CHAPTER 10

Geometry

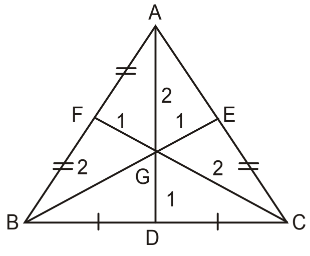
Triangle and Its Various Kinds of Centres

There are 4 very important ways of viewing the center of a triangle. We can look at its CENTROID, ORTHOCENTER, CIRCUMCENTER, and the INCENTER.

Centroid

The centroid is the first center and is obtained by locating the intersection of the three medians of the triangle. The median of a triangle is obtained by joining each vertex with the midpoint of the opposite side.

- It is the point of the intersection of the three median of the triangle. It is denoted by G.
- A centroid divides the area of the triangle in exactly three parts.



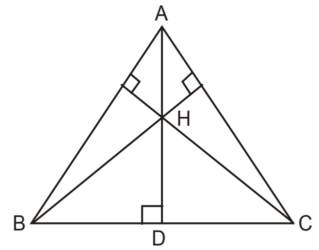
Medians:

- A line segment joining the midpoint of the side with the opposite vertex is called median.
- Median bisects the opposite side as well as divide the area of the triangle in two equal parts.

Orthocenter

The second center of a triangle is the orthocenter. It is obtained by finding the intersection of the 3 altitudes of the triangle. An altitude is found by joining each vertex with the point on the opposite side that creates a perpendicular line with the opposite side.

It is the point of intersection of all the three altitudes of the triangle.



$\angle BHC = 180 - \angle A$

$\angle AHB = 180 - \angle C$

$\angle AHC = 180 - \angle B$

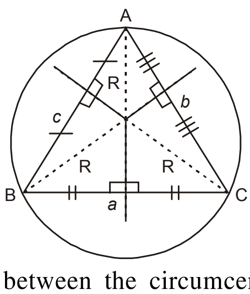
Position of orthocentre inside the triangle:

Acute angled triangle: lies inside the triangle.

Obtuse angle triangle: lies outside the triangle on the backside of the obtuse angle. Orthocentre and circumcentre lie opposite to each other in obtuse angle triangle.

Circumcenter

The third center is the circumcenter. The circumcenter is the intersection of the perpendicular bisectors of each side of the triangle. We can also think of this center as the point that is equidistant from each of the vertices. Since it is equidistant from each vertex, we can also construct a circle that passes through each vertex with the center being the circumcenter.



The distance between the circumcentre and the three vertices of a triangle is always equal.

OA = OB = OC = R (circumradius) = *abc*/4A

$\angle BOC = 2\angle A$

$\angle AOC = 2\angle B$

$\angle AOB = 2\angle C$

Location of circumcentre in various types of triangle:

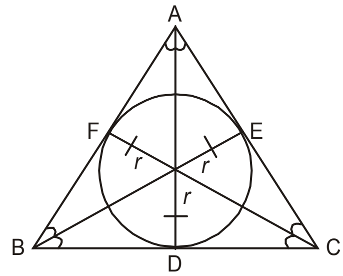
Acute angle triangle: Lies inside the triangle

Obtuse angle triangle: Lies outside the triangle

Right angle triangle: Lies at the midpoint of the hypotenuse.

Incenter

The last center is the incenter. The incenter is found by first constructing the angle bisectors of each of the three angles. The incenter is the intersection of these 3 segments.



The distance of the in-centre from the all the three sides is equal (ID = IE = IF = inradius "*r*")

In-radius (*r*) = Area of triangle/Semiperimetre = A/S

$\angle BIC = 90 + \angle A/2$

$\angle AIC = 90 + \angle B/2$

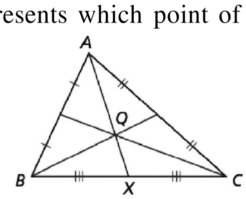
$\angle AIB = 90 + \angle C/2$

EXERCISE

1. In a triangle, a segment connecting the midpoints of two sides of the triangle is called a _____.

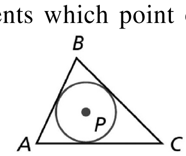
A. shortcut C. centroid
B. midsegment D. vertex

2. Point Q represents which point of concurrency?



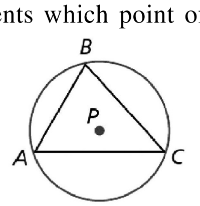
A. centroid C. orthocenter
B. incenter D. circumcenter

3. Point P represents which point of concurrency?



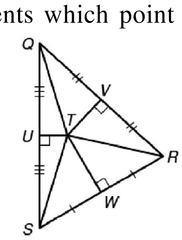
A. centroid C. orthocenter
B. incenter D. circumcenter

4. Point P represents which point of concurrency?



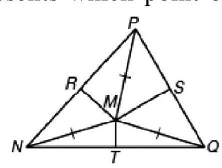
A. centroid B. incenter
C. orthocenter D. circumcenter

5. Point T represents which point of concurrency?



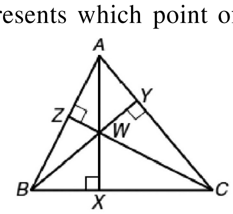
A. centroid B. incenter
C. orthocenter D. circumcenter

6. Point M represents which point of concurrency?



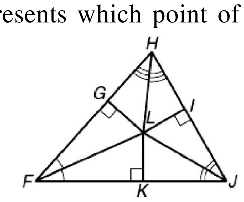
A. centroid B. incenter
C. orthocenter D. circumcenter

7. Point M represents which point of concurrency?



A. centroid B. incenter
C. orthocenter D. circumcenter

8. Point L represents which point of concurrency?



A. centroid B. incenter
C. orthocenter D. circumcenter

9. Which point of concurrency is the intersection of the medians of the triangle?

A. centroid B. incenter
C. orthocenter D. circumcenter

10. Which point of concurrency is the intersection of the altitudes of the triangle?

A. centroid B. incenter
C. orthocenter D. circumcenter

11. Which point of concurrency is the intersection of the angle bisectors of the triangle?
A. centroid B. incenter
C. orthocenter D. circumcenter

12. Which point of concurrency is the intersection of the perpendicular bisectors of the triangle?
A. centroid B. incenter
C. orthocenter D. circumcenter

13. Which point of concurrency is equidistant from the three sides of a triangle?
A. centroid B. incenter
C. orthocenter D. circumcenter

14. Which point of concurrency is equidistant from the three verticies of a triangle?
A. centroid B. incenter
C. orthocenter D. circumcenter

15. Which point of concurrency is the center of gravity of a triangle?
A. centroid B. incenter
C. orthocenter D. circumcenter

ANSWERS

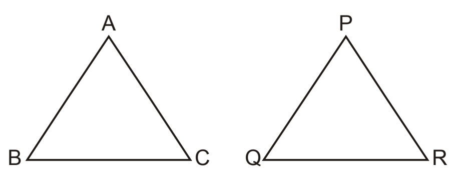
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	A	B	D	D	D	C	B	A	C
11	**12**	**13**	**14**	**15**					
B	D	B	D	A					

Congruence and Similarity of Triangles

Congruence of triangle: Two triangles having same shape and measurement but different positions are called congruent triangle.

Congruence criterion

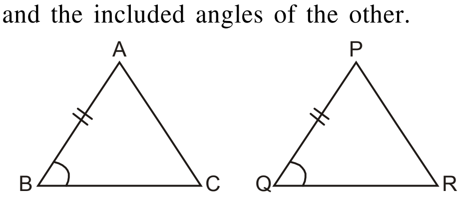
(*i*) **Side-Side-Side (SSS) Congruence criterion:** Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.



