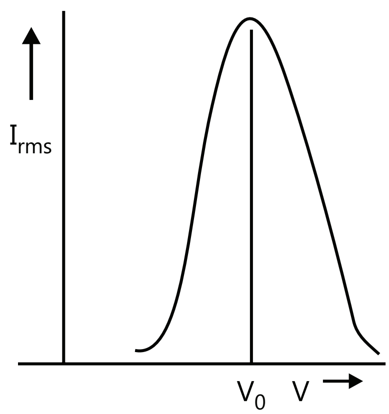


Figure 23.16

PLANCESS CONCEPTS

The idea of resonance is used in TV channelsfor clarity: a particular frequency is assignedto a channel and when this frequency is received by the receiver,the current corresponding to this frequency becomes maximum. This helps in maximum possible separation of channels, thus increasing their individual clarity.

It is also used by intelligence agencies to intercept the signals of anti-social elements. They generally use frequency of a very high order.

Nivvedan (JEE 2009, AIR 113)

Illustration 9: In the circuit shown in the Fig. 23.17, find **(JEE MAIN)**

(a) The reactance of the circuit

(b) Impedance of the circuit

(c) The current

(d) Reading of the ideal AC voltmeters

(These are hot wire instruments and read RMS values)

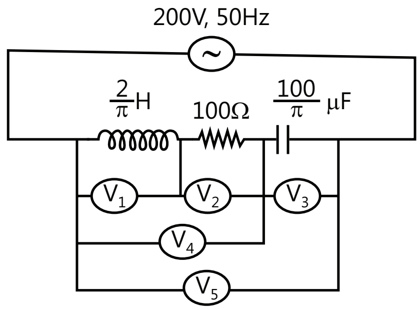


Figure 23.17

Sol: In series LCR circuit, the impedance is $Z=\sqrt{R^2+\left(X_C-X_L\right)^2}$ where X_C and X_L are the capacitive reactance and inductive reactance respectively.

(a) $X_L=2\pi fL=2\pi\times 50\times\frac{2}{\pi}=200\Omega$ $X_C=\frac{1}{2\pi 50\frac{100}{\pi}\times 10^{-6}}=100\,\Omega$

∴ The reactance of the circuit $X=X_L-X_C=200-100=100\,\Omega$

Since $X_L>X_C$, the circuit is called inductive.

(b) Impedance of circuit Z= $\sqrt{R^2+X^2}=\sqrt{100^2+100^2}=100\sqrt{2}\ \Omega$

(c) The current $I_{rms}=\frac{V_{rms}}{Z}=\frac{200}{100\sqrt{2}}=\sqrt{2}A$

(d) Readings of ideal voltage

$V_1: I_{rms}X_L=200\sqrt{2}\ V$

$V_2: I_{rms}R=100\sqrt{2}\ V$

$V_3: I_{rms}X_c=100\sqrt{2}\ V$

$V_4: I_{rms}\sqrt{R^2+X_L{}^2}=100\sqrt{10}\ \ V$

$V_4: I_{rms}Z=200\ \ V$, , which also happens to be the voltage of source.

Illustration 10: A resistance R, inductance L and a capacitor C all are connected in series with ac supply. The resistance of R is 16 Ω and for a given frequency, the inductive reactance of L is 24 Ω and capacitive reactance of C is 12 Ω. If the current in the circuit is 5 amp, find: **(JEE MAIN)**

(a) The potential difference across R, L and C

(b) The impedance of the circuit

(c) The voltage of ac supply

(d) Phase angle

Sol: In series LCR circuit, the impedance is $Z=\sqrt{R^2+\left(X_C-X_L\right)^2}$ where X_C and X_L are the capacitive reactance and inductive reactance respectively. The phase angle between voltage and current is given by $\phi=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)$.

(a) Potential difference across resistance: $V_R = iR = 5\times16 = 80\ V$

Potential difference across inductance: $V_L = i\times(\omega L) = 5\times24 = 120\ V$

Potential difference across capacitor: $V_c = i\times(1/\omega C) = 5\times12 = 60\ V$

(b) $$Z = \sqrt{\left[R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2\right]} = \sqrt{(16)^2 + (12)^2} = 20\Omega$$

(c) The voltage of ac supply is given by: $V = IZ = 5\times20 = 100\ V$

(d) $$\phi = \tan^{-1}\left(\frac{\omega L - (1/\omega c)}{R}\right) = \tan^{-1}\left(\frac{24-12}{16}\right) = \tan^{-1}(0.75) = 36^0 46^{''}$$

Illustration 11: An oscillating voltage drives an alternating current through a resistor, an inductor, and a capacitor that are all connected in series. Calculate the RMS voltage across each another by multiplying the reactance or resistance of each element by the RMS current. To calculate the RMS current, divide the RMS voltage by the impedance. **(JEE ADVANCED)**

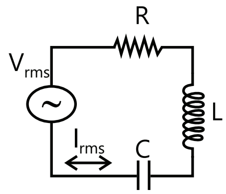


Figure 23.18

Sol: In series LCR circuit, the impedance is $Z = \sqrt{R^2 + (X_C - X_L)^2}$ where X_c and X_L are the capacitive reactance and inductive reactance respectively. The phase angle between voltage and current is given by $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$. Find the current in the series circuit, and multiply the resistance or reactance of each element with the currrent to find the voltage drop across it.

1. Calculate X_C; $X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60.0Hz)0.15\mu F} = 17.68\ k\Omega$

2. Calculate X_L; $X_L = \omega L = 2\pi(60.0Hz)(25mH) = 9.42\pi\ \Omega$

3. Calculate the impedance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(9.9k\Omega)^2 + (0.00942k\Omega - 17.68k\Omega)^2} = 20.25\ K\Omega$$

4. Divide the voltage by the impedance: $I_{rms} = \frac{V_{rms}}{Z} = \frac{115\ V}{20.25\ k\Omega} = 5.7\ mA$

5. Multiply the current by the resistance: $V_{rms.R} = I_{rms}R = 5.68\ mA(9.9k\Omega) =$ 56 V

6. Multiply the current by the inductive reactance: $V_{rms.L} = I_{rms}X_L = 5.68\ mA(9.42k\Omega) =$ 54 V

7. Multiply the current by the capacitive reactance:

$V_{rms.C} = I_{rms}X_C = 5.68\ m\ A(17.68k\Omega) = 100V =$ 0.10 KV

6.5 Parallel RCL Circuits

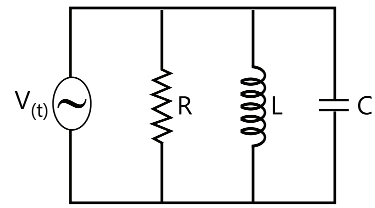


Figure 23.19 Parallel LRC circuit

Consider the parallel RLC circuit illustrated in Fig. 23.19.

The voltage source is V (t) =V_0 sin ωt .

Unlike the series RLC circuit, the instantaneous voltage acrossall three circuit elements R, L, and C are the same, and each voltage is in phase with the current through the resistor. However, the current through each element will be different.

In analysing this circuit, we make use of the results derived before. The current in the resistor is $I_R(t) = \frac{V(t)}{R} = \frac{V_0}{R} = \sin\omega t = I_{R0}\sin\omega t$ ... (i)

Where $I_{R0} = V_0/R$. The voltage across the inductor is $V_L(t) = V(t) = V_0\sin\omega t = L\frac{dI_L}{dt}$ ... (ii)

which gives $I_L(t) = \int_0^t \frac{V_0}{L}\sin\omega t'dt' = \frac{V_0}{\omega L}\cos\omega t = \frac{V_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) = I_{L0}\sin\left(\omega t - \frac{\pi}{2}\right)$ ... (iii)

where $I_{L0} = V_0 / X_L$ and $X_L = \omega L$ is the inductive reactance.

Similarly, the voltage across the capacitor is V_c (t)=V_0 sin ωt =Q(t)/c, which implies

$$I_C(t) = \frac{dQ}{dt} = \omega CV_0\cos\omega t = \frac{V_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right) = I_{C0}\sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{... (iv)}$$

where $I_{C0} = V_0 / X_C$ and $X_C = 1/\omega L$ is the capacitive reactance.

Using Kirchhoff's junction rule, the total current is simply the sum of all three currents.

$$I(t) = I_R(t) + I_L(t) + I_c(t) = I_{R0}\sin\omega t + I_{L0}\sin\left(\omega t - \frac{\pi}{2}\right) + I_{C0}\sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{... (v)}$$

The current can be represented with the phasor diagram shown in Fig. 23.20

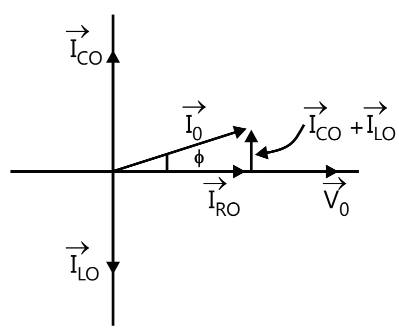


Figure 23.20: Phase difference between current and voltage

From the phasor diagram, we see that. $\vec{I}_0 = \vec{I}_{R0} + \vec{I}_{L0} + \vec{I}_{C0}$ ... (vi)

And the maximum amplitude of the total current, I_0, can be obtained as

$$\vec{I}_0 = \left|\vec{I}_0\right| = \left|\vec{I}_{R0} + \vec{I}_{L0} + \vec{I}_{C0}\right| = \sqrt{I^2_{R0} + (I_{c0} - I_{L0})^2} = V_0\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = V_0\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2} \quad \text{... (vii)}$$

Note however, since $I_R(t)$, $I_L(t)$ and $I_C(t)$ are not in phase with one another, I_0 is not equal to the sum of the maximum amplitudes of the three currents: $I_0 \neq I_{R0} + I_{L0} + I_{C0}$ ... (viii)

With $I_0 = V_0/Z$, the (inverse) impedance of the circuit is given by:

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2} \quad \text{... (ix)}$$

The relationship between Z, R, X_L and X_C is shown in Fig. 23.21 which shows a relationship between Z, R, X_L and X_C in a parallel RLC circuit.

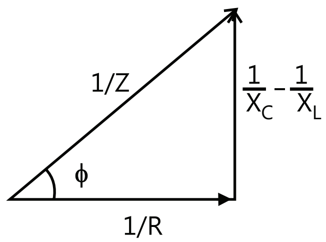


Figure 23.21: Impedance triangle

From the phasor diagram, we see that the phase can be obtained as: $\tan\phi = \left(\frac{I_{C0} - I_{L0}}{I_{R0}}\right) = \frac{\frac{V_0}{X_C} - \frac{V_0}{X_L}}{\frac{V_0}{R}} = R\left(\frac{V_0}{X_C} - \frac{V_0}{X_L}\right)$
$= R\left(\omega t - \frac{\pi}{2}\right)$ ... (x)

The resonance condition for the parallel RLC circuit is given by $\phi = 0$, which implies:

$$\frac{1}{X_C} = \frac{1}{X_L} \quad \text{... (xi)}$$

The resonant frequency is: $\omega_0 = \frac{1}{\sqrt{LC}}$ ... (xii)

which is the same as for the series RLC circuit. From Eq. (xii), we readily see that 1/Z is minimum (or Z is maximum) at resonance. The current in the inductor exactly cancels out the current in the capacitor, so that the total current in the circuit reaches minimum, and is equal to the current in the resistor: $I_0 = \frac{V_0}{R}$ ... (xiii)

As in the series RLC circuit, power is dissipated only through the resistor. The average power is

$$\langle P(t)\rangle = \langle I_R(t)V(t)\rangle = \langle I_R^2(t)R\rangle = \frac{V_0^2}{R}\langle \sin^2 \omega t\rangle = \frac{V_0^2}{2R} = \frac{V_0^2}{2Z} = \left(\frac{Z}{R}\right) \quad \text{... (xiv)}$$

Thus, the power factor in this case is

$$\text{Power factor} = \frac{\langle P(t)\rangle}{V_0^2/2Z} = \frac{Z}{R} = \frac{1}{\sqrt{1 + \left(R\omega C - \frac{R}{\omega L}\right)^2}} = \cos\phi \quad \text{... (xv)}$$

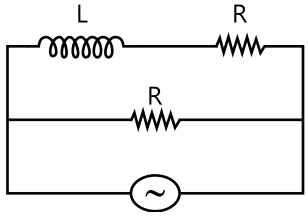


Figure 23.22

Illustration 12: The image shows an inductor (L=0.22 mH) in series with a 15Ω resistor. These elements are in parallel with a second 15Ω resistor. An AC generator powers the circuit with an RMS voltage of 65V.

In the limit of high frequency, the inductor behaves like a very large resistor. In such a case, nearly all of the current flows through the branch with the lone resistor. Calculate the current by dividing the RMS voltage by the single resistor.

In the limit of low frequency, the reactance of the inductor approaches zero. In such a case, the current flows through each resistor equally. Calculate the equivalent resistor and divide the voltage by the equivalentresistance to determine the current. **(JEE ADVANCED)**

Sol: For very high source frequency, the reactance of the inductor becomes practically infinite so that the current doesn't flow through the inductor. Thus, the inductor acts as an open circuit. For very low source frequency, the reactance of the inductor becomes practically zero, and theinductor behaves as a short circuit.

1. Calculate the current at high frequency: $I_{rms} = \frac{V_{rms}}{R} = \frac{65\text{ V}}{15\ \Omega} = 4.3\text{ A}$

2. Calculate the equivalent resistance at low frequency: $R_{eq} = \left(\frac{1}{R}+\frac{1}{R}\right)^{-1} = \frac{R}{2} = \frac{15\Omega}{2} = 7.5\Omega$ 3.

Divide the voltage by the equivalent resistance: $I_{rms} = \frac{V_{rms}}{R_{eq}} = \frac{65V}{7.5\Omega} = 8.7\text{ A}$

Illustration 13: For the circuit shown in Fig. 23.23, current in inductance is 0.8 A while its capacitance is 0.6A. What is the current drawn from the source? **(JEE ADVANCED)**

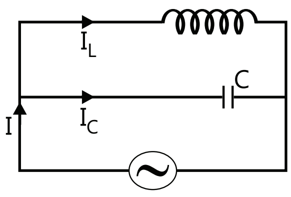


Figure 23.23

Sol: For LC circuit, total current in the circuit is $I = I_0\sin(\omega t + \phi) = I_L + I_C$. The current in the inductor lags the applied voltage by phase difference of $\frac{\pi}{2}$ while in capacitor, the current leads applied voltage by $\frac{\pi}{2}$ In parallel ac circuit, $V = V_0 \sin\omega t$ is applied across both the inductorand capacitor, current in inductor lags the applied voltage while current in capacitor leads the applied voltage.

So, $I_L = \frac{V}{X_C}\sin\left(\omega t - \frac{\pi}{2}\right) = -0.8\cos\omega t$; $I_C = \frac{V}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right) = 0.6\cos\omega t$

So, the current drawn from the source, $I = I_L + I_C = -0.2\cos\omega t$, i.e. $|I_0| = 0.2\text{ A}$

7. MORE ON POWER FACTOR

(a) The factor $\cos\phi$ present in the relation for average power of an ac circuit is called power factor.

So, $\cos\phi = \frac{P_{ac}}{E_{rms}I_{rms}} = \frac{P_{avg}}{p_V}$. Thus, ratio of average power and virtual power in the circuit is equal to power factor.

(b) Power factor is also equal to the ratio of the resistance and the impedance of the ac circuit.

Thus, $\cos\phi = \frac{R}{Z}$

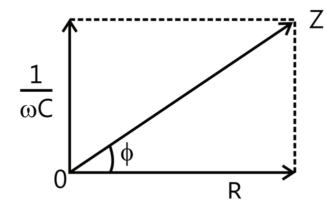


Figure 23.24

(c) Power factor depends upon the nature of the components used in the circuit.(d) If a pure resistor is connected in the ac circuit then,

$$\phi = 0, \cos\phi = 1; \qquad p_{av} = \frac{E_0 I_0}{2} = \frac{E_0^2}{2R} = \text{Erms } I_{rms}$$

Thus, the power loss is maximum and electrical energy is converted in the form of heat.

(e) If a pure inductor or capacitor are connected in the ac circuit, then

$\phi \neq 90^o$, $\cos\phi = 0$ $\therefore$ $P_{av} = 0$ (minimum)

Thus is no loss of power.

(f) If a resistor and an inductor or a capacitor are connected in an ac circuit, then $\phi \neq 0$ or $\phi \neq 90^o$. Thus ϕ is in between 0 & 90^o.

(g) If the components L, C and R are connected in series in a circuit, then

$$\tan\phi = \frac{X}{R} = \frac{(\omega L - 1/\omega C)}{R} \text{ and } \cos\phi = \frac{R}{Z} = \frac{R}{\left[R^2 (\omega L - 1/\omega C)^2\right]^{1/2}}; \text{ Power factor } \cos\phi = \frac{R}{Z}$$

(h) Power factor is a unit less quantity.

(i) If there is only an inductance coil in the circuit, there will be no loss of power, and energy will be stored in the magnetic field.

(j) If a capacitor is only connected in the circuit, there will also be no loss of power, and energy will be stored in the electrostatic field.

(k) In reality, an inductor and capacitor do have some resistance. So, there is always some loss of power.

(l) In the state of resonance, the power factor is one.

8. WATTLESS CURRENT

(a) The component of current whose contribution to the average power is nil, is called wattless current.

(b) The average wattle of power iszero because the average of the second component of instantaneous power for a full cycle will be

$$E_0 \sin\omega t (I_0 \sin\phi) \sin(\omega t - \pi/2) = 0$$

(c) The component of current associated with this part is called Wattless current. Thus the current

$(I_0 \sin\phi) \sin(\omega t - \pi/2)$ is a wattless current whose amplitude is $I_0 \sin\phi$.

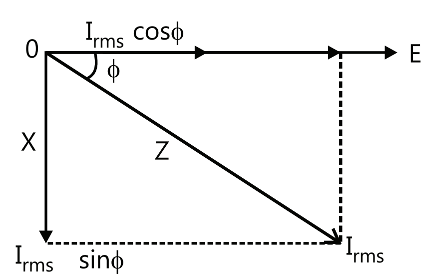


Figure 23.25

(c) If RMS value of current in the circuit is I_{rms}, then the RMS value of a wattless current will be $I_{rms} \sin\phi$. A wattless current lags or leads the e.m.f. by an angle $\pi/2$. RMS value of wattless current:

$I_{rms}\sin\phi = \frac{I_0}{\sqrt{2}}\sin\phi ; = \frac{I_0}{\sqrt{2}}\frac{X}{Z}$. Since $\sin\phi = \frac{X}{Z}$, where X is the resultant reactance of the circuit.

9. TRANSFORMERS

A transformer is a device used to convert low alternating voltage at higher current into high alternating voltage at lower current, and vice-versa. In other words, a transformer is an electrical device used to increase or decrease alternating voltage.

9.1 Types of Transformers

(a) Step-up transformers: The transformerwhich converts low alternating voltage at higher current into a high alternating voltage at lower current is called a step-up transformer.

(b) Step-down transformers: The transformer which converts high alternating voltage at lower current into a low alternating voltage athigher current is called a step-down transformer.

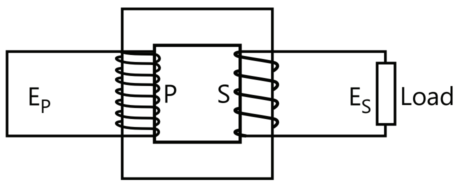


Figure 23.26

Principle: A transformer is based on the principle of mutual induction. An e.m.f. is induced in a coil, when a changing current flows through its nearby coil.

Construction: Itconsists of two separate coils of insulated wires wound on the same iron core. One of the coil connected to a.c. input is called primary (p) and the other winding giving output is called secondary (S) winding or coil.

Theory: When an alternating source of e.m.f. E_p is connected to the primary coil, an alternating current flows through it. Due to the flow of alternating current in the primary coil, an alternating magnetic flux induces an alternating e.m.f. in the secondary coil (E_s). Let N_p and N_s be the number of turns in the primary and secondary coil respectively. The iron core is capable of coupling the whole of the magnetic flux ϕ produced by the turns of the primary coil with the secondary coil.

According to Faraday's law of electromagnetic induction, the induced e.m.f in the primary coil,

$$E_P = -N_P\frac{d\phi}{dt} \quad \text{... (i)}$$

The induced e.m.f in the secondary coil. $E_S = -N_S\frac{d\phi}{dt}$ ... (ii)

Dividing (ii) by (i), we get $\frac{E_S}{E_p} = \frac{N_S}{N_P}$; Where $\frac{N_S}{N_P}$ =K the transformation ratio or ratio.

Then, $\frac{E_S}{E_p} = \frac{N_S}{N_P} = K$

K< 1 for step down transformer. In this case, $N_S < N_P$ and $E_S < E_p$ i.e. E_p, and output alternating voltage <input alternating voltage.

K>1 for step up transformer. In this case, $N_S > N_P$ and $E_S > E_p$ i.e., output alternating voltage is greater than the input alternating voltage.

For an ideal transformer (in which there in no energy losses), output power= input power .... (iii)

Let I_p and I_s be the current in the primary and secondary coil respectively.

Then output power= $E_s\,I_S$; input power= $E_p\,I_p$; from equation (iii) $E_p = E_s$ or $\frac{E_s}{E_p} = \frac{I_P}{I_S}$; In general, $E \propto \frac{1}{I}$. For same power transfer, voltage increases with the decrease in current and vice-versa. Thus, whatever is gained in voltage ratio is lost in the current ratio and viceversa. So, astep-up transformer increases the alternating voltage by decreasing the alternating current, and a step- down transformer decreases the alternating voltage by increasing the alternating current.

For a transformer, efficiency, n= $\frac{\text{ouputpower}}{\text{inputpower}} = \frac{E_s I_S}{E_P I_P}$ For an ideal transformer, efficiency, n is 100%. But in a real transformer, the efficiency varies from 90-99%. This indicates that there are some energy losses in the transformer.

10. CHOKING COIL

Let us consider a choke coil of large inductance L and low resistance R. Then, the power factor of the given circuit will be given by $\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{R}{\omega L}$ (as $R << \omega L$)

Now, as we know that $R<<\omega L$, the power factor is small and hence the power absorbed will be very small. And also, on account of its large impedance (large inductance), current passing through the coil is very small. Hence, such a coil is preferred in electrical circuits for the purpose of adjusting the current to any desired value without having a significant energy waste.

Illustration 14: An ac circuit consists of a 220 Ω resistance and a 0.7 H choke. Find the power absorbed from a 220V and 50 Hz source connected in this circuit if the resistance and choke are joined, (a) in series (b) in parallel

(JEE ADVANCED)

Sol: For a seriesLR circuit, impedance is $Z = \sqrt{R^2 + \omega^2 L^2}$ and average power dissipated in circuit is calculated as $P = V_{rms} I_{rms} \cos\phi$.

In parallel LR circuit $\frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{\omega^2 L^2}$. But for a choke, L is very large, so $\frac{1}{\omega^2 L^2} \approx 0$.

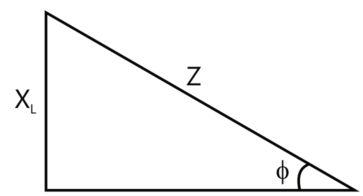


Figure 23.27

(a) in series the impedance of the circuit is:

$$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2} = \sqrt{(220)^2 + (2\times 3.14\times 50\times 0.7)^2} = 311\Omega$$

$$\therefore I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{311} = 0.707\text{A}, \ \cos\phi = \frac{R}{Z} = \frac{220}{311} = 0.707$$

and the power absorbed in the circuit, $P = V_{rms} i_{rms} \cos\phi = (220)(0.707)(0.707) = 110.08$ W

(b) When the resistance and choke are in parallel, the entire power is absorbed in resistance, as the choke (having zero resistance) absorbs no power. $\therefore P = \frac{V^2_{rms}}{R} = \frac{(220)^2}{220}$ =220W

PROBLEM-SOLVING TACTICS

(a) In this chapter, we have seen how a phasor provides a powerful tool for analysing the AC circuits.

Below are some important tips:

1. Keep in mind the phase relationship for simple circuits.

(i) For a resistor, the voltage and phase are always in phase.

(ii) For an inductor, the current lags the voltage by90^0.

(iii) For a capacitor, the current leads the voltage by 90^0.

(b) When circuit elements are connected in series, the instantaneous current is the same for allelements, and instantaneous voltages across the elements are out of phase. On the otherhand, when circuit elements are connected in parallel, the instantaneous voltage is the same for all elements, and the instantaneous currents across the elements are out of phase.

(c) For a series connection, draw a phasor diagram for the voltage. The amplitude of the voltage drop across all the circuit elements involved should be represented with phasors. In Fig. 23.28, the phasor diagram for a series RLC circuit is shown for both the inductive case $X_L > X_C$ and the capacitive case $X_L < X_C$. Below is a phasor diagram for the series RLC circuit for (a) $X_L > X_C$ (b) $X_L < X_C$.

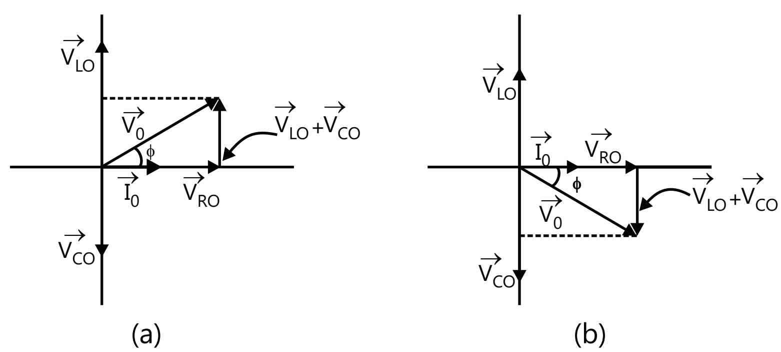


Figure 23.28: Phase angle between applied voltage and current (a) in RC circuit, (b) in LC circuit

From Fig. 23.28(a), we see that $V_{L0} > V_{C0}$ in the inductive case and $\bar{V}_0$ leads $\bar{I}_0$ by a phase ϕ. On the other hand, in the capacitive case shown in Fig. 23.28(b), $V_{C0} > V_{L0}$ and $\bar{I}_0$ leads $\bar{V}_0$ by a phase ϕ.

