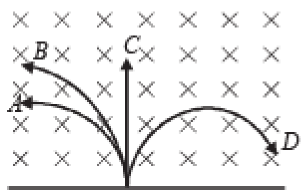
Moving Charges and Magnetism

Fill Ups, True/False

Q.1. A neutron, a proton, and an electron and an alpha particle enter a region of constant magnetic field with equal velocities.The magnetic field is along the inward normal to the plane of the paper. The tracks of the particles are labelled in fig.The electron follows track ...... and the alpha particle follows track .....



Ans. D, B

Solution. According to Fleming's left hand rule, the force on electrons will be towards right (D).

Also, by the same rule we find that the force on proton and a-particle is towards left. Now since the magnetic force will behave as centripetal force, therefore

$$\therefore \quad \frac{mv^2}{r} = qvB$$

$$\therefore \quad \frac{mv}{qB} = r \qquad \text{or} \qquad r \propto \frac{m}{q}$$

For proton $r \propto \frac{1}{1} = 1$; For α-particle $r \propto \frac{4}{2} = 2$

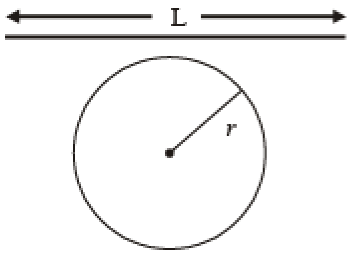
∴ radius will be more for α-particle

∴α-particle will take path B.

Q.2. A wire of length L metre, carrying a current i ampere is bent in the form of a circle. The magnitude of its magnetic moment is ....in MKS units.

Ans. $\frac{iL^2}{4\pi}$

Solution.



Wire of length L is bent in the form of a circle. Then the perimeter of the circle

$$2\pi r = L \Rightarrow r = \frac{L}{2\pi}$$

$$\therefore \quad \text{Area of the circle} = \pi r^2 = \frac{\pi L^2}{4\pi^2} = \frac{L^2}{4\pi}$$

Magnetic moment of a loop in which current i flows is given by

$$M = iA = \frac{iL^2}{4\pi}.$$

Q.3. In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making 10^{16}revolutions per second. The magnetic moment associated with the orbital motion of the electron is ......

Ans. 1.25×10^{-23} Am^2

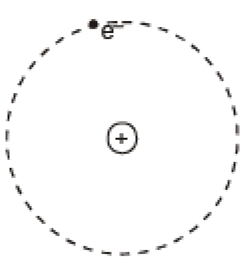
Solution.

$$i = \frac{q}{t} = \frac{ne}{t} = \frac{10^{16}}{1} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-3}\,A.$$

$$M = i \times A = i \times \pi r^2$$

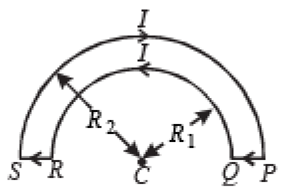
$$= 1.6 \times 10^{-3} \times 3.14 \times 0.5 \times 10^{-10} \times 0.5 \times 10^{-10}$$

$$= 1.25 \times 10^{-23}\,Am^2$$



Q.4. The wire loop PQRSP formed by joining two semicircular wires of radii R_1 and R_2carries a current I as shown. The magnitude of the magnetic

induction at the centre C is ....

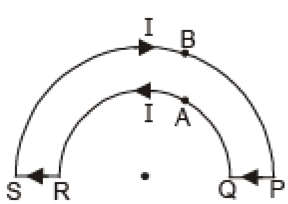


Ans. $\frac{\mu_0 I}{4}\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

Solution. The magnetic field at C due to current in PQ and RS is zero. Magnetic field due to current in semi-circular arc QAR

$$=\frac{1}{2}\left[\frac{\mu_0}{2}\frac{I}{R_1}\right]$$

directed towards reader perpendicular to the plane of paper.



Magnetic field due to current in semi-circular arc

$$SBP=\frac{1}{2}\left[\frac{\mu_0}{2}\frac{I}{R_2}\right]$$

directed away from reader perpendicular to the plane of paper.

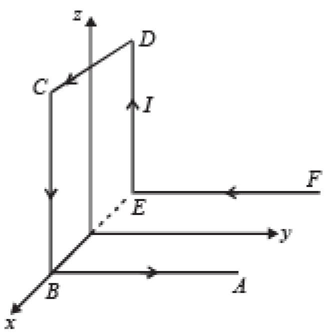
$$\therefore \quad \text{Net Magnetic field}=\frac{1}{2}\left[\frac{\mu_0}{2}\frac{I}{R_1}\right]-\frac{1}{2}\left[\frac{\mu_0}{2}\frac{I}{R_2}\right]$$

(directed towards the reader perpendicular to plane of paper).

$$= \frac{\mu_0 I}{4}\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

Q.5. A wire ABCDEF (with each side of length L) bent as shown in figure and carrying a current I is placed in a uniform magnetic induction B parallel to the positive y-direction.

The force experienced by the wire is .... in the .... direction.



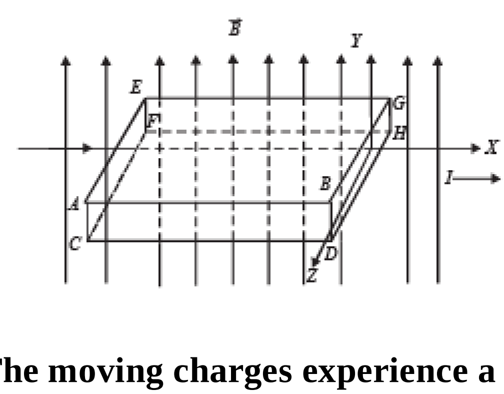
Ans. IlB; +Z direction

Solution. We may assume current to be flowing in segment EB in both directions.

Net force on the loop EDCBE will be zero. Also force due to segment FE and BA will be zero. Force due to segment EB

$$\vec{F} = I[L\hat{i} \times B\hat{j}] = ILB\hat{k}$$

Q.6. A metallic block carrying current I is subjected to a uniform magnetic induction $\vec{B}$ as shown in Figure .



The moving charges experience a force $\vec{F}$ given by .... which results in the lowering of the potential of the face .... Assume the speed of the carriers to be v.

Ans. evB; ABCD

Solution. $\vec{F} = q(\vec{v} \times \vec{B}) = (-e)(-v\hat{i} \times B\hat{j}) = evB\,\hat{k}$

NOTE : The direction of flow of electrons is opposite to that of current.

Q.7. No net force acts on a rectangular coil carrying a steady current when suspended freely in a uniform magnetic field.

Ans. T

Solution. A current carrying coil is a magnetic dipole. The net force on a magnetic dipole placed in uniform magnetic field is zero.

Q.8.There is no change in the energy of a charged particle moving in a magnetic field although a magnetic force is acting on it.

Ans. T

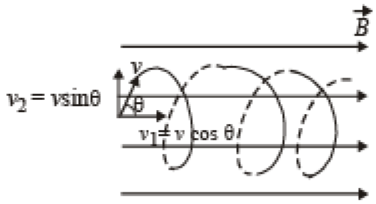
Solution.

NOTE : The magnetic force acts in a direction perpendicular to the direction of velocity and hence it cannot change the speed of the charged particle. Therefore, the kinetic energy $\left(= \frac{1}{2}mv^2\right)$ does not change.

Q.9. A charged particle enters a region of uniform magnetic field at an angle of 85° to the magnetic line of force . The path of the particle is a circle.

Ans. F

Solution. The velocity component v_2 will be responsible in moving the charged particle in a circle.



The velocity component v_1 will be responsible in moving the charged particle in horizontal direction. Therefore the charged particle will travel in a helical path.

Q.10. An electron and a proton are moving with the same kinetic energy along the same direction. When they pass through a uniform magnetic field perpendicular to the direction of their motion, they describe circular paths of the same radius.

Ans. F

Solution. When a charged particle passes through a uniform magnetic field perpendicular to the direction of motion, a force acts on the particle perpendicular to the velocity. This force acts as a centripetal force

$$\therefore \quad \frac{mv^2}{r} = qvB$$

$$r = \frac{\sqrt{2mK}}{qB} \qquad \left[\text{Where } K = \frac{P^2}{2m}\right]$$

$$\therefore \quad r \propto \frac{\sqrt{m}}{q} \qquad [\text{for const. K.E. and } B]$$

Here, q is same for electron and proton

$$\therefore \quad r \propto \sqrt{m}$$

Radius of proton will be more.

Subjective Questions

Q.1. A bar magnet with poles 25 cm apart and of strength 14.4 amp-m rests with centre on a frictionless pivot. It is held in equilibrium at an angle of 60° with respect to a uniform magnetic field of induction 0.25 Wb/m^2, by applying a force F at right angles to its axis at a point 12 cm from pivot.Calculate F. What will happen if the force F is removed?