Here, AB = PQ, BC = QR and AC = PR

∴ $\Delta ABC \cong \Delta PQR$

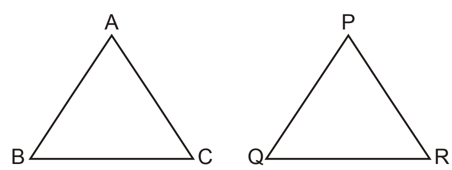
(*ii*) **Side-Angle-Side (SAS) Congruence criterion:** Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angles of the other.



Here, AB = PQ, $\angle B = \angle Q$ and BC = QR

∴ $\Delta ABC \cong \Delta PQR$

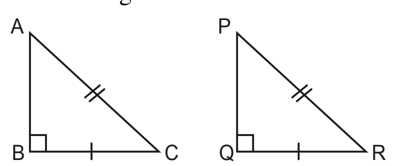
(*iii*) **Angle-Side-Angle (ASA) Congruence criterion:** Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the triangle.



∵ $\angle B = \angle Q$, $\angle C = \angle R$ and BC = QR

∴ $\Delta ABC \cong \Delta PQR$

(*iv*) **Right Angle-Hypotenuse-Side (RHS) congruence criterion:** Two right triangles are congruent. If the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.



∵ $\angle B = \angle Q = 90°$, AC = PR, BC = QR

∴ $\Delta ABC \cong \Delta PQR$.

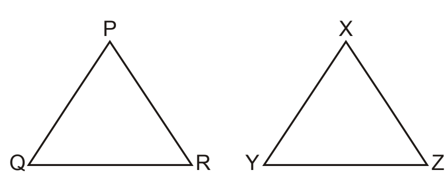
Similar Triangles: Two triangles having same shape but not necessarily of the same size, called similar triangle.

Basic properties of similar triangles:

(*i*) The corresponding angles of similar triangles are equal.

(*ii*) The corresponding sides of similar triangles are proportional.

The symbol for similar or is similar to is ~



$\angle P = \angle X$, $\angle Q = \angle Y$ and $\angle R = \angle Z$

Then, $\frac{PQ}{QR} = \frac{XY}{YZ}$ or, $\frac{PR}{PQ} = \frac{XZ}{XY}$

Thus, $\Delta PQR \sim \Delta XYZ$

→ Congruent triangles are always similar but similar triangles may or may not be congruent.

→ Any two equilateral triangles are always similar triangles.

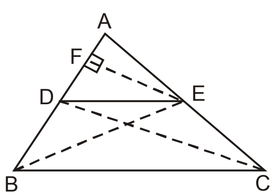
SOME IMPORTANT THEOREMS

Basic Proportionality Theorem: (Thalse Theorem)

Theorem 1: *In a triangle, a line drawn parallel to one side to intersect the other sides in distinct points divides the two sides in the same ratio.*

Given: A ΔABC in which DE is drawn parallel to side BC.

To prove. $\frac{AD}{DB} = \frac{AE}{EC}$.



Const.: Join BE and CD. Draw EF ⊥ AB.

Proof: ar.(ΔBDE) = ar.(ΔCDE) ...(1)

[∵ Δs on the same base DE and between the same parallels are equal in area]

$$\frac{ar.(\Delta ADE)}{ar.(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF}$$

[∵ Area of $\Delta = \frac{1}{2}$ base × height]

$$\Rightarrow \frac{ar.(\Delta ADE)}{ar.(\Delta BDE)} = \frac{AD}{DB} \quad ...(2)$$

Similarly, $\frac{ar.(\Delta ADE)}{ar.(\Delta CDE)} = \frac{AE}{EC}$ ...(3)

$$\therefore \frac{ar.(\Delta ADE)}{ar.(\Delta BDE)} = \frac{AE}{EC}$$

[from (1) and (3)] ...(4)

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[from (2) and (4)]}$$

Corollary 1: $\frac{AD}{DB} = \frac{AE}{EC}$

Adding 1 to both sides, we get

$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \frac{AD + DB}{DB} = \frac{AE + EC}{EC}$$

Corollary 2: $\frac{AD}{DB} = \frac{AE}{EC}$

$$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

Adding 1 to both sides, we get

$$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$

$$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}.$$

Converse of Basic Proportionality Theorem

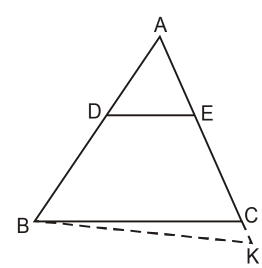
Theorem 2: *If a line divides any two sides of a triangle in the same ratio, prove that it is parallel to the third side.*

Given: A ΔABC, DE is st. line such that

$$\frac{AD}{DB} = \frac{AE}{EC}$$

To Prove. DE ∥ BC

Const.: If DE is not ∥ BC, draw BK ∥ DE meeting AC produce in.



Proof: In DABK, DE ∥ BK

$$\therefore \frac{AD}{DB} = \frac{AE}{EK} \quad ...(1)$$

[∵ A line drawn ∥ to one side of a Δ divides the other two sides in the same ratio]

But, $\frac{AD}{DB} = \frac{AE}{EC}$ (given) ...(2)

From (1) and (2), we get

$\frac{AE}{EK} = \frac{AE}{EC}$ or, EK = EC

which is possible only when C and K coincides.

Hence, DE ∥ BC.

EXERCISE

1. Two similar triangles have
A. equal sides
B. equal areas
C. equal angles
D. None of these

2. Two congruent triangles have
A. proportional sides
B. equal sides
C. equal corresponding sides
D. equal corresponding angles

3. Which of the following is false for two congruent triangles?
A. Corresponding angles are equal.
B. Two sides and included angles are equal.
C. Corresponding sides are equal.
D. Two angles and one side are equal.

4. If the sides of a triangle are 8 cm, 12 cm and 15 cm then the angle is
A. Right angle
B. Obtuse angle
C. Acute angle
D. None of these

5. If two triangles are on the same base and between the parallel lines then they will be
A. equilaterals
B. right angled
C. equal in area
D. congruent

6. If the three heights of a traingle are equal then it is
A. right angled triangle
B. obtuse angled triangle
C. equilateral triangle
D. None of these

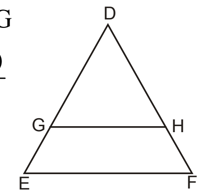
7. If two corresponding sides and the angle between them of a traingle are equal to another triangle. Then the angles are :
A. congruent but not similar
B. similar but not congruent
C. neither congruent nor similar
D. congruent and similar.

8. Ratio of areas of two similar triangles is equal to :
A. ratio of squares of the corresponding altitudes
B. ratio of squares of corresponding medians.
C. Either (A) or (B)
D. (A) and (B) both

9. If the areas of two similar triangles are equal then the triangles :
A. are congruent
B. have equal length of corresponding sides
C. (A) and (B)
D. None of these

10. Two isosceles triangles have equal vertical angles and their areas are in the ratio of 9 : 25 then the ratio between their corresponding heights is :
A. 5 : 3
B. 25 : 9
C. 3 : 5
D. 16 : 9

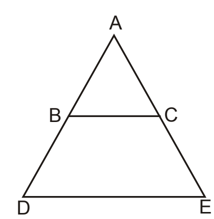
11. If in a Δ DEF, GH ∥ EF and DG : EG = 2 : 3 then the value of $\frac{ar(\Delta DGH)}{ar(\Delta DEF)}$ is :



A. $\frac{2}{15}$
B. $\frac{4}{15}$
C. $\frac{4}{25}$
D. $\frac{2}{25}$

12. Sides of two similar triangles are in the ratio of 5 : 11 then ratio of their areas is :
A. 25 : 11
B. 25 : 121
C. 125 : 121
D. 121 : 25

13. If ΔABC ~ ΔADE and ar (Δ ADE) = 9 ar (Δ ABC) then $\frac{BC}{DE}$ is equal to :



A. $\frac{1}{3}$
B. $\frac{1}{9}$
C. $\frac{3}{4}$
D. None of these

14. In a triangle a line is drawn from the mid-point of one side of and parallel to another side then
A. the line bisects the whole traingle
B. bisects the third side
C. bisects the opposite angle
D. None of these

