Quadratic Equations

Q1. Solve the following quadratic equation: $x^2 + 4x - 8 = 0$

Give your answer correct to one decimal place. (Use mathematical tables if necessary.) [2023]

Answer: x= 1.5, -5.5

Step-by-step explanation:

$$x^2 + 4x - 8 = 0$$

comparing the above equation with $ax^2 + bx + c = 0$, we have,

a = 1, b = 4, c = -8

By Quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-8)}}{2 \times 1}$$

$$x = \frac{-4 \pm \sqrt{16 + 32}}{2}$$

$$x = \frac{-4 \pm \sqrt{16 + 32}}{2}$$

$$x = \frac{-4 \pm \sqrt{48}}{2}$$

$$x = \frac{-4 \pm 6.928}{2}$$

Q2. If 3 is a root of the quadratic equation $x^2 - px + 3 = 0$ then p is equal to:

(a) 4

(b) 3

(c) 5

(d) 2 [2023]

Answer: (a) 4

Step-by-step explanation:

 $x^2 - px + 3 = 0$ Given, 3 is a root of the above equation. Therefore, $(3)^2 - p \times 3 + 3 = 0$ 9 - 3p + 3 = 0 - 3p + 12 = 0 - 3p = -12p = 4

Q3. One of the roots of the quadratic equation $x^2 - 8x + 5 = 0$ is 7.3166. The root of the equation correct to 4 significant figures is: [1]

- (a) 7.3166
- (b) 7.317
- (c) 7.316
- (d) 7.32 [2021 Semester-1]

Answer: (b) 7.317

Step-by-step explanation:

 $x^2 - 8x + 5 = 0$ Given, 7.3166 is a root of the above equation. Therefore, The root of the equation correct to 4 significant figures is 7.317

Q4. Which of the following quadratic equations has 2 and 3 as its roots? [1]

- (a) $x^2 5x + 6 = 0$
- (b) $x^2 + 5x + 6 = 0$
- (c) $x^2 5x 6 = 0$
- (d) $x^2 + 5x 6 = 0$ [2021 Semester-1]
- **Answer:** (a) x²-5x+6=0

$$x^{2} - 5x + 6 = 0$$

$$x^{2} - 3x - 2x + 6 = 0$$

$$x(x - 3) - 2(x - 3) = 0$$

$$(x - 2)(x - 3) = 0$$
Either $x - 3 = 0$ OR $x - 2 = 0$
There fore, $x = 3$ OR $x = 2$

Q5. Solve the following Quadratic Equation:

 $x^2 - 7x + 3 = 0$ Give your answer correct to two decimal places. [2020]

Answer: x = 6.54, 0.46

Step-by-step Explanation:

 $\begin{aligned} x^2 - 7x + 3 &= 0\\ Comparing the given equation with <math>ax^2 + bx + c &= 0 \text{ we have},\\ a &= 1, b = -7, c = 3\\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ x &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times 3}}{2 \times 1}\\ x &= \frac{7 \pm \sqrt{49 - 12}}{2}\\ x &= \frac{7 \pm \sqrt{37}}{2}\\ x &= \frac{7 \pm \sqrt{37}}{2}\\ x &= \frac{7 \pm 6.083}{2}\\ x &= \frac{7 \pm 6.083}{2}\\ x &= \frac{7 \pm 6.083}{2}, \frac{7 - 6.083}{2}\\ x &= \frac{13.083}{2}, \frac{0.917}{2}\\ x &= 6.5416, 0.4585\\ x &= 6.54, 0.46\end{aligned}$

Q6. Solve for x the quadratic equation $x^2 - 4x - 8 = 0$

Give your answer correct to three significant figures. [2019]

