11

Conic Sections

TOPIC 1 Circles

1. If the length of the chord of the circle, $x^2 + y^2 = r^2 (r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to : **[Sep. 05, 2020 (II)]**

(a) $\frac{9}{5}$ (b) 12 (c) $\frac{24}{5}$ (d) $\frac{12}{5}$

2. The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point: **[Sep. 04, 2020 (II)]**

(a) $(-1, 3)$ (b) $(-3, 6)$ (c) $(-3, 1)$ (d) $(1, -3)$

3. Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is ______________. **[NA Sep. 04, 2020 (II)]**

4. The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x = 3$ and $y = 2$, is __________. **[NA Sep. 03, 2020 (I)]**

5. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is________. **[NA Sep. 02, 2020 (I)]**

6. A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle? **[Jan. 9, 2020 (I)]**

(a) $4x - 3y + 17 = 0$ (b) $3x - 4y - 24 = 0$
(c) $3x + 4y - 6 = 0$ (d) $4x + 3y - 8 = 0$

7. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is ______. **[NA Jan. 9, 2020 (II)]**

8. If a line, $y = mx + c$ is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then: **[Jan. 8, 2020 (II)]**

(a) $c^2 - 7c + 6 = 0$ (b) $c^2 + 7c + 6 = 0$
(c) $c^2 + 6c + 7 = 0$ (d) $c^2 - 6c + 7 = 0$

9. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to: **[Jan. 7, 2020 (II)]**

(a) $\frac{52}{5}$ (b) $\frac{56}{5}$ (c) $\frac{64}{5}$ (d) $\frac{32}{5}$

10. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is : **[April 12, 2019 (I)]**

(a) $\frac{13}{5}$ (b) $\frac{120}{13}$ (c) $\frac{60}{13}$ (d) $\frac{13}{2}$

11. A circle touching the x-axis at $(3, 0)$ and making an intercept of length 8 on the y-axis passes through the point : **[April 12, 2019 (II)]**

(a) $(3, 10)$ (b) $(3, 5)$
(c) $(2, 3)$ (d) $(1, 5)$

12. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in \mathbf{R})$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for: **[April 10, 2019 (I)]**

(a) infinitely many values of K
(b) no value of K.
(c) exactly two values of K
(d) exactly one value of K

13. The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is: **[April 10, 2019 (I)]**

(a) 3 (b) $2\sqrt{2}$ (c) 2 (d) $3\sqrt{2}$

14. The locus of the centres of the circles, which touch the circle, $x^2+y^2=1$ externally, also touch the y-axis and lie in the first quadrant, is: **[April 10, 2019 (II)]**

(a) $x=\sqrt{1+4y}, y\geq 0$ (b) $y=\sqrt{1+2x}, x\geq 0$

(c) $y=\sqrt{1+4x}, x\geq 0$ (d) $x=\sqrt{1+2y}, y\geq 0$

15. All the points in the set $S=\left\{\frac{\alpha+i}{\alpha-1}:\propto\in R\right\}(i=\sqrt{-1})$ lie on a: **[April 09, 2019 (I)]**

(a) straight line whose slope is 1.

(b) circle whose radius is 1.

(c) circle whose radius is $\sqrt{2}$.

(d) straight line whose slope is –1.

16. If a tangent to the circle $x^2+y^2=1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is: **[April 09, 2019 (I)]**

(a) $x^2+y^2-4x^2y^2=0$ (b) $x^2+y^2-2xy=0$

(c) $x^2+y^2-16x^2y^2=0$ (d) $x^2+y^2-2x^2y^2=0$

17. The common tangent to the circles $x^2+y^2=4$ and $x^2+y^2+6x+8y-24=0$ also passes through the point: **[April 09, 2019 (II)]**

(a) $(4,-2)$ (b) $(-6,4)$ (c) $(6,-2)$ (d) $(-4,6)$

18. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2+y^2=16$, by the lines, $x+y=n$, $n\in N$, where N is the set of all natural numbers, is : **[April 08, 2019 (I)]**

(a) 320 (b) 105 (c) 160 (d) 210

19. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is : **[Jan. 12, 2019 (II)]**

(a) $\left(x^2+y^2\right)^2=4R^2x^2y^2$

(b) $\left(x^2+y^2\right)^3=4R^2x^2y^2$

(c) $\left(x^2+y^2\right)^2=4Rx^2y^2$

(d) $\left(x^2+y^2\right)(x+y)=R^2xy$

20. Let C_1 and C_2 be the centres of the circles $x^2+y^2-2x-2y-2=0$ and $x^2+y^2-6x-6y+14=0$ respectively. If P and Q are the points of intersection of these circles then, the area (in sq. units) of the quadrilateral PC_1QC_2 is : **[Jan. 12, 2019 (I)]**

(a) 8 (b) 6 (c) 9 (d) 4

21. If a variable line, $3x+4y-\lambda=0$ is such that the two circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-2y+78=0$ are on its opposite sides, then the set of all values of λ is the interval : **[Jan. 12, 2019 (I)]**

(a) $(2,17)$ (b) $[13,23]$

(c) $[12,21]$ (d) $(23,31)$

22. A square is inscribed in the circle $x^2+y^2-6x+8y-103=0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is : **[Jan. 11, 2019 (I)]**

(a) 6 (b) $\sqrt{137}$ (c) $\sqrt{41}$ (d) 13

23. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is : **[Jan. 11, 2019 (I)]**

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) $\sqrt{2}$

24. A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is : **[Jan. 11, 2019 (II)]**

(a) a hyperbola (b) an ellipse

(c) a straight line (d) a parabola

25. If a circle C passing through the point (4, 0) touches the circle $x^2+y^2+4x-6y=12$ externally at the point $(1,-1)$, then the radius of C is: **[Jan 10, 2019 (I)]**

(a) $2\sqrt{5}$ (b) 4 (c) 5 (d) $\sqrt{57}$

26. If the area of an equilateral triangle inscribed in the circle, $x^2+y^2+10x+12y+c=0$ is $27\sqrt{3}$ sq. units then c is equal to: **[Jan. 10, 2019 (II)]**

(a) 13 (b) 20 (c) –25 (d) 25

27. Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then: **[Jan 09, 2019 (I)]**

(a) $\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}$ (b) $\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{c}}$

(c) a, b, c are in A.P (d) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

28. If the circles $x^2+y^2-16x-20y+164=r^2$ and $(x-4)^2+(y-7)^2=36$ intersect at two distinct points, then: **[Jan. 09, 2019 (II)]**

(a) $r>11$ (b) $0<r<1$

(c) $r=11$ (d) $1<r<11$

29. The straight line $x+2y=1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is : **[Jan. 11, 2019 (I)]**

(a) $\frac{\sqrt{5}}{2}$ (b) $2\sqrt{5}$ (c) $\frac{\sqrt{5}}{4}$ (d) $4\sqrt{5}$

30. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is : **[2018]**

(a) 185 (b) 85 (c) 95 (d) 195

31. If a circle C, whose radius is 3, touches externally the circle, $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point (2, 2), then the length of the intercept cut by this circle c, on the x-axis is equal to **[Online April 16, 2018]**

(a) $\sqrt{5}$ (b) $2\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{5}$

32. A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to **[Online April 15, 2018]**

(a) $\sqrt{5}$ (b) 1 (c) $\sqrt{2}$ (d) 2

33. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. if the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is? **[Online April 15, 2018]**

(a) $3(x+y)+4=0$ (b) $8(2x+y)+3=0$

(c) $4(x+y)+3=0$ (d) $x+2y+3=0$

34. The tangent to the circle $C_1 : x^2 + y^2 - 2x - 1 = 0$ at the point (2, 1) cuts off a chord of length 4 from a circle C_2 whose centre is (3, – 2). The radius of C_2 is **[Online April 15, 2018]**

(a) $\sqrt{6}$ (b) 2 (c) $\sqrt{2}$ (d) 3

35. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is : **[2017]**

(a) $4\left(\sqrt{2}+1\right)$ (b) $2\left(\sqrt{2}+1\right)$

(c) $2\left(\sqrt{2}-1\right)$ (d) $4\left(\sqrt{2}-1\right)$

36. The equation

$\text{Im}\left(\frac{iz-2}{z-i}\right)+1=0, z \in C, z \neq i$ represents a part of a circle having radius equal to : **[Online April 9, 2017]**

(a) 2 (b) 1 (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

37. A line drawn through the point P(4, 7) cuts the circle $x^2 + y^2 = 9$ at the points A and B. Then PA·PB is equal to : **[Online April 9, 2017]**

(a) 53 (b) 56 (c) 74 (d) 65

38. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is : **[Online April 9, 2017]**

(a) 12.5 (b) 13.2 (c) 12 (d) 13

39. Let $z \in C$, the set of complex numbers. Then the equation, $2|z+3i| - |z-i| = 0$ represents : **[Online April 8, 2017]**

(a) a circle with radius $\frac{8}{3}$.

(b) a circle with diameter $\frac{10}{3}$.

(c) an ellipse with length of major axis $\frac{16}{3}$.

(d) an ellipse with length of minor axis $\frac{16}{9}$

40. If a point P has co–ordinates (0, –2) and Q is any point on the circle, $x^2 + y^2 - 5x - y + 5 = 0$, then the maximum value of $(PQ)^2$ is : **[Online April 8, 2017]**

(a) $\frac{25+\sqrt{6}}{2}$ (b) $14+5\sqrt{3}$

(c) $\frac{47+10\sqrt{6}}{2}$ (d) $8+5\sqrt{3}$

41. If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is : **[Online April 8, 2017]**

(a) $\frac{4}{\sqrt{7}}$ (b) $\frac{8}{\sqrt{7}}$ (c) $\frac{8}{7}$ (d) $\frac{16}{7}$

42. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is: **[2016]**

(a) 5 (b) 10 (c) $5\sqrt{2}$ (d) $5\sqrt{3}$

43. Equation of the tangent to the circle, at the point (1, –1) whose centre is the point of intersection of the straight lines $x - y = 1$ and $2x + y = 3$ is : **[Online April 10, 2016]**

(a) $x + 4y + 3 = 0$ (b) $3x - y - 4 = 0$

(c) $x - 3y - 4 = 0$ (d) $4x + y - 3 = 0$

44. A circle passes through (–2, 4) and touches the y-axis at (0, 2). Which one of the following equations can represent a diameter of this circle ? **[Online April 9, 2016]**

(a) $2x - 3y + 10 = 0$ (b) $3x + 4y - 3 = 0$

(c) $4x + 5y - 6 = 0$ (d) $5x + 2y + 4 = 0$

45. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0, k \in \mathbf{R}$, is a : **[2015]**

(a) circle of radius $\sqrt{2}$.

(b) circle of radius $\sqrt{3}$.

(c) straight line parallel to x-axis

(d) straight line parallel to y-axis

46. The number of common tangents to the circles $x^2 + y^2 - 4x - 6x - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is: **[2015]**

(a) 3 (b) 4 (c) 1 (d) 2

47. If the incentre of an equilateral triangle is (1, 1) and the equation of its one side is $3x + 4y + 3 = 0$, then the equation of the circumcircle of this triangle is : **[Online April 11, 2015]**

(a) $x^2 + y^2 - 2x - 2y - 14 = 0$
(b) $x^2 + y^2 - 2x - 2y - 2 = 0$
(c) $x^2 + y^2 - 2x - 2y + 2 = 0$
(d) $x^2 + y^2 - 2x - 2y - 7 = 0$

48. If a circle passing through the point (–1, 0) touches y-axis at (0, 2), then the length of the chord of the circle along the x-axis is : **[Online April 11, 2015]**

(a) $\frac{3}{2}$ (b) 3 (c) $\frac{5}{2}$ (d) 5

49. Let the tangents drawn to the circle, $x^2 + y^2 = 16$ from the point P(0, h) meet the x-axis at point A and B. If the area of ΔAPB is minimum, then h is equal to : **[Online April 10, 2015]**

(a) $4\sqrt{2}$ (b) $3\sqrt{3}$ (c) $3\sqrt{2}$ (d) $4\sqrt{3}$

50. If $y + 3x = 0$ is the equation of a chord of the circle, $x^2 + y^2 - 30x = 0$, then the equation of the circle with this chord as diameter is : **[Online April 10, 2015]**

(a) $x^2+y^2+3x+9y=0$ (b) $x^2+y^2+3x-9y=0$
(c) $x^2+y^2-3x-9y=0$ (d) $x^2+y^2-3x+9y=0$

51. The largest value of r for which the region represented by the set $\{\omega \in C | |\omega - 4 - i| \le r\}$ is contained in the region represented by the set $(z \in c / |z-1| \le |z+i|)$, is equal to: **[Online April 10, 2015]**

(a) $\frac{5}{2}\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\frac{3}{2}\sqrt{2}$ (d) $\sqrt{17}$

52. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to **[2014]**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

53. The equation of circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as diameter is: **[Online April 19, 2014]**

(a) $x^2 + y^2 + 3x + y - 11 = 0$
(b) $x^2 + y^2 + 3x + y + 1 = 0$
(c) $x^2 + y^2 + 3x + y - 2 = 0$
(d) $x^2 + y^2 + 3x + y - 22 = 0$

54. For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$, there is/are **[Online April 12, 2014]**

(a) one pair of common tangents
(b) two pair of common tangents
(c) three pair of common tangents
(d) no common tangent

55. The set of all real values of λ for which exactly two common tangents can be drawn to the circles
$x^2 + y^2 - 4x - 4y + 6 = 0$ and
$x^2 + y^2 - 10x - 10y + \lambda = 0$ is the interval: **[Online April 11, 2014]**

(a) (12, 32) (b) (18, 42)
(c) (12, 24) (d) (18, 48)

56. If the point (1, 4) lies inside the circle $x^2 + y^2 - 6x - 10y + P = 0$ and the circle does not touch or intersect the coordinate axes, then the set of all possible values of P is the interval: **[Online April 9, 2014]**

(a) (0, 25) (b) (25, 39)
(c) (9, 25) (d) (25, 29)

57. Let a and b be any two numbers satisfying $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$. Then, the foot of perpendicular from the origin on the variable line, $\frac{x}{a} + \frac{y}{b} = 1$, lies on: **[Online April 9, 2014]**

(a) a hyperbola with each semi-axis = $\sqrt{2}$
(b) a hyperbola with each semi-axis = 2
(c) a circle of radius = 2
(d) a circle of radius = $\sqrt{2}$

58. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point **[2013]**

(a) (–5, 2) (b) (2, –5) (c) (5, –2) (d) (–2, 5)

59. If a circle of unit radius is divided into two parts by an arc of another circle subtending an angle 60° on the circumference of the first circle, then the radius of the arc is: **[Online April 25, 2013]**

(a) $\sqrt{3}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

60. **Statement 1**: The only circle having radius $\sqrt{10}$ and a diameter along line $2x + y = 5$ is $x^2 + y^2 - 6x + 2y = 0$.
Statement 2 : $2x + y = 5$ is a normal to the circle $x^2 + y^2 - 6x + 2y = 0$. **[Online April 25, 2013]**

(a) Statement 1 is false; Statement 2 is true.
(b) Statement 1 is true; Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is true; Statement 2 is false.
(d) Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

61. If the circle $x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$ touches the axis of x, then a equals. **[Online April 23, 2013]**

(a) 0 (b) ±4 (c) ±2 (d) ±3

62. If a circle C passing through (4, 0) touches the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ externally at a point (1, –1), then the radius of the circle C is : **[Online April 22, 2013]**

(a) 5 (b) $2\sqrt{5}$ (c) 4 (d) $\sqrt{57}$

63. If two vertices of an equilateral triangle are A $(-a, 0)$ and B $(a, 0)$, $a > 0$, and the third vertex C lies above x-axis then the equation of the circumcircle of ΔABC is : **[Online April 22, 2013]**

(a) $3x^2 + 3y^2 - 2\sqrt{3}ay = 3a^2$

(b) $3x^2 + 3y^2 - 2ay = 3a^2$

(c) $x^2 + y^2 - 2ay = a^2$

(d) $x^2 + y^2 - \sqrt{3}ay = a^2$

64. If each of the lines $5x + 8y = 13$ and $4x - y = 3$ contains a diameter of the circle
$x^2 + y^2 - 2(a^2 - 7a + 11)\ x - 2(a^2 - 6a + 6)\, y + b^3 + 1 = 0$, then **[Online April 9, 2013]**

(a) $a = 5$ and $b \notin (-1,1)$ (b) $a = 1$ and $b \notin (-1,1)$

(c) $a = 2$ and $b \notin (-\infty,1)$ (d) $a = 5$ and $b \in (-\infty,1)$

65. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3) is: **[2012]**

(a) $\frac{10}{3}$ (b) $\frac{3}{5}$ (c) $\frac{6}{5}$ (d) $\frac{5}{3}$

66. The number of common tangents of the circles given by $x^2 + y^2 - 8x - 2y + 1 = 0$ and $x^2 + y^2 + 6x + 8y = 0$ is **[Online May 26, 2012]**

(a) one (b) four (c) two (d) three

67. If the line $y = mx + 1$ meets the circle $x^2 + y^2 + 3x = 0$ in two points equidistant from and on opposite sides of x-axis, then **[Online May 19, 2012]**

(a) $3m + 2 = 0$ (b) $3m - 2 = 0$

(c) $2m + 3 = 0$ (d) $2m - 3 = 0$

68. If three distinct points A, B, C are given in the 2-dimensional coordinate plane such that the ratio of the distance of each one of them from the point (1, 0) to the distance from $(-1, 0)$ is equal to $\frac{1}{2}$, then the circumcentre of the triangle ABC is at the point **[Online May 19, 2012]**

(a) $\left(\frac{5}{3}, 0\right)$ (b) $(0, 0)$

(c) $\left(\frac{1}{3}, 0\right)$ (d) $(3, 0)$

69. The equation of the circle passing through the point (1, 2) and through the points of intersection of
$x^2 + y^2 - 4x - 6y - 21 = 0$ and $3x + 4y + 5 = 0$ is given by **[Online May 7, 2012]**

(a) $x^2 + y^2 + 2x + 2y + 11 = 0$

(b) $x^2 + y^2 - 2x + 2y - 7 = 0$

(c) $x^2 + y^2 + 2x - 2y - 3 = 0$

(d) $x^2 + y^2 + 2x + 2y - 11 = 0$

70. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is - **[2011 RS]**

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$

(b) $x^2 + y^2 - x - y = 0$

(c) $x^2 + y^2 + 2x + 2y - 7 = 0$

(d) $x^2 + y^2 + x + y - 2 = 0$

71. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ $(c > 0)$ touch each other if **[2011]**

(a) $|a| = c$ (b) $a = 2c$

(c) $|a| = 2c$ (d) $2|a| = c$

72. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[2010]**

(a) $-35 < m < 15$ (b) $15 < m < 65$

(c) $35 < m < 85$ (d) $-85 < m < -35$

73. If P and Q are the points of intersection of the circles
$x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and
$x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P, Q and (1, 1) for: **[2009]**

(a) all except one value of p

(b) all except two values of p

(c) exactly one value of p

(d) all values of p

74. Three distinct points A, B and C are given in the 2-dimensional coordinates plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point: **[2009]**

(a) $\left(\frac{5}{4}, 0\right)$ (b) $\left(\frac{5}{2}, 0\right)$

(c) $\left(\frac{5}{3}, 0\right)$ (d) $(0, 0)$

75. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is **[2008]**

(a) $(3, -4)$ (b) $(-3, 4)$

(c) $(-3, -4)$ (d) $(3, 4)$

76. Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval **[2007]**

(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$

(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$

77. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its center is

(a) $x^2+y^2=\frac{3}{2}$ (b) $x^2+y^2=1$ **[2006]**

(c) $x^2+y^2=\frac{27}{4}$ (d) $x^2+y^2=\frac{9}{4}$

78. If the lines $3x-4y-7=0$ and $2x-3y-5=0$ are two diameters of a circle of area 49π square units, the equation of the circle is **[2006]**

(a) $x^2+y^2+2x-2y-47=0$

(b) $x^2+y^2+2x-2y-62=0$

(c) $x^2+y^2-2x+2y-62=0$

(d) $x^2+y^2-2x+2y-47=0$

79. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its centre is **[2005]**

(a) $x^2+y^2-3ax-4by+(a^2+b^2-p^2)=0$

(b) $2ax+2by-(a^2-b^2+p^2)=0$

(c) $x^2+y^2-2ax-3by+(a^2-b^2-p^2)=0$

(d) $2ax+2by-(a^2+b^2+p^2)=0$

80. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then **[2005]**

(a) $3a^2-10ab+3b^2=0$

(b) $3a^2-2ab+3b^2=0$

(c) $3a^2+10ab+3b^2=0$

(d) $3a^2+2ab+3b^2=0$

81. If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line $5x+by-a=0$ passes through P and Q for **[2005]**

(a) exactly one value of a

(b) no value of a

(c) infinitely many values of a

(d) exactly two values of a

82. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is **[2004]**

(a) $2ax-2by-(a^2+b^2+4)=0$

(b) $2ax+2by-(a^2+b^2+4)=0$

(c) $2ax-2by+(a^2+b^2+4)=0$

(d) $2ax+2by+(a^2+b^2+4)=0$

83. A variable circle passes through the fixed point $A(p,q)$ and touches x-axis. The locus of the other end of the diameter through A is **[2004]**

(a) $(y-q)^2=4px$ (b) $(x-q)^2=4py$

(c) $(y-p)^2=4qx$ (d) $(x-p)^2=4qy$

84. If the lines $2x+3y+1=0$ and $3x-y-4=0$ lie along diameter of a circle of circumference 10π, then the equation of the circle is **[2004]**

(a) $x^2+y^2+2x-2y-23=0$

(b) $x^2+y^2-2x-2y-23=0$

(c) $x^2+y^2+2x+2y-23=0$

(d) $x^2+y^2-2x+2y-23=0$

85. Intercept on the line $y=x$ by the circle $x^2+y^2-2x=0$ is AB. Equation of the circle on AB as a diameter is **[2004]**

(a) $x^2+y^2+x-y=0$ (b) $x^2+y^2-x+y=0$

(c) $x^2+y^2+x+y=0$ (d) $x^2+y^2-x-y=0$

86. If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct point, then **[2003]**

(a) $r>2$ (b) $2<r<8$

(c) $r<2$ (d) $r=2$.

87. The lines $2x-3y=5$ and $3x-4y=7$ are diameters of a circle having area as 154 sq.units.Then the equation of the circle is **[2003]**

(a) $x^2+y^2-2x+2y=62$

(b) $x^2+y^2+2x-2y=62$

(c) $x^2+y^2+2x-2y=47$

(d) $x^2+y^2-2x+2y=47$.

88. If the chord $y = mx + 1$ of the circle $x^2+y^2=1$ subtends an angle of measure 45° at the major segment of the circle then value of m is **[2002]**

(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$

(c) $-1 \pm \sqrt{2}$ (d) none of these

89. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2=25$. The locus of any point in the set is **[2002]**

(a) $4 \le x^2+y^2 \le 64$ (b) $x^2+y^2 \le 25$

(c) $x^2+y^2 \ge 25$ (d) $3 \le x^2+y^2 \le 9$

90. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2+y^2=9$ is **[2002]**

(a) $\left(\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},-\sqrt{2}\right)$ (c) $\left(\frac{3}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},\frac{3}{2}\right)$

91. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is **[2002]**

(a) $x^2+y^2=9a^2$ (b) $x^2+y^2=16a^2$

(c) $x^2+y^2=4a^2$ (d) $x^2+y^2=a^2$

TOPIC 2 Parabola

92. Let L_1 be a tangent to the parabola $y^2=4(x+1)$ and L_2 be a tangent to the parabola $y^2=8(x+2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line : **[Sep. 06, 2020 (I)]**

(a) $x+3=0$ (b) $2x+1=0$

(c) $x+2=0$ (d) $x+2y=0$

93. The centre of the circle passing through the point (0, 1) and touching the parabola $y=x^2$ at the point (2,4) is: **[Sep. 06, 2020 (II)]**

(a) $\left(\frac{-53}{10},\frac{16}{5}\right)$ (b) $\left(\frac{6}{5},\frac{53}{10}\right)$

(c) $\left(\frac{3}{10},\frac{16}{5}\right)$ (d) $\left(\frac{-16}{5},\frac{53}{10}\right)$

94. If the common tangent to the parabolas, $y^2=4x$ and $x^2=4y$ also touches the circle, $x^2+y^2=c^2$, then c is equal to: **[Sep. 05, 2020 (I)]**

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

95. Let P be a point on the parabola, $y^2=12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then : **[Sep. 03, 2020 (I)]**

(a) $PN=4$ (b) $MQ=\frac{1}{3}$

(c) $MQ=\frac{1}{4}$ (d) $PN=3$

96. Let the latus ractum of the parabola $y^2=4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is : **[Sep. 03, 2020 (II)]**

(a) $8\sqrt{5}$ (b) 8 (c) 12 (d) $4\sqrt{5}$

97. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2=8x$, with one of its vertices on the vertex of this parabola, is : **[Sep. 02, 2020 (II)]**

(a) $64\sqrt{3}$ (b) $256\sqrt{3}$

(c) $192\sqrt{3}$ (d) $128\sqrt{3}$

98. If one end of a focal chord AB of the parabola $y^2=8x$ is at $A\left(\frac{1}{\sqrt{2}},-2\right)$, then the equation of the tangent to it at B is: **[Jan. 9, 2020 (II)]**

(a) $2x+y-24=0$ (b) $x-2y+8=0$

(c) $x+2y+8=0$ (d) $2x-y-24=0$

99. The locus of a point which divides the line segment joining the point (0, –1) and a point on the parabola, $x^2=4y$, internally in the ratio 1 : 2, is: **[Jan. 8, 2020 (I)]**

(a) $9x^2-12y=8$ (b) $9x^2-3y=2$

(c) $x^2-3y=2$ (d) $4x^2-3y=2$

100. Let a line $y=mx$ $(m>0)$ intersect the parabola, $y^2=x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q, If area $(\Delta OPQ)=4$ sq. units, then m is equal to _______. **[NA Jan. 8, 2020 (II)]**

101. If $y=mx+4$ is a tangent to both the parabolas, $y^2=4x$ and $x^2=2by$, then b is equal to: **[Jan. 7, 2020 (I)]**

(a) –32 (b) –64 (c) –128 (d) 128

102. The tangents to the curve $y=(x-2)^2-1$ at its points of intersection with the line $x-y=3$, intersect at the point : **[April 12, 2019 (II)]**

(a) $\left(\frac{5}{2},1\right)$ (b) $\left(-\frac{5}{2},-1\right)$ (c) $\left(\frac{5}{2},-1\right)$ (d) $\left(-\frac{5}{2},1\right)$

103. If the line $ax+y=c$, touches both the curves $x^2+y^2=1$ and $y^2=4\sqrt{2}x$, then $|c|$ is equal to **[April 10, 2019 (II)]**

(a) 2 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$

104. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2=4x$ at the point (1, 2) and the x-axis is: **[April 09, 2019 (II)]**

(a) $8\pi(2-\sqrt{2})$ (b) $4\pi(2-\sqrt{2})$

(c) $4\pi(3+\sqrt{2})$ (d) $8\pi(3-2\sqrt{2})$

105. If one end of a focal chord of the parabola, $y^2 = 16x$ is at $(1, 4)$, then the length of this focal chord is: **[April 09, 2019 (I)]**

(a) 25 (b) 22 (c) 24 (d) 20

106. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is : **[April 08, 2019 (I)]**

(a) 2 (b) $\frac{7}{8}$ (c) $\frac{7}{4\sqrt{2}}$ (d) $\frac{11}{4\sqrt{2}}$

107. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (a, b) are perpendicular to each other, then a^2 is equal to : **[April 08, 2019 (I)]**

(a) $\frac{128}{17}$ (b) $\frac{64}{17}$ (c) $\frac{4}{17}$ (d) $\frac{2}{17}$

108. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point : **[April 08, 2019 (II)]**

(a) $\left(-\frac{1}{3}, \frac{4}{3}\right)$ (b) $\left(\frac{1}{4}, \frac{3}{4}\right)$

(c) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (d) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

109. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is : **[Jan. 12, 2019 (II)]**

(a) $y = x\tan\theta + 2\cot\theta$ (b) $y = x\tan\theta - 2\cot\theta$
(c) $x = y\cot\theta + 2\tan\theta$ (d) $x = y\cot\theta - 2\tan\theta$

110. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is : **[Jan. 11, 2019 (I)]**

(a) $x + y + 1 = 0$ (b) $x - 2y + 4 = 0$
(c) $x + 2y + 4 = 0$ (d) $4x + 2y + 1 = 0$

111. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is : **[Jan. 11, 2019 (II)]**

(a) $5\sqrt{5}$ (b) $5(2^{1/3})$ (c) $(10)^{2/3}$ (d) 5

112. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a, b, c)? **[Jan 10, 2019 (I)]**

(a) $\left(\frac{1}{2}, 2, 3\right)$ (b) $(1, 1, 3)$

(c) $\left(\frac{1}{2}, 2, 0\right)$ (d) $(1, 1, 0)$

113. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is: **[Jan. 10, 2019 (II)]**

(a) $3\sqrt{2}$ (b) $2\sqrt{11}$ (c) $8\sqrt{2}$ (d) $6\sqrt{3}$

114. Axis of a parabola lies along x-axis. If its vertex and focus are at distance 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it? **[Jan 09, 2019 (I)]**

(a) $(5, 2\sqrt{6})$ (b) $(8, 6)$

(c) $(6, 4\sqrt{2})$ (d) $(4, -4)$

115. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is : **[Jan 09, 2019 (I)]**

(a) $2\sqrt{3}y = 12x + 1$ (b) $\sqrt{3}y = x + 3$

(c) $2\sqrt{3}y = -x - 12$ (d) $\sqrt{3}y = 3x + 1$

116. Let A $(4, -4)$ and B $(9, 6)$ be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is: **[Jan. 09, 2019 (II)]**

(a) $31\frac{1}{4}$ (b) $30\frac{1}{2}$ (c) 32 (d) $31\frac{3}{4}$

117. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is: **[2018]**

(a) 2 (b) 3 (c) $\frac{4}{3}$ (d) $\frac{1}{2}$

118. Tangents drawn from the point $(-8, 0)$ to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to **[Online April 15, 2018]**

(a) 48 (b) 32 (c) 24 (d) 64

119. If $y = mx + c$ is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then $|c|$ is equal to : **[Online April 9, 2017]**

(a) $2\sqrt{3}$ (b) $8\sqrt{3}$ (c) $10\sqrt{3}$ (d) $16\sqrt{3}$

120. If the common tangents to the parabola, $x^2 = 4y$ and the circle, $x^2 + y^2 = 4$ intersect at the point P, then the distance of P from the origin, is : **[Online April 8, 2017]**

(a) $\sqrt{2} + 1$ (b) $2(3 + 2\sqrt{2})$

(c) $2(\sqrt{2} + 1)$ (d) $3 + 2\sqrt{2}$

121. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is: **[2016]**

(a) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$
(b) $x^2 + y^2 - 4x + 9y + 18 = 0$
(c) $x^2 + y^2 - 4x + 8y + 12 = 0$
(d) $x^2 + y^2 - x + 4y - 12 = 0$

122. P and Q are two distinct points on the parabola, $y^2 = 4x$, with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is : **[Online April 10, 2016]**

(a) 8 (b) 4 (c) 6 (d) 2

123. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then locus of P is : **[2015]**

(a) $y^2 = 2x$ (b) $x^2 = 2y$
(c) $x^2 = y$ (d) $y^2 = x$

124. Let PQ be a double ordinate of the parabola, $y^2 = -4x$, where P lies in the second quadrant. If R divides PQ in the ratio 2 : 1 then the locus of R is : **[Online April 11, 2015]**

(a) $3y^2 = -2x$ (b) $3y^2 = 2x$
(c) $9y^2 = 4x$ (d) $9y^2 = -4x$

125. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is **[2014]**

(a) $\frac{1}{8}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

126. A chord is drawn through the focus of the parabola $y^2 = 6x$ such that its distance from the vertex of this parabola is $\frac{\sqrt{5}}{2}$, then its slope can be: **[Online April 19, 2014]**

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{3}}$

127. Two tangents are drawn from a point $(-2, -1)$ to the curve, $y^2 = 4x$. If α is the angle between them, then $|\tan \alpha|$ is equal to: **[Online April 12, 2014]**

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 3

128. Let L_1 be the length of the common chord of the curves $x^2 + y^2 = 9$ and $y^2 = 8x$, and L_2 be the length of the latus rectum of $y^2 = 8x$, then: **[Online April 11, 2014]**

(a) $L_1 > L_2$ (b) $L_1 = L_2$
(c) $L_1 < L_2$ (d) $\frac{L_1}{L_2} = \sqrt{2}$

129. **Given :** A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement-2 : If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$. **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

130. The point of intersection of the normals to the parabola $y^2 = 4x$ at the ends of its latus rectum is : **[Online April 23, 2013]**

(a) (0, 2) (b) (3, 0) (c) (0, 3) (d) (2, 0)

131. **Statement-1:** The line $x - 2y = 2$ meets the parabola, $y^2 + 2x = 0$ only at the point $(-2, -2)$.