15. If in a ΔABC, ∠B = ∠C and BD = CE then which of the following is true
A. $DE = \frac{1}{2} BC$
B. DE ∥ BC
C. (A) and (B)
D. None of these

ANSWERS

1	2	3	4	5	6	7	8	9	10
C	C	D	C	C	C	C	D	C	C
11	**12**	**13**	**14**	**15**					
C	B	A	B	B					

EXPLANATORY ANSWERS

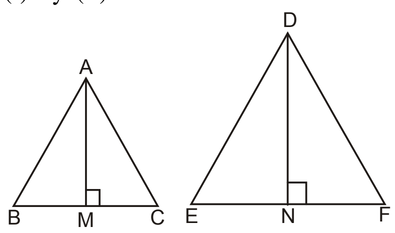
10. According to equestion

$\angle A = \angle D$ and $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{9}{25}$

Since, $AB = AC$ (given) ....(*i*)

$DE = DF$ (given) ....(*ii*)

Dividing (i) by (ii)



$$\frac{AB}{DE} = \frac{AC}{DF} \quad ...(iii)$$

Hence, $\Delta ABC \sim \Delta DEF$

[By SAS criterion of similar Δs]

In ΔAMC and ΔDNF

$\angle AMC = \angle DNF = 90°$

$\angle C = \angle F$ (because $\Delta ABC \sim \Delta DEF$)

$\therefore \angle AMC \sim DNF$

(By AA Criterion of similar Δs)

$\therefore \quad \dfrac{AC}{DF} = \dfrac{AM}{DN}$

and, $\dfrac{ar(\Delta ABC)}{ar(DEF)} = \dfrac{AC^2}{DF^2} = \dfrac{AM^2}{DN^2}$

$\therefore \quad \dfrac{AM^2}{DN^2} = \dfrac{9}{25}$

$\Rightarrow \quad \dfrac{AM}{DN} = \dfrac{3}{5}$

Hence required ratio = 3 : 5

11. $\dfrac{DG}{EG} = \dfrac{2}{3} \quad \therefore \quad \dfrac{DG}{DE} = \dfrac{2}{2+3} = \dfrac{2}{5}$

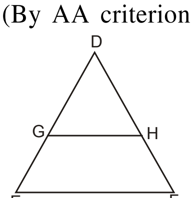
In Δ DGH and Δ DEF, $\angle D = \angle D$

$\angle DGH = \angle DEF$

(Since, GH $\parallel$ EF and DE is the transversal among corresponding angles)

$\therefore \quad \Delta DGH \sim \Delta DEF$

(By AA criterion of similar triangles)



$\therefore \quad \dfrac{ar(\Delta DGH)}{ar(\Delta DEF)} = \dfrac{DG^2}{DE^2} = \dfrac{2^2}{5^2} = \dfrac{4}{25}$

12. Since, ratio of area of two similar traingles = ratio of square of corresponding sides

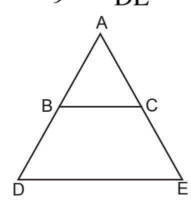
ratio of sides = 5 : 11

$\therefore$ ratio of their areas = $(5)^2 : (11)^2 = 25 : 121$.

13. From question,

$ar(\Delta ADE) = 9\ ar(\Delta ABC)$

or, $\dfrac{ar(\Delta ABC)}{ar(\Delta ADE)} = \dfrac{1}{9} = \dfrac{BC^2}{DE^2}$



Hence, $\dfrac{BC}{DE} = \sqrt{\dfrac{1}{9}} = \dfrac{1}{3}$

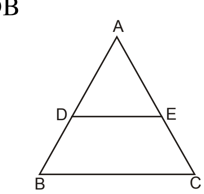
14. From question, In ΔABC, D is the mid-point of AB and DE is drawn parallel to BC and it meets at E on AC.

Since, DE $\parallel$ BC (By B.P. Theorem)

$$\frac{AD}{DB} = \frac{AE}{EC} \quad ...(i)$$

also, AD = DB ($\because$ D is mid-point)

$\therefore \quad \dfrac{AD}{DB} = 1$



Now, $\dfrac{AE}{EC} = 1$ (From ...(*i*))

$\Rightarrow$ AE = EC

Hence, the line bisects the third line

15. According to questions, $\angle B = \angle C$ (given)

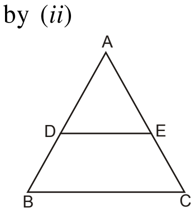
$\Rightarrow$ AB = AC (*i*)

and, BD = CE (*ii*)

Subtracting (*ii*) from (*i*), AB – BD = AC – CE

$\Rightarrow$ AD = AE (*iii*)

Dividing (*iii*) by (*ii*)



$$\frac{AD}{BD} = \frac{AE}{CE}$$

$\therefore$ DE $\parallel$ BC [By converse of BP Theorem]

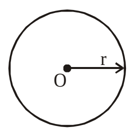
Hence, option (B) is true.

Circle and Its Chord

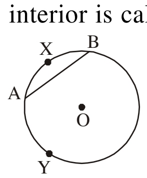
A circle is a set of those points in a plane that are at a given constant distance from a given fixed point in the plane. The fixed point is called the **centre of the circle** and the constant distance of every point on the circle from its centre of called the **radius of the circle**.

Things to Remember

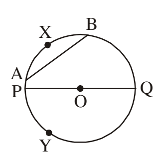
1. Locus of a point moving in a plane such that its distance from a fixed point in the same plane remains constant is called a circle.



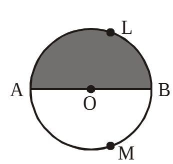
2. The fixed point O is called its centre and the constant distance *r* is called the radius.
3. The path traced by the moving point as described above is called the circumference of the circle.
4. The part of the plane containing the circle that consists the circle and its interior is called the circular region.



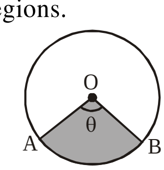
5. Line segment joining any two points on the circumference is called a chord of the circle. In the above figure AB is a chord of the circle.



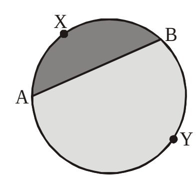
6. The chord passing through the centre of the circle is called its diameter.
   In the above figure PQ is a diameter of the circle.
7. Diameter is the longest chord of the circle.
8. Any fraction of the circumference is called an arc of the circle.
9. The remaining part of the circumference is called the alternate segment of the circle with respect to the arc.
10. An arc of a circle smaller in length than the semicircle is called a minor arc. In the above figure arc AXB is a minor arc.
11. An arc of a circle more in length than the semicircle is called a major arc. In the above figure arc AYB is a major arc.



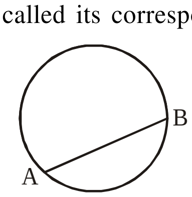
12. Diameter of a circle divides its circumference into two equal arcs. Each of them is called a semicircle.
13. The part of the plane containing the semicircle that consists the semi-circle, the enclosing diameter and its interior is called the semicircular region.
    In the above figure region ALB and region BMA are semicircular regions.



14. The angle formed by joining the end points of an arc to the centre of the circle is said to be the angle subtended by an arc at the centre of the circle. In the above figure, θ is the degree measure of arc AB.
15. The part of the plane containing the circle that consists of the arc enclosing radii and its interior is called the sector of the circle. In the above figure AOB is sector of the circle.
16. A chord of the circle other than a diameter divides the circular region into two unequal parts.
    Each of these parts is called a segment. The larger part is called the major segment and the smaller is called the minor segment.



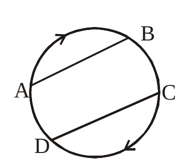
    In the above figure segment AXB is a minor segment and segment BYA is the major segment.
17. Two circles in a plane are said to be concentric if they have same centre but different radii.
18. The chord formed by joining the end points of an arc of a circle is called its corresponding chord.