(d) Students should directly learn the formula for reactance, impedance, etc.to solve any problem easily.

(e) For parallel connection, draw a phasor diagram for the currents. The amplitudes of the current across all the circuit elements involved should be represented with phasors. In the following Fig. 23.29, the phasor diagram for a parallel RLC circuit is shown for both the inductive case $X_L > X_C$ and the capacitive case $X_L < X_C$.

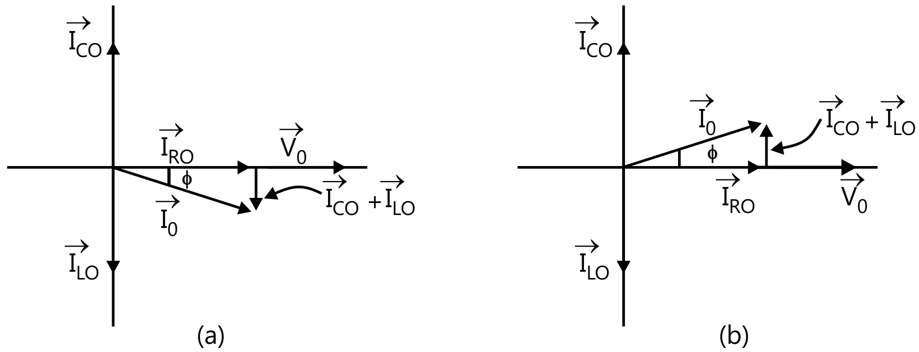


Figure 23.29

(f) Phasor diagram for the parallel RLC circuit for (a) $X_L > X_C$ And (b) $X_L < X_C$: From Fig. 23.29(a), we see that $I_{L0} > I_{C0}$ in the inductive case and $\overline{V_0}$ lead $\overline{I_0}$ by a phase ϕ.On the other hand, in the capacitive case shown in Fig. 23.29 (b), $I_{C0} > I_{L0}$ and $\overline{I_0}$ leads $\overline{V_0}$ by a phase ϕ.

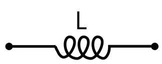
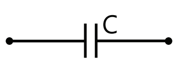
FORMULAE SHEET

(a) In an AC circuit, sinusoidal voltage source of amplitude V_0 is represented as:V(t) =V_0 sinwt.

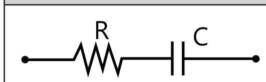
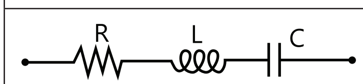
The current in the circuit has amplitude I_0 and lags the applied voltage by phase angle ϕ.

Current is represented as: I(t) = I_0 sin $(\omega t - \phi)$

(b) For a single-element circuit (a resistor, a capacitor or an inductor) connected to the AC voltage source, we summarise the results in the below table:

Circuit elements	Resistance/Reactance	Current Amplitude	Phase angel ϕ
	R	$I_{R_0} = \frac{V_0}{R}$	0
	Inductive Reactance $X_L = \omega L$	$I_{L_0} = \frac{V_0}{X_L}$	$(\pi/2)$ i.e.,current lags voltage by 90^0
	Capacitive Reactance $X_C = \frac{1}{\omega C}$	$I_{C_0} = \frac{V_0}{X_C}$	$(-\pi/2)$ i.e. current leads voltage by 90^0

(c) For a circuit having more than one circuit element connected ina series,we summarise the results in the below table:

Circuit elements	Impedance Z	Current amplitude	Phase angle ϕ
	$\sqrt{R^2 + X_L^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_L^2}}$	$0 < \phi < \left(\frac{\pi}{2}\right)$
	$\sqrt{R^2 + X_C^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_C^2}}$	$\left(-\frac{\pi}{2}\right) < \phi < 0$
	$\sqrt{R^2 + (X_L - X_C)^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}$	$\phi > 0$ if $X_L > X_C$ $\phi < 0$ if $X_L < X_C$

(d) For series LCR circuit,

(i) the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$

(ii) the current lags the voltage by phase angle $\phi = \tan^{-1}\frac{(X_L - X_C)}{R}$

(iii) the resonant frequency is $\omega_0 = \sqrt{\frac{1}{LC}}$.

At resonance, the current in the series LCR circuit is maximum, while that in parallel LCR circuit is minimum.

(e) Impedance for parallel LCR circuit, is given by

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

The phase angle by which the current lags the voltage is

$$\phi = \tan^{-1} R\left(\frac{1}{X_L} - \frac{1}{X_C}\right) = \tan^{-1} R\left(\frac{1}{\omega L} - \omega C\right)$$

(f) The RMS (root mean square) value of voltage and current in an AC circuit are given as

$V_{rms} = \frac{V_0}{\sqrt{2}}$, and $I_{rms} = \frac{I_0}{\sqrt{2}}$

(g) Average power of an AC circuit is $\langle P(t) \rangle = I_{rms} V_{rms} \cos\phi$ where $\cos\phi = \frac{R}{Z}$ is the power factor of the circuit.

(h) Quality factor Q of LCR circuit is $Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

(i) For a transformer, the ratio of secondary coil voltage to that of primary coil voltage is $\frac{V_2}{V_1} = \frac{N_2}{N_1}$

where N_1 is number of turns in primary coil, and N_2 is number of turns in secondary coil.

For the step-up transformer, $N_2 > N_1$; for step down transformer, $N_2 < N_1$.

Solved Examples

JEE Main/Boards

Example1: A resistance R, inductance L and a capacitor C all are connected ina series with an AC supply. The resistance of R is 16 Ω, and for a given frequency, the inductive reactance of L is 24 Ω,and capacitive reactance of C is 12 Ω. If the current in the circuit is 5 A, find

(a) The potential difference across R, L and C

(b) The impedance of the circuit

(c) The voltage of AC supply

(d) Phase angle

Sol: In a series LCR circuit, the impedance of circuit is $Z=\sqrt{R^2+\left(X_C-X_L\right)^2}$ where X_C and X_L are the capacitive and inductive reactances respectively. Phase difference between voltage and current is $\phi=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)$.

Potential drop across resistance is IR and that across reactance is IX.

(a) Potential difference across

(i) Resistance $V_R=I\times R=5\times 16=80\text{ V}$

(ii) Inductor $V_L=I\times(\omega L)=5\times 24=120\text{ V}$

(iii) Capacitor $V_C=I\times(1/\omega C)=5\times 12=60\text{ V}$

(b) The impedance of the circuit

$$Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}=\sqrt{(16)^2+(24-12)^2}$$

$=20\ \Omega$

(c) The voltage of AC supply is given by

$E=I\times Z=5\times 20=100\text{ V}$

(d) Phase angle between voltage & current is

$$\phi=\tan^{-1}\left[\frac{\omega L-(1/\omega C)}{R}\right]=\tan^{-1}\left[\frac{24-12}{16}\right]$$

$=\tan^{-1}(0.75)=36^0\,52'$

Example 2: A circuit draws a power of 550 W from a source of 220 V, 50Hz. The power factor of the circuit is 0.8 and the current lags in phase behind the potential difference. To make the power factor of circuit as 1.0, what capacitance will be connected in the circuit?

Sol: In series LR circuit, the current lags the applied voltage by angle ϕ and the power factor of circuit is $\cos\phi=\frac{R}{\sqrt{R^2+\omega^2L^2}}$. When capacitor is connected in series in the circuit, the impedance of the circuit is $Z=\sqrt{R^2+\left(X_C-X_L\right)^2}$ and the power factor of the circuit is $\cos\phi=\frac{R}{\sqrt{R^2+\left[\left(X_L-X_C\right)^2\right]}}$.

We want to find the value of the capacitor to make the circuit's power factor 1.0

(A) Find the value resistance and inductive reactance.

For a LR circuit, current lags behind voltage in phase.

The power in AC circuit is given as

$$P=\frac{V^2_{rms}\times\cos\phi}{Z} \qquad \text{... (i)}$$

$$\Rightarrow Z=\frac{V^2_{rms}\times\cos\phi}{P}=\frac{(220)^2\times 0.8}{550}=70.4\ \Omega$$

Power factor $\cos\phi=\frac{R}{Z}$, so we get value of resistance as

$R=Z\times\cos\phi=70.4\times 0.8=56.32\ \Omega$

Inductive Reactance is

$\omega L=\sqrt{\left(Z^2-R^2\right)}=\sqrt{(70.4)^2-(56.32)^2}$

$\omega L=42.2\ \Omega$

(B) Capacitance needed to be connected in circuit to make power factor = 1.0

When the capacitor is connected in the circuit.

Impedance

$$Z=\sqrt{R^2+\left[\left(\omega L-\frac{1}{\omega C}\right)^2\right]} \qquad \text{...(ii)}$$

and power factor is given by

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left[\left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$$

When $\cos\phi = 1,\ \omega L = \frac{1}{\omega C}$ ...(iii)

From (iii) we get $C = \frac{1}{\omega(\omega L)} = \frac{1}{2\pi f(\omega L)}$

$$= \frac{1}{(2\times 3.14\times 50)\times(42.2)} = 75\times 10^{-6}F$$

$= 75\mu F.$

Therefore to make a circuit with power factor = 1, 75 μF capacitor is to be connected in a series with resistance and inductor.

Example 3: A 750 Hz, 20 V source is connected to a resistance of 100 ohm, an inductance of 0.1803 Henry and a capacitance of 10 microfarad all in series. Calculate the time in which the resistance (thermal capacity 2J/°C) will get heated by 10°C.

Sol: For an LCR circuit, the average power dissipated as heat is $P_{av} = \frac{V_{rms}^2}{Z^2}\times R$, where Z is the impedance of the circuit.

Product of power and time equals the heat generated.

$X_L = \omega L = 2\pi f L = 2\pi\times 750\times 0.1803 = 849.2\ \Omega$ and

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi\times 750\times 10^{-5}} = 21.2\Omega$$

So $X = X_L - X_C = 849.2 - 21.2 = 828\ \Omega$

And hence $Z = \sqrt{R^2 + X^2} = \sqrt{(100)^2 + (828)^2} = 834\Omega$

But as in case of ac,

$$P_{av} = V_{rms} I_{rms}\cos\phi = V_{rms}\times\frac{V_{rms}}{Z}\times\frac{R}{Z}$$

i.e. $P_{av} = \left(\frac{V_{rms}}{Z}\right)^2\times R = \left(\frac{20}{834}\right)^2\times 100 = 0.00575W$ And

as, $U = P\times t = mc\Delta\theta = (TC)\Delta\theta$;

$$t = \frac{(TC)\times\Delta\theta}{P} = \frac{2\times 10}{0.0575} = 348\text{ sec} = 5.8\text{ min}$$

Example 4: A 100 V ac source of frequency 500 Hz is connected to a series LCR circuit with L=8.1 mH, C = 12.5 μF and R= 10Ω. Find the potential different across the resistance.

Sol: For LRC circuit, total potential difference is

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}.$$

Inductive reactance,

$$X_L = 2\pi\times 500\times 8.1\times 10^{-3} = 25.45\Omega$$

Capacitive reactance,

$$X_C = \frac{10^6}{2\pi\times 500\times 12.5} = 25.45\Omega$$

$\Rightarrow X_L = X_C$

This is the condition of resonance. This means that total potential drop occurs across the resistance only.

$$\therefore\ V = \sqrt{V_R^2 + (V_L - V_C)^2} = V_R = 100\text{ V}$$

The total potential difference across resistance is the same as the applied voltage across circuit.

Example 5: A 0.21 H inductor and a 12 Ω resistor are connected ina series to a 20 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

Sol: In series LR circuit, the current lags voltage by phase angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$. And RMS value of the current is $I_{rms} = \frac{V_{rms}}{Z}$ where Z is impedance of the circuit.

Impedance $Z = \sqrt{R^2 + (\omega L)^2}$;

$$\sqrt{12^2 + (2*3.14*50*0.21)^2}$$

$$= \sqrt{(12^2) + (65.94)^2} = 67\Omega$$

Current $I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{67} = 3.28A$

Phase angle ϕ

$\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{65.94}{12}\right)$;

$\tan = (5.495)$ = 78.69 °

Example 6: A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 22 V, 50 rad/sec ac source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also, find the power developed in the circuit if a 2500 μF condenser is connected in a series with the coil.

Sol: For dc supply, the coil is purely resistive; inductance does not come into picture. For AC voltage source,the reactance of the inductor is non-zero. When a capacitor is connected in a series in a circuit, the impedance of circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

The real power in the circuit is

$P = I^2R = \frac{V^2}{Z^2}R$.

Resistance of the coil, $R = \frac{12}{4} = 3\ \Omega$

($\because$ Reactance of inductor in dc circuit is zero)

Impedance of coil, $Z = \frac{12}{2.4} = 5\ \Omega$;

Now, $Z^2 = R^2 + \omega^2 L^2$;

or $L = \frac{\sqrt{Z^2 - R^2}}{\omega} = \frac{4}{50} = 0.08$ H

Reactance of the capacitor

$$X_C = \frac{1}{\omega L} = \frac{1}{50 \times 2500 \times 10^{-6}} = 8\ \Omega$$

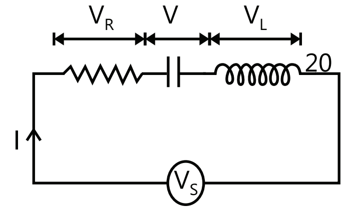
$\therefore$ When the capacitor is connected in series,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8-4)^2} = 5\ \Omega$$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{3}{5}$;

Power developed $P = I_{rms}^2 Z\cos\phi = (2.4)^2 \times 3$ =17.28 W.

Example 7: A resistance R, an inductance L, and capacitor C are connected in series with an AC supply where R=16Ω. Inductive reactance $X_L = 24\ \Omega$ and capacitive reactance $X_C = 12\ \Omega$. If the current in the circuit is 5 A, find



(a) P.D. across R,L and C

(b) Impedance of circuit

(c) Voltage of AC supply and

(d) Phase angle

Sol: For the LCR circuit, impedance is

$Z = \sqrt{R^2 + (X_C - X_L)^2}$.

The phase angle between voltage and current is given by $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$.

(a) P.D. across each component is found below

$V_R = 5 \times 16 = 80$ V $V_L = IX_L = 5 \times 24 = 120$ V, $V_C = IX_C = 5 \times 12 = 60$ V

(b) Using the formula of Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(16)^2 + (24-12)^2} = 20\ \Omega$$

(c) Voltage of AC source is

E = IZ = $5 \times 20 = 100$ V

(d) Phase angle is

$$\Phi = \tan^{-1}\frac{(X_L - X_C)}{R} = \tan^{-1}\left(\frac{24-12}{16}\right)$$

$= \tan^{-1}(0.75) = 36^0\ 87'$

Example 8: A coil of resistance 20Ω and inductance 0.5H is switched to dc 200 V supply. Calculate the rate of increase of current:

(a) At the instant of closing the switch

(b) After one time constant

(c) Find the steady state current in the circuit

Sol: The current in the LR circuit attains constant value over a long period of time. Generally, the current in the circuit is given by $i = i_0\left(1 - e^{-t/\tau}\right)$ where τ is one time constant.

(a) Current at any time is given by:

$$i = i_0\left(1 - e^{-\frac{Rt}{L}}\right) \quad \text{... (i)}$$

Differentiating above equation w.r.t. t, we get

$$\frac{dI}{dt} = \left(\frac{V}{R}.\frac{R}{L}\right)e^{-\frac{Rt}{L}} \quad \left(\therefore i_0 = \frac{V}{R}\right) \quad \text{... (ii)}$$

$$\text{At } t = 0, \ \frac{dI}{dt} = \frac{V}{L} = \frac{200}{0.5} = 400\ A/s$$

(b) Current after one time constant $\tau = \frac{L}{R}$

From equation (ii)

$$\frac{dI}{dt} = 400\ e^{-1} = 147.15\ \ A/s$$

(c) For steady state t = ∞

So from (i) we get $i(\infty) = i_0 = 400\,A$

Example 9: What is average and RMS current over half cycle if instantaneous current is given by i=4 $\sin\omega t + 3\cos\omega t$. ?

Sol: Reduce the given expression of current in standard form $i = i_0 \sin\left(\omega t + \phi\right)$, where i_0 is the maximum current in the circuit.

Given i = $4\sin\omega t + 3\cos\omega t$.

$$= 5\left(\frac{4}{5}\sin\omega t + \frac{3}{5}\cos\omega t\right) = 5\sin\left(\omega t + \alpha\right)$$

where cos $\alpha = \frac{4}{5}$ and $\sin\alpha = \frac{3}{5}$;

Comparing with $i = i_0 \sin\left(\omega t + \phi\right)$

$$i_0 = 5\ A;\ \Rightarrow \quad i_{rms} = \left(\frac{5}{\sqrt{2}}\right)A;\ i_{avg} = \left(\frac{10}{\pi}\right)A$$

JEE Advanced/Boards

Example 1: A sinusoidal voltage V(t) = (200 V) $\sin\omega t$ is applied to a series LCR circuit with L=10.0 mH, C=100 nF and R=20.0$\,\Omega$. Find the following quantities:

(a) The resonant frequency

(b) The amplitude of current at resonance

(c) The quality factor Q of the circuit

(d) The amplitude of the voltage across the inductor at the resonant frequency.

Sol: When the LCR circuit is set to resonance, the resonant frequency is $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$.

Quality factor is $Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$.

(a) Using formula of resonant frequency

The resonant frequency, for the circuit is given by

$$f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{\left(10 \times 10^{-3}H\right)\left(100 \times 10^{-9}F\right)}} = 5033Hz$$

(b) At resonance current is Maximum i.e. I_0

$$I_0 = \frac{V_0}{R} = \frac{200}{20.0\Omega} = 10.0\,A$$

(c) The quality factor Q of the circuit is given by

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi\left(5033s^{-1}\right)\left(10.0 \times 10^{-3}H\right)}{\left(20.0\Omega\right)}$$

$$= 15.8$$

(d) At resonance, the amplitude of the voltage across the inductor is

$$V_{L_0} = I_0 X_L = I_0\omega_0 L$$

$$= \left(10.0A\right)2\pi\left(5033\ s^{-1}\right)\left(10.0 \times 10^{-3}\,H\right)$$

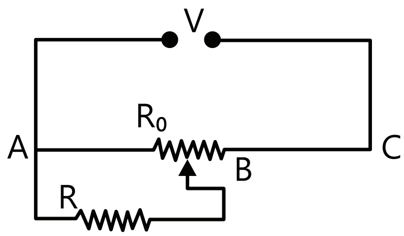
$$= 3.16 \times 10^3\,V$$

Example 2: Consider the circuit shown in figure. The sinusoidal voltage source is V (t) = $V_0\sin\omega t$. If both switches s_1 and s_2 are closed initially, find the following quantities, ignoring the transient effect and assuming that R, L, V_0 and ω are known:

(a) The current I(t)as a function of time

(b) The average power delivered to the circuit

(c) The current as a function of time, a long time after only S_1 is opened



(d) The capacitance C if both s_1 and s_2 are opened for a long time, with the current and voltage in phase.

(e) The impedance of circuit when both s_1 and s_2 are opened.

(f) The maximum energy stored in the capacitor during oscillations.

(g) The maximum energy stored in the inductor during oscillations.

(h) The phase difference between the current and the voltage if the frequency of V (t) is doubled.

(i) The frequency at which the inductive reactance X_L is equal to half the capacitive reactance X_C.

Sol: In LCR circuit explained above, when the switches are closed, the current follows path of least resistance i.e., L and C are short-circuited. Impedance of series LCR circuit is $Z=\sqrt{R^2+\left(X_C-X_L\right)^2}$. The energy stored in inductor is $U_L=\frac{1}{2}LI^2$ and that stored in capacitor is $U_C=\frac{1}{2}CV_c^{\,2}$.

(a) When both switches s_1 and s_2 are closed, the current goes through only the generator and the resistor, so the total impedance of the circuit is R and the current is $I_R(t)=\frac{V_0}{R}\sin\omega t$

(b) The average power is given by:

$$\langle P(t)\rangle=\langle I_R(t)V(t)\rangle=\frac{V^2{}_0}{R}\langle\sin^2\omega t\rangle=\frac{V^2{}_0}{2R}$$

(c) If only S_1 is opened, after a long time a current will pass through the generator, the resistor and the inductor. For this RL circuit, the impedance becomes

$$Z=\frac{1}{\sqrt{R^2+X_L^2}}=\frac{1}{\sqrt{R^2+\omega^2L^2}}$$

And the phase angle ϕ is $\phi=\tan^{-1}\left(\frac{\omega L}{R}\right)$

Thus, the current as a function of time is $I(t)=I_0\sin(\omega t-\phi)=\frac{V_0}{\sqrt{R^2+\omega^2L^2}}\sin\left(\omega t-\tan^{-1}\frac{\omega L}{R}\right)$

Note that in the limit of vanishing resistance R=0, $\phi=\pi/2$, and we recover the expected result for a purely inductive circuit.

(d) If both the switches are opened, then this would be a driven RLC circuit, with the phase angle ϕ given by tan $\phi=\frac{X_L-X_C}{R}=\frac{\omega L-\frac{1}{\omega c}}{R}$

If the current and voltage are in phase, lthen= ϕ, implying $\tan\phi$ =0. Let the corresponding angular frequency be ω_0; we then obtain. $\omega_0 L=\frac{1}{\omega_0 c}$ And the capacitance is $C=\frac{1}{\omega_0^2 L}$

(e) From (d), we see that both switches are opened; the circuit is at resonance with $X_L = X_C$. Thus, the impedance of the circuit becomes

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}=R$$

(e) The electric energy stored in the capacitor is $U_E=\frac{1}{2}CV_C^2=\frac{1}{2}C\left(IX_C\right)^2$ It attains maximum when the current in at its maximum I_0:

$$U_{C,max}=\frac{1}{2}CI_0^2X_C^2=\frac{1}{2}C\left(\frac{V_0}{R}\right)^2\frac{1}{\omega_0^2C^2}=\frac{V_0^2L}{2R^2}$$

Where we have used $\omega_0^2=1/LC$.

(g) The maximum energy stored in the inductor is given by.

$$U_{L,max}=\frac{1}{2}LI_0^2=\frac{LV_0^2}{2R^2}$$

(h) If the frequency of the voltage source is double, i.e., $\omega=2\omega_0=1/\sqrt{LC}$, then the phase becomes

$$\phi=\tan^{-1}\left(\frac{\omega L-1/\omega C}{R}\right)$$

$$=\tan^{-1}\left(\frac{\left(2/\sqrt{LC}\right)L-\left(\sqrt{LC}/2C\right)}{R}\right)$$

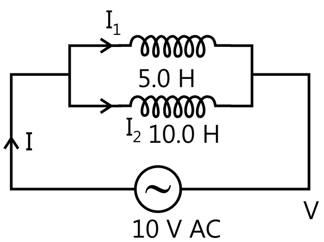
$$= \tan^{-1}\left(\frac{3}{2\pi}\sqrt{\frac{L}{C}}\right)$$

(i) If the inductive reactance in one-half the capacitive reactance,

$$X_L = \frac{1}{2}X_C \;; \qquad \Rightarrow \omega L = \frac{1}{2}\left(\frac{1}{\omega C}\right);$$

Then $\omega = \dfrac{1}{\sqrt{2LC}} = \dfrac{\omega_0}{\sqrt{2}}$

Example 3: Two inductances of 5.0 H and 10.0 H are connected in parallel circuit. Find the equivalent inductance and RMScurrent in each inductor and in mains circuit when connected to source of 10 V AC.



Sol: When two inductors are connected in parallel, the net inductance is $L = \dfrac{L_1L_2}{L_1 + L_2}$. If V is the RMS value of applied voltage, then RMScurrent through inductor is $I = \dfrac{V}{X_L}$.

Let $E = E_0 \sin\omega t$, then current drawn from supply is,

$I = I_0 \sin\left(\omega t - \dfrac{\pi}{2}\right) = \dfrac{E_0}{\omega L}\sin\left(\omega t - \dfrac{\pi}{2}\right)$ (Since current lags by $\dfrac{\pi}{2}$)

Where L is equivalent inductance of circuit.

$$\because I = I_1 + I_2 = \frac{E_0}{\omega L_1}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= \frac{E_0}{\omega L_1}\sin\left(\omega t - \frac{\pi}{2}\right) + \frac{E_0}{\omega L_2}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\Rightarrow \frac{I}{L} = \frac{I}{L_1} + \frac{I}{L_2} = \frac{1}{5} + \frac{1}{10} = \frac{15}{50} = \frac{3}{10};$$

$$\Rightarrow L = \frac{10}{3}H$$

$$I_{rms} in L_1 = \frac{V}{\omega L_1} - = \frac{10}{2\pi \times 50 \times 5} = \frac{1}{50\pi};$$

$$I_{rms} in L_2 = \frac{V}{\omega L_2} - = \frac{10}{2\pi \times 50 \times 10} = \frac{1}{100\pi};$$

$$I_{rms} in circuit = \frac{1}{50\pi} + \frac{1}{100\pi} = \frac{3}{100\pi}$$

Example 4: A series LCR circuit containing a resistance of 120 Ω has angular frequency 4 × 10^5 $rads^{-1}$. At resonance, the voltage across resistance and inductance are 60 V and 40 V respectively. Find the value of L and C. At what frequency does the current lag the voltage by 45°?