Ans. 25.98 N

Solution. 2l = 0.25 m

Also, m × 2l = 14.4 $\Rightarrow\ m = \frac{14.4}{0.25} = 57.6\,\text{A-m}^2$

Torque due to magnetic field

$= p_m \times B \times \sin 60° = 14.4 \times 0.25 \times \frac{\sqrt{3}}{2}$

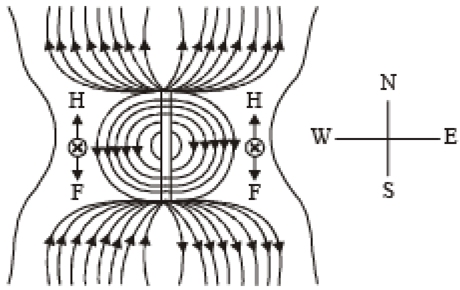
The torque due to the force = F × 0.12

For equilibrium F × 0.12 = 14.4 × 0.25 $\times \frac{\sqrt{3}}{2} \Rightarrow F = 25.98\,\text{N}$

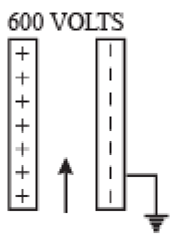
If the force F is removed, the torque due to magnetic field will move the bar magnet. It will start oscillating about the mean position where the angle between $\vec{p}_m$ and $\vec{B}$ is 0.

Q.2. A bar magnet is placed with its north pole pointing north and its south pole pointing south. Draw a figure to show the location of neutral points.

Solution.

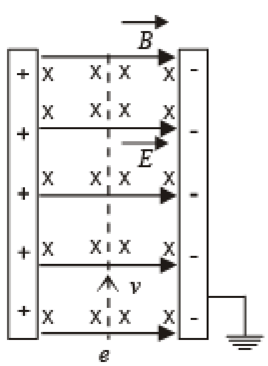


Q.3. A potential difference of 600 volts is applied across the plates of a parallel plate condenser. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 2 × 10^6 m/sec moves undeflected between the plates. Find the magnitude and direction of the magnetic field in the region between the condenser plates. (Neglect the edge effects). (Charge of the electron = – 1.6 × 10^{-19}coulomb)



Ans. 0.1T, Directed perpendicular to the plane of paper inwards.

Solution. The force on electron will be towards the left plate due to electric field and will be equal to F_e= e_E



NOTE : For the electron to move undeflected between the plates there should be a force (magnetic) which is equal to the electric force and opposite in direction. The force should be directed towards the right as the electric force is towards the left.

On applying Fleming's left hand rule we find the magnetic field should be directed perpendicular to the plane of paper inwards. Therefore, Force due to electric field = Force due to magnetic field. eE = evB

$$\therefore \quad B = \frac{E}{v} = \frac{V/d}{v} \qquad \left[\because E = \frac{V}{d}\right]$$

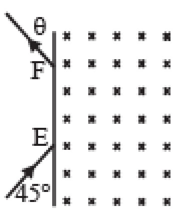
where V = p.d. between plates

d = distance between plates

$$\therefore \quad B = \frac{600/3\times10^{-3}}{2\times10^{6}} = \frac{600}{3\times10^{-3}\times2\times10^{6}}$$

B = 0.1 tesla

Q.4. A particle of mass m = 1.6 × 10^{-27} kg and charge q = 1.6 × 10^{-19} C enters a region of uniform magnetic field of strength 1 tesla along the direction shown in fig. The speed of the particle is 10^{7} m/ s. (i) The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ. (ii) If the direction of the field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.



Ans. (i) 0.1414 m, 45°   (ii)   4.71 × 10^{-8} sec.

Solution.

m = 1.6 × 10^{-27} kg, q = 1.6 × 10^{-19} C

B = 1 T

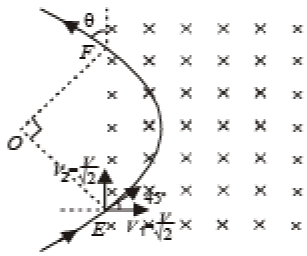
v = 10^{7} m/s

F = q . v B sin α

(acting towards O by Fleming's left hand rule)

⇒ F = qvB [∴α = 90°]

But F = ma



$$\therefore qvB = ma \quad \therefore a = \frac{qvB}{m}$$

$$= \frac{1.6 \times 10^{-19} \times 10^{7} \times 1}{1.6 \times 10^{-27}}$$

= 10^{15} m/s^2

∠ OEF = 45° (∴ OE act as a radius)

By symmetry ∠ OEF = 45°

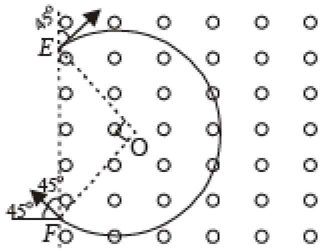
∴ ∠EOF = 90° (by Geometry)

This is the centripetal acceleration

$$\therefore \quad \frac{v^2}{r} = 10^{15} \quad \Rightarrow r = \frac{10^{14}}{10^{15}} = 0.1\text{m}.$$

Therefore EF = 0.141 m.

If the magnetic field is in the outward direction and the particle enters in the same way at E, then according to Fleming's left hand rule, the particle will turn towards clockwise direction and cover 3/4th of a circle as shown in the figure.

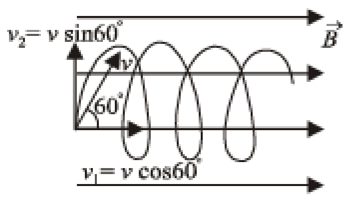


$\therefore$ Time required $= \frac{3}{4} \times \left[\frac{2\pi r}{v}\right] = 4.71 \times 10^{-8}$ sec.

Q.5. A beam of protons with a velocity 4 × 10⁵ m/sec enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation).

Ans. 0.012 m, 0.044 m

Solution.



v_1 is responsible for horizontal motion of proton

v_2 is responsible for circular motion of proton

$$\therefore \quad \frac{mv_2^2}{r} = qv_2B$$

$$r = \frac{mv_2}{qB} = \frac{1.76\times10^{-27}\times4\times10^5\times\sqrt{3}}{1.6\times10^{-19}\times0.3\times2} = 0.012\text{ m}$$

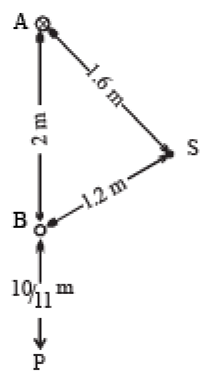
Pitch of helix = $v_1 \times T$

where $T = \frac{2\pi r}{v_2} = \frac{2\pi r}{v \sin\theta}$

$\Rightarrow$ Pitch of helix $= v\cos\theta \times \frac{2\pi r}{v\sin\theta}$

$= 2\pi r \cot\theta = 2 \times 3.14 \times 0.012 \times \cot 60^\circ = 0.044\,\text{m}$

Q.6. Two long straight parallel wires are 2 metres apart, perpendicular to the plane of the paper (see figure). The wire A carries a current of 9.6 amps, directed into the plane of the paper.The wire B carries a current such that the magnetic field of induction at the point P, at a distance of 10/11 metre from the wire B, is zero.



Find :

(i) The magnitude and direction of the current in B .

(ii) The magnitude of the magnetic field of induction at the point S.

(iii) The force per unit length on the wire B.

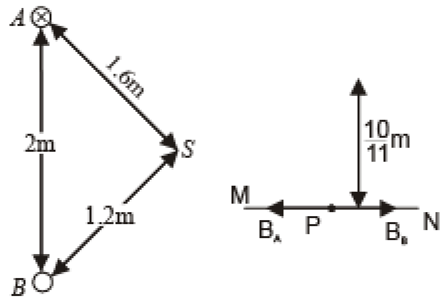
Ans. (i) 3A, upward direction (ii) 1.3 × 10–6 T (iii) 28.8 × 10–7 N

Solution. (i) The magnetic field at P due to current in wire A.

$$B_A = \frac{\mu_0}{4\pi}\frac{2I_A}{r_{AP}} = \frac{\mu_0}{4\pi} \times \frac{2 \times 9.6}{\left(2 + \frac{10}{11}\right)} \text{ (Direction } P \text{ to } M) \quad ...(i)$$

NOTE : The current in wire B should be in upward direction so as to cancel the magnetic field due to A at P. (By right hand Thumb rule)

The magnetic field at P due to current in wire B



$$B_B = \frac{\mu_0}{4\pi} \times \frac{2I_B}{\left(\frac{10}{11}\right)} \quad \text{...(ii)}$$

From (i) and (ii)

$$\frac{\mu_0}{4\pi} \times \frac{2 \times 9.6}{\left(2 + \frac{10}{11}\right)} = \frac{\mu_0}{4\pi} \times \frac{2I_B}{\left(\frac{10}{11}\right)}$$

$$\Rightarrow \quad \frac{9.6 \times 11}{32} = \frac{I_B \times 11}{10} \quad \Rightarrow I_B = \frac{96}{32} = 3\text{A}$$

