Chapter 5

Information Theory and Coding

CHAPTER HIGHLIGHTS

- Information Theory
- Uncertainty of Information
- Entropy of Binary Memory Less Source
- Shannon's Source Coding Theorem
- Discrete Memory Less Channel
- Deterministic Channel
- Binary Symmetric Channel
- Conditional Entropy
- Mutual Information
- Channel Capacity
- Shannon's Channel Coding Theorem
- Differential Entropy
- Information Capacity Theorem for AWGN Channel
- Shannon's Limit
- Cyclic Redundancy Check Codes
- Channel Coding
- Hamming Code

UNCERTAINTY OF INFORMATION

If a source is emitting a symbol S_i with probability p_i, the amount of information gained after observing S_i is $\log_2\left(\frac{1}{P_i}\right)$

Similarly, $\log_2\left(\frac{1}{P_i}\right)$ is the amount of uncertainty resolved after observing the symbol S_i.

Entropy

If a source is emitting the symbols $S_0, S_1 \ldots S_{N-1}$ with probabilities $P_0, P_1, \ldots\ldots P_{N-1}$.

Entropy is defined as average information per symbol of the source.

$$H = \sum_{K=0}^{N-1} P_K \; I(S_K)$$

$$H = \sum_{K=0}^{N-1} P_K \; \log_2\left(\frac{1}{P_K}\right)$$

The entropy H is maximum, if all the symbols occur with equal probability.

$$P_0 = P_1 = P_2 \ldots\ldots\ldots\ldots\ldots P_{N-1} = \frac{1}{N}$$

In this case, the maximum entropy is given by

$$H = \log_2{}^{N}$$

$$0 \le H \le \log^{N}{}_2$$

$H = 0$, if the probability of one symbol is unity and the probability of all remaining symbols is zero.

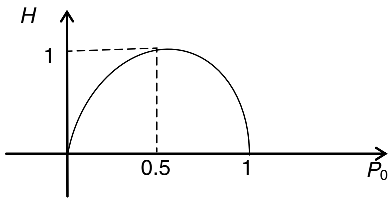
Entropy of Binary Memory Less Source

If the source emitting symbols 0 and 1 with probability P_0 and $1 - P_0$.

$$H = P_0 \log_2 \frac{1}{P_0} + (1 - P_0)\log_2 \frac{1}{1 - P_0}$$

If $P_0 = \frac{1}{2}$ $H = 1$ bit

If $P_0 \to 1$ or 0, $H = 0$ bits



SHANNON'S SOURCE CODING THEOREM

Given a discrete memory less source with entropy H, then the average code word length $\bar{L} \ge H$,

where H represents a fundamental limit on the average number of bits per source symbol (code word length).

The coding efficiency of a particular source code is given by

$$\eta = \frac{H}{\bar{L}}$$

One important variety of source coding technique is prefix coding.

Prefix coding guarantees

$$H \le \bar{L} \le H + 1$$

Huffman coding and Lempel-Ziv coding are the examples of prefix coding techniques. Both Huffman and Lempel-Ziv coding are used for data compaction to remove redundancy in data.

Discrete Memory less Channel

If $x = \{x_0, x_1, \ldots\ldots\ldots\ldots x_{J-1}\}$ is input alphabet to a channel and $Y = \{y_0, y_1 \ldots\ldots y_{K-1}\}$ is the output alphabet of the channel, then a discrete memory-less channel is defined by the matrix,

$$P = \begin{bmatrix} p(y_0 / x_0)\; P(y_1 / x_0) \ldots\ldots\ldots P(y_{k-1} / x_0) \\ p(y_0 / x_1)\; P(y_1 / x_1) \ldots\ldots\ldots P(y_{k-1} / x_1) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ P(y_0 / x_{J-1})\; P(y_1 / x_{J-1}) \ldots\ldots\ldots P(y_{K-1} / x_{J-1}) \end{bmatrix}$$

Every element in the discrete memory-less channel matrix is the conditional probability $P(y_k/x_j)$.

$P(y_k/x_j)$ is the probability of receiving y_k if we transmit x_j.

The DMC matrix satisfies

$$\sum_{K=0}^{K-1} P(y_k / x_j) = 1 \text{ for all } j$$

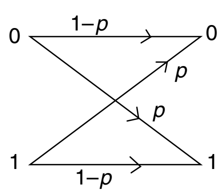
$$P(y_k) = \sum_{j=0}^{j-1} P(y_k / x_j) P(x_j) \text{ for all } K.$$

Deterministic Channel

A channel whose behaviour is entirely determined by its initial states or inputs are known as deterministic channel. If the same input information is passed from this channel over and over again, then the output will be always same.

Binary Symmetric Channel

In BSC, the input to the channel is either 0 or 1 and output of BSC is again either 0 or 1.