Sol: At resonance, $X_L = X_C$. The phase angle by which the current lags the voltage is $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right)$

For resistance $V_R = I_{rms}R$;

or $I_{rms} = \dfrac{V_R}{R} = \dfrac{60}{120} = 0.5$ A

For inductor $V_L = I_{rms}\omega_0 L$

$$40 = 0.5 \times 4 \times 10^5 \times L \Rightarrow L = 2 \times 10^{-4} H$$

At resonance, $X_L = X_C$ i.e. $\omega_0 L = \dfrac{1}{\omega_0 C}$

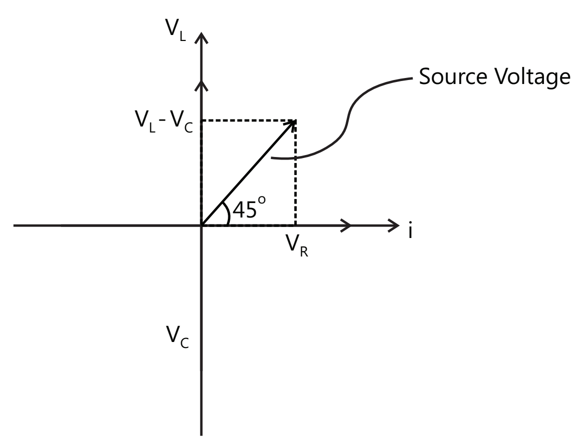
$$C = \frac{1}{\omega_0^2 L} = \frac{1}{\left(4 \times 10^5\right)^2 \times 2 \times 10^{-4}} = \frac{1}{32}\mu F$$

When the current lags behind the voltage by = 45°, using $\tan\phi = \dfrac{X_L - X_C}{R}$, gives

$$1 = \frac{\omega L - \dfrac{1}{\omega C}}{R} \Rightarrow R = \omega L - \frac{1}{\omega C} = \omega L - \left(\frac{\omega_o^2 L}{\omega}\right)$$

$$\therefore \omega R = \omega^2 L - \omega_o^2 L$$

$$120\,\omega = 2 \times 10^{-4}\left(\omega^2 - (4 \times 10^5)^2\right)$$



On solving the above equation,we get

$\omega = 8\times10^5$ or $\omega = -2\times10^5$

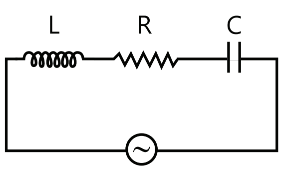
$\because$ Frequency can't be negative

$\therefore$ Ignoring negative root we have $\omega = 8\times10^5$ Hz

Example 5: An inductor of 20mH, a capacitor 100 μF and a resistor 50Ω are connected in a series across a source of e.m.f. V=10 sin (314t). Find the energy dissipated in the circuit in 20 minutes. If resistance is removed from the circuit and the value of inductance is doubled, then find the variation of current with time in the new circuit.

Sol: For the LCR circuit, the energy dissipated over a long time is $U = \left(V_{rms}I_{rms}\cos\phi\right)t$. When resistance is removed,the circuit becomes LC circuit, the impedance and hence current changes.

The circuit is as shown in figure. One time cycle T= $\frac{2\pi}{\omega} = \frac{2\pi}{314} = 0.02s$. So, we have to calculate the average energy at time t>>T.



Energy dissipated in time t

$$U = \left(V_{rms}I_{rms}\cos\phi\right)t = \left(\frac{I_0}{\sqrt{2}}\times\frac{V_0}{\sqrt{2}}\times\frac{R}{Z}\right)t$$

$$\therefore U = \frac{V_0^2R}{2Z^2}t \quad \left(\because I_0 = \frac{V_0}{Z}\right)$$

$$\therefore\ U = \frac{10^2\times50\times20\times60}{2\times3153.7} = 864.2\,J$$

When resistance is removed,and inductance is doubled, then $\cos\phi = 0 \Rightarrow \phi = \pi/2$

Value of impedance is

$$Z' = \frac{1}{\omega C} - \omega L' = \frac{1}{314\times10^{-4}} - 314\times40\times10^{-3}\Omega$$

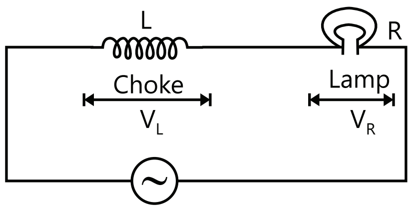
$=19.3\,\Omega$

And the current in the circuit is found to be

$$I = \frac{V_0}{Z}\sin\left(\omega t+\phi\right) = \frac{10}{19.3}\sin\left(314t+\pi/2\right)$$

$= 0.52\ \cos\ 314t$

Example 6: A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The lamp has an effective resistance of $5\,\Omega$ when running at 10 A (RMS). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (dc), what additional resistance is required? Compare the power losses in both cases.



$V = V_0 \sin\omega t$

Sol: Choke coil has large inductance and low internal resistance, sothere is no power loss in the choke coil. Hence, when alamp of some resistance is connected in series with the coil, the net RMS voltage in circuit is $\left(V_{rms}\right)^2 = \left(V_{rms}\right)_R^2 + \left(V_{rms}\right)_L^2$. When the same lamp is operated on dc, additional resistance in a series is required to limit the current in the lamp to 10 A.

Voltage drop across the lamp is

$\left(V_{rms}\right)_R = \left(I_{rms}\right)\left(R\right) = 10\times5 = 50$ V Voltage drop across choke coil is

$$\therefore \left(V_{rms}\right)_L = \sqrt{\left(V_{rms}\right)^2 - \left(V_{rms}\right)_R^2}$$

$$= \sqrt{\left(160^2\right) - \left(50\right)^2} = 152\ V$$

As $\left(V_{rms}\right)_L = \left(i_{rms}\right)X_L = \left(i_{rms}\right)\left(2\pi fL\right)$;

$$\therefore L = \frac{\left(V_{rms}\right)_L}{\left(2\pi f\right)\left(i_{rms}\right)}$$

Substituting the values

$$L = \frac{152}{\left(2\pi\right)\left(50\right)\left(10\right)} = 4.84\times10^{-2}H$$

When lamp is operated on DC supply with a resistance R′ in series, then voltage drop across the circuit is

$V = i\left(R+R'\right)$ or 160=10(5+R′);

$\therefore R' = 11\Omega$

Choke coil has no resistance.Therefore,for ac circuit power loss in choke coil is zero, while in case of dc, the loss due to additional resistance R' is

$P = i^2R' = (10)^2(11) = 1100$ W

Example 7: A series AC circuit contains an inductor (20 mH), a capacitor (100 μF) and resistance (50 Ω). AC source of 12 V (RMS), 50 Hz is applied across the circuit. Find the energy dissipated in the circuit in 1000 s.

Sol: The average power dissipated in series LCR circuit is $P_{av} = V_{rms}I_{rms}\cos\phi$. For time $t \gg T$, the energy dissipated is $U = P_{av}t$.

The time period of the source is,

T=1/f=20 ms.

and $t = 1000\text{ s} \gg T$

The average power dissipated is

$$P_{av} = V_{rms}\frac{V_{rms}}{Z}\frac{R}{Z} = \frac{RV_{rms}^2}{Z^2} = \frac{(50\Omega)(12V)^2}{Z^2}$$

$$P_{av} = \frac{7200}{Z^2} \quad \text{... (i)}$$

The capacitive reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\times50\times100\times10^{-6}}\Omega = \frac{100}{\pi}\Omega$$

The inductive reactance

$$X_L = \omega L = 2\pi\times50\times20\times10^{-3}\ \Omega = 2\pi\ \Omega.$$

The net reactance is $X = \frac{1}{\omega C} - \omega L$

$$= \frac{100}{\pi}\ \Omega - 2\pi\ \Omega = 25.5\ \Omega$$

Thus, $Z^2 = (50\ \Omega)^2 + (25.5\ \Omega)^2 = 3150\ \Omega^2$

From (i), average power

$$P_{av} = \frac{7200}{3150} = 2.3\text{ W}$$

∴ The energy dissipated in t = 1000s is

$$U = P_{av}\times1000\text{ s} = 2.3\times10^3\text{ J}$$

JEE Main/Boards

Exercise 1

Q.1 The resistance of coil for direct current (dc)is $10\,\Omega$. When alternating current (ac) is sent through it; will its resistance increase, decrease or remain the same?

Q.2 Prove that an ideal inductor does not dissipate power in an A.C. circuit.

Q.3 What is impedance? Derive a relation for it in an A.C. Series LCR circuit. Show it by a vector.

Q.4 An A.C. supply $E = E_0\sin\omega t$ is connected to a series combination of L, C and R. Calculate the impedance of the circuit and discuss the phase relation between voltage and current.

Q.5 What is the relation between peak value and root mean square value of alternating e.m.f?

Q.6 Is there any device which may control the direct current without dissipation of energy?

Q.7 What is the phase relationship between current and voltage in an inductor?

Q.8 Find the reactance of a capacitance C at f Hz.

Q.9 Prove that an ideal capacitor connected to an A.C. source does not dissipate power.

Q.10 State the principle of an A.C. generator.

Q.11 How are the energy losses reduced in a transformer?

Q.12 Discusses the principle, working and use of a transformer for long distance transmission of electrical energy.

Q.13(a) What will be instantaneous voltage for A.C. supply of 220 V and 50 Hz?

(b) In an A.C. circuit, the rms voltage is $100\sqrt{2V}$, find the peak value of voltage and its mean value during a positive half cycle.

Q.14 What should be the frequency of alternating 200 V so as to pass a maximum current of 0.9 A through an inductance of 1 H?

Q.15 An alternating e.m.f of 100 V (r.m.s), 50 Hz is applied across a capacitor of 10 μF and a resistor of 100 W in series.Calculate (a) The reactance of the capacitor; (b) The current flowing (c) the average power supplied.

Q.16 The effective value of current in a 50 cycle A.C. circuit 5.0 A. What is the value of current 1/300s after it is zero?

Q.17 A pure capacitor is connected to an ac source of 220 V, 50 Hz, what will be the phase difference between the current and applied emf in the circuit?

Q.18 A 100 Ω resistance is connected to a 220 V, 50 Hz A.C. supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Q.19 A pure inductance of 1 H is connected across a 110V, 70 Hz source, find (a) reactance (b) current (c) peak value of current.

Q.20 A series circuit contains a resistor of 10Ω, a capacitor, an ammeter of negligible resistance. It is connected to a source 220V-50 Hz, if the reading of an ammeter is 2.0 A, calculate the reactance of the capacitor.

Q.21 A series LCR circuit connected to a variable frequency 230V source and $L=5.0\text{ H}, C=80\,\mu\text{F}, R=40\,\Omega$.

(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of the current at the resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Q.22 A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to 230 V, 50Hz supply. The resistance of the circuit is negligible. (a) Obtain the current amplitude and rms values. (b) Obtain the rms value of potential drops across each element, (c) What is the average transferred to the inductor? (d) What is the average power transferred to the capacitor? (e) What is the total average power absorbed by the circuit? [‘average’ ‘implies’ averaged over one cycle;].

Q.23 Answer the following questions: (a) in any A.C. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltage across the series element of the circuit? Is the same true for rms voltage? (b) A capacitor is used in the primary circuit of an inductor coil. (c) A supplied voltage signal consists of a super position of a D.C voltage and A.C. voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the D.C. signal will appear across C and the A.C. signal across L. (c) An applied voltage signal consists of a superposition of a D.C. voltage and an A.C. Voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the D.C. signal will appear across C and the A.C. signal across L. (e) Why is choke coil needed in the use of florescent tubes with A.C. mains? Why can we not use an ordinary resistor instead of the choke coil?

Q.24 An inductance of negligible resistance, whose reactance is 22 Ω at 200 Hz is connected to a 220 V, 50 hertz power line, what is the value of the inductance and reactance?

Q.25 An electric lamp market 220 V D.C. consumes a current of 10 A. It is connected to 250 V-50 Hz A.C. main through a choke. Calculate the inductance of the choke required.

Q.26 A 2 μF capacitor, 100 Ω resistor and 8H inductor are connected in series with an A.C. source. What should be the frequency of this A.C source, for which the current drawn in the circuit is maximum? If the peak value of e.m.f of the source is 200 V, find for maximum current, (i) The inductive and capacitive reactance of the circuit; (ii) Total impedance of the circuit; (iii) Peak value of current in the circuit ; (iv) The phase relation between voltages across inductor and resistor; (v) The phase difference between voltage across inductor and capacitor.

Q.27 A step-down transformer converts a voltage of 2200 V into 220 V in the transmission line. Number of turns in primary coil is 5000. Efficiency of the transformer is 90% and its output power is 8 kW. Calculate (i) Number of turns in the secondary coil (ii) input power.

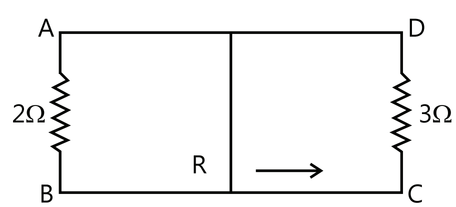
Q.28 What will be the effect on inductive reactance X_L and capacitive X_C, if frequency of ac source is increased?

Q.29 The frequency of ac is doubled, what happens to (i) Inductive reactance (ii) Capacitive reactance?

Exersice 2

Single Correct Choice Type

Q.1 A rectangular loop with a sliding connector of length 10 cm is situated in uniform magnetic field perpendicular to plane of loop. The magnetic induction is 0.1 tesla and resistance of connecter (R) is 1 Ω. The sides AB and CD have resistance 2 Ω and 3 Ωrespectively. Find the current in the connecter during its motion with constant velocity of 1 meter/sec.



(A) $\frac{1}{110}$ A (B) $\frac{1}{220}$ A (C) $\frac{1}{55}$ A (D) $\frac{1}{440}$ A

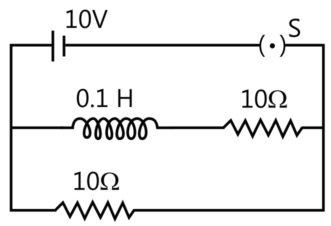
Q.2 For L-R circuit, the time constant is equal to (A) Twice the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

(B) Ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

(C) Half the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

(D) Square of the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

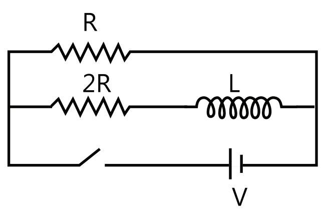
Q.3 In the adjoining circuit, initially the switch S is open. The switch's' is closed at t=0. The difference between and minimum current that can flow in the circuit is



(A) 2 Amp (B) 3 Amp

(C) 1 Amp (D) Nothing can be concluded

Q.4 The ratio of time constant in build-up and decay in the circuit shown in figure is

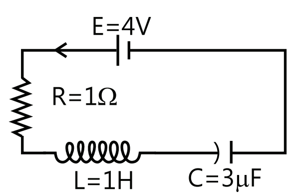


(A) 1:1 (B) 3:2 (C) 2:3 (D) 1:3

Q.5 A current of 2A is increased at a rate of 4 A/s through a coil of inductance 2H. The energy stored in the inductor per unit time is

(A) 2 J/s (B) 1 J/s (C) 16 J/s (D) 4 J/s

Q.6 The current in the given circuit is increased with a rate a=4 A/s. The charge on the capacitor at an instant when the current in the circuit is 2 amp will be:



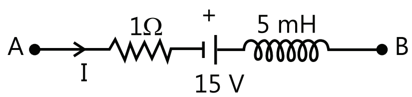
(A) 4 μC (B) 5 μC

(C) 6 μC (D) None of these

Q.7 A coil of inductance 5H is joined to a cell of emf 6 V through a resistance 10Ω at time t=0. The emf across the coil at time t= $\sqrt{2}$ s is:

(A) 3V (B) 1.5V (C) 0.75V (D) 4.5V

Q.8 The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5A and it is decreasing at a rate of $10^3 As^{-1}$ then V_B-V_A equals.

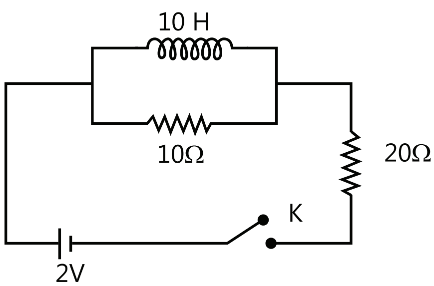


(A) 20 V (B) 15 V (C) 10 (D) 5 V

Q.9 In the previous question, if I is reversed in direction, then V_B-V_A equals

(A) 5 V (B) 10 V (C) 15 V (D) 20 V

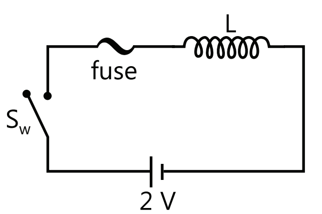
Q.10 Two resistors of 10Ω and 20Ω and an ideal inductor of 10 H are connected to a 2 V battery as shown in figure. The key K is inserted at time t=0. The initial (t=0) and final (t>=00) current through battery are



(A) $\frac{1}{15}$A, $\frac{1}{10}$A (B) $\frac{1}{10}$A, $\frac{1}{15}$A

(C) $\frac{2}{15}$A, $\frac{1}{10}$A (D) $\frac{1}{15}$A, $\frac{2}{25}$A

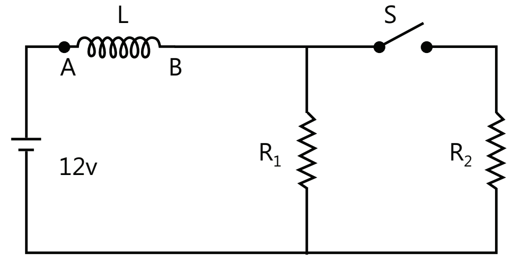
Q.11 In the circuit shown, the cell is ideal. The coil has an inductance of 4H and zero resistance. F is a fuse zero resistance and will blow when the current through it reaches 5A. The switch is closed at t=0. The fuse will blow



(A) Just after t=0 (B) After 2

(C) After 5s (D) After 10s

Q.12 The circuit shown has been operating for a long time. The instant after the switch in the circuit labeled S is opened, what is the voltage across the inductor V_L and which labeled point (A or B) of the inductor is at a higher potential? Take R_1=4.0Ω, R_2=8.0Ω and L= 2.5 H.



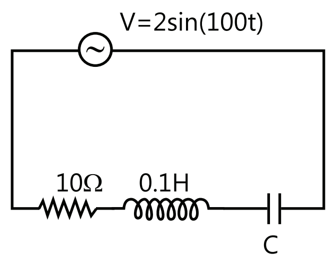
(A) V_L=12 V; point A is at the higher potential

(B) V_L=12 V; point B is at the higher potential

(C) V_L=6 V; point A is at the higher potential

(D) V_L=6 V; point B is at the higher potential

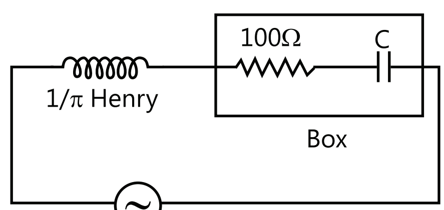
Q.13 The power factor of the circuit shown in figure is $1/\sqrt{2}$. The capacitance of the circuit is equal to



(A) 400 μF (B) 300 μF

(C) 500 μF (D) 200 μF

Q.14 In the circuit, as shown in the figure, if the value of R.M.S current is 2.2 ampere, the power factor of the box is



V_{rms}=220 volt, ω-100 π s^{-1}

(A) $\frac{1}{\sqrt{2}}$ (B) 1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

Q.15 When 100 V DC is applied across a solenoid, a current of 1 A flows in it. When 100 V AC is applied across the same coil, the current drops to 0.5 A. If the frequency of the AC source is 50 Hz, the impedance and inductance of the solenoid are:

(A) 100Ω, 0.93 H (B) 200Ω, 1.0 H

(C) 10Ω, 0.86 H (D) 200Ω, 0.55 H

Q.16 An ac current is given by $I = I_0 + I_1 \sin \omega t$ then its rms value will be

(A) $\sqrt{I_0^2 + 0.5I_1^2}$ (B) $\sqrt{I_0^2 + 0.5I_0^2}$

(C) 0 (D) $I_0/\sqrt{2}$

Q.17 The phase difference between current and voltage in an AC circuit is $\pi/4$ radians. If the frequency of AC is 50 Hz, then the phase difference is equivalent to the time difference:

(A) 0.78 s (B) 15.7 ms

(C) 0.25 s (D) 2.5 ms

Q.18 Power factor an L-R series circuit is 0.6 and that of a C-R series circuit is 0.5. If the element (L, C, and R) of the two circuits are joined in series, the power factor of this circuit is found to be 1. The ratio of the resistance in the L-R circuit to the resistance in the C-R circuit is

(A) 6/5 (B) 5/6 (C) $\frac{4}{3\sqrt{3}}$ (D) $\frac{3\sqrt{3}}{4}$

Q.19 The effective value of current i=2 sin100π t+2 sin (100π t+30^0) is:

(A) $\sqrt{2}$A (B) $2\sqrt{2+\sqrt{3}}$

(C) 4 (D) None of these

Q.20 In a series R-L-C circuit, the frequency of the source is half of the resonance frequency. The nature of the circuit will be

(A) Capacitive

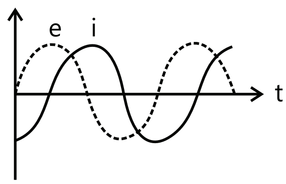
(B) Inductive

(C) Purely resistive

(D) Data insufficient

Previous Years' Questions

Q.1 When an AC source of emf e=E_0sin (100 t) is connected across a circuit, the phase difference between the emf and the current i in the circuit is observed to be $\frac{\pi}{4}$ ahead, as shown in the figure. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements: ***(2003)***



(A) R=1 KΩ,C=10μF (B) R=1 KΩ,C=1μF

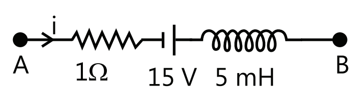
(C) R=1 KΩ,L=10H (D) R=1 KΩ,L=1H

Q.2 The current I_4 through the resistor and voltage v_C across the capacitor are compared in the two cases. Which of the following is/are true? ***(2011)***

(a) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$

(C) $I_C^A > I_C^B$ (D) $I_C^A < I_C^B$

Q.3 The network shown in Figure is part of a complete circuit. If at a certain instant the current (I) is 5A and is decreasing at a rate of 10^3 A/s then $V_B - V_A =$ ....V ***(1997)***



Q.4 An arc lamp requires a direct current of 10 A and 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to: ***(2016)***

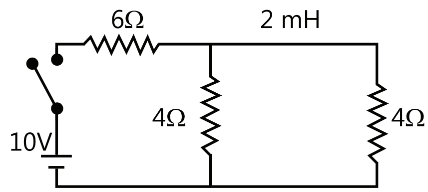
(A) 0.08 H (B) 0.044 H

(C) 0.065 H (D) 80 H

JEE Advanced/Boards

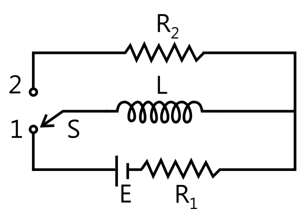
Exercise 1

Q.1 In the given circuit, find the ratio of i_1 to i_2 where i_1 is the initial current (at t=0), $i_2$is steady state (at t=∞) current through the battery.

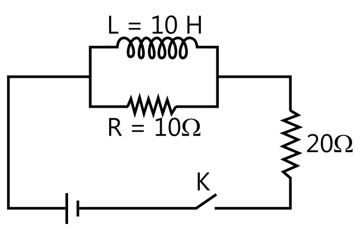


Q.2 Find the dimension of the quantity $\frac{L}{RCV}$, where symbols have usual meaning.

Q.3 In the circuit shown, initially the switch is in position 1 for a long time. Then the switch is shifted to position 2 for long time. Find the total heat produced in R_2.

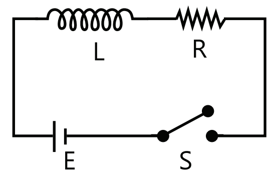


Q.4 Two resisters of 10Ω and 20 Ω and an ideal inductor of 10 H are connected to a 2V battery as shown in figure. The key K is shorted at time t=0. Find the initial (t=0) and final (t->∞) current through battery.



Q.5 An emf of 15 V is applied in a circuit containing 5 H inductance and 10Ω resistance. Find the ratio of the current at time t=∞ and t=1 second.

Q.6 In the circuit in shown in figure, switch S is closed at time t=0. Find the charge which passes through the battery in one time constant.



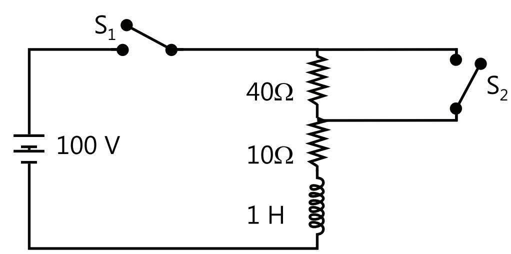
Q.7 Two coils, 1 & 2, have a mutual inductance = M and resistance R each. A current flows in coils 1, which varies with time as: $I_1 = kt_2$, where k is constant 't' is time. Find the total charge that has flown through coil 2, between t = 0 and t = T.

Q.8 Find the value of an inductance which should be connected in series with a capacitor of 5 F, resistance of 10 Ω and an ac source of 50 Hz so that the power factor of the circuit is unity.

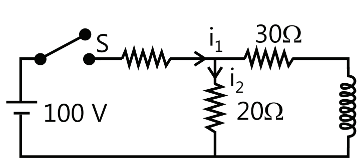
Q.9 In an L-R series A.C circuit the potential difference across an inductance and resistance joined in series are respectively 12 V and 16 V. Find the total potential difference across the circuit.

Q.10 A 50W, 100V lamp is to be connected to an ac mains of 200V, 50Hz. What capacitance is essential to be put in series with lamp.

Q.11 In the circuit shown in the figure, the switched S_1 and S_2 are closed at time t=0. After time t = (0.1) In 2sec, switch S_2 is opened. Find the current in the circuit at time t = (0.2) In 2sec.



Q.12 Find the value of $i_1$and i_2



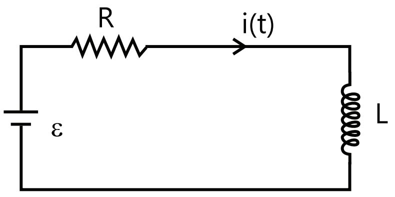
(i) Immediately after the switch S is closed.

(ii) Long time later, with S closed.

(iii) Immediately after switch S is open

(iv) Long time after S is opened.

Q.13 Suppose the emf of the battery in the circuit shown varies with time t so the current is given by i(t) = 3+5t, where i is in amperes & t is in seconds. Take R=4Ω, L=6H & find an expression for the battery emf as a function of time.



Q.14 An LCR series circuit with 100 Ω resistance is connected to an ac source of 200 V and angular frequency 300rad/s. When only the capacitance is removed, the current lags behind the voltage by 60^0. When only the inductance is removed, the current leads the voltage by 60^0. Calculate the current and the power dissipated in the LCR circuit.