(ii) The dimensions given shows that

$$SA^2 + SB^2 = AB^2 \Rightarrow \quad \angle ASB = 90^\circ$$

Magnetic field due to A at S

$$B_{SA} = \frac{\mu_0}{4\pi} \cdot \frac{2I_A}{r_{SA}} = \frac{\mu_0}{4\pi} \times \frac{2 \times 9.6}{1.6} \quad \text{(Directed } S \text{ to } B\text{)}$$

Magnetic field due to B at S

$$B_{SB} = \frac{\mu_0}{4\pi} \cdot \frac{2I_B}{r_{SB}} = \frac{\mu_0}{4\pi} \frac{2 \times 3}{1.2} \quad \text{(Directed } S \text{ to } A\text{)}$$

The resultant magnetic field

$$B = \sqrt{B_{SA}^2 + B_{SB}^2} = \frac{\mu_0}{4\pi}\sqrt{\left(\frac{9.6}{0.8}\right)^2 + \left(\frac{3}{0.6}\right)^2}$$

$$= 10^{-7} \times 13 = 1.3 \times 10^{-6}\,\text{T}$$

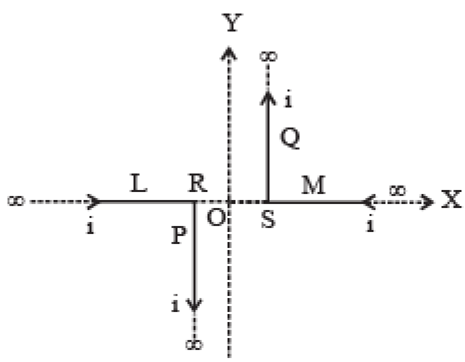
(iii) Force per unit length on wire B

$$= \frac{\mu_0}{4\pi}\frac{2I_A I_B}{r_{AB}}$$

$$= \frac{10^{-7} \times 2 \times 9.6 \times 3}{2} = 28.8 \times 10^{-7}\,\text{N/m}$$

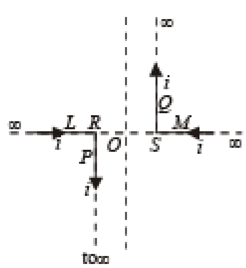
This force will be repulsive in nature.

Q.7. A pair of stationary and infinitely long bent wires are placed in the XY plane as shown in fig. The wires carry currents of i = 10 amperes each as shown. The segments L and M are along the X-axis. The segments P and Q are parallel to the Yaxis such that OS = OR = 0.02 m. Find the magnitude and direction of the magnetic induction at the origin O.



Ans. 10^{-4} tesla; directed towards the reader perpendicular to the plane of paper.

Solution.



∴ Magnetic field due to current carrying conductor P at point O is

$$B_1 = \frac{\mu_0}{4\pi}\frac{i}{(OR)}$$

directed towards the reader perpendicular to the plane of paper.

Magnetic field due to current carrying conductor Q at point O is directed towards the reader perpendicular to the plane of paper.

$$B_2 = \frac{\mu_0}{4\pi}\frac{i}{(OS)}$$

Magnetic field due to current carrying conductors L and M at O is zero.

∴ Resultant magnetic field at O

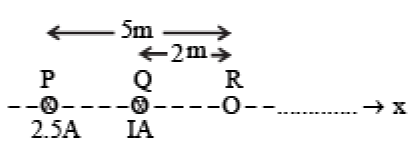
$B = B_1 + B_2$

(directed towards the reader perpendicular to the plane of paper)

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{i}{OR} + \frac{\mu_0}{4\pi}\frac{i}{OS} = \frac{\mu_0}{4\pi} i \left[\frac{1}{OR} + \frac{1}{OS}\right]$$

$$= 10^{-7} \times 10 \times \left[\frac{1}{0.02} + \frac{1}{0.02}\right] = 10^{-4} \text{ tesla.}$$

Q.8. Two long parallel wires carrying current 2.5 amperes and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 metres and 2 metres respectively from a collinear point R (see figure)



(i) An electron moving with a velocity of 4×10^5 m/s along the positive x – direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

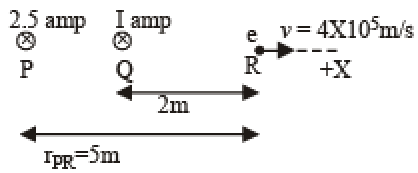
(ii) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 amperes may be placed so that the magnetic induction at R is zero.

Ans. (i) 4 A. (ii) r = 1m where r is the distance from R

Solution. The magnetic field (due to current in wire P) at R

$$= \frac{\mu_0}{4\pi} \times \frac{2I_P}{r_{PR}} = \frac{\mu_0}{4\pi} \times \frac{2 \times 2.5}{5}$$

$= \frac{\mu_0}{4\pi}$ [in the plane of paper downwards]



Similarly, the magnetic field (due to current is wire Q) at R

$$= \frac{\mu_0}{4\pi} \times \frac{2 \times I}{2} = \frac{\mu_0}{4\pi} I$$

[in the plane of paper downwards]

The total magnetic field at R [due to P and Q]

$$B = \frac{\mu_0}{4\pi} + \frac{\mu_0}{4\pi} I = \frac{\mu_0}{4\pi}(1 + I)$$

[in the plane of paper downwards]

The force experienced by the electron

F = qvBsinθ

$$= evB \sin 90° = 1.6 \times 10^{-19} \times 4 \times 10^5 \times \frac{\mu_0}{4\pi}(1 + I)$$

But $F = 3.2 \times 10^{-20}$ N (Given)

$\therefore \quad 3.2 \times 10^{-20} = 1.6 \times 10^{-19} \times 4 \times 10^{5} \times 10^{-7}(1 + I)$

$\Rightarrow \quad I = 4$ amp.

(ii) Let us consider a position between Q and R. The magnetic field produced should be equal to 5×10^{-7}T in the plane of paper acting upwards.

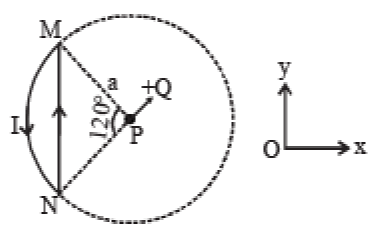
For this let the wire having current 2.5 amp be placed at a distance r from R and current flowing outwards the plane of paper.

$$\therefore \quad 5 \times 10^{-7} = \frac{\mu_0}{4\pi} \times \frac{2 \times 2.5}{r} \text{ or } r = 1 \text{ m}$$

Let us consider another position beyond R collinear with P, Q and R. Let it be placed at a distance r' from R, having current in the plane of paper.

$$\therefore \quad 5 \times 10^{-7} = \frac{\mu_0}{4\pi} \times \frac{2 \times 2.5}{r'} \text{ or } r' = 1 \text{ m}$$

Q.9. A wire loop carrying a current I is placed in the x-y plane as shown in fig.



(a) If a particle with charge +Q and mass m is placed at the centre P and given a velocity $\vec{v}$ along NP (see figure), find its instantaneous acceleration.

(b) If an external uniform magnetic in duction field $\vec{B} = B\hat{i}$ is applied, find the force and the torque acting on the loop due to this field.

Ans.(a) $\frac{0.11\mu_0 IQv}{ma}$ directed 30° with the negative X-axis (b) zero, 0.614 $BIa^2\,\hat{j}$

Solution.

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2I\sqrt{3}}{a}(-\hat{k}); \quad \vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2\pi I}{3a}\hat{k}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\frac{2}{3} - 2\sqrt{3}\right]\hat{k} = \frac{-\mu_0}{4\pi}\frac{2I}{a}(1.4)(\hat{k});$$

$$\vec{v} = v\cos 60\hat{i} + v\sin 60\hat{j}$$

$$\vec{F} = Q(\vec{V}\times\vec{B}) = Q\left[\frac{v}{2}\hat{i} + \frac{\sqrt{3}}{2}v\hat{j}\right]\times\left[\frac{-\mu_0}{4\pi}\frac{2.8I}{a}\hat{k}\right]$$

Now apply $\vec{a} = \frac{\vec{F}}{m}$

(b) KEY CONCEPT : The torque acting on the loop in the magnetic field is given by

$\vec{\tau} = \vec{M}\times\vec{B}$ where $M = IA$

A = (area of PMQNP) – (area of triangle PMN)

$$= \frac{1}{3}(\pi a^2) - \frac{1}{2}\times MN \times PS$$

$$= \frac{\pi a^2}{3} - \frac{1}{2}\times\sqrt{3}a\times\frac{a}{2} = a^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]$$

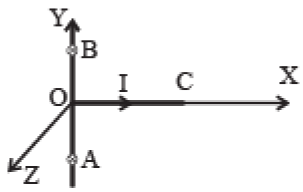
$$\vec{A} = a^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{k}$$

$$\therefore \quad \vec{\tau} = Ia^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{k}\times\hat{i}B$$

$$\vec{\tau} = BIa^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)\hat{j} = 0.614\,BIa^2\hat{j}$$

The force acting on the loop is zero.