BSC can be mentioned as $\begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$

Conditional Entropy

Conditional entropy $H(X/Y)$ represents the amount of uncertainty remaining about the channel input after the channel output has been observed.

$$H(X/Y) = \sum_{k=0}^{K-1} \sum_{j=0}^{J-1} P(x_j, y_k) \log_2 \left[\frac{1}{P(x_j / y_k)} \right]$$

Mutual Information

Mutual information $I(X; Y)$ is the uncertainty about input X resolved by the channel.

$$I(X\,;\,Y) = H(X) - H(X/Y)$$

Properties of mutual information:

1. $I(X, Y) = I(Y; X)$
2. $I(X\,;\,Y) \ge 0$
3. If x and y are independent $H(X/Y) = H(X)$
   $I(X; Y) = 0$

Channel Capacity

Channel capacity = max I $(X\,;Y)$ $\{p(x_j)\}$

Channel Capacity of BSC

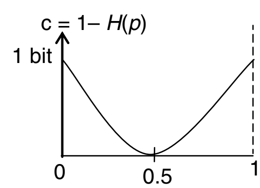
$$C = \max I(X\,;\,Y) = \max\,[H(X) - H(X/Y)]$$

For BSC $\begin{bmatrix} 1-P & P \\ P & 1-P \end{bmatrix}$

$$H(X/Y) = P \log\frac{1}{P} + (1-P)\log\frac{1}{1-P} = H(P)$$

Max $H(x) = 1$ bit

$\therefore\; C = 1 - H(P)$



Shannon's Channel Coding Theorem

Let H is the entropy of a discrete memory less source and the source is producing a symbol for every T_s.

Let C be the capacity of a discrete memory less channel and the channel is used once for every T_c. Then, reliable data transmission is possible if

$$\frac{H(\Im)}{T_S} \le \frac{C}{T_C}$$

For a binary symmetric channel, reliable data transmission is possible if rate of transmission $R \le C$.

Differential Entropy

Differential entropy is used to calculate the entropy for continuous sources.

$$h(X) = \int_{-\infty}^{\infty} f_X(x) \log_2 \left(\frac{1}{f_X(x)} \right) dx$$

For a given variance, the differential entropy is maximum if $f_X(x)$ is Gaussian distributed.
For the given σ_x^2, the maximum value of differential entropy is given by

$$h(X) = \frac{1}{2}\log_2\left(2\pi\, e\sigma_x^2\right)$$

$$I(x\,;y) = h(x) - h(x/y)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{xy}(x,y)\log_2\left[\frac{f_X(x/y)}{f_X(x)}\right]dx\,dy$$

Information Capacity Theorem for AWGN Channel

The information capacity of a continuous channel of bandwidth BHz, perturbed by additive white Gaussian noise of power spectral density $\frac{N_0}{2}$ and limited in bandwidth to B is given by

$$C = B\log_2\left(1+\frac{P}{N_0 B}\right)\text{bits/s}$$

$\frac{P}{N_0 B}$ is the SNR of the channel.

$\therefore\ C = B\log_2(1 + \text{SNR})$ bits/s

where P is the average transmitted power of signal.

Shannon's Limit

$$C = B\log_2\left(1+\frac{P}{N_0 B}\right)$$

$$P = E_b C$$

$$\frac{C}{B} = \log_2\left(1+\frac{E_b}{N_0}\cdot\frac{C}{B}\right)$$

$$\therefore\ \frac{E_b}{N_0} = \frac{2^{C/B}-1}{C/B}$$

For infinite bandwidth, $\frac{C}{B}\to 0$

$$\frac{E_b}{N_0} = \frac{C}{B}\to 0\,\frac{2^{C/B}-1}{C/B} = \log_e^2 = 0.69$$

$$= -1.6\text{ dB}$$

−1.6 dB is the minimum E_b/N_0 required to transmit, provided we have infinite bandwidth. This limiting value is called Shannon's limit.

Cyclic Redundancy Check Codes

CRC codes are used for error detection in the transmission. Binary (n, k) CRC codes are capable of detecting the following error patterns:

1. All error bursts (continuous sequence of bits in error) of length $(n - K)$ or less.
2. A fraction of error bursts of length equal or greater than $n - K + 1$. The fraction equal to $1 - 2^{-(n-K-1)}$
3. All combinations of $d_{min} - 1$ (or fewer) errors.
4. All error patterns of odd number of errors, if the generator polynomial $g(X)$ has an even number of coefficients.

Solved Examples

Example 1

A source produces four symbols with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ and $\frac{1}{8}$. For this source, a particular coding scheme has an average code word length of 3 bits/symbol. The efficiency of this code is

(A) 0.9 (B) 0.8 (C) 0.6 (D) 0.4

Solution

$$\text{Efficiency of code} = \frac{H}{\bar{L}}$$

$$H = \frac{1}{2}.1+\frac{1}{4}.2+\frac{1}{8}.3+\frac{1}{8}.3 = \frac{14}{8}$$

$$\eta = \frac{14/8}{3} = \frac{14}{24} = 60\%\text{ or }0.6$$

Example 2

An image uses 1,024 × 1,024 pixel elements. Each of the pixel can take any of the eight distinguishable intensity levels. The maximum entropy of the abovementioned image will be