    In the given figures chord AB is the corresponding chord of arc AB.

Important Theorems on Circles

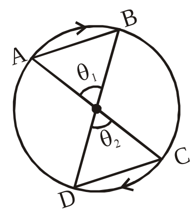
1.



(*a*) If two arcs of a circle are equal then their corresponding chords are also equal.
AB = CD
$\Leftrightarrow$ arcAB = arcCD.

(*b*) If two chords of a circle are equal then their corresponding arcs are also equal.

2.

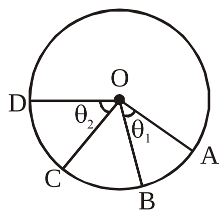


(*a*) If two chords of a circle are equal then they subtend equal angles at the centre of the circle.

(*b*) If two chords of a circle subtend equal angles at the centre of a circle then they are equal

$AB = CD \Leftrightarrow \theta_1 = \theta_2$

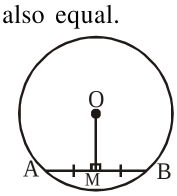
3.



(*a*) If two arcs of a circle have same degree measure then they are equal.
arc AB = arc CD
$\Leftrightarrow \theta_1 = \theta_2$

(*b*) If two arcs of a circle are equal then their degree measures are also equal.

4.

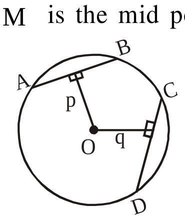


(*a*) Perpendicular to a chord from the centre of the circle bisects the chord.

(*b*) Line segment joining the centre of the circle to the mid point of the chord is perpendicular to the chord.

$OM \perp AB \Leftrightarrow M$ is the mid point of AB.

5.

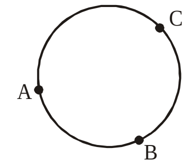


(*a*) If two chords of a circle are equidistant from the centre of the circle, they are equal.

(*b*) If two chords of a circle are equal then they are equidistant from the centre of the circle

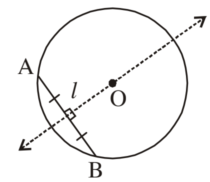
$AB = CD \Leftrightarrow p = q$

6.



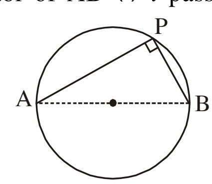
Through any three non collinear points, passes one and only one circle. A, B and C are three non-collinear points in a plane then one and only one circle passes through them.

7. Perpendicular bisector of a chord of a circle passes through the centre of the circle.



l is the $\perp$ bisector of AB $\Leftrightarrow$ l passes through O.

8.

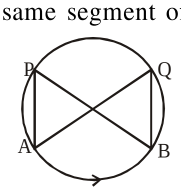


(*a*) **Theorem of Thales:** Angle in a semicircle is a right angle.

(*b*) If an arc subtends a right angle at any point in its alternate segment then it is a semi-circle.

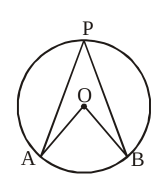
$\overset{\frown}{APB}$ is a semicircle $\Leftrightarrow \angle APB = 90°$

9. Angles in the same segment of a circle are equal



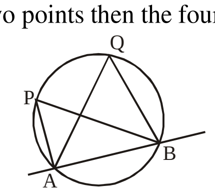
In the above figure $\angle APB = \angle AQB$

10.



Degree Measure Theorem : Degree measure of an arc is twice the angle subtended by it at any point on the alternate segment of the circle with respect to the arc. In the above figure $\angle AOB = 2\angle APB$

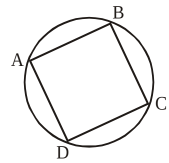
11. If a line segment joining two points subtends equal angles at two other points on same side of the line containing the two points then the four points are concyclic.



In the above figure $\angle APB = \angle AQB$, then A,B,Q and P are concylic.

Definition

A quadrilateral is said to be cyclic if all of its four vertices lie on the circle, ABCD is a cyclic quadrilateral.



12. If in a quadrilateral the pair of opposite angles is supplementary then the quadrilateral is cyclic.

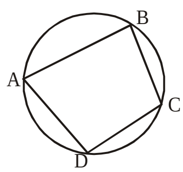
In the above figure

$\angle A + \angle C = 180°$

or $\angle B + \angle D = 180°$.

$\Rightarrow$ ABCD is cyclic quadrilateral.

13. Sum of each pair of opposite angles of a cyclic quadrilateral is 180°.



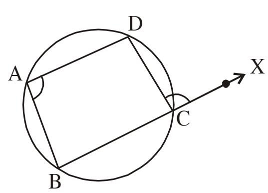
In the above figure

ABCD is cyclic quadrilateral

$\Rightarrow \angle A + \angle C = 180°$

and $\angle B + \angle D = 180°$.

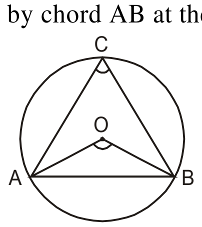
14. Exterior angle of a cyclic quadrilateral is equal to its opposite interior angle.



In the above figure, $\angle DCX = \angle A$

Angle subtended by a Chord

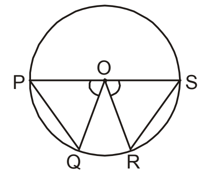
The angle subtended by the chord AB at a point C (not on the chord AB) on the circumference of the circle is $\angle ACB$. The angle subtended by chord AB at the centre O is $\angle AOB$.



Activity: Let us find out the relation between the size of the chord and the angle subtended by it at the centre.

We draw many chords and angles subtended by them at the centre. We observe that the longer is the chord the bigger will be the angle subtended by it at the centre. We draw two equal chords and measure the angles subtended by them at the centre. We find that the angles subtended by them at the centre are equal.

Theorem 1. Equal chords of a circle subtend equal angles at the centre.



Given. A circle such that

Chord PQ = Chord RS

To prove, $\angle POQ = \angle ROS$

Proof. In ΔPOQ and ΔROS,

$PQ = RS$

$OP = OS$ = radius

$OQ = OR$ = radius

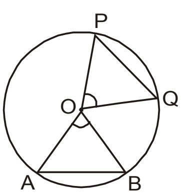
$\therefore \quad \Delta POQ \cong \Delta ROS$

(By SSS Theorem of congruence)

$\Rightarrow \quad \angle POQ = \angle ROS$ (C.P.C.T.)

$\Rightarrow$ Angles subtended by equal chords are equal.

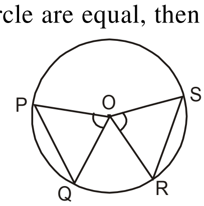
Activity, We draw a circle with centre O and draw an angle AOB, where A and B are the points on the circle. Draw another angle $\angle POQ$ at the centre equal to $\angle AOB$. Separate these two angles AOB and POQ. If we put segment AOB on the segment POQ of the circle, we find that they cover each other.



So, $AB = PQ$

Repeat this activity for other equal angles. We find the chords are equal in each case.

Theorem 2. If the angles subtended by the two chords at the centre of a circle are equal, then the chords are equal.



Given. Two chords PQ and RS of a circle in which,

$\angle POQ = \angle ROS$; $PQ = RS$

To prove.

Proof. In ΔPOQ and ΔROS

$OP = OR$ = radius

$\angle POQ = \angle ROS$

$OQ = OS$ = radius

$\therefore \quad \Delta POQ \cong \Delta ROS$

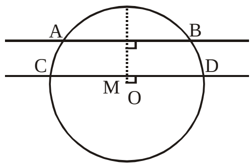
[By SAS Theorem of congruence]

$\Rightarrow \quad PQ = RS$ (C.P.C.T.)

Hence, Chords are equal.

