Chapter 3

Beams

CHAPTER HIGHLIGHTS

- *Introduction*
- *Types of sections*
- *Classification of cross-sections*
- *Bending (flexural) strength*
- *Shear strength of laterally supported beam*
- *Web crippling*
- *Web buckling*
- *Built-up beams (plated beams)*

Introduction

A beam is a structural member which is subjected to transverse loads, i.e., loads perpendicular to its longitudinal axis. A beam may be classified as a joists (a small beam to support roofs in a building), girder (a number of joists are supported by a large beam), spandrels (exterior beams at floor level to carry part of floor load and the exterior wall), purlins (beams which carry roof loads in trusses) and lintels (beams which support the loads from masonry over the openings).

Types of Sections

- Usually, the most efficient and economical shapes are I-shapes, either rolled steel or built up.
- Angles and T-sections are weak in bending, while channels can only be used for light loads.
- When laterally unsupported length of girders is quite large, closed sections, like square or rectangular hollow sections are most efficient due to high torsional stiffness.
- A built up I-section is preferred for heavy loads in case of bridges.
- Double angles, T-sections are generally used for lintels.
- Selection of beam section is based upon section modulus.

Classification of Cross-sections

Based on yield and plastic moments and rotational capacities, the following are the different classes of cross-sections.

Plastic Section

- Used in intermediate frames.
- Exhibits sufficient ductility ($\theta_2 > 6\ \theta_1$, where θ_1 is the rotation at the onset of plasticity, and θ_2 is the lower limit of rotation for treatment as a plastic section).
- Can fully develop plastic hinges, and failure of structure by formation of a plastic mechanism.
- Stress distribution for these sections is rectangular.

Compact Section

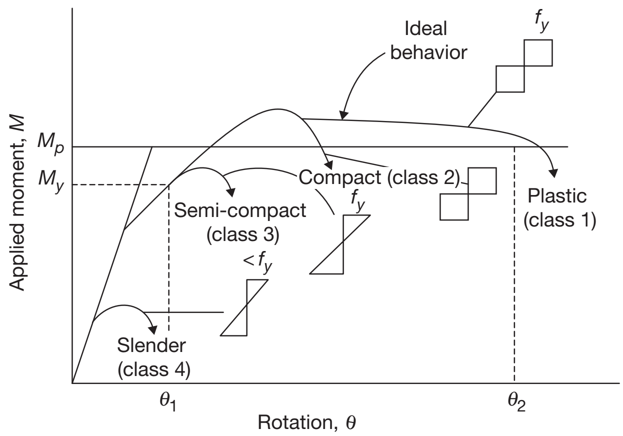
- It is suitable for simply supported beams which fail after reaching plastic moment at one section.
- Have comparatively lower deformation capacity compared to plastic section, but capable of reaching their full plastic moment values.

Semi-compact Section

- It can be used for elastic design, where the section fails by reaching the yield moment, M_y, at the extreme fibers of beam.

Slender Section

- Used in cold-formed members.
- Cross-sections in which local or lateral buckling occurs in elastic range.



Plastic: $M_d = Z_p f_y$
Compact: $M_d = Z_p f_y$
Semi-compact: $M_d = Z_e f_y$
Slender: $M_d = Z_{ee} f_y$
Z_p = Plastic section modulus
Z_e = Elastic section modulus
Z_{ee} = Effective elastic section modulus

Bending (Flexural) Strength

- Yield stress is governing criteria in design bending strength of a beam for laterally supported beam.
- Lateral torsional buckling controls the design in case of a laterally unsupported beam.

Laterally Supported Beams

- If $\frac{d}{t_w} \leq 67\varepsilon$, shear buckling does not occurs, and factored design moment M at any of its section due to external action should be less than or equal to the design bending strength M_d of the section.
- If $\frac{d}{t_w} > 67\varepsilon$, shear buckling occurs in web before yielding.
- The design bending strength determination is based on the following two cases:

Case 1: $V \leq 0.6\ V_d$ (Low shear case)

Where

V_d = Design shear strength $= \frac{V_n}{\gamma_{mo}}$

V_n = Nominal shear resistance = Plastic shear strength of a beam (V_P).

- In this case, the bending strength of beam is not influenced by shear force.
- Design bending strength is given by,

$$M_d = \beta_b Z_P \frac{f_y}{\gamma_{mo}}$$

$$\leq 1.2 Z_e \frac{f_y}{\gamma_{mo}} \text{ (for simply supported beams)}$$

$$\leq 1.5 Z_e \frac{f_y}{\gamma_{mo}} \text{ (for cantilever beams)}$$

Where

$\beta_b = 1.0$ (for plastic and compact sections)

$= \frac{Z_e}{Z_p}$ (for semi-compact sections)

Z_e, Z_p = Elastic and plastic section moduli of the cross-section

f_y = Yield stress of the material.