(A) 3×2^{10} bits (B) 2×2^{20} bits
(C) 3×2^{20} bits (D) 2×2^{13} bits

Solution

The maximum entropy of the pixel = $\log^8_2$ = 3 bits. Maximum entropy of entire picture =

$$1{,}024\times 1{,}024\times 3 = 3\times 2^{20}\text{ bits}$$

Example 3

A source generates three symbols with probabilities 0.25, 0.5, and 0.25 at a rate of 5,000 symbols/s. The most efficient source encoder would have average bit rate as

(A) 5,000 bits /s (B) 7,500 bits/s
(C) 10,000 bits/s (D) 2,500 bits/s

Solution

Average bit rate = 5,000 × $H(\rceil)$

$$H(\rceil) = \frac{1}{4}.2+\frac{1}{2}.1+\frac{1}{4}.2$$

$$= 1.5\text{ bits}$$

Average bit rate = 7,500 bits

Example 4

If a source emitting four alphabets with probabilities P_1, P_2, P_3, P_4 and the entropy of this source is $H(x_1)$. If another source emitting four symbols with probabilities $P_1 + \delta$, $P_1 - \delta$, P_3, P_4 and the entropy of this source is $H(x_2)$. Which of the following statement is correct if δ is a small positive constant?

(A) $H(x_1) = H(X_2)$ (B) $H(x_1) > H(x_2)$
(C) $H(x_1) < H(x_2)$ (D) $H(x_1) \le H(x_2)$

Solution

The entropy is maximum if, the probability of symbols is equal, that is, the variation in the probability of different symbols is zero.

In X_2, the variation in probabilities are more when compared with X_1. Thus, $H(x_1) > H(x_2)$

Example 5

If X is a Gaussian random variable with mean zero and variance 10. The differential entropy of X is

(A) 1.52 bits (B) 3.8 bits
(C) 2.0 bits (D) 5.0 bits

Solution

Differential entropy

$$h(X) = \frac{1}{2}\log_2\left(2\pi e\sigma^2\right)$$

$$= \frac{1}{2}\log_2\left(2\times 3.14\times 2.7\times 10\right) = 3.8 \text{ bits}$$

Example 6

An AWGN channel is used to transmit the symbols at an SNR of 30 dB. The channel capacity per symbol in this channel is

(A) 2 bits (B) 4 bits (C) 5 bits (D) 8 bits

Solution

$$\text{SNR} = 30 \text{ dB} = 10^3 = 1{,}000$$

$$\text{Channel capacity } C = \frac{1}{2}\log(1+SNR)$$

$$= \frac{1}{2}\log_2(1+1{,}000)$$

$$= \frac{1}{2}\log_2(1{,}001) = 5 \text{ bits}$$

Channel Coding

Channel codes are used to correct the transmission errors and also to get a trade-off between power required to transmit and bandwidth.

Examples of channel codes are Hamming code, BCH codes, RS codes, convolution codes, turbo codes, low density parity check codes, etc.

Channel encoder converts as k-bit message block into n-bit code block by adding $n - k$ redundant bits. This code is called (n,k,d) code, where d is the minimum distance between any two-code words. If 'd' is the minimum distance of the code, the code can correct up to $\frac{d-1}{2}$ errors and detect up to $\frac{d}{2}$ errors.

Channel coding is performed by a $k \times n$ generator matrix G to generate linear block code.

$$C = mG$$

$$C = i \times n \text{ matrix}$$

$$G = k \times n$$

The correctness of a received word can be checked by a parity check matrix H.

Every code word 'C' satisfies the equation

$$HC^{T} = 0$$

The order of H is $(n - k) \times n$. The minimum distance of code = the number of linearly independent columns in the parity check matrix + 1.

For linear block code, the linear combinations of rows of generator matrix spans the code book.

Hamming Code

Hamming code is as single-error correcting channel code with minimum distance three.

For this code,
n = number of code bits = $2^m - 1$
k = number of message bits = $2^m - m - 1$
$d_{min} = 3$
Example of Hamming codes are as follows:

1. For $m = 10$, (1023, 1013, 3)
2. For $m = 8$, (255, 247,3)

Exercises

Practice Problems I

Direction for questions 1 to 16: Select the correct alternative from the given choices.

1. A data packet of 100-bit size is passed through a binary symmetric channel with cross-over probability 0.1. The data packet is including Hamming code with one-bit error correcting capability. What is the probability of reliable reception of packet?

   (A) $(0.9)^{99}$ (B) $(0.9)^{99} + (0.9)^{100}$
   (C) $10 \times (0.9)^{98}$ (D) $2 \times (0.9)^{99}$

2. Entropy of a random variable X is given by 3 bits. The entropy of a new random variable $Y = 2X$ is given by

   (A) 6 bits (B) 1.5 bits
   (C) 4.5 bits (D) 3.0 bits

3. A source x emits eight symbols with equal probability. The entropy of the source X is
(A) 4 bits (B) 2 bits
(C) 3 bits (D) 2.5 bits

4. If U and V are identical distributed independent random variables taking the values 1 and 2 with equal probability. The entropy of $H(U + V)$ is
(1) 1 bits (B) 1.5 bits
(C) 2 bits (D) 2.5 bits