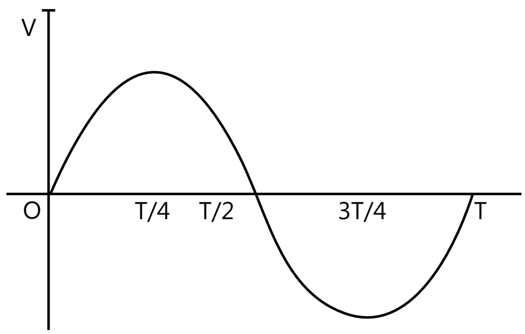
Q.15 A box P and a coil Q are connected is series with an ac source of variable frequency. The emf source at 10V. Box P contains a capacitance of 1μF in series with a resistance of 32Ω. Coil Q has a self-inductance 4.9 mH and a resistance of 68Ω series. The frequency adjusted so that the maximum current flows in P and Q. Find the impedance of P and Q atthis frequency. Also find the voltage across P and Q respectively.

Q.16 A series LCR circuit containing a resister of 120 Ω has angularresonance frequency 4×10^5 rad s^{-1}. At resonance, the voltage across resistance and inductance are 60V and 40V respectively. Find the values of L and C. At what frequency current in the circuit lags the voltage by 45^o?

Q.17 In an LR series circuit, a sinusoidal voltage $V=V_0 \sin\omega t$ is applied. It is given that

$$L = 35\text{mH}, R = 11\Omega, V_{rms} = 220\text{V}, \quad \frac{\omega}{2\pi} = 50\text{Hz}$$

And $\pi = 22/7$.

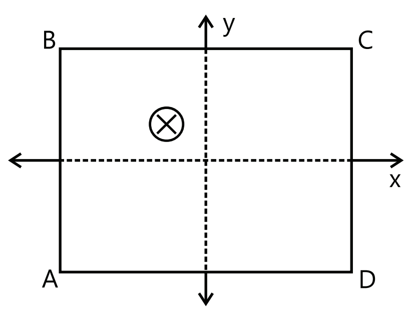


Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph.

Exercise 2

Single Correct Choice Type

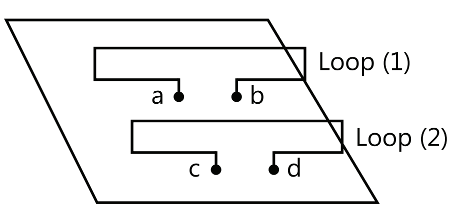
Q.1 A square coil ABCD is placed in x-y plane with its centre at origin. A long straight wire, passing through origin, carries a current in negative Z-direction. Current in this wire increases with time. The induced current in the coil is



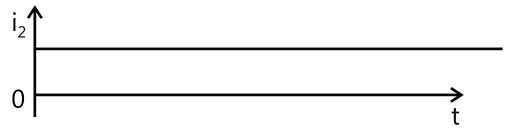
(A) Clock wise (B) Anti clockwise

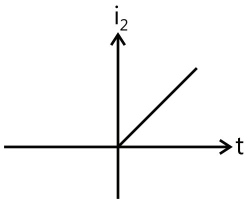
(C) Zero (D) Alternating

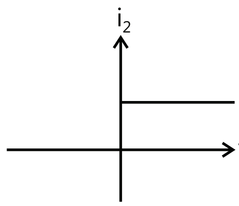
Q.2 An electric current i_1 can flow in either direction through loop (1) and induced current i_2 in loop (2). Positive i_1 is when current is from 'a' to 'b' in loop (1) and positive i_2 is when the current is from 'c' to 'd' in loop

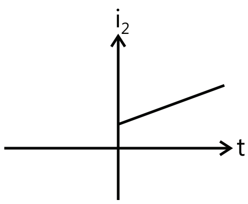


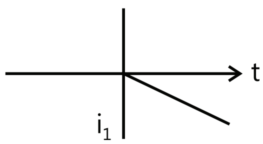
(2) In an experiment, the graph of i_2 against time 't' is as shown below by Figure which one (s) of the following graphs could have caused i_2 to behave as give above.



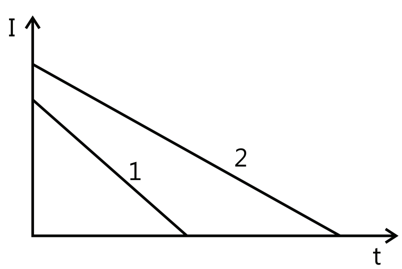
(A) 

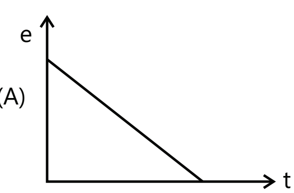
(B) 

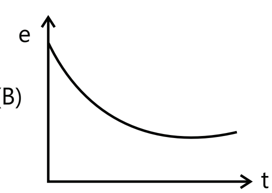
(C) 

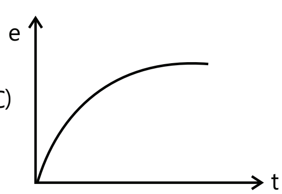
(D) 

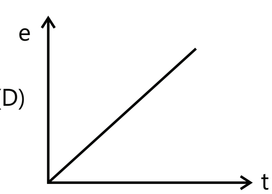
Q.3 In an L-R circuit connected to a battery of constant e.m.f. E, switch S is closed at time t = 0. If e denotes the magnitude of induced e.m.f. across inductor and i the current in the circuit at anytime t. Then which of the following graphs shows the variation of e with i?



(A) 

(B) 

(C) 

(D) 

Q.4 Two identical inductances carry currents that vary with time according to linear laws (see in figure). In which of the inductances is the self-inductance emf greater?

(A) 1 (B) 2

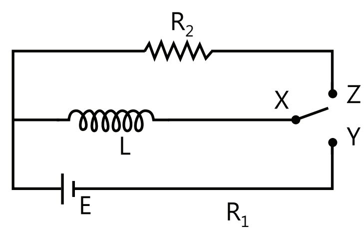
(C) Same (D) Data is insufficient to decide

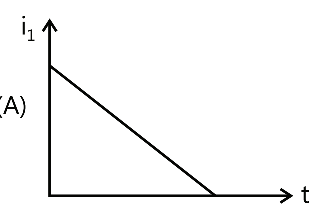
Q.5 L, C and R represents physical quantities inductance, capacitance and resistance. The combination which has the dimensions of frequency?

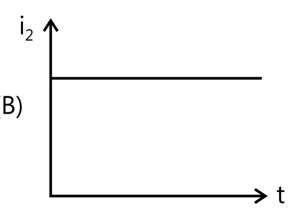
(A) $\frac{1}{RC}$ and $\frac{R}{L}$ (B) $\frac{1}{\sqrt{RC}}$ and $\sqrt{\frac{R}{L}}$

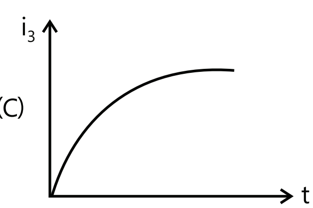
(C) $\frac{1}{\sqrt{LC}}$ (D) $\frac{C}{L}$

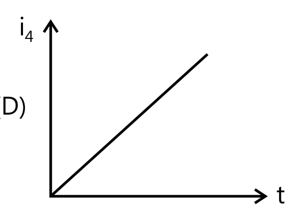
Q.6 In the circuit shown, X is joined to Y for a long time, and then X is joined to Z, the total heat produced in R_2 is:



(A) 

(B) 

(C) 

(D) 

(A) $\frac{LE^2}{2R_1^2}$ (B) $\frac{LE^2}{2R_2^2}$ (C) $\frac{LE^2}{2R_1R_2}$ (D) $\frac{LE^2R_2}{2R_1^2}$

Q.7 An induction coil stores 32 joules of magnetic energy and dissipates energy as heat at the rate of 320 watt when a current of 4 amperes is passed through it. Find the time constant of the circuit when the coil is joined across a battery.

(A) 0.2s (B) 0.1s (C) 0.3s (D) 0.4s

Q.8 In an L-R decay circuit, the initial current at t=0 is 1. The total charge that has inductor has reduced to one-fourth of its initial value is

(A) LI/R (B) $LI/2R$ (C) $LI/\sqrt{2}R$ (D) None

Q.9 An inductor coil stores U energy when i current is passed through it and dissipates energy at the rate of P. The time constant of the circuit, when the coil is connected across a battery of zero internal resistance is

(A) $\frac{4U}{P}$ (B) $\frac{U}{P}$ (C) $\frac{2U}{P}$ (D) $\frac{2P}{U}$

Q.10 When a resistance R is connected in series with an element A, the electric current is found to be lagging behind the voltage by angle θ_1. When the same resistance is connected in series with element B, current leads voltage by θ_2. When R, A, B, are connected in series, the current now leads voltage by θ. Assume same AC source in used in all cases. Then:

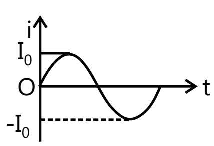
(A) $\theta = \theta_1 - \theta_2$ (B) $\tan\theta = \tan\theta_2 - \tan\theta_1$

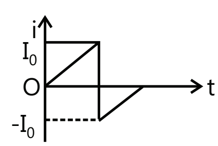
(C) $\theta = \frac{\theta_1 + \theta_2}{2}$ (D) None of these

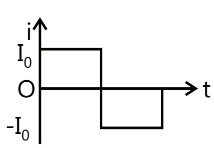
Q.11 The power in ac circuit is given by $P = E_{rms} I_{rms} \cos\phi$. The value of $\cos\phi$ in series LCR circuit at resonance is:

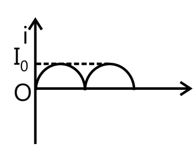
(A) Zero (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

Q.12 If I_1, I_2, I_3 and I_4 are the respective r.m.s values of the time varying current as shown in figure the four cases I.II,III and IV in. Then identify the correct relations.

(I)


(II)


(III)


(IV)


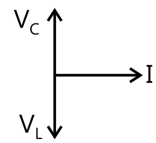
(A) $I_1 = I_2 = I_3 = I_4$ (B) $I_3 > I_1 = I_2 > I_4$

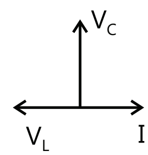
(C) $I_3 > I_4 > I_2 = I_1$ (D) $I_3 > I_2 > I_1 > I_4$

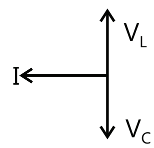
Q.13 In series LR circuit $X_L = 3R$. Now a capacitor with $X_C = R$ is added in series. Ratio of new to old power factor is

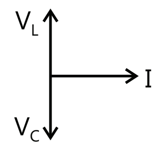
(A) 1 (B) 2 (C) $\frac{1}{\sqrt{2i}}$ (D) $\sqrt{2}$

Q.14 The current I, potential difference V_L across the inductor and potential difference V_C across the capacitor in circuit as shown in the figure are best represented vectorially as.

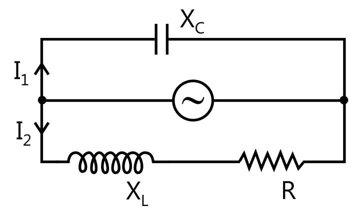
(A)


(B)


(C)


(D)


Q.15 In the shown AC circuit in figure, phase difference between current I_1 and I_2 is



(A) $\frac{\pi}{2} - \tan^{-1}\frac{X_L}{R}$ (B) $\tan^{-1}\frac{X_L - X_C}{R}$

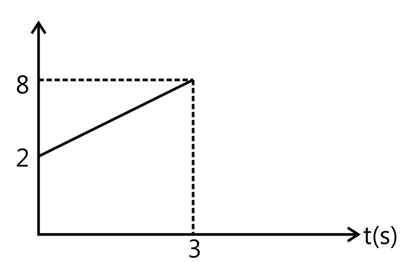
(C) $\frac{\pi}{2} + \tan^{-1}\frac{X_L}{R}$ (D) $\tan^{-1}\frac{X_L - X_C}{R} + \frac{\pi}{2}$

Multiple Correct Choice Type

Q.16 A circuit element is placed in a closed box. At time t=0, constant current generator supplying a current of 1 amp, is connected across the box. Potential difference across the box varies according to graph shown in Figure. The element in the box is:

(A) Resistance of 2 Ω (B) Battery of emf 6V

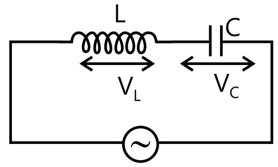
(C) Inductance of 2H (D) Capacitance of 0.5F



Q.17 For L-R circuit, the time constant is equal to

(A) Twice the ratio of the energy stored in the magnetic field to the rate of the dissipation of energy in the resistance

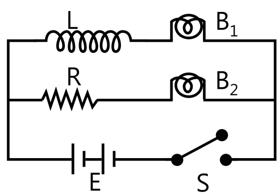
(B) The ratio of the energy stored in the magnetic field to the rate of the dissipation of energy in the resistance.



(C) Half of the ratio of the energy stored in the magnetic field to the rate of the dissipation of energy in the resistance.

(D) Square of the ratio of the energy stored in the magnetic field to the rate of the dissipation of energy in the resistance.

Q.18 An inductor L, a resistor R and two identical bulbs B_1 and B_2 are connected to a battery through a switch S as shown in the figure. The resistance of the coil having inductance L is also R. Which of the following statement gives the correct description of the happening when the switch S is closed?



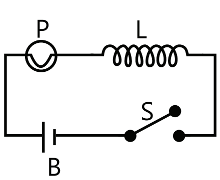
(A) The bulb B_2 lights up earlier then B_1 and finally both the bulbs shine equally bright.

(B) B_1 lights up earlier and finally both the bulbs acquire brightness.

(C) B_2 lights up earlier and finally B_1 shines brighter than B_2.

(D) B_1 and B_2 lights up together with equal brightness all the time.

Q.19 In figure, a lamp P is in series with an iron-core inductor L. When the switch S is closed, the brightness of the lamp rises relatively slowly to its full brightness than it would to without the inductor. This is due to



(A) The low resistance of P

(B) The induced-emf in L

(C) The low resistance of L

(D) The high voltage of the battery B

Q.20 Two different coils have a self-inductanceof 8mH and 2mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same instant of time. The power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are I_1 V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are I_2, V_2 and W_2 respectively . Then:

(A) $\frac{I_1}{I_2}=\frac{1}{3}$ (B) $\frac{I_1}{I_2}=4$

(C) $\frac{W_1}{W_2}=4$ (D) $\frac{V_2}{V_1}=\frac{1}{4}$

Q.21 The symbol L, C, R represents inductance, capacitance and resistance respectively. Dimension of frequency is given by the combination.

(A) 1/RC (B) R/L (C) $\frac{1}{\sqrt{LC}}$ (D) C/L

Q.22 An LR circuit with a battery is connected at t=0. Which of the following quantities is not zero just after the circuit is closed?

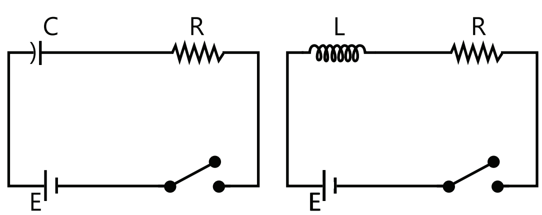
(A) Current in the circuit

(B) Magnetic field

(C) Power delivered by the battery

(D) Emf induced in the inductor

Q.23 The switches in figure (a) and (b) are closed at t=0



(a) (b)

(A) The charge on C just after t=0 is EC.

(B) The charge on C long after t=0 is EC.

(C) The charge on L just after t=0 is E/R.

(D) The charge on L long after t=0 is EC.

Q.24 Two coils A and B have coefficient of mutual inductance M=2H. The Magnetic flux passing through coil A changes by 4 Weber in 10 seconds due to the change in current in B. Then

(A) Change in current in B in this time interval is 0.5 A

(B) The change in current in B in this time interval is 2A

(C) The change in current in B in this time interval is 8A

(D) A change in current of 1A in coil A will produce a change in flux passing through B by 4 Weber.

Assertion Reasoning Type

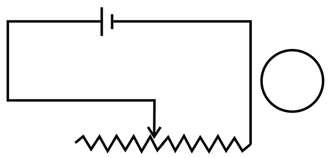
(A) Statement-I is true, statement-II is true and statement-II is correct explaining for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is not correct explaining for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.25 Statement-I: when resistance of rheostat is increased, clockwisecurrent is induced in the ring. **Statement-II:** Magnetic flux through the ring is out of the phase and decreasing.



Q.26 Statement-I: Peak voltage across the resistance can be greater than the peak voltage of the source in a series LCR circuit.

Statement-II: Peak voltage across the inductor can be greater than the peak voltage of the source in a series LCR circuit.

Q.27 Statement-I: when a circuit having large inductance is switched off, sparking occurs at the switch.

Statement-II: Emf induced in an inductor is given by $|e| = L\left|\frac{di}{dt}\right|$ (A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

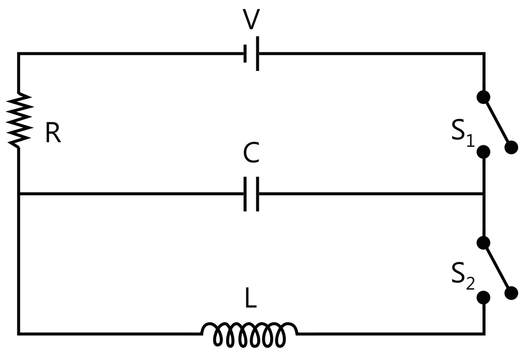
(B) Statement-I is true, statement-II is true and statement-II is not the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Comprehension Type Question

Paragraph 1: A capacitor of capacitance C can be charged (with the help of a resistance R) by a voltage source V, by closing switch s_1 while keeping switch s_2 open. The capacitor can be connected in series with an inductor 'L' by closing switch S_2 and opening S_1.



Q.28 After the capacitor gets fully charged, s_1 is opened and S_2 is closed so that the inductor is connected in series with the capacitor. Then,

(A) At t=0, energy stored in the circuit is purely in the form of magnetic energy.

(B) At any time t>0, current in the circuit is in the same direction.

(C) At t>0, there is no exchange of energy between the inductor and capacitor.

(D) At any time t>0, instantaneous current in the circuit is $V\sqrt{\frac{C}{L}}$

Q.29 If the total charge stored in the LC circuit is Q_0 then for t>=0

(A) The charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{LC}}\right)$

(B) The charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{LC}}\right)$

(C) The charge on the capacitor is $Q = LC\frac{d^2Q}{dt^2}$

(D) The charge on the capacitor is $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

Paragraph 2: In a series L-R circuit, connected with a sinusoidal ac source, the maximum potential difference across L and R are respectively 3 volts and 4 volts

Q.30 At an instant, the potential difference across resistor is 2 V. The potential difference in volt, across the inductor at the same instant will be:

(A) 3 cos30° (B) 3 cos60°

(C) 3 cos45° (D) None of these

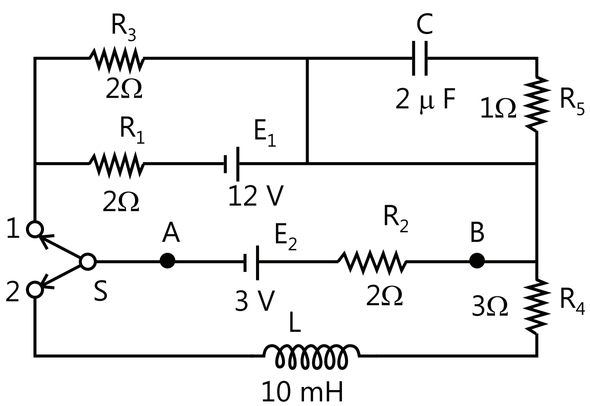
Q.31 At the same instant, the magnitude of the potential difference in volt, across the ac source may be

(A) $4+3\sqrt{3}$ (B) $\frac{4+3\sqrt{3}}{2}$

(C) $1+\frac{\sqrt{3}}{2}$ (D) $2+\frac{\sqrt{3}}{2}$

Previous Years' Questions

Q.1 A circuit containing a two position switch S is shown in Figure.



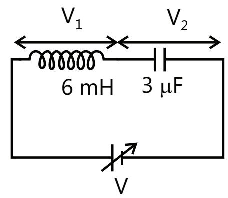
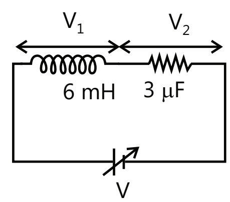
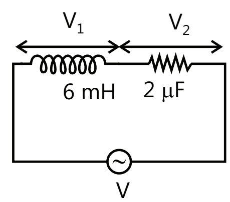
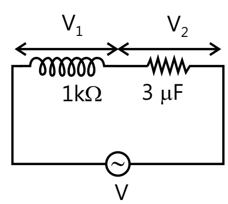
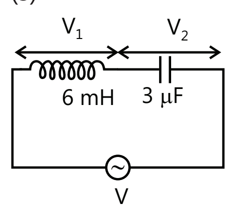
(a) The switch S is in two position 1. Find the potential difference $V_A - V_B$ and the rate production of joule heat in R_1.

(b) If Now The switch S is put in position 2 at t=0. Find:

(i) Steady current in R_4 and(ii) The time when current in R_4 is half the steady value. Also calculate the energy stored in the inductor L at that time. ***(1991)***

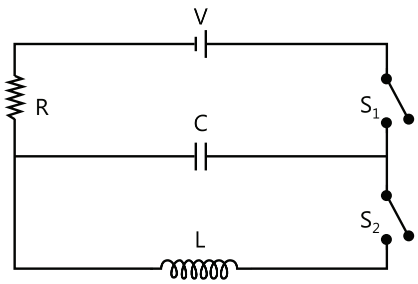
Q.2 Match the Columns

You are given many resistances, capacitors and inductors. They are connected to a variable DC voltage source (the first two circuits) or in AC voltage source of 50 Hz frequency (the next three circuits) in difference ways as shown in column II. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in column I. ***(2010)***

Column I	Column II
(A) $I \neq 0, V_1$ is Proportional to I	(p) 
(B) $I \neq 0, V_2 > V_1$	(q) 
(C) $V_1 = 0, V_2 = V$	(r) 
(D) $I \neq 0, V_2$ is Proportional to I	(t) 
	(s) 

Paragraph 1 (Q.3 to Q.8)

The capacitor of capacitance C can be charged(with the help of resistance R) by a voltage source V, by closing switch S_1 while keeping switch S_2 open. The capacitor can be connected in series with an inductor L by closing switch S_2 and opening S_1.



Q.3 Initially, the capacitor was uncharged. Now switch s_1 is closed and S_2 is kept open. If time constant of this circuit is τthen **(2006)**

(A) After time interval τ, charge on the capacitor is CV/2

(B) After time interval 2τ, charge on the capacitor is CV $(1-e^{-2})$

(C) The work done by voltage source will be half of the heat dissipated when the capacitor is fully charged

(D) After time interval 2τ, charge on the capacitor is CV $(1-e^{-1})$

Q.4 After capacitor gets fully charged, S_1 is opened and S_2 is closed so that the inductor isconnectedin series with the capacitor, then **(2006)**

(A) At t=0, energy stored in the circuit is purely in the form of magnetic energy.

(B) At any time t>0, current in the circuit is in the same direction.

(C) At t>0, there is no exchange of energy between the inductor and capacitor.

(D) At any time t>0, instantaneous current in the circuit may $V\sqrt{\frac{C}{L}}$

Q.5 If the total charge stored in the LC circuit is Q_0 then for $t \geq 0$ **(2006)**

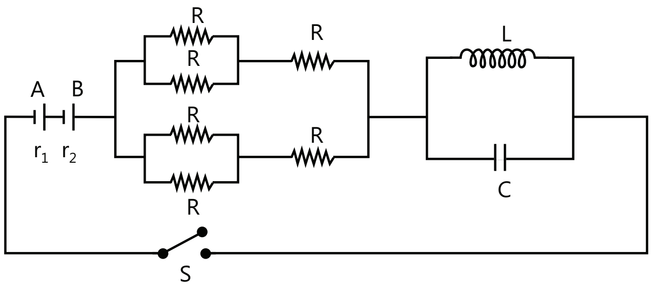
(A) The charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{LC}}\right)$

(B) The charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} - \frac{t}{\sqrt{LC}}\right)$

(C) The charge on the capacitor is $Q = LC\frac{d^2Q}{dt^2}$

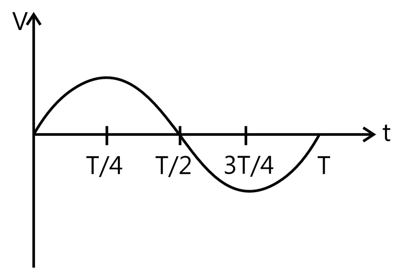
(D) The charge on the capacitor is $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

Q.6 In the circuit shown, A and B are two cells of same emf E but different internal resistance r_1 and r_2 ($r_1 > r_2$) respectively find the value of R such that the potential difference across the terminals of cell A is zero a long time after the key K is closed **(2004)**



Q.7 In an L-R series circuit, a sinusoidal voltage $V = V_0 \sin\omega t$ is applied. It is given that L=35 mH, R=11 Ω, $V_{rms} = 220V$, $\omega / 2\pi = 50Hz$ and $\pi = 22/7$.

Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph. **(2004)**



Q.8 What is the maximum energy of the anti-neutrino ? **(2012)**

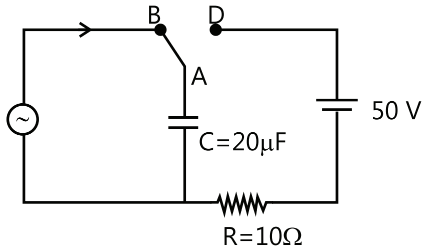
(A) Zero

(B) Much less than 0.8×10^6 eV

(C) Nearly 0.8×10^6 eV

(D) Much larger than 0.8×10^6 eV

Q.9 At time t = 0 terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with I_0 = 1A and $\omega = 500$ rad/s starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20 μF, $R = 10\Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s). **(2014)**



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is $1\times10^{-3}C$

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10 A.

(D) $Q = 2\times10^{-3}C$

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 15 Q.21 Q.22
Q.23 Q.27

Exercise 2

Q. 1 Q.3 Q. 11
Q.12

JEE Advanced/Boards

Exercise 1

Q. 3 Q.4 Q.7
Q.14 Q.15 Q.16

Exercise 2

Q.2 Q.3 Q.12
Q.14 Q.22 Q.23
Q.28 Q.28 Q.29
Q.30 Q.31

Answer Key

JEE Main/Boards

Exercise 1

Q.5. $V_{rms} = \left(\frac{V_0}{\sqrt{2}}\right)$

Q.6 No

Q.7 The current lags behind the voltage by phase angle $\pi/2$.