Answer: x= 5.46 , -1.46

Step-by-step Explanation:

$$x^{2} - 4x - 8 = 0$$
Comparing the given equation with $ax^{2} + bx + c = 0$, we have,
 $a = 1, b = -4, c = -8$
By formula,
 $x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$
 $x = \frac{-(-4) \pm \sqrt{(-4)^{2} - 4 \times 1 \times (-8)}}{2 \times 1}$
 $x = \frac{4 \pm \sqrt{16 + 32}}{2}$
 $x = \frac{4 \pm \sqrt{16 + 32}}{2}$
 $x = \frac{4 \pm 6.928}{2}$
 $x = \frac{4 \pm 6.928}{2}$, $\frac{4 - 6.928}{2}$
 $x = \frac{4 \pm 6.928}{2}$, $\frac{4 - 6.928}{2}$
 $x = \frac{10.928}{2}$, $\frac{-2.928}{2}$
 $x = 5.464$, -1.464
 $x = 5.464$, -1.464

Q7. Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. [4] [2018]

Answer: 0.89, -7.89

$$\begin{aligned} x^2 + 7x &= 7\\ x^2 + 7x - 7 &= 0\\ \end{aligned}$$
Comparing the given equation with $ax^2 + bx + c = 0$, we have,
 $a = 1, b = 7, c = -7\\ By \ formula,\\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ x &= \frac{-7 \pm \sqrt{(7)^2 - 4 \times 1 \times (-7)}}{2 \times 1}\\ x &= \frac{-7 \pm \sqrt{49 + 28}}{2}\\ x &= \frac{-7 \pm \sqrt{77}}{2}\\ x &= \frac{-7 \pm \sqrt{77}}{2}\\ x &= \frac{-7 \pm 8.775}{2}\\ x &= \frac{-7 + 8.775}{2}, \frac{-7 - 8.775}{2}\\ x &= \frac{1.775}{2}, \frac{-15.775}{2}\\ x &= 0.8875, -7.8875\\ x &= 0.89, -7.89\end{aligned}$

Q8. Find the value of k for which the following equation has equal roots. [3]

 $x^{2} + 4kx + (k^{2} - k + 2) = 0$ [2018]

Answer: k= -1 or 2/3

$$\begin{aligned} x^2 + 4kx + (k^2 - k + 2) &= 0\\ Comparing the equation with $ax^2 + bx + c = 0 \text{ we have},\\ a &= 1, b = 4k, c = (k^2 - k + 2)\\ Given, b^2 - 4ac &= 0\\ Therefore, (4k)^2 - 4 \times 1 \times (k^2 - k + 2) &= 0\\ 16k^2 - 4k^2 + 4k - 8 &= 0\\ 12k^2 + 4k - 8 &= 0\\ 4(3k^2 + k - 2) &= 0\\ 3k^2 + k - 2 &= 0\\ 3k^2 + 3k - 2k - 2 &= 0\\ 3k(k + 1) - 2(k + 1) &= 0\\ (k + 1)(3k - 2) &= 0\\ Either \ k + 1 &= 0 \ or \ 3k - 2 &= 0\\ Either \ k &= -1 \ or \ \frac{2}{3}\end{aligned}$$$

Q9. Solve the equation $4x^2 - 5x - 3 = 0$ and give your answer correct to two decimal places. [4] [2017]

Answer: x = 1.69 , -0.44

$$4x^{2} - 5x - 3 = 0$$
Comparing the equation with $ax^{2} + bx + c = 0$ we have,
 $a = 4, b = -5, c = -3$
By formula,
 $x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$
 $x = \frac{-(-5) \pm \sqrt{(-5)^{2} - 4 \times 4 \times (-3)}}{2 \times 4}$
 $x = \frac{5 \pm \sqrt{25 + 48}}{8}$
 $x = \frac{5 \pm \sqrt{73}}{8}$
 $x = \frac{5 \pm 8.544}{8}$, $\frac{5 - 8.544}{8}$
 $x = \frac{5 \pm 8.544}{8}$, $\frac{5 - 8.544}{8}$
 $x = \frac{13.544}{8}$, $\frac{-3.544}{8}$
 $x = 1.693$, -0.443
 $x = 1.69$, -0.44

Q10. Solve the quadratic equation $x^2 - 3(x + 3) = 0$; Give your answer correct to two significant figures. [3] [2016]