Q.10. A straight segment OC (of length L meter) of a circuit carrying a current I amp is placed along the x-axis (Fig.). Two infinetely long straight wires A and B, each extending from z = - ∞ to +∞ , are fixed at y = – a meter and y = + a meter respectively,, as shown in the figure.



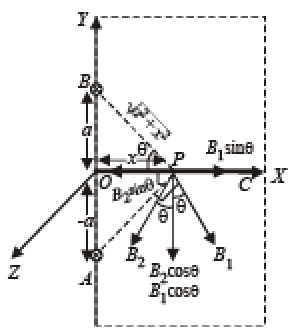
If the wires A and B each carry a current I amp into the plane of the paper, obtain the expression for the force acting on the segment OC. What will be the force on OC if the current in the wire B is reversed ?

Ans. $F = \frac{\mu_0}{4\pi} 2I^2 \left[\log_e \frac{a^2 + L^2}{a^2} \right]$ directed toward –Z direction, zero.

Solution. The magnetic field produced at different points on OC will be different. Let us consider an arbitrary point P on OC which is at a distance x from the origin. Let the magnetic field due to currents in A and B at P be B_1 and B_2 respectively, both being in the X-Y plane.

Let $\angle BPO = \angle APO = \theta$

$$|\vec{B}_1| = \frac{\mu_0}{4\pi} \frac{2I}{\sqrt{a^2 + x^2}} = |\vec{B}_2|$$



On resolving B_1 and B_2 we get xq B1sinc that the sin q components cancel out and the cos q components add up.

Therefore, the total magnetic field at P is

$B = 2B_1 \cos\theta$

$$= \frac{2\mu_0}{4\pi} \frac{2I}{\sqrt{a^2 + x^2}} \times \frac{x}{\sqrt{a^2 + x^2}} = \frac{\mu_0}{4\pi} \frac{4Ix}{(a^2 + x^2)}$$

(towards – Y direction) Let us consider a small portion of wire OC at P of length dx.

The small amount of force acting on that small portion

$$\vec{d}F = I(\vec{d}x \times \vec{B}) \quad \therefore\ dF = I\,dx\,B\sin 90°$$

$$\Rightarrow\ dF = I\,dx \times \frac{\mu_0}{4\pi} \times \frac{4Ix}{(a^2 + x^2)}$$

$$\Rightarrow\ dF = \frac{\mu_0}{4\pi} 4I^2 \frac{xdx}{(a^2 + x^2)}$$

The total force

$$F = \frac{\mu_0}{4\pi} \times 4I^2 \int_0^L \frac{xdx}{(a^2 + x^2)}$$

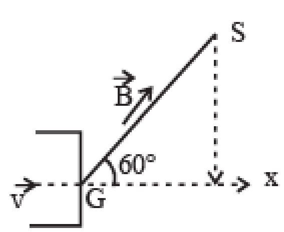
$$= \frac{\mu_0}{4\pi} \times 4I^2 \left[\frac{1}{2}\log_e(a^2 + x^2)\right]_0^L$$

$$\Rightarrow\ F = \frac{\mu_0}{4\pi} \times 2I^2 \left[\log_e \frac{a^2 + L^2}{a^2}\right]$$

To find the direction of force we can use Fleming's left hand rule. The direction of $\vec{F}$ is towards – Z direction.

When the current in wire B is reversed, the resultant magnetic field at any arbitrary point P on OC will be in the X-direction. Since the current is also in X-direction, therefore force acting will be zero (F = I lB sin θ and θ = 180°).

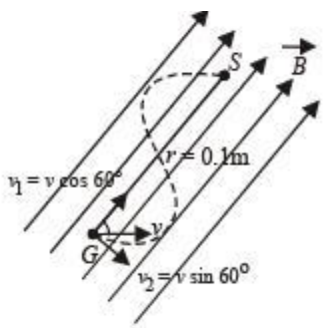
Q.11. An electron gun G emits electrons of energy 2keV travelling in the positive x-direction. The electrons are required to hit the spot S where GS = 0.1m, and the line GS makes an angle of 60° with the x-axis as shown in the fig. A uniform magnetic field $\vec{B}$ parallel to GS exists. Find $\vec{B}$ parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electrons hit S.



Ans. 4.737×10^{-3} T

Solution. (a) Let us resolve the velocity into two rectangular components v_1 (= vcos 60°) and v_2 (= vsin 60°). v_1 component of velocity is responsible to move the charge particle in the direction of the magnetic field whereas v_2 component is responsible for revolving the charged particle in circular motion. The overall path is helical. The condition for the charged particle to strike S with minimum value of B is that Pitch of Helix = GS

$$T \times v_1 = GS \Rightarrow \frac{2\pi m}{qB} \times v\cos 60° = 0.1$$



$$B = \frac{2\pi m v \cos 60°}{q \times 0.1}$$

$$\text{But } \frac{1}{2}mv^2 = E \Rightarrow v = \sqrt{\frac{2E}{m}}$$

$$\therefore \quad B = \frac{2\pi m}{q \times 0.1} \times \sqrt{\frac{2E}{m}} \times \cos 60°$$

$$= \frac{2\pi}{q \times 0.1} \times \sqrt{2mE} \times \cos 60° = \frac{2 \times 3.14}{1.6 \times 10^{-19} \times 0.1}$$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 10^3 \times 1.6 \times 10^{-19}} \times \frac{1}{2}$$

$$= \frac{149.8}{10^{-19}} \times 0.316 \times 10^{-23} = 47.37 \times 10^{-4}$$

$= 4.737 \times 10^{-3}$ T

Q.12. A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20A, is in equilibrium at a height of 0.01 m over

another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30A, as shown in figure . Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations.

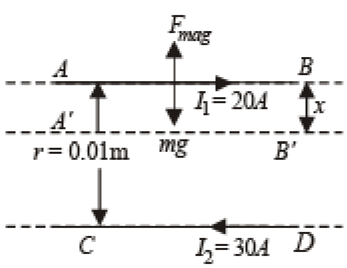
Ans. 0.2 sec.

Solution. When AB is steady,

Weight per unit length = Force per unit length

Weight per unit length = $\frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$ ... (i)

NOTE : When the rod is depressed by a distance x, then the force acting on the upper wire increases and behaves as a restoring force



Restoring force/length = $\frac{\mu_0}{4\pi}\frac{2I_1I_2}{r-x} - \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$

$= \frac{\mu_0}{4\pi}2I_1I_2\left[\frac{1}{r-x} - \frac{1}{r}\right]$

⇒ Restoring force/length = $\frac{\mu_0}{4\pi}2I_1I_2\left[\frac{r-(r-x)}{(r-x)r}\right]$

$= \frac{\mu_0}{4\pi}\frac{2I_1I_2x}{r(r-x)}$

When x is small i.e., x $<< r$ then $r = x \approx r$

Restoring force/length F = $\frac{\mu_0}{4\pi}\frac{2I_1I_2}{r^2}x$

Since, F ∝ x and directed to equilibrium position.

∴ The motion is simple harmonic

$\therefore \quad \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r^2} =$ (mass per unit length) ω^2 ... (ii)

From (i), (Mass per unit length) × g = $\frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$

Mass per unit length = $\frac{\mu_0}{4\pi}\frac{2I_1I_2}{rg}$ ... (iii)

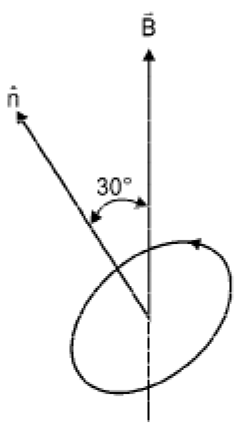
From (ii) and (iii)

$$\frac{\mu_0}{4\pi}\frac{2I_1I_2}{r^2} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{rg} \times \omega^2 \Rightarrow \omega = \sqrt{\frac{g}{r}}$$

$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{r}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{0.01}{9.8}} = 0.2\,\text{sec}$$

Q.13. An electron in the ground state of hydrogen atom is revolving in anticlock-wise direction in a circular orbit of radius R.



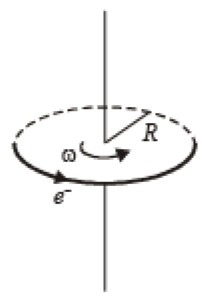
(i) Obtain an expression for the orbital magnetic dipole moment of the electron.

(ii) The atom is placed in a uniform magnetic induction $\vec{B}$ such that the plane-normal of the electron-orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron.

Ans. (i) $M = \frac{he}{4\pi m}$ (ii) $\frac{heB}{8\pi m}$ directed perpendicular to the plane containing $\hat{n}$ and $\vec{B}$.