EXERCISE

1. AB and CD are two parallel chords of a circle such that AB = 5 cm and CD = 11 cm. If distance between them is 3 cm then radius of the circle is



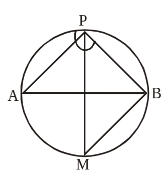
A. $2\sqrt{146}$ cm   B. $\sqrt{146}$ cm
C. $\frac{\sqrt{146}}{2}$ cm   D. None of these

2. The maximum number of common tangents to any pair of circles in the same plane is :
A. 4   B. 5
C. 2   D. 3

3. Find the length of a chord which is at a distance of 3 cm from the centre of a circle of radius 5 cm.
A. 2 cm   B. 6 cm
C. 8 cm   D. 10 cm

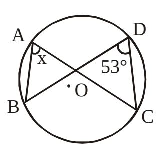
4. If AB and AC are two chords of a circle of radius 5 cm such that AB = AC = $4\sqrt{5}$ cm then the length of the chord BC is :
A. 8 cm   B. 8.4 cm
C. 9 cm   D. None of these

5. In the given figure AB is the diameter of the circle, PM bisects ∠APB then the measure of ∠ABM is :



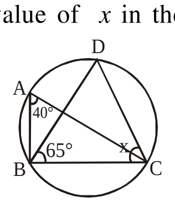
A. 45°   B. 30°
C. 15°   D. 60°

6. Determine the value of x in the figure given below.



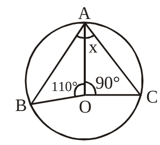
A. 53°   B. 106°
C. 26.5°   D. 100°

7. Determine the value of x in the figure given below.



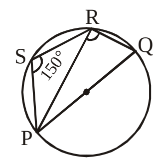
A. 40°   B. 75°
C. 65°   D. 105°

8. Determine the value of x in the figure given below.



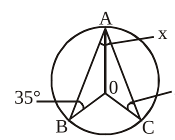
A. 80°   B. 100°
C. 90°   D. 110°

9. PQ is a diameter and PQRS is a cyclic quadrilateral. If ∠PSR = 150°, then measure of ∠RPQ is :



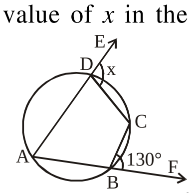
A. 90º   B. 60º
C. 30º   D. None of these

10. Determine the value of x in the figure given below.



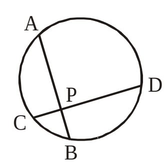
A. 35°   B. 25°
C. 30°   D. 60°

11. Determine the value of x in the figure given below.



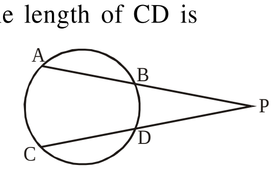
A. 65°   B. 60°
C. 50°   D. 80°

12. Chords AB and CD of a circle meet inside the circle at D. If PA = 4 cm, AB = 7 cm and PD = 6 cm then length of CD is :



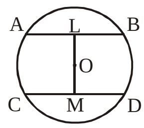
A. 8 cm   B. 6 cm
C. 2 cm   D. None of these

13. Chords AB and CD of a circle when product meet out side the circle at P, If AB = 4 cm; BP = 3 cm and CP = 14 cm then the length of CD is



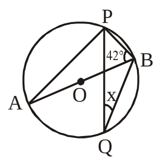
A. 3 cm   B. 4.5 cm
C. 6 cm   D. None of these

14. AB and CD are two parallel chords of circle such that AB = 10 cm and CD = 24 cm. If LM = 17 cm then the diameter of the circle is



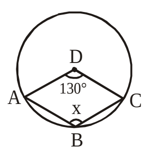
A. 13 cm B. 26 cm
C. 14 cm D. None of these

15. In the given figure determine the value of *x*.



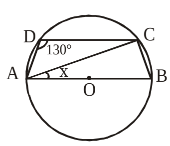
A. 48° B. 42°
C. 60° D. 38°

16. In the given figure determine the value of *x*.



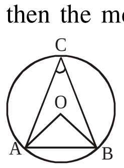
A. 90° B. 115°
C. 130° D. 65°

17. Find *x*



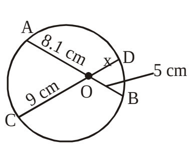
A. 40° B. 50°
C. 30° D. 60°

18. If O is the centre of the circle and ΔAOB is an equilateral triangle, then the measure of ΔACB is



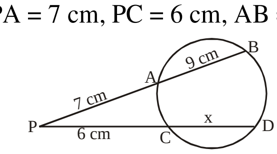
A. 60° B. 30°
C. 90° D. 75°

19. Find *x* if AO = 8.1 cm, BO = 5 cm, OC = 9 cm and OD = *x* cm.



A. 4.1 cm B. 4.1 cm
C. 4.2 cm D. 4.5 cm

20. Find *x* if PA = 7 cm, PC = 6 cm, AB = 9 cm and CD = *x*



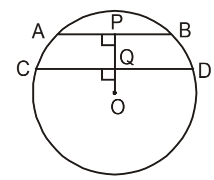
A. 12.0 cm B. 12.6 cm
C. 12.66 cm D. 12.67 cm

21. Two chords AB, CD of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.

A. $\frac{\sqrt{146}}{2}$ B. $\frac{\sqrt{150}}{2}$

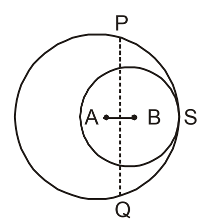
C. $\frac{150}{3}$ D. $\frac{100}{2}$

22. O is the centre of the circle of radius 5 cm, OP ⊥ AB, OQ ⊥ CD, AB ∥ CD, AB = 6 cm and CD = 8 cm. Determine PQ.



A. 5 cm B. 10 cm
C. 1 cm D. 2 cm

23. Two circle with centres A and B and of radii 5 cm and 3 cm, respectively touch each other internally. If the perpendicular bisector of segment AB meets the bigger circle in P and Q. Find the length of PQ.



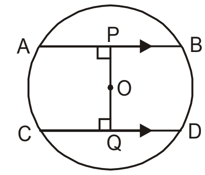
A. $4\sqrt{6}$ cm B. $3\sqrt{2}$ cm

C. $5\sqrt{4}$ cm D. $3\sqrt{5}$ cm

24. AB and CD are two parallel chords of a circle, which are an opposite sides of the centre, such that AB = 10 cm, CD = 24 cm and the distance between AB and CD is 17 cm. Find the radius of the circle.

A. 15 cm B. 13 cm
C. 14 cm D. 20 cm

25. O is the centre of the circle with radius 5 cm. OP ⊥ AB, OQ ⊥ CD, AB ∥ CD, AB = 8 cm and CD = 6 cm. Determine PQ.



A. 3 cm B. 4 cm
C. 5 cm D. 7 cm

ANSWERS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	A	C	A	A	A	B	A	B	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	A	D	B	A	B	A	B	D	C
21	**22**	**23**	**24**	**25**					
A	C	A	B	D					

EXPLANATORY ANSWERS

1. Let OM = x

Then $x^2+\left(\frac{11}{2}\right)^2=r^2$ and $(x+3)^2+\left(\frac{5}{2}\right)^2=r^2$

Then, $x^2+6x+9+\frac{25}{4}=x^2+\frac{121}{4}$

$\Rightarrow$ $6x = 15 \Rightarrow x = 2.5$ cm

Thus, $r^2 = (2.5)^2+\frac{121}{4}=\frac{146}{4} \Rightarrow r=\frac{\sqrt{146}}{2}$ cm

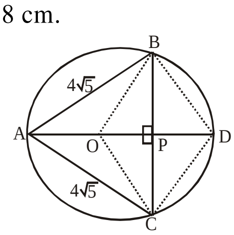
3. Length of chord = $2.\sqrt{5^2-3^2}=8$ cm.

4. Let OP = x and BP = y

Then $x^2+y^2=25$ and $(5+x)^2+y^2=80$

Solving these two, we get, y = BP = 4 cm

Thus, BC = 8 cm.