Q.8 Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fc}$

Q.10 It is based up on the principle of electromagnetic induction.

Q.11 (i) By using laminated iron core, we minimize loss of energy due to eddy current.

(ii) By selecting a suitable materials for the core of a transformer, the hysteresis loss can be minimized.

Q.13 (a) $\approx 311\sin 314t$ (b) 200V, 127.4V

Q.14 50Hz

Q.15 (a) 318.31 Ω (b) 0.527 A (c) 9 W

Q.16 6.124A

Q.18 (a) 2.20A, (b) 484 W

Q.19 0.354A

Q.20 109.5 A

Q.21 (a) 50 rad s^{-1}, (b) 40 Ω, 8.1A, (c) V_{Lcms}=1437.5

V,V_{vcrms}=1437.5V,V_{Rms}=230 $V_{LCrms} = I_{rms}\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0$

Q.22 (a) For $V = V_0 \sin\omega_t$

$$I = \frac{V_0}{\left|\omega L - \frac{1}{\omega C}\right|}\sin\left(\omega t + \frac{\pi}{2}\right); \text{If } R = 0$$

Where- sign appears if $\omega L > I/\omega C$, and+sign appears if $\omega L < I\,\omega C$.

$I_0 = 11.6A, I_{rms} = 8.24A$

(b) V_{LCrms} =207V, V_{Crms} =437 V

(c) Whatever be the current I in L, actual voltage leads current by $\pi/2$. Therefore, average power consumed by L is zero.

(d) For C, voltage lags by $\pi/2$. Again average power consumed by C is zero.

(e) Total average power absorbed is zero.

Q.23 (a) Yes. The same is not true for rms voltage, because voltage across different element may not be in phase.

(b) The high induced voltage, when the circuit is broken, is used to change the capacitor, thus avoiding sparks, etc.

(c) For dc, impedance of L is negligible and C very high (infinite), so the D.C. signal appears across C. For frequency ac, impedance of L is high and that of C is low. So, the A.C. signal appears across L.

(e) A choke coil reduces voltage across the tube without wasting power. A resister would waste power as heat.

Q.24 1.75×10^{-2} H; 5.5Ω

Q.25 0.04H

Q.26 Resonant frequency=39.79 Hz

(i) 2000 Ω (ii) 100 Ω (iii) 2A

(iv) 90^0 (v)180^0

Q.27 (i) 500; (ii) 8.9kW

Exercise 2

Q.1 B	**Q.2** A	**Q.3** C	**Q.4** B	**Q.5** C	**Q.6** C
Q.7 A	**Q.8** B	**Q.9** C	**Q.10** A	**Q.11** D	**Q.12** D
Q.13 C	**Q.14** A	**Q.15** D	**Q.16** A	**Q.17** D	**Q18** D
Q.19 B	**Q.20** A				

Previous Years' Questions

Q.1 A	**Q.2** B, C	**Q.3** 15V	**Q.4** C

JEE Advanced/Boards

Exercise 1

Q.1 0.8

Q.2 $[I]^{-1}$

Q.3 $\frac{LE^2}{2R_1^2}$

Q.4 $\frac{1}{15A}, \frac{1}{10A}$

Q.5 $\frac{e^2 - 1}{e^2}$

Q.6 $\frac{EL}{eR^2}$

Q.7 $q = \frac{KLt^2}{R}$ C

Q.8 $\frac{20}{\pi^2} \cong 2H$

Q.10 $C = 9.2$ .F

Q.11 6.94 A

Q.12 (i) $i_1 = i_2 = 10/3A$, (ii) $i_1 = 0$, $i_2 = 30/11A$, (iv) $i_1 = i_2 = 0$

Q.13 42+20t V

Q.14 2A, 400W

Q.15 $Z = 100\Omega$, $V_Q = 9.8$ V

Q.16 0.2 mH, $\frac{1}{32}\mu F, 8\times10^5 rad/s$

Q.17 20A, $\frac{\pi}{4}$, $\therefore$ Steady state current=$20\sin\pi\left(100t-\frac{1}{4}\right)$

Exercise 2

Single Correct Choice Type

Q.1 C	**Q.2** D	**Q.3** A	**Q.4** A	**Q.5** A	**Q.6** A
Q.7 A	**Q.8** B	**Q.9** C	**Q.10** B	**Q.11** B	**Q.12** B
Q.13 D	**Q.14** D	**Q.15** A			

Multiple Correct Choice Type

Q.16 D	**Q.17** D	**Q.18** A	**Q.19** B	**Q.20** B, C, D	**Q.21** A, B, C
Q.22 D	**Q.23** B, D	**Q.24** D			

Assertion Reasoning Type

Q.25 C **Q.26** D **Q.27** A

Comprehension Type

Paragraph 1: **Q.28** D **Q.29** C

Paragraph 2: **Q.30** D **Q.31** B

Previous Years' Questions

Q.1 (a) -5v, 24.5w (b) (i) 0.6A (ii) 1.386×10^{-3}s, 4.5×10^{-4} J

Q.2 A $\rightarrow$ r, s, t; B $\rightarrow$ q, r, s, t; C $\rightarrow$q, p; D $\rightarrow$ q, r, s, t

Q.3 B **Q.4** D **Q.5** C **Q.6** $R = \frac{4}{3}(r_1 - r_2)$

Q.7 Amplitude = 20A, phase difference = $\frac{\pi}{4}$

Q.8 C

Q.9 C, D

Solutions

JEE Main/Boards

Exercise 1

Sol 1: In a resistance coil, when an alternating current is flown, there will be a magnetic field generated across the coil and so there will be an inductance induced into the coil. Hence it will have more impedance compared to the one withDC current.

Sol 2: We know that power dissipated = VI cosθ.

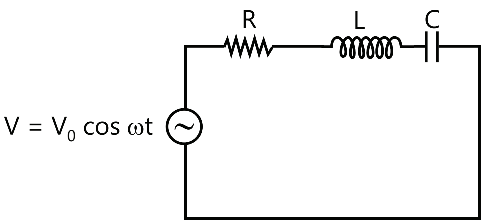
$\cos\theta = \left(\frac{R}{Z}\right) \Rightarrow$ power factor

now for an ideal inductor, $Z = \omega L$ and $R = 0$

$\therefore \cos\theta = 0$

Hence power = VI (0) = 0

Sol 3: Impedance is the effective resistance of an electric circuit or component to alternating current, arising from the combined effect of ohmic resistance and reactance.



Now let 'i' (iota) be the complex number, square root of –1.

Now, Impedance of resistance 'R' = $R \equiv Z_R$

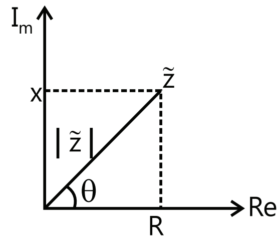
Impedance of Inductor 'L' = $i\,\omega L \equiv Z_L$

Impedance of capacitor 'C' = $\left(\frac{-i}{\omega C}\right) \equiv Z_C$

now net Impedance of the circuit (figure (i)) is

$Z_{net} = Z_R + Z_C + Z_L$

$= R - \frac{i}{\omega C} + i\omega L = R + i\left(\omega L - \frac{1}{\omega C}\right)$



Sol 4: As derived above,

$Z_R = R$

$Z_L = i\omega L$

$Z_C = -i/\omega C$

$z_{net} = Z_R + Z_L + Z_C$ (Since they all are in series)

Now we can write any quantity in phasor notation,

for $V = V_0 \cos(\omega t + \theta)$

we write this quantity in phasor notation as,

$V = |V| \angle\theta$

$\Rightarrow V = V_0\angle\theta$. [θ is the phase angle].

This is very helpful for us.

Now for the given potential, $V = V_0 \sin\omega t$

$V = V_0 \cos\left(\omega t - \frac{\pi}{2}\right)$

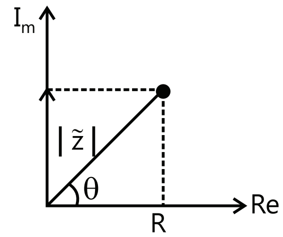
$\therefore \tilde{V} = V_0\angle -\frac{\pi}{2}$ ... (i)

We got $Z_{net} = Z_R + Z_L + Z_C = R + i\,\omega L - \frac{i}{\omega L}$

$Z_{net} = R + i\left(\omega L - \frac{1}{\omega C}\right)$

now $|Z_{net}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$\tan\theta = \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$



With this we can write

$$\boxed{Z_{net} = |Z_{net}| \angle\theta} \quad \text{... (ii)}$$

Now we known that

$\tilde{V} = \tilde{I} \times \tilde{Z}$ [∵ V = I R]

$$\tilde{I} = \left(\frac{\tilde{V}}{\tilde{Z}}\right); \tilde{I} = \frac{v_0\angle -\frac{\pi}{2}}{Z_0\angle\theta}$$

$$\tilde{I} = \left(\frac{V_0}{Z_0}\right)\angle -\frac{\pi}{2} - \theta$$

$$\tilde{I} = I_0\angle -\left(\frac{\pi}{2} + \theta\right) \quad \text{... (iii)}$$

Phase of current = $-\left(\frac{\pi}{2} + \theta\right)$

Phase of voltage = $-\frac{\pi}{2}$

∴ Depending upon the 'θ' we can speak more about the relation between ϕ_V and ϕ_I.

Sol 5: Let V = V_0 sin (ωt + θ) be an ac voltage source. Then

$$V_{rms} = \left[\frac{\int_0^T V^2 dt}{\int_0^T dt}\right]^{1/2}$$

$$V_{rms} = \left[\frac{\int_0^T V_0^2 \sin^2(\omega t + \theta)}{T}\right]^{1/2}$$

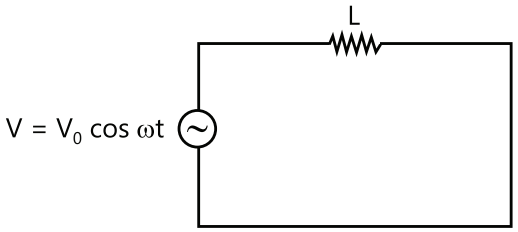
now for simplifying the calculation,

∴ We put θ = 0, and solve;

we get $V_{rms} = \left(\frac{V_0}{\sqrt{2}}\right)$

Sol 6: No nothing is perfect. It is impossible to make a perpetual machine.

Sol 7: Using the notation used in Q.4 and Q.5;



In phasor notation: $V_0 = V_0\angle 0$

$Z_L = i\omega L \Leftrightarrow Z_L = |Z_L| \angle\frac{\pi}{2}$

[∵ use complex analysis in maths.]

$\Leftrightarrow Z_L = \omega L \angle\frac{\pi}{2}$.

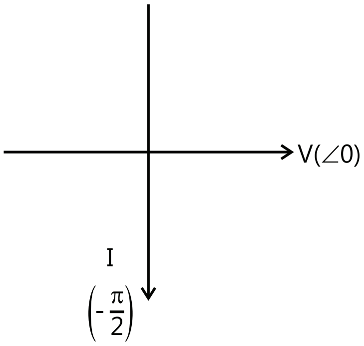
Now we know that $\tilde{V} = \tilde{I}\,\tilde{Z}_L$

$$\frac{V_0\angle 0}{\omega L\angle\frac{\pi}{2}} = \tilde{I}$$

$$\tilde{I} = \frac{V_0}{\omega L}\angle -\frac{\pi}{2}$$

$$\Rightarrow \tilde{I} = I_0\angle -\frac{\pi}{2}$$

Phase of voltage = ∠ 0 = zero



Phase of current = $\angle -\frac{\pi}{2} = -\frac{\pi}{2}$

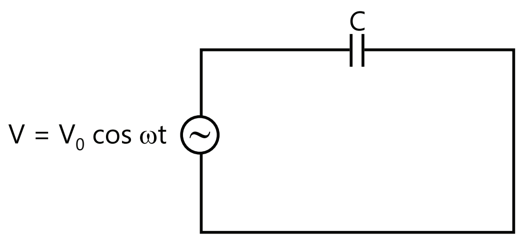
Hence current lags behind the voltage by an angle of $\left(\frac{\pi}{2}\right)$.

Sol 8: ω = 2πf

Now as derived in Q.4;

$Z_C = \frac{-i}{\omega C} = \frac{-i}{2\pi fC}$

Sol 9: $\tilde{V} = V_0\angle 0$ [In phasor] .....(i)



$Z_C = \frac{-i}{\omega C} = \left(\frac{1}{\omega C}\right)\angle -\frac{\pi}{2}$

$\left[\begin{array}{l} Z_C = |Z_C|\angle\theta;\text{ for } \quad i \to \angle -\frac{\pi}{2} \\ \qquad\qquad\qquad\qquad -i \to \angle -\frac{\pi}{2} \end{array}\right]$

Now $\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = \frac{V_0\angle 0}{\left(\frac{1}{\omega C}\right)\angle -\frac{\pi}{2}}$

$\tilde{I} = V_0\omega C\angle\frac{\pi}{2}$

$\tilde{I} = I_0\angle\frac{\pi}{2}$ .....(ii)

Now power dissipated $P = \tilde{V}\,\tilde{I}\Big|$ standard notation get familiar with this

$P = (V_0\angle 0)\left(I_0\angle\frac{\pi}{2}\right)$

$P = V_0 I_0\angle 0 + \frac{\pi}{2}$

$P = V_0 I_0\angle\frac{\pi}{2}$

And $\cos\frac{\pi}{2}$ = zero

Hence P = 0.

Sol 10: Refer to theory.

Sol 11: Refer to theory.

Sol 12: Refer to theory.

Sol 13: (a) Instantaneous voltage $V = V_0\sin\omega t$ now V_0 is the maximum possible voltage (or amplitude)

220 V given is the RMS value of voltage

$\therefore v_{rms} = \frac{V_0}{\sqrt{2}}$

$V_0 = (v_{rms})\sqrt{2}$

$V_0 = (220)\ (\sqrt{2})$

$V_0 = 311$ V.

And given f = 50 $H_{Z;}$

ω = 2πf= 2π(50) = 100 p

ω = 314

∴ v = 311 sin (314 t)

(b) Given $V_{rms} = 100\sqrt{2}V$;

We know that $V_{rms} = \frac{V_0}{\sqrt{2}}$

Comparing both of them;

$V_0 = 200$ V

V = 200 sin (ωt)

V = 200 sin (314 t)

Now; $\omega = \frac{2\pi}{T}$

$\Rightarrow V = 200\sin\left(\frac{2\pi t}{T}\right)$

$\text{Average} = \frac{\int_0^{T/2} 200\sin\left(\frac{2\pi t}{T}\right)dt}{\int_0^{T/2} dt} = 127\text{ V.}$

Sol 14: Let 'f' be the required frequency

ω = 2πf

now $V = V_0\cos(2\pi ft)$

we are given $V_{rms} = 200$ V

$\therefore V_{rms} = \frac{V_0}{\sqrt{2}}$

$V_0 = 200\sqrt{2}\,V$

$\Rightarrow \tilde{V} = 200\sqrt{2}\angle 0$ ....(i)

$Z_L = i\omega L = i(\omega)$ (i)

$= i\omega \equiv i2\pi f$

$\tilde{Z}_L = 2\pi f\angle\frac{\pi}{2}$

now $\tilde{I} = \frac{\tilde{V}}{\tilde{Z}}$

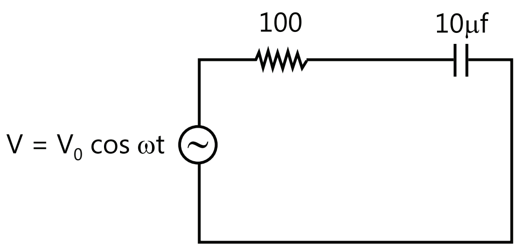
$$\tilde{I} = \frac{200\sqrt{2}\angle 0}{2\pi f \angle \frac{\pi}{2}}$$

$$\tilde{I} = I_0 \angle -\frac{\pi}{2}$$

we want $I_0 = \frac{200\sqrt{2}}{2\pi f} = 0.9$

$\therefore f = \frac{200\sqrt{2}}{2\pi(0.9)}$ $H_Z \equiv 50$ Hz

Sol 15: $V_0 = V_{rms} \cdot \sqrt{2}$



(a) $V_0 = 100\sqrt{2}$

$\omega = 2\pi\ (50) = 100$ p

$\therefore\ \tilde{V} = 100\sqrt{2}\ \cos(100\ \pi t) = 100\sqrt{2} \angle 0$

$Z_R = R = 100$

$$Z_C = \left(\frac{-i}{\omega C}\right) = \frac{-i}{(100\pi)(10\times10^{-6})} = -i\ (318)\ \Omega$$

$\therefore$ Resistance of capacitor is $|Z_C| \approx 318\ \Omega$

(b) now $Z_{net} = Z_R + Z_C$

$Z_{net} = 100 - i\ (318)$

$$Z_{net} = \sqrt{(100)^2 + (318)^2} \angle \tan^{-1}\left(\frac{-318}{100}\right)$$

$Z_{net} = 334 \angle -72.5°$

$$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = \frac{100\sqrt{2}\angle 0}{334\angle -72.5} = 0.42 \angle 72.5 = 0.527 \text{ A}$$

(c) $P_{avg} = V_{rms}\ I_{rms} \cos\phi$

$$= (100)\left(\frac{0.42}{\sqrt{2}}\right) . \cos(72.5) = 29.9 \cos(72.5)$$

$P_{avg} = 9$ watt

Sol 16: $f = 50$ Hz $\therefore\ \omega = 2\pi \times 50 = 100\ \pi$

$I_{rms} = 5.0$ A

$\therefore\ I_{max} = 5.0\sqrt{2}$ A

Let $\therefore\ I = 5\sqrt{2}\sin(100\pi t)$

when $t = \frac{1}{300}$ sec

then

$$I = 5\sqrt{2}\sin\frac{\pi}{3} = 5\sqrt{2} \times \frac{\sqrt{3}}{2} = 2.5\sqrt{6}\ \text{A}$$

Sol 17: $V_{rms} = 220$

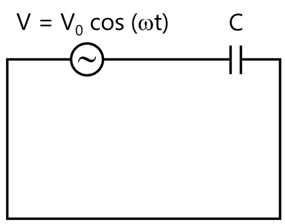
$V_0 = \sqrt{2}\ (V_{rms})$

$V_0 = 220\sqrt{2}$

$\omega = 2\pi f$

$V = 220\sqrt{2}\ \cos(2\pi ft)$

$V = 220\sqrt{2} \angle 0$



$$Z_C = \frac{-i}{\omega C} = \frac{1}{\omega C} \angle -\frac{\pi}{2} \text{ [In phasor notation]}$$

$$\tilde{Z}_C = \left(\frac{1}{\omega C}\right) \angle -\frac{\pi}{2}$$

$$\text{Now } \tilde{I} = \frac{\tilde{V}}{\tilde{Z}_C} = \frac{V_0\angle 0}{\left(\frac{1}{\omega C}\right)\angle -\frac{\pi}{2}}$$

$$\tilde{I} = V_0\omega C \angle \frac{\pi}{2} + 0$$

$$\tilde{I} = V_0\omega C \angle \frac{\pi}{2}$$

$\therefore$ Phase of current $= \frac{\pi}{2}$

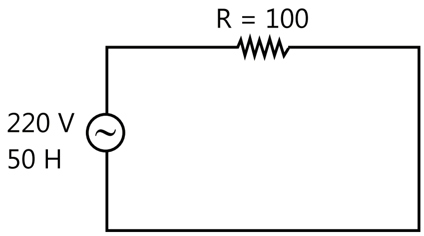
Phase of voltage = 0

$\therefore\ \phi_I - \phi_v = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

Sol 18: $V = 220\sqrt{2}\ \cos(50\ (2\pi)\ t)$

$V = 220\sqrt{2}\ \cos(100\ \pi t)$

$V = 220\sqrt{2}\angle 0$



(a) $Z_R = R = 100 \Leftrightarrow Z_R = 100 \angle 0$

$\tilde{V} = \tilde{I}\,\tilde{Z}$

$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = \frac{220\sqrt{2}\angle 0}{100\angle 0}$

$\tilde{I} = 2.2\sqrt{2}\angle 0$

$\Rightarrow I = (2.2)\sqrt{2}\cos(100\pi t)$

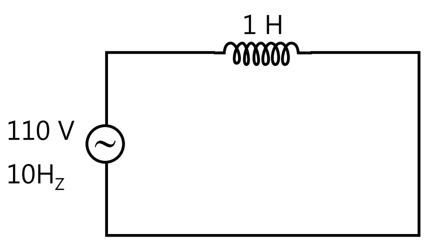
now $I_0 = (2.2)\sqrt{2}$

$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{(2.2)(\sqrt{2})}{\sqrt{2}} = 2.2$ Amp.

(b) Net power over a full cycle

$= \frac{(V_{rms})^2}{R} = \frac{(220)^2}{100} = 484$ watt

Sol 19: $\tilde{V} = 110\sqrt{2}\cos(2\pi(70)t)$



$\tilde{V} = 110\sqrt{2}\cos(140\pi t) = 110\sqrt{2}\angle 0$ ...(i)

$Z_L = i\omega L = i(140\pi) = i(140\pi)$

$|Z_L| = 440$ W

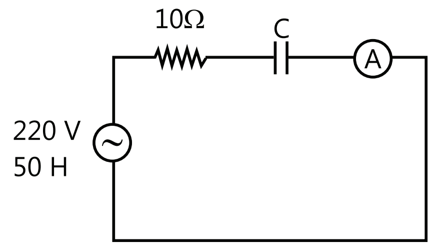
$Z_L = 440\angle\frac{\pi}{2}$ ...(ii)

$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = \frac{110\sqrt{2}\angle 0}{440\angle\frac{\pi}{2}}$

$\tilde{I} = \frac{1}{2\sqrt{2}}\angle -\frac{\pi}{2} = \left(\frac{1}{2\sqrt{2}}\right)\cos\left(140\pi t - \frac{\pi}{2}\right)$

$I_0 = \frac{1}{2\sqrt{2}} = 0.354$ Amp.

Sol 20: $V = 220\sqrt{2}\cos(2\pi(50)t)$



$V = 220\sqrt{2}\cos(100\pi t)$

$\tilde{V} = 220\sqrt{2}\angle 0$ ... (i)

Now let 'C' be the capacitance of the circuit;

$Z_C = \frac{-i}{\omega C} = \frac{-i}{2\pi fc} = \left(\frac{1}{2\pi fc}\right)\angle -\frac{\pi}{2}$ ... (ii)

$Z_R = R = 10\Omega = 10\angle 0$ ... (iii)

Now $Z_{net} = Z_R + Z_C$

$Z_{net} = (10 + Z_C) = 10 - \frac{1}{2\pi fc}\,i$

$|Z_{net}| = \sqrt{(10)^2 + \left(\frac{1}{2\pi fc}\right)^2}$

$\tan\theta = \left(\frac{-\frac{1}{2\pi fc}}{R}\right) = \left(-\frac{1}{2\pi fcR}\right)$

$\theta = \tan^{-1}\left(\frac{-1}{2\pi fRC}\right)$

$\therefore \tilde{Z} = \sqrt{(10)^2 + (X_C)^2}\angle\tan^{-1}\left(\frac{-1}{2\pi fRC}\right)$ ... (iv)

Now $\tilde{V} = \tilde{I}\tilde{Z}$

$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}}$

$\tilde{I} = \frac{220\sqrt{2}}{\sqrt{100 + X_C^2}}\angle 0 - \tan^{-1}\left(\frac{-1}{2\pi fRC}\right)$

Now $I_0 = \frac{220\sqrt{2}}{100 + X_C^2}$

$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{220}{\sqrt{100 + X_C^2}}$

$I_{rms} = 2A$ (Given)

$\Rightarrow 2 = \frac{220}{\sqrt{100 + X_C^2}}$

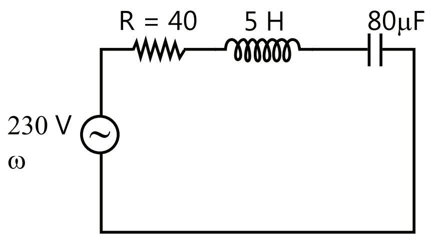
$100 + X_C^2 = (110)^2$

$X_C = 109.5\ A$

Sol 21: $Z = Z_R + Z_L + Z_C = 40 + i\omega L - \dfrac{1}{\omega C}$

$Z = 40 + i\left(\omega L - \dfrac{1}{\omega C}\right)$

Now condition for resonance is Imaginary part of Impedance is zero



$$V = 230\sqrt{2}\cos(\omega t)$$

$$\tilde{V} = 230\sqrt{2} < 0 \ \rightarrow \qquad \text{... (i)}$$

$\therefore\ \omega L - \dfrac{1}{\omega C} = 0$

$\Rightarrow \omega^2 = \dfrac{1}{LC}$, $\omega C = \dfrac{1}{\sqrt{LC}}$

$\omega = \dfrac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \dfrac{1}{20 \times 10^{-3}}$

$= \dfrac{1000}{20} = 50$ rad/s

$f = \dfrac{w}{2\pi} = \dfrac{50}{2\pi} = \dfrac{25}{\pi} \simeq 8H_Z$

$Z = 40 + i(0) = 40$

$\Leftrightarrow Z = 40 < 0 \rightarrow \qquad \text{... (ii)}$

Now from (i) and (ii), $\tilde{I} = \dfrac{\tilde{V}}{\tilde{Z}}$

$\tilde{I} = \dfrac{230\sqrt{2}\angle 0}{40\angle 0} = \dfrac{23\sqrt{2}}{4}\angle 0$

$\tilde{I} = \dfrac{23\sqrt{2}}{4}\cos(50t)$

$I = 8.13\cos(50\,t)$

Now potential drop across

(a) Resistance:

$V = -\left(\tilde{I}R\right)$

$V = -(813\angle\ 0)(40)$

$\tilde{V} = -325\angle 0$

$\Leftrightarrow V = -325\cos(50\,t)$

$V_{rms} = \dfrac{-325}{\sqrt{2}} = -230$

(b) Inductance:

$V = -\left(\tilde{I}\tilde{Z}_L\right) = -(8.13\ \angle\ 0)\left(50 \times 5\angle\dfrac{\pi}{2}\right)$

$V = -\left(2033\angle\dfrac{\pi}{2}\right) \rightarrow (x_1)$

$V = -\left[2033\cos\left(50t + \dfrac{\pi}{2}\right)\right]$

(c) Capacitor:

$V = -\left(\tilde{I}\,\tilde{Z}_c\right)$

$V_C = -\left[(8.13\angle 0)\left(\dfrac{1}{50 \times 80 \times 10^{-6}}\angle -\dfrac{\pi}{2}\right)\right]$

$V_C = -\left[\dfrac{8.13}{50 \times 80 \times 10^{-6}}\angle -\dfrac{\pi}{2}\right]$

$V_C = -\left[2033\angle -\dfrac{\pi}{2}\right] \rightarrow (x_2)$

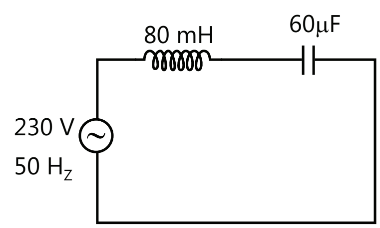
$\Leftrightarrow V_C = -\left[2033\cos\left(50t - \dfrac{\pi}{2}\right)\right]$

Now from equations (x_1) and (x_2)

we get $V_L + V_C = 0$.