Answer: x = 5.9 , -0.85

$$\begin{aligned} x^2 - 3(x+3) &= 0 \\ x^2 - 3x - 9 &= 0 \\ \end{aligned}$$
Comparing the equation with $ax^2 + bx + c = 0$ we have,
 $a = 1, \ b = -3, \ c = -9 \\ By \ formula, \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-9)}}{2 \times 1} \\ x &= \frac{3 \pm \sqrt{9 + 36}}{2} \\ x &= \frac{3 \pm \sqrt{9 + 36}}{2} \\ x &= \frac{3 \pm \sqrt{45}}{2} \\ x &= \frac{3 \pm 6.708}{2} \\ x &= \frac{4.854}{2}, \ -3.708 \\ x &= 4.854, \ -1.854 \\ x &= 4.9, \ -1.9 \end{aligned}$

Q11. Find the value of 'K' for which x = 3 is a solution of the quadratic equation, $(K + 2) x^2 - Kx + 6 = 0$. Thus find the other root of the equation. [2015]

Answer: k = -4, other root = -1

 $(K + 2) x^2 - Kx + 6 = 0$ x = 3 is a solution of the above equation. Therefore, $(k+2)(3)^2 - k \times 3 + 6 = 0$ 9k + 18 - 3k + 6 = 06k + 24 = 06k = -24k = -4putting k = -4 in the given equation, $-2x^2 + 4x + 6 = 0$ $-2(x^2-2x-3)=0$ $x^2 - 2x - 3 = 0$ $x^2 - 3x + x - 3 = 0$ x(x-3) + 1(x-3) = 0(x-3)(x+1) = 0*Either* x + 1 = 0 *or* x - 3 = 0 $x = -1 \ or \ 3$ So, other root of the equation is -1.

Q12. Solve for x using the quadratic formula. Write your answer correct to two significant figures,

 $(x-1)^2 - 3x + 4 = 0.[3][2014]$

Answer: 3.6, 1.4

$$egin{array}{rcl} {(x-1)}^2 &-3x \;+\; 4 \;=\; 0 \ \Rightarrow x^2 &-2x +1 -3x +4 =0 \ \Rightarrow x^2 &-5x +5 =0 \end{array}$$

comparing the given equation with $ax^2 + bx + c = 0$, we have,

$$a = 1, \ b = -5, \ c = 5$$

By formula,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$x = \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$x = \frac{5 \pm \sqrt{5}}{2}$$

$$x = \frac{5 \pm \sqrt{5}}{2}$$

$$x = \frac{5 \pm 2.24}{2}$$

$$x = \frac{5 \pm 2.24}{2}$$

or $\frac{5 - 2.24}{2}$
$$x = \frac{7.24}{2} \text{ or } \frac{5 - 2.24}{2}$$

$$x = 3.62 \text{ or } 1.38$$

$$x = 3.6, \ 1.4$$

Q13. Solve the following equation and calculate the answer correct to two decimal places:

 $x^2 - 5x - 10 = 0$ [3] [2013]

Answer: 6.53 , -1.53

$$x^{2} - 5x - 10 = 0$$
comparing the given equation with $ax^{2} + bx + c = 0$, we have,
 $a = 1, b = -5, c = -10$
By formula,
 $x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$
 $x = \frac{-(-5) \pm \sqrt{(-5)^{2} - 4 \times 1 \times (-10)}}{2 \times 1}$
 $x = \frac{5 \pm \sqrt{25 + 40}}{2}$
 $x = \frac{5 \pm \sqrt{65}}{2}$
 $x = \frac{5 \pm 8.062}{2}$
 $x = \frac{13.062}{2}$ or $\frac{5 - 8.062}{2}$
 $x = 6.531$ or -1.531
 $x = 6.53, -1.53$

Q14. Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots: $x^{2} + (p-3)x + p = 0$ [2013]