5. The requirement in an AWGN communication channel is to transmit at the rate $\frac{C}{B} = 2$ bits. The minimum ratio of $\frac{E_b}{N_0}$ required for reliable transmission is
(A) 2.5 (B) 2.0 (C) 1.5 (D) 4.0

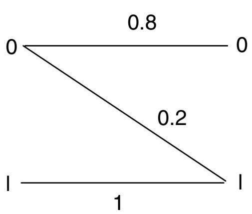
6. A binary symmetric channel is characterized with a cross-over probability of 0.1. If $H(P)$ is the entropy of a binary source with symbol probabilities P and $1 - P$, the channel capacity of abovementioned binary symmetric channel is
(A) $1 + H(0.1)$ (B) $1 + H(0.2)$
(C) $1 - H(0.2)$ (D) $1 - H(0.1)$

7. $X(t)$ is a continuous Gaussian source with mean 2 and variance 10. The difference entropy of the source $X(t)$ is given by
(A) 3.2 bits (B) 4.3 bits
(C) 3.7 bits (D) 2.5 bits

8. An AWGN channel is used at an SNR of 20 dB. The maximum number of bits transmitted per symbol in this channel is
(A) 2.5 bits (B) 8.2 bits
(C) 4.7 bits (D) 3.3 bits

9. Hamming code is used to correct single error in the channel. If the block length after coding in Hamming code is 1,023 bits, the code rate approximately is
(A) 0.99 (B) 0.96 (C) 0.5 (D) 0.82

10. A binary communication channel is defined as in the following figure.



If symbol 0 and 1 transmitted with equal probability, the conditional entropy $H(Y/X)$ is given by
(A) 0.7 bits (B) 0.82
(C) 0.48 (D) 0.38

11. Parity check matrix of an error correcting code is of the order of 3×7. The number of code words in the code book are
(A) 8 (B) 16 (C) 32 (D) 128

12. Minimum distance of a linear block code is 10. Consider the following two statements. S1: The code can correct up to 4 errors S2: The code can detect up to 5 errors
(A) S1 is true, S2 is not true
(B) Both S1 and S2 are true
(C) S1 is not true, S2 is true
(D) Both S1 and S2 are not true

13. Two sources X and Y emit the symbols 1, 2, 3, 4, and 5 with the following probabilities?

	1	2	3	4	5
X	0.2	0.3	0.1	0.4	0
Y	0.2	0.3	0.1	0.2	0.2

Which of the following statement is true?
(A) $H(x) > H(y)$ (B) $H(x) = H(y)$
(C) $H(x) < H(y)$ (D) $H(x) \geq H(y)$

14. A 10-kHz binary symmetric channel with cross-over probability zero is used for transmission of baseband data. The maximum data rate is
(A) 10 kbps (B) 20 kbps
(C) 40 kbps (D) 5 kbps

15. For a (3,1) repetition code, the parity check matrix H is given by

(A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

16. If X, Y, and Z are three independent random variable taking the values -1 and 1 with equal probability the joint entropy $H(X, Y + Z)$ is given by
(A) 2 bits (B) 3 bits
(C) 4 bits (D) 2.5 bits

Practice Problems 2

Direction for questions 1 to 15: Select the correct alternative from the given choices.

1. A TV image consists 500 lines with 600 pixels each line. Each pixel takes eight different values with equal probability. The maximum information contained in one image is
(A) 2.5 mb (B) 24 mb
(C) 1.5 mb (D) 0.3 mb

2. Differential entropy of a zero mean Gaussian random variable X is 5 bits. The differential entropy of as new random variable $y = 4x$ is
(A) 5 bits (B) 10 bits
(C) 20 bits (D) 7 bits

3. If X and Y are independent random variables, the value of conditional entropy $H(X/Y)$ is
(A) Zero (B) $H(X)$
(C) $H(Y)$ (D) $H(X) - H(Y)$

4. A parity check matrix of a block code have an order of 4×10. The rate of the code is
(A) 0.4 (B) 0.6 (C) 0.8 (D) 0.25

5. A source is emitting three symbols $S1$, $S2$, and $S3$ with probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. If a binary code of $\{1,01,001\}$ is used to represent the symbols $\{S1, S2, S3\}$, the efficiency of code is
(A) 86% (B) 74% (C) 56% (D) 92%

6. A source alphabet of 16 symbols are required to transmit in a noise-free communication channel. The channel capacity of the channel is
(A) 4 bits
(B) 16 bits
(C) 8 bits
(D) given data is not adequate

7. In a binary symmetric channel, the probability of error is 0.5. The channel capacity of this channel is
(A) 1 bit (B) 0.5 bits
(C) 0.25 bits (D) 0

8. A Gaussian random variable X is having differential entropy of 10 bits. The entropy of the new random variable $X + 4$ is
(A) 10 bits (B) 12 bits (C) 14 bits (D) 20 bits

9. A generator matrix of linear block code is given by

$$G = \begin{bmatrix} 1\,0\,0\,1 \\ 0\,1\,0\,1 \\ 0\,0\,1\,1 \end{bmatrix}$$

Which of the following statement is correct?
(A) The code corrects one-bit error
(B) The code detects one-bit error
(C) The code detects up to two-bit errors
(D) The code neither corrects nor detects any errors.