Statement-2: The line $y = mx - \frac{1}{2m} (m \neq 0)$ is tangent to the parabola, $y^2 = -2x$ at the point $\left(-\frac{1}{2m^2}, -\frac{1}{m}\right)$. **[Online April 22, 2013]**

(a) Statement-1 is true; Statement-2 is false.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.
(c) Statement-1 is false; Statement-2 is true.
(d) Statement-1 a true; Statement-2 is true; Statement-2 is **not** a correct explanation for statement-1.

132. The normal at $\left(2, \frac{3}{2}\right)$ to the ellipse, $\frac{x^2}{16} + \frac{y^2}{3} = 1$ touches a parabola, whose equation is **[Online May 26, 2012]**

(a) $y^2 = -104x$ (b) $y^2 = 14x$
(c) $y^2 = 26x$ (d) $y^2 = -14x$

133. The chord PQ of the parabola $y^2 = x$, where one end P of the chord is at point $(4, -2)$, is perpendicular to the axis of the parabola. Then the slope of the normal at Q is **[Online May 26, 2012]**

(a) -4 (b) $-\frac{1}{4}$ (c) 4 (d) $\frac{1}{4}$

134. **Statement 1:** $y = mx - \frac{1}{m}$ is always a tangent to the parabola, $y^2 = -4x$ for all non-zero values of m.

Statement 2: Every tangent to the parabola, $y^2 = -4x$ will meet its axis at a point whose abscissa is non-negative. **[Online May 7, 2012]**

(a) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.
(b) Statement 1 is false, Statement 2 is true.
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

135. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is **[2011]**

(a) $\frac{3\sqrt{2}}{8}$ (b) $\frac{8}{3\sqrt{2}}$ (c) $\frac{4}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{4}$

136. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is **[2010]**

(a) $2x + 1 = 0$ (b) $x = -1$
(c) $2x - 1 = 0$ (d) $x = 1$

137. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at **[2008]**

(a) $(0, 2)$ (b) $(1, 0)$
(c) $(0, 1)$ (d) $(2, 0)$

138. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is **[2007]**

(a) $(2, 4)$ (b) $(-2, 0)$ (c) $(-1, 1)$ (d) $(0, 2)$

139. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is **[2006]**

(a) $xy = \frac{105}{64}$ (b) $xy = \frac{3}{4}$
(c) $xy = \frac{35}{16}$ (d) $xy = \frac{64}{105}$

140. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is **[2005]**

(a) $y^2 - 4x + 2 = 0$ (b) $y^2 + 4x + 2 = 0$
(c) $x^2 + 4y + 2 = 0$ (d) $x^2 - 4y + 2 = 0$

141. A circle touches the x- axis and also touches the circle with centre at (0,3) and radius 2. The locus of the centre of the circle is **[2005]**

(a) an ellipse (b) a circle
(c) a hyperbola (d) a parabola

142. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then **[2004]**

(a) $d^2 + (3b - 2c)^2 = 0$ (b) $d^2 + (3b + 2c)^2 = 0$
(c) $d^2 + (2b - 3c)^2 = 0$ (d) $d^2 + (2b + 3c)^2 = 0$

143. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then **[2003]**

(a) $t_2 = t_1 + \frac{2}{t_1}$ (b) $t_2 = -t_1 - \frac{2}{t_1}$
(c) $t_2 = -t_1 + \frac{2}{t_1}$ (d) $t_2 = t_1 - \frac{2}{t_1}$

144. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are **[2002]**

(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$
(c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$

TOPIC 3 Ellipse

145. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci? **[Sep. 06, 2020 (I)]**

(a) $(-2, \sqrt{3})$ (b) $(-1, \sqrt{2})$
(c) $(-1, \sqrt{3})$ (d) $(1, 2)$

146. If the normal at an end of a latus rectum of an ellipse passes through an extermity of the minor axis, then the eccentricity e of the ellipse satisfies: **[Sep. 06, 2020 (II)]**

(a) $e^4 + 2e^2 - 1 = 0$ (b) $e^2 + e - 1 = 0$
(c) $e^4 + e^2 - 1 = 0$ (d) $e^2 + 2e - 1 = 0$

147. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to : **[Sep. 05, 2020 (I)]**

(a) 16 (b) 8 (c) 6 (d) 9

148. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, – 4), then PQ^2 is equals to : **[Sep. 05, 2020 (I)]**

(a) 36 (b) 48 (c) 21 (d) 29

149. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to: **[Sep. 04, 2020 (I)]**

(a) 145 (b) 116 (c) 126 (d) 135

150. Let $x = 4$ be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta), \beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is : **[Sep. 04, 2020 (II)]**

(a) $4x - 3y = 2$ (b) $8x - 2y = 5$
(c) $7x - 4y = 1$ (d) $4x - 2y = 1$

151. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does **not** pass through which of the following points? **[Sep. 03, 2020 (I)]**

(a) $\left(\frac{1}{\sqrt{2}}, 0\right)$ (b) $\left(-\sqrt{\frac{3}{2}}, 1\right)$
(c) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (d) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$

152. Area (in sq. units) of the region outside $\frac{|x|}{2}+\frac{|y|}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is : **[Sep. 02, 2020 (I)]**

(a) $6(\pi-2)$ (b) $3(\pi-2)$

(c) $3(4-\pi)$ (d) $6(4-\pi)$

153. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola, $\frac{x^2}{9}-\frac{y^2}{4}=1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2+3y^2=k$, then k is equal to **[Jan. 9, 2020 (I)]**

(a) 16 (b) 17 (c) 15 (d) 14

154. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x+6y=8$; then its eccentricity is: **[Jan. 9, 2020 (II)]**

(a) $\frac{1}{2}\sqrt{\frac{11}{3}}$ (b) $\sqrt{\frac{5}{6}}$ (c) $\frac{1}{2}\sqrt{\frac{5}{3}}$ (d) $\frac{1}{3}\sqrt{\frac{11}{3}}$

155. Let the line $y=mx$ and the ellipse $2x^2+y^2=1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}},0\right)$ and $(0, \beta)$, then β is equal to: **[Jan. 8, 2020 (I)]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\frac{\sqrt{2}}{3}$

156. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is: **[Jan. 7, 2020 (I)]**

(a) $\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\frac{3}{\sqrt{2}}$ (d) $2\sqrt{3}$

157. If $3x+4y=12\sqrt{2}$ is *a* tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ for some $a \in R$, then the distance between the foci of the ellipse is: **[Jan. 7, 2020 (II)]**

(a) $2\sqrt{7}$ (b) 4 (c) $2\sqrt{5}$ (d) $2\sqrt{2}$

158. If the normal to the ellipse $3x^2+4y^2=12$ at a point P on it is parallel to the line, $2x+y=4$ and the tangent to the ellipse at P passes through Q (4,4) then PQ is equal to : **[April 12, 2019 (I)]**

(a) $\frac{5\sqrt{5}}{2}$ (b) $\frac{\sqrt{61}}{2}$ (c) $\frac{\sqrt{221}}{2}$ (d) $\frac{\sqrt{157}}{2}$

159. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points ? **[April 12, 2019 (II)]**

(a) $(\sqrt{2},2)$ (b) $(2,\sqrt{2})$

(c) $(2,2\sqrt{2})$ (d) $(1,2\sqrt{2})$

160. If the line $x-2y=12$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3,\frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is : **[April 10, 2019 (I)]**

(a) 9 (b) $12\sqrt{2}$ (c) 5 (d) $8\sqrt{3}$

161. The tangent and normal to the ellipse $3x^2+5y^2=32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is : **[April 10, 2019 (II)]**

(a) $\frac{34}{15}$ (b) $\frac{14}{3}$ (c) $\frac{16}{3}$ (d) $\frac{68}{15}$

162. If the tangent to the parabola $y^2=x$ at a point (α, β), $(\beta>0)$ is also a tangent to the ellipse, $x^2+2y^2=1$, then α is equal to: **[April 09, 2019 (II)]**

(a) $\sqrt{2}-1$ (b) $2\sqrt{2}-1$

(c) $2\sqrt{2}+1$ (d) $\sqrt{2}+1$

163. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is: **[April 08, 2019 (II)]**

(a) 10 (b) 5 (c) 8 (d) 6

164. Let S and S′ be the foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS)=8$ sq. units, hen the length of a latus rectum of the ellipse is : **[Jan. 12, 2019 (II)]**

(a) 4 (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) 2

165. If tangents are drawn to the ellipse $x^2+2y^2=2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve : **[Jan. 11, 2019 (I)]**

(a) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (b) $\frac{x^2}{4}+\frac{y^2}{2}=1$

(c) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (d) $\frac{x^2}{2}+\frac{y^2}{4}=1$

166. Two sets A and B are as under :

$A = \{(a, b) \in R \times R : |a-5| < 1 \text{ and } |b-5| < 1\};$

$B = \{(a,b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \le 36\}$. Then : **[2018]**

(a) $A \subset B$

(b) $A \cap B = \phi$ (an empty set)

(c) neither $A \subset B$ nor $B \subset A$

(d) $B \subset A$

167. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is? **[Online April 16, 2018]**

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

168. The eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, –1) and (–2, 2) is : **[Online April 9, 2017]**

(a) $\frac{1}{2}$ (b) $\frac{2}{\sqrt{5}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{4}$

169. Consider an ellipse, whose centre is at the origin and its major axis is along the x–axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse, is : **[Online April 8, 2017]**

(a) 8 (b) 32 (c) 80 (d) 40

170. If the tangent at a point on the ellipse $\frac{x^2}{27} + \frac{y^2}{3} = 1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is : **[Online April 9, 2016]**

(a) $3\sqrt{3}$ (b) $\frac{9}{2}$ (c) 9 (d) $\frac{9}{\sqrt{3}}$

171. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is : **[2015]**

(a) $\frac{27}{2}$ (b) 27 (c) $\frac{27}{4}$ (d) 18

172. If the distance between the foci of an ellipse is half the length of its latus rectum, then the eccentricity of the ellipse is: **[Online April 11, 2015]**

(a) $\frac{2\sqrt{2}-1}{2}$ (b) $\sqrt{2}-1$

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{2}-1}{2}$

173. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is **[2014]**

(a) $(x^2 + y^2)^2 = 6x^2 + 2y^2$

(b) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

(c) $(x^2 - y^2)^2 = 6x^2 + 2y^2$

(d) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

174. A stair-case of length l rests against a vertical wall and a floor of a room. Let P be a point on the stair-case, nearer to its end on the wall, that divides its length in the ratio 1 : 2. If the stair-case begins to slide on the floor, then the locus of P is: **[Online April 11, 2014]**

(a) an ellipse of eccentricity $\frac{1}{2}$

(b) an ellipse of eccentricity $\frac{\sqrt{3}}{2}$

(c) a circle of radius $\frac{1}{2}$

(d) a circle of radius $\frac{\sqrt{3}}{2}l$

175. If OB is the semi-minor axis of an ellipse, F_1 and F_2 are its foci and the angle between F_1B and F_2B is a right angle, then the square of the eccentricity of the ellipse is: **[Online April 9, 2014]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{4}$

176. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0, 3) is **[2013]**

(a) $x^2 + y^2 - 6y - 7 = 0$

(b) $x^2 + y^2 - 6y + 7 = 0$

(c) $x^2 + y^2 - 6y - 5 = 0$

(d) $x^2 + y^2 - 6y + 5 = 0$

177. A point on the ellipse, $4x^2 + 9y^2 = 36$, where the normal is parallel to the line, $4x - 2y - 5 = 0$, is : **[Online April 25, 2013]**

(a) $\left(\frac{9}{5}, \frac{8}{5}\right)$ (b) $\left(\frac{8}{5}, -\frac{9}{5}\right)$

(c) $\left(-\frac{9}{5}, \frac{8}{5}\right)$ (d) $\left(\frac{8}{5}, \frac{9}{5}\right)$

178. Let the equations of two ellipses be

$E_1 : \frac{x^2}{3} + \frac{y^2}{2} = 1$ and $E_2 : \frac{x^2}{16} + \frac{y^2}{b^2} = 1$,

If the product of their eccentricities is $\frac{1}{2}$, then the length of the minor axis of ellipse E_2 is : **[Online April 22, 2013]**

(a) 8 (b) 9 (c) 4 (d) 2

179. Equation of the line passing through the points of intersection of the parabola $x^2 = 8y$ and the ellipse $\frac{x^2}{3} + y^2 = 1$ is : **[Online April 9, 2013]**

(a) $y - 3 = 0$ (b) $y + 3 = 0$

(c) $3y + 1 = 0$ (d) $3y - 1 = 0$

180. If P_1 and P_2 are two points on the ellipse $\frac{x^2}{4} + y^2 = 1$ at which the tangents are parallel to the chord joining the points (0, 1) and (2, 0), then the distance between P_1 and P_2 is **[Online May 12, 2012]**

(a) $2\sqrt{2}$ (b) $\sqrt{5}$ (c) $2\sqrt{3}$ (d) $\sqrt{10}$

181. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is **[2011]**

(a) $5x^2 + 3y^2 - 48 = 0$ (b) $3x^2 + 5y^2 - 15 = 0$

(c) $5x^2 + 3y^2 - 32 = 0$ (d) $3x^2 + 5y^2 - 32 = 0$

182. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is : **[2009]**

(a) $x^2 + 12y^2 = 16$ (b) $4x^2 + 48y^2 = 48$

(c) $4x^2 + 64y^2 = 48$ (d) $x^2 + 16y^2 = 16$

183. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is **[2008]**

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

184. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is **[2006]**

(a) $\frac{3}{5}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{\sqrt{5}}$

185. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is **[2005]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

186. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x = 4$, then the equation of the ellipse is: **[2004]**

(a) $4x^2 + 3y^2 = 1$ (b) $3x^2 + 4y^2 = 12$

(c) $4x^2 + 3y^2 = 12$ (d) $3x^2 + 4y^2 = 1$

TOPIC 4 Hyperbola

187. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true? **[Sep. 05, 2020 (II)]**

(a) $c^2 = 369$ (b) $5m = 4$

(c) $4c^2 = 369$ (d) $8m + 5 = 0$

188. Let $P(3, 3)$ be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to : **[Sep. 04, 2020 (I)]**

(a) $\left(\frac{9}{2}, 3\right)$ (b) $\left(\frac{3}{2}, 2\right)$ (c) $\left(\frac{9}{2}, 2\right)$ (d) $(9, 3)$

189. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ $(b < 5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1 e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to : **[Sep. 03, 2020 (II)]**

(a) $(8, 12)$ (b) $\left(\frac{20}{3}, 12\right)$

(c) $\left(\frac{24}{5}, 10\right)$ (d) $(8, 10)$

190. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to : **[Sep. 02, 2020 (I)]**

(a) 6 (b) 8 (c) 10 (d) 5

191. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2 \sec^2 \theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2 \sec^2 \theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is : **[Sep. 02, 2020 (II)]**

(a) $2\sqrt{6}$ (b) $\sqrt{30}$

(c) $\frac{2\sqrt{5}}{3}$ (d) $\frac{4\sqrt{5}}{3}$

192. If a hyperbola passes through the point $P(10,16)$ and it has vertices at $(\pm 6,0)$, then the equation of the normal to it at P is: **[Jan. 8, 2020 (II)]**

(a) $3x + 4y = 94$ (b) $2x + 5y = 100$

(c) $x + 2y = 42$ (d) $x + 3y = 58$

193. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and hyperbola $8x^2 - y^2 = 8$. If S and S′ denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS′ in a ratio : **[April 12, 2019 (I)]**

(a) 13 : 11 (b) 14 : 13 (c) 5 : 4 (d) 2 : 1

194. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$, is : **[April 12, 2019 (II)]**

(a) $x - y + 4 = 0$ (b) $x + y + 4 = 0$

(c) $x - 2y + 16 = 0$ (d) $2x - y + 2 = 0$

195. If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then : **[April 10, 2019 (I)]**

(a) $4e^4 - 24e^2 + 27 = 0$ (b) $4e^4 - 12e^2 - 27 = 0$

(c) $4e^4 - 24e^2 + 35 = 0$ (d) $4e^4 + 8e^2 - 35 = 0$

196. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is : **[April 10, 2019 (II)]**

(a) $(5, 0)$ (b) $\left(-\frac{5}{3}, 0\right)$ (c) $\left(\frac{5}{3}, 0\right)$ (d) $(-5, 0)$

197. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is : **[April 09, 2019 (I)]**

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{15}}{2}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{3}{\sqrt{5}}$

198. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is : **[April. 08, 2019 (II)]**

(a) $x - 2y + 8 = 0$ (b) $2x - 3y + 10 = 0$

(c) $2x - y - 2 = 0$ (d) $3x - 2y = 0$

199. If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola? **[Jan. 12, 2019 (I)]**

(a) $\left(-6, 2\sqrt{10}\right)$ (b) $\left(2\sqrt{6}, 5\right)$

(c) $\left(4, \sqrt{15}\right)$ (d) $\left(6, 5\sqrt{2}\right)$

200. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is : **[Jan. 11, 2019 (II)]**

(a) $\frac{13}{12}$ (b) 2 (c) $\frac{13}{6}$ (d) $\frac{13}{8}$

201. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it? **[Jan. 11, 2019 (II)]**

(a) $\left(4\sqrt{2}, 2\sqrt{2}\right)$ (b) $\left(4\sqrt{3}, 2\sqrt{2}\right)$

(c) $\left(4\sqrt{3}, 2\sqrt{3}\right)$ (d) $\left(4\sqrt{2}, 2\sqrt{3}\right)$

202. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is: **[Jan 10, 2019 (I)]**

(a) $x - y + 1 = 0$ (b) $x - y + 7 = 0$

(c) $x - y + 9 = 0$ (d) $x - y - 3 = 0$

203. Let

$$S = \left\{(x, y) \in \mathbf{R}^2 : \frac{y^2}{1 + r} - \frac{x^2}{1 - r} = 1\right\},$$

where $r \neq \pm 1$ Then S represents: **[Jan. 10, 2019 (II)]**

(a) a hyperbola whose eccentricity is $\frac{2}{\sqrt{1 - r}}$, when $0 < r < 1$

(b) an ellipse whose eccentricity is $\sqrt{\frac{2}{r + 1}}$, when $r > 1$

(c) a hyperbola whose eccentricity is $\frac{2}{\sqrt{r + 1}}$, when $0 < r < 1$

(d) an ellipse whose eccentricity is $\frac{1}{\sqrt{r + 1}}$, when $r > 1$

204. Let $0<\theta<\frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$ is greater than 2, then the length of its latus rectum lies in the interval: **[Jan 09, 2019 (I)]**

(a) $(3,\infty)$ (b) $(3/2,2]$ (c) $(2,3]$ (d) $(1,3/2]$

205. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the *x*-axis. Then the eccentricity of the hyperbola is : **[Jan. 09, 2019 (II)]**

(a) $\frac{3}{2}$ (b) $\sqrt{3}$ (c) 2 (d) $\frac{2}{\sqrt{3}}$

206. Tangents are drawn to the hyperbola $4x^2-y^2=36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is : **[2018]**

(a) $54\sqrt{3}$ (b) $60\sqrt{3}$ (c) $36\sqrt{5}$ (d) $45\sqrt{5}$

207. The locus of the point of intersection of the lines, $\sqrt{2}x-y+4\sqrt{2}k=0$ and $\sqrt{2}kx+ky-4\sqrt{2}=0$ (*k* is any non-zero real parameter) is. **[Online April 16, 2018]**

(a) A hyperbola with length of its transverse axis $8\sqrt{2}$

(b) An ellipse with length of its major axis $8\sqrt{2}$

(c) An ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$

(d) A hyperbola whose eccentricity is $\sqrt{3}$

208. A hyperbola passes through the point $P\left(\sqrt{2},\sqrt{3}\right)$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point : **[2017]**

(a) $\left(-\sqrt{2},-\sqrt{3}\right)$ (b) $\left(3\sqrt{2},2\sqrt{3}\right)$

(c) $\left(2\sqrt{2},3\sqrt{3}\right)$ (d) $\left(\sqrt{3},\sqrt{2}\right)$

209. The locus of the point of intersection of the straight lines,
$tx-2y-3t=0$
$x-2ty+3=0\ (t\in R)$, is : **[Online April 8, 2017]**

(a) an ellipse with eccentricity $\frac{2}{\sqrt{5}}$

(b) an ellipse with the length of major axis 6

(c) a hyperbola with eccentricity $\sqrt{5}$

(d) a hyperbola with the length of conjugate axis 3

210. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **[2016]**

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{4}{3}$ (d) $\frac{4}{\sqrt{3}}$

211. A hyperbola whose transverse axis is along the major axis of the conic, $\frac{x^2}{3}+\frac{y^2}{4}=4$ and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does NOT lie on it ? **[Online April 10, 2016]**

(a) $(\sqrt{5},2\sqrt{2})$ (b) $(0,2)$

(c) $(5,2\sqrt{3})$ (d) $(\sqrt{10},2\sqrt{3})$

212. Let a aand b respectively be the semitransverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2-18e+5=0$. If S(5, 0) is a focus and $5x=9$ is the corresponding directrix of this hyperbola, then a^2-b^2 is equal to : **[Online April 9, 2016]**

(a) –7 (b) –5 (c) 5 (d) 7

113. An ellipse passes through the foci of the hyperbola, $9x^2-4y^2=36$ and its major and minor axes lie along the transverse and conjugate axes of the hyperbola respectively. If the product of eccentricities of the two conics is $\frac{1}{2}$, then which of the following points **does not** lie on the ellipse? **[Online April 10, 2015]**

(a) $\left(\sqrt{\frac{13}{2}},\sqrt{6}\right)$ (b) $\left(\frac{\sqrt{39}}{2},\sqrt{3}\right)$

(c) $\left(\frac{1}{2}\sqrt{13},\frac{\sqrt{3}}{2}\right)$ (d) $\left(\sqrt{13},0\right)$

114. The tangent at an extremity (in the first quadrant) of latus rectum of the hyperbola $\frac{x^2}{4}-\frac{y^2}{5}=1$, meet x-axis and y-axis at A and B respectively. Then $(OA)^2-(OB)^2$, where O is the origin, equals: **[Online April 19, 2014]**

(a) $-\frac{20}{9}$ (b) $\frac{16}{9}$ (c) 4 (d) $-\frac{4}{3}$

115. Let P $(3\sec\theta, 2\tan\theta)$ and Q $(3\sec\phi, 2\tan\phi)$ where $\theta+\phi=\frac{\pi}{2}$, be two distinct points on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$. Then the ordinate of the point of intersection of the normals at P and Q is: **[Online April 11, 2014]**

(a) $\frac{11}{3}$ (b) $-\frac{11}{3}$ (c) $\frac{13}{2}$ (d) $-\frac{13}{2}$

216. A common tangent to the conics $x^2 = 6y$ and $2x^2 - 4y^2 = 9$ is: **[Online April 25, 2013]**

(a) $x - y = \frac{3}{2}$ (b) $x + y = 1$

(c) $x + y = \frac{9}{2}$ (d) $x - y = 1$

217. A tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ meets x-axis at P and y-axis at Q. Lines PR and QR are drawn such that OPRQ is a rectangle (where O is the origin). Then R lies on: **[Online April 23, 2013]**

(a) $\frac{4}{x^2} + \frac{2}{y^2} = 1$ (b) $\frac{2}{x^2} - \frac{4}{y^2} = 1$

(c) $\frac{2}{x^2} + \frac{4}{y^2} = 1$ (d) $\frac{4}{x^2} - \frac{2}{y^2} = 1$

218. If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ coincide with the foci of the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$, then b^2 is equal to **[Online May 19, 2012]**

(a) 8 (b) 10 (c) 7 (d) 9

219. If the eccentricity of a hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$, which passes through $(k, 2)$, is $\frac{\sqrt{13}}{3}$, then the value of k^2 is **[Online May 7, 2012]**

(a) 18 (b) 8 (c) 1 (d) 2

220. The equation of the hyperbola whose foci are $(-2, 0)$ and $(2, 0)$ and eccentricity is 2 is given by : **[2011RS]**

(a) $x^2 - 3y^2 = 3$ (b) $3x^2 - y^2 = 3$
(c) $-x^2 + 3y^2 = 3$ (d) $-3x^2 + y^2 = 3$

221. For the Hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies = ? **[2007]**

(a) abscissae of vertices (b) abscissae of foci
(c) eccentricity (d) directrix.

222. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is **[2005]**

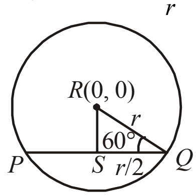
(a) an ellipse (b) a circle
(c) a parabola (d) a hyperbola

223. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is **[2003]**

(a) 9 (b) 1 (c) 5 (d) 7

Hints & Solutions

1. **(d)** In right ΔRSQ, $\sin 60° = \frac{RS}{r}$



$\Rightarrow RS = r \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}r}{2}$

Now equation of PQ is $y - 2x - 3 = 0$

$\therefore \frac{\sqrt{3}r}{2} = \frac{|0+0-3|}{\sqrt{5}}$

$\Rightarrow \frac{\sqrt{3}r}{2} = \frac{3}{\sqrt{5}} \Rightarrow r = \frac{2\sqrt{3}}{5} \Rightarrow r^2 = \frac{12}{5}$

2. **(b)** We know family of circle be $S_1 + \lambda S_2 = 0$

$x^2 + y^2 - 6x + \lambda(x^2 + y^2 - 4y) = 0$

$\Rightarrow (1+\lambda)x^2 + (1+\lambda)y^2 - 6x - 4\lambda y = 0$ ...(i)

Centre $(-g, -f) = \left(\frac{3}{1+\lambda}, \frac{2\lambda}{\lambda+1}\right)$

Centre lies on $2x - 3y + 12 = 0$, then

$\frac{6}{\lambda+1} - \frac{6\lambda}{\lambda+1} + 12 = 0 \Rightarrow \lambda = -3$

Equation of circle (i),

$-2x^2 - 2y^2 - 6x + 12y = 0$

$\Rightarrow x^2 + y^2 + 3x - 6y = 0$ ...(ii)

Only $(-3, 6)$ satisfy equation (ii).