Solution. (i) KEY CONCEPT : Orbital magnetic dipole moment M = IA where I is the current due to orbital motion of electron and A is the area of loop made by electron.

$$\Rightarrow \; M = \frac{e}{T} \times \pi R^2 \;\Rightarrow\; M = \frac{e\omega}{2\pi} \times \pi R^2$$



$$\Rightarrow \; M = \frac{1}{2} e\omega R^2$$

But according to Bohr's postulate

$$mR\omega^2 = \frac{nh}{2\pi} \Rightarrow R\omega^2 = \frac{nh}{2\pi m}$$

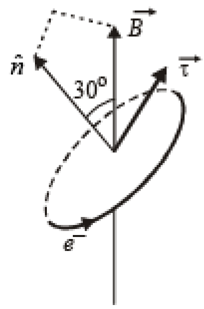
$$\Rightarrow \; M = \frac{e}{2} \times \frac{nh}{2\pi m} = \frac{nhe}{4\pi m} = \frac{eh}{4\pi m} \;(\because n = 1 \text{ for ground state})$$

NOTE : The direction of magnetic momentum is same as the direction of area vector, i.e., perpendicular to the plane of orbital motion.

(ii) KEY CONCEPT : We know that torque

$\vec{\tau} = \vec{M} \times \vec{B} \Rightarrow \tau = MB\sin\theta$

where θ is the angle between M and B



$$\Rightarrow \tau = \frac{he}{4\pi m} \times B\sin 30° = \frac{heB}{8\pi m}$$

NOTE : The direction of torque can be found by right hand thumb rule.

The direction of torque is perpendicular to the plane containing $\hat{n}$ and $\vec{B}$ as shown.

Q.14. Three infinitely long thin wires, each carrying current i in the same direction, are in the x-y plane of a gravity free space. The central wire is along the y-axis while the other two are along x = ± d.

(i) Find the locus of the points for which the magnetic field B is zero.

(ii) If the central wire is displaced along the Z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wires is λ, find the frequency of oscillation.

Ans. **(i)** $\pm\frac{d}{\sqrt{3}}$ **(ii)** $n = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$

Solution. (i) KEY CONCEPT : Magnetic field due to an infinitely long current carrying wire at distance r is given by

$$B = \frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right)$$

The direction of B is given by right hand palm rule.

Hence, in case of three identical wires, resultant field can be zero only if the point P is between the two wires, otherwise field B due to all the wires will be in the same direction and so resultant B cannot be zero. Hence, if point P is at a distance x from the central wire as shown in figure, then,

$$\vec{B}_P = \vec{B}_{PA} + \vec{B}_{PB} + \vec{B}_{PC}$$

where $\vec{B}_{PA}$ = magnetic field at P due to A

$\vec{B}_{PB}$ = Magnetic field at P due to B

$\vec{B}_{PC}$ = Magnetic field at P due to C.

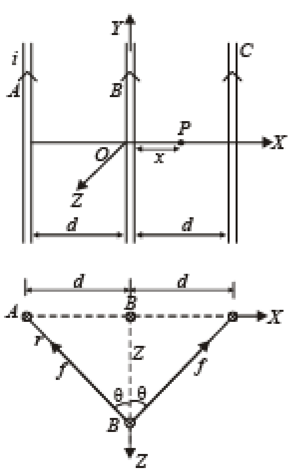
$$\vec{B}_P = \frac{\mu_0}{4\pi} 2i \left[\frac{1}{d+x} + \frac{1}{x} - \frac{1}{d-x} \right] (-\hat{k}).$$

For $\vec{B}_P = 0$, we get $x = \pm\, d/\sqrt{3}$

(ii) KEY CONCEPT : The force per unit length between two parallel current carrying wires is given by

$$\frac{\mu_0}{4\pi} \frac{2 i_1 i_2}{r} = f \text{(say)}$$

and is attractive if currents are in the same direction.



So, when the wire B is displaced along Z-axis by a small distance Z, the restoring force per unit length F/l on the wire B due to wires A and C will be

$$\frac{F}{\ell} = 2f\cos\theta = 2\frac{\mu_0}{4\pi}\frac{2i_1i_2}{r}\times\frac{z}{r} \quad \left[\text{as}\cos\theta = \frac{z}{r}\right]$$

or $$\frac{F}{\ell} = \frac{\mu_0}{4\pi}\cdot\frac{4i^2z}{(d^2+z^2)} \quad [\text{as } I_1 = I_2 \text{ and } r^2 = d^2+z^2]$$

or $$\frac{F}{\ell} = -\frac{\mu_0}{4\pi}\left(\frac{2i}{d}\right)^2 z \quad [\text{as d} >> \text{z and F is opposite to } z] \quad ...(1)$$

Since F ∝ – z, the motion is simple harmonic.

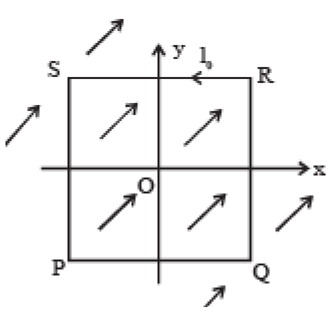
Comparing eq. (1) with the standard equation of S.H.M. which is

$$F = -m\omega^2 z \quad \text{i.e.,} \quad \frac{F}{\ell} = -\frac{m}{\ell}\omega^2 z$$
$$= -\lambda\,\omega^2 z, \text{ we get}$$

$$\lambda\,\omega^2 = \frac{\mu_0}{4\pi}\times\frac{4i^2}{d^2} \Rightarrow \omega = \sqrt{\frac{\mu_0 i^2}{\pi d^2\lambda}}$$

$$\Rightarrow 2\pi n = \frac{i}{d}\sqrt{\frac{\mu_0}{\pi\lambda}} \Rightarrow n = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$

Q.15.A uniform, constant magnetic field B is directed y at an angle of 45° to the x axis in the xy-plane. PQRS is a rigid, square wire frame carrying a steady current I_0, with its centre at the origin O. At time t = 0, the frame is at rest in the position as shown in Figure, with its sides parallel to the x and y axes. Each side of the frame is of mass M and length L.



(a) What is the torque τ about O acting on the frame due to the magnetic field?

(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about this rotation occurs. (Δt is so

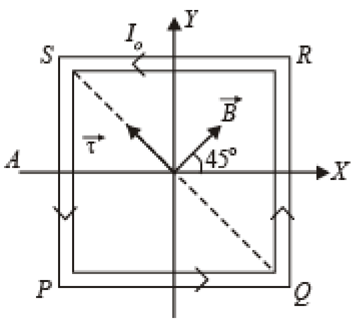
short that any variation in the torque during this interval may be neglected.) Given : the moment of inertia of the frame about an axis through its centre perpendicular to its plane is 4/3 ML^2.

Ans. (a) $\frac{I_0L^2B}{\sqrt{2}}(\hat{j}-\hat{i})$ (b) $\frac{3}{4}\frac{I_0B}{M}\Delta t^2$

Solution. (a) As the magnetic field $\vec{B}$ is in x – y plane and subtends an angle of 45° with the x-axis, hence,

$B_x = B \cos 45° = B/\sqrt{2}$ and

$B_y = B \sin 45° = B/\sqrt{2}$



So, in vector from

$$\vec{B} = \hat{i}\left(\frac{B}{\sqrt{2}}\right) + \hat{j}\left(\frac{B}{\sqrt{2}}\right) \text{ and } \vec{M} = I = I_0L^2\hat{k}$$

$$\text{So, } \vec{\tau} = \vec{M} \times \vec{B} = I_0L^2\hat{k} \times \left(\frac{B}{\sqrt{2}}\hat{i} + \frac{B}{\sqrt{2}}\hat{j}\right) = \frac{I_0L^2B}{\sqrt{2}}(\hat{j}-\hat{i})$$

i.e., torque has magnitude $I_0^2L_2B$ and is directed along line QS from Q to S.

(b) According to th e theorem of perpendicular axes, moment of inertia of the frame about QS.

$$I_{QS} = \frac{1}{2}I_z = \frac{1}{2}\left(\frac{4}{3}ML^2\right) = \frac{2}{3}ML^2$$

Also $\tau = I\alpha$,

$$\therefore \quad \alpha = \frac{\tau}{I} = \frac{I_0L^2B \times 3}{2ML^2} = \frac{3}{2}\frac{I_0B}{M}$$

Here α is constant, therefore we can apply

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \text{ with } \omega_0 = 0, \text{ we have}$$

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{3I_0B}{2M}\right)(\Delta t)^2$$

$$\text{or} \quad \theta = \frac{3}{4}\frac{I_0B}{M}(\Delta t)^2$$

Q.16. The region between x = 0 and x = L is filled with uniform, steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\hat{i}$ travels along x-axis and enters the region of the magnetic field. Neglect gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at angle 30° to its initial velocity.