$\gamma_{mo} = 1.1$, the partial safety factor.

Slender Sections

$$\boxed{M_d = Z_{ee} f_y}$$

Where, Z_{ee} = Effective elastic section modulus.

NOTES

1. For most of I-sections and channel section, the ratio $\frac{Z_{pz}}{Z_{ez}}$ is less than 1.2 and the plastic moment capacity governs the design.
2. For sections having $\frac{Z_{pz}}{Z_{ez}} > 1.2$, the constant 1.2 may be replaced by the ratio of factored load/service load, i.e., by γ_f.

Case 2: $V > 0.6\ V_d$ (High shear case)

- Due to both bending and shear, the moment capacity of section is reduced (denoted by M_{dv}).
- Design bending strength, $M_d = M_{dv}$.

Plastic or Compact Section

- If the applied moment is uniform over the length and if design shear force is high, the following equation is used:

$$\boxed{\begin{array}{c} M_{dv} = M_d - \beta(M_d - M_{fd}) \\ \leq 1.2\ Z_e f_y / \gamma_{mo} \end{array}}$$

Where

M_d = Plastic design moment of the whole section disregarding high shear force effect, but considering web buckling effect.

M_{fd} = Plastic design strength of the area of the cross-section excluding the shear area, considering partial safety factor, γ_{mo}.

$$\beta = \left(2\frac{V}{V_d} - 1\right)^2$$

V_d = Design shear strength as governed by web yielding or web buckling.

V = Factored applied shear force.

Z_e = Elastic section modulus of the whole section.

Semi-compact Section

Moment capacity is given by:

$$M_d = Z_e f_y / \gamma_{mo}$$

Laterally Unsupported Beams

The bending strength of laterally supported beam is given by:

$$M_d = \beta_b Z_p f_{bd}$$

Where

β_b = 1.0 (for plastic and compact sections)

= Z_e/Z_p (for semi-compact sections)

Where

Z_e = Elastic section modulus

Z_p = Plastic section modulus

f_{bd} = Design bending compressive stress

$$= x_{LT} \frac{f_y}{\gamma_{mo}}$$

γ_{mo} = Partial safety factor for material = 1.10

x_{LT} = Bending stress reduction factor to account for lateral torsional buckling

$$x_{LT} = \frac{1}{\phi_{LT} + \left(\phi_{LT}^2 - \lambda_{LT}^2\right)^{0.5}} \le 1.0$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2\right]$$

α_{LT} = Imperfection factor

= 0.21 (for rolled section)

= 0.49 (for welded section)

λ_{LT} = Non-dimensional slenderness ratio

$$= \sqrt{\beta_b Z_p \frac{f_y}{M_{cr}}} \le \sqrt{1.2 Z_e \frac{f_y}{M_{cr}}}$$

$$= \sqrt{\frac{f_y}{f_{cr,b}}}$$

$$M_{cr} = \sqrt{\frac{\pi^2 EI_y}{(L_{LT})^2}\left[GI_t + \frac{\pi^2 EI_w}{(L_{LT})^2}\right]} = \beta_b Z_p f_{cr,b}$$

Where

M_{cr} = Elastic critical moment corresponding to lateral-torsional buckling of beam.

I_y = Moment of inertia about minor axis.

I_w = Warping constant = $(1 - \beta_f)\, \beta_f I_y h_f^2$

I_y = St. Venant's constant

$$= \sum \frac{b_i t_i^3}{3} = 2\frac{b_f f_f^3}{3} + \frac{b_f t_w^3}{3}, \text{for open sections}$$

G = Shear modulus

Shear Strength of Laterally Supported Beam

Nominal shear strength of section:

$$v_n = \frac{A_v f_{yw}}{\sqrt{3}}$$

Design shear strength of section:

$$v_d = \frac{V_n}{\gamma_{mo}}$$

$$v_d = \frac{A_v f_{yw}}{\sqrt{3}\gamma_{mo}}$$

Where

f_{yw} = Yield strength of web

A_v = Shear area

γ_{mo} = Partial safety factor against shear = 1.1

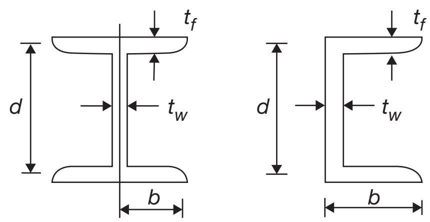
Shear Area (A_v)