Answer: p= 1 or 9

$$x^{2} + (p-3)x + p = 0$$

comparing the given equation with $ax^{2} + bx + c = 0$, we have,
 $a = 1, b = (p-3), c = p$
 $Given, b^{2} - 4ac = 0$
 $\Rightarrow (p-3)^{2} - 4 \times 1 \times p = 0$
 $\Rightarrow p^{2} - 6p + 9 - 4p = 0$
 $\Rightarrow p^{2} - 10p + 9 = 0$
 $\Rightarrow p^{2} - 9p - p + 9 = 0$
 $\Rightarrow p(p-9) - 1(p-9) = 0$
 $\Rightarrow (p-9)(p-1) = 0$
 $Either p - 9 = 0 \ OR \ p - 1 = 0$
 $p = 9 \ or 1$

Q15. Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots. $x^{2} + 2 (m - 1) x + (m + 5) = 0 [3] [2012]$

Answer: m = -1, 4

$$x^{2} + 2 (m - 1) x + (m + 5) = 0$$

comparing the given equation with $ax^{2} + bx + c = 0$
 $a = 1, b = 2(m - 1), c = (m + 5)$
 $given, b^{2} - 4ac = 0$
 $[2(m - 1)]^{2} - 4 \times 1 \times (m + 5) = 0$
 $4(m^{2} - 2m + 1) - 4m - 20 = 0$
 $4m^{2} - 8m + 4 - 4m - 20 = 0$
 $4m^{2} - 8m + 4 - 4m - 20 = 0$
 $4m^{2} - 12m - 16 = 0$
 $4(m^{2} - 3m - 4) = 0$
 $m^{2} - 3m - 4 = 0$
 $m^{2} - 4m + m - 4 = 0$
 $m(m - 4) + 1(m - 4) = 0$
 $(m + 1)(m - 4) = 0$
 $Either (m + 1) = 0 \text{ or } (m - 4) = 0$
 $m = -1 \text{ or } 4$

Q16. Solve the following equation and give your answer correct to 3 significant figures:

 $5x^2 - 3x - 4 = 0$ [3] [2012]

Answer: 1.24, -0.643

Step-by-step Explanation:

 $5x^{2} - 3x - 4 = 0$ comparing the given equation with $ax^{2} + bx + c = 0$ a = 5, b = -3, c = -4by quadratic formula, $x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ $x = \frac{-(-3) \pm \sqrt{(-3)^{2} - 4 \times 5 \times (-4)}}{2 \times 5}$ $x = \frac{3 \pm \sqrt{9 + 80}}{10}$ $x = \frac{3 \pm \sqrt{9 + 80}}{10}$ $x = \frac{3 \pm 9.434}{10}$ $x = \frac{3 \pm 9.434}{10}$, $\frac{3 - 9.434}{10}$ $x = \frac{12.434}{10}$, $\frac{-6.434}{10}$ x = 1.2434, -0.6434x = 1.24, -0.643

Q17. Solve the following equation:

$$x - \frac{18}{x} = 6$$

Give your answer correct to two significant figures. [3] [2011]

Answer: 8.2, -2.2

Step-by-step Explanation:

$$\begin{aligned} x - \frac{18}{x} &= 6\\ \Rightarrow \frac{x^2 - 18}{x} &= 6\\ \Rightarrow x^2 - 18 &= 6x\\ \Rightarrow x^2 - 6x - 18 &= 0\\ comparing the given equation with $ax^2 + bx + c = 0\\ a &= 1, b = -6, c = -18\\ by quadratic formula,\\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times (-18)}}{2 \times 1}\\ x &= \frac{6 \pm \sqrt{36 + 72}}{2}\\ x &= \frac{6 \pm \sqrt{108}}{2}\\ x &= \frac{6 \pm \sqrt{108}}{2}\\ x &= \frac{6 \pm 10.392}{2}\\ x &= \frac{6 \pm 10.392}{2}\\ x &= \frac{16.392}{2}, \frac{-4.392}{2}\\ x &= 8.196, -2.196\\ x &= 8.2, -2.2\end{aligned}$$$

Q18. Without solving the following quadratic equation, find the value of 'p' for which the roots are equal. $px^2 - 4x + 3 = 0$ [3] [2010]