10. The generator matrix of a linear block code is given in the following matrix.

$$G = \begin{bmatrix} 1\,1\,0\,0 \\ 0\,0\,1\,1 \\ 1\,1\,1\,1 \end{bmatrix}$$

The number of code words in this code is
(A) 8 (B) 4 (C) 6 (D) 12

11. If X and Y are two random variables, which of the following statement is true?
(A) $H(X) \le H(X/Y)$ (B) $H(X) \ge H(X)Y)$
(C) $H(X) > H(X/Y)$ (D) $H(X) < H(X/Y)$

12. If X & Y are two random variables, which of the following statement is true?
(A) $H(X+Y) = H(X) + H(Y)$
(B) $H(X+Y) > H(X) + H(Y)$
(C) $H(X+Y) \le H(X)+H(Y)$
(D) $H(X+Y) \ge H(X) + H(Y)$

13. The generator matrix of a linear block code is mentioned as

$$G = \begin{bmatrix} 1\,0\,1\,0\,1 \\ 1\,1\,1\,0\,0 \\ 1\,0\,0\,0\,1 \end{bmatrix}$$

The minimum distance of the code is
(A) 1 (B) 2 (C) 3 (D) 4

14. A source is emitting four symbols with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ and $\frac{1}{8}$. The entropy of the source is
(A) 2 bits (B) 1.5 bits
(C) 1.75 bits (D) 1.25 bits

15. A Hamming code is used with the block length 255. The rate of this code is
(A) 0.944 (B) 0.926
(C) 0.967 (D) 0.984

Previous Years' Questions

1. Consider a binary digital communications system with equally likely 0s and 1s. When binary 0 is transmitted, the voltage at the detector, input can lie between the levels−0.25 V and +0.245 V with equal probability. When binary 1 is transmitted, the voltage at the detector can be any value between 0 and 1 V with equal probability. If the detector has a threshold of 0.2 V (i.e., if the received signal is greater than 0.2 V, the bit is taken as 1), the average bit error probability is **[2004]**
(A) 0.15 (B) 0.2 (C) 0.05 (D) 0.5

2. A source generates three symbols with probabilities 0.25, 0.25, and 0.50 at a rate of 3,000 symbols/s. Assuming independent generation of symbols, the most efficient source encoder would have average bit rate as **[2006]**

(A) 6,000 bits/s (B) 4,500 bits/s
(C) 3,000 bits/s (D) 1,500 bits/s

3. A source generates three symbols with probabilities 0.25, 0.25, and 0.50 at a rate of 3,000 symbols/s. Assuming independent generation of symbols, the most efficient source encoder would have average bit rate as **[2007]**
(A) 6,000 bits/s (B) 4,500 bits/s
(C) 3,000 bits/s (D) 1,500 bits/s

4. A memory less source emits n symbols each with a probability p. The entropy of the source as a function of n **[2008]**
(A) increases as log n
(B) decreases as log $(1/n)$
(C) increases as n
(D) increases as n log n

5. Consider a Binary Symmetric Channel (BSC) with probability of error being p. To transmit a bit, say 1, we transmit a sequence of three 1s. The receiver will interpret the received sequence to represent 1 if at least two bits are 1. The probability that the transmitted bit will be received in error is **[2008]**
(A) $p^3 + 3p^2(1-p)$ (B) p^3
(C) $(1-p)^3$ (D) $p^3 + p^2(1-p)$

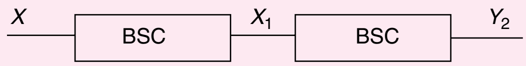
6. A communication channel with AWGN operating at a signal to noise ratio SNR >> 1 and bandwidth B has capacity C_1. If the SNR is doubled keeping B constant, the resulting capacity C_2 is given by **[2009]**
(A) $C_2 \approx 2C_1$ (B) $C_2 \approx C_1 + B$
(C) $C_2 \approx C_1 + 2B$ (D) $C_2 \approx C_1 + 0.3B$

7. A source alphabet consists of N symbols with the probability of the first two symbols being the same. A source encoder increases the probability of the first symbol by a small amount ε and decreases that of the second by ε. After encoding, the entropy of the source **[2012]**
(A) increases (B) remains the same
(C) increases only if $N = 2$ (D) decreases

8. A binary symmetric channel (BSC) has a transition probability of 1/8. If the binary transmit symbol X is such that $P(X = 0) = 9/10$, then the probability of error for an optimum receiver will be **[2012]**
(A) 7/80 (B) 63/80
(C) 9/10 (D) 1/10

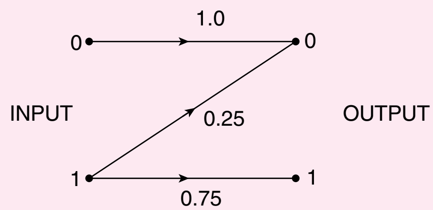
9. The capacity of a Binary Symmetric Channel (BSC) with cross-over probability 0.5 is _______ **[2014]**

10. A binary random variable X takes the value of 1 with probability 1/3. X is input to a cascade of two independent identical binary symmetric channels (BSCs) each with crossover probability ½. The output of BSCs are the random variables Y_1 and Y_2, as shown in the figure.