Study more effectively on Resonance conditions.

Sol 22: $Z_L = i\omega L = i\,(100\,\pi)\,(80 \times 10^{-3})$



$$V = 230\sqrt{2}\cos(2\pi(50)t)$$

$$V = 230\sqrt{2}\cos(100\pi t)$$

$$\tilde{V} = 230\sqrt{2} < 0$$

$Z_L = i\,(8\pi)$

$Z_L \Leftrightarrow 8\pi < \dfrac{\pi}{2} \qquad \text{... (i)}$

$$Z_C = \frac{-i}{\omega C} = \frac{-i}{100\pi \times 60 \times 10^{-6}} = -\frac{500i}{3\pi}$$

$$Z_C \Leftrightarrow \frac{500}{3\pi} \angle -\frac{\pi}{2} \quad \text{... (ii)}$$

$$Z_{net} = Z_L + Z_C = 8\pi i - \frac{500i}{3\pi}$$

$$Z_{net} = \left(8\pi - \frac{500}{3\pi}\right) i \Rightarrow Z_{net} = -28\ i$$

$$\Leftrightarrow Z_{net} = 28 \angle -\frac{\pi}{2} \quad \text{... (iii)}$$

Now $\tilde{V} = \tilde{I} Z_{net} \Rightarrow \frac{\tilde{V}}{Z_{net}} = \tilde{I}$

$$\Rightarrow \tilde{I} = \frac{230\sqrt{2}\angle 0}{28\angle -\frac{\pi}{2}}$$

$$\tilde{I} = \frac{230\sqrt{2}}{28} \angle \frac{\pi}{2}$$

$$\tilde{I} = 6 \angle \frac{\pi}{2} \quad \text{... (iv)}$$

$$\Rightarrow I = 6 \cos\left(100\pi t + \frac{\pi}{2}\right)$$

$I_0 = 6$ and $I_{rms} = \frac{11.6}{\sqrt{2}} = 8.2$ amp

Potential drop across;

(a) Inductor;

$$V_L = \tilde{I}.\ Z_L = \left(11.6\angle\frac{\pi}{2}\right)\left(8\pi\angle\frac{\pi}{2}\right)$$

$V_L = (11\ 6 \times 8\pi) < \pi$

$V_L = 290\ \angle \pi \rightarrow (x_1)$

$V_L = 290 \cos(100\ \pi t + \pi)$

$$V_{LO} = 290;\ \left(V_{LO}\right)_{rms} = \frac{290}{\sqrt{2}} = 205\,V$$

(b) Capacitor

$$V_C = \left(\tilde{I}\right)\left(\tilde{Z}_C\right) = \left(11.6\angle\frac{\pi}{2}\right)\left(\frac{500}{2\pi}\angle -\frac{\pi}{2}\right)$$

$$V_C = \frac{11.6 \times 500}{3\pi} \angle 0$$

$V_C = 616\ \angle\ 0 \rightarrow (x_2)$

$\Leftrightarrow V_C = 616 \cos(100\ \pi t + 0)$

$(V_C)_O = 616\ (V_C)_{rms} = \frac{616}{\sqrt{2}} = 4$

Power transferred to Inductor

$= \left(\tilde{V}_L\right)\left(\tilde{I}\right) = (290 < \pi)$

$\left(11.6\angle\frac{\pi}{2}\right)$ | From (i) and (ii)

$= (290 \times 6)\ \angle\frac{3\pi}{2}$

$= 290 \times 6 \cos\left(\frac{3\pi}{2}\right)$

= Zero

Similarly zero for the capacitor to.

Total power absorbed by the circuit is

$$P = \left(\tilde{V}\right)\left(\tilde{I}\right) = \left(230\sqrt{2}\ \angle 0\right)\left(11.6\ \angle\frac{\pi}{2}\right)$$

$$P = \left(230\sqrt{2} \times 11.6\right)\angle\frac{\pi}{2}$$

$$P = \left(230\sqrt{2} \times 11.6\right)\cos\frac{\pi}{2}$$

P = zero

Sol 23: Explained in the key.

Sol 24: Initially

$X_L = 22$ at $f_1 = 200\,H_Z$

$[\omega_1 = 2\pi \times 200]$

$(X_L)_A = \omega_1 L = 22$

$\Rightarrow 2\pi \times 200\ L = 22$ ... (i)

$L = \frac{22}{400\pi} = 1.75 \times 10^{-2}$ H and finally;

$f_2 = 50H_z$

$\omega_2^2\ 2\pi\ (50)$

$X_2 = \omega_2 L$ ... (ii)

$$\frac{(i)}{(ii)} \Rightarrow \frac{x_1}{x_2} = \frac{2\pi \times 200 \times L}{2\pi \times 50 \times L}$$

$$\frac{x_1}{x_2} = 4$$

$$x_2 = \frac{x_1}{4} = \frac{22}{4} = 5.5 \text{ ohm.}$$

Sol 25: Resistance of the lamp

$= \frac{220\,V}{10}$ = 22 ohm.

Let 'L' be the Inductance of the lamp;

$X_L = \omega L = (100\,\pi)\,L$

$Z_{net} = 22 + i\,(100\,\pi L)$

$Z_{net} = \sqrt{(22)^2 + (100\pi L)^2} \angle \tan^{-1}\left(\frac{100\pi L}{22}\right)$ Now

$$\tilde{I} = \frac{\tilde{V}}{Z_{net}} = \frac{250\sqrt{2}\angle 0}{\sqrt{(22)^2 + (100\pi L)^2} \angle \tan^{-1}\left(\frac{100\pi L}{22}\right)}$$

$$\tilde{I} = \frac{250\sqrt{2}}{\sqrt{484 + (100\pi L)^2}} \angle -\tan^{-1}\left(\frac{100\pi L}{22}\right)$$

$I_0 = \frac{250\sqrt{2}}{\sqrt{484 + (100\pi L)^2}}$ and $I_{rms} = \frac{I_0}{\sqrt{2}}$

$\Rightarrow I_{rms} = \frac{250}{\sqrt{484 + (100\pi L)^2}}$

Put we are given that I_{rms} = 10 A;

$\therefore\ 10 = \frac{250}{\sqrt{484 + (100\pi L)^2}}$

$484 + (100\,\pi L)^2 = 625$

$100\,\pi L = \sqrt{141} \Rightarrow L = \frac{\sqrt{141}}{100\pi}$

$\Rightarrow L = \frac{11.9}{100\pi}$ L = 0.04 H.

Sol 26: Current drawn in circuit is maximum when the circuit is in Resonance i.e. the Imaginary part of the circuit is zero.

Now solve this question exactly as solved in Q. 21.

Sol 27: $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$\frac{220}{2200} = \frac{N_S}{5000}$

N_S = 500 turns.

n (efficiency) = $\frac{\text{Output power}}{\text{Imput power}}$

$x = \frac{8\text{ kW}}{P_i}$

$\frac{90}{100} = \frac{8\,kW}{P_i} \Rightarrow P_i = \frac{8 \times 100}{90}$ kW

$\Rightarrow P_i = \frac{80}{9}$ kW $\Rightarrow P_i$ = 8.9 kW.

Sol 28: $X_L = \omega L$; $X_C = \frac{-i}{\omega C}$

Now as ω is increased, both X_L and X_C increase.

Sol 29: $X_L = \omega L$

$\frac{x_1}{x_2} = \frac{\omega_1}{\omega_2} \Rightarrow x_2 = \frac{\omega_2}{\omega_1}\,.\,x_1$

$\Rightarrow x_2 = 2x$

$x_c = \left(\frac{-1}{\omega C}\right)$

$\frac{x_1}{x_2} = \frac{w_2}{w_1} \Leftrightarrow x_2 = \left(\frac{x_1}{2}\right)$

Phasor method:-

Let $V = V_0 \cos(\omega t + \theta_1)$ be the emf of an AC-source, then can write this is phasor method as,

$\tilde{V} = |V| \angle \theta_1 \Leftrightarrow \tilde{V} = V_0 \angle \theta_1$

Now for $I = I_0 \cos(\omega t + \theta_2)$

$\Leftrightarrow \tilde{I} = I_0 \angle \theta_2$

Now let Impedance (Z);

$Z_{Resistance} = R$

$Z_{capacitor} = \frac{-i}{\omega C}$

(i is iota; complex number)

$Z_{inductor} = i\omega L$

Now in a circuit with series RCL;

$Z_{net} = Z_R = Z_C + Z_L = R = \frac{i}{\omega C} + i\omega L$

$Z_{net} = R + i\left(\omega L - \frac{1}{\omega C}\right) \rightarrow$ ... (i)

Now let us write this in phasor notation,

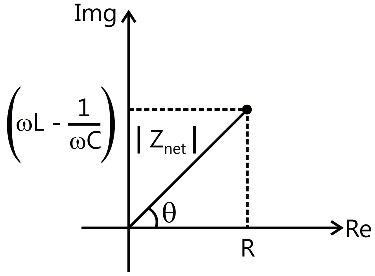
$Z_{net} = |Z_{net}| \angle \theta$

$|Z_{net}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$$\theta = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$\therefore \tilde{Z}_{net} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\ \angle \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Now for a source of emf $V = V_0 \cos(\omega t + \theta_1)$



$$\Leftrightarrow \tilde{V} = V_0 < \theta_1$$

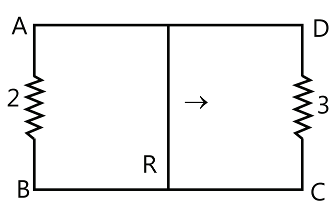
$$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}_{net}} = \frac{V_0 \angle \theta_1}{|Z_{net}| \angle \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)}$$

$$\tilde{I} = \underbrace{\boxed{\frac{V_0}{|Z_{net}|}}}_{I_0} \angle \theta_1 - \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

For resonance, imaginary part in eq. (i) is zero!

Exercise 2

Sol 1: (B) Emf induced in rod = BLv



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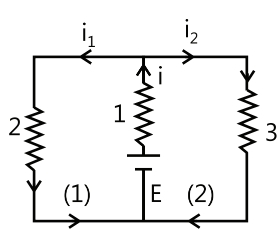


$E = (0.1)(0.1)\,1 \Rightarrow E = 10^{-2}$ V

now applying KVL in mesh (i)

$E - i\,(i) - i_1\,(2) = 0$

$E = i + 2i_1$ ... (i)

In mesh-(ii);

$\Rightarrow E - i\,(i) - 3i_2 = 0$

$\Rightarrow E = i + 3i_2$ ... (ii)

$\Rightarrow i = i_1 + i_2$ ... (iii)

From this we get $i = \frac{1}{220}$A.

Sol 2: (A) For an L–R circuit,

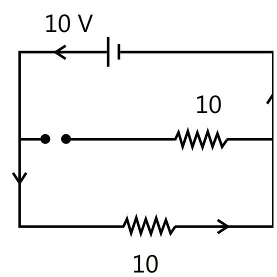
T (time constant) = $\left(\frac{L}{R}\right)$

Now energy stored in magnetic field is $\frac{1}{2}LI^2$ and rate of dissipation of energy is I^2R.

Sol 3: (C) At t = 0, inductor is open circuited

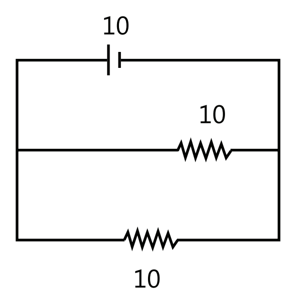
At t = ∞, inductor is short circuited

At t = 0,



$i = \frac{10}{10} = 1$ amp

At t = ∞



$i = \frac{10V}{R_{net}} = \frac{10V}{5} = 2$ amp.

∴ Difference = (2 – 1) amp= 1 amp.

Sol 4: (B) T_1 (time constant) during build up = $\left(\frac{L}{2R}\right)$

T_2 during decay = $\left(\frac{L}{3R}\right)$

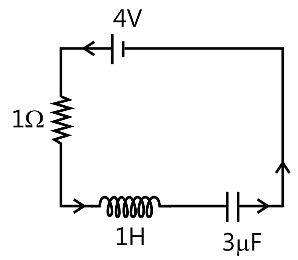
$\therefore \frac{T_1}{T_2} = \frac{3}{2}$

Sol 5: (C) Energy stored per unit time = $Li\frac{di}{dt}$

= 2 (2) (4) = 16 J/s.

Sol 6: (C) i = 2 amp $\frac{di}{dt}$ = 4 amp/s.

Applying KVL,



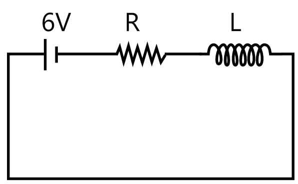
$\Rightarrow 4 - i\,(1) - L\frac{di}{dt} - \frac{Q}{C} = 0$

$\Rightarrow 4 - 2\,(1) - 1\,(4) - \frac{Q}{C} = 0$

$\Rightarrow Q = -2 \times 3$

$\Rightarrow Q = 6C.$

Sol 7: (A) $i = i_0\left(1 - e^{\frac{-Rt}{L}}\right)$



$i_0 = \frac{6v}{10} = 0.6$

$i = 0.6\left(1 - e^{\frac{-10t}{5}}\right) \Rightarrow i = 0.6\,(1 - e^{-2t})$

Put $t = \ln\sqrt{2}$

$\Rightarrow i = 0.6\left(1 - e^{2\,\ell n\sqrt{2}}\right)$

$\Rightarrow i = 0.6\left(1 - e^{\ell n\,2^{-1}}\right) \Rightarrow i = 0.6\left(1 - \frac{1}{2}\right)$

$\Rightarrow i = 0.6\left(\frac{1}{2}\right) \Rightarrow i = 0.3$ amp

Emf across coil = $L\frac{di}{dt}$

$\frac{di}{dt} = i_0\,(-(-2)\,e^{-2t}) \Rightarrow \frac{di}{dt} = 2\,i_0\,e^{-2t}$

Emf = $2L\,(0.6)\,e^{-2t}$

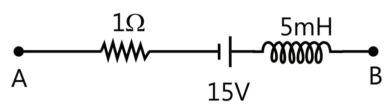
$\Rightarrow E = 6\,e^{-2t} \Rightarrow E = 6\,e^{-2\ln\sqrt{2}}$

$E = 6\,e^{\ln 2^{-1}} \Rightarrow E = 6 \times \frac{1}{2}$ E = 3V

Sol 8: (B) i = 5 amp

$\frac{di}{dt} = -10^3$ A/S

[Since decreasing; –ve sign]



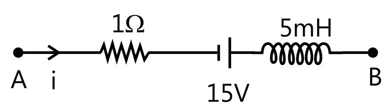
$V_A - i(1) + 15 - L\frac{di}{dt} = V_B$

$V_A - V_B = i - 15 + L\frac{di}{dt}$

$V_A - V_B = 5 - 15 + 5 \times 10^{-3}\,(-10^{+3})$

$V_A - V_B = 5 - 15 - 5 \Rightarrow V_A - V_B = -15$ V

Sol 9: (C) When 'i' is reversed,



$V_A + i\,(1) + 15 - L\left(\frac{di}{dt}\right) = V_B$

$V_A - V_B = -i - 15 + L\,\frac{di}{dt}$

$= -5 - 15 + 5\,(+10^{-3}) \times 10^3$

[i is decreasing against the direction of KVL. Hence $\frac{di}{dt}$ = 10^3].

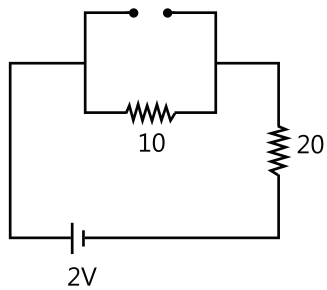
$V_A - V_B = -5 - 15 + 5$

$V_A - V_B = -15$ V

Sol 10: (A) At t = 0, inductor is open circuited,

at t = ∞, it is short circuited

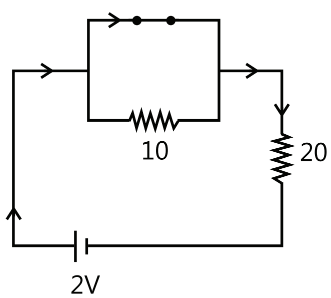
at t = 0,



$$i = \frac{2V}{R_{net}} \Rightarrow i_1 = \frac{2}{10+20}$$

$$\Rightarrow i_1 = \frac{2}{30} = \frac{1}{15} \text{ amp.}$$

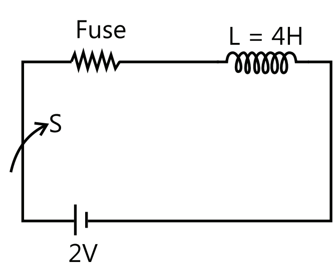
Finally; at t = ∞



$$i_2 = \frac{2V}{R_{net}} \Rightarrow i_2 = \frac{2}{20} \text{ amp}$$

$$i_2 = \frac{1}{10} \text{ amp.}$$

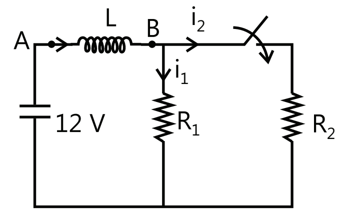
Sol 11: (D) At t = 0, no current flows in the circuit.



As time starts, current starts flowing and at t = ∞, current in the circuit is infinity.

Hence at t = 10, i → ∞ so the fuse will get blown [∵ Infinity is just an unknown number !]

Sol 12: (D) Just before the switch is opened, let us find the currents,

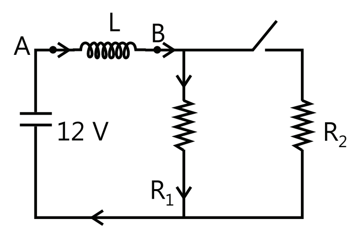


$$i = \frac{I_2\ V}{R_{net}}$$

$$R_{net} = \left(\frac{R_1R_2}{R_1+R_L}\right) = \frac{4\times 8}{12} = \frac{8}{3}\Omega$$

$$i = \frac{12}{\frac{8}{3}} \Rightarrow i = \frac{9}{2} \text{ amp.}$$

Now just at the instant switch is opened, i would remain same



$$\therefore\ V_{R_1} = i\,R_1 = \frac{9}{2}\times 4 \quad V_{R_1} = 18V$$

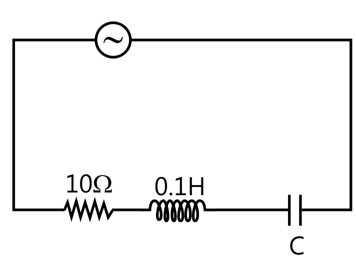
Now applying KVL;

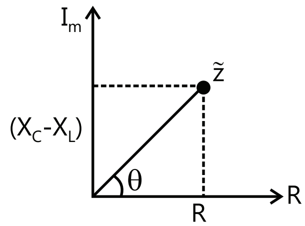
12 + (V_B – V_A) – 18 = 0 ⇒ V_B – V_A = 6 V.

Sol 13: (C) Power factor,

$$\cos\phi = \left[\frac{R}{\sqrt{(X_C - X_L)^2 + R^2}}\right]$$

V=2sin (100t)





$\cos\phi = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

$\therefore \tan\phi = \frac{|X_C - X_L|}{R} \Rightarrow |X_C - X_L| = R$

$X_L = \omega L = (0.1)(100) \Rightarrow X_L = 10\ \Omega.$

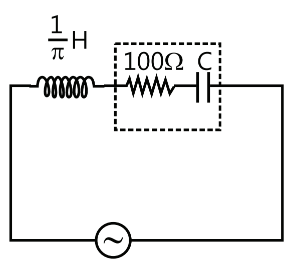
$\Rightarrow |X_C - X_L| = R$

$\frac{1}{\omega C} = R + X_L$

$C = \frac{1}{\omega(R + X_L)} \Rightarrow C = \frac{1}{100(20)}$

$C = \frac{1}{2}\times 10^{-3} \Rightarrow C = 500\ \mu F.$

Sol 14: (A) $Z_L = i\omega L = \frac{1}{\pi} \times 100\pi = i\,100\ W$



$Z_R = 100\ W$

$Z_C = \frac{-i}{\omega C} = \frac{-i}{100\pi C}$

$Z_{net} = Z_R + Z_L + Z_C$

$Z_{net} = 100 + i(100) - \frac{i}{100\pi C}$

$Z_{net} = \sqrt{(100)^2 + \left(100 - \frac{1}{100\pi C}\right)^2}$

$\tan^{-1}\left(\frac{100 - \frac{1}{100\pi C}}{100}\right)$

$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = \frac{220\sqrt{2}}{\sqrt{(100)^2 + \left(100 - \frac{1}{100\pi C}\right)^2}}$

$-\tan^{-1}\left(\frac{100 - \frac{1}{100\pi C}}{100}\right)$

$i_{rms} = \frac{220}{\sqrt{(100)^2 + \left(100 - \frac{1}{100\pi C}\right)^2}} \equiv 2.2$

$\frac{220}{2.2} = \sqrt{(100)^2 + \left(100 - \frac{1}{100\pi C}\right)^2}$

$\therefore (100)^2 = 100^2 + \left(100 - \frac{1}{100\pi C}\right)^2$

$\Rightarrow 100 - \frac{1}{100\pi C} = 0$

$\therefore X_C = -100 \left[\therefore X_C = \frac{-1}{\omega C}\right]$

Now power factor; $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

$\phi = \tan^{-1}\left(\frac{-100}{100}\right) \Rightarrow \phi = -\frac{\pi}{4}$

Power factor; $\cos\phi = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

Sol 15: (D) For 100 V D.C. source, i =1 amp.

Hence, $R = \frac{100}{1} = 100\Omega$

Now for AC source of 100 V

$i = \frac{100}{Z_{net}} \Rightarrow \frac{1}{2} = \frac{100}{Z_{net}}$

$\Rightarrow Z_{net} = 200$

$Z_{net} = \sqrt{R^2 + X_L^2}$

$\therefore R^2 + X_L^2 = (200)^2 \Rightarrow X_L^2 = (200)^2 - (100)^2$

$X_L = 174\ W$

$\omega L = 174$

$L = \frac{174}{100\pi} \Rightarrow L = 0.55$ H.

Sol 16: (A) $I = I_0 + I_1 \sin \omega t$

$$I^2_{rms} = \frac{\int_O^T I^2 dt}{\int_O^T dt} \quad T = \frac{2\pi}{\omega}$$

$$= \frac{\int_O^T (I_0^2 + I_1^2 \sin^2 \omega t + 2I_0 I_1 \sin\omega t)dt}{\int_O^T dt}$$

$$I^2_{rms} = \frac{I_0^2 T + \frac{I_1^2 T}{2} + 0}{T} \Rightarrow I_{rms} = \sqrt{I_0^2 + \frac{I_1^2}{2}}$$

Sol 17: (D) $\frac{\pi}{4} = \omega t\,; \frac{\pi}{4} = 100\pi t\,; t = \frac{1}{400}$ s.

Sol 18: (D) for LR circuit;

$$\cos\theta_1 = \left(\frac{R_1}{\sqrt{R_1^2 + X_L^2}}\right) = 0.6 \quad \text{... (i)}$$

for CR circuit;

$$\cos\theta_2 = \left(\frac{R_2}{\sqrt{R_2^2 + X_C^2}}\right) = 0.5 \quad \text{... (ii)}$$

Now when L, C, R of two circuits are joined;

$$\cos\theta = \left(\frac{R_1 + R_2}{\sqrt{(R_1 + R_2)^2 + (X_C - X_L)^2}}\right)$$

Given that $\cos\theta = 1$

$\therefore X_C = X_L = X$

$$\tan\theta_1 = \left(\frac{X_L}{R_1}\right)$$

$$\tan\theta_2 = \left(\frac{X_C}{R_2}\right)$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{X_L}{R_1} \cdot \frac{R_2}{X_C} \equiv \left(\frac{R_2}{R_1}\right)$$

$$\tan\theta_1 = \frac{4}{3}$$

$$\tan\theta_2 = \sqrt{3}$$

$$\therefore \frac{R_1}{R_2} = \frac{3\sqrt{3}}{4}$$

(*) Don't run to catch $\cos\theta$.

Use $\tan\theta$ and simplify!