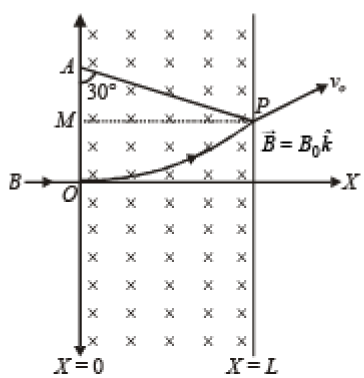
(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now extends up to 2.1L.

$$L = \frac{mv_0}{2qB} \quad \textbf{(b)} \ -v_0\hat{i}, \frac{\pi m}{qB_0}$$

Ans.

Solution. KEY CONCEPT : This question involves a simple understanding of the motion of charged particle in a magnetic field.

(a)



Let the particle emerge out from the region of magnetic field at point P. Then the velocity vector $\vec{v}_0$ makes an angle 30° with x-axis. The normal to circular path at P intersects the negative y-axis at point A.

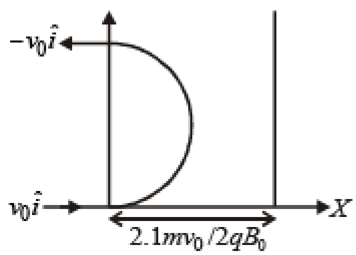
Hence, AO = AP = R = radius of circular path, which can be found as

$$\frac{mv_0^2}{R} = B_0 q v_0 \Rightarrow R = \frac{mv_0}{qB_0} \quad \text{...(i)}$$

In ΔAPM, R sin 30° = L $\Rightarrow \frac{R}{2} = L$ ...(ii)

From (i) and (ii), $L = mv_0/2qB_0$

(b)



As the new region of magnetic field is 2.1 L

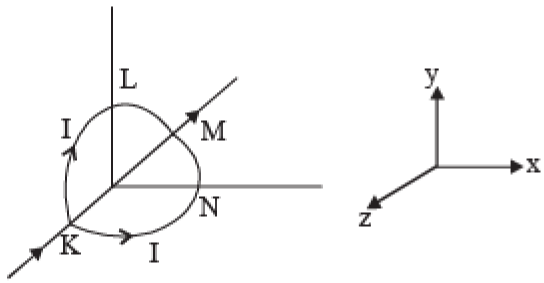
$= \frac{2.1R}{2}$ which is obviously > R.

Thus, the required velocity $= -v_0\hat{i}$.

Since the time period for complete revolution = $2\pi m/qB_0$.

The time taken by the particle to cross the region of magnetic field = $\pi m/qB_0$.

Q.17. A circular loop of radius R is bent along a diameter and given a shape as shown in the figure. One of the semicircles (KNM) lies in the x-z plane and the other one (KLM) in the yz plane with their centres at the origin. Current I is flowing through each of the semi circles as shown in figure.



(a) A particle of charge q is released at the origin with a velocity $\vec{v} = -v_0 \hat{i}$. Find the instantaneous force $\vec{F}$ on the particle. Assume that space is gravity free.

(b) If an external uniform magnetic field $B_o \hat{j}$ is applied, determine the force $\vec{F_1}$ and $\vec{F_2}$ on the semicircles KLM and KNM due to the field and the net force $\vec{F}$ on the loop.

Ans. (a) $\left(\frac{-\mu_0 q v_0 I}{4R}\right)\hat{k}$ (b) $\vec{F_1} = 2BIR\hat{i}$, $\vec{F_2} = 2BIR\hat{i}$, $4BIR\hat{i}$

Solution. (a) Magnetic field $(\vec{B})$ at the origin = Magnetic field due to semicircle KLM + Magnetic field due to other semicircle KNM.

Therefore, $\vec{B} = \frac{\mu_0 I}{4R}(-\hat{i}) + \frac{\mu_0 I}{4R}(\hat{j})$

$$\Rightarrow \quad \vec{B} = -\frac{\mu_0 I}{4R}\hat{i} + \frac{\mu_0 I}{4R}\hat{j} = \frac{\mu_0 I}{4R}(-\hat{i} + \hat{j})$$

[NOTE : The magnetic field $(\vec{B})$ due to a circular current

carrying loop is $\frac{\mu_0 I}{2R}$ ∴For semicircle it is half]

Therefore, magnetic force acting on the particle.

$$\vec{F} = q(\vec{v} \times \vec{B}) = q\left\{(-v_0\hat{i}) \times (-\hat{i} + \hat{j}) \times \frac{\mu_0 I}{4R}\right\}$$

$= \frac{-\mu_0 q v_0 I}{4R} \hat{k}$

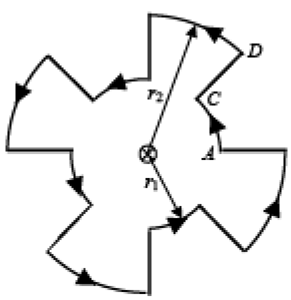
(b) $\vec{F}_{KLM} = \vec{F}_{KNM} = \vec{F}_{KM}$

and $\vec{F}_{KM} = BI(2R)\hat{i} = 2BIR\hat{i}$

Therefore, $\vec{F}_1 = \vec{F}_2 = 2BIR\hat{i}$ or total force on the loop,

$\vec{F} = \vec{F}_1 + \vec{F}_2 \Rightarrow \vec{F} = 4BIR\hat{i}$

Q.18. A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii r_1 = 0.08 m and r_1 = 0.12 m. Each arc subtends the same angle at the center.



(a) Find the magnetic field produced by this circuit at the center.

(b) An infinitely long straight wire carrying a current of 10 A is passing through the center of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the center due to the current in the circuit?

What is the force acting on the arc AC and the straight segment CD due to the current at the center?

Ans. (a) 6.54×10^{-5}T (b) 0, Force on arc AC = 0, 8.1×10^{-6}N

Solution. For finding the magnetic field produced by this circuit at the centre we can consider it to contain two semicircles of radii, r_1 = 0.08 m and r_2 = 0.12 m. Since current is flowing in the same direction, the magnetic field created by circular arcs will be in the same direction and therefore will be added.

$\therefore \quad B_1 = \frac{\mu_0 i}{4r_1}$ and $B_2 = \frac{\mu_0 i}{4r_2} \quad \therefore \quad B = \frac{\mu_0 i}{4}\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$

$\therefore \quad B = (6.54 \times 10^{-5})\,T$ Directed outwards. (Right hand thumb rule)

(b) Force acting on a current carrying conductor placed in a magnetic field is given by

$\vec{F} = I(\vec{\ell} \times \vec{B}) = I\ell B \sin\theta$

(i) For force acting on the wire at the centre In this case θ = 180°

∴ F = 0

(ii) On arc AC due to current at the centre

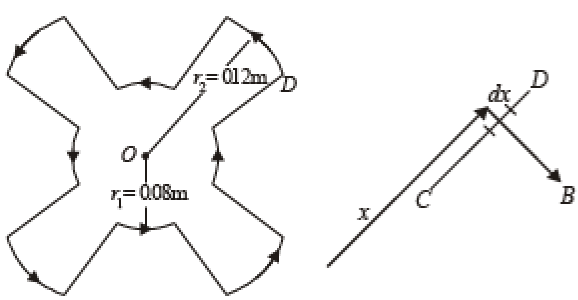
$|\vec{B}|$ on AC will be B = $\frac{\mu_0 I}{2\pi r_1}$

The direction of this magnetic field on any small segment of AC will be tangential

$\therefore \quad \theta = 180° \Rightarrow F = 0$

(iii) On segment CD.

Force on a small segment dx distant r from O



dF = I dxB

$$= 10 \times dx \times \frac{\mu_0 I}{2\pi x} = \frac{5\mu_0 I}{\pi}\frac{dx}{x}$$

On integrating

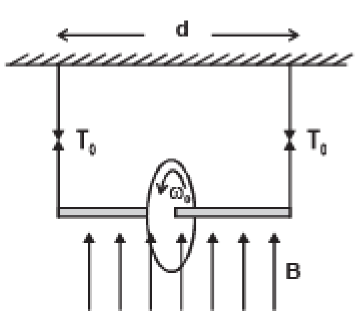
$$\therefore \quad F = \frac{5\mu_0 I}{\pi}\int_{r_1}^{r_2}\frac{dx}{x} \quad \therefore \quad F = \frac{5\mu_0 I}{\pi}\left[\log_e x\right]_{r_1}^{r_2}$$

$$\therefore \quad F = \frac{5\mu_0 I}{\pi}\log_e\frac{r_2}{r_1} = \frac{5\mu_0 \times 10}{\pi}\log_e\left(\frac{0.12}{0.08}\right)$$

$= 8.1 \times 10^{-6}$ N

directed downwards (By Fleming left hand rule).