Answer: p = 4/3

$$px^2 - 4x + 3 = 0$$

comparing the above equation with $ax^2 + bx + c = 0$, we have,
 $a = p$, $b = -4$, $c = 3$
given, $b^2 - 4ac = 0$
 $(-4)^2 - 4 \times p \times 3 = 0$
 $16 - 12p = 0$
 $-12p = -16$
 $p = \frac{16}{12}$

 $p=\frac{4}{3}$

Q19. A man covers a distance of 100 km, travelling with a uniform speed of x km/hr. Had the speed been 5 km/hr more it would have taken 1 hour less. Find x the original speed. [2023]

Answer: 20 km/h

Step-by-step Explanation:

Original speed = x km/hr

Distance = 100 km

Therefore, Time taken = 100/x hr

Now, if speed = (x+5) km/hr

Then, by the problem,

$$\frac{100}{x+5} = \frac{100}{x} - 1$$

$$\Rightarrow \frac{100}{x+5} - \frac{100}{x} = -1$$

$$\Rightarrow \frac{100x - 100(x+5)}{x(x+5)} = -1$$

$$\Rightarrow \frac{100x - 100x - 500}{x^2 + 5x} = -1$$

$$\Rightarrow -500 = -x^2 - 5x$$

$$\Rightarrow x^2 + 5x - 500 = 0$$

$$\Rightarrow x^2 + 25x - 20x - 500 = 0$$

$$\Rightarrow x(x+25) - 20(x+25) = 0$$

$$\Rightarrow (x+25)(x-20) = 0$$

either $(x+25) = 0$ or $(x-20) = 0$
 $x = -25$ or 20

As speed cannot be negative, therefore original speed is 20 km/h

Q20. The difference of two natural numbers is 7 and their product is 450. Find the numbers. [2020]

Answer: 18 and 25

Step-by-step Explanation:

Let the two numbers be x and (x-7).

by the problem,

$$x(x-7) = 450$$

- or, $x^2 7x 450 = 0$
- or, $x^2 25x + 18x 450 = 0$
- or, x(x 25) + 18(x 25) = 0
- or, (x 25)(x + 18) = 0

Either (x - 25) = 0 or (x + 18) = 0

x = 25 or -18

As natural numbers cannot be negative, therefore

One number is 25 and

the other number is (25-7) = 18.

Q21. The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers. [3] [2019]

Answer: 27 and 30

Step-by-step Explanation:

Let the two numbers be 3x and 3(x+1).

by the problem,

$$3x \times 3(x + 1) = 810$$

$$\Rightarrow 3x(3x + 3) = 810$$

$$\Rightarrow 9x^{2} + 9x = 810$$

$$\Rightarrow 9(x^{2} + x) = 810$$

$$\Rightarrow x^{2} + x = 90$$

$$\Rightarrow x^{2} + x - 90 = 0$$

$$\Rightarrow x^{2} + 10x - 9x - 90 = 0$$

$$\Rightarrow x(x + 10) - 9(x + 10) = 0$$

$$\Rightarrow (x + 10)(x - 9) = 0$$

either $(x + 10) = 0$ or $(x - 9) = 0$
 $x = -10$ or 9
As natural numbers cannot be negative, therefore,
 $x = 9$
Therefore the two numbers are $3x = 27$ and
 $3(x + 1) = 30$

Q22. ₹ 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹ 100 more. Find the original number of children. [2018]

Answer: 50

Step-by-step Explanation:

Let the original number of children be x. Each child gets Rs. $\frac{7500}{x}$. By the problem, $\frac{7500}{x-20} - \frac{7500}{x} = 100$ $\Rightarrow \frac{7500x - 7500(x - 20)}{x(x - 20)} = 100$ $\Rightarrow \frac{7500x - 7500x + 150000}{x^2 - 20x} = 100$ $\Rightarrow 100x^2 - 2000x = 150000$ $\Rightarrow 100x^2 - 2000x - 150000 = 0$ $\Rightarrow 100(x^2 - 20x - 1500) = 0$ $\Rightarrow x^2 - 20x - 1500 = 0$ $\Rightarrow x^2 - 50x + 30x - 1500 = 0$ $\Rightarrow x(x - 50) + 30(x - 50) = 0$ $\Rightarrow (x + 30)(x - 50) = 0$ either (x + 30) = 0 or (x - 50) = 0x = -30 or 50