1. For **channel** and **I-section:**
   - Major axis bending
   - Hot rolled: $A_v = ht_w$
   - Welded: $A_v = dt_w$

   Minor axis bending

   Hot rolled or welded: $A_v = 2bt_f$

2. For **rectangular hollow section** of uniform thickness:
   - Load parallel to depth (h): $A_v = \dfrac{Ah}{(b+h)}$
   - Load parallel to width (b): $A_v = \dfrac{Ab}{(b+h)}$



Rolled beams **Rolled channels**

Deflection Limit

- Vertical Deflection for simple span (in industrial building):

Supporting	Maximum Deflection
Elastic cladding	Span/240
Brittle cladding	Span/300

- Vertical deflection for cantilever span (in industrial building):

Supporting	Maximum Deflection
Elastic cladding	Span/120
Brittle cladding	Span/150

- Vertical deflection for purlins and girts (in industrial building):

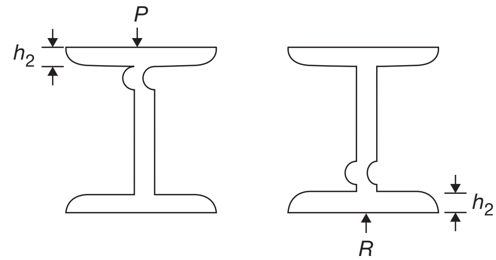
Supporting	Maximum Deflection
Elastic cladding	Span/150
Brittle cladding	Span/180

NOTE

Deflections can be reduced by increasing the depth of the beam section, reducing the span and providing greater end restraints.

Web Crippling

- It is a type of local buckling phenomenon which occurs due to stress concentration at the junction of web and flange. Due to this, the web near the portion of stress concentration tends to fold over the flange called 'crippling' or 'crimpling of web'.



Web failure by local crippling

- Below concentrated loads and above the reactions from support, the webs may be subjected to large amount of stresses.
- To keep bearing stress within permissible limit, the concentrated load should be transferred from flanges to web on large bearing areas.
- Bearing strength is given by:

$$F_w = A_e \frac{f_{yw}}{\gamma_{mo}}$$

Where

f_{yw} = The design strength of web.

A_e = Effective area of web = $b_1 t_w$.

- The angle of dispersion of the load is assumed to be 1 : 2.5. Bearing length:

$$b_1 = b + 2n_1$$

(Under concentrated loads.)

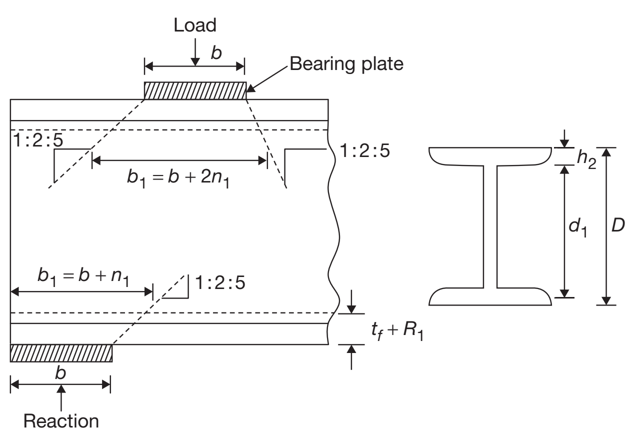
$$b_1 = b + n_1$$

(Under reactions at support.)

Where

$n_1 = 2.5(t_f + R_1)$

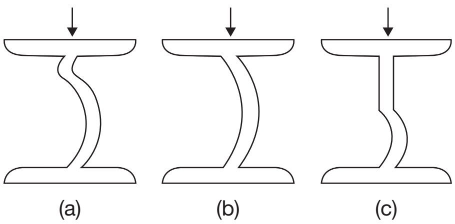
- If the bearing calculated is greater than the concentrated load, then it is safe under crippling.



Bearing length

Web Buckling

- Web buckling occurs when the intensity of vertical compressive stress near the centre of section becomes greater than the critical buckling stress for the web acting as a column.
- In rolled beam sections, vertical buckling is not a problem.
- Web buckling occurs in deep plate girders having thin webs.