3. **(7)**

Let $P(3\cos\theta, 3\sin\theta)$, $Q(-3\cos\theta, -3\sin\theta)$

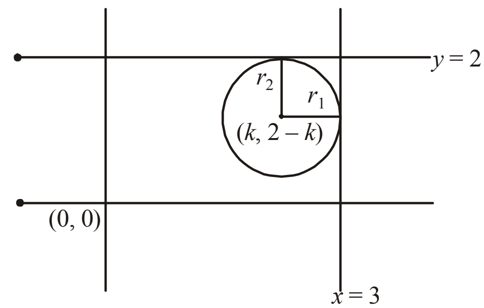
$\alpha = \left|\frac{3\cos\theta + 3\sin\theta - 2}{\sqrt{2}}\right|$, $\beta = \left|\frac{-3\cos\theta - 3\sin\theta - 2}{\sqrt{2}}\right|$

$\alpha\beta = \left|\frac{(3\cos\theta + 3\sin\theta)^2 - 4}{2}\right| = \left|\frac{5 + 9\sin 2\theta}{2}\right|$

$\alpha\beta$ is max. when $\sin 2\theta = 1$

$\therefore \alpha\beta|_{max} = \frac{5+9}{2} = 7.$

4. **(3)**



$\Rightarrow$ Radius $(r_1) = 3 - k$

$\because$ Centre lies on $x + y = 2$

Let $x = k$

$\therefore y = 2 - k$

$\Rightarrow$ Centre $= (k, 2-k)$

Also, radius $(r_2) = 2 - (2 - k)$

$\therefore 3 - k = 2 - (2 - k)$

$\Rightarrow k = \frac{3}{2}$

$r = 3 - \frac{3}{2} = \frac{3}{2}$

Hence, diameter $= 3$.

5. **(9)**

The given circle is $x^2 + y^2 - 2x - 4y + 4 = 0$

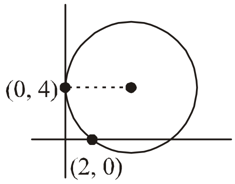
$\therefore$ Centre of circle $(1, 2)$, $r = 1$.

If line cuts circle then $p < r$, where $p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

$\Rightarrow \left|\frac{3 + 8 - k}{5}\right| < 1 \Rightarrow k \in (6, 16)$

$k = 7, 8, 9, 10, 11, 12, 13, 14, 15$

6. **(d)**



Equation of family of circle

$(x-0)^2+(y-4)^2+\lambda x=0$

Passes through the point (2, 0) then

$4+16+2\lambda=0 \Rightarrow \lambda=-10$

Hence, the equation of circle

$x^2+y^2-10x-8y+16=0$

$\Rightarrow (x-5)^2+(y-4)^2=25$

Centre (5, 4).

$$R=\sqrt{\frac{1}{2}\text{coeff. of } x+\frac{1}{2}\text{coeff. of } y-\text{constant}}$$

$$=\sqrt{25+16-16}=5$$

Perpendicular distance of $4x+3y-8=0$ from the centre of circle

$$=\left|\frac{20+16-8}{\sqrt{16+9}}\right|=\frac{28}{5}\neq 5$$

Hence, $4x+3y-8=0$ can not be tangent to the circle.

7. **(36)** The given equation of circle

$x^2-6x+y^2+8=0$

$(x-3)^2+y^2=1$ ...(i)

So, centre of circle (i) is $C_1(3, 0)$ and radius $r_1=1$.

And the second equation of circle

$x^2-8y+y^2+16-k=0\ (k>0)$

$x^2+(y-4)^2=\left(\sqrt{k}\right)^2$ ...(ii)

So, centre of circle (ii) is $C_2(0, 4)$ and radius $r_2=\sqrt{k}$

Two circles touches each other when

$C_1C_2=|r_1\pm r_2| \Rightarrow 5=\left|1\pm\sqrt{k}\right|$

Distance between $C_2(3, 0)$ and $C_1(0, 4)$ is

either $\sqrt{k}+1$ or $\left|\sqrt{k}-1\right|$ $(C_1C_2=5)$

$\Rightarrow \sqrt{k}+1=5$ or $\left|\sqrt{k}-1\right|=5$

$\Rightarrow k=16$ or $k=36$

Hence, maximum value of k is 36

The given equation of circles

$x^2-6x+y^2+8=0$

$\Rightarrow (x-3)^2+y^2=1$

8. **(c)** Slope of tangent of $x^2+y^2=1$ at $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

$$\frac{1}{\sqrt{2}}x+\frac{1}{\sqrt{2}}y-1=0$$

$x+y\sqrt{2}=0$, which is perpendicular to $x-y+c=0$

At $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ which is tangent of $(x-3)^2+y^2=1$

So, $m=1 \Rightarrow y=x+c$

Now, distance of (3, 0) from $y=x+c$ is

$$\left|\frac{c+3}{\sqrt{2}}\right|=1$$

$\Rightarrow c=-3\pm\sqrt{2}$

$\Rightarrow (c+3)^2=2$

$\Rightarrow c^2+6c+9=2$

$\therefore c^2+6c+7=0$

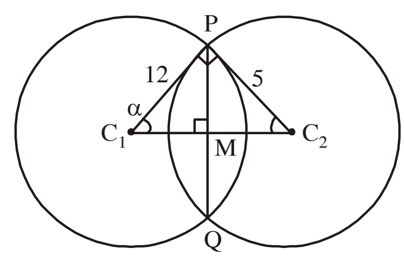
9. **(c)** $L=\sqrt{S_1}=\sqrt{16}=4$

$R=\sqrt{16+4-16}=2$

Length of chord of contact

$$=\frac{2LR}{\sqrt{L^2+R^2}}=\frac{2\times4\times2}{\sqrt{16+4}}=\frac{16}{\sqrt{20}}$$ Square of length of chord of contact $=\frac{64}{5}$

10. **(b)**



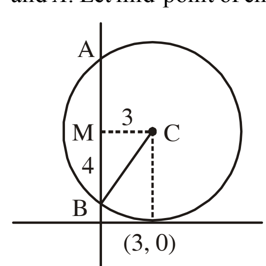
According to the diagram,

In ΔPC_1C_2, $\tan\alpha=\frac{5}{12} \Rightarrow \sin\alpha=\frac{5}{13}$

In ΔPC_1M, $\sin\alpha=\frac{PM}{12} \Rightarrow \frac{5}{13}=\frac{PM}{12} \Rightarrow PM=\frac{60}{13}$

Hence, length of common chord $(PQ)=\frac{120}{13}$

11. **(a)** Let centre of circle is C and circle cuts the y-axis at B and A. Let mid-point of chord BA is M.



$CB = \sqrt{MC^2 + MB^2}$

$\sqrt{3^2+4^2} = 5 =$ radius of circle

$\therefore$ equation of circle is,

$(x-3)^2 + (y-5)^2 = 5^2$

(3, 10) satisfies this equation.

Although there will be another circle satisfying the same conditions that will lie below the x-axis having equation

$(x-3)^2 + (y-5)^2 = 5^2$

12. **(b)** $S_1 \equiv x^2 + y^2 + 5Kx + 2y + K = 0$

$S_2 \equiv x^2 + y^2 + Kx + \frac{3}{2}y - \frac{1}{2} = 0$

Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow 4Kx + \frac{y}{2} + K + \frac{1}{2} = 0$ ...(i)

Equation of the line passing through the intersection points P & Q is,

$4x + 5y - K = 0$ ...(ii)

Comparing (i) and (ii),

$\frac{4K}{4} = \frac{1}{10} = \frac{2K+1}{-2K}$ ...(iii)

$\Rightarrow K = \frac{1}{10}$ and $-2K = 20K + 10$

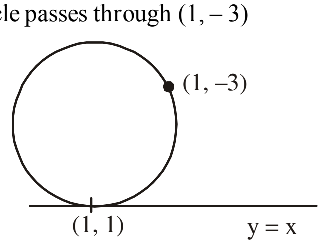
$\Rightarrow 22K = -10 \Rightarrow K = \frac{-5}{11}$

$\because K = \frac{1}{10}$ or $\frac{-5}{11}$ is not satisfying equation (3)

$\therefore$ No value of K exists.

13. **(b)** Equation of circle which touches the line $y = x$ at (1, 1) is, $(x-1)^2 + (y-1)^2 + \lambda(y-x) = 0$

This circle passes through (1, – 3)



$\therefore 0 + 16 + \lambda(-3-1) = 0$

$\Rightarrow 16 + \lambda(-4) = 0 \Rightarrow \lambda = 4$

Hence, equation of circle will be,

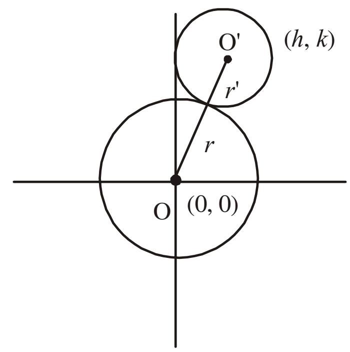
$(x-1)^2 + (y-1)^2 + 4y - 4x = 0$

$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

$\therefore$ Radius $= \sqrt{9+1-2} = 2\sqrt{2}$

14. **(b)** Let centre of required circle is (h, k).

$\therefore$ OO' $= r + r'$ [By the diagram]



$\Rightarrow \sqrt{h^2+k^2} = 1 + h$

$\Rightarrow h^2 + k^2 = 1 + h^2 + 2h$

$\Rightarrow k^2 = 1 + 2h$

$\therefore$ locus is $y = \sqrt{1+2x}$, $x \geq 0$

15. **(b)** Let $z \in S$ then $z = \frac{\alpha + i}{\alpha - i}$

Since, z is a complex number and let $z = x + iy$

Then, $x + iy = \frac{(\alpha+i)^2}{\alpha^2+1}$ (by rationalisation)

$\Rightarrow x + iy = \frac{(\alpha^2-1)}{\alpha^2+1} + \frac{i(2\alpha)}{\alpha^2+1}$

Then compare both sides

$x = \frac{\alpha^2-1}{\alpha^2+1}$ ...(i)

$y = \frac{2\alpha}{\alpha^2+1}$ ...(ii)

Now squaring and adding equations (i) and (ii)

$\Rightarrow x^2 + y^2 = \frac{(\alpha^2-1)^2}{(\alpha^2+1)^2} + \frac{4\alpha^2}{(\alpha^2+1)^2} = 1$

16. **(a)** Let any tangent to circle $x^2 + y^2 = 1$ is

$x\cos\theta + y\sin\theta = 1$

Since, P and Q are the point of intersection on the co-ordinate axes.

Then $P \equiv \left(\frac{1}{\cos\theta}, 0\right)$ & $Q \equiv \left(0, \frac{1}{\sin\theta}\right)$

$\therefore$ mid-point of PQ be $M \equiv \left(\frac{1}{2\cos\theta}, \frac{1}{2\sin\theta}\right) \equiv (h, k)$

$\Rightarrow \cos\theta = \frac{1}{2h}$ ...(i)

$\sin\theta = \frac{1}{2k}$ ...(ii)

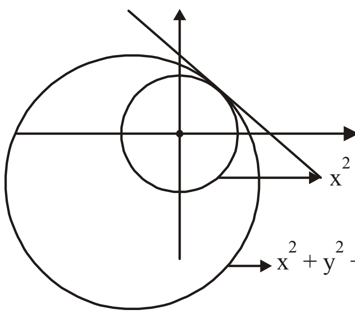
Now squaring and adding equation (i) and (ii)

$\frac{1}{h^2} + \frac{1}{k^2} = 4$

$\therefore$ locus of M is : $x^2 + y^2 = 4x^2y^2$

$\Rightarrow h^2 + k^2 = 4h^2k^2$

17. (c) By the diagram, $d_{c_1c_2} = |r_1 - r_2|$



Equation of common tangent is,

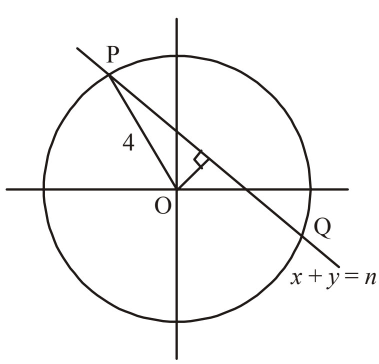
$S_1 - S_2 = 0$

$6x + 8y - 20 = 0 \Rightarrow 3x + 4y - 10 = 0$

Hence $(6, -2)$ lies on it.

18. (d) Let the chord $x + y = n$ cuts the circle $x^2 + y^2 = 16$ at P and Q Length of perpendicular from O on PQ

$= \left|\frac{0+0-n}{\sqrt{1^2+1^2}}\right| = \frac{n}{\sqrt{2}}$



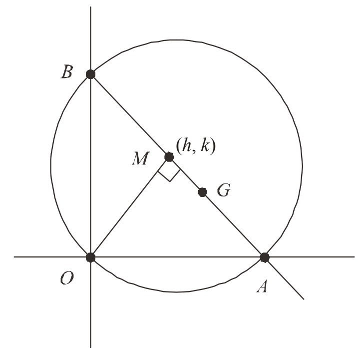
Then, length of chord $PQ = 2\sqrt{4^2 - \left(\frac{n}{\sqrt{2}}\right)^2} = 2\sqrt{16 - \frac{n^2}{2}}$

Thus only possible values of n are 1, 2, 3, 4, 5.

Hence, the sum of squares of lengths of chords

$= \sum_{n=1}^{5} 4\left(16 - \frac{n^2}{2}\right) = 64 \times 5 - 2.\frac{5\times6\times11}{6} = 210$

19. (b) As $\angle AOB = 90°$



Let AB diameter and $M(h, k)$ be foot of perpendicular, then

$M_{AB} = \frac{-h}{k}$

Then, equation of AB

$(y - k) = \frac{-h}{k}(x - h)$

$\Rightarrow hx + ky = h^2 + k^2$

Then, $A\left(\frac{h^2+k^2}{h}, 0\right)$ and $B\left(0, \frac{h^2+k^2}{k}\right)$

$\because AB$ is the diameter, then

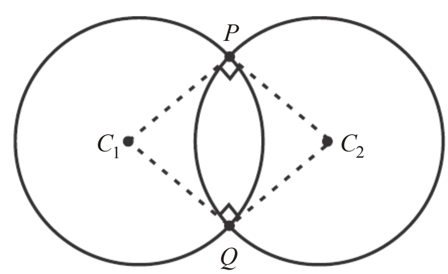
$AB = 2R$

$\Rightarrow AB^2 = 4R^2$

$\Rightarrow \left(\frac{h^2+k^2}{h}\right)^2 + \left(\frac{h^2+k^2}{k}\right) = 4R^2$

Hence, required locus is $(x^2 + y^2)^3 = 4R^2 x^2 y^2$

20. (d)



$2g_1g_2 + 2f_1f_2 = 2(-1)(-3) + 2(-1)(-3) = 12$

$c_1 + c_2 = 14 - 2 = 12$

Since, $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Hence, circles intersect orthogonally

$\therefore$ Area of the quadrilateral PC_1QC_1

$= 2\left(\frac{1}{2}(C_1P)(C_2P)\right)$

$= 2 \times \frac{1}{2} r_1 r_2 = (2)(2) = 4$ sq, units

21. **(c)** **Condition 1:** The centre of the two circles are (1, 1) and (9, 1). The circles are on opposite sides of the line $3x + 4y - \lambda = 0$.

Put $x = 1, y = 1$ in the equation of line,

$3(1) + 4(1) - \lambda = 0 \Rightarrow 7 - \lambda = 0$

Now, put $x = 9, y = 1$ in the equation of line,

$3(9) + 4(1) - \lambda = 0$

Then, $(7 - \lambda)(27 + 4 - \lambda) < 0$

$\Rightarrow (\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7, 31)$ …(i)

Condition 2: Perpendicular distance from centre on line $\geq$ radius of circle.

For $x^2 + y^2 - 2x - 2y = 1$,

$\Rightarrow \frac{|3 + 4 - \lambda|}{5} \geq 1$

$\Rightarrow |\lambda - 7| \geq 5$

$\Rightarrow \lambda \geq 12$ or $\lambda \Rightarrow 2$ ...(ii)

For $x^2 + y^2 - 18x - 2y + 78 = 0$

$\frac{|27 + 4 - \lambda|}{5} \geq 2$

$\Rightarrow \lambda \geq 41$ or $\lambda \leq 21$ ...(iii)

Intersection of (1), (2) and (3) gives $\lambda \in [12, 21]$.

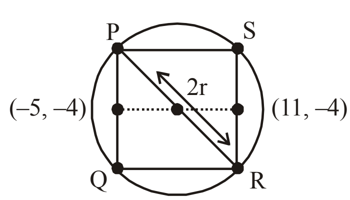
22. **(c)** The equation of circle is,

$x^2 + y^2 - 6x + 8y - 103 = 0$

$\Rightarrow (x - 3)^2 + (y + 4)^2 = (8\sqrt{2})^2$

$C(3, -4), r = 8\sqrt{2}$

$\Rightarrow$ Length of side of square $= \sqrt{2}r = 16$



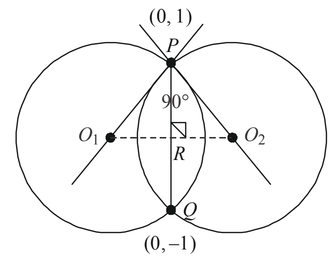
$\Rightarrow P(-5, 4), Q(-5, -12)$

$R(11, -12), S(11, 4)$

$\Rightarrow$ Required distance $= OP$

$= \sqrt{(-5-0)^2 + (-4-0)^2} = \sqrt{25 + 16} = \sqrt{41}$

23. **(b)** $\because$ Two circles of equal radii intersect each other orthogonally. Then R is mid point of PQ.

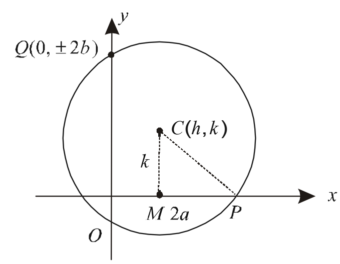


and $PR = O_1R = O_2R$

$PR = \frac{1}{2}\sqrt{(0-0)^2 + (1+1)^2} = 1$

$\therefore$ Distance between centres $= 1 + 1 = 2$.

24. **(d)**



Let centre be $C(h, k)$

$CQ = CP = r$

$\Rightarrow CQ^2 = CP^2$

$(h - 0)^2 + (k \pm 0)^2 = CM^2 + MP^2$

$h^2 + (k \pm 2b)^2 = k^2 + 4a^2$

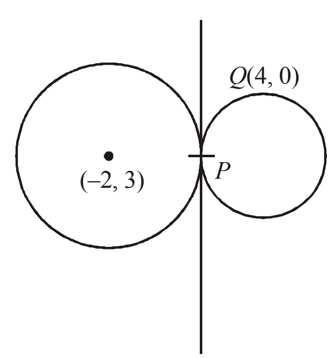
$h^2 + k^2 + 4b^2 \pm 4bk = k^2 + 4a^2$

Then, the locus of centre $C(h, k)$

$x^2 + 4b^2 \pm 4by = 4a^2$

Hence, the above locus of the centre of circle is a parabola.

25. **(c)** The equation of circle $x^2 + y^2 + 4x - 6y = 12$ can be written as $(x + 2)^2 + (y - 3)^2 = 25$



Let $P=(1,-1)$ & $Q=(4,0)$

Equation of tangent at $P(1,-1)$ to the given circle :

$x(1)+y(-1)+2(x+1)-3(y-1)-12=0$

$3x-4y-7=0$ ...(i)

The required circle is tangent to (1) at $(1,-1)$.

$\therefore\ (x-1)^2+(y+1)^2+\lambda(3x-4y-7)=0$ ...(ii)

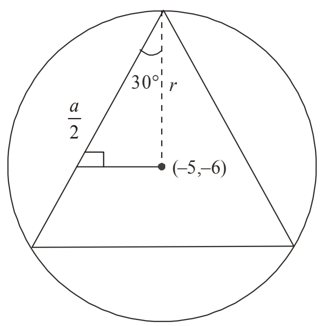
Equation (ii) passes through $Q(4,0)$

$\Rightarrow\ 3^2+1^2+\lambda(12-7)=0 \Rightarrow 5\lambda+10=0 \Rightarrow \lambda=-2$

Equation (2) becomes $x^2+y^2-8x+10y+16=0$

radius $=\sqrt{(-4)^2+(5)^2-16}=5$

26. **(d)**



Let the sides of equilateral Δ inscribed in the circle be a,

then $\cos 30° = \dfrac{a}{2r}$

$\dfrac{\sqrt{3}}{2}=\dfrac{a}{2r} \Rightarrow a=\sqrt{3}r$

Then, area of the equilateral triangle $=\dfrac{\sqrt{3}}{4}a^2$

$=\dfrac{\sqrt{3}}{4}\left(\sqrt{3}r\right)^2=\dfrac{3\sqrt{3}}{4}r^2$

But it is given that area of equilateral triangle $=27\sqrt{3}$

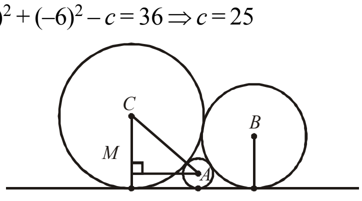
Then, $27\sqrt{3}=\dfrac{3\sqrt{3}}{4}r^2$

$r^2=36 \Rightarrow r=6$

But $\left(-\dfrac{1}{2}\text{ coeff. of } x\right)^2+\left(-\dfrac{1}{2}\text{ coeff. of } y\right)^2$ – constant term $=r^2$

$(-5)^2+(-6)^2-c=36 \Rightarrow c=25$

27. **(a)**



$AM^2=AC^2-MC^2$

$=\ (a+c)^2-(a-c)^2=4ac$

$\Rightarrow\ AM^2=XY^2=4ac$

$\Rightarrow\ XY=2\sqrt{ac}$

Similarly, $YZ=2\sqrt{ba}$ and $XZ=2\sqrt{bc}$

Then, $XZ=XY+YZ$

$\Rightarrow\ 2\sqrt{bc}=2\sqrt{ac}+2\sqrt{ba}$

$\Rightarrow\ \dfrac{1}{\sqrt{a}}=\dfrac{1}{\sqrt{b}}+\dfrac{1}{\sqrt{c}}$

28. **(d)** Consider the equation of circles as,

$x^2+y^2-16x-20y+164=r^2$

i.e. $(x-8)^2+(y-10)^2=r^2$ ...(i)

and $(x-4)^2+(y-7)^2=36$ ...(ii)

Both the circles intersect each other at two distinct points.

Distance between centres

$=\sqrt{(8-4)^2+(10-7)^2}=5$

$\therefore\ |r-6|<5<|r+6|$

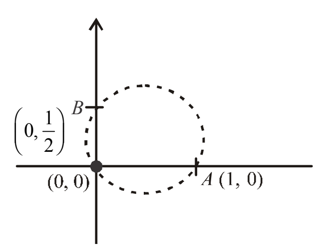
$\therefore$ If $|r-6|<5 \Rightarrow r\in(1,11)$ ...(iii)

and $|r+6|>5 \Rightarrow r\in(-\infty,-11)\cup(-1,\infty)$ ...(iv)

From (iii) and (iv),

$r\in(1,11)$

29. **(a)**



Let equation of circle be $x^2+y^2+2gx+2fy=0$

As length of intercept on x axis is $1=2\sqrt{g^2-c}$

$\Rightarrow\ |g|=\dfrac{1}{2}$

length of intercept on y-axis $=\dfrac{1}{2}=2\sqrt{f^2-c}$

$\Rightarrow\ |f|=\dfrac{1}{4}$

Equation of circle that passes through given points is

$x^2+y^2-x-\dfrac{y}{2}=0$

Tangent at (0, 0) is,

$$(y-0)=\left(\frac{dy}{dx}\right)_{(0,0)}^{(x-0)}\cdot(x-0)$$

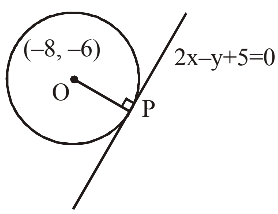
$\Rightarrow\ 2x+y=0$

Perpendicular distance from $B(1, 0)$ on the tangent to the circle $=\frac{\frac{1}{2}}{\sqrt{5}}$

Perpendicular distance from $B\left(0,\frac{1}{2}\right)$ on the tangent to the circle $=\frac{2}{\sqrt{5}}$

Sum of perpendicular distance $=\frac{\frac{1}{2}+2}{\sqrt{5}}=\frac{\sqrt{5}}{2}$.

30. **(c)** Equation of tangent at (1, 7) to $x^2=y-6$ is $2x-y+5=0$.



Now, perpendicular from centre O(–8, –6) to $2x-y+5=0$ should be equal to radius of the circle

$$\therefore\quad \left|\frac{-16+6+5}{\sqrt{5}}\right|=\sqrt{64+36-C}$$

$\Rightarrow\ \sqrt{5}=\sqrt{100-c}\ \Rightarrow c=95$

31. **(d)** Given circle is:

$x^2+y^2+2x-4y-4=0$

$\therefore$ its centre is (– 1, 2) and radius is 3 units.

Let $A=(x, y)$ be the centre of the circle C

$$\therefore\ \frac{x-1}{2}=2\ \Rightarrow\ x=5 \text{ and } \frac{y+2}{2}=2\ \Rightarrow\ y=2$$

So the centre of C is (5, 2) and its radius is 3

$\therefore$ equation of centre C is:

$x^2+y^2-10x-4y+20=0$

$\therefore$ The length of the intercept it cuts on the x-axis

$=2\sqrt{g^2-c}=2\sqrt{25-20}=2\sqrt{5}$

32. **(c)** Equation of the line passing through the points (2, 3) and (4, 5) is

$$y-3=\left(\frac{5-3}{4-2}\right)x-2\Rightarrow\ x-y+1=0 \quad \text{..... (i)}$$

Equation of the perpendicular line passing through the midpoint (3, 4) is $x+y-7=0$ ..... (ii)

Lines (1) and (2) intersect at the center of the circle. So, the center of the circle is (3, 4)

Therefore, the radius is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(2-3)^2+(3-4)^2}=\sqrt{2}$$

units.

33. **(c)** As origin is the only common point to x-axis and y-axis, so, origin is the common vertex

Let the equation of two of parabolas be $y^2=4ax$ and $x^2=4by$

Now latus rectum of both parabolas = 3

$$\therefore\ 4a=4b=3\Rightarrow a=b=\frac{3}{4}$$

$\therefore$ Two parabolas are $y^2=3x$ and $x^2=3y$

Suppose $y=mx+c$ is the common tangent.

$\therefore\ y^2=3x\Rightarrow(mx+c)^2=3x\Rightarrow m^2x^2+(2mc-3)x+c^2=0$

As, the tangent touches at one point only

So, $b^2-4ac=0$

$\Rightarrow(2mc-3)^2-4m^2c^2=0$

$\Rightarrow\ 4m^2c^2+9-12mc-4m^2c^2=0$

$$\Rightarrow c=\frac{9}{12m}=\frac{3}{4m} \quad \text{.... (i)}$$

$\therefore\ x^2=3y\Rightarrow x^2=3(mx+c)\Rightarrow x^2-3mx-3c=0$

Again, $b^2-4ac=0$

$\Rightarrow 9m^2-4(1)(-3c)=0$

$\Rightarrow 9m^2=-12c$ ..... (ii)

Form (i) and (ii)

$$m^2=\frac{-4c}{3}=\frac{-4}{3}\left(\frac{3}{4m}\right)$$

$$\Rightarrow m^3=-1\Rightarrow m=-1\Rightarrow c=\frac{-3}{4}$$

Hence, $y=mx+c=-x-\frac{3}{4}$

$\Rightarrow 4(x+y)+3=0$

34. **(a)** Here, equation of tangent on C_1 at (2, 1) is:

$2x+y-(x+2)-1=0$

Or $x+y=3$

If it cuts off the chord of the circle C_2 then the equation of the chord is:

$x+y=3$

$\therefore$ distance of the chord from (3, – 2) is :

$$d=\left|\frac{3-2-3}{\sqrt{2}}\right|=\sqrt{2}$$

Also, length of the chord is $l=4$

$$\therefore\ \text{radius of } C_2=r=\sqrt{\left(\frac{l}{2}\right)^2+d^2}$$

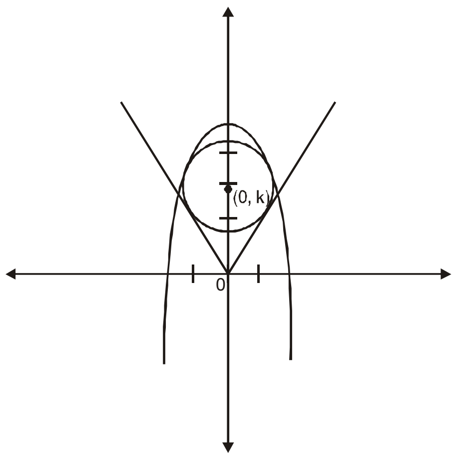
$$=\sqrt{(2)^2+(\sqrt{2})^2}=\sqrt{6}$$

35. **(None)**

(Let the equation of circle be

$x^2+(y-k)^2=r^2$

It touches $x-y=0$



$\Rightarrow \left|\frac{0-k}{\sqrt{2}}\right|=r$

$\Rightarrow k=r\sqrt{2}$

$\therefore$ Equation of circle becomes

$x^2+(y-k)^2=\frac{k^2}{2}$ ...(i)

It touches $y=4-x^2$ as well

$\therefore$ Solving the two equations

$\Rightarrow 4-y+(y-k)^2=\frac{k^2}{2}$

$\Rightarrow 1y^2-y(2k+1)+\frac{k^2}{2}+4=0$

It will give equal roots $\therefore D=0$

$\Rightarrow (2k+1)^2=4\left(\frac{k^2}{2}+4\right)$

$\Rightarrow 2k^2+4k-15=0$

$\Rightarrow k=\frac{-2+\sqrt{34}}{2}$

$\therefore r=\frac{k}{\sqrt{2}}=\frac{-2+\sqrt{34}}{2\sqrt{2}}$

Which is not matching with any of the option given here.