Sol 19: (B) $i = 2 \sin 100\pi t + 2 \sin (100\pi t + 30°)$

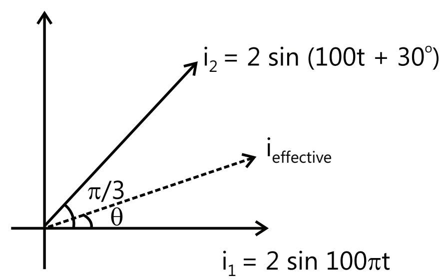
It is similar to superimposition of two vectors with an angle of 30° in between them

$i_{net} = i_0 \sin (100\pi t + \theta)$

$$i_0 = \sqrt{2^2 + 2^2 + 2(2)(2)\cos(30°)}$$

$$i_0 = \sqrt{8 + 8\sqrt{3}} \Rightarrow i_0 = 2\sqrt{2 + \sqrt{3}}$$

Phase diagram will be shown as



Sol 20: (A) We can speak on nature by observing the phase of final Impedance. If the phase of Impedance is negative then it is capacitive, else it is inductive.

$$\therefore \omega' = \frac{\omega}{2} = \frac{1}{2\sqrt{LC}}$$

$$\therefore Z_R = R$$

$$Z_L = i\omega L = i \cdot \frac{1}{2\sqrt{LC}} \cdot L = i\,\frac{1}{2}\sqrt{\frac{L}{C}}$$

$$Z_C = \frac{-i}{\omega C} = \frac{-i}{\frac{1}{2\sqrt{LC}} \cdot C} = -2i\sqrt{\frac{L}{C}}$$

$$\therefore Z_L + Z_C = -\frac{3i}{2}\sqrt{\frac{L}{C}}\,;\; Z_{net} = R - \frac{3i}{2}\sqrt{\frac{L}{C}}$$

$$Z_{net} = Z_0 \angle \tan^{-1}\left(\frac{-3i}{2R}\sqrt{\frac{L}{R}}\right)$$

$\therefore$ -ve phase

Hence capacitive.

Previous Years' Questions

Sol 1: (A) As the current i leads the emf e by $\frac{\pi}{4}$, it is an R–C circuit.

$\tan\phi = \frac{X_C}{R}$ or $\tan\frac{\pi}{4} = \frac{\frac{1}{\omega C}}{R}$ $\therefore \omega CR = 1$

As ω = 100 rad/s

The product of C–R should be $\frac{1}{100}s^{-1}$

Sol 2: (B, C) $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

In case (b) capacitance C will be more. Therefore, impedance Z will be less. Hence, current will be more.

$\therefore$Option (b) is correct.

Further,

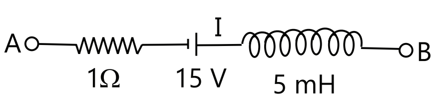
$V_C = \sqrt{V^2 - V_R^2}$

$= \sqrt{V^2 - (IR)^2}$

In case (b), since current I is more.

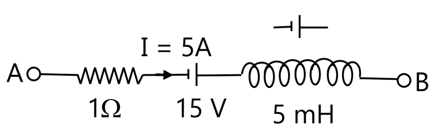
Therefore, Vc will be less.

Sol 3: $\frac{dI}{dt} = 10^3$ A/s



$\therefore$Induced emf across inductance, $|e| = L\frac{di}{dt}$ $|e|$ = $(5 \times 10^{-3})(10^3)$ V = 5 V

Since, the current is decreasing, the polarity of this emf would be so as to increase the existing current. The circuit can be redrawn as



Now $V_A - 5 + 15 + 5 = V_B$

$\therefore V_A - V_B = -15$ V

or $V_B - V_A = 15$ V

Sol 4: (C) For the lamp with direct current,

V = IR

$\Rightarrow R = 8\Omega$ and $P = 80 \times 10 = 800$ W

For ac supply

$P = I_{rms}^2 R = \frac{E_{rms}^2}{Z^2}R$

$\Rightarrow \quad Z^2 = \frac{(220)^2 \times 8}{800}$

$\Rightarrow \quad Z = 22\Omega$

$\Rightarrow \quad R^2 + \omega^2 L^2 = (22)^2$

$\Rightarrow \omega L = \sqrt{420}$

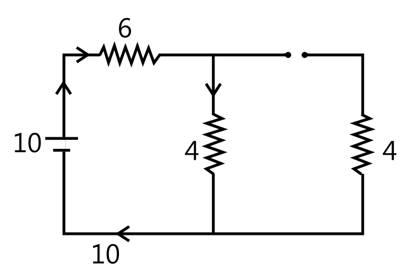
$\Rightarrow L = 0.065$ H

JEE Advanced/Boards

Exercise 1

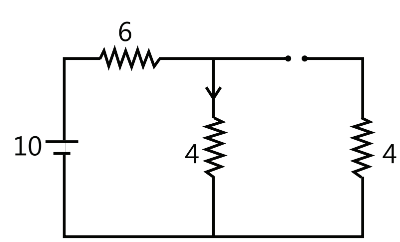
Sol 1: At t = 0, we can replace the inductor by open circuit and at t = ∞, the inductor can be short circuited

at t = 0,



$i_1 = \frac{10}{10}$ = 1 amp.

At t = ∞,



$i_2 = \frac{10}{R_{eff}} = \frac{10}{8}$amp

$\frac{i_1}{i_2} = \frac{1}{\frac{10}{8}} = \frac{8}{10}$ = 0.8 amp

Sol 2: $\frac{L}{RCV}$

$V = IR \Rightarrow \frac{L}{RC(IR)} \Rightarrow \frac{L}{R(RC)I}$

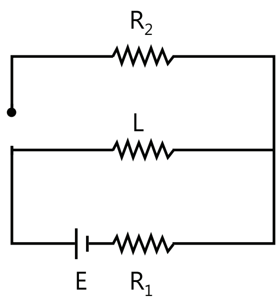
Now {RC} = time constant in RC circuit

$\because$ [RC] = [T] and $\left[\frac{L}{R}\right]$ = time constant in LR circuit

$\because \left[\frac{L}{R}\right] = [T]$

$\therefore \left[\frac{L}{RCV}\right] = \frac{[T]}{[T]\,[I]} = [I]^{-1}$.

Sol 3: Let us calculate the total energy stored in the inductor before switch is shifted.



Energy stored in inductor = $\frac{1}{2}LI^2$

$= \frac{1}{2}L\,[I_{at\ t\ =\ \infty}]^2$

$I_{t=\infty} = \left(\frac{E}{R_1}\right)$

$\therefore E = \frac{1}{2}L\left(\frac{E}{R_1}\right)^2$

$E = \frac{LE^2}{2R_1^2}$

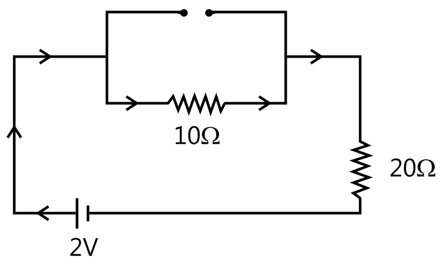
Now this is the total heat produced in R_2.

Sol 4: This is similar to the Questions 1 (Ex. I).

At t = 0; Inductor is open circuited,

At t = ∞, Inductor is short circuited.

At t = 0;

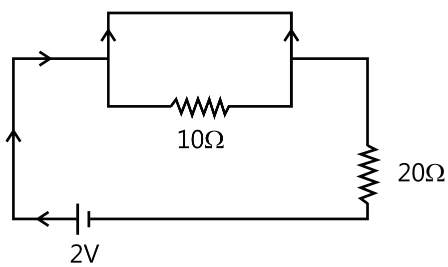


$I = \frac{2}{R_{net}}$

$I_1 = \frac{2}{10+20}$

$I_1 = \frac{2}{30}$ amp. ... (i)

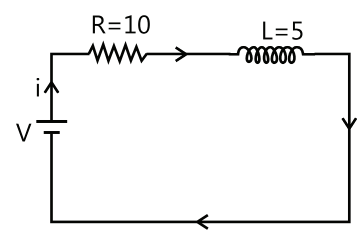
at t = ∞,



Here the resistor 10 Ω is shorted.

$I_2 = \frac{2}{R_{net}} = \frac{2}{20} = \frac{1}{10}$ amp.

Sol 5: Let us now derive the current in the circuit as a function of time



at time t = t; current = i amp;

using KVL;

$V - iR - L\frac{di}{dt} = 0 \Rightarrow V - iR = L\frac{di}{dt}$

$\Rightarrow \frac{1}{L}\,dt = \frac{di}{V - iR}$

Integrating;

$\frac{1}{L}\int_0^t dt = \int_{i_0}^{i}\frac{di}{V - iR} \Rightarrow i = i_0\left(1 - e^{-Rt/L}\right)$

At t = 0, i = zero

At t = ∞, i = i_0 = constant

Now R = 10Ω, L = 5

$$i = i_0\left(1 - e^{-2t}\right)$$

At t = 1 sec

$$i = i_0\,(1 - e^{-2}) \Rightarrow \frac{i}{i_0} = (1 - e^{-2})$$

$$\left(\frac{i}{i_0}\right) = \frac{e^2 - 1}{e^2}$$

Sol 6: $i = i_0\,(1 - e^{-Rt/L})$

$$i = \frac{dq}{dt} \Rightarrow q = \int i\,dt$$

$$q = \int i_0\left(1 - e^{\frac{-Rt}{L}}\right)dt$$

$$q = i_0\int_{t_0}^{t} 1 - e^{\frac{-RT}{L}}\Big)dt$$

$$\Rightarrow q = i_0\left[t - \left(-\frac{L}{R}\right)e^{\frac{-Rt}{L}}\right]_0^t$$

$$\Rightarrow q = i_0\left[t + \frac{L}{R}\,e^{\frac{-Rt}{L}} - \left(0 + \frac{L}{R}\right)\right]$$

$$\Rightarrow q = i_0\left[t - \frac{L}{R}\left(1 - e^{\frac{-Rt}{L}}\right)\right]$$

$$\Rightarrow q = i_0 t - \frac{i_0 L}{R}\left(1 - e\frac{-Rt}{L}\right)$$

One time constant $\Rightarrow t = \left(\frac{L}{R}\right)$

$$\Rightarrow q = i_0 \cdot \frac{L}{R} - \frac{i_0 L}{R}\left(1 - e^{-1}\right)$$

$$q = \frac{i_0 L}{R} - \frac{i_0 L}{R}\left(1 - \frac{1}{e}\right) \Rightarrow q = \frac{i_0 L}{Re}$$

$$q = \frac{EL}{R^2 e}$$

Sol 7: Given mutual inductance between coils = M

And $I_1 = kt^2$

∴ EMF induced in second coil $= L\frac{dI}{dt}$ = L [2kt]

E = 2kLt

Current in the coil II is $\frac{E}{R} = \left(\frac{2kL}{R}\right)t$

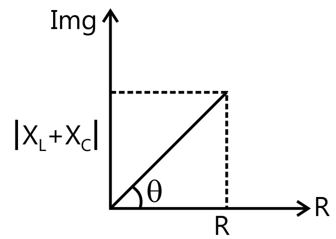
$$i = \frac{dq}{dt}$$

$$q = \int_{t=0}^{t} i\,dt \Rightarrow q = \int\left(\frac{2KL}{R}\right)t\,dt$$

$$q = \left(\frac{2KL}{R}\right).\ \frac{t^2}{2}\Bigg|_0^t \Rightarrow q = \frac{2KL}{2R}\left(t^2\right)$$

$$q = \frac{KLt^2}{R}\ \ C$$

Sol 8: Power factor is cos (θ)



Given that cos θ = 1 ⇒ θ = 0

∴ $|X_L + X_C| = 0 \Rightarrow X_L = -X_C$

$X_L = \omega L$

$$X_L = \frac{-1}{\omega C} \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega^2 C}$$

$$\omega = 2\pi\,(50) = 100\,\pi$$

$$L = \frac{1}{(100\pi)^2 C} = \frac{20}{\pi^2} = 2H.$$

Sol 9: We know that V_R and V_L will have a phase difference of $\frac{\pi}{2}$.

$$V_{net} = \sqrt{V_R^2 + V_C^2} = \sqrt{16^2 + 12^2} = 20V.$$

Sol 10: Resistance of Lamp = R

$$R = \left(\frac{V^2}{P}\right) = \left(\frac{100 \times 100}{50}\right) = 200\Omega$$

Maximum current the lamp can sustain,

$$i_{max} = \frac{P_{max}}{V}$$

$$i_{max} = \frac{50}{100} = \frac{1}{2} \text{ amp.}$$

Now in the given conditions;

(200 V, 50 H_Z)

$i = \frac{200V}{200\Omega}$ = 1 amp which is greater than 0.5 amp.
Hence we need to increase the Impedance by using a capacitor of capacitance 'C'. Such that ' I' will be equal to $\frac{1}{2}$ amp.

$$\therefore Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$I = \frac{200}{\sqrt{R^2 + \left(\frac{1}{100}\right)^2}}$$

$$I = \frac{1}{2} \text{ amp} \Rightarrow \frac{1}{2} = \frac{200}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

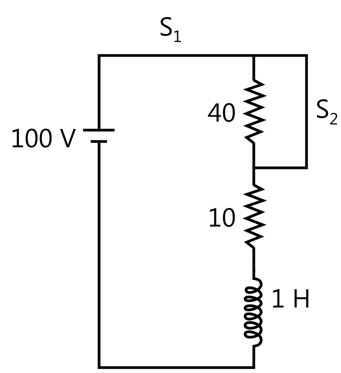
$$R^2 + \left(\frac{1}{\omega C}\right)^2 = (400)^2$$

$$(200)^2 + \left(\frac{1}{\omega C}\right)^2 = (400)^2$$

ω = 2π (50) = 100 π

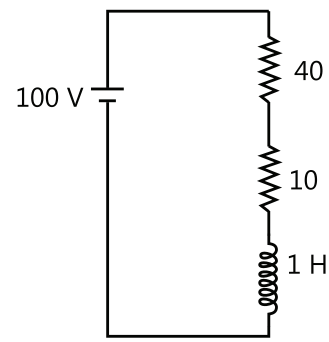
Solving this will give the value of 'C'.

Sol 11: $i = \frac{100}{10}\left(1 - e^{\frac{-10t}{e}}\right)$



$$i = 10\left(1 - e^{-10t}\right) \quad \text{... (i)}$$

now at t = 0.1 ln 2, S_2 is open;



$$\therefore i_{new} = \frac{100}{50}\left(1 - e^{-50t}\right)$$

$$i_{new} = 2\left(1 - e^{-50t}\right) \quad \text{... (ii)}$$

But this equation; at t' = 0, we get i_{new} = 0

But this is not true; Since there is a current flowing in the circuit at that instant.

Also t' = 0 $\Rightarrow$ t = 0.1 ln 2 sec.

$\therefore$ t' = t – 0.1 ln 2

$$\therefore i_{new} = i_0\left[1 - e^{-50(t-0.1\ln 2)}\right]; t \geq 0.1 \ln 2 \quad \text{... (iii)}$$

$$i_0 = \frac{100}{50} = 2 \text{ amp.}$$

using equation (iii) at time t = 0.1 ln 2, i = 0

But this is not true, since there is a current flowing in the circuit guided by the equation,

i = 10 (1 – e^{-10t}) [from eq.(i)]

now at t = 0.1 ln 2

i = 10 (1 – $e^{-10t(0.1)\ln 2}$)

$$i = 10\left(1 - \frac{1}{2}\right) \Rightarrow i = 5 \text{ amp.}$$

$$\therefore i_{new} = 5 + 2\left[1 - e^{-50(t-0.1\ln 2)}\right]$$

At time t = 0.2 ln2

$$i_{new} = 5 + 2\left[1 - e^{-50(t-0.1\ln 2)}\right] = 5 + 2\left[1 - e^{\ln 2^{-5}}\right]$$

$$= 5 + 2\left[1 - \frac{1}{2^5}\right]$$

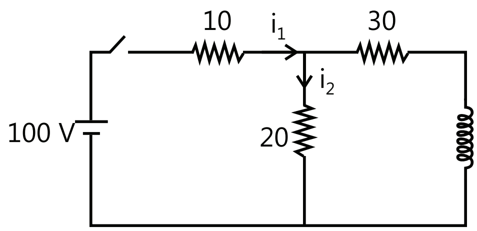
$$i_{new} = 5 + 2\left[\frac{31}{32}\right]$$

$$i_{new} = \left(5 + \frac{31}{16}\right) \text{ amp.} = 6.94 \text{ amp.}$$

Sol 12: After switch is closed;

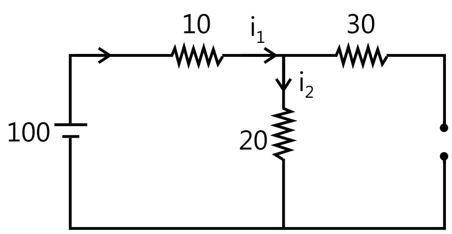
(i) t = 0; open circuiting the inductor;

$i_1 = i_2 = \frac{100}{30} = \frac{10}{3}$ amp.



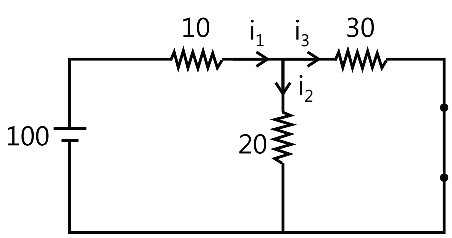
(ii) now at t = ∞ ;

inductor is short circuited,



$i_1 = \frac{100}{R_{net}}$

$i_1 = \frac{100}{22} = \frac{50}{11}$ amp



and $i_1 = i_2 + i_3$

$2i_2 = 3i_3 \Rightarrow i_3 = \frac{2i_2}{3}$

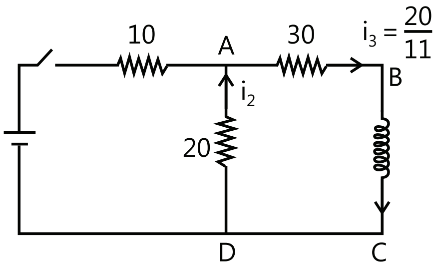
$i_1 = i_2 + \frac{2i_2}{3} \Rightarrow i_1 = \frac{5i_2}{3}$

$i_2 = \frac{3}{5}\ i_1 = \frac{3}{5}\left(\frac{50}{11}\right) = \frac{30}{11}$ amp.

$i_3 = \frac{20}{11}$ amp.

(iii) Now when switch is open

(a) Immediately after that, current through inductor will be same as just before

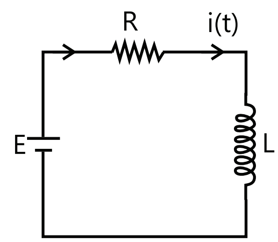


∴ Hence the current in loop ABCD will be $\frac{20}{11}$ amp.

And this current will start decaying to zero

∴ At t = ∞, i = zero.

Sol 13: Applying KVL;



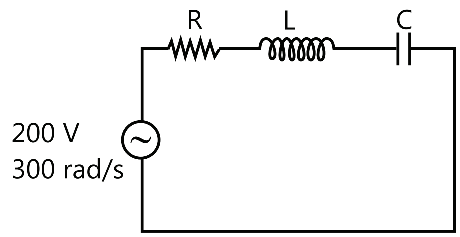
$E - i\,(t)\,R - L\frac{di}{dt} = 0$

$i(t) = 3 + 5t \Rightarrow \frac{di}{dt} = 5$

$E = R\,i(t) + L(5) \Rightarrow E = 4(3 + 5t) + 5(6)$

$E = 42 + 20t$

Sol 14: Now when capacitance is removed;



$\tilde{V} = 200\sqrt{2}\cos(300t)$

$\tilde{V} = 200\sqrt{2}\angle 0$

$Z_{net} = Z_R + Z_L$

$Z_{net} = R + i\omega L$

$Z_{net} = \sqrt{R^2\omega^2L^2}\angle\tan^{-1}\left(\frac{\omega L}{R}\right)$

$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}_{net}}$

$$\tilde{I} = \frac{200\sqrt{2}\angle 0}{\sqrt{R^2+\omega^2L^2}\angle \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

$$\tilde{I} = \frac{200\sqrt{2}}{\sqrt{R^2+\omega^2L^2}}\angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Now given that current lags behind voltage by 60°,

$$\therefore \tan^{-1}\left(\frac{\omega L}{R}\right) = 60$$

$$\therefore \frac{\omega L}{R} = \sqrt{3} \Leftrightarrow X_L = R\sqrt{3} \rightarrow (x_1)$$

$$L = \frac{R\sqrt{3}}{\omega} \Rightarrow L = \frac{100\sqrt{3}}{300}$$

$$L = \frac{1}{\sqrt{3}}\ H.$$

Now when the inductance is removed;

By intuition we can say that

$$\tan^{-1}\left(\frac{X_C}{R}\right) = \frac{\pi}{3}$$

$$\frac{X_C}{R} = \sqrt{3} \Rightarrow X_C = \sqrt{3}R \rightarrow (x_2)$$

$$\frac{1}{\omega C} = R\sqrt{3} \Rightarrow C = \frac{1}{R\sqrt{3}\omega}$$

$$C = \frac{1}{100.\sqrt{3}(300)} \Rightarrow C = \frac{100}{3\sqrt{3}}\mu F$$

Now when all together are present

$$Z_{net} = Z_R + Z_L + Z_C = 100 + iR\sqrt{3} - iR\sqrt{3}$$

[From X_1 and X_2]

$$Z_{net} = 100$$

$$Z_{net} = 100\angle 0$$

$$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}_{net}} = \frac{200\sqrt{2}\angle 0}{100\angle 0} \Rightarrow \tilde{I} = 2\sqrt{2}\angle 0$$

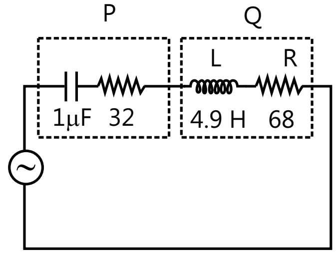
$$\text{power} = \tilde{V}\tilde{I} = (200\sqrt{2})\,(2\sqrt{2})\cos(0)$$

P = 800 W

$$P_{avg} = V_{rms} \cdot I_{rms} = \left(\frac{200\sqrt{2}}{\sqrt{2}}\right)\left(\frac{2\sqrt{2}}{\sqrt{2}}\right)$$

P_{avg} = 400 W.

Sol 15: Maximum current flows when the circuit is in resonance



$$\tilde{V} = 10\sqrt{2}\cos(\omega t)$$

$$\tilde{V} = 10\sqrt{2}\angle 0$$

i.e. $\omega = \frac{1}{\sqrt{LC}}$

$$\omega = \frac{1}{\sqrt{1\times10^{-6}\times4.9\times10^{-3}}} \Rightarrow \omega = \frac{1}{\sqrt{49\times10^{-10}}}$$

$$\omega = \frac{1}{7}\times10^5 \text{ rad/s.}$$

Impedance of Box P is $\sqrt{(32)^2+(X_C)^2}$

$$X_C = \frac{-1}{\omega C} = \frac{-1}{\frac{1}{7}\times10^5\times10^{-6}} = -70W$$

$$\therefore Z_P = \sqrt{(32)^2+(70)^2}$$

$|Z_P|$ = 77 ohm,

And impedance of coil Q is $\sqrt{(68)^2+(X_L)^2}$

$$X_L = \omega L = \frac{1}{7}\times10^5\times4.9\times10^{-3}$$

X_L = 70 W

$$\therefore \text{Impedance} = \sqrt{(68)^2+(70)^2}$$

$|Z_Q|$ = 98 W

Z_{net} = 32 – 70 i + 68 + 70 i

Z_{net} = 100 W

$$\tilde{I} = \frac{10\sqrt{2}}{100}\angle 0 \Rightarrow \tilde{I} = \frac{\sqrt{2}}{10}\angle 0$$

Voltage across P; $V_P = (I_{rms})\,(|Z_P|)$

$$= \frac{\left(\frac{\sqrt{2}}{10}\right)}{\sqrt{2}}.(77)$$

V_P = 7.7 V

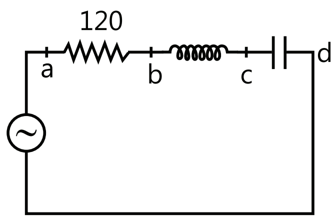
Voltage across Q; $V_Q = (I_{rms})\left(|Z_P|\right) = \frac{1}{10}$ (98)

$V_Q = 9.8$ V.

Sol 16: $\omega_r = 4 \times 10^5$ rad/s.

Given $V_a - V_b = 60$ V

and $V_b - V_C = 40$ V



We know that during resonance,

$V_L + V_C = 0$

$V_C = -40$ V $\because V_c - V_d = -40$V

$(V_a - V_b) = i_{rms} R$

$60 = i_{rms} . 120 \Rightarrow i_{rms} = \frac{1}{2}$ amp.

Now, $V_b - V_C = (i_{rms}) . Z_L$

$40 = (i_{rms}) . (Z_L) \Rightarrow Z_L = \frac{40}{i_{rms}} = \frac{40}{\frac{1}{2}} = 80\Omega$

$\omega L = 80$

$(4 \times 10^5) L = 80$; $L = 0.2$ mH

Now $V_c - V_d = -40$

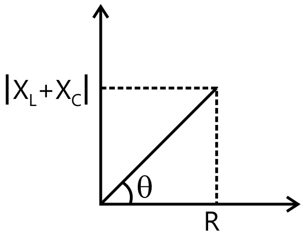
i.e. $i_{rms} . Z_c = -40$

$Z_c = -80 ; \frac{-1}{\omega C} = -80$

$C = \frac{1}{80\omega} ; C = \frac{1}{80 \times 4 \times 10^5}$

$C = \frac{1}{32} \mu F$.

$\tan\theta = \left(\frac{|X_L - X_C|}{R}\right)$



Now at $\theta = \frac{\pi}{4}$

$|X_L - X_C| = R$; $\left|\omega L - \frac{1}{\omega C}\right| = R$

$\frac{\omega^2 L - 1}{\omega C} = R$; $\omega^2 L - \omega CR - 1 = 0$

Solving this would give us

$\omega = 8 \times 10^5$ rad/s.

Sol 17: $V = 220\sqrt{2}\sin(100\pi t)$

$\tilde{V} = 220\sqrt{2}\angle 0$

$Z_{net} = Z_L + Z_R = i(100\pi \times 35 \times 10^{-3}) + 11$

$Z_{net} = 11i + 11$

$Z_{net} = \sqrt{11^2 + 11^2}\angle \tan^{-1}\left(\frac{11}{11}\right)$

$Z_{net} = 11\sqrt{2}\angle\frac{\pi}{4}$

$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = \frac{220\sqrt{2}\angle 0}{11\sqrt{2}\angle\frac{\pi}{4}}$; $\tilde{I} = 20\angle -\frac{\pi}{4}$

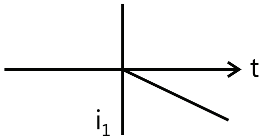
$\Leftrightarrow I = 20\sin\left(100\pi t - \frac{\pi}{4}\right)$

Exercise 2

Single Correct Choice Type

Sol 1: (C) Current is induced by varying magnetic flux. Here there is no such phenomena as flux linked with the coil is zero. Hence induced current is zero.