The value of $H(Y_1) + H(Y_2)$ in bits is __________. **[2014]**

11. Consider the Z-channel given in the figure. The input is 0 or 1 with equal probability.



If the output is 0, the probability that the input is also 0 equals _____________ **[2014]**

12. A sinusoidal signal of amplitude A is quantized by a uniform quantizer. Assume that the signal utilizes all the representation levels of the quantizer. If the signal to quantization noise ratio is 31.8 dB, the number of levels in the quantizer is _________. **[2015]**

13. Consider a discrete memory less source with alphabet $S = \{s_0, s_1, s_2, s_3, s_4, \ldots\}$ and respective probabilities of occurrence $P = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots\right\}$. The entropy of the source (in bits) is __________ . **[2016]**

14. A digital communication system uses a repetition code for channel encoding/decoding. During transmission, each bit is repeated three times (0 is transmitted as 000, and 1 is transmitted as 111). It is assumed that the source puts out symbols independently and with equal probability. The decoder operates as follows:

In a block of three received bits, if the number of zeros exceeds the number of ones, the decoder decides in favor of a 1. Assuming a binary symmetric channel with crossover probability $p = 0.1$, the average probability of error is __________ . **[2016]**

15. A discrete memoryless source has an alphabet $\{a_1, a_2, a_3, a_4\}$ with corresponding probabilities $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right\}$. The minimum required average codeword length in bits to represent this source for error-free reconstruction is __________. **[2016]**

16. An information source generates a binary sequence $\{\alpha_n\}$. α_n can take one of the two possible values −1 and +1 with equal probability and are statistically independent and identically distributed. This sequence is precoded to obtain another sequence $\{\beta_n\}$, as $\beta_n = \alpha_n + k\,\alpha_{n-3}$.

The sequence $\{\beta_n\}$ is used to modulate a pulse $g(t)$ to generate the base band signal:

$$X(t) = \sum_{n=-\infty}^{\infty} \beta_n g(t - nT) \text{where}$$

$$g(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}.$$

If there is a null at $f = \frac{1}{3T}$ in the power spectral density of $X(t)$, then k is ________. **[2016]**

17. A binary communication system makes use of the symbols "zero" and "one". There are channel errors. Consider the following events:

x_0: a "zero" is transmitted

x_1: a "one" is transmitted

y_0: a "zero" is received

y_1: a "one" is received

The following probabilities are given: $P(x_0) = \frac{1}{2}$ $P(y_0/x_0) = \frac{3}{4}$ and $P(y_0/x_1) = \frac{1}{2}$. The information in bits that you obtain when you learn which symbol has been received (while you know that a "zero" has been transmitted) is ________. **[2016]**

0.81 bits per symbol

18. An analog baseband signal, bandlimited to 100Hz, is sampled at the Nyquist rate. The samples are quantized into four message symbols that occur independently with probabilities $P_1 = P_4 = 0.125$ and $P_2 = P_3$. The information rate (bits/sec) of the message source is ___________ . **[2016]**

19. The bit error probability of a momoryless binary symmetric channel is 10^{-5}. If 10^5 bits are sent over this channel, then the probability that not more than one bit will be in error is ___________. **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. C	**2.** D	**3.** C	**4.** B	**5.** C	**6.** D	**7.** C	**8.** D	**9.** A	**10.** D
11. B	**12.** B	**13.** C	**14.** B	**15.** A	**16.** D				

Practice Problems 2

1. C	**2.** D	**3.** B	**4.** B	**5.** A	**6.** A	**7.** D	**8.** A	**9.** B	**10.** B
11. B	**12.** C	**13.** A	**14.** C	**15.** C					

Previous Years' Questions

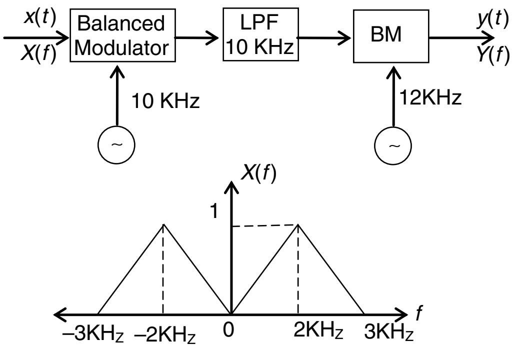
1. A	**2.** B	**3.** B	**4.** A	**5.** A	**6.** C	**7.** D	**8.** A	**9.** −0.01 to 0.01
10. 2	**11.** 0.8	**12.** 32	**13.** 2 bits	**14.** 0.028	**15.** 1.75	**16.** −1	**17.** 0.81 bits per symbol	
18. 362.3 bits/sec		**19.** 0.735						

COMMUNICATIONS

Time: 60 Minutes

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. An antenna current of an AM transmitter is 5 amp when the carrier is not modulated, and it is increased to 6 amp when the carrier is modulated. The efficiency of AM transmitter is -
   (A) 20% (B) 30.5%
   (C) 41.3% (D) 33.3%
2. Consider the signal shown in figure.