36. **(c)** Let $z=x+y$

$$\text{Im}\left[\left(\frac{ix-y-2}{x+(y-1)i}\right)\left(\frac{x-(y-1)i}{x-(y-1)i}\right)\right]+1=0$$

On solving, we get:

$2x^2+2y^2-y-1=0$

$\Rightarrow x^2+y^2-1/2y-1/2=0$

$\Rightarrow x^2+\left(y-\frac{1}{4}\right)^2=\frac{9}{16}$

$\Rightarrow r=\frac{3}{4}$

37. **(b)** P (4, 7). Here, $x=4$, $y=7$

$x-y=-3$

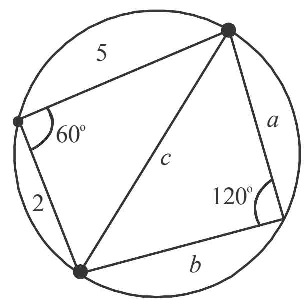
$\therefore PA\times PB=PT^2$

Also; $PT=\sqrt{x^2+y^2-(x-y)^2}$

$\Rightarrow PT=\sqrt{16+49-9}=\sqrt{56}$

$\Rightarrow PT^2=56 \therefore PA\times PB=56$

38. **(c)**



Here; $\cos\theta=\frac{a^2+b^2-c^2}{2ab}$

and $\theta=60^\circ$

$\Rightarrow \cos 60^\circ=\frac{4+25-c^2}{2.2.5}$

$\Rightarrow 10=29-c^2$

$\Rightarrow c^2=19$

$\Rightarrow \boxed{c=\sqrt{19}}$

also; $\cos\theta=\frac{a^2+b^2-c^2}{2ab}$

and $\theta=120^\circ$

$\Rightarrow -\frac{1}{2}=\frac{a^2+b^2-19}{2ab}$

$\Rightarrow a^2+b^2-19=-ab$

$\Rightarrow a^2+b^2+ab=19$

$\therefore \text{Area}=\frac{1}{2}\times 2\times 5\sin 60+\frac{1}{2}ab\sin 120^\circ=4\sqrt{3}$

$\Rightarrow \frac{5\sqrt{3}}{2}+\frac{ab\sqrt{3}}{4}=4\sqrt{3}$

$\Rightarrow \frac{ab}{4} = 4 - \frac{5}{2} = \frac{3}{2}$

$\Rightarrow \boxed{ab\ =\ 6}$

$\therefore\ a^2 + b^2 = 13$

$\Rightarrow a = 2,\ b = 3$

Perimeter = Sum of all sides = 2 + 5 + 2 + 3 = 12

39. **(a)** Let z = x + iy

$\Rightarrow\ 2\,|x + i\,(y+3)| = |x + i\,(y-1)|$

$\Rightarrow\ 2\sqrt{x^2+(y+3)^2} = \sqrt{x^2+(y-1)^2}$

$\Rightarrow\ 4x^2 + 4(y+3)^2 = x^2 + (y-1)^2$

$\Rightarrow\ 3x^2 = y^2 - 2y + 1 - 4y^2 - 24y - 36$

$\Rightarrow\ 3x^2 + 3y^2 + 26y + 35 = 0$ (which is a circle)

$\Rightarrow\ x^2 + y^2 + \frac{26}{3}y + \frac{35}{3} = 0$

$\Rightarrow\ r = \sqrt{0 + \frac{169}{9} - \frac{35}{3}}$

$\Rightarrow\ r = \sqrt{\frac{64}{9}} = \frac{8}{3}$

40. **(b)** Given that $x^2 + y^2 - 5x - y + 5 = 0$

$\Rightarrow\ (x - 5/2)^2 - \frac{25}{4} + (y - 1/2)^2 - 1/4 = 0$

$\Rightarrow\ (x - 5/2)^2 + (y - 1/2)^2 = 3/2$

on circle $\left[Q \equiv \left(5/2 + \sqrt{3/2}\cos Q, \frac{1}{2} + \sqrt{3/2}\sin Q\right)\right]$

$\Rightarrow\ PQ^2 = \left(\frac{5}{2} + \sqrt{3/2}\cos Q\right)^2 + \left(\frac{5}{2} + \sqrt{3/2}\sin Q\right)^2$

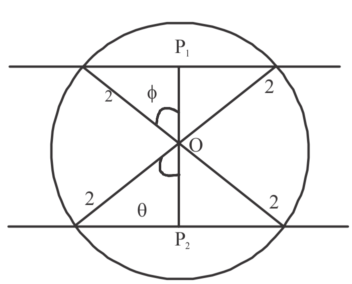
$\Rightarrow\ PQ^2 = \frac{25}{2} + \frac{3}{2} + 5\sqrt{3/2}\left(\cos Q + \sin Q\right)$

$= 14 + 5\sqrt{3/2}\ \ (\cos Q + \sin Q)$

$\therefore$ Maximum value of PQ^2

$= 14 + 5\sqrt{3/2} \times \sqrt{2}\ = 14 + 5\sqrt{3}$

41. **(b)**



Since $\cos 2\theta\ = 1/7 \Rightarrow\ 2\cos^2 Q - 1 = 1/7$

$\Rightarrow\ 2\cos^2\theta = 8/7$

$\Rightarrow\ \cos^2\theta = 4/7$

$\Rightarrow\ \cos^2\theta = \frac{4}{7}$

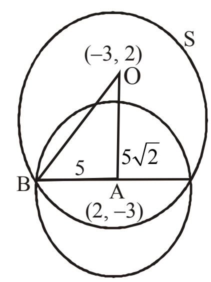
$\Rightarrow\ \cos^2\phi = \frac{2}{\sqrt{7}}$

Also, $\sec^2\phi = 7 = \frac{1}{2\cos^2\phi - 1} = 7$

$= \cos^2\phi - 1 = \frac{1}{7} = 2\cos^2\phi = \frac{8}{7} = \cos\phi = \frac{2}{\sqrt{7}}$

$P_1P_2 = r\cos\theta + r\cos\phi = \frac{4}{\sqrt{7}} + \frac{4}{\sqrt{7}} = \frac{8}{\sqrt{7}}$

42. **(d)**



Given, centre of S is O (–3, 2) and centre of given circle is A(2, –3) and radius is 5.

$OA\ = 5\sqrt{2}$

Also AB = 5 ($\because$ AB = radius of the given circle)

Using pythagoras theorem in $\triangle$OAB

$r = 5\sqrt{3}$

43. **(a)** Point of intersection of lines

$x - y = 1$ and $2x + y = 3$ is $\left(\frac{4}{3}, \frac{1}{3}\right)$

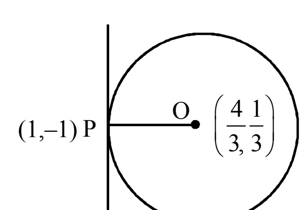
Slope of OP $= \frac{\frac{1}{3}+1}{\frac{4}{3}-1} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4$

Slope of tangent $= -\frac{1}{4}$

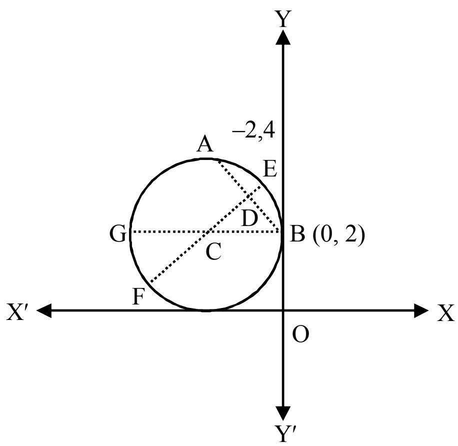
Equation of tangent $\quad y + 1 = -\frac{1}{4}(x - 1)$

$4y + 4 = -x + 1$

$x + 4y + 3 = 0$



44. (a)



EF = perpendicular bisector of chord AB
BG = perpendicular to y-axis
Here C = centre of the circle
mid-point of chord AB, D = (– 1, 3)

slope of AB $= \frac{4-2}{-2-0} = -1$

$\because$ EF $\perp$ AB

$\therefore$ Slope of EF = 1

Equation of EF, $y - 3 = 1(x + 1)$

$\Rightarrow \quad y = x + 4$ ...(i)

Equation of BG

$y = 2$ ...(ii)

From equations (i) and (ii)

$x = -2, y = 2$

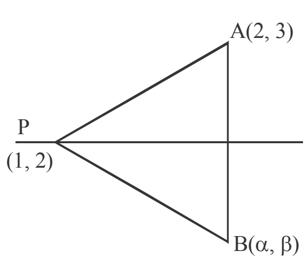
since C be the point of intersection of EF and BG, therefore centre, C = (–2, 2)

Now coordinates of centre C satisfy the equation

$2x - 3y + 10 = 0$

Hence $2x - 3y + 10 = 0$ is the equation of the diameter

45. (a) Intersection point of $2x - 3y + 4 = 0$ and $x - 2y + 3 = 0$ is (1, 2)



Let image of $A(2, 3)$ is $B(\alpha, \beta)$.

Since, P is the fixed point for given family of lines

So, PB = PA

$(\alpha - 1)^2 + (\beta - 2)^2 = (2 - 1)^2 + (3 - 2)^2$

$(\alpha - 1)^2 + (\beta - 2)^2 = 1 + 1 = 2$

$(x - 1)^2 + (y - 2)^2 = (\sqrt{2})^2$

Compare with

$(x - a)^2 + (y - b)^2 = r^2$

Therefore, given locus is a circle with centre (1, 2) and radius $\sqrt{2}$.

46. (a) $x^2 + y^2 - 4x - 6y - 12 = 0$ ...(i)

Centre, $C_1 = (2, 3)$

Radius, $r_1 = 5$ units

$x^2 + y^2 + 6x + 18y + 26 = 0$ ...(ii)

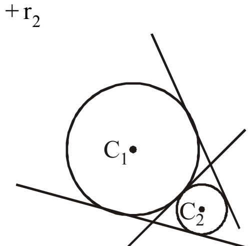
Centre, $C_2 = (-3, -9)$

Radius, $r_2 = 8$ units

$C_1C_2 = \sqrt{(2+3)^2 + (3+9)^2} = 13$ units

$r_1 + r_2 = 5 + 8 = 13$

$\therefore C_1 C_2 = r_1 + r_2$



Therefore there are three common tangents.

47. (a) Let radius of circumcircle be

According to the question,

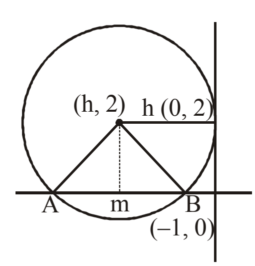
$\frac{r}{2} = \frac{10}{5} \Rightarrow r = 4$

So equation of required circle is

$(x - 1)^2 + (y - 1)^2 = 16$

$\Rightarrow x^2 + y^2 - 2x - 2y - 14 = 0$

48. (b) Let 'h' be the radius of the circle and since circle touches y–axis at (0, 2) therefore centre = (h, 2)



Now, eqn of circle is

$(h + 1)^2 + 2^2 = h^2$

$\Rightarrow 2h + 5 = 0$

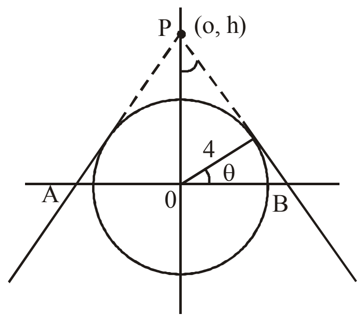
$h = -\frac{5}{2}$

From the figure, it is clear that AB is the chord along x–axis

$\therefore\ AB = 2\,(AM) = 2\sqrt{\dfrac{25}{4} - 4} = 2\left(\dfrac{3}{2}\right) = 3$

49. **(a)** $OP = \dfrac{4}{\sin\theta}$

$OB = \dfrac{4}{\cos\theta}$



$\text{Area} = OP \times OB = \dfrac{16}{\sin\theta\cos\theta} = \dfrac{32}{\sin 2\theta}$

least value $\sin 2\theta = 1;\ \theta = 45°$

So, $h = \dfrac{4}{\sin 45°} = 4\sqrt{2}$

50. **(d)** Given that $y + 3x = 0$ is the equation of a chord of the circle then

$y = -3x$ ....(i)

$(x^2) + (-3x)^2 - 30x = 0$

$10x^2 - 30x = 0$

$10x(x - 3) = 0$

$x = 0,\ y = 0$

so the equation of the circle is

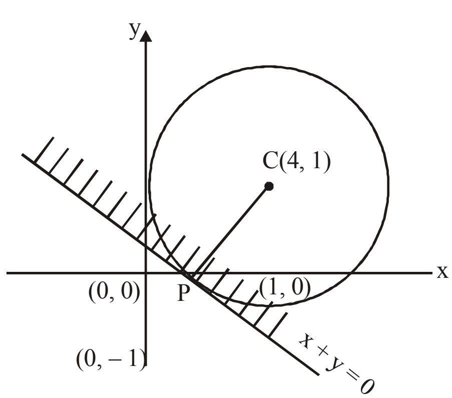
$(x - 3)(x - 0) + (y + 9)(y - 0) = 0$

$x^2 - 3x + y^2 + 9y = 0$

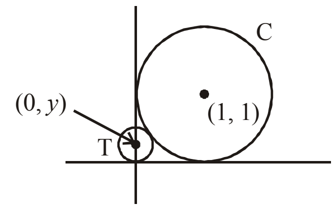
$x^2 + y^2 - 3x + 9y = 0$

51. **(a)** Radius

$CP = \dfrac{4+1}{\sqrt{2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5}{2}\sqrt{2}$



52. **(b)**



Equation of circle

$C \equiv (x-1)^2 + (y-1)^2 = 1$

Radius of T = $|y|$

T touches C externally therefore,

Distance between the centres = sum of their radii

$\Rightarrow\ \sqrt{(0-1)^2 + (y-1)^2} = 1 + |y|$

$\Rightarrow\ (0-1)^2 + (y-1)^2 = (1 + |y|)^2$

$\Rightarrow\ 1 + y^2 + 1 - 2y = 1 + y^2 + 2|y|$

$2|y| = 1 - 2y$

If $y > 0$ then $2y = 1 - 2y \Rightarrow y = \dfrac{1}{4}$

If $y < 0$ then $-2y = 1 - 2y \Rightarrow 0 = 1$ (not possible)

$\therefore\ y = \dfrac{1}{4}$

53. **(a)** Given circle is $x^2 + y^2 - 16 = 0$

Eqn of chord say AB of given circle is $3x + y + 5 = 0$.

Equation of required circle is

$x^2 + y^2 - 16 + \lambda(3x + y + 5) = 0$

$\Rightarrow\ x^2 + y^2 + (3\lambda)x + (\lambda)y + 5\lambda - 16 = 0$ ...(1)

Centre $C = \left(\dfrac{-3\lambda}{2}, \dfrac{-\lambda}{2}\right)$.

If line AB is the diameter of circle (1), then

$C\left(\dfrac{-3\lambda}{2}, \dfrac{-\lambda}{2}\right)$ will lie on line AB.

i.e. $3\left(\dfrac{-3\lambda}{2}\right) + \left(\dfrac{-\lambda}{2}\right) + 5 = 0$

$\Rightarrow\ -\dfrac{9\lambda - \lambda}{2} + 5 = 0 \Rightarrow \lambda = 1$

Hence, required eqn of circle is

$x^2 + y^2 + 3x + y + 5 - 16 = 0$

$\Rightarrow\ x^2 + y^2 + 3x + y - 11 = 0$

54. **(d)** Let, $x^2 + y^2 = 16$ or $x^2 + y^2 = 4^2$

radius of circle $r_1 = 4$, centre C_1 (0, 0)

we have, $x^2 + y^2 - 2y = 0$

$\Rightarrow x^2 + (y^2 - 2y + 1) - 1 = 0$ or $x^2 + (y - 1)^2 = 1^2$

Radius 1, centre C_2 (0, 1)

$|C_1C_2| = 1$

$|r_2 - r_1| = |4 - 1| = 3$

$|C_1C_2| < |r_2 - r_1|$

55. **(b)** The equations of the circles are

$x^2 + y^2 - 10x - 10y + \lambda = 0$ ...(1)

and $x^2 + y^2 - 4x - 4y + 6 = 0$ ...(2)

C_1 = centre of (1) = (5, 5)

C_2 = centre of (2) = (2, 2)

d = distance between centres

$= C_1C_2 = \sqrt{9+9} = \sqrt{18}$

$r_1 = \sqrt{50-\lambda}$, $r_2 = \sqrt{2}$

For exactly two common tangents we have

$r_1 - r_2 < C_1C_2 < r_1 + r_2$

$\Rightarrow \sqrt{50-\lambda} - \sqrt{2} < 3\sqrt{2} < \sqrt{50-\lambda} + \sqrt{2}$

$\Rightarrow \sqrt{50-\lambda} - \sqrt{2} < 3\sqrt{2}$ or $3\sqrt{2} < \sqrt{50-\lambda} + \sqrt{2}$

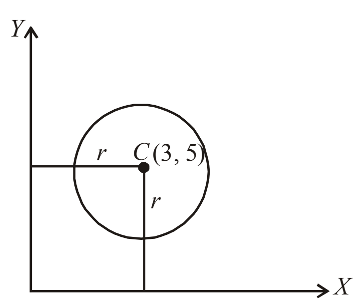
$\Rightarrow \sqrt{50-\lambda} < 4\sqrt{2}$ or $2\sqrt{2} < \sqrt{50-\lambda}$

$\Rightarrow 50 - \lambda < 32$ or $8 < 50 - \lambda$

$\Rightarrow \lambda > 18$ or $\lambda < 42$

Required interval is (18, 42)

56. **(d)**



The equation of circle is

$x^2 + y^2 - 6x - 10y + P = 0$ ...(i)

$(x-3)^2 + (y-5)^2 = (\sqrt{34-P})^2$

Centre (3, 5) and radius 'r' $= \sqrt{34-P}$

If circle does not touch or intersect the x-axis then radius $x < y$ - coordiante of centre C

or $\sqrt{34-P} < 5$

$\Rightarrow 34 - P < 25$

$\Rightarrow P > 9$ ...(ii)

Also if the circle does not touch or intersect x-axis the radius $r < x$-coordinate of centre C.

or $\sqrt{34-P} < 3 \Rightarrow 34 - P < 9 \Rightarrow P > 25$ ... (iii)

If the point (1, 4) is inside the circle, then its distance from centre C < r.

or $\sqrt{[(3-1)^2 + (5-4)^2]} < \sqrt{34-P}$

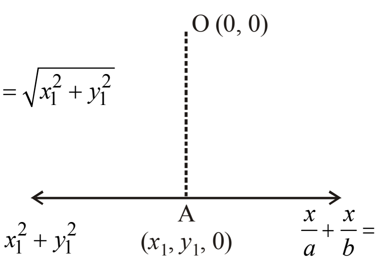
$\Rightarrow 5 < 34 - K$

$\Rightarrow P < 29$ ... (iv)

Now all the conditions (ii), (iii) and (iv) are satisfied if 25 < P < 29 which is required value of P.

57. **(c)** Let the foot of the perpendicular from (0, 0) on the variable line $\frac{x}{a} + \frac{y}{b} = 1$ is $(x_1 > y_1)$

Hence, perpendicular distance of the variable line $\frac{x}{a} + \frac{y}{b} = 1$ from the point O (0, 0) = OA



$\Rightarrow \frac{|-1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \sqrt{x_1^2 + y_1^2}$

$\Rightarrow \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}} = x_1^2 + y_1^2$

$\Rightarrow 4 = x_1^2 + y_1^2 \quad \left[\because \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}\right]$,

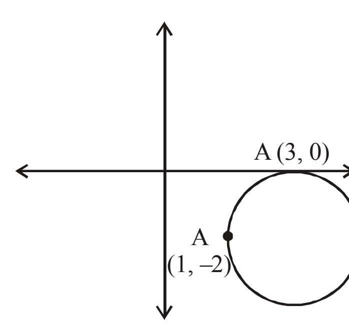
which is equation of a circle with radius 2.

Hence (x_1, y_1) i.e., the foot of the perpendicular from the point (0, 0) to the variable line $\frac{x}{a} + \frac{x}{b} = 1$ is lies on a circle with radius = 2

58. **(c)** Since circle touches x-axis at (3, 0)

$\therefore$ The equation of circle be

$(x-3)^2 + (y-0)^2 + \lambda y = 0$



As it passes through (1, –2)

$\therefore$ Put $x = 1, y = -2$

$\Rightarrow (1-3)^2 + (-2)^2 + \lambda(-2) = 0$

$\Rightarrow \lambda = 4$

$\therefore$ equation of circle is

$(x-3)^2 + y^2 - 8 = 0$

Now, from the options (5, –2) satisfies equation of circle.

59. **(*)** Given information is incomplete in the question.

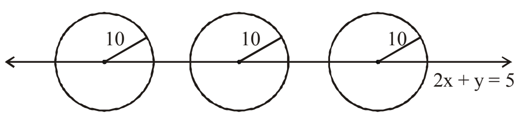
60. **(a)** Circle : $x^2 + y^2 - 6x + 2y = 0$ ...(i)

Line : $2x + y = 5$ ...(ii)

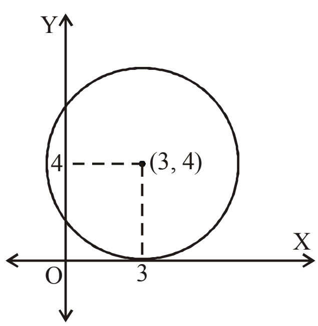
Centre = (3, – 1)

Now, 2 × 3 – 1 = 5, hence centre lies on the given line. Therefore line passes through the centre. The given line is normal to the circle.

Thus statement-2 is true, but statement-1 is not true as there are infinite circle according to the given conditions.



61. **(b)**

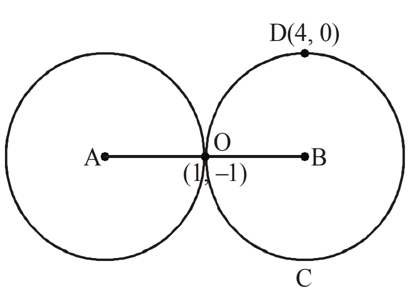


$x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$

Radius = $4 = \sqrt{9 + 16 + (25 - a^2)}$

$\Rightarrow a = \pm 4$

62. **(a)** Let A be the centre of given circle and B be the centre of circle C.



$x^2 + y^2 + 4x - 6y - 12 = 0$

$\therefore$ A = (– 2, 3) and B = (g, f)

Now, from the figure, we have

$\frac{-2+g}{2} = 1$ and $\frac{3+f}{2} = -1$ (By mid point formula)

$\Rightarrow g = 4$ and $f = -5$

63. **(a)** Let C = (x, y)

Now, $CA^2 = CB^2 = AB^2$

$\Rightarrow (x+a)^2 + y^2 = (x-a)^2 + y^2 = (2a)^2$

$\Rightarrow x^2 + 2ax + a^2 + y^2 = 4a^2$ ...(i)

and $x^2 - 2ax + a^2 + y^2 = 4a^2$ ...(ii)

From (i) and (ii), $x = 0$ and $y = \pm\sqrt{3}a$

Since point C(x, y) lies above the x-axis and $a > 0$, hence $y = \sqrt{3}a$

$\therefore$ C = $(0, \sqrt{3}a)$

Let the equation of circumcircle be

$x^2 + y^2 + 2gx + 2fy + C = 0$

Since points A$(-a, 0)$, B$(a, 0)$ and C$(0, \sqrt{3}a)$ lie on the circle, therefore

$a^2 - 2ga + C = 0$ ...(iii)

$a^2 + 2ga + C = 0$ ...(iv)

and $3a^2 + 2\sqrt{3}af + C = 0$ ...(v)

From (iii), (iv), and (v)

$g = 0, c = -a^2, f = -\frac{a}{\sqrt{3}}$

Hence equation of the circumcircle is

$x^2 + y^2 - \frac{2a}{\sqrt{3}}y - a^2 = 0$

$\Rightarrow x^2 + y^2 - \frac{2\sqrt{3}ay}{3} - a^2 = 0$

$\Rightarrow 3x^2 + 3y^2 - 2\sqrt{3}ay = 3a^2$

64. **(d)** Point of intersection of two given lines is (1, 1). Since each of the two given lines contains a diameter of the given circle, therefore the point of intersection of the two given lines is the centre of the given circle.

Hence centre = (1, 1)

$\therefore\ a^2-7a+11=1 \Rightarrow a=2,5$ ...(i)

and $a^2-6a+6=1 \Rightarrow a=1,5$ ...(ii)

From both (i) and (ii), $a=5$

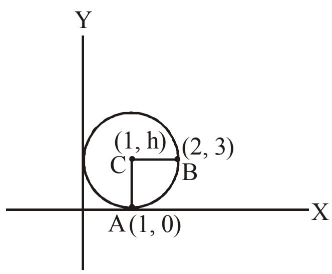
Now on replacing each of $(a^2-7a+11)$ and (a^2-6a+6) by 1, the equation of the given circle is $x^2+y^2-2x-2y+b^3+1=0$

$\Rightarrow (x-1)^2+(y-1)^2+b^3=1$

$\Rightarrow b^3=1-[(x-1)^2+(y-1)^2]$

$\therefore\ b\in(-\infty,1)$

65. **(a)** Since, circle touches , the x-axis at (1, 0). So, let centre of the circle be (1, h)



Given that circle passes through the point B (2,3)

$\therefore\ CA=CB$ (radius)

$\Rightarrow CA^2=CB^2$

$\Rightarrow (1-1)^2+(h-0)^2=(1-2)^2+(h-3)^2$

$\Rightarrow h^2=1+h^2+9-6h$

$\Rightarrow h=\frac{10}{6}=\frac{5}{3}$

$\therefore$ Length of the diameter $=\frac{10}{3}$

66. **(c)** Given circles are

$x^2+y^2-8x-2y+1=0$

and $x^2+y^2+6x+8y=0$

Their centres and radius are

$C_1(4,1), r_1=\sqrt{16}=4$

$C_2(-3,-4), r_2=\sqrt{25}=5$

Now, $C_1C_2=\sqrt{49+25}=\sqrt{74}$

$r_1-r_2=-1, r_1+r_2=9$

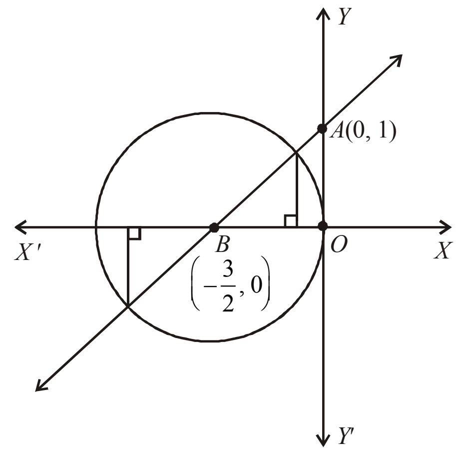
Since, $r_1-r_2<C_1C_2<r_1+r_2$

$\therefore$ Number of common tangents = 2

67. **(b)** Circle : $x^2+y^2+3x=0$

Centre, $B=\left(-\frac{3}{2},0\right)$

Radius $=\frac{3}{2}$ units.



Line : $y=mx+1$

y-intercept of the line = 1

$\therefore$ A = (0, 1)

Slope of line, $m=\tan\theta=\frac{OA}{OB}$

$\Rightarrow m=\frac{1}{\frac{3}{2}}=\frac{2}{3}$

$\Rightarrow 3m-2=0$

68. **(a)** Let $P(1,0)$ and $Q(-1,0)$, $A(x,y)$

Given: $\frac{AP}{AQ}=\frac{BP}{BQ}=\frac{CP}{CQ}=\frac{1}{2}$

$\Rightarrow 2AP=AQ$

$\Rightarrow 4(AP)^2=AQ^2$

$\Rightarrow 4[(x-1)^2+y^2]=(x+1)^2+y^2$

$\Rightarrow 4(x^2+1-2x)+4y^2=x^2+1+2x+y^2$

$\Rightarrow 3x^2+3y^2-8x-2x+4-1=0$

$\Rightarrow 3x^2+3y^2-10x+3=0$

$\Rightarrow x^2+y^2-\frac{10}{3}x+1=0$ ...(1)

$\therefore$ A lies on the circle given by (1). As B and C also follow the same condition.

$\therefore$ Centre of circumcircle of ΔABC = centre of circle given by (1) $=\left(\frac{5}{3},0\right)$.

69. **(d)** Point (1, 2) lies on the circle $x^2+y^2+2x+2y-11=0$, because coordinates of point (1, 2) satisfy the equation $x^2+y^2+2x+2y-11=0$

Now, $x^2+y^2-4x-6y-21=0$ ...(i)

$x^2+y^2+2x+2y-11=0$ ...(ii)

$3x+4y+5=0$ ...(iii)

From (i) and (iii),

$$x^2+\left(-\frac{3x+5}{4}\right)^2-4x-6\left(-\frac{3x+5}{4}\right)-21=0$$

$\Rightarrow\ 16x^2+9x^2+30x+25-64x+72x+120-336=0$

$\Rightarrow\ 25x^2+38x-191=0$ ...(iv)

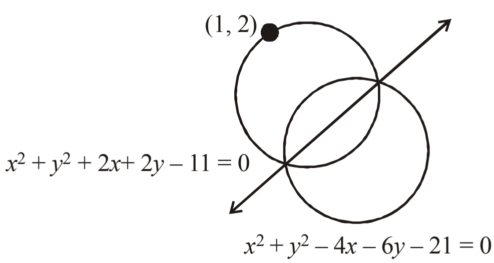
From (ii) and (iii),

$$x^2+\left(-\frac{3x+5}{4}\right)^2+2x+2\left(-\frac{3x+5}{4}\right)-11=0$$

$\Rightarrow\ 16x^2+9x^2+30x+25+32x-24x-40-176=0$

$\Rightarrow\ 25x^2+38x-191=0$ ...(v)

Thus we get the same equation from (ii) and (iii) as we get from equation (i) and (iii). Hence the point of intersections of (ii) and (iii) will be same as the point of intersections of (i) and (iii). Therefore the circle (ii) passing through the point of intersection of circle(i) and point (1, 2) also as shown in the figure.