5. $\angle ABM=\angle APM=\frac{1}{2}\angle APB=\frac{1}{2}\times 90^\circ=45^\circ$

6. As angle in the same segment are equal.

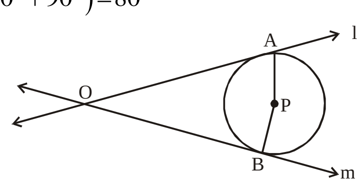
$x = 53^\circ$

7. $x+65^\circ+40^\circ=180^\circ$

$x=75^\circ$

8. $x=\frac{1}{2}\left[(180^\circ-110^\circ)+(180^\circ-90^\circ)\right]$

$=\frac{1}{2}(70^\circ+90^\circ)=80^\circ$



Let P be the centre of a circle touching l and m. Clearly, AP = BP.

Hence, P lies on angle bisector of $\angle$AOB. Hence, locus of P will be the angle bisectors of the angles formed by these intersecting lines.

9. $\angle PQR=180^\circ-150^\circ=30^\circ$

$\angle PRQ=90^\circ$ (Angle of a semicircle)

$\angle RPQ+90^\circ+30^\circ=180^\circ$

$\Rightarrow \angle RPQ=60^\circ$

10. $\angle OAB=\angle OBA\ (\because OA=OB)$

$\angle OAB=35^\circ$

Similarly, $\angle AOC=25^\circ$

$\therefore\ \angle x=35^\circ+25^\circ=60^\circ$

12. Use $PA\times PB=PC\times PD$

$4\times(7-4)=6\times PC$

$\Rightarrow$ PC = 2 cm

CD = PC + PD = 2 + 6 = 8 cm.

13. Use $PA\times PB=PC\times PD$

$\Rightarrow (4+3)\times 3=14\times PD \Rightarrow PD = 1.5$ cm

$\Rightarrow CD=PC-PD=14-1.5=12.5$ cm.

14. Let OL = x.

Then OM$(17-x)$

Also LB = 5 cm,

MD = 12 cm.

Then $x^2+25=r^2$ and $(17-x)^2+144=r^2$

Solving these we get

x = 12 cm and r = 13 cm

Thus, diameter = 2 × 13 = 26 cm.

15. $x+42^\circ+90^\circ=180^\circ \Rightarrow x=48^\circ$

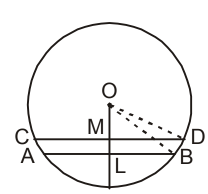
18. $\angle ACB=\frac{1}{2}\angle AOB=\frac{1}{2}\times 60^\circ=30^\circ$

21. Given, chord AB = 5 cm, chord CD = 11 cm and AB ∥ CD

Perpendicular distance ML between AB and CD = 3 cm

To find. Radius of the circle.

Construction, Join OB, OD and draw perpendicular bisectors OL of AB and OM of CD



Procedure. In rt. ΔOMD,

$$OD^2 = MD^2 + OM^2$$

(By Pythagoras Theorem)

Let, $OM = x$ cm

$$\therefore \quad r^2 = \left(\frac{11}{2}\right)^2 + x^2 \quad ...(1)$$

And In rt. ΔOLB, $OB^2 = BL^2 + OL^2$

$$\Rightarrow \quad r^2 = \left(\frac{5}{2}\right)^2 + (3+x)^2 \quad ...(2)$$

From (1) and (2), we get,

$$r^2 = \left(\frac{11}{2}\right)^2 + x^2 = \left(\frac{5}{2}\right)^2 + (3+x)^2$$

$$\Rightarrow \quad \frac{121}{4} + x^2 = \frac{25}{4} + 9 + x^2 + 6x$$

$$\Rightarrow \quad x^2 - x^2 + 6x = \frac{121}{4} - \frac{25}{4} - 9$$

$$\Rightarrow \quad 6x = \frac{121 - 25 - 36}{4} = \frac{60}{4} = 15$$

$$\Rightarrow \quad x = \frac{15}{6} \text{ cm} = \frac{5}{2} \text{ cm}$$

From (1), $r^2 = \left(\frac{11}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{121+25}{4} = \frac{146}{4}$

or $r = \frac{\sqrt{146}}{2}$ cm.

22. Given. A circle with centre O, such that chord AB = 6 cm, chord CD = 8 cm

AB ∥ CD and radius of the circle = 5 cm

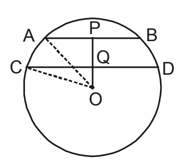
OP ⊥ AB and OQ ⊥ CD

To find. PQ

Construction. Join OA and OC

Procedure. AP = 1/2, AB = 3 cm, CQ = 1/2, CD = 4 cm

[Perpendicular from the centre of the circle bisects the chord]



In right Δ OPA, $OA^2 = OP^2 + AP^2$

[By Pythagoras Theorem]

$$\Rightarrow \quad 5^2 = OP^2 + 3^2$$

$$\Rightarrow \quad 5^2 - 3^2 = OP^2 \quad \Rightarrow \quad 25 - 9 = OP^2$$

$$\Rightarrow \quad 16 = OP^2 \quad \Rightarrow \quad 4 = OP$$

In right ΔOQC, $OC^2 = OQ^2 + CQ^2$

[By Pythagoras Theorem]

$$5^2 = OQ^2 + 4^2$$

$$\Rightarrow \quad 5^2 - 4^2 = OQ^2 \quad \Rightarrow \quad 25 - 16 = OQ^2$$

$$\Rightarrow \quad 9 = OQ^2 \quad \Rightarrow \quad 3 = OQ$$

$$\therefore \quad PQ = OP - OQ = 4 - 3 = 1 \text{ cm.}$$

23. Given. Two circles touch internally at S, A and B be the centres of the bigger and smaller circles, respectively. The perpendicular bisector PQ bisects AB and meets the circle at P and Q.

To find PQ

Construction. Join PA, ABS

Procedure. With given radii, we find

AS = 5 cm

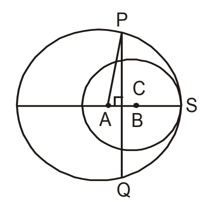
BS = 3 cm

AB = 5 – 3 = 2 cm and AC = 1 cm

[⊥ bisector bisects the chord]

PA = radius of bigger circle = 5 cm

In the right triangle ACP,



$$PC^2 = PA^2 - AC^2$$

[By Pythagoras Theorem]

$$\Rightarrow \quad PC^2 = (5)^2 - (1)^2$$

$$\Rightarrow \quad PC^2 = 25 - 1 = 24$$

$$\Rightarrow \quad PC = \sqrt{24} \Rightarrow PC = 2\sqrt{6}$$

$$\therefore \quad PQ = 2PC = 4\sqrt{6} \text{ cm.}$$

24. Given, AB and CD are two parellel chords of a circle, which are on opposite sides of the centre.

AB = 10 cm, CD = 24 cm

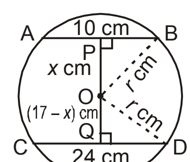
Distance between AB = CD = 17 cm

To find. Radius = ?

Construction. Draw OP ⊥ AB and OQ ⊥ CD. Join OB and OD.

Procedure. Since AB ∥ CD and OP ⊥ AB, OQ ⊥ CD

∴ Points P, O and Q are collinear.