   The positive frequencies where $X(f)$ have spectral peaks
   (A) 3 kHz and 20 kHz (B) 4 kHz and 20 kHz
   (C) 2 kHz and 18 kHz (D) 5 kHz and 20 kHz
3. An AM signal with modulation index = 5 can be detected by _____
   (A) square law detector (B) envelope detector
   (C) synchronous detector (D) None of these
4. Match the following List-I with List-II

List-I		List-II	
i	AM	(1)	Broad casting
ii	SSB	(2)	Radio
iii	FM	(3)	Voice communication
iv	VSB	(4)	Video

   (A) i – 1, ii – 3, iii – 2, iv – 4
   (B) i – 1, ii – 4, iii – 3, iv – 2
   (C) i – 4, ii – 3, iii – 2, iv – 1
   (D) i – 4, ii – 2, iii – 1, iv – 3
5. The PDF of a Gaussian random variable X is given by
   $$p_x(x) = \frac{1}{2\sqrt{2\pi}}\exp\left[-\frac{(x-2)^2}{18}\right]$$
   The probability of the event $\{X = 2\}$ is _____
   (A) $\frac{1}{2}$ (B) $\frac{1}{2\sqrt{2\pi}}$
   (C) 2 (D) 0
6. Bit error occurs independently with probability 'p', during the transmission over a communication channel. If a block of n-bits is transmitted, the probability of at most two-bit error is equal to
   (A) $(n-2)\,p\,(1-p)^{n-2}$
   (B) $1-(1-p)^{n-1}$
   (C) $p+(n-2)(1-p)$
   (D) ${}^nC_0(1-p)^n + {}^nC_1\,(p)^1\,(1-p)^{n-1} + {}^nC_2\,(p)^2\,(1-p)^{n-2}$
7. A frequency-modulated signal 5 cos $[2\pi\times10^5t + \sin(2\pi\times2{,}000t) + 2.5\sin(2\pi\times500\,t)]$ with carrier frequency is 10^5 Hz. The modulation index is
   (A) 1.6 (B) 1.8 (C) 12.5 (D) 15
8. A 5 MHz carrier is frequency modulated by a sinusoidal signal of 1 kHz, the maximum frequency deviation being 40 kHz. The bandwidth required, as given by the Carson's rule is ________
   (A) 80 kHz (B) 82 kHz
   (C) 81 kHz (D) 42 kHz
9. A carrier is phase modulated with frequency deviation of 20 kHz by a single-tone frequency of 500 Hz. If the single-tone frequency is decreased to 300 Hz, and phase deviation is remains unchanged, the band width of PM signal is
   (A) 40.3 kHz (B) 20 kHz
   (C) 24.6 kHz (D) 12.3 kHz
10. The signal $\cos(\omega_ct) + 0.75\cos(\omega_{mt})\cos(\omega_ct)$ is
    (A) FM only (B) AM only
    (C) both AM and FM (D) neither AM nor FM
11. The PSD (power spectral density) and the power of a signal $x(t)$ are $S_x(\omega)$ and P_x, respectively. The PSD and power of a signal $4x(t)$ are _____
    (A) 16 $S_x(\omega)$ and 4 P_x (B) 8 $S_x(\omega)$ and 8 P_x
    (C) 16 $S_x(\omega)$ and 16 P_x (D) 4 $S_x(\omega)$ and 4 P_x
12. An energy signal is $f(t) = 8\,\delta(t)$. Its energy density spectrum is
    (A) 16 (B) 8 (C) 1 (D) 64
13. Highest value of autocorrelation of a function 20 $\sin(10\pi)$ is equal to
    (A) 0 (B) 200
    (C) 100 (D) depends upon time t
14. The joint probability density of random variables X and Y is
    $$f(x, y) = \frac{1}{2}e^{x-|y|}, -\infty < x < \infty, -\infty < y < \infty$$
    probability that $X \le 0,\ Y \le 1$

(A) $\frac{2-e^{-1}}{2}$ (B) $\frac{1}{2}$

(C) $\frac{e^{-1}}{2}$ (D) $\frac{1-e^{-1}}{2}$

15. A random process, $x(t) = A\cos(\omega t + \theta)$ where θ is uniform random variable in the range $[-\pi, \pi]$ and A, ω are constant. Then, $x(t)$ is
(A) wide sense stationary
(B) Ergodic random process
(C) Multiple random process
(D) None of these

16. pdf $f(x) = ae^{-2|x|}$, where X is a random variable and $-\infty < X < \infty$
The probability that X lies between 2 and 4.
(A) 1 (B) 0
(C) $\frac{a}{2}[e^{-8} - e^{-4}]$ (D) $\frac{a}{2}[e^{-4} - e^{-8}]$