Hence equation(ii) i.e.

$x^2+y^2+2x+2y-11=0$ is the equation of required circle.

70. **(b)** Given circle whose diametric end points are (1,0) and (0,1) will be of smallest radius. Equation of this smallest circle is

$(x-1)(x-0)+(y-0)(y-1)=0$

$\Rightarrow\ x^2+y^2-x-y=0$

71. **(a)** If the two circles touch each other and centre (0, 0) of $x^2+y^2=c^2$ is lies on circle $x^2+y^2=ax$ then they must touch each other internally.

So, $\frac{|a|}{2}=c-\frac{|a|}{2}\ \Rightarrow |a|=c$

72. **(a)** Given equation of circle is

$x^2+y^2-4x-8y-5=0$

Centre = (2, 4), Radius = $\sqrt{4+16+5}=5$

Given circle is intersecting the line $3x-4y=m$, at two distinct points.

$\Rightarrow$ length of perpendicular from centre to the line < radius

$\Rightarrow \frac{|6-16-m|}{5}<5\ \Rightarrow |10+m|<25$

$\Rightarrow -25<m+10<25 \Rightarrow -35<m<15$

73. **(a)** The given circles are

$S_1\equiv x^2+y^2+3x+7y+2p-5=0$....(1)

$S_2\equiv x^2+y^2+2x+2y-p^2=0$ ....(2)

$\therefore$ Equation of common chord PQ is

$S_1-S_2=0$ [From (i) and (ii)]

$\Rightarrow\ L\equiv x+5y+p^2+2p-5=0$

$\Rightarrow$ Equation of circle passing through P and Q is

$S_1+\lambda L=0$

$\Rightarrow (x^2+y^2+3x+7y+2p-5)$

$+\lambda(x+5y+p^2+2p-5)=0$

Given that it passes through (1, 1), therefore

$(7+2p)+\lambda(2p+p^2+1)=0$

$\Rightarrow \lambda=-\frac{2p+7}{(p+1)^2}$

which does not exist for p = – 1

74. **(a)** Given that $P(1,0)$, $Q(-1,0)$

and $\frac{AP}{AQ}=\frac{BP}{BQ}=\frac{CP}{CQ}=\frac{1}{3}$

$\Rightarrow 3AP=AQ$

Let $A=(x,y)$ then

$3AP=AQ \Rightarrow 9AP^2=AQ^2$

$\Rightarrow 9(x-1)^2+9y^2=(x+1)^2+y^2$

$\Rightarrow 9x^2-18x+9+9y^2=x^2+2x+1+y^2$

$\Rightarrow 8x^2-20x+8y^2+8=0$

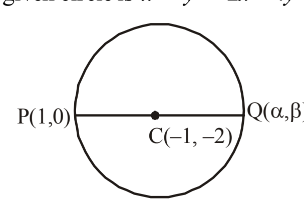
$\Rightarrow x^2+y^2-\frac{5}{3}x+1=0$ ....(1)

$\therefore$ A lies on the circle given by eq (1). As B and C also follow the same condition, they must lie on the same circle.

$\therefore$ Centre of circumcircle of ΔABC

= Centre of circle given by (1) = $\left(\frac{5}{4},0\right)$

75. **(c)** The given circle is $x^2+y^2+2x+4y-3=0$



Centre $(-g,-f)=(-1,-2)$

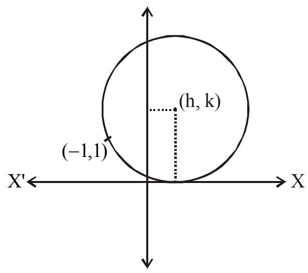
Let $Q(h,k)$ be the point diametrically opposite to the point $P(1,0)$,

then $\frac{1+h}{2}=-1$ and $\frac{0+k}{2}=-2$

$\Rightarrow$ h = –3, k = – 4

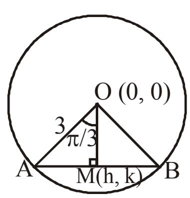
So, Q is (–3, –4)

76. **(d)** Equation of circle whose centre is (h, k) and touch the x-axis
i.e $(x-h)^2+(y-k)^2=k^2$



(radius of circle = k because circle is tangent to x-axis)
$\because$ Equation of circle passing through $(-1, 1)$
$\therefore\ (-1-h)^2+(1-k)^2=k^2$
$\Rightarrow\ 1+h^2+2h+1+k^2-2k=k^2$
$\Rightarrow\ h^2+2h-2k+2=0$
$D \geq 0$
$\therefore\ (2)^2-4\times 1.(-2k+2)\geq 0$
$\Rightarrow\ 4-4(-2k+2)\geq 0 \Rightarrow 1+2k-2\geq 0 \Rightarrow k\geq \frac{1}{2}$

77. **(d)** Given that centre of circle be $(0, 0)$ and radius is 3 unit
Let $M(h, k)$ be the mid point of chord AB where $\angle AOB=\frac{2\pi}{3}$



$\therefore\ \angle AOM=\frac{\pi}{3}$. Also $OM=3\cos\frac{\pi}{3}=\frac{3}{2}$

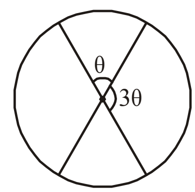
$\Rightarrow \sqrt{h^2+k^2}=\frac{3}{2} \Rightarrow h^2+k^2=\frac{9}{4}$

$\therefore$ Locus of (h, k) is $x^2+y^2=\frac{9}{4}$

78. **(d)** On solving we get point of intersection of $3x-4y-7=0$ and $2x-3y-5=0$ is $(1,-1)$ which is the centre of the circle
Area of circle $=\pi r^2=49\pi$
$\therefore$ radius = 7
$\therefore$ Equation is $(x-1)^2+(y+1)^2=49$
$\Rightarrow\ x^2+y^2-2x+2y-47=0$

79. **(d)** Let the centre variable circle be (α, β)
$\because$ It cuts the circle $x^2+y^2=p^2$ orthogonally
$\therefore$ Using $2g_1g_2+2f_1f_2=c_1+c_2$, we get
$2(-\alpha)\times 0+2(-\beta)\times 0=c_1-p^2$
$\Rightarrow c_1=p^2$
Let equation of circle is $x^2+y^2-2\alpha x-2\beta y+p^2=0$
$\because$ It passes through (a, b)
$\Rightarrow a^2+b^2-2\alpha a-2\beta b+p^2=0$
$\therefore$ Locus of (α, β) is
$\therefore 2ax+2by-(a^2+b^2+p^2)=0$.

80. **(d)**



Given that area of one sector
= 3 × area of another sector
$\Rightarrow$ Angle at centre by one sector $=3\times$ angle at centre by another sector
Let one angle be θ then other $=3\theta$
Clearly $\theta+3\theta=180 \Rightarrow \theta=45^\circ$ (Linear pair)
$\therefore$ Angle between the diameters represented by pair of equation
$ax^2+2(a+b)xy+by^2=0$ is 45°

$\therefore$ Using $\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}$

we get, $\tan 45^\circ=\frac{2\sqrt{(a+b)^2-ab}}{a+b}$

$\Rightarrow 1=\frac{2\sqrt{a^2+b^2+ab}}{a+b}$

$\Rightarrow (a+b)^2=4\left(a^2+b^2+ab\right)$

$\Rightarrow a^2+b^2+2ab=4a^2+4b^2+4ab$

$\Rightarrow 3a^2+3b^2+2ab=0$

81. **(b)** Given that
$s_1=x^2+y^2+2ax+cy+a=0$
$s_2=x^2+y^2-3ax+dy-1=0$
Equation of common chord PQ of circles s_1 and s_2 is given by $s_1-s_2=0$
$\Rightarrow 5ax+(c-d)y+a+1=0$

Given that $5x + by - a = 0$ passes through P and Q

$\therefore$ The two equations should represent the same line

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{a}{1} = \frac{c-d}{b} = \frac{a+1}{-a} \Rightarrow a+1 = -a^2$

$a^2 + a + 1 = 0$ [$\because$ D=–3]

$\therefore$ No real value of a.

82. **(b)** Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ......(1)

It passes through (a, b)

$\therefore a^2 + b^2 + 2ga + 2fb + c = 0$ ......(2)

Circle (1) cuts $x^2 + y^2 = 4$ orthogonally

Two circles intersect orthogonally if $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\therefore 2(g \times 0 + f \times 0) = c - 4 \Rightarrow c = 4$

$\therefore$ from (2) $a^2 + b^2 + 2ga + 2fb + 4 = 0$

$\therefore$ Locus of centre $(-g, -f)$ is

$a^2 + b^2 - 2ax - 2by + 4 = 0$

or $2ax + 2by = a^2 + b^2 + 4$

83. **(d)** Let the variable circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(1)

Since it passes through (p, q)

$\therefore p^2 + q^2 + 2gp + 2fq + c = 0$ ....(2)

Circle (1) touches x-axis,

$\therefore g^2 - c = 0 \Rightarrow c = g^2$. From (2)

$p^2 + q^2 + 2gp + 2fq + g^2 = 0$ ....(3)

Let the other end of diameter through (p, q) be (h, k), then

$\frac{h+p}{2} = -g$ and $\frac{k+q}{2} = -f$

Putting value of g and f in (3), we get

$p^2 + q^2 + 2p\left(-\frac{h+p}{2}\right) + 2q\left(-\frac{k+q}{2}\right) + \left(\frac{h+p}{2}\right)^2 = 0$

$\Rightarrow h^2 + p^2 - 2hp - 4kq = 0$

$\therefore$ locus of (h, k) is

$x^2 + p^2 - 2xp - 4yq = 0$

$\Rightarrow (x-p)^2 = 4qy$

84. **(d)** Two diameters are along

$2x + 3y + 1 = 0$ and $3x - y - 4 = 0$

On solving we get centre $(1, -1)$

Circumference of circle $= 2\pi r = 10\pi$

$\therefore r = 5$.

Required circle is, $(x-1)^2 + (y+1)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 2x + 2y - 23 = 0$

85. **(d)** Solving $y = x$ and the circle

$x^2 + y^2 - 2x = 0$, we get

$x = 0, y = 0$ and $x = 1, y = 1$

$\therefore$ Extremities of diameter of the required circle are A (0, 0) and B (1, 1). Hence, the equation of circle is

$(x-0)(x-1) + (y-0)(y-1) = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$

86. **(b)** $\because$ Given two circles intersect at two points

$\therefore |r_1 - r_2| < C_1C_2$

$\Rightarrow r - 3 < 5 \Rightarrow 0 < r < 8$ .... (1)

and $r_1 + r_2 > C_1C_2$, $r + 3 > 5 \Rightarrow r > 2$ .... (2)

From (1) and (2), $2 < r < 8$.

87. **(d)** Area of circle $= \pi r^2 = 154 \Rightarrow r = 7$

For centre, solving equation

$2x - 3y = 5$ & $3x - 4y = 7$ we get, $x = 1, y = -1$

$\therefore$ centre = $(1, -1)$

Equation of circle, $(x-1)^2 + (y+1)^2 = 7^2$

$x^2 + y^2 - 2x + 2y = 47$

88. **(c)** Given equation of circle $x^2 + y^2 = 1 = (1)^2$

$\Rightarrow x^2 + y^2 = (y - mx)^2$

$\Rightarrow x^2 = m^2x^2 - 2\,mxy$;

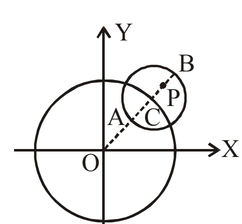
$\Rightarrow x^2(1 - m^2) + 2mxy = 0$. Which represents the pair of lines between which the angle is 45°.

$\therefore \tan 45 = \pm \frac{2\sqrt{m^2 - 0}}{1 - m^2} = \frac{\pm 2m}{1 - m^2}$;

$\Rightarrow 1 - m^2 = \pm 2m \Rightarrow m^2 \pm 2m - 1 = 0$

$\Rightarrow m = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$.

89. **(a)** $\because$ The centre C of circle of radius 3 lies on circle of radius 5. Let $P(x, y)$ in the smaller circle.



we should have

$OA \le OP \le OB$

$\Rightarrow (5-3) \le \sqrt{x^2+y^2} \le 5+3$

$\Rightarrow 4 \le x^2+y^2 \le 64$

90. **(b)** Let the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Since it passes through (0, 0) and (1, 0)

On putting these values, we get

$\Rightarrow c = 0$ and $g = -\frac{1}{2}$

Points (0, 0) and (1, 0) lie inside the circle $x^2+y^2=9$, so two circles touch internally

$\Rightarrow c_1c_2 = r_1 - r_2$

$\therefore \sqrt{g^2+f^2} = 3 - \sqrt{g^2+f^2} \Rightarrow \sqrt{g^2+f^2} = \frac{3}{2}$

Squaring both side, we get

$\Rightarrow f^2 = \frac{9}{4} - \frac{1}{4} = 2 \qquad \therefore f = \pm\sqrt{2}$.

Hence, the centres of required circle are

$\left(\frac{1}{2}, \sqrt{2}\right)$ or $\left(\frac{1}{2}, -\sqrt{2}\right)$

91. **(c)** Let ABC be an equilateral triangle, whose median is AD.

In equilateral triangle median is also altitude

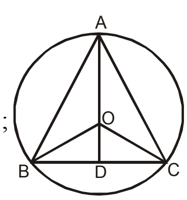
So, $AD \perp BC$

Given $AD = 3a$.

Let $AB = BC = AC = x$.

In ΔABD, $AB^2 = AD^2 + BD^2$;

$\Rightarrow x^2 = 9a^2 + (x^2/4)$



$\frac{3}{4}x^2 = 9a^2 \Rightarrow x^2 = 12a^2$.

In ΔOBD, $OB^2 = OD^2 + BD^2$

$\Rightarrow r^2 = (3a-r)^2 + \frac{x^2}{4}$

$\Rightarrow r^2 = 9a^2 - 6ar + r^2 + 3a^2$

$\Rightarrow 6ar = 12a^2$

$\Rightarrow r = 2a$

So equation of circle is $x^2 + y^2 = 4a^2$

92. **(a)** $L_1 : y = m_1(x+1) + \frac{1}{m_1}$ [Tangent to $y^2 = 4(x+1)$]

$L_2 : y = m_2(x+2) + \frac{2}{m_2}$ [Tangent to $y^2 = 8(x+2)$]

$m_1^2(x+1) - ym_1 + 1 = 0$ ...(i)

$m_2^2(x+2) - ym_2 + 2 = 0$ ...(ii)

$\because m_2 = -\frac{1}{m_1} \qquad (\because L_1 \perp L_2)$

[From (ii)]

$\Rightarrow 2m_1^2 + ym_1 + (x+2) = 0$ ...(iii)

From (i) and (iii),

$\frac{x+1}{2} = \frac{-y}{y} = \frac{1}{x+2} \Rightarrow x+3 = 0$

93. **(d)** Circle passes through $A(0, 1)$ and $B(2, 4)$. So its centre is the point of intersection of perpendicular bisector of AB and normal to the parabola at (2, 4).

Perpendicular bisector of AB;

$y - \frac{5}{2} = -\frac{2}{3}(x-1) \Rightarrow 4x + 6y = 19$ ...(i)

Equation of normal to the parabola at (2, 4) is,

$y - 4 = -\frac{1}{4}(x-2) \Rightarrow x + 4y = 18$ ...(ii)

$\therefore$ From (i) and (ii), $x = -\frac{16}{5}$, $y = \frac{53}{10}$

$\therefore$ Centre of the circle is $\left(-\frac{16}{5}, \frac{53}{10}\right)$

94. **(b)** Equation tangent to parabola $y^2 = 4x$ with slope m be:

$y = mx + \frac{1}{m}$ ...(i)

$\because$ Equation of tangent to $x^2 = 4y$ with slope m be :

$y = mx - am^2$ ...(ii)

From eq. (i) and (ii),

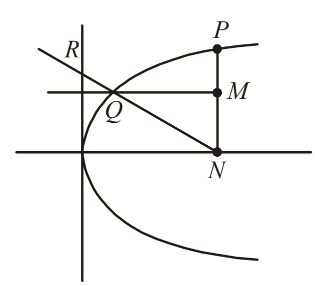
$\frac{1}{m} = -m^2 \Rightarrow m = -1$

$\therefore$ Equation tangent : $x + y + 1 = 0$

It is tangent to circle $x^2 + y^2 = c^2$

$\Rightarrow c = \frac{1}{\sqrt{2}}$

95. **(c)**



$\because y^2 = 12x$

$\therefore a = 3$

Let $P(at^2, 2at)$

$\Rightarrow N(at^2, 0) \Rightarrow M(at^2, at)$

$\because$ Equation of QM is $y = at$

So, $y^2 = 4ax \Rightarrow x = \dfrac{at^2}{4}$

$\Rightarrow Q\left(\dfrac{at^2}{4}, at\right)$

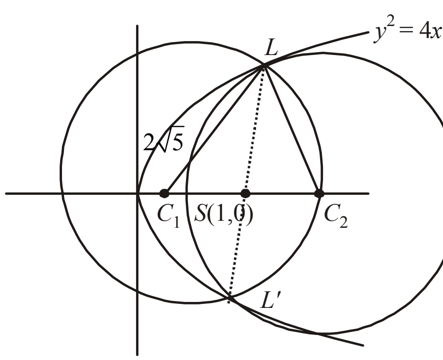
$\Rightarrow$ Equation of QN is $y = \dfrac{-4}{3t}(x - at^2)$

$\because$ QN passes through $\left(0, \dfrac{4}{3}\right)$, then

$\dfrac{4}{3} = -\dfrac{4}{3t}(-at^2) \Rightarrow at = 1 \Rightarrow t = \dfrac{1}{3}$

Now, $MQ = \dfrac{3}{4}at^2 = \dfrac{1}{4}$ and $PN = 2at = 2$

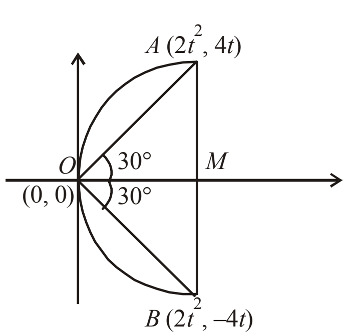
96. **(b)**



Distance between the centres

$= C_1C_2 = 2C_1S = 2\sqrt{20-4} = 8.$

97. **(c)** Let $A = (2t^2, 4t)$ and $B = (2t^2, -4t)$



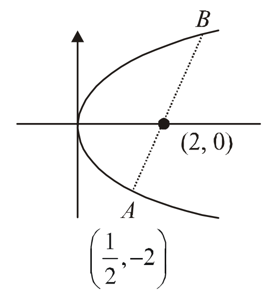
For equilateral triangle $(\angle AOM = 30°)$

$\tan 30° = \dfrac{4t}{2t^2} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{4t}{2t^2} \Rightarrow t = 2\sqrt{3}$

Area $= \dfrac{1}{2} \cdot 8(2\sqrt{3}) \cdot 2 \cdot 24 = 192\sqrt{3}.$

98. **(b)** Let parabola $y^2 = 8x$ at point $\left(\dfrac{1}{2}, -2\right)$ is $(2t^2, 4t)$

$\Rightarrow t = \dfrac{-1}{2}$



Parameter of other end of focal chord is 2

So, coordinates of B is (8, 8)

$\Rightarrow$ Equation of tangent at B is $8y - 4(x + 8) = 0$

$\Rightarrow 2y - x = 8$

$\Rightarrow x - 2y + 8 = 0$

99. **(a)** Let point P be $(2t, t^2)$ and Q be (h, k)

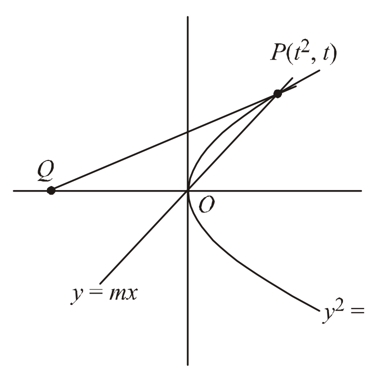
Using section formula,

$h = \dfrac{2t}{3}, k = \dfrac{-2 + t^2}{3}$

Hence, locus is $3k + 2 = \left(\dfrac{3h}{2}\right)^2$

$\Rightarrow 9x^2 = 12y + 8$

100. **(0.5)** Let the coordinates of $P = P(t^2, t)$



Tangent at $P(t^2, t)$ is $ty = \dfrac{x + t^2}{2}$

$\Rightarrow 2ty = x + t^2$

$Q(-t^2, 0), O(0, 0)$

$\therefore$ Area of $\Delta OPQ = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = 4$

$\Rightarrow |t|^3 = 8$

$t = \pm 2\ (t > 0)$

$\therefore$ $4y = x + 4$ is a tangent

$\therefore$ P is (4, 2)

Now, $y = mx \quad \therefore m = \frac{1}{2}$

101. (c) $y = mx + 4$ ...(i)

Tangent of $y^2 = 4x$ is

$\Rightarrow y = mx + \frac{1}{m}$ ...(ii)

[$\because$ Equation of tangent of $y^2 = 4ax$ is $y = mx + \frac{a}{m}$]

From (i) and (ii)

$4 = \frac{1}{m} \Rightarrow m = \frac{1}{4}$

So, line $y = \frac{1}{4}x + 4$ is also tangent to parabola

$x^2 = 2by$, so solve both equations.

$x^2 = 2b\left(\frac{x+16}{4}\right)$

$\Rightarrow 2x^2 - bx - 16b = 0$

$\Rightarrow D = 0$ [For tangent]

$\Rightarrow b^2 - 4 \times 2 \times (-16b) = 0$

$\Rightarrow b^2 + 32 \times 4b = 0$

$b = -128, b = 0$ (not possible)

102. (c) Tangent to the curve $y = (x-2)^2 - 1$ at any point (h, k) is,

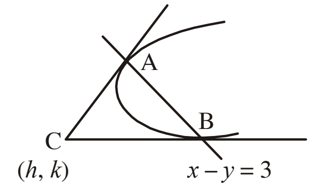
$\Rightarrow \frac{1}{2}(y+k) = (x-2)(h-2) - 1$

$\Rightarrow \frac{y+k}{2} = xh - 2x - 2h + 3$

$\Rightarrow (2h-4)x - y - 4h + 6 - k = 0$

Given line, $x - y - 3 = 0$

$\Rightarrow \frac{2h-4}{1} = \frac{4h-6+k}{3} = 1$



$\Rightarrow h = \frac{5}{2}, k = -1$

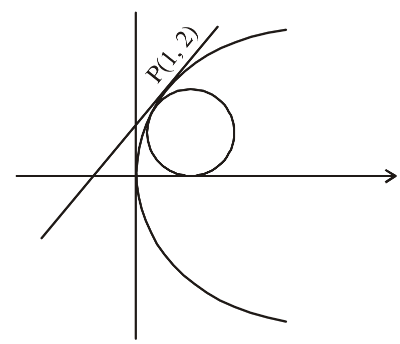
103. (d) Equation of tangent on $y^2 = 4\sqrt{2}x$ is $yt = x + \sqrt{2}t^2$

This is also tangent on circle

$\therefore \left|\frac{\sqrt{2}t^2}{\sqrt{1+t^2}} = 1\right| \Rightarrow 2t^4 = 1 + t^2 \Rightarrow t^2 = 1$

Hence, equation is $\pm y = x + \sqrt{2} \Rightarrow |c| = \sqrt{2}$

104. (d) The circle and parabola will have common tangent at P (1, 2).



So, equation of tangent to parabola is,

$y \times (2) = \frac{4(x+1)}{2} \Rightarrow 2y = 2x + 2 \Rightarrow y = x + 1$

Let equation of circle (by family of circles) is

$(x - x_1)^2 + (y - y_1)^2 + \lambda T = 0$

$\Rightarrow c \equiv (x-1)^2 + (y-2)^2 + \lambda(x - y + 1) = 0$

$\because$ circles touches x-axis.

$\therefore$ y-coordinate of centre = radius

$\Rightarrow c = x^2 + y^2 + (\lambda - 2)x + (-\lambda - 4)y + (\lambda + 5) = 0$

$\frac{\lambda+4}{2} = \sqrt{\left(\frac{\lambda-2}{2}\right)^2 + \left(\frac{-\lambda-4}{2}\right)^2 - (\lambda+5)}$

$\Rightarrow \frac{\lambda^2 - 4\lambda + 4}{4} = \lambda + 5 \Rightarrow \lambda^2 - 4\lambda + 4 = 4\lambda + 20$

$\Rightarrow \lambda^2 - 8\lambda - 16 = 0 \Rightarrow \lambda = 4 \pm 4\sqrt{2}$

$\Rightarrow \lambda = 4-4\sqrt{2}$ ($\because \lambda = 4+4\sqrt{2}$ forms bigger circle)

Hence, centre of circle $\left(2\sqrt{2}-2, 4-2\sqrt{2}\right)$ and radius

$= 4-2\sqrt{2}$

$\therefore$ area $= \pi\left(4-2\sqrt{2}\right)^2 = 8\pi\left(3-2\sqrt{2}\right)$

105. (a) $\because\ y^2 = 16x$

$\Rightarrow a = 4$

One end of focal chord of the parabola is at (1, 4)

y – coordinate of focal chord is 2at

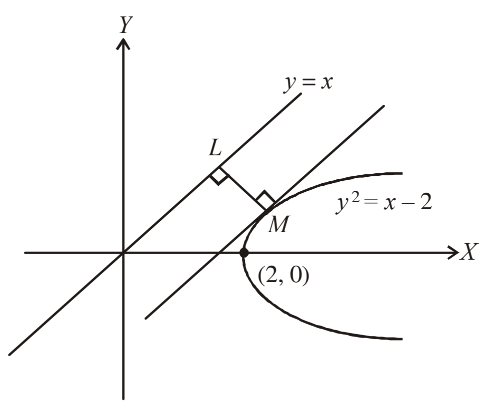
$\therefore 2at = 4$

$\Rightarrow t = \dfrac{1}{2}$

Hence, the required length of focal chord

$= a\left(t+\dfrac{1}{t}\right)^2 = 4\times\left(2+\dfrac{1}{2}\right)^2 = 25$

106. (c) The shortest distance between line $y = x$ and parabola = the distance LM between line $y = x$ and tangent of parabola having slope 1.



Let equation of tangent of parabola having slope 1 is,

$y = m(x-2) + \dfrac{a}{m}$

Here $m = 1$ and $a = \dfrac{1}{4}$

$\therefore$ equation of tangent is: $y = x - \dfrac{7}{4}$

Distance between the line $y - x = 0$ and $y - x + \dfrac{7}{4} = 0$

$= \left|\dfrac{\frac{7}{4}-0}{\sqrt{1^2+1^2}}\right| = \dfrac{7}{4\sqrt{2}}$

107. (d) Since (a, b) touches the given ellipse $4x^2 + y^2 = 8$

$\therefore\ 4a^2 + b^2 = 8$ ...(i)

Equation of tangent on the ellipse at the point A (1, 2) is:

$4x + 2y = 8 \Rightarrow 2x + y = 4 \Rightarrow y = -2x + 4$

But, also equation of tangent at $P(a, b)$ is:

$4ax + by = 8 \Rightarrow y = \dfrac{-4a}{b} + \dfrac{8}{b}.$

Since, tangents are perpendicular to each other.

$\Rightarrow \dfrac{-4a}{b} = \dfrac{-1}{2} \Rightarrow b = 8a$ ...(ii)

from (1) & (2) we get:

$\Rightarrow a = \pm\dfrac{2}{\sqrt{34}} \Rightarrow a^2 = \dfrac{2}{17}$

108. (c) To find intersection point of $x^2 + y^2 = 5$ and $y^2 = 4x$, substitute $y^2 = 4x$ in $x^2 + y^2 = 5$, we get

$x^2 + 4x - 5 = 0 \Rightarrow x^2 + 5x - x - 5 = 0$

$\Rightarrow x(x+5) - 1(x+5) = 0$

$\therefore x = 1, -5$

Intersection point in 1st quadrant be (1, 2).

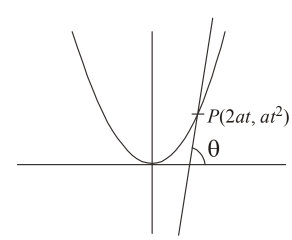
Now, equation of tangent to $y^2 = 4x$ at (1, 2) is

$y \times 2 = 2(x+1) \Rightarrow y = x + 1$

$\Rightarrow x - y + 1 = 0$ ...(i)

Hence, $\left(\dfrac{3}{4}, \dfrac{7}{4}\right)$ lies on (i)

109. (c) $x^2 = 8y$



Then, equation of tangent at P

$tx = y + at^2$

$\Rightarrow y = tx - at^2$

Then, slope $t = \tan\theta$

Now, $y = \tan\theta x - 2\tan^2\theta$

$\Rightarrow \cot\theta y = x - 2\tan\theta$

$x = y\cot\theta + 2\tan\theta$

110. (c) Equation of a tangent to parabola $y^2 = 4x$ is:

$y = mx + \dfrac{1}{m}$

This line is a tangent to $xy = 2$

$\therefore \quad x\left(mx+\frac{1}{m}\right)=2 \Rightarrow mx^2+\frac{1}{m}x-2=0$

$\because$ Tangent is common for parabola and hyperbola.