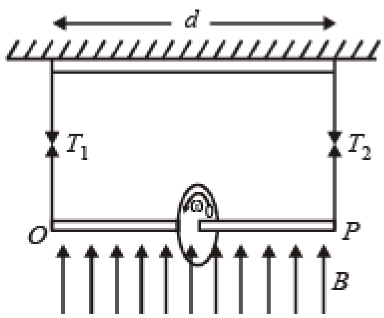
Q.19. A wheel of radius R having charge Q, uniformly distributed on the rim of the wheel is free to rotate about a light horizontal rod. The rod is suspended by light inextensible strings and a magnetic field B is applied as shown in the figure. The initial tensions in the strings are T_0. If the breaking tension of the strings are $\frac{3T_0}{2}$ find the maximum angular velocity ω_0 with which the wheel can be rotated.



Ans. $\omega = \frac{DT_0}{BQr^2}$

Solution. When the ring is not rotating Wt. of ring = Tension in string mg = $2T_0$

$\therefore T_0 = mg/2$.... (i)



When the ring is rotating, we can treat it as a current carrying loop. The magnetic moment of this loop

$$M = iA = \frac{Q}{T} \times \pi r^2 = \frac{Q}{2\pi}\omega \times \pi R^2$$

This current carrying loop will create its own magnetic field which will interact with the given vertical magnetic field in such a way that the tensions in the strings will become unequal. Let the tensions in the strings be T_1 and T_2.

For translational equilibrium

$T_1 + T_2 = mg$ ... (ii)

Torque acting on the ring about the centre of ring

$$\vec{\tau} = \vec{M} \times \vec{B}$$
$$\tau = M \times B \times \sin 90°$$

$$= \frac{Q}{2\pi}\omega \times \pi R^2 \times B = \frac{Q\omega BR^2}{2}$$

NOTE : For rotational equilibrium, the torque about the centre of ring should be zero.

$$\therefore \quad T_1 \times \frac{D}{2} - T_2 \times \frac{D}{2} = \frac{Q\omega BR^2}{2}$$

$$\Rightarrow \quad T_1 - T_2 = \frac{Q\omega BR^2}{D} \qquad \text{... (iii)}$$

On solving (ii) and (iii), we get

$$T_1 = \frac{mg}{2} + \frac{Q\omega BR^2}{2D}$$

But the maximum tension is $3T_0/2$

$$\therefore \quad \frac{3T_0}{2} = T_0 + \frac{Q\omega_{max} BR^2}{2D} \qquad \left[\because T_0 = \frac{mg}{2}\right]$$

$$\therefore \quad \omega_{max} = \frac{DT_0}{BQR^2}$$

Q.20. A proton and an α-particle are accelerated with same potential difference and they enter in the region of constant magnetic field B perpendicular to the velocity of particles. Find the ratio of radius of curvature of proton to the radius of curvature of α - particle.

Ans. $1/\sqrt{2}$

Solution. KEY CONCEPT :

$$eV = \frac{1}{2} m v_p^2 \text{ and } eV = \frac{1}{2} m v_\alpha^2$$

V is the potential difference

v_p = velocity of proton

v_α = velocity of α-particle

m = mass of proton, mass of a-particle = 4 m

$$\Rightarrow \quad v_p = \sqrt{\frac{2eV}{m}}, v_\alpha = \sqrt{\frac{2eV}{4m}}$$

Now when the particles enter in magnetic field, the force on proton is

$$ev_pB = \frac{mv_p^2}{r_p} \text{ or } r_p = \frac{mv_p}{eB} \Rightarrow r_\alpha = \frac{m}{eB}$$

$$\sqrt{\frac{2eV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{e}} \quad \text{and } r_\alpha = \frac{1}{B}\sqrt{\frac{4mV}{e}}$$

$$\therefore \quad \frac{r_p}{r_\alpha} = \frac{1}{\sqrt{2}}$$

Q.21. In a moving coil galvanometer, torque on the coil can be expressed as τ= ki , where i is current through the wire and k is constant. The rectangular coil of the galvanometer having number of turns N, area A and moment of inertia I is placed in magnetic field B. Find

(a) k in terms of given parameters N, I, A and B

(b) the torsion constant of the spring, if a current i_0 produces a deflection of π/2 in the coil.

(c) the maximum angle through which the coil is deflected, if charge Q is passed through the coil almost instantaneously. (ignore the damping in mechanical oscillations).

Ans. (a) k = NAB **(b)** $\frac{2Ni_0AB}{\pi}$ **(c)** $Q\sqrt{\frac{NAB\pi}{2Ii_0}}$

Solution. (a) The torque acting on a rectangular coil placed in a uniform magnetic field is given by,

$$\vec{\tau} = \vec{M} \times \vec{B} \quad \Rightarrow \tau = MB\sin\theta$$

But M = N i A and θ = 90° (for moving coil galvanometer)

$$\therefore \quad \tau = NiAB\sin 90°$$
$$\Rightarrow \quad \tau = NiAB$$

But τ = k i (given)

$$\therefore \quad ki = NiAB$$
$$\Rightarrow \quad k = NAB$$

(b) The torsion constant is given by

$$C = \frac{\tau}{\theta} = \frac{NiAB}{\theta}$$

Here given that when $i = i_0$, $\theta = \pi/2$

$$\therefore \quad C = \frac{2N i_0 AB}{\pi} \qquad \text{...(i)}$$

(c) We know that angular Impulse

$$= \int \tau dt = \int NiAB\, dt = NAB \int i\, dt$$
$$= NABQ \qquad \text{...(ii)}$$

This angular impulse creates an angular momentum

$$\int \tau\, dt = I\omega \qquad \text{...(iii)}$$

From (ii) and (iii)

$$I\omega = NABQ \Rightarrow \omega = \frac{NABQ}{I}$$

This is the instantaneous angular momentum due to which the coil starts rotating. Let us apply the law of energy conservation to find the angle of rotation.

Rotational kinetic energy of coil

$$= \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{I N^2 A^2 B^2 Q^2}{I^2} = \frac{N^2 A^2 B^2 Q^2}{2I}$$

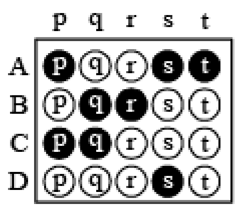
$$\frac{1}{2} C\theta_{max}^2 = \frac{N^2 A^2 B^2 Q^2}{2I}$$

$$\Rightarrow \quad \theta_{max}^2 = \frac{N^2 A^2 B^2 Q^2}{CI} = \frac{N^2 A^2 B^2 Q^2}{2Ni_0 ABI} \times \pi$$

$$\Rightarrow \quad \theta_{max}^2 = \frac{\pi NABQ^2}{2i_0 I} \quad \Rightarrow \quad \theta_{max} = Q\sqrt{\frac{NAB\pi}{2Ii_0}}.$$

Match the Following

DIRECTIONS (Qs. 1-3) : Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example : If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q.1.1. Match the following columns :

Column I	Column II
(A) Dielectric ring uniformly charged	(p) Constant electrostatic field out of system
(B) Dielectric ring uniformly charged rotating with angular velocity ω	(q) Magnetic field strength
(C) Constant current in ring i	(r) Electric field (induced)
(D) $i = i_0 \cos\omega t$	(s) Magnetic dipole moment

Ans. A-p; B-q, s; C-q, s; D-q, r, s

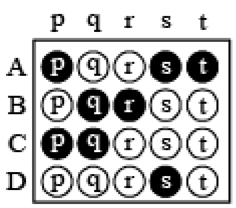
Solution. (A) Charge on ring will create electric field which is time independent.

(B) The rotating charge is like a current. This will create a magnetic field and a magnetic moment.

(C) Since net charge is zero there will be no time independent electric field. The current produces magnetic field and magnetic moment.

(D) A changing magnetic field will be produced. This will create a induced electric field. Also a changing magnetic moment will be produced.

Q. Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example : If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q.2.Column I gives certain situations in which a straight metallic wire of resistance R is used and Column II gives some resulting effects. Match the statements in Column I with the statements in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) A charged capacitor is connected to the ends of the wire	(p) A constant current flows through the wire
(B) The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	(q) Thermal energy is generated in the wire
(C) The wire is placed in a constant electric field that has a direction along the length of the wire	(r) A constant potential difference develops between the ends of the wire

(D) A battery of constant emf is connected to the ends of the wire.	(s) charges of constant magnitude appear at the ends of the wire

Ans. A-q; B-r, s; C-s; D-p, q, r

Solution. A : q

Reason : When a charged capacitor is connected to the ends of the wire, a variable current (decreasing in magnitude with time) passes through the wire (shown as resistor) and thermal energy is generated. The potential difference across the wire also decreases with time. The charge on the capacitor plate also decreases with time.

B : r. s

Reason : e = Blv

When B, l,v are constant, e is constant

⇒ A constant potential difference develops across the ends of the wire and charges of constant magnitude appear at the ends of the wire.

C : s

Reason : The free electrons move under the influence of electric field opposite to the direction of electric field. This movement of e^- continues till the electric field inside the wire is zero.

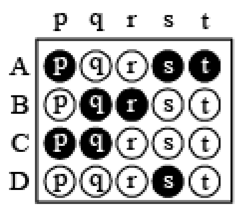
⇒ Changes of constant magnitude appear at the ends of the wire.