17. A modulating signal $m(t) = 5\sin(4\pi \times 10^3 t)$ is used to modulated a carrier of frequency 10^7 Hz. The band width when amplitude of modulating signal is halved and phase modulation is used ($\beta_p = 10$) and ($\beta_p = K_p A_m$)
(A) 24 kHz (B) 48 kHz
(C) 44 kHz (D) 20 kHz

18. If a pdf is given as

$$p(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1/2\pi & \text{for } 0 \le x \le 2\pi \\ 0 & \text{for } \ge 2\pi \end{cases}$$

Then the average value $E\{x\}$ is
(A) π (B) 2π (C) 3π (D) 2

19. For a random variable X, the probability density function is given as

$$p(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{elsewhere} \end{cases}$$

The variance σx^2 for random variable X is

(A) $\frac{1}{2}(b^3 - a^3) + \frac{1}{4}(b-a) - \frac{1}{3}(b^2 - a^2)$

(B) $\frac{(a-b)^2}{12}$

(C) $\frac{1}{(b-a)}$

(D) $\frac{1}{3}(b^2 - a^2) - \frac{1}{2}(b-a)$

20. A sinusoidal $A\sin(\omega_o t + \phi)$ is generated through a random process called $X(t)$ where $X(t) = A\sin(\omega_o t + \phi)$ and ϕ is random phase angle then
(A) Autocorrelation and autocovariance functions are identical.
(B) Autocorrelation and autocovariance are identical, that is $\phi = 0$.
(C) Autocorrelation and autocovariance never be identical.
(D) None of these

21. A deterministic energy signal

$f(t) = \text{rect}\left(\frac{t}{\tau}\right)$ is given

The ESD for this signal is
(A) Sampling square function
(B) Triangular function
(C) Impulse function
(D) Gate function

22. A Gaussian white noise is passes through an ideal low-pass filter. The power at output of the ideal LPF is if f_c is cut-off frequency of LPF
(A) $2\eta f_c$ (B) $\eta f_c/2$
(C) $\eta f_c/4$ (D) None of these

23. An LTI system with unknown impulse response $h(t)$ is excited with a white noise random process $n_i(t)$ with power spectral density $S_{nini} = \eta/2$. The output noise is $n_o(t)$. Then, the crosscorrelation between input and output process is given by ________

(A) $(2\eta)h(\tau)$ (B) $\left(\eta^2/2\right)h(\tau)$

(C) $(\eta)h(\tau)$ (D) $\left(\eta/2\right)h(\tau)$

24. A process which is not purely indeterministic and process $X(t) = \cos(\omega_o t + \phi)$.
So the autocovariance of the process $X(t)$ is
Where ϕ is random variable.

(A) 0 (B) $\frac{1}{4}\cos^2(\omega_0 k)$

(C) $1/2\cos(\omega_0 k)$ (D) $1/4\cos(\omega_0 k)$

25. The values of constant k, so that $f(x) = ke^{-x}$ is a probability density function, on [0, 1]
(A) 1.5 (B) 1.6 (C) 1.52 (D) 1

26. If pdf $f(x) = \frac{2}{x^2}$ on the interval [1, 2], then

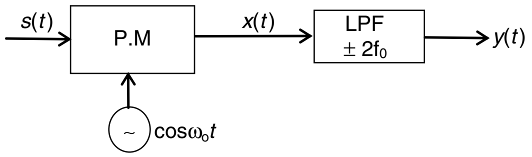
(A) $f(x) \ge 0$, for every x

(B) $\int_1^2 f(x)dx = 1$

(C) $f(x)$ is positive in interval [1, 2]
(D) All of above

Direction for questions 27 and 28:

Signal $s(t) = \cos(\omega_o t) + 2\sin(3\omega_o t) + 0.75\sin(4\omega_o t)$ is applied at the product modulator and then pass through a RC – low-pass filter having cut-off frequency $2f_0$.



27. The output $y(t)$ is

(A) $\frac{1}{2} + \frac{1}{2}\cos(2\omega_o t) + \sin(2\omega_o t)$

(B) $\frac{1}{2}$

(C) $\frac{1}{2}\cos 2\omega_o t$

(D) $2\cos 2\omega_o t + \sin 2\omega_o t$

28. The input $s(t)$ signal power is

(A) 2.78 W (B) 2.75 W

(C) 3.75 W (D) 2 W

Direction for questions 29 and 30:

29. An AM signal $s(t) = 10[1 + 0.75 \text{ Cos } 2\pi\, 10^4\, t] \text{ Cos } 2\pi 10^6 t$ is radiated into free space using an antenna having a resistance of 16Ω. The total power of the modulated signal is ________

(A) 1 W (B) 2 W

(C) 3 W (D) 4 W

30. Modulation efficiency is

(A) 21.95% (B) 22%

(C) 23% (D) 24%

Answer Keys

1. B	**2.** B	**3.** C	**4.** A	**5.** B	**6.** D	**7.** A	**8.** B	**9.** C	**10.** B
11. C	12. D	13. A	14. A	**15.** A	**16.** D	**17.** A	**18.** A	**19.** B	**20.** A
21. A	**22.** D	**22.** D	**24.** C	**25.** B	**26.** D	**27.** A	**28.** A	**29.** D	**30.** A