$\therefore \quad D=\left(\frac{1}{m}\right)^2-4\cdot m\cdot(-2)=0$

$\frac{1}{m^2}+8m=0$

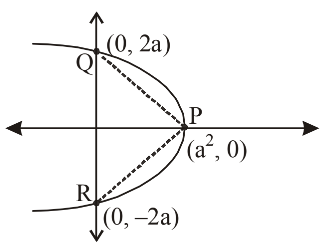
$1+8m^3=0$

$m^3=-\frac{1}{8} \Rightarrow m=-\frac{1}{2}$

$\therefore$ Equation of common tangent: $y=-\frac{1}{2}x-2$

$\Rightarrow \quad 2y=-x-4 \Rightarrow x+2y+4=0$

111. (d) $y^2=-4(x-a^2)$



Area $=\frac{1}{2}(4a)(a^2)=2a^3$

Since $2a^3=250 \Rightarrow a=5$

112. (a, b, c, d)

Normal to $y^2=8ax$ is

$y=mx-4am-2am^3$ ...(i)

and normal to $y^2=4b(x-c)$ with slope m is

$y=m(x-c)-2bm-bm^3$ ...(ii)

Since, both parabolas have a common normal.

$\therefore \quad 4am+2am^3=cm+2bm+bm^3$

$\Rightarrow \quad 4a+2am^2=c+2b+bm^2$ or $m=0$

$\Rightarrow \quad (4a-c-2b)=(b-2a)m^2$

or (X-axis is common normal always)

Since, x-axis is a common normal. Hence all the options are correct for $m=0$.

113. (d) Let intersection points be $P(x_1, y_1)$ and $Q(x_2, y_2)$

The given equations

$x^2=4y$ ...(i)

$x-\sqrt{2}y+4\sqrt{2}=0$ ...(ii)

Use eqn (i) in eqn (ii)

$x-\sqrt{2}\frac{x^2}{4}+4\sqrt{2}=0$

$\sqrt{2}x^2-4x-16\sqrt{2}=0$

$x_1+x_2=2\sqrt{2}, x_1x_2=-16, (x_1-x_2)^2=8+64=72$

Since, points P and Q both satisfy the equations (ii), then

$x_1-\sqrt{2}y_1+4\sqrt{2}=0$

$x_1-\sqrt{2}y_2+4\sqrt{2}=0$

$(x_2-x_1)=\sqrt{2}(y_2-y_1) \Rightarrow (x_2-x_1)^2=2(y_2-y_1)^2$

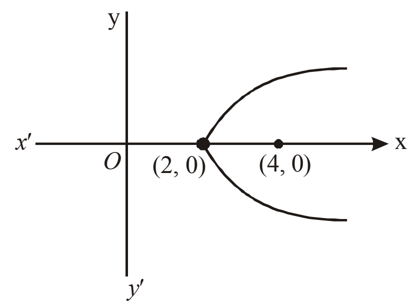
$\Rightarrow \quad PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(x_2-x_1)^2+\frac{(x_2-y_1)^2}{2}}$

$=|x_2-x_1|\cdot\frac{\sqrt{3}}{\sqrt{2}}=6\sqrt{2}\times\frac{\sqrt{3}}{\sqrt{2}}=6\sqrt{3}$

Hence, length of chord $=6\sqrt{3}$.

114. (b) Since, vertex and focus of given parabola is (2, 0) and (4, 0) respectively



Then, equation of parabola is

$(y-0)^2=4\times 2(x-2)$

$\Rightarrow y^2=8x-16$

Hence, the point (8, 6) does not lie on given parabola.

115. (b) Since, the equation of tangent to parabola $y^2=4x$ is

$y=mx+\frac{1}{m}$ ...(i)

The line (i) is also the tangent to circle

$x^2+y^2-6x=0$

Then centre of circle = (3, 0)

radius of circle = 3

The perpendicular distance from centre to tangent is equal to the radius of circle

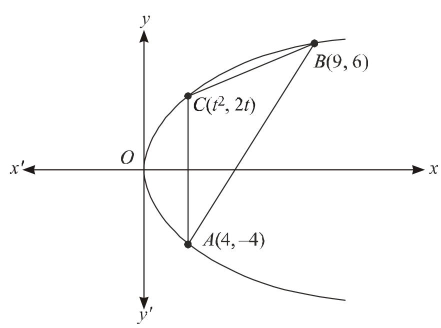
$\therefore \dfrac{\left|3m+\dfrac{1}{m}\right|}{\sqrt{1+m^2}}=3 \Rightarrow \left(3m+\dfrac{1}{m}\right)^2=9\left(1+m^2\right)$

$\Rightarrow m=\pm\dfrac{1}{\sqrt{3}}$

Then, from equation (i): $y=\pm\dfrac{1}{\sqrt{3}}x\pm\sqrt{3}$

Hence, $\sqrt{3}y=x+3$ is one of the required common tangent.

116. (a)



Let the coordinates of C is $(t^2, 2t)$.
Since, area of ΔACB

$$=\frac{1}{2}\left\|\begin{matrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{matrix}\right\|$$

$$=\frac{1}{2}|t^2(6+4)-2t(9-4)+1(-36-24)|$$

$$=\frac{1}{2}|10t^2-10t-60|$$

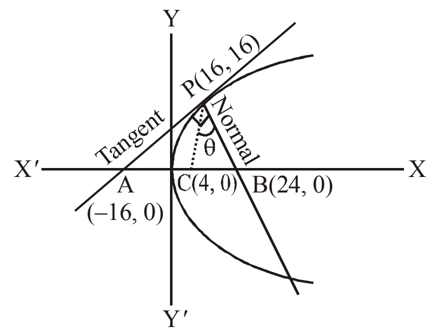
$$=5|t^2-t-6|$$

$$=5\left|\left(t-\frac{1}{2}\right)^2-\frac{25}{4}\right| \qquad [\text{Here, } t\in(0,3)]$$

For maximum area, $t=\dfrac{1}{2}$

Hence, maximum area $=\dfrac{125}{4}=31\dfrac{1}{4}$ sq. units

117. (a) Equation of tnagent at P(16, 16) is given as:
$x-2y+16=0$



Slope of PC $(m_1)=\dfrac{4}{3}$

Slope of PB $(m_2)=-2$

Hence, $\tan\theta=\left|\dfrac{m_1-m_2}{1+m_1\cdot m_2}\right|=\left|\dfrac{\dfrac{4}{3}+2}{1-\dfrac{4}{3}\cdot 2}\right|$

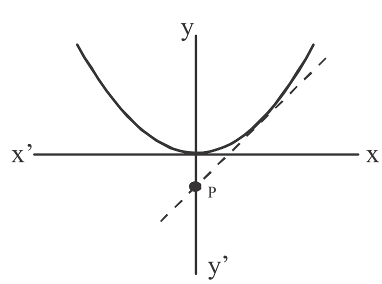
$\Rightarrow \tan\theta=2$

118. (a) Equation of the chord of contact PQ is given by:
$T=0$
or $T \equiv yy_1-4(x+x_1)$, where $(x_1, y_1)\equiv(-8, 0)$
$\therefore$ Equation becomes: $x=8$
& Chord of contact is $x=8$
$\therefore$ Coordinates of point P and Q are (8, 8) and (8, – 8)
and focus of the parabola is $F(2, 0)$

$\therefore$ Area of triangle $PQF=\dfrac{1}{2}\times(8-2)\times(8+8)=48$ sq. units

119. (c) $c=-29m-9m^3$
$a=2$
Given $(at^2-a)^2+4a^2t^2=64$
$\Rightarrow (a(t^2+1))=8$
$\Rightarrow t^2+1=4$
$\Rightarrow t^2=3$
$\Rightarrow t=\sqrt{3}$
$\therefore c=2at(2+t^2)=2\sqrt{3}(5)$
$|c|=10\sqrt{3}$

120. (c)



Tangent to $x^2+y^2=4$ is

$y=mx\pm 2\sqrt{1+m^2}$

Also, $x^2=4y$

$x^2=4mx+8\sqrt{1+m^2}$ or $x^2=4mx-8\sqrt{1+m^2}$

For D = 0

we have; $16m^2+4.8\sqrt{1+m^2}=0$

$\Rightarrow\ m^2+2\sqrt{1+m^2}=0$

$\Rightarrow\ m^2=-2\sqrt{1+m^2}$

$\Rightarrow\ m^4=4+4m^2$

$\Rightarrow\ m^4-4m^2-4=0$

$\Rightarrow\ m^2=\dfrac{4\pm\sqrt{16+16}}{2}$

$\Rightarrow\ m^2=\dfrac{4\pm4\sqrt{2}}{2}$

$\Rightarrow\ m^2=2+2\sqrt{2}$

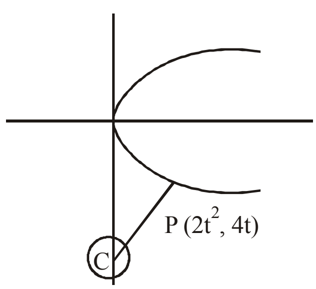
121. (c) Minimum distance $\Rightarrow$ perpendicular distance

Eq^n of normal at $p(2t^2, 4t)$

$y=-tx+4t+2t^3$

It passes through C(0, –6)

$t^3+2t+3=0\Rightarrow t=-1$



Centre of new circle = $P(2t^2, 4t)=P(2,-4)$

Radius = PC $=\sqrt{(2-0)^2+(-4+6)^2}=2\sqrt{2}$

$\therefore$ Equation of circle is :

$(x-2)^2+(y+4)=\left(2\sqrt{2}\right)^2$

$\Rightarrow\ x^2+y^2-4x+8y+12=0$

122. (a) $t_1=-t-\dfrac{2}{t}$

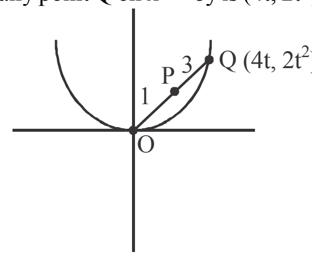
$t_1^2=t^2+\dfrac{4}{t^2}+4$

$t^2+\dfrac{4}{t^2}\geq 2\sqrt{t^2.\dfrac{4}{t^2}}=4$

Minimum value of $t_1^2=8$

123. (b) Let P(h, k) divides OQ in the ratio 1 : 3

Let any point Q on $x^2=8y$ is $(4t, 2t^2)$.



Then by section formula

$\Rightarrow\ k=\dfrac{t^2}{2}$ and $h=t$

$\Rightarrow\ 2k=h^2$

Required locus of P is $x^2=2y$

124. (d) Let $P(-at_1^2, 2at_1), Q(-at_1^2, -2at_1)$ and R (h, k)

By using section formula, we have

$h=-at_1^2,\ k=\dfrac{-2at_1}{3}$

$k=-\dfrac{2at_1}{3}$

$\Rightarrow\ 3k=-2at_1$

$\Rightarrow\ 9k^2=4a^2t_1{}^2=4a(-h)$

$\Rightarrow\ 9k^2=-4ah$

$\Rightarrow\ 9k^2=-4h\Rightarrow 9y^2=-4x$

125. (c) Given parabolas are

$y^2=4x$ ...(1)

$x^2=-32y$ ...(2)

Let m be slope of common tangent

Equation of tangent of parabola (1)

$y=mx+\dfrac{1}{m}$ ...(i)

Equation of tangent of parabola (2)

$y=mx+8m^2$ ...(ii)

(i) and (ii) are identical

$\Rightarrow\ \dfrac{1}{m}=8m^2\Rightarrow m^3=\dfrac{1}{8}\Rightarrow \boxed{m=\dfrac{1}{2}}$

ALTERNATIVE METHOD:

Let tangent to $y^2=4x$ be $y=mx+\dfrac{1}{m}$

Since this is also tangent to $x^2=-32y$

$\therefore\ x^2=-32\left(mx+\dfrac{1}{m}\right)$

$\Rightarrow\ x^2+32mx+\dfrac{32}{m}=0$

Now, D = 0

$$(32)^2 - 4\left(\frac{32}{m}\right) = 0$$

$$\Rightarrow m^3 = \frac{4}{32} \quad \Rightarrow m = \frac{1}{2}$$

126. **(a)** Equation of parabola, $y^2 = 6x$

$$\Rightarrow y^2 = 4 \times \frac{3}{2}x$$

$$\therefore \text{ Focus } = \left(\frac{3}{2}, 0\right)$$

Let equation of chord passing through focus be

$ax + by + c = 0$ ...(1)

Since chord is passing through $\left(\frac{3}{2}, 0\right)$

$\therefore$ Put $x = \frac{3}{2}$, $y = 0$ in eqn (1), we get

$$\frac{3}{2}a + c = 0$$

$$\Rightarrow c = -\frac{3}{2}a \text{ ...(2)}$$

distance of chord from origin is $\frac{\sqrt{5}}{2}\ \frac{\sqrt{5}}{2}$

$$= \left|\frac{a(0) + b(0) + c}{\sqrt{a^2 + b^2}}\right| = \frac{c}{\sqrt{a^2 + b^2}}$$

Squaring both sides

$$\frac{5}{4} = \frac{c^2}{a^2 + b^2}$$

$$\Rightarrow a^2 + b^2 = \frac{4}{5}c^2$$

Putting value of c from (2), we get

$$a^2 + b^2 = \frac{4}{5} \times \frac{9}{4}a^2$$

$$b^2 = \frac{9}{5}a^2 - a^2 = \frac{4}{5}a^2$$

$$\frac{a^2}{b^2} = \frac{5}{4}, \frac{a}{b} = \pm\frac{\sqrt{5}}{2}$$

Slope of chord, $\frac{dy}{dx} = -\frac{a}{b} = -\left(\frac{\pm\sqrt{5}}{2}\right) = \mp\frac{\sqrt{5}}{2}$

127. **(d)** The locus of the point of intersection of tangents to the parabola $y^2 = 4ax$ inclined at an angle α to each other is

$\tan^2\alpha\,(x + a)^2 = y^2 - 4ax$

Given equation of Parabola $y^2 = 4x$ $\{a = 1\}$

Point of intersection $(-2, -1)$

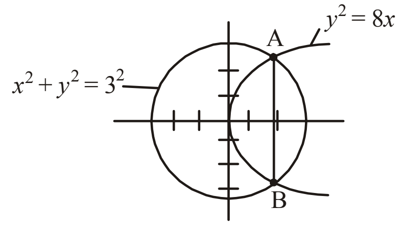
$\tan^2\alpha\,(-2 + 1)^2 = (-1)^2 - 4 \times 1 \times (-2)$

$\Rightarrow \tan^2\alpha = 9$

$\Rightarrow \tan\alpha = \pm 3$

$\Rightarrow |\tan\alpha| = 3$

128. **(c)**



We have

$x^2 + (8x) = 9$

$x^2 + 9x - x - 9 = 0$

$x(x + 9) - 1(x + 9) = 0$

$(x + 9)(x - 1) = 0$

$x = -9, 1$

for $x = 1$, $y = \pm 2\sqrt{2x} = \pm 2\sqrt{2}$

L_1 = Length of AB = $\sqrt{(2\sqrt{2} + 2\sqrt{2})^2 + (1-1)^2} = 4\sqrt{2}$

L_2 = Length of latus rectum = $4a = 4 \times 2 = 8$

$L_1 < L_2$

129. **(b)** Let common tangent be

$$y = mx + \frac{\sqrt{5}}{m}$$

Since, perpendicular distance from centre of the circle to the common tangent is equal to radius of the circle, therefore

$$\frac{\frac{\sqrt{5}}{m}}{\sqrt{1 + m^2}} = \sqrt{\frac{5}{2}}$$

On squaring both the side, we get

$m^2(1 + m^2) = 2$

$\Rightarrow m^4 + m^2 - 2 = 0$

$\Rightarrow (m^2 + 2)(m^2 - 1) = 0$

$\Rightarrow m = \pm 1 \quad (\because m^2 \neq -2)$

$y = \pm\left(x + \sqrt{5}\right)$, both statements are correct as $m = \pm 1$ satisfies the given equation of statement-2.

130. **(b)** We know that point of intersection of the normal to the parabola $y^2 = 4ax$ at the ends of its latus rectum is $(3a, 0)$

Hence required point of intersection = (3, 0)

131. **(b)** Both statements are true and statement-2 is the correct explanation of statement-1

$\therefore$ The straight line $y = mx + \frac{a}{m}$ is always a tangent to the parabola $y^2 = 4ax$ for any value of m.

The co-ordinates of point of contact $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

Now, required radius = OB = $\sqrt{9 + 16} = \sqrt{25} = 5$

132. (a) Ellipse is $\frac{x^2}{16}+\frac{y^2}{3}=1$

Now, equation of normal at (2, 3/2) is

$$\frac{16x}{2}-\frac{3y}{3/2}=16-3$$

$\Rightarrow$ $8x-2y=13$

$\Rightarrow$ $y=4x-\frac{13}{2}$

Let $y=4x-\frac{13}{2}$ touches a parabola

$y^2=4ax$.

We know, a straight line $y=mx+c$ touches a parabola $y^2=4ax$ if $a-mc=0$

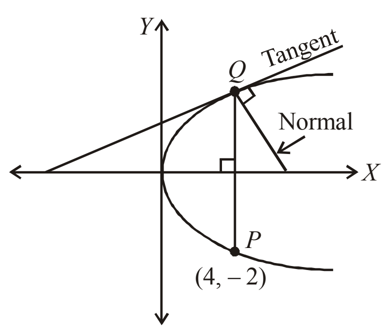
$\therefore$ $a-(4)\left(-\frac{13}{2}\right)=0 \Rightarrow a=-26$

Hence, required equation of parabola is

$y^2=4(-26)x=-104x$

133. (a) Point P is (4, –2) and $PQ \perp x$-axis

So, Q = (4, 2)



Equation of tangent at (4, 2) is

$yy_1=\frac{1}{2}(x+x_1)$

$\Rightarrow 2y=\frac{1}{2}(x+2) \Rightarrow 4y=x+2$

$\Rightarrow y=\frac{x}{4}+\frac{1}{2}$

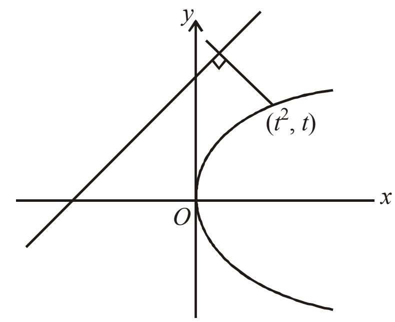
So, slope of tangent = $\frac{1}{4}$

$\therefore$ Slope of normal = – 4

134. (d) Both the given statements are true.

Statement - 2 is not the correct explanation for statement - 1.

135. (a)



Let (t^2, t) be point on parabola from that line have shortest distance.

$\therefore$ Distance $=\left|\frac{t^2-t+1}{\sqrt{2}}\right|$

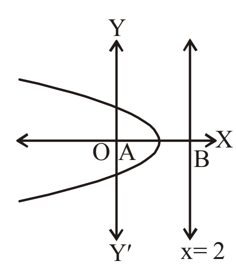
$$=\frac{1}{\sqrt{2}}\left[\left(t-\frac{1}{2}\right)^2+\frac{3}{4}\right]$$

Distance is minimum when $t-\frac{1}{2}=0$

$\therefore$ Shortest distance $=\frac{1}{\sqrt{2}}\left[0+\frac{3}{4}\right]=\frac{3\sqrt{2}}{8}$

136. (b) We know that the locus of perpendicular tangents is directrix i.e., $x=-a; x=-1$

137. (b) We know that vertex of a parabola is the mid point of focus and the point

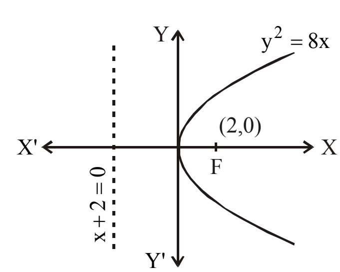


where directrix meets the axis of the parabola.

Given that focus is $O(0, 0)$ and directrix meets the axis at $B(2, 0)$

$\therefore$ Vertex of the parabola is $\left(\frac{0+2}{2},0\right)=(1,0)$

138. (b) Given that parabola $y^2=8x$



We know that the locus of point of intersection of two perpendicular tangents to a parabola is its directrix.
Point must be on the directrix of parabola
$\because$ Equation of directrix $x+2=0$
$\Rightarrow x=-2$
Hence the point is (–2, 0)

139. (a) Given that family of parabolas is

$$y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$$

$$\Rightarrow y=\frac{a^3}{3}\left(x^2+\frac{3}{2a}x+\frac{9}{16a^2}\right)-\frac{3a}{16}-2a$$

$$\Rightarrow y+\frac{35a}{16}=\frac{a^3}{3}\left(x+\frac{3}{4a}\right)^2$$

$\therefore$ Vertex of parabola is $\left(\frac{-3}{4a},\frac{-35a}{16}\right)$

To find locus of this vertex,

$$x=\frac{-3}{4a} \text{ and } y=\frac{-35a}{16}$$

$$\Rightarrow a=\frac{-3}{4x} \text{ and } a=-\frac{16y}{35}$$

$$\Rightarrow \frac{-3}{4x}=\frac{-16y}{35} \quad \Rightarrow 64xy=105$$

$\Rightarrow xy=\frac{105}{64}$ which is the required equation of locus.

140. (a) Given $P=(1,0)$, let $Q=(h,k)$
Since Q lies on $y^2=8x$

$\therefore K^2=8h$ ...(i)

Let (α,β) be the midpoint of PQ

$$\therefore \alpha=\frac{h+1}{2},\ \beta=\frac{k+0}{2}$$

$2\alpha-1=h \qquad 2\beta=k.$

Putting value of h and k in (i)

$(2\beta)^2=8(2\alpha-1) \Rightarrow \beta^2=4\alpha-2$

$\Rightarrow y^2-4x+2=0.$

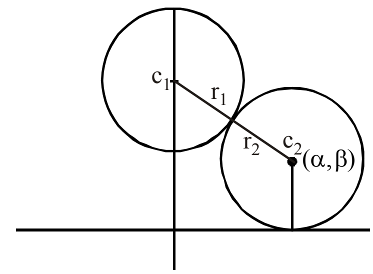
141. (d) Equation of circle with centre (0, 3) and radius 2 is

$x^2+(y-3)^2=4$

Let locus of the centre of the variable circle is (α,β)

$\because$ It touches x - axis.

$\therefore$ It's equation is $(x-\alpha)^2+(y-\beta)^2=\beta^2$



Circle touch externally $\Rightarrow c_1c_2=r_1+r_2$

$\therefore \sqrt{\alpha^2+(\beta-3)^2}=2+\beta$

$\alpha^2+(\beta-3)^2=\beta^2+4+4\beta$
$\alpha^2+\beta^2-6\beta+9=\beta^2+4+4\beta$

$\Rightarrow \quad \alpha^2=10(\beta-1/2)$

$\therefore$ Locus is $x^2=10\left(y-\frac{1}{2}\right)$

Which is equation of parabola.

142. (d) Solving equations of parabolas
$y^2=4ax$ and $x^2=4ay$, we get (0, 0) and $(4a, 4a)$
Putting in the given equation of line
$2bx+3cy+4d=0$, we get
$d=0$ and $2b+3c=0$
$\Rightarrow d^2+(2b+3c)^2=0$

143. (b) Equation of the normal to a parabola $y^2=4bx$ at point $\left(bt_1^2, 2bt_1\right)$ is
$y=-t_1x+2bt_1+bt_1^3$
Given that, it also passes through $\left(bt_2^2, 2bt_2\right)$ then
$2bt_2=-t_1bt_2^2+2bt_1+bt_1^3$
$\Rightarrow 2t_2-2t_1=-t_1\left(t_2^2-t_1^2\right)$
$\Rightarrow 2(t_2-t_1)=-t_1(t_2+t_1)(t_2-t_1)$
$\Rightarrow 2=-t_1(t_2+t_1) \Rightarrow t_2+t_1=-\frac{2}{t_1}$
$\Rightarrow t_2=-t_1-\frac{2}{t_1}$

144. (b) The equation of any tangent to the parabola $y^2=8ax$ is
$y=mx+\frac{2a}{m}$ ...(i)
If (i) is also a tangent to the circle, $x^2+y^2=2a^2$ then,
$\sqrt{2}a=\pm\frac{2a}{m\sqrt{m^2+1}}$
$\Rightarrow m^2(1+m^2)=2 \Rightarrow (m^2+2)(m^2-1)=0 \Rightarrow m=\pm 1.$
Putting the value of m in eqn (i), we get
$y=\pm(x+2a).$

145. (c) We know that the locus of the feet of the perpendicular draw from foci to any tangent of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is the auxiliary circle $x^2+y^2=a^2$

$\therefore$ Auxiliary circle : $x^2+y^2=4$

$\therefore (-1, \sqrt{3})$ satisfies the given equation.

146. (c) Normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $\left(ae, \frac{b^2}{a}\right)$ is

$$\frac{a^2x}{ae}-\frac{b^2y}{b^2/a}=a^2-b^2$$

$$\Rightarrow x-ey=\frac{e(a^2-b^2)}{a} \quad ...(i)$$

$\because (0, -b)$ lies on equation (i), then

$$be=\frac{e(a^2-b^2)}{a}$$

$$\Rightarrow ab=a^2e^2 \Rightarrow b=ae^2 \Rightarrow \frac{b^2}{a^2}=e^4$$

$$\therefore 1-e^2=e^4 \Rightarrow e^4+e^2-1=0$$

147. (b) Ellipse : $\frac{x^2}{16}+\frac{y^2}{9}=1,$

$$a=4, b=3, c=\sqrt{16-9}=\sqrt{7}$$

$\therefore (\pm\sqrt{7}, 0)$ are the foci of given ellipse. So for any point P on it; $PA+PB=2a$

$$\Rightarrow PA+PB=2(4)=8.$$

148. (a) Ellipse $\equiv \frac{x^2}{5}+\frac{y^2}{4}=1$

Let a point on ellipse be $(\sqrt{5}\cos\theta, 2\sin\theta)$

$$\therefore PQ^2=(\sqrt{5}\cos\theta)^2+(-4-2\sin\theta)^2$$

$$=5\cos^2\theta+4\sin^2\theta+16+16\sin\theta$$

$$=21+16\sin\theta-\sin^2\theta$$

$$=21+64-(\sin\theta-8)^2=85-(\sin\theta-8)^2$$

PQ^2 to be maximum when $\sin\theta=1$

$$\therefore PQ^2_{\max}=85-49=36.$$

149. (c) The given ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b)$$

Length of latus rectum $=\frac{2b^2}{a}$

$$\Rightarrow \frac{2b^2}{a}=10 \Rightarrow b^2=5a \quad ...(i)$$

Now $\phi(t)=\frac{5}{12}+t-t^2$

$$\phi'(t)=1-2t=0 \Rightarrow t=\frac{1}{2}$$

$$\phi''(t)=-2<0 \Rightarrow \text{maximum}$$

$$\Rightarrow \phi(t)_{\max}=\frac{5}{12}+\frac{1}{2}-\frac{1}{4}=\frac{8}{12}=\frac{2}{3}$$

Since, $\phi(t)_{\max.}=$ eccentricity

$$\Rightarrow e=\frac{2}{3}$$

Now, $b^2=a^2(1-e^2)$

$$5a=a^2\left(1-\frac{4}{9}\right) \Rightarrow 5a=\frac{5a^2}{9} \Rightarrow a^2-9a=0$$

$$\Rightarrow a=9 \Rightarrow a^2=81 \text{ and } b^2=45$$

$$\therefore a^2+b^2=81+45=126$$

150. (d) $\frac{a}{e}=4 \Rightarrow a=4\times\frac{1}{2}=2$

Now, $b^2=a^2(1-e^2)$

$$\Rightarrow b^2=4\left(1-\frac{1}{4}\right)=4\times\frac{3}{4}=3$$

So, equation $\frac{x^2}{4}+\frac{y^2}{3}=1$

$$\Rightarrow 3x^2+4y^2=12 \quad ...(i)$$

Now, $P(1, \beta)$ lies on it

$$\Rightarrow 3+4\beta^2=12 \Rightarrow \beta=\frac{3}{2}$$

So, equation of normal at $P\left(1, \frac{3}{2}\right)$

$$\Rightarrow \frac{a^2x}{1} - \frac{b^2y}{3/2} = a^2 - b^2 \Rightarrow 4x - 2y = 1$$

151. (d) The given ellipse :

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$\because c = \sqrt{a^2 - b^2} = \sqrt{4-3} = 1$

$\therefore$ Foci = $(\pm 1, 0)$

Now for hyperbola :

Given : $2a = \sqrt{2} \Rightarrow a = \dfrac{1}{\sqrt{2}}$

$\because c^2 = a^2 + b^2 \Rightarrow 1 = \dfrac{1}{2} + b^2 \Rightarrow b = \dfrac{1}{\sqrt{2}}$

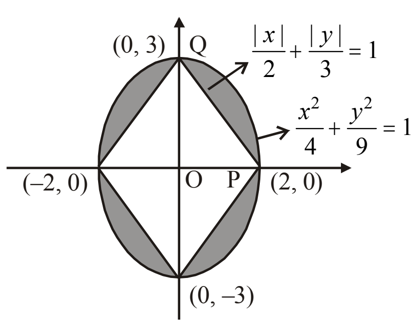
So, equation of hyperbola is

$$\frac{x^2}{\frac{1}{2}} - \frac{y^2}{\frac{1}{2}} = 1$$

$\Rightarrow 2x^2 - 2y^2 = 1$

So, option (d) does not satisfy it.