As number of children cannot be negative, therefore, x = 50.Therefore, original number of children is 50. Q23. Two cars X and Y use 1 litre of diesel to travel x km and (x + 3) km respectively. If both the cars covered a distance of 72 km, then:

i. The number of litres of diesel used by car X is: [1]

$$(a) \frac{72}{x-3} litres$$
$$(b) \frac{72}{x+3} litres$$
$$(c) \frac{72}{x} litres$$
$$(d) \frac{12}{x} litres$$

ii. The number of litres of diesel used by car Y is: [1]

$$(a) \frac{72}{x-3} litres$$
$$(b) \frac{72}{x+3} litres$$
$$(c) \frac{72}{x} litres$$
$$(d) \frac{12}{x+3} litres$$

iii. If car X used 4 litres of diesel more than car Y in the journey, then: [1]

(a)
$$\frac{72}{x-3} - \frac{12}{x} = 4$$

(b) $\frac{72}{x+3} - \frac{72}{x} = 4$
(c) $\frac{72}{x} - \frac{72}{x+3} = 4$
(d) $\frac{72}{x-3} - \frac{72}{x+3} = 4$

iv. The amount of diesel used by the car X is: [1]

(a) 6 litres

(b) 12 litres

(c) 18 litres

(d) 24 litres [2021 Semester-1]

Answer: i. (c) , ii. (b) , iii. (c) , iv. (b)

$$By the problem,
\frac{72}{x} - \frac{72}{x+3} = 4
\Rightarrow \frac{72(x+3) - 72x}{x(x+3)} = 4
\Rightarrow \frac{72x + 216 - 72x}{x^2 + 3x} = 4
\Rightarrow 4x^2 + 12x = 216
\Rightarrow 4x^2 + 12x - 216 = 0
\Rightarrow 4(x^2 + 3x - 54) = 0
\Rightarrow x^2 + 3x - 54 = 0
\Rightarrow x^2 + 9x - 6x - 54 = 0
\Rightarrow x(x+9) - 6(x+9) = 0
\Rightarrow (x+9)(x-6) = 0$$

either (x + 9) = 0 or (x - 6) = 0 x = -9 or 6 therefore, x = 6 litres.

Therefore, The amount of diesel used by the car X is

$$\frac{72}{x} = \frac{72}{6} = 12 \ litres.$$
option (b).

Q24. The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages. [4] [2017]

Answer: 25 years, 22 years

Step-by-step Explanation:

Let the age of Vivek be x years and that of his younger brother be (47-x) years.

By the problem,

x(47 - x) = 550

or, $47x - x^2 = 550$

or, $x^2 - 47x + 550 = 0$

or, $x^2 - 25x - 22x + 550 = 0$

or, x(x - 25) - 22(x - 25) = 0

or, (x - 25)(x - 22) = 0

either (x - 25) = 0 or (x - 22) = 0

x = 25 or 22

Therefore, Vivek's age is 25 years and his younger brother's age is 22 years.

Q25. A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed gets reduced by 10 km/h and as such it takes two hours longer to cover the total distance. Assuming the uniform speed to be 'x' km/h, form an equation and solve it to evaluate 'x'. [3] [2016]

Answer: 40 km/h

Step-by-step Explanation:

 $Uniform \ speed = x \ km/h$ $dis \tan ce = 240 \ km$ $time = \frac{Dis \tan ce}{speed} = \frac{240}{x} \ hours$ $Due \ to \ heavy \ rain \ speed = (x - 10) \ km/h$ $Now \ time = \frac{240}{x - 10} \ hours$