D : p, q, r

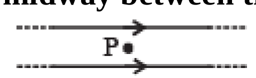
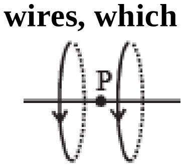
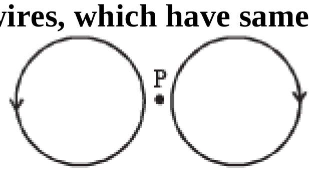
Reason : Since, E, R are constant, a constant current flows in the wire. Due to heating effect of current, thermal energy is generated in the wire. Also a constant potential difference develops between the ends of the wire.

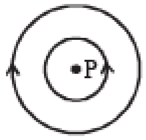
Q Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these

questions have to be darkened as illustrated in the following example : If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q.3. Two wires each carrying a steady current I are shown in four configurations in Column I. Some of the resulting effects are described in Column II. Match the statements in Column I with the statements in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) Point P is situated midway between the wires. 	**(p) The magnetic fields (B) at P due to the currents**
(B) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	**(q) The magnetic fields (B) at P due to the currents in the wires are in opposite directions.**
(C) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	**(r) There is no magnetic field at P.**

(D) Point P is situated at the common center of the wires. 	**(s) The wires repel each other.**

Ans. A-q, r; B-p; C-q, r; D-q, s

Solution. A : q, r

Reason : The magnetic field at P due to current flowing in AB is perpendicular to the plane of paper acting vertically downward. And the magnetic field at P due to current flowing in CD is perpendicular to the plane of paper acting vertically upwards.

Therefore, q is correct.

As P is the mid point, the two magnetic fields, cancel out each other. Therefore, r is correct.

B : p
Reason : The magnetic field at P due to current in loop A is along the axial line towards right. Similarly, the magnetic field at P due to current in loop B is also along the axial line towards right.

C : q, r
Reason : The magnetic field due to current in loop A at P is equal and opposite to the magnetic field due to current in loop B at P.

D : q, s
Reason : The direction of magnetic field at P due to current in loop A is perpendicular to the plane of paper directed vertically upwards.

The direction of magnetic field at P due to current in loop B is perpendicular to the plane of paper directed vertically downward.

Since the current are in opposite direction the wires repel each other. But net force on each wire is zero.

Integer Value of Moving

Q.1. A steady current I goes through a wire loop PQR having shape of a right angle triangle with PQ = 3x, PR = 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k.

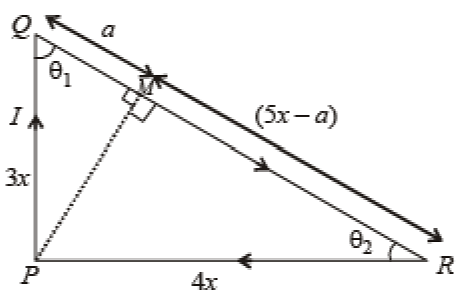
Ans. 7

Solution.

The right angled triangle is shown in the figure. Let us drop a perpendicular from P on QR which cuts QR at M.

The magnatic field due to currents in PQ and RP at P is zero.

The magnetic field due to current in QR at P is



$$B = \frac{\mu_0}{4\pi}\frac{I}{PM}(\cos\theta_1 + \cos\theta_2) \quad \dots(i)$$

In ΔPQM,

$$9x^2 = PM^2 + a^2 \quad \dots(ii)$$

In ΔPRM,

$16x^2 = PM^2 + (5x - a)^2 \dots$ (iii)

$\Rightarrow 7x^2 = 25x^2 - 10xa \Rightarrow 10xa = 18x^2$

$\Rightarrow a = 1.8x \dots$ (iv)

From (ii) & (iv),

$9x^2 = PM^2 + (1.8x)^2$

$$\Rightarrow \quad PM = \sqrt{9x^2 - 3.24x^2} = \sqrt{5.76x^2} = 2.4x \ \ldots \text{(v)}$$

$$\text{Also } \cos\theta_1 = \frac{a}{3x} = \frac{1.8x}{3x} = 0.6 \qquad \ldots \text{(vi)}$$

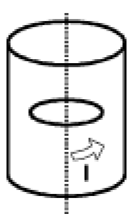
$$\cos\theta_2 = \frac{5x - a}{4x} = \frac{5x - 1.8x}{4x} = \frac{3.2}{4} = 0.8 \qquad \ldots \text{(vii)}$$

From (i), (v), (vi) and (vii),

$$B = \frac{\mu_0}{4\pi} \times \frac{I}{2.4x}[0.6 + 0.8] = \frac{\mu_0}{4\pi} \times \frac{I}{2.4x} \times 1.4 = 7\left[\frac{\mu_0 I}{48\pi x}\right]$$

Comparing it with B = $k\left[\frac{\mu_0 I}{48\pi x}\right]$, we get, $k = 7$.

Q.2. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as I = I_0cos(300 t) where I_0 is constant. If the magnetic moment of the loop is N$\mu_0 I_0$sin (300 t), then 'N' is

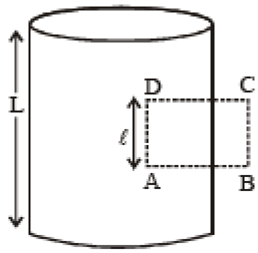


Ans. 6

Solution. Let us consider an amperian loop ABCD which is a rectangle as shown in the figure. Applying ampere's circuital law we get

$\oint \vec{B}.\overrightarrow{d\ell} = \mu_0 \times$ (current passing through the loop)

$$\therefore \quad \oint \vec{B}.\overrightarrow{d\ell} = \mu_0\left(\frac{I}{L}\right) \times \ell$$



$$\therefore \quad B \times \ell = \mu_o \frac{I}{L} \times \ell$$

$$\therefore \quad B = \frac{\mu_o I}{L} = \frac{\mu_o}{L} I_o \cos(300\,t)$$

The magnetic moment of the loop = (current in the loop) × πr^2

$$= \frac{1}{R}\left(-\frac{d\phi}{dt}\right) \times \pi r^2$$

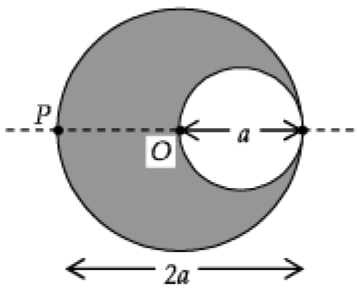
$$= -\frac{1}{R}\left[\frac{d}{dt}(B \times \pi r^2)\right] \times \pi r^2 = -\frac{\pi^2 r^4}{R}\frac{dB}{dt}$$

$$= \left[\frac{\pi^2 r^4}{R} \times \frac{\mu_o}{L} I_o \sin(300t)\right] \times 300$$

Comparing it with the expression given in the question we get

$$N = \frac{300\pi^2 r^4}{R} \times \frac{1}{L} = \frac{300(3.14)^2 \times (0.1)^4}{0.005 \times 10} = 6$$

Q.3.A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinity long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



Ans.5

Solution. Current density J = $\frac{\text{current}}{\text{area}} = \frac{I}{\pi(2a)^2} = \frac{I'}{\left(\pi a^2\right)}$

$$\Rightarrow \quad I' = \frac{I}{4}$$

Let us consider the cavity to have current I' flowing in both the directions.

The magnetic field at P due to the current flowing through the cylinder

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I}{a}$$

The magnetic field at P due to the current (I') flowing in opposite direction is

$$B_2 = \frac{\mu_0}{4\pi}\frac{3I'}{3a/2} = \frac{\mu_0}{4\pi}\frac{2(I/4)}{3a/2} = \frac{\mu_0}{4\pi}\frac{I}{3a}$$

∴ The net magnetic field is

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi}\frac{I}{a}\left[2 - \frac{1}{3}\right] = \frac{\mu_0}{4\pi}\frac{I}{a} \times \frac{5}{3}$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{J\pi a^2}{a} \times \frac{5}{3} = \mu_0 \frac{5Ja}{12}$$

Q.4. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each

other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$ **the value of** $\frac{R_1}{R_2}$ **is**

Ans. 3

Solution. $R = \frac{mv}{qB}$

$$\frac{R_1}{R_2} = \frac{B_2}{B_1} \qquad [\because m, q, v \text{ are the same}]$$

$$\frac{R_1}{R_2} = \frac{\frac{\mu_0}{4\pi} \times 2I\left[\frac{1}{X_1} + \frac{1}{X_0 - X_1}\right]}{\frac{\mu_0}{4\pi} \times 2I\left[\frac{1}{X_1} - \frac{1}{X_0 - X_1}\right]}$$

$$= \frac{X_0 - X_1 + X_1}{X_0 - X_1 - X_1} = \frac{X_0}{X_0 - 2X_1}$$

$$\therefore \frac{R_1}{R_2} = \frac{\frac{X_0}{X_1}}{\frac{X_0}{X_1} - 2} = \frac{3}{3 - 2} = 3$$