Sol 2: (D)



Current i_2 is constant and positive i.e. from 'c' to 'd' have i_1 has to be from 'b' to 'a'. Hence negative

Also $i_2 = \frac{L\frac{di}{dt}}{R_L}$

$\therefore \frac{di}{dt} = \text{constant}$

Hence i_1 versus t is as shown.

Sol 3: (A) Emf induced across inductor $= L\frac{di}{dt}$

$i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$

$\frac{di}{dt} = i_0\left(-\left(-\frac{R}{L}\right)e^{-\frac{Rt}{L}}\right) \Rightarrow \frac{di}{dt} = \frac{i_0 R}{L}\left(e^{-\frac{Rt}{L}}\right)$

$e = i_0 R \,.\, e^{-\frac{Rt}{L}}$ ... (i)

$i = i_0 - i_0 e^{-\frac{Rt}{L}}$

$i = i_0 - \frac{e}{R}$

$\frac{e}{R} = -i + i_0$

$e = R(-i + i_0)$ [$y = -mx + c$]

Hence graph A.

Sol 4: (A) Self-induction Emf = $-L\frac{di}{dt}$

$\frac{di_1}{dt} < \frac{di_2}{dt} \Rightarrow -\frac{di_1}{dt} > -\frac{di_2}{dt}$

$E_1 > E_2$.

Sol 5: (A) We know that RC and $\frac{L}{R}$ will have dimensions of time. Hence $\frac{1}{RC}$ and $\frac{R}{L}$ will have dimensions of frequency.

Sol 6: (A) Refer to Questions – 3 (Ex –I JEE Advanced)

Sol 7: (A) $\frac{1}{2}LI^2 = 32J$ ... (i)

$I^2R = 320$ ... (ii)

$\frac{(1)}{(2)} = \frac{L}{R} = \frac{2 \times 32}{320}$

$\tau = \frac{L}{R} = 0.2\,s.$

Sol 8: (B) In an L-R decay circuit, the initial current at t=0 is 1. The total charge that has inductor has reduced to onefourth of its initial value is LI/2R

Sol 9: (C) $\frac{1}{2}LI^2 = U$

$I^2R = P$

$T = \frac{L}{R} = \frac{2U}{P}$

Sol 10: (B) Let Z_A be the Impedance of element A, and Z_B be that of element B.

Initially; when R is connected to A;

$Z_{net} = R + Z_A$.

$\Leftrightarrow Z_{net} = \sqrt{Z_A^2 + R^2}\angle \tan^{-1}\left(\frac{Z_A}{R}\right)$

$\tilde{i} = \frac{\tilde{V}}{\tilde{Z}}$

$\tilde{i} = \frac{V}{\sqrt{Z_A^2 + R^2}}\angle -\tan^{-1}\left(\frac{Z_A}{R}\right)$

Given that current is lagging behind voltage by angle 'θ_1'

$\therefore \tan^{-1}\left(\frac{Z_A}{R}\right) = \theta_1$ ... (i)

When R is connected to B

$\tilde{Z} = \sqrt{Z_B^2 + R^2}\angle \tan^{-1}\left(\frac{Z_B}{R}\right)$

$\tilde{i} = \frac{V}{\sqrt{Z_B^2 + R^2}}\angle -\tan^{-1}\left(\frac{Z_B}{R}\right)$

Given that current leads voltage by 'θ_2'

$\because \theta_2 = -\tan^{-1}\left(\frac{Z_B}{R}\right)$ ... (ii)

Using same method, when R, A, B are connected,

$\theta = -\tan^{-1}\left(\frac{Z_A + Z_B}{R}\right)$ ... (iii)

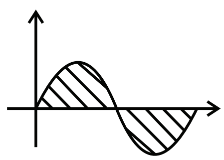
$\tan(-\theta) = \tan(-\theta_2) + \tan\theta_1$

$\tan\theta = \tan\theta_2 - \tan\theta_1$

Sol 11: (B) Resonance is a condition of maximum power

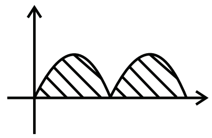
Hence $\cos\phi = 1$.

Sol 12: (B) In calculating the rms value, we square each value.



A

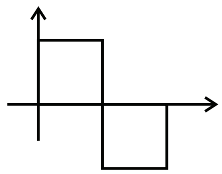
Hence both A and B have same square value at every point.



B

Hence $i_{rmsA} = i_{rmsB}$

Here we have every value greater than that of I_{rms} in graph A or graph B.



$\therefore (i_{rms})_C > I_A = I_B$.

Sol 13: (D) Initially in LR circuit;

$$\cos\theta_1 = \left(\frac{R}{\sqrt{R^2+9R^2}}\right) \Rightarrow \cos\theta_1 = \left(\frac{R}{R\sqrt{10}}\right)$$

$$P_1 = \frac{1}{\sqrt{10}}$$

Now finally

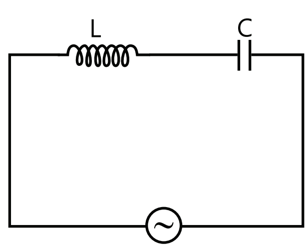
$X_L - X_C = 3R - R = 2R$

$$\cos\theta_2 = \left(\frac{R}{\sqrt{R^2+4R^2}}\right)$$

$$P_2 = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{1}{\sqrt{10}}.\sqrt{5} \Rightarrow \frac{P_2}{P_1} = \sqrt{2}$$

Sol 14: (D) $Z_{net} = Z_L + Z_C$



$V = V_0 \cos \omega t$

$$Z_{net} = i\omega L + \left(\frac{-i}{\omega C}\right)$$

$$\Rightarrow Z_{net} = i\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z_{net} = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2} \angle \left|\frac{\pi}{2}\right|$$

$$\tilde{i} = \frac{\tilde{V}}{\tilde{Z}_{net}} \Rightarrow \frac{V_0}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2}} \angle - \left|\frac{\pi}{2}\right|$$

$$V_L = \tilde{i}\,\tilde{Z}_L$$

$$V_L = \left(\frac{V_0}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2}} \angle - \left|\frac{\pi}{2}\right|\right) \omega L \angle \frac{\pi}{2}$$

$$V_L = V^1 \angle \frac{\pi}{2} - \left|\frac{\pi}{2}\right|$$

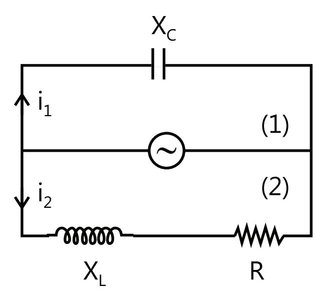
$$V_C = \tilde{i}\,\tilde{Z}_C$$

$$V_C = V^1 \angle - \frac{\pi}{2} - \left|\frac{\pi}{2}\right|$$

$$V_C = V^1 \angle - \left(\frac{\pi}{2} + \left|\frac{\pi}{2}\right|\right)$$

Hence phase difference between V_L and V_C will be π and between V_L and I will be $\pm\frac{\pi}{2}$. Graph D satisfies all the conditions.

Sol 15: (A) Let us consider mesh (1);



$$\tilde{V} = V_0 < 0$$

$$\tilde{Z}_1 = \tilde{Z}_C = \frac{1}{\omega C} \angle - \frac{\pi}{2}$$

$$\tilde{i}_1 = \frac{\tilde{V}}{\tilde{Z}_1} = \frac{V_0 \angle 0}{\frac{1}{\omega C} \angle - \frac{\pi}{2}}$$

$$i_1 = V_0 \omega C \angle \frac{\pi}{2} \qquad \text{... (i)}$$

Now in mesh (2)

$$\tilde{Z}_2 = Z_R + Z_L = R + i\,\omega L$$

$$\tilde{Z}_2 = \sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\tilde{i}_2 = \frac{\tilde{V}}{\tilde{Z}_2} = \frac{V\angle 0}{\sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

$$\tilde{i}_2 = i_0^1 \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Phase difference between i_1 and $i_2 = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L}{R}\right)$

$= \frac{\pi}{2} - \tan^{-1}\left(\frac{X_L}{R}\right)$

Multiple Correct Choice Type

Sol 16: (D) Using intuition;

Let us go for capacitance in the box

$\therefore$ Q = CV

$$\frac{dq}{dt} = C\frac{dv}{dt}$$

Given $i = \frac{dq}{dt}$ = constant

$\therefore$ $\frac{dv}{dt}$ = constant

$\therefore$ Graph looks like a straight line.

$$i = C\frac{dv}{dt}$$

Slope of the graph = $\frac{8-2}{3} = 2$

$\because$ i = 2C = 1 amp

$C = \frac{1}{2}$ C = 0.5 C.

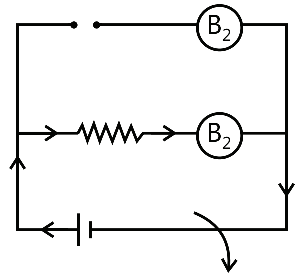
Sol 17: (D) Time constant $\tau = \left(\frac{L}{R}\right)$

Energy stored in magnetic field = $\frac{1}{2}LI^2$

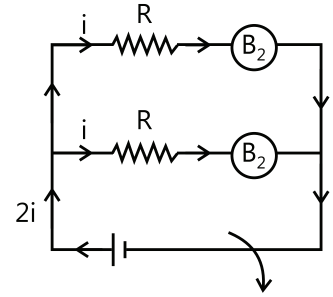
Power dissipated in resistor = I^2R

$$\therefore\ 2\left[\frac{\frac{1}{2}LI^2}{I^2R}\right] = \tau$$

Sol 18: (A) At t = 0;

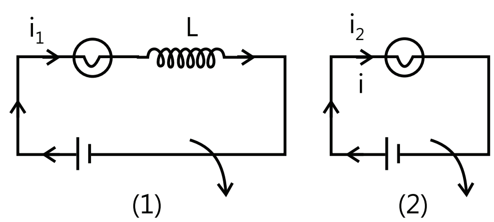


At t = ∞;



Hence B_2 lights up early; but finally both B_1 and B_2 shine with equal brightness.

Sol 19: (B)



Just after switch is closed, Inductor tries to oppose the current 'i_1'. Hence $i_1 < i_2$. As time goes on, the opposition given by inductor reduces.

This opposition is due to the induced EMF in 'L'.

Sol 20: (B, C, D) Emf induced in coil 1 = $L_1\frac{di_1}{dt}$

$$E_2 = L_2\frac{di_2}{dt}$$

Given that $\frac{di_1}{dt} = \frac{di_2}{dt}$

$$\therefore\ \frac{E_1}{E_2} = \frac{L_1}{L_2} = 4$$

$$\therefore\ \frac{V_2}{V_1} = \frac{1}{4}$$

And also given that power given to the two coils is same,

$\therefore$ $V_i i_1 = V_2 i_2$

$$\frac{i_1}{i_2} = \frac{V_2}{V_1} \Rightarrow \frac{i_1}{i_2} = \frac{1}{4}$$

$$W_1 = \frac{1}{2}L_1I_1^2 \text{ and } W_2 = \frac{1}{2}L_2I_2^2$$

$$\frac{W_1}{W_2} = \left(\frac{L_1}{L_2}\right)\left(\frac{I_1}{I_2}\right)^2 \Rightarrow \frac{W_1}{W_2} = \left(\frac{8}{2}\right)\left(\frac{1}{4}\right)^2$$

$$\therefore \frac{W_1}{W_2} = \frac{1}{4}.$$

Sol 21: (A, B, C) RC and $\frac{L}{R}$ will have the dimensions of time and hence $\frac{1}{RC}$ and $\frac{R}{L}$ will have dimensions of frequency.

Sol 22: (D) When just after battery is connected, current is zero in the circuit, and hence will follow magnetic field energy $\left(\frac{1}{2}LI^2\right)$ and power delivered (I^2R) is also zero.

EMF induced is $\left(L\frac{di}{dt}\right)$. Hence there is a finite value.

Sol 23: (B, D) At time t = 0, capacitor is short circuited,

Inductor is open circuited.

At t = ∞, capacitor is open circuited,

Inductor is short circuited.

Hence both the options follow from this.

Sol 24: (D) $M.\frac{di_B}{dt} = \frac{d\phi_A}{dt}$

$$M\frac{\Delta i_B}{\Delta t} = \frac{\Delta\phi_A}{\Delta t}$$

$$\therefore \Delta i_B = \frac{\Delta\phi_A}{M}$$

$$\Delta I_B = \frac{4}{2}$$

ΔI_B = 2A

$$\frac{\Delta\phi_B}{\Delta t} = M\frac{\Delta i_A}{\Delta t}$$

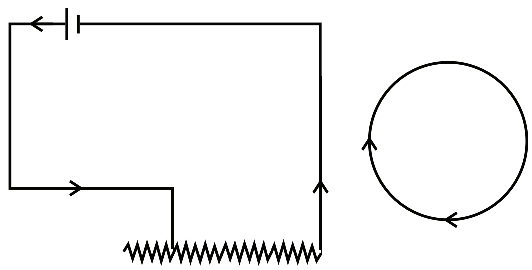
$\Delta\phi_B$ = 2(1) = 2

But given the values of 4 weber.

Hence options D isn't true.

Assertion Reasoning Type

Sol 25: (C) Magnetic field is into the page



As resistance is increasing, current decreases

∴ Magnetic field decreases.

Hence there will b e a clockwise current in the ring.

Sol 26: (D) In an LCR circuit,

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$i_{max} = \frac{V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$(V_R)_{max} = \frac{R \cdot V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}};$$

$$(V_L)_{max} = \frac{\omega L . V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Now $(V_R)_{max} = V_{max}$; at resonance condition, $(X_L - X_C = 0)$,

now for $(V_L)_{max}$; we can set conditions,

(a) R ?0 and (b) $X_L = X_C$;

This will lead to $(V_L)_{max} > V_{max}$.

Sol 27: (A) When circuit is suddenly switched off, there will be a change in current, and it will lead to induced EMF.

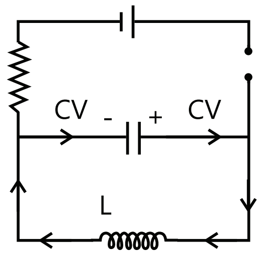
$$|E| = L\left|\frac{di}{dt}\right|$$

Now for large 'L', E is also high.

Comprehension Type

Paragraph 1

Sol 28: (D) Now when S_1 is opened and S_2 is closed



At t = 0; energy stored is purely in capacitor.In this type of circuits, charge and current will be in the form of sin or cos. Thus oscillatory.

$$q = Q_0 \cos\left(\frac{1}{\sqrt{LC}}t\right) \; ; Q_0 = CV$$

$$i = \frac{-1}{\sqrt{LC}} Q_0 \sin \omega t$$

$$i = \frac{Q_0}{\sqrt{LC}} = \frac{CV}{\sqrt{LC}} = V\sqrt{\frac{C}{L}}$$

Hence option D.

Sol 29: (C) $q = Q_0 \cos\left(\frac{1}{\sqrt{LC}}t\right)$

$$\frac{dq}{dt} = \frac{-Q_0}{\sqrt{LC}}\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

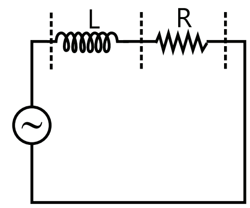
$$\frac{d^2q}{dt^2} = \frac{-Q_0}{LC}\cos\left(\frac{1}{\sqrt{LC}}t\right)$$

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q,$$

Hence option 'C'.

Paragraph 2

Sol 30: (D)



$V = V_0 \cos \omega t$

$$Z_{net} = R + i\omega L$$

$$|Z_{net}| = \sqrt{R^2 + \omega^2 L^2}\; ; \; \tilde{Z}_{net} = |\tilde{Z}_{net}| \angle \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = \frac{V_0\angle 0}{|Z_{net}| \angle \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

$$\tilde{I} = \frac{\tilde{V}}{|Z_{net}|}\angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Now potential difference across resistance,

$$V_R = \tilde{i} \times \tilde{Z}_R$$

$$= \left[\frac{V_0}{|Z_{net}|}\angle -\tan^{-1}\left(\frac{\omega L}{R}\right)\right]\left[R\angle 0\right]$$

$$V_R = \frac{V_0 R}{|Z_{net}|}\angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$(V_R)_{max} = \frac{V_0 R}{\sqrt{R^2 + X_L^2}} \equiv 4 \text{ volts (given)} \qquad \text{... (i)}$$

$$\left(\tilde{V}_L\right) = \left(\tilde{i}\right)\left(\tilde{Z}_L\right)$$

$$= \left[\frac{V_0}{|Z_{net}|}\angle -\tan^{-1}\left(\frac{X_L}{R}\right)\right]\left[\omega L\angle \frac{\pi}{2}\right]$$

$$\left(\tilde{V}_L\right) = \left[\frac{V_0 X_L}{\sqrt{R^2 + X_L^2}}\angle \frac{\pi}{2} - \tan^{-1}\left(\frac{X_L}{R}\right)\right]$$

$$(V_L)_{max} = \frac{V_0 \; X_L}{\sqrt{R^2 + X_L^2}} \equiv 3 \text{ V} \qquad \text{... (ii)}$$

$$\frac{(i)}{(ii)} = \frac{R}{X_L} = \frac{4}{3}$$

$$\therefore \; \frac{R}{X_L} = \frac{4}{3} \Rightarrow X_L = \frac{3R}{4} \qquad \text{... (iii)}$$

$$R^2 + X_L^2 = R^2 + \frac{9R^2}{16} = \frac{25R^2}{16}$$

$$\sqrt{R^2 + X_L^2} = \frac{5R}{4} \quad \text{... (iv)}$$

In equation (i)

$$\frac{V_0 R}{\sqrt{R^2 + X_L^2}} = 4\,; \quad \frac{V_0 R}{\frac{5R}{4}} = 4$$

$V_0 = 1$ V

you can just start from here if you understand how I wrote them

$$\tilde{V}_R = 4\angle - \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\Leftrightarrow V_R = 4\cos\left(\omega t - \tan^{-1}\left(\frac{X_L}{R}\right)\right)$$

$$\tilde{V}_L = 3\angle\frac{\pi}{2} - \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\Leftrightarrow V_L = 3\cos\left(\omega t + \frac{\pi}{2} - \tan^{-1}\left(\frac{X_L}{R}\right)\right)$$

$$V_L \equiv 3\sin\left(\omega t - \tan^{-1}\frac{X_L}{R}\right)$$

Given $V_R = 2$

$$\therefore 2 = 4\cos\left(\omega t - \tan^{-1}\left(\frac{X_L}{R}\right)\right)$$

$$\frac{1}{2} = \cos\left(\omega t - \tan^{-1}\left(\frac{X_L}{R}\right)\right)$$

$$\therefore \omega t - \tan^{-1}\left(\frac{X_L}{R}\right) = \frac{\pi}{3} \rightarrow (X_1)$$

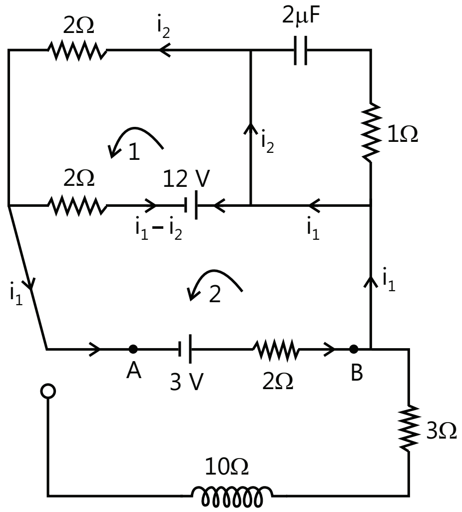
Now $V_L = 3\sin\left(\frac{\pi}{3}\right)$; $\quad V_L = 3\sin 60° = \frac{3\sqrt{3}}{2}$

Sol 31: (B) $V_{source} = V_L + V_R = \frac{3\sqrt{3}}{2} + 2$

$$V_{source} = \frac{4 + 3\sqrt{3}}{2}$$

Previous Years' Questions

Sol 1: In steady state no current will flow through capacitor. Applying Kirchhoff's second law in loop 1:



$-2i_2 + 2(i_1 - i_2) + 12 = 0$

$\therefore 2i_1 - 4i_2 = -12$

or $i_1 - 2i_2 = -6$ ...(i)

Applying Kirchhoff's second law in loop 2:

$-12 - 2(i_1 - i_2) + 3 - 2i_1 = 0$

$4i_1 - 2i_2 = -9$ ...(ii)

Solving Equations (i) and (ii), we get

$i_2 = 2.5$ A and $i_1 = -1$A

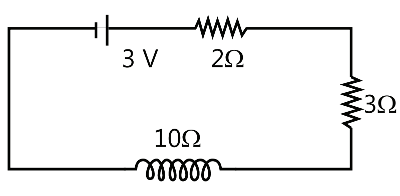
Now, $V_A + 3 - 2i_1 = V_B$

or $V_A - V_B = 2i_1 - 3$

$= 2(-1) - 3 = -5$V

$P_{R_1} = (i_1 - i_2)^2 R_1 = (-1 - 2.5)^2 (2) = 24.5$ W

(b) In position 2: Circuit is as under



Steady current in R_4:

$$i_0 = \frac{3}{3+2} = 0.6 \text{ A}$$

Time when current in R_4 is half the steady value

$$t_{1/2} = \tau_L (\ln 2) = \frac{L}{R}\ \ell n\ (2) = \frac{(10\times10^{-3})}{5}\ \ell n\ (2)$$

$= 1.386 \times 10^{-4}$ s

$$U = \frac{1}{2}Li^2 = \frac{1}{2}(10 \times 10^{-3})\ (0.3)^2 = 4.5 \times 10^4 J$$

Sol 2: In circuit (p): I can't be non-zero in steady state.

In circuit (q): $V_1 = 0$ and $V_2 = 2I = V$ (also)

In circuit (r): $V_1 = X_L I = (2\pi fL)I$

$= (2\pi \times 50 \times 6 \times 10^{-3})I = 1.88I$

$V_2 = 2I$

In circuit (s): $V_1 = X_L I = 1.88\ I$

$$V_2 = X_C I = \left(\frac{1}{2\pi fC}\right)\ell$$

$$= \left(\frac{1}{2\pi\times50\times3\times10^{-6}}\right) = I = (1061)\ I$$

In circuit (t): $V_1 = IR = (1000)\ I$

$V_2 = X_C I = (1061)I$

Therefore the correct options are as under

(A) → r, s, t (B) → q, r, s, t

(C) → q or p, q (D) → q, r, s, t

Sol 3: (B) Charge on capacitor at time t is

$q = q_0\ (1 - e^{-t/\tau})$

Here $q_0 = CV$ & $t = 2t$

Here $q_0 = CV(1 - e^{-2\tau/\tau}) = CV\ (1 - e^{-2})$

Sol 4: (B) From conservation of energy,

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV^2 \therefore I_{max} = V\sqrt{\frac{C}{L}}$$

Sol 5: (C) Comparing the LC oscillations with normal SHM, we get

$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

Here, $\omega^2 = \frac{1}{LC}$

$$\therefore Q = -LC\frac{d^2Q}{dt^2}$$

Sol 6: After a long time, resistance across an inductor becomes zero while resistance across capacitor becomes infinite. Hence, net external resistance,

$$R_{net} = \frac{\frac{R}{2}+R}{2} = \frac{3R}{4}$$

Current through the batteries, $i = \dfrac{2E}{\frac{3R}{4}+r_1+r_2}$

Given that potential across the terminals of cell A is zero.

$\therefore E - ir_i = 0$

or $E - \left(\dfrac{2E}{3R/4 + r_1 + r_2}\right)r_1 = 0$

Solving this equation, we get, $R = \frac{4}{3}(r_1 - r_2)$

Sol 7: Inductive reactance $X_L = \omega L$

$= (50)\ (2\pi)\ (35 \times 10^{-3}) = 11W$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2 + 11)^2}$

$= 11\sqrt{2}\ \ \Omega$

Given $V_{rms} = 220$ V

Hence, amplitude of valtage $V_0 = \sqrt{2}\ \ V_{rms}$

$= 220\sqrt{2}\ \ V$

$\therefore$ Amplitude of current $i_0 = \dfrac{V_0}{Z} = \dfrac{220\sqrt{2}}{11\sqrt{2}}$

or $i_0 = 20$ A

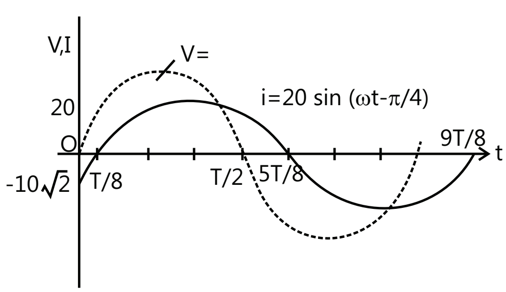
Phase difference $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right) = \tan^{-1}\left(\dfrac{11}{11}\right)$

$\phi = \dfrac{\pi}{4}$

In L-R circuit voltage leads the current. Hence, instantaneous current in the circuit is,

$i = (20\ A)\ \sin\ (\omega t - \pi/4)$

Corresponding i-t graph is shown in figure.



Sol 8: (C) When e^- has zero kinetic energy total energy is shared by antineutrino and proton. This time energy of antineutrino is its maximum possible kinetic energy.

As antineutrino is very light mass in comparison to proton so it will have almost contribution in total energy.

$\therefore$ Its energy is almost 0.8×10^6 eV

Sol 9: (C, D) As current leads voltage by $\pi/2$ in the given circuit initially, then ac voltage can be represented as

$V = V_0 \sin \omega t$

$\therefore \quad q = CV_0 \sin \omega t = Q \sin \omega t$

Where, $Q = 2\times10^{-3}C$

- At $t = 7\pi/6\omega$; $I = -\frac{\sqrt{3}}{2}I_0$ and hence current is anticlockwise

- Current 'i' immediately after $t = \frac{7\pi}{6\omega}$ is

$i = \frac{V_c + 50}{R} = 10$ A

Charge flow $= Q_{final} - Q_{(7\pi/6\omega)} = 2\times10^{-6}C$