152. (a)



$\because$ Area of ellipse $= \pi ab = \pi \times 2 \times 3 = 6\pi$

$\therefore$ Required area = Area of ellipse -4 (Area of triangle OPQ)

$$= 6\pi - 4\left(\frac{1}{2} \times 2 \times 3\right)$$

$$= 6\pi - 12 = 6(\pi - 2) \text{ sq. units}$$

153. (a) Eccentricity of ellipse

$$e_1 = \sqrt{1 - \frac{4}{18}} = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$$

Eccentricity of hyperbola

$$e_2 = \sqrt{1 + \frac{4}{9}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$$

Since, the point (e_1, e_2) is on the ellipse

$15x^2 + 3y^2 = k.$

Then, $15e_1^2 + 3e_2^2 = k$

$\Rightarrow k = 15\left(\dfrac{7}{9}\right) + 3\left(\dfrac{13}{9}\right)$

$\Rightarrow k = 16$

154. (a) Let $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1; a > b$

$2b = \dfrac{4}{\sqrt{3}} \Rightarrow b = \dfrac{2}{\sqrt{3}}$

Equation of tangent $\equiv y = mx \pm \sqrt{a^2m^2 + b^2}$

Comparing with $\equiv y = \dfrac{-x}{6} + \dfrac{4}{3}$

$m = \dfrac{-1}{6}$ and $a^2m^2 + b^2 = \dfrac{16}{9}$

$\Rightarrow \dfrac{a^2}{36} + \dfrac{4}{3} = \dfrac{16}{9} \Rightarrow \dfrac{a^2}{36} = \dfrac{16}{9} - \dfrac{4}{3} = \dfrac{4}{9}$

$\Rightarrow a^2 = 16 \Rightarrow a = \pm 4$

Now, eccentricity of ellipse $(e) = \sqrt{1 - \dfrac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \dfrac{4}{3 \times 16}} = \sqrt{\dfrac{11}{12}} = \dfrac{1}{2}\sqrt{\dfrac{11}{3}}$

155. (d) Let P be (x_1, y_1).

So, equation of normal at P is

$$\frac{x}{2x_1} - \frac{y}{y_1} = -\frac{1}{2}$$

It passes through $\left(-\dfrac{1}{3\sqrt{2}}, 0\right)$

$\Rightarrow \dfrac{-1}{6\sqrt{2}x_1} = -\dfrac{1}{2} \Rightarrow x_1 = \dfrac{1}{3\sqrt{2}}$

So, $y_1 = \dfrac{2\sqrt{2}}{3}$ (as P lies in I[st] quadrant)

So, $\beta = \dfrac{y_1}{2} = \dfrac{\sqrt{2}}{3}$

156. (b) $2ae = 6$ and $\dfrac{2a}{e} = 12$

$\Rightarrow ae = 3$ ...(i)

and $\frac{a}{e}=6 \Rightarrow e=\frac{a}{6}$ ...(ii)

$\Rightarrow a^2=18$ [From (i) and (ii)]

$\Rightarrow b^2=a^2-a^2e^2=18-9=9$

$\therefore$ Latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{3\sqrt{2}}=3\sqrt{2}$

157. (a) $3x+4y=12\sqrt{2}$

$\Rightarrow 4y=-3x+12\sqrt{2}$

$\Rightarrow y=-\frac{3}{4}x+3\sqrt{2}$

Now, condition of tangency, $c^2=a^2m^2+b^2$

$\therefore 18=a^2.\frac{9}{16}+9 \Rightarrow a^2.\frac{9}{16}=9$

$\Rightarrow a^2=16 \Rightarrow a=4$

Eccentricity $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

$\therefore ae=\frac{\sqrt{7}}{4}.4=\sqrt{7}$

$\therefore$ Focus are $(\pm\sqrt{7},0)$

$\therefore$ Distance between foci of ellipse $=2\sqrt{7}$

158. (a) Slope of tangent on the line $2x+y=4$ at point P is $\frac{1}{2}$.

Given ellipse is,

$3x^2+4y^2=12 \Rightarrow \frac{x^2}{2^2}+\frac{y^2}{(\sqrt{3})^2}=1$

Let point $P(2\cos\theta, \sqrt{3}\sin\theta)$

$\therefore$ equation of tangent on the ellipse, at P is,

$\frac{x}{2}\cos\theta+\frac{y}{\sqrt{3}}\sin\theta=1$

$\Rightarrow m_T=-\frac{\sqrt{3}}{2}\cot\theta$

$\because$ both the tangents are parallel $\Rightarrow -\frac{\sqrt{3}}{2}\cot\theta=\frac{1}{2}$

$\Rightarrow \tan\theta=-\sqrt{3} \Rightarrow \theta=\pi-\frac{\pi}{3}$ or $\theta=2\pi-\frac{\pi}{3}$

Case-1: $\theta=\frac{2\pi}{3}$, then point $P\left(-1,\frac{3}{2}\right)$ and $PQ=\frac{5\sqrt{5}}{2}$

Case-2: $\theta=\frac{5\pi}{3}$, then tangent does not pass through $Q(4,4)$.

159. (a) Let the equation of ellipse :

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Given that length of minor axis is 4 i.e. $a=4$.

Also given be = 2

$\because a^2=b^2(1-e^2) \Rightarrow 4=b^2-4 \Rightarrow b=2\sqrt{2}$

Hence, equation of ellipse will be $\frac{x^2}{4}+\frac{y^2}{8}=1$

$\because (\sqrt{2},2)$ satisfies this equation.

$\therefore$ ellipse passes through $(\sqrt{2},2)$.

160. (a) Equation of tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $\left(3,-\frac{9}{2}\right)$ is,

$\frac{3x}{a^2}-\frac{9y}{2b^2}=1$

But given equation of tangent is, $x-2y=12$

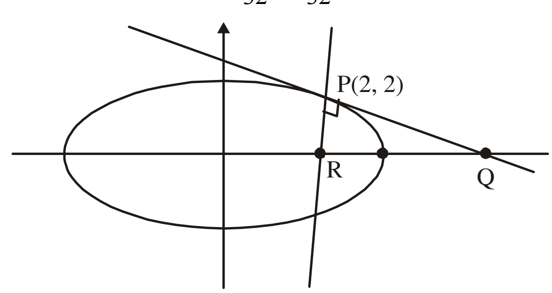
$\therefore \frac{3}{a^2}=\frac{-9}{2b^2.(-2)}=\frac{1}{12}$ (On comparing)

$\Rightarrow a^2=3\times 12$ and $b^2=\frac{9\times 12}{4}$

$\Rightarrow a=6$ and $b=3\sqrt{3}$

Therefore, latus rectum $=\frac{2b^2}{a}=\frac{2\times 27}{6}=9$

161. (d) $3x^2+5y^2=32 \Rightarrow \frac{3x^2}{32}+\frac{5y^2}{32}=1$



Tangent on the ellipse at P is

$\frac{3(2)x}{32}+\frac{5(2)y}{32}=1 \Rightarrow \frac{3x}{16}+\frac{5y}{16}=1$

$\therefore$ co-ordinates of Q will be $\left(\frac{16}{3},0\right)$

Now, normal at P is $\frac{32}{3(2)} - \frac{32y}{5(2)} = \frac{32}{3} - \frac{32}{5}$

$\therefore$ co-ordinates of R will be $\left(\frac{4}{5}, 0\right)$

Hence, area of $\Delta PQR = \frac{1}{2}(PQ)(PR)$

$= \frac{1}{2}\sqrt{\frac{136}{9}} \cdot \sqrt{\frac{136}{25}} = \frac{68}{15}$

162. (d) Let tangent to parabola at point $\left(\frac{1}{4m^2}, -\frac{1}{2m}\right)$ is

$y = mx + \frac{1}{4m}$

and tangent to ellipse is, $y = mx \pm \sqrt{m^2 + \frac{1}{2}}$

Now, condition for common tangency,

$\frac{1}{4m} = \pm\sqrt{m^2 + \frac{1}{2}} \Rightarrow \frac{1}{16m^2} = m^2 + \frac{1}{2}$

$\Rightarrow 16m^4 + 8m^2 - 1 = 0 \Rightarrow m^2 = \frac{-8 \pm \sqrt{64+64}}{2(16)}$

$= \frac{-8 \pm 8\sqrt{2}}{2(16)} = \frac{\sqrt{2}-1}{4}$

$\alpha = \frac{1}{4m^2} = \frac{1}{4\frac{\sqrt{2}-1}{4}} = \sqrt{2}+1$

163. (b) Given that focus is $(0, 5\sqrt{3}) \Rightarrow |b| > |a|$

Let $b > a > 0$ and foci is $(0, \pm be)$

$\because a^2 = b^2 - b^2e^2 \Rightarrow b^2e^2 = b^2 - a^2$

$be = \sqrt{b^2 - a^2} \Rightarrow b^2 - a^2 = 75$ ...(i)

$\because 2b - 2a = 10 \Rightarrow b - a = 5$ ...(ii)

From (i) and (ii)

$b + a = 15$ ...(iii)

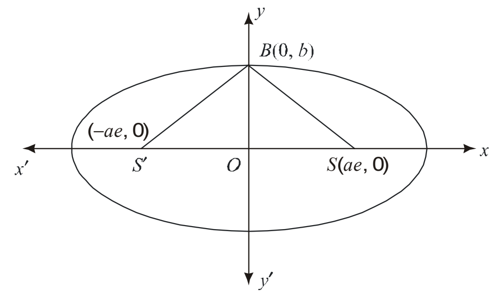
On solving (ii) and (iii), we get

$\Rightarrow b = 10, a = 5$

Now, length of latus rectum $= \frac{2a^2}{b} = \frac{50}{10} = 5$

164. (a) $\because \Delta S'BS$ is right angled triangle, then

(Slope of BS) $\times$ (Slope of BS') $= -1$



$\frac{b}{-ae} \times \frac{b}{ae} = -1 \Rightarrow b^2 = a^2e^2$ ...(i)

Since, area of $\Delta S'BS = 8$

$\Rightarrow \frac{1}{2} \cdot 2ae \cdot b = 8 \Rightarrow b^2 = 8$ ...(ii)

From eq^n (i)

$a^2e^2 = 8$

Also, $e^2 = 1 - \frac{b^2}{a^2}$

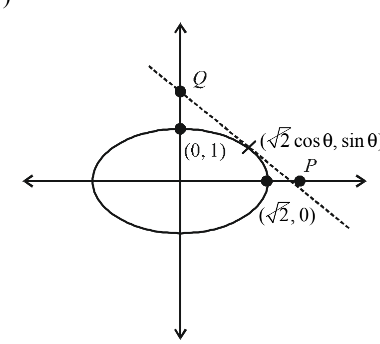
$\Rightarrow a^2e^2 = a^2 - b^2 \Rightarrow 8 = a^2 - 8 \Rightarrow a^2 = 16$

Hence, required length of latus rectum $= \frac{2b^2}{a} = \frac{2(8)}{4}$

$= 4$ units

165. (c) Given the equation of ellipse,

$\frac{x^2}{(\sqrt{2})^2} + y^2 = 1$



$\frac{\sqrt{2}\cos\theta\, x}{2} + y\sin\theta = 1$

$P\left(\frac{\sqrt{2}}{\cos\theta}, 0\right)$ and $Q\left(0, \frac{1}{\sin\theta}\right)$

Let mid point be (h, k)

$\Rightarrow h = \frac{1}{\sqrt{2}\cos\theta}, k = \frac{1}{2\sin\theta}$

As $\cos^2\theta + \sin^2\theta = 1$

$\therefore \quad \frac{1}{2h^2}+\frac{1}{4k^2}=1$

Locus is $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

166. (a) $A=\{(a,b)\in R\times R: |a-5|<1, |b-5|<1\}$

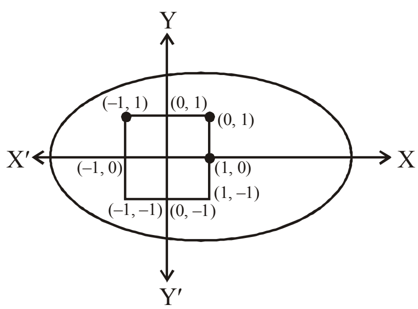
Let $a-5,=x,\ b-5=y$

Set A contains all points inside $|x|<1, |y|<1$

$B=\{(a,b)\in R\times R: 4(a-6)^2+9(B-5)^2\le 36\}$

Set B contains all points inside or on

$\frac{(x-1)^2}{9}+\frac{y^2}{4}=1$



$\therefore \quad (\pm1,\pm1)$ lies inside the ellipse.

Hence, $A\subset B$.

167. (d) Let for ellipse coordinates of focus and vertex are $(ae, 0)$ and $(a, 0)$ respectively.

$\therefore$ Distance between focus and vertex $= a(1-e)=\frac{3}{2}$ (given)

$\Rightarrow a-\frac{3}{2}=ae$

$\Rightarrow a^2+\frac{9}{4}-3a=a^2e^2 \qquad$ ...(i)

Length of latus rectum $=\frac{2b^2}{a}=4$

$\Rightarrow b^2=2a \qquad$ ...(ii)

$e^2=1-\frac{b^2}{a^2}$

$\Rightarrow e^2=1-\frac{2a}{a^2} \qquad$ (from (ii))

$\Rightarrow e^2=1-\frac{2}{a} \qquad$ ... (iii)

Substituting the value of e^2 in eq. (i) we get;

$\Rightarrow a^2+\frac{9}{4}-3a=a^2\left(1-\frac{2}{a}\right)$

$\Rightarrow a=\frac{9}{4}$

$\therefore$ from eq. (iii) we get;

$e^2=1-\frac{2}{a}=1-\frac{8}{9}=\frac{1}{9}$

$\Rightarrow e=\frac{1}{3}$

168. (c) Centre at (0, 0)

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

at point (4, –1)

$\frac{16}{a^2}+\frac{1}{b^2}=1$

$\Rightarrow 16b^2+a^2=a^2b^2 \qquad$ ...(i)

at point (–2, 2)

$\frac{4}{a^2}+\frac{4}{b^2}=1$

$\Rightarrow 4b^2+4a^2=a^2b^2 \qquad$ ...(ii)

$\Rightarrow 16b^2+a^2=4a^2+4b^2$

From equations (i) and (ii)

$\Rightarrow 3a^2=12b^2 \Rightarrow \boxed{a^2=4b^2}$

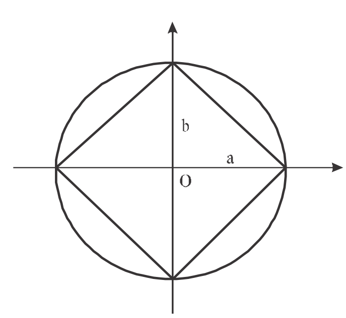
$b^2=a^2(1-e^2)$

$\Rightarrow e^2=\frac{3}{4} \Rightarrow e=\frac{\sqrt{3}}{2}$

169. (d) $e=3/5$ & $2ae=6\Rightarrow a=5$

$\because \ b^2=a^2(1-e^2)$

$\Rightarrow b^2=25(1-9/25)$



$\Rightarrow b=4$

$\therefore$ area of required quadrilateral $=4(1/2\ ab)=2ab=40$

170. (c) Equation of tangent to ellipse

$$\frac{x}{\sqrt{27}}\cos\theta+\frac{y}{\sqrt{3}}\sin\theta=1$$

Area bounded by line and co-ordinate axis

$$\Delta=\frac{1}{2}\cdot\frac{\sqrt{27}}{\cos\theta}\cdot\frac{\sqrt{3}}{\sin\theta}=\frac{9}{\sin 2\theta}$$

Δ = will be minimum when $\sin 2\theta=1$

$\Delta_{\min}=9$

171. (b) The end point of latus rectum of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in first quadrant is $\left(ae,\frac{b^2}{a}\right)$ and the tangent at this point intersects x-axis at $\left(\frac{a}{e},0\right)$ and y-axis at (0, a).

The given ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$

Then $a^2=9, b^2=5$

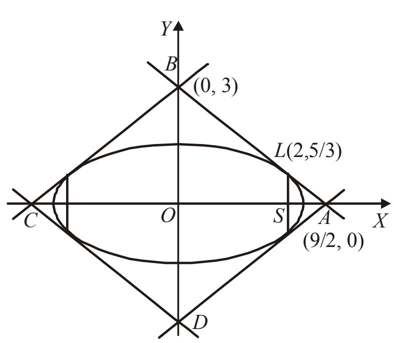
$\Rightarrow\ e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$

$\therefore$ End point of latus rectum in first quadrant is $L\,(2, 5/3)$

Equation of tangent at L is $\frac{2x}{9}+\frac{y}{3}=1$

[$\because$ It meets x-axis at $A\,(9/2, 0)$ and y-axis at $B\,(0, 3)$]

$\therefore$ Area of ΔOAB $=\frac{1}{2}\times\frac{9}{2}\times 3=\frac{27}{4}$



By symmetry area of quadrilateral

$=4\times(\text{Area }\Delta OAB)=4\times\frac{27}{4}=27$ sq. units.

172. (b) Focus of an ellipse is given as (± ae, 0)

Distance between them = 2ae

According to the question, $2ae=\frac{b^2}{a}$

$\Rightarrow 2a^2e=b^2=a^2(1-e^2)$

$\Rightarrow 2e=1-e^2\Rightarrow(e+1)^2=2\Rightarrow e=\sqrt{2}-1$

173. (a) Given equation of ellipse can be written as

$$\frac{x^2}{6}+\frac{y^2}{2}=1$$

$\Rightarrow\ a^2=6, b^2=2$

Now, equation of any variable tangent is

$$y=mx\pm\sqrt{a^2m^2+b^2}\qquad\text{...(i)}$$

where m is slope of the tangent

So, equation of perpendicular line drawn from centre to tangent is

$$y=\frac{-x}{m}\qquad\text{...(ii)}$$

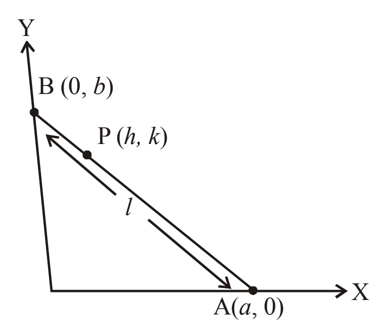
Eliminating m, we get

$$(x^4+y^4+2x^2y^2)=a^2x^2+b^2y^2$$

$$\Rightarrow\ (x^2+y^2)^2=a^2x^2+b^2y^2$$

$$\Rightarrow\ (x^2+y^2)^2=6x^2+2y^2$$

174. (b) Let point A $(a, 0)$ is on x-axis and B $(0, b)$ is on y-axis.



Let P (h, k) divides AB in the ratio 1 : 2.

So, by section formula

$$h=\frac{2(0)+1(a)}{1+2}=\frac{a}{3}$$

$$k=\frac{2(b)+1(0)}{3}=\frac{2b}{3}$$

$\Rightarrow a=3h$ and $b=\frac{3k}{2}$

Now, $a^2+b^2=l^2$

$$\Rightarrow 9h^2+\frac{9k^2}{4}=l^2$$

$$\Rightarrow \frac{h^2}{\left(\frac{l}{3}\right)^2}+\frac{k^2}{\left(\frac{2l}{3}\right)^2}=1$$

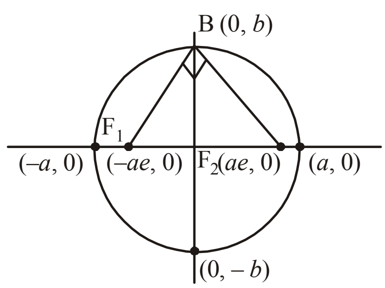
Now $e=\sqrt{1-\left(\frac{l^2}{9}\times\frac{9}{4l^2}\right)}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$

Thus, required locus of P is an ellipse with eccentricity $\frac{\sqrt{3}}{2}$.

175. (a) Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be the equation of ellipse.

Given that F_1B and F_2B are perpendicular to each other.
Slope of F_1B × slope of F_2B = – 1

$$\left(\frac{0-b}{-ae-0}\right)\times\left(\frac{0-b}{ae-0}\right)=-1$$



$$\left(\frac{b}{ae}\right)\times\left(\frac{-b}{ae}\right)=-1$$

$$b^2=a^2e^2$$

$$e^2=\frac{b^2}{a^2} \qquad \left\{\because e^2=1-\frac{b^2}{a^2}\right\}$$

$$1-\frac{b^2}{a^2}=\frac{b^2}{a^2}$$

$$1=2\frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2}=\frac{1}{2}$$

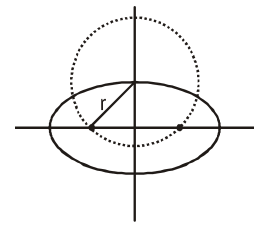
$$e^2=1-\frac{b^2}{a^2}=1-\frac{1}{2}=\frac{1}{2}$$

$$e^2=\frac{1}{2}$$

No common tangents for these two circles.

176. (a) From the given equation of ellipse, we have

$$a=4, b=3, e=\sqrt{1-\frac{9}{16}}$$



$$\Rightarrow e=\frac{\sqrt{7}}{4}$$

Now, radius of this circle $=a^2=16$

$\Rightarrow$ Focii $=(\pm\sqrt{7},0)$

Now equation of circle is

$(x-0)^2+(y-3)^2=16$

$x^2+y^2-6y-7=0$

177. (c) Given ellipse is $4x^2+9y^2=36$

$$\Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$$

Normal at the point is parallel to the line

$4x-2y-5=0$

Slope of normal = 2

Slope of tangent $=\frac{-1}{2}$

Point of contact to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

and line is $\left(\frac{a^2m}{\sqrt{a^2m^2+b^2}},\frac{b}{\sqrt{a^2m^2+b^2}}\right)$

Now, $a^2=9,\ b^2=4$

$\therefore$ Point $=\left(\frac{-9}{5},\frac{8}{5}\right)$

178. (c) Given equations of ellipses

$E_1: \frac{x^2}{3}+\frac{y^2}{2}=1$

$$\Rightarrow e_1=\sqrt{1-\frac{2}{3}}=\frac{1}{\sqrt{3}}$$

and $E_2: \frac{x^2}{16}+\frac{y^2}{b^2}=1$

$$\Rightarrow e_2=\sqrt{\frac{1-b^2}{16}}=\sqrt{\frac{16-b^2}{4}}$$

Also, given $e_1\times e_2=\frac{1}{2}$

$$\Rightarrow \frac{1}{\sqrt{3}}\times\sqrt{\frac{16-b^2}{4}}=\frac{1}{2} \Rightarrow 16-b^2=12$$

$\Rightarrow \quad b^2 = 4$

$\therefore$ Length of minor axis of

$E_2 = 2b = 2 \times 2 = 4$

179. (d) $x^2 = 8y$ ...(i)

$\frac{x^2}{3} + y^2 = 1$ ...(ii)

From (i) and (ii),

$\frac{8y}{3} + y^2 = 1 \Rightarrow y = -3, \frac{1}{3}$

When $y = -3$, then $x^2 = -24$, which is not possible.

When $y = \frac{1}{3}$, then $x = \pm\frac{2\sqrt{6}}{3}$

Point of intersection are

$\left(\frac{2\sqrt{6}}{3}, \frac{1}{3}\right)$ and $\left(-\frac{2\sqrt{6}}{3}, \frac{1}{3}\right)$

Required equation of the line,

$y - \frac{1}{3} = 0 \Rightarrow 3y - 1 = 0$

180. (d) Any tangent on an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by

$y = mx \pm \sqrt{a^2m^2 + b^2}$

Here $a = 2, b = 1$

$m = \frac{1-0}{0-2} = -\frac{1}{2}$

$c = \sqrt{4\left(-\frac{1}{2}\right)^2 + 1^2} = \sqrt{2}$

So, $y = -\frac{1}{2}x \pm \sqrt{2}$

For ellipse : $\frac{x^2}{4} + \frac{y^2}{1} = 1$

We put $y = -\frac{1}{2}x + \sqrt{2}$

$\therefore \frac{x^2}{4} + \left(-\frac{x}{2} + \sqrt{2}\right)^2 = 1$

$\frac{x^2}{4} + \left(\frac{x^2}{4} - 2\left(\frac{x}{2}\right)\sqrt{2} + 2\right) = 1$

$\Rightarrow x^2 + 2\sqrt{2}x + 2 = 0$

or $x^2 - 2\sqrt{2}x + 2 = 0$

$\Rightarrow x = \sqrt{2}$ or $-\sqrt{2}$

If $x = \sqrt{2}, y = \frac{1}{\sqrt{2}}$ and $x = -\sqrt{2}, y = -\frac{1}{\sqrt{2}}$

$\therefore$ Points are $\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right), \left(-\sqrt{2}, -\frac{1}{\sqrt{2}}\right)$

$\therefore P_1P_2 = \sqrt{\left\{\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right\}^2 + \left\{\sqrt{2} - \left(-\sqrt{2}\right)\right\}^2}$

$= \sqrt{\left(\frac{2}{\sqrt{2}}\right)^2 + \left(2\sqrt{2}\right)^2} = \sqrt{2+8} = \sqrt{10}$

181. (d) Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given it passes through (–3, 1) so

$\frac{9}{a^2} + \frac{1}{b^2} = 1$ ...(i)

Also, we know that

$b^2 = a^2(1-e^2) = a^2(1-2/5)$

$\Rightarrow 5b^2 = 3a^2$ ...(ii)

Solving (i) and (ii) we get $a^2 = \frac{32}{3}, b^2 = \frac{32}{5}$

So, the equation of the ellipse is

$3x^2 + 5y^2 = 32$

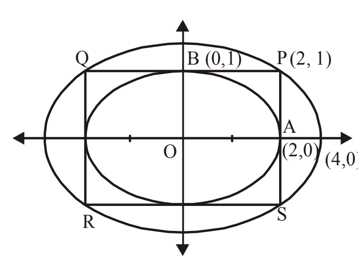
182. (a) The given equation of ellipse is

$\frac{x^2}{4} + \frac{y^2}{1} = 1$

So, $A = (2, 0)$ and $B = (0, 1)$

If $PQRS$ is the rectangle in which it is inscribed, then $P = (2, 1)$.

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the ellipse circumscribing the rectangle $PQRS$.



Then it passed through P (2,1)

$\therefore \frac{4}{a^2} + \frac{1}{b^2} = 1$ ....(i)

Also, given that, it passes through (4, 0)

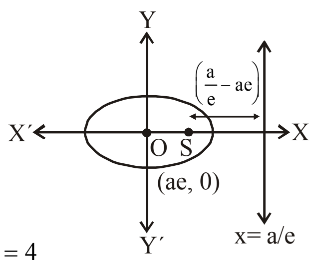
$\therefore \dfrac{16}{a^2} + 0 = 1 \Rightarrow a^2 = 16$

$\Rightarrow b^2 = 4/3$ [putting $a^2 = 16$ in eq^n (i)]

$\therefore$ The required equation of ellipse is $\dfrac{x^2}{16} + \dfrac{y^2}{4/3} = 1$

or $x^2 + 12y^2 = 16$

183. (a) Perpendicular distance of directrix $x = \pm\dfrac{a}{e}$ from focus $(\pm ae, 0)$



$= \dfrac{a}{e} - ae = 4$

$\Rightarrow a\left(2 - \dfrac{1}{2}\right) = 4$

$\Rightarrow a = \dfrac{8}{3}$

$\therefore$ Semi major axis = 8/3

184. (a) Given that distance between foci is $2ae = 6 \Rightarrow ae = 3$ and length of minor axis is $2b = 8 \Rightarrow b = 4$

we know that $b^2 = a^2(1 - e^2)$

$\Rightarrow 16 = a^2 - a^2e^2 \Rightarrow a^2 = 16 + 9 = 25 \Rightarrow a = 5$

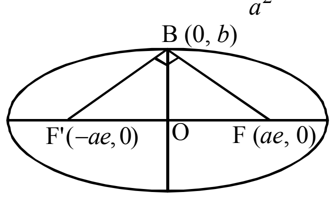
$\therefore e = \dfrac{3}{a} = \dfrac{3}{5}$

185. (a) Given that $\angle FBF' = 90°$

$\Rightarrow FB^2 + F'B^2 = FF'^2$

$\therefore \left(\sqrt{a^2e^2 + b^2}\right)^2 + \left(\sqrt{a^2e^2 + b^2}\right)^2 = (2ae)^2$

$\Rightarrow 2(a^2e^2 + b^2) = 4a^2e^2 \Rightarrow e^2 = \dfrac{b^2}{a^2}$ ...(i)



We know that $e^2 = 1 - b^2/a^2 = 1 - e^2$ [from (i)]

$\Rightarrow 2e^2 = 1,\ e = \dfrac{1}{\sqrt{2}}$.

186. (b) Given that $e = \dfrac{1}{2}$. Directrix, $x = \dfrac{a}{e} = 4$

$\therefore a = 4 \times \dfrac{1}{2} = 2 \quad \therefore b = 2\sqrt{1 - \dfrac{1}{4}} = \sqrt{3}$

Equation of ellipse is

$\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1 \Rightarrow 3x^2 + 4y^2 = 12$

187. (c) General tangent to hyperbola in slope form is

$y = mx \pm \sqrt{100m^2 - 64}$

and the general tangent to the circle in slope form is

$y = mx \pm 6\sqrt{1 + m^2}$

For common tangent,

$36(1 + m^2) = 100m^2 - 64$

$\Rightarrow 100 = 64m^2 \Rightarrow m^2 = \dfrac{100}{64}$

$\therefore c^2 = 36\left(1 + \dfrac{100}{64}\right) = \dfrac{164 \times 36}{64} = \dfrac{369}{4}$

$\Rightarrow 4c^2 = 369$

188. (a) $\because$ The equation of hyperbola is

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\because$ Equation of hyperbola passes through (3, 3)

$\dfrac{1}{a^2} - \dfrac{1}{b^2} = \dfrac{1}{9}$ ...(i)

Equation of normal at point (3, 3) is :

$\dfrac{x - 3}{\frac{1}{a^2} \cdot 3} = \dfrac{y - 3}{-\frac{1}{b^2} \cdot 3}$

$\because$ It passes through (9, 0)

$\dfrac{6}{\frac{1}{a^2}} = \dfrac{-3}{-\frac{1}{b^2}}$

$\therefore \dfrac{1}{b^2} = \dfrac{1}{2a^2}$ ...(ii)

From equations (i) and (ii),

$a^2 = \dfrac{9}{2},\ b^2 = 9$

$\because$ Eccentricity $= e$, then $e^2 = 1+\frac{b^2}{a^2} = 3$

$\therefore (a^2, e^2) = \left(\frac{9}{2}, 3\right)$

189. (d) Equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{b^2}=1$

Then, $e_1 = \sqrt{1-\frac{b^2}{25}}$

The equation of hyperbola, $\frac{x^2}{16}+\frac{y^2}{b^2}=1$

Then, $e_2 = \sqrt{1+\frac{b^2}{16}}$

$e_1 e_2 = 1$

$\Rightarrow (e_1e_2)^2 = 1 \Rightarrow \left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right)=1$

$\Rightarrow 1+\frac{b^2}{16}-\frac{b^2}{25}-\frac{b^4}{25\times16}=1$

$\Rightarrow \frac{9}{16\cdot25}b^2-\frac{b^4}{25\cdot16}=0 \Rightarrow b^2=9$

$\therefore e_1 = \sqrt{1-\frac{9}{25}}=\frac{4}{5}$

And, $e_2 = \sqrt{1+\frac{9}{16}}=\frac{5}{4}$

Distance between focii of ellipse

$= \alpha = 2ae_1 = 2(5)(e_1) = 8$

Distance between focii of hyperbola

$= \beta = 2ae_2 = 2(4)(e_2) = 10$

$\therefore (\alpha, \beta) = (8, 10)$

190. (a) The tangent to the hyperbola at the point (x_1, y_1) is,

$xx_1 - 2yy_1 - 4 = 0$

The given equation of tangent is

$2x - y = 0$

$\Rightarrow \frac{x_1}{2y_1} = 2$

$\Rightarrow x_1 = 4y_1$ ...(i)

Since, point (x_1, y_1) lie on hyperbola.