By the problem,

$$\frac{240}{x-10} - \frac{240}{x} = 2$$

$$\Rightarrow \frac{240x - 240(x-10)}{x(x-10)} = 2$$

$$\Rightarrow \frac{240x - 240x + 2400}{x^2 - 10x} = 2$$

$$\Rightarrow 2x^2 - 20x = 2400$$

$$\Rightarrow 2x^2 - 20x - 2400 = 0$$

$$\Rightarrow 2(x^2 - 10x - 1200) = 0$$

$$\Rightarrow x^2 - 10x - 1200 = 0$$

$$\Rightarrow x^2 - 40x + 30x - 1200 = 0$$

$$\Rightarrow x(x-40) + 30(x-40) = 0$$

$$\Rightarrow (x-40)(x+30) = 0$$
either $(x-40) = 0$ or $(x+30) = 0$
 $x = 40$ or -30
As speed cannot be negative, therefore,
 $x = 40$
Therefore, speed of bus is 40 km/h.

Q26. Sum of two natural numbers is 8 and the difference of their reciprocal is 2 / 15. Find the numbers. [3] [2015]

Answer: 3 and 5

Step-by-step Explanation:

Let the two natural numbers be x and (8-x).

Now, by the problem,

$$\frac{1}{x} - \frac{1}{8-x} = \frac{2}{15}$$

$$\Rightarrow \frac{8-x-x}{x(8-x)} = \frac{2}{15}$$

$$\Rightarrow \frac{8-2x}{8x-x^2} = \frac{2}{15}$$

$$\Rightarrow 120 - 30x = 16x - 2x^2$$

$$\Rightarrow 2x^2 - 46x + 120 = 0$$

$$\Rightarrow 2(x^2 - 23x + 60) = 0$$

$$\Rightarrow x^2 - 23x + 60 = 0$$

$$\Rightarrow x^2 - 3x - 20x + 60 = 0$$

$$\Rightarrow x(x-3) - 20(x-3) = 0$$

$$\Rightarrow (x-3)(x-20) = 0$$
either $(x-3) = 0$ or $(x-20) = 0$

$$x = 3, 20$$
20 is not possible.

Therefore, the two numbers are 3 and (8-3) = 5.

Q27. A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. [4] [2014]

Answer: 23

Step-by-step Explanation:

Let the tens' digit be x and unit's digit be $\frac{6}{x}$. Therefore the two digit number = $10x + \frac{6}{x}$.

According to the problem,

$$10x + \frac{6}{x} + 9 = (10 \times \frac{6}{x}) + x$$

$$\Rightarrow 10x + \frac{6}{x} - \frac{60}{x} - x = -9$$

$$\Rightarrow \frac{10x^2 + 6 - 60 - x^2}{x} = -9$$

$$\Rightarrow 9x^2 + 6 - 60 - x^2 = -9x$$

$$\Rightarrow 9x^2 - 54 = -9x$$

$$\Rightarrow 9x^2 + 9x - 54 = 0$$

$$\Rightarrow 9(x^2 + x - 6) = 0$$

$$\Rightarrow x^2 + 3x - 2x - 6 = 0$$

$$\Rightarrow x(x + 3) - 2(x + 3) = 0$$

$$\Rightarrow (x + 3)(x - 2) = 0$$

Either $(x + 3) = 0$ or $(x - 2) = 0$

$$x = -3, 2$$

Since number is positive, therefore

x=2

Therefore, the two – digit number is

$$10x + \frac{6}{x}$$
$$20 + \frac{6}{2}$$
$$23.$$

Q28. A shopkeeper purchases a certain number of books for Rs. 960. If the cost per book was 8 less, the number of books that could be purchased for Rs. 960 would be 4 more. Write an equation, taking the original cost of each book to be Rs. x, and solve it to find the original cost of the books.[4] [2013]