Let, $OP = x$ cm

Then, $OQ = (17 - x)$ cm

$$PB = \frac{10}{2} = 5 \text{ cm}$$

(∵ ⊥ from the centre bisects the chord)

$$QD = \frac{24}{2} = 12 \text{ cm}$$

(∵ ⊥ from the centre bisects the chord)

In rt. ΔOPB, $r^2 = x^2 + 5^2$ ...(1)

(By Pythagoras Theorem)

In rt. ΔOQD, $r^2 = (17 - x)^2 + 12^2$ ...(2)

From (1) and (2), we have

$x^2 + 25 = (17 - x)^2 + 12^2$

$\Rightarrow x^2 + 25 = 289 + x^2 - 34x + 144$

$\Rightarrow x^2 - x^2 + 34x = 289 + 144 - 25$

$\Rightarrow 34x = 408$

$\Rightarrow x = 12$ cm

Using the value of x in (1), we get,

$r^2 = 12^2 + 5^2 = 144 + 25 = 169$

$\therefore$ Radius, $r = 13$ cm. ($\because$ radius can't be –ve)

25. Given, AB and CD are two parallel chords. AB = 8 cm, CD = 6 cm, radius = 5 cm.

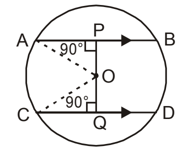
To find. PQ

Construction. Join OA, OC, where O is the centre of the circle

Procedure. AP = PB = 4 cm ...[$\because$ AB = 8 cm]

CQ = QD = 3 cm ...[$\because$ CD = 6 cm]

[$\because$ $\perp$ from the centre of a circle on any chord of the circle bisects it]



In rt. ΔOAP, OA = OC = radii of the circle = 5 cm

$OP^2 = OA^2 - AP^2$ [By Pythagoras Theorem]

$= (5)^2 - (4)^2 = 25 - 16 = 9$

$\Rightarrow$ OP = 3

In rt. ΔOCQ, $OQ^2 = OC^2 - CQ^2$ [By Pythagoras Theorem]

$(5)^2 - (3)^2 = 25 - 9 = 16$

$\Rightarrow$ OQ = 4

$\therefore$ PQ = PO + OQ = 3 + 4 = 7 cm.

Tangent

Secant

A line which interesects a circle in two distinct points is called a **secant** of the circle. In the fig. the line l intersects the circle in two distinct points A and B. The line l is a secant to the circle.

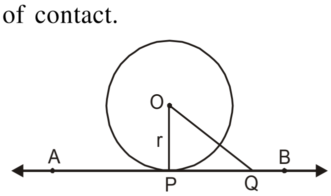
Tangent

A tangent to a circle is a line that intersects the circle at exactly one point. The point at which it meets the circle is called its point of contact and the line (tangent) is said to touch the circle at this point. In the figure, the line l meets the circle at only point A. Here A is the point of contact.

Some Important theorems on tangent

Theorem 1: *A tangent to a circle is perpendicular to the radius throught the point of contact.*

Given: A tangent AB to a circle C(O, r) with the point P as its point of contact.



To prove: OP $\perp$ AB.

Construction: Let Q be any point other that P, an AB. join OQ.

Proof: $\because$ Q is a point on the tangent AB other than the point of contact P.

$\therefore$ Q lies in the exterior of the circle.

$\therefore$ OQ > OP

i.e., OP < OQ

Thus, of all the segments that can be drawn from the centre O to any point on the line AB, OP is the shortest.

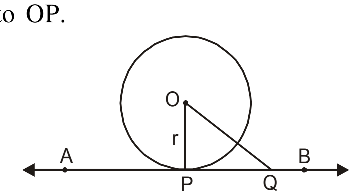
We know that the shortest segment that can be drawn from a given point to a given line perpendicular from the given point to the given line.

Hence OP $\perp$ AB.

Theorem 2: (Converse of theorem 1)

A line drawn through the end of a radius and perpendicular on it is tangent to the circle.

Given: A radius OP of a circle C(O, r) and a line APB perpendicular to OP.



To prove: AB is the tangent to the circle at A.

Proof: Take a point Q, different from P, on line AB, since OP $\perp$ AB, OQ > OP.

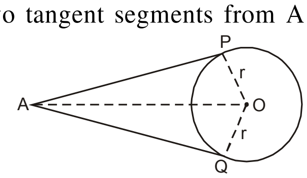
$\therefore$ The point Q lies outside the circle (since OP is the shortest line segment from O to AB).

Thus every point on the line AB, other P, lies outside the circle and therefore AB meets the circle only at the point P.

Hence AB is a tangent to the circle.

Theorem 3: *The lengths of the two tangents drawn from an external point to a circle are equal.*

Given: A is an external point to the circle C(O, r). AP and AQ are two tangent segments from A to the circle.



To prove: AP = AQ.

Construction: Draw line segments AP, OP and OQ.

Proof: A tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ $\angle OPA = \angle OQA = 90°$

Now in right Δs OPA and OQA,

OP = OQ [each = r]

OA = OA [common]

$\therefore$ $\Delta OPA \cong \Delta OQA$ [by RHS congruence rule]

$\therefore$ AP = AQ [c.p.c.t.c.]

Common Tangents to Two circles

(*i*) Where the two circles neither intersect nor touch each other, there are FOUR common tangents, two of them are transverse & the others are direct common tangents.

(*ii*) When they intersect there are two common tangents, both of them being direct.

(*iii*) When they touch each other:

(***a***) **Externally:** there are three common tangents, two direct and one is the tangent at the point of contact.

(***b***) **Internally:** only one common tangent possible at their point of contact.

(*iv*) Length of an external common tangent & internal common tangent to the two circles is given by:

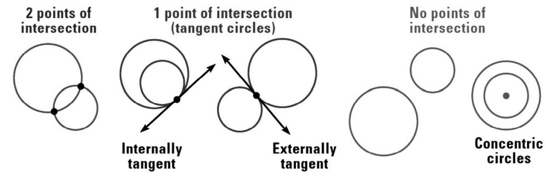
$$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2} \text{ \& } L_{int} = \sqrt{d^2 - (r_1 + r_2)^2}$$

Where d = distance between the centres of the two circles. r_1 & r_2 are the radii of the two circles.

(*v*) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii.

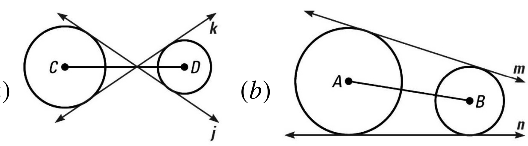
Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles.** Coplanar circles that have a common center are called **concentric.**



A line or segment that is tangent to two coplanar circles is called a **common tangent.** A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

Example: Tell whether the common tangents are internal or external.

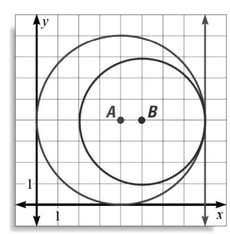


Solution: (*a*) The lines *j* and *k* intersect $\overline{CD}$, so they are common internal tangents.

(*b*) The lines m and n do not intersect $\overline{AB}$, so they are common external tangents.

In a plane, the **interior of a circle** consists of the points that are inside the circle. The **exterior of a circle** consists of the points that are outside the circle.

Example: Give the center and the radius of each circle. Describe the intersection of the two circles and describe all common tangents.



Solution: The center of $\odot A$ is A(4, 4) and its radius is 4. The center of $\odot B$ is B(5, 4) and its radius is 3. The two circles have only one point of intersection. It is the point (8, 4). The vertical line x = 8 is the only common tangent of the two circles.