$\therefore \frac{x_1^2}{4}-\frac{y_1^2}{2}-1=0$ ...(ii)

On solving eqs. (i) and (ii)

$y_1^2 = \frac{2}{7}, x_1^2 = \frac{32}{7}$

$\therefore x_1^2+5y_1^2 = \frac{32}{7}+5\times\frac{2}{7}=6$

191. (d) Hyperbola : $\frac{x^2}{10}-\frac{y^2}{10\cos^2\theta}=1 \Rightarrow e_1 = \sqrt{1+\cos^2\theta}$

and Ellipse : $\frac{x^2}{5\cos^2\theta}+\frac{y^2}{5}=1$

$\Rightarrow e_2 = \sqrt{1-\cos^2\theta} = \sin\theta$

According to the question, $e_1 = \sqrt{5}e_2$

$\Rightarrow 1+\cos^2\theta = 5\sin^2\theta \Rightarrow \cos^2\theta = \frac{2}{3}$

Now length of latus rectum of ellipse

$= \frac{2a^2}{b} = \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$

192. (b) Let the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

If a hyperbola passes through vertices at (± 6, 0), then

$\therefore a = 6$

As hyperbola passes through the point $P(10, 16)$

$\therefore \frac{100}{36}-\frac{256}{b^2}=1 \Rightarrow b^2 = 144$

$\therefore$ Required hyperbola is $\frac{x^2}{36}-\frac{y^2}{144}=1$

Equation of normal is $\frac{36x}{10}+\frac{144y}{16}=36+144$

$\therefore$ At P(10, 16) normal is

$\frac{36x}{10}+\frac{144y}{16}=36+144$

$\therefore 2x+5y=100.$

193. (c) Equation of tangent to $y^2 = 12x$ is $y = mx+\frac{3}{m}$

Equation of tangent to

$\frac{x^2}{1}-\frac{y^2}{8}=1$ is $y = mx \pm\sqrt{m^2-8}$

$\because$ parabola and hyperbola have common tangent.

$\therefore \quad \frac{3}{m} = \pm\sqrt{m^2-8} \Rightarrow \frac{9}{m^2} = m^2 - 8$

Put $m^2 = u$

$u^2 - 8u - 9 = 0 \Rightarrow \quad u^2 - 9u + u - 9 = 0$

$\Rightarrow \quad (u+1)(u-9) = 0$

$\because u = m^2 \geq 0 \Rightarrow u = m^2 = 9 \Rightarrow m = \pm 3$

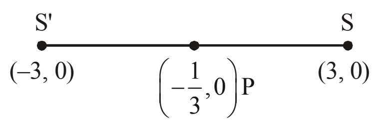
$\therefore$ equation of tangent is $y = 3x + 1$

or $y = -3x - 1$

$\therefore$ intersection point is $P\left(-\frac{1}{3}, 0\right)$.

$$e = \sqrt{1+\frac{b^2}{a^2}} \Rightarrow e = \sqrt{1+\frac{8}{1}} \Rightarrow e = 3$$

$\therefore$ foci $(\pm 3, 0)$



$$\frac{SP}{SP'} = \frac{3+\frac{1}{3}}{3-\frac{1}{3}} = \frac{10}{8} = \frac{5}{4}$$

194. (a) Given curves, $y^2 = 16x$ and $xy = -4$

Equation of tangent to the given parabola;

$$y = mx + \frac{4}{m}$$

$\because$ This is common tangent.

So, put $y = mx + \frac{4}{m}$ in $xy = -4$.

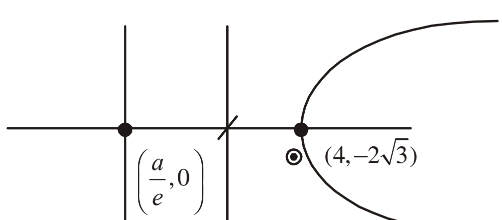
$$x\left(mx + \frac{4}{m}\right) + 4 = 0 \Rightarrow mx^2 + \frac{4}{m}x + 4 = 0$$

$$D = 0 \Rightarrow \frac{16}{m^2} = 16m \Rightarrow m^3 = 1 \Rightarrow m = 1$$

$\therefore$ equation of common tangent is $y = x + 4$

195. (c) $\because$ directrix of a hyperbola is,

$$5x = 4\sqrt{5} \Rightarrow x = \frac{4}{\sqrt{5}} \Rightarrow \frac{a}{e} = \frac{4}{\sqrt{5}}$$



Now, hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes throug $(4, -2\sqrt{3})$

$$\therefore \frac{16}{a^2} - \frac{12}{a^2e^2 - a^2} = 1$$

$$\left[\because e^2 = 1 + \frac{b^2}{a^2} \Rightarrow a^2e^2 - a^2 = b^2\right]$$

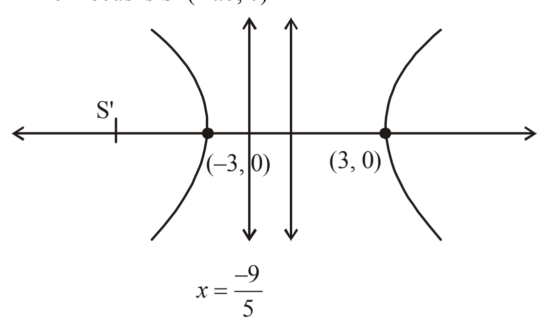
$$\Rightarrow \frac{4}{a^2}\left[\frac{4}{1} - \frac{3}{e^2-1}\right] = 1 \Rightarrow 4e^2 - 4 - 3 = (e^2-1)\left(\frac{a^2}{4}\right)$$

$$\Rightarrow 4(4e^2 - 7) = (e^2 - 1)\left(\frac{4e}{\sqrt{5}}\right)^2$$

$$\Rightarrow 4e^4 - 24e^2 + 35 = 0$$

196. (d) $16x^2 - 9y^2 = 144 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

Then focus is S' $(-ae, 0)$



$$a = 3, b = 4 \Rightarrow e^2 = 1 + \frac{16}{9} = \frac{25}{9} \quad \left[\because e = \sqrt{1+\frac{b^2}{a^2}}\right]$$

$$\therefore \text{ the focus } S' \equiv \left(3 - \times\frac{5}{3}, 0\right) \equiv (-5, 0)$$

197. (c) Since, $lx + my + n = 0$ is a normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

then $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

but it is given that $mx - y + 7\sqrt{3}$ is normal to hyperbola

$$\frac{x^2}{24} - \frac{y^2}{18} = 1$$

then $\frac{24}{m^2} - \frac{18}{(-1)^2} = \frac{(24+18)^2}{(7\sqrt{3})^2} \Rightarrow m = \frac{2}{\sqrt{5}}$

198. (c) Let equation of hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad \text{...(i)}$$

$$\because e=\sqrt{1+\frac{b^2}{a^2}} \Rightarrow b^2=a^2(e^2-1)$$

$$e=2 \Rightarrow b^2=3a^2 \quad \text{...(ii)}$$

Equation (i) passes through (4, 6),

$$\therefore \frac{16}{a^2}-\frac{36}{b^2}=1 \quad \text{...(iii)}$$

On solving (i) and (ii), we get

$a^2=4, b^2=12$

Now equation of hyperbola is $\frac{x^2}{4}-\frac{y^2}{12}=1$

Now equation of tangent to the hyperbola at (4, 6) is

$$\frac{4x}{4}-\frac{6y}{12}=1 \Rightarrow x-\frac{y}{2}=1 \Rightarrow 2x-y=2$$

199. (d) Let the points are,

$A(2, 0), A'(-2, 0)$ and $S(-3, 0)$

$\Rightarrow$ Centre of hyperbola is $O(0, 0)$

$AA'=2a \Rightarrow 4=2a \Rightarrow a=2$

$\because$ Distance between the centre and foci is ae.

$\therefore$ $OS=ae \Rightarrow 3=2e \Rightarrow e=\frac{3}{2}$

$\Rightarrow$ $b^2=a^2(e^2-1)=a^2e^2-a^2=9-4=5$

$\Rightarrow$ Equation of hyperbola is $\frac{x^2}{4}-\frac{y^2}{5}=1$ ...(i)

$\because$ (6, 552) does not satisfy eq (i).

$\therefore$ $(6, 5\sqrt{2})$ does not lie on this hyperbola.

200. (a) $\therefore$ Conjugate axis = 5

$\therefore$ $2b=5$

Distance between foci =13

$2ae=13$

Then, $b^2=a^2(e^2-1)$

$\Rightarrow$ $a^2=36$

$\therefore$ $a=6$

$$ae=\frac{13}{2} \Rightarrow e=\frac{13}{12}$$

201. (b) Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Then, $\frac{2b^2}{a}=8,\ 2ae=b^2$ and $b^2=a^2(1-e^2)$

$\Rightarrow$ $a=8, b^2=32$

Then, the equation of the ellipse

$$\frac{x^2}{64}+\frac{y^2}{32}=1$$

Hence, the point $(4\sqrt{3}, 2\sqrt{2})$ lies on the ellipse.

202. (a) Given, the equation of line,

$x-y=2 \Rightarrow y=x-2$

$\therefore$ its slope $=m=1$

Equation of hyperbola is:

$$\frac{x^2}{5}-\frac{y^2}{4}=1 \Rightarrow a^2=5, b^2=4$$

The equation of tangent to the hyperbola is,

$y=mx\pm\sqrt{a^2m^2-b^2}$

$=x\pm\sqrt{5-4} \Rightarrow y=x\pm 1$

203. (b) Since, $r\neq\pm1$, then there are two cases, when $r>1$

$$\frac{x^2}{r-1}+\frac{y^2}{r+1}=1 \text{ (Ellipse)}$$

Then,

$$(r-1)=(r+1)(1-e^2) \Rightarrow 1-e^2=\frac{(r-1)}{(r+1)}$$

$$\Rightarrow e^2=1-\frac{(r-1)}{(r+1)}=\frac{2}{(r+1)}$$

$$\Rightarrow e=\sqrt{\frac{2}{(r+1)}}$$

When $0<r<1$, then

$$\frac{x^2}{1-r}-\frac{y^2}{1+r}=-1 \text{ (Hyperbola)}$$

Then,

$$(1-r)=(1+r)(e^2-1) \Rightarrow e^2=1+\frac{(r-1)}{(r+1)}=\frac{2r}{(r+1)}$$

$$\Rightarrow e=\sqrt{\frac{2r}{r+1}}$$

204. (a) $\because$ $a^2=\cos^2\theta, b^2=\sin^2\theta$

and $e>2 \Rightarrow e^2>4 \Rightarrow 1+b^2/a^2>4$

$\Rightarrow$ $1+\tan^2\theta>4$

$$\Rightarrow \sec^2\theta>4 \Rightarrow \theta\in\left(\frac{\pi}{3},\frac{\pi}{2}\right)$$

Latus rectum,

$LR = \frac{2b^2}{a} = \frac{2\sin^2\theta}{\cos\theta} = 2(\sec\theta - \cos\theta)$

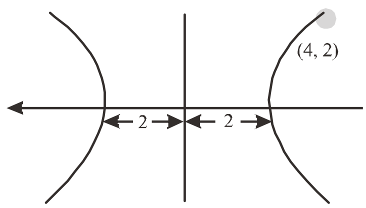
$\Rightarrow \frac{d(LR)}{d\theta} = 2(\sec\theta\tan\theta + \sin\theta) > 0 \ \forall\theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$\therefore \min(LR) = 2\left(\sec\frac{\pi}{3} - \cos\frac{\pi}{3}\right) = 2\left(2 - \frac{1}{2}\right) = 3$

max (LR) tends to infinity as $\theta \to \frac{\pi}{2}$

Hence, length of latus rectum lies in the interval $(3, \infty)$

205. (d)



Consider equation of hyperbola

$\frac{x^2}{2^2} - \frac{y^2}{b^2} = 1$

$\because$ (4, 2) lies on hyperbola

$\therefore \frac{16}{4} - \frac{4}{b^2} = 1$

$\therefore b^2 = \frac{4}{3}$

Since, eccentricity $= \sqrt{1 + \frac{b^2}{a^2}}$

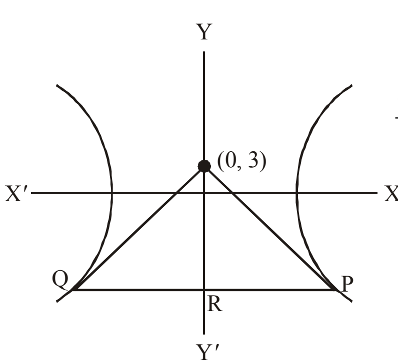
Hence, eccentricity $= \sqrt{1 + \frac{\frac{4}{3}}{4}} = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$

206. (d) Here equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{36} = 1$

Now, PQ is the chord of contant

$\therefore$ Equation of PQ is : $\frac{x(0)}{9} - \frac{y(3)}{36} = 1$

$\Rightarrow y = -12$



$\frac{x^2}{9} - \frac{y^2}{36} = 1$

$\therefore$ Area of $\Delta PQT = \frac{1}{2} \times TR \times PQ$

$\because$ $P \equiv (3\sqrt{5}, -12)$ $\therefore$ $TR = 3 + 12 = 15$,

$\therefore$ Area of $\Delta PQT = \frac{1}{2} \times 15 \times 6\sqrt{5} = 45\sqrt{5}$ sq. units

207. (a) Here, lines are:

$\sqrt{2}x - y + 4\sqrt{2}k = 0$

$\Rightarrow \sqrt{2}x + 4\sqrt{2}k = y$ ...(i)

and $\sqrt{2}kx + ky - 4\sqrt{2} = 0$ ...(ii)

Put the value of y from (i) in (ii) we get;

$\Rightarrow 2\sqrt{2}kx + 4\sqrt{2}\left(k^2 - 1\right) = 0$

$\Rightarrow x = \frac{2(1 - k^2)}{k}, y = \frac{2\sqrt{2}(1 + k^2)}{k}$

$\therefore \left(\frac{y}{4\sqrt{2}}\right)^2 - \left(\frac{x}{4}\right)^2 = 1$

$\therefore$ length of transverse axis

$2a = 2 \times 4\sqrt{2} = 8\sqrt{2}$

Hence, the locus is a hyperbola with length of its transverse axis equal to $8\sqrt{2}$

208. (c) Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

foci is $(\pm 2, 0) \Rightarrow ae = \pm 2 \Rightarrow a^2e^2 = 4$

Since $b^2 = a^2(e^2 - 1)$

$b^2 = a^2e^2 - a^2 \ \therefore a^2 + b^2 = 4$ ...(i)

Hyperbola passes through $(\sqrt{2}, \sqrt{3})$

$\therefore \frac{2}{a^2} - \frac{3}{b^2} = 1$ ...(ii)

$\frac{2}{4 - b^2} - \frac{3}{b^2} = 1$ [from (i)]

$\Rightarrow b^4 + b^2 - 12 = 0$

$\Rightarrow (b^2 - 3)(b^2 + 4) = 0$

$\Rightarrow b^2 = 3$

$b^2 = -4$ (Not possible)

For $b^2 = 3$

$\Rightarrow a^2 = 1 \ \therefore \frac{x^2}{1} - \frac{y^2}{3} = 1$

Equation of tangent is $\frac{\sqrt{2}x}{1} - \frac{\sqrt{3}y}{3} = 1$

Clearly $(2\sqrt{2}, 3\sqrt{3})$ satisfies it.

209. (d) Here, $tx-2y-3t=0$ & $x-2ty+3=0$
On solving, we get;

$$y=\frac{6t}{2t^2-2}=\frac{3t}{t^2-1} \text{ \& } x=\frac{3t^2+3}{t^2-1}$$

Put t = tan θ

$\therefore$ $x=-3\sec 2\theta$ & $2y=3(-\tan 2\theta)$

$\because$ $\sec^2 2\theta-\tan^2 2\theta=1$

$\Rightarrow \frac{x^2}{9}-\frac{y^2}{9/4}=1$

which represents a hyperbola

$\therefore$ $a^2=9$ & $b^2=9/4$

$\lambda(\text{T.A.})=6;\ e^2=1+\frac{9/4}{9}=1+\frac{1}{4}\Rightarrow e=\frac{\sqrt{5}}{2}$

210. (a) $\frac{2b^2}{a}=8$ and $2b=\frac{1}{2}(2ae)$

$\Rightarrow 4b^2=a^2e^2\Rightarrow 4a^2(e^2-1)=a^2e^2$

$\Rightarrow 3e^2=4 \Rightarrow e=\frac{2}{\sqrt{3}}$

211. (c) $\frac{x^2}{12}+\frac{y^2}{16}=1$

$e=\sqrt{1-\frac{12}{16}}=\frac{1}{2}$

Foci $(0, 2)$ & $(0, -2)$

So, transverse axis of hyperbola $=2b=4\Rightarrow b=2$

& $a^2=1^2(e^2-1)$

$\Rightarrow a^2=4\left(\frac{9}{4}-1\right)$

$\Rightarrow a^2=5$

$\therefore$ It's equation is $\frac{x^2}{5}-\frac{y^2}{4}=-1$

The point $(5, 2\sqrt{3})$ does not satisfy the above equation.

212. (a) $S(5, 0)$ is focus $\Rightarrow ae=5$ (focus) ——— (a)

$x=\frac{a}{5}\Rightarrow\frac{a}{e}=\frac{9}{5}$ (directrix) ——— (b)

(a) & (b) $\Rightarrow a^2=9$

(a) $\Rightarrow (e)=\frac{5}{3}$

$b^2=a^2(e^2-1)\Rightarrow b^2=16$

$a^2-b^2=9-16=-7$

213. (c) Equation of hyperbola is

$\frac{x^2}{4}-\frac{y^2}{9}=1$

Its Foci $=\left(\pm\sqrt{13},0\right)$

$e=\frac{\sqrt{13}}{2}$

If e, be the eccentricity of the ellipse, then

$e_1\times\frac{\sqrt{13}}{2}=\frac{1}{2} \Rightarrow e_1=\frac{1}{\sqrt{13}}$

Equation of ellipse is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Since ellipse passes through the foci $(\pm\sqrt{13}, 0)$ of the hyperbola, therefore

$a^2=13$

Now $\sqrt{a^2-b^2}=ae_1$

$\therefore 13-b^2=1$

$\Rightarrow b^2=12$

Hence, equation of ellipse is

$\frac{x^2}{13}+\frac{y^2}{12}=1$

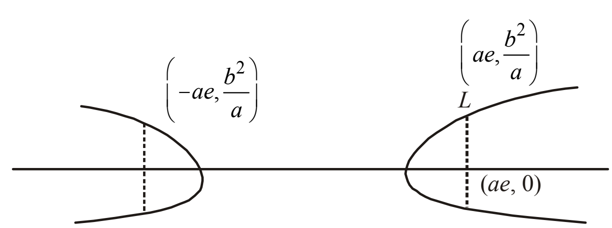
Now putting the coordinate of the point $\left(\frac{\sqrt{13}}{2},\frac{\sqrt{3}}{2}\right)$ in the equation of the ellipse, we get

$\frac{13}{4\times13}+\frac{3}{4\times12}=1$

$\Rightarrow \frac{1}{4}+\frac{1}{16}=1$, which is not true,

Hence the point $\left(\frac{\sqrt{13}}{2},\frac{\sqrt{3}}{2}\right)$ does not lie on the ellipse.

214. (a)



Given $\frac{x^2}{4}-\frac{y^2}{5}=1$

$\Rightarrow$ $a^2=4,\ b^2=5$

$$e=\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{4+5}{4}}=\frac{3}{2}$$

$$L=\left(2\times\frac{3}{2},\frac{5}{2}\right)=\left(3,\frac{5}{2}\right)$$

Equation of tangent at (x_1, y_1) is

$$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$$

Here $x_1=3,\ y_1=\frac{5}{2}$

$$\Rightarrow \frac{3x}{4}-\frac{y}{2}=1 \Rightarrow \frac{x}{\frac{4}{3}}+\frac{y}{-2}=1$$

x-intercept of the tangent, OA $=\frac{4}{3}$

y-intercept of the tangent, OB $=-2$

$$OA^2-OB^2=\frac{16}{9}-4=-\frac{20}{9}$$

215. **(d)** Let the coordinate at point of intersection of normals at P and Q be (h, k)

Since, equation of normals to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ At point (x_1, y_1) is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

therefore equation of normal to the hyperbola $\frac{x^2}{3^2}-\frac{y^2}{2^2}=1$ at point P $(3\sec\theta, 2\tan\theta)$ is

$$\frac{3^2x}{3\sec\theta}+\frac{2^2y}{2\tan\theta}=3^2+2^2$$

$$\Rightarrow \boxed{3x\cos\theta+2y\cot\theta=3^2+2^2} \quad ...(1)$$

Similarly, Equation of normal to the hyperbola $\frac{x^2}{3^2}-\frac{y^2}{2^2}$ at point Q $(3\sec\phi, 2\tan\phi)$ is

$$\frac{3^2x}{3\sec\phi}+\frac{2^2y}{2\tan\phi}=3^2+2^2$$

$$\Rightarrow \boxed{3x\cos\phi+2y\cot\phi=3^2+2^2} \quad ...(2)$$

Given $\theta+\phi=\frac{\pi}{2}\Rightarrow\phi=\frac{\pi}{2}-\theta$ and these passes through (h, k)

$\therefore$ From eq. (2)

$$3x\cos\left(\frac{\pi}{2}-\theta\right)+2y\cot\left(\frac{\pi}{2}-\theta\right)=3^2+2^2$$

$$\Rightarrow \boxed{3h\sin\theta+2k\tan\theta=3^2+2^2} \quad ...(3)$$

and $\boxed{3h\cos\theta+2k\cot\theta=3^2+2^2}$ ...(4)

Comparing equation (3) & (4), we get

$$3h\cos\theta+2k\cot\theta = 3h\sin\theta+2k\tan\theta$$

$$3h\cos\theta-3h\sin\theta = 2k\tan\theta-2k\cot\theta$$

$$3h(\cos\theta-\sin\theta) = 2k(\tan\theta-\cot\theta)$$

$$3h(\cos\theta-\sin\theta) = 2k\frac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta\cos\theta}$$

$$\text{or, } 3h=\frac{-2k(\sin\theta+\cos\theta)}{\sin\theta\cos\theta} \quad ...(5)$$

Now, putting the value of equation (5) in eq. (3)

$$\frac{-2k(\sin\theta+\cos\theta)\cancel{\sin\theta}}{\cancel{\sin\theta}\cos\theta}+2k\tan\theta=3^2+2^2$$

$$\Rightarrow 2k\cancel{\tan\theta}-2k+2k\cancel{\tan\theta}=13$$

$$-2k=13\Rightarrow k=\frac{-13}{2}$$

Hence, ordinate of point of intersection of normals at P and Q is $\frac{-13}{2}$

216. **(a)** $x^2-6y=0$ ...(i)

$2x^2-4y^2=9$ ...(ii)

Consider the line,

$$x-y=\frac{3}{2} \quad ...(iii)$$

On solving (i) and (iii), we get only

$$x=3, y=\frac{3}{2}$$

Hence $\left(3,\frac{3}{2}\right)$ is the point of contact of conic (i), and line (iii)

On solving (ii) and (iii), we get only $x=3,\ y=\frac{3}{2}$

Hence $\left(3, \frac{3}{2}\right)$ is also the point of contact of conic (ii) and line (iii).

Hence line (iii) is the common tangent to both the given conics.

217. (d) Equation of the tangent at the point 'θ' is

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$$

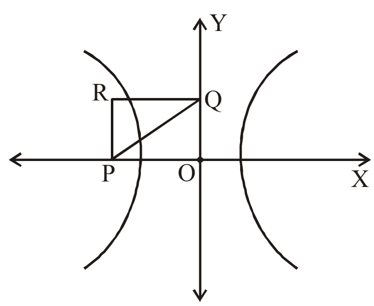
$\Rightarrow P = (a\cos\theta, 0)$ and $Q = (0, -b\cot\theta)$

Let R be (h, k) $\Rightarrow$ h = a cos θ, k = –b cot θ

$$\Rightarrow \frac{k}{h} = \frac{-b}{a\sin\theta} \Rightarrow \sin\theta = \frac{-bh}{ak} \text{ and } \cos\theta = \frac{h}{a}$$

By squaring and adding,

$$\frac{b^2h^2}{a^2k^2} + \frac{h^2}{a^2} = 1$$



$$\Rightarrow \frac{b^2}{k^2} + 1 = \frac{a^2}{h^2} \quad \Rightarrow \quad \frac{a^2}{h^2} - \frac{b^2}{k^2} = 1$$

Now, given eq^n of hyperbola is $\frac{x^2}{4} - \frac{y^2}{2} = 1$

$\Rightarrow a^2 = 4, b^2 = 2$

$\therefore$ R lies on $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$ i.e., $\frac{4}{x^2} - \frac{2}{y^2} = 1$

218. (c) Given equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$$

eccentricity $= e = \sqrt{1 - \frac{b^2}{16}}$

foci: $\pm ae = \pm 4\sqrt{1 - \frac{b^2}{16}}$

Equation of hyperbola is $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$$\Rightarrow \frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$$

$$\text{eccentricity} = e = \sqrt{1 + \frac{81}{25} \times \frac{25}{144}} = \sqrt{1 + \frac{81}{144}}$$

$$= \sqrt{\frac{225}{144}} = \frac{15}{12}$$

foci: $\pm ae = \pm\frac{12}{5} \times \frac{15}{12} = \pm 3$

Since, foci of ellipse and hyperbola coincide

$$\therefore \pm 4\sqrt{1 - \frac{b^2}{16}} = \pm 3 \Rightarrow b^2 = 7$$

$$\frac{K^2}{9} - \frac{4}{4} = 1 \qquad (\because b = \pm 2)$$

$\Rightarrow K^2 = 18$

219. (a) Given hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{b^2} = 1$$

Since this passes through (K, 2), therefore

$$\frac{K^2}{9} - \frac{4}{b^2} = 1 \qquad \text{...(1)}$$

Also, given $e = \sqrt{1 + \frac{b^2}{a^2}} = \frac{\sqrt{13}}{3}$

$$\Rightarrow \sqrt{1 + \frac{b^2}{9}} = \frac{\sqrt{13}}{3} \Rightarrow 9 + b^2 = 13$$

$\Rightarrow b = \pm 2$

Now, from eq^n (1), we have

$$\frac{K^2}{9}-\frac{4}{4}=1 \qquad (\because b=\pm 2)$$

$\Rightarrow K^2=18$

220. (b) Given that $ae=2$ and $e=2$

$\therefore \quad a=1$

We know, $b^2=a^2\left(e^2-1\right)$

$b^2=1(4-1)$

$b^2=3$

$\therefore$ Equation of hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow \frac{x^2}{1}-\frac{y^2}{3}=1$

$3x^2-y^2=3$

221. (b) Given, equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$$

Compare with equation of hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, we get $a^2=\cos^2\alpha$ and

$b^2=\sin^2\alpha$

We know that, $b^2=a^2(e^2-1)$

$\Rightarrow \sin^2\alpha=\cos^2\alpha(e^2-1)$

$\Rightarrow \sin^2\alpha+\cos^2\alpha=\cos^2\alpha.e^2$

$\Rightarrow e^2=\sec^2\alpha$

$\Rightarrow e=\sec\alpha$

$\therefore \quad ae=\cos\alpha\,.\,\frac{1}{\cos\alpha}=1$

Co-ordinates of foci are $(\pm ae,0)$

i.e. $(\pm 1,0)$

Hence, abscissae of foci remain constant when α varies.

222. (d) We know that tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$y=mx \pm \sqrt{a^2m^2-b^2}$

Given that $y=\alpha x+\beta$ is the tangent of hyperbola.

$\Rightarrow m=\alpha$ and $a^2m^2-b^2=\beta^2$

$\therefore \quad a^2\alpha^2-b^2=\beta^2$

Locus is $a^2x^2-y^2=b^2$ which is hyperbola.

223. (d) $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$

$a=\sqrt{\frac{144}{25}}=\frac{12}{5}, b=\sqrt{\frac{81}{25}}=\frac{9}{5},$

$e=\sqrt{1+\frac{81}{144}}=\frac{15}{12}=\frac{5}{4}$

$\therefore$ Foci $=(\pm ae,0)=(\pm 3,0)$

$\therefore$ foci of ellipse = foci of hyperbola

$\therefore$ for ellipse $ae=3$ but $a=4$,

$\therefore \quad e=\frac{3}{4}$

Then, $b^2=a^2(1-e^2)$

$\Rightarrow b^2=16\left(1-\frac{9}{16}\right)=7$