Answer: Rs. 48

Step-by-step Explanation:

original cost of each book is Rs x.
Then, the no. of books =
$$\frac{960}{x}$$
.
Also, cost of each books = Rs (x - 8)
Therefore, by the problem,
 $\frac{960}{x-8} - \frac{960}{x} = 4$
 $\Rightarrow \frac{960x - 960(x-8)}{x(x-8)} = 4$
 $\Rightarrow \frac{960x - 960x + 7680}{x^2 - 8x} = 4$
 $\Rightarrow 4x^2 - 32x = 7680$
 $\Rightarrow 4x^2 - 32x = 7680$
 $\Rightarrow x^2 - 8x - 1920 = 0$
 $\Rightarrow x^2 + 40x - 48x - 1920 = 0$
 $\Rightarrow (x - 48)(x + 40) = 0$
Either $x - 48 = 0$ or $x + 40 = 0$
 $x = 48$ or -40
 \therefore Original cost of book = Rs 48.

Q29. A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car. [4] [2012]

Answer: 48 km/h

Let the original speed be $x \ km/h$.

$$Then \ Time = \frac{400}{x} \ hrs$$

$$If \ speed = x + 12, \ then$$

$$Time = \frac{400}{x + 12}$$

$$By \ the \ problem,$$

$$\frac{400}{x} - \frac{400}{x + 12} = 1hr + \frac{40}{60}$$

$$\Rightarrow \frac{400(x + 12) - 400x}{x(x + 12)} = \frac{5}{3}$$

$$\Rightarrow \frac{400x + 4800 - 400x}{x^2 + 12x} = \frac{5}{3}$$

$$\Rightarrow \frac{400x + 4800 - 400x}{x^2 + 12x} = \frac{5}{3}$$

$$\Rightarrow 5x^2 + 60x = 14400$$

$$\Rightarrow 5x^2 + 60x - 14400 = 0$$

$$\Rightarrow x^2 + 12x - 2880 = 0$$

$$\Rightarrow x^2 + 60x - 48x - 2880 = 0$$

$$\Rightarrow x(x + 60) - 48(x + 60) = 0$$

$$\Rightarrow (x + 60)(x - 48) = 0$$
Either $x + 60 = 0 \ or \ x - 48 = 0$

$$x = -60 \ or \ 48$$
Speed cannot be negative. therefore, speed of the car is 48 km/h.

Q30. Rs.480 is divided equally among 'x' children. If the number of children was 20 more, then each would have got Rs. 12 less. Find 'x'. [3] [2011]

Answer: 20

Given, Number of children = xTherefore, money received by each child = $Rs. \frac{480}{2}$ Now, if Number of children = x + 20, then, by the problem, $\frac{480}{x} - \frac{480}{x+20} = 12$ $\Rightarrow rac{480(x+20)-480x}{x(x+20)} = 12$ $\Rightarrow \frac{480x + 9600 - 480x}{x^2 + 20x} = 12$ $\Rightarrow 12x^2 + 240x = 9600$ $\Rightarrow x^2 + 20x - 800 = 0$ $\Rightarrow x^{2} + 40x - 20x - 800 = 0$ $\Rightarrow x(x+40) - 20(x+40) = 0$ $\Rightarrow (x+40)(x-20) = 0$ *Either* x + 40 = 0 *or* x - 20 = 0x = -40 or 20.Children cannot be in negative. Therefore, number of children is 20.

Q31. A positive number is divided into two parts such that the sum of the squares of the two parts is 20. The square of the larger part is 8 times the smaller part. Taking x as the smaller part of the two parts, find the number. [4] [2010]

Answer: 6

Then,
$$(larger part)^2 = 8x$$
 Therefore, $larger part = \sqrt{8x}$
By the problem,
 $x^2 + 8x = 20$
 $\Rightarrow x^2 + 8x - 20 = 0$
 $\Rightarrow x^2 + 10x - 2x - 20 = 0$
 $\Rightarrow x(x + 10) - 2(x + 10) = 0$
 $\Rightarrow (x + 10)(x - 2) = 0$
Either $(x + 10) = 0$ or $(x - 2) = 0$
 $x = -10$ or 2.
As the number is positive, therefore,
smaller part is 2 and
larger part is $\sqrt{8 \times 2} = 4$
Therefore the number is $4 + 2 = 6$.