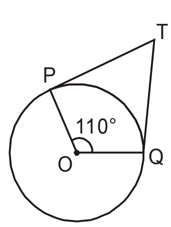
EXERCISE

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance from the centre is 25 cm. The radius of the circle is

A. 7 cm B. 12 cm

C. 15 cm D. 24.5 cm

2. In the given figure, if TP and TQ are the two tangents to a circle with centre O and that $\angle POQ = 110°$, then $\angle PTQ$ is equal to



A. 60° B. 70°

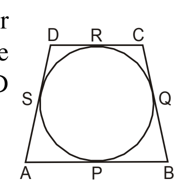
C. 80° D. 90°

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle POA$ is equal to

A. 50° B. 60°

C. 70° D. 80°

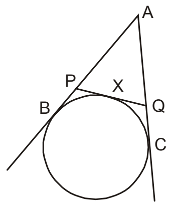
4. In Fig., a circle touches all the four sides of a quadrilateral ABCD whose sides AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD



A. 2 cm B. 5 cm

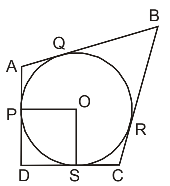
C. 3 cm D. 4 cm

5. If AB, AC, PQ are tangents in the figure and AB = 5 cm. The perimeter of ΔAPQ is



A. 8 cm B. 6 cm
C. 10 cm D. 5 cm

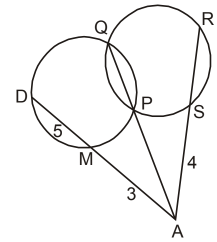
6. In Fig., a circle is inscribed within a quadrilateral ABCD. Given that BC = 38 cm, BQ = 27 cm and DC = 25 cm and that AD is perpendicular to DC. The radius of the circle is



A. 10 cm B. 14 cm
C. 8 cm D. 12 cm

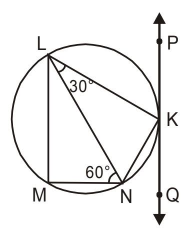
7. Two tangents are drawn to a circle from an external point A, touching the circle at the points P and Q. A third tangent intersects segment AP at B and segment AQ at C and touches the circle at R. If AQ = 10 units, then the perimeter of ΔABC (in units) is
A. 22.0 B. 20.5
C. 20.0 D. 40.0

8. In Fig., two circles intersect each other at points P and Q. From A on line PQ, secant AMD for one circle and secant ASR for the second one are drawn. If AM = 3, MD = 5 and AS = 4, determine SR.



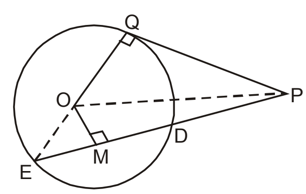
A. 2 cm B. 4 cm
C. 3 cm D. 1 cm

9. In the figure, KLMN is a cyclic quadrilateral and PQ is a tangent to the circle at K. If LN is a diameter of the circle. ∠KLN = 30° and ∠MNL = 60°. Determine ∠QKN



A. 30° B. 60°
C. 90° D. 100°

10. In the given Fig., PQ is tangent and O is the centre of the circle. Find EP is OP = 21, OQ = 9 and OM $\sqrt{80}$.



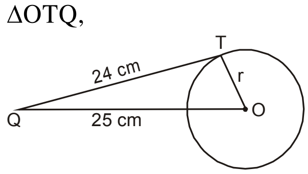
A. 10 B. 20
C. 30 D. 40

ANSWERS

1	2	3	4	5	6	7	8	9	10
A	B	A	C	C	B	C	A	A	B

EXPLANATORY ANSWERS

1. In rt. ∠*d* ΔOTQ,

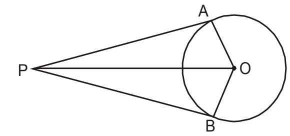


$$OT = \sqrt{OQ^2 - OT^2}$$

$$= \sqrt{25^2 - 24^2} = \sqrt{625 - 576} = \sqrt{49} = 7 \text{ cm.}$$

2. Since ∠POQ + ∠PTQ = 180°
[∵ ∠OPT = 90°, ∠OQT = 90°]
⇒ 110° + ∠PTQ = 180°
⇒ ∠PTQ = 180° – 110° = 70°.

3. Since ∠APB = 80°
∠AOB = 180° – 80° = 100°
∠PAO = ∠PBO = 90°
Since OP bisects ∠AOB



$$\angle AOP = \frac{1}{2}(100°) = 50° \text{ i.e. } \angle POA = 50°.$$

4. AD = AS + DS = AP + DR

[∵ AS = AP and DS = DR]

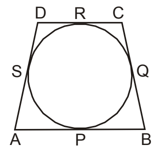
= (AB − BP) + (CD − RC)

= AB + CD − (BP + RC)

= AB + CD − (BQ + CQ)

[∵ BP = BQ, RC = CQ]

= AB + CD − BC = 6 + 4 − 7 = 3 cm.



5. Since AB and AC are the tangents from the same point A

∴ AB = AC = 5 cm

Similarly, BP = PX and XQ = QC

Perimeter of ΔAPQ

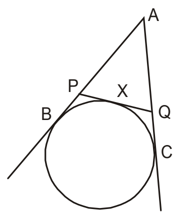
= AP + AQ + PQ

= AP + AQ + (PX + XQ)

= (AP + PX) + (AQ + XQ)

= (AP + BP) + (AQ + QC)

= AB + AC = 5 + 5 = 10 cm.



6. ∵ Lengths of tangents drawn from an external point to a circle are equal

∴ BQ = BR = 27 cm

Let x be the radius of the circle

∴ OP = OS = PD = x

CR = BC − BR

= 38 − 27 = 11 cm

Now CS = CR = 11 cm

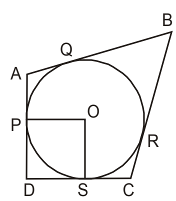
[∵ Length of tangents from an external point to a circle are equal]

∴ DS = CD − CS

= 25 cm − 11 cm = 14 cm

But DS = OP = x = 14 cm.

Hence radius = OP = 14 cm.



7. Since the tangents to a circle from an external point are equal in length

AP = AQ = 10 units

and BP = BR, CQ = CR

Adding these, we get,

BP + CQ = BR + CR = BC

Adding AB and AC to both sides, we get

BP + CQ + AB + AC = BC + AB + AC

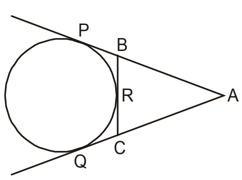
⇒ (BP + AB) + (CQ + AC) = AB + BC + AC

⇒ AP + AQ = Perimeter of ΔABC

⇒ (10 + 10) units = Perimeter of ΔABC

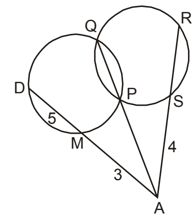
∴ Perimeter of ΔABC = 20 units.

∴ (*iii*) is the correct answer.



8. In the larger circle, chord QP and chord DM intersect each other at the point A outside the circle

∴ AM × AD = AP ...(1)



Again in the smaller circle, chord QP and chord RS intersect each other at the point A outside the circle.

∴ AS × AR = AP × AQ ...(2)

∴ AM × AD = AS × AR ...(3)

[From (1) and (2)]

Now AD = AM + MD

= 3 + 5 = 8

∴ From (3), 3 × 8 = 4 × AR

$$\Rightarrow \quad AR = \frac{24}{4} = 6$$

But AS + SR = AR

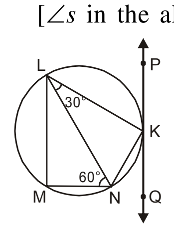
∴ 4 + SR = 6,

SR = 6 − 4 = 2.

9. KN is a chord through the point of contact K.

∴ ∠QKN = ∠KLN = 30°

[∠*s* in the alt. segment are equal]



10. Join OQ and OE

In ΔOPQ, *m* ∠OQP = 90°

∴ $OQ^2 + QP^2 = OP^2$ [By Pythagoras Theorem]

⇒ $(9)^2 + QP^2 = (21)^2$

⇒ $QP^2 = (21)^2 - (9)^2 = 360$

In rt. ∠*d* ΔOME,

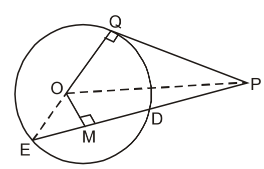
$OM^2 + EM^2 = OE^2$

⇒ $80 + (EM)^2 = (9)^2$

⇒ $EM^2 = 81 - 80 = 1$

⇒ $EM = +\sqrt{1} = 1$

∴ ED = 2EM = 2 × 1 = 2



Let EP = x

Then $PQ^2 = PD \times PE$

[Tangent secant second point theorem]

∴ $360 = (x - 2)x$

⇒ $x^2 - 2x - 360 = 0$

⇒ $(x - 20)(x + 18) = 0$

⇒ $x = 20, -18$

But $x \neq -18$

Hence EP = 20.