Buckling failure due to vertical loading

- Web buckling strength:
  At support:

$$F_{wb} = B_1 t_w f_{cd}$$

Below concentrated load

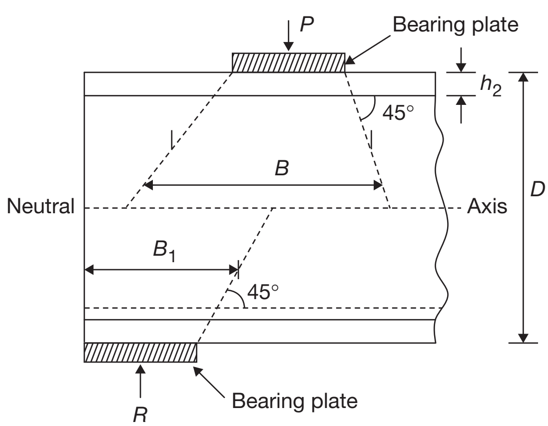
$$F_{wb} = B_1 t_w f_{cd}$$

Where

f_{cd} = Allowable compressive stress corresponding to the assumed web strut according to buckling 'c'.

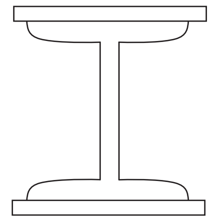
B = Length of stiff portion plus additional length given by dispersion at 45° to the level of neutral axis.

The following figure shows the bearing length for buckling consideration.



Built-up Beams (Plated Beams)

- I rolled beam sections do not have sufficient strength to resist the external bending moment, built up beams are used.
- In case of restriction of depth due to head room requirement, built-up beams are used.



Built-up beam

Design of Cover Plates

Let $I_{P,\text{req}}$ = Plastic moment of inertia of the section to resist the total bending moment.

I_{zz} = Moment of inertia of desirable beam section available.

I_a = Additional moment of inertia of the section required from plate sections.

- By using the following equation, the moment of inertia of plates can be calculated.

$$I_a = I_{p,\text{req}} - I$$

- Let us assume that the centre of gravity of the compression hanger lies at the junction of flange plate and cover plate. The above equation can be rewritten as:

$$\frac{I_a}{h/2} = \frac{I_{p,\text{req}}}{h/2} - \frac{I}{h/2}$$

Or, $Z_a = Z_{p,\text{req}} - Z_p$

Or, $Z_a = A_a h$

Where

h = The overall depth of the I-section.

Z_p = Plastic section modulus of I-section $= \dfrac{M}{f_y}$.

M = Factored moment

Z_a = Plastic section modulus of plates required

$A_a = Bt_{fp}$

Where, B is width of flange plate and t_{fp} is the thickness of the flange cover plate.

EXERCISES

1. A steel supporting loads from the floors slab as well as from wall is termed as
(A) stringer beam.
(B) lintel beam.
(C) spandrel beam.
(D) header beam.

2. When designing steel structures, one must ensure that local buckling in webs does not take place. This check may not be very critical when using rolled steel sections because
(A) quality control at the time of manufacture of rolled sections is very good.
(B) web depths available are small.
(C) web stiffeners are in-built in rolled sections.
(D) depth to thickness ratios (of the web) are appropriately adjusted.

3. An ISMB 500 is used as a beam in a multi-storey construction. From the view point of structural design, it can be considered to be 'laterally restrained' when,
(A) the tension flange is 'laterally restrained'.
(B) the compression flange is 'laterally restrained'.
(C) the web is adequately stiffened.
(D) the conditions in (A) and (C) are met.

4. Generally the maximum deflection/span ratio of a steel member should not exceed

(A) $\frac{1}{750}$ (B) $\frac{1}{500}$

(C) $\frac{1}{325}$ (D) $\frac{1}{250}$

5. Which one of the following factors does not affect the lateral buckling strength of a steel I-section undergoing bending about its major axis?
(A) Boundary conditions at the ends.
(B) Radius of gyration about the minor axis of the section.
(C) Radius of gyration about the major axis of the section.
(D) Laterally unsupported length of the compression flange.

6. Match List I (Type of connection) with List II (Type of beams) and select the correct answer using the code given below the lists:

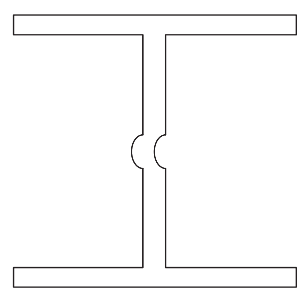
	List I		List II
a.	Semi-rigid connection	**1.**	To permit large angles of rotation and to transmit negligible moment.
b.	Framed connection	**2.**	To allow small end rotation and transmit appreciable moment.
c.	Flexible connection	**3.**	When a beam is connected to a beam or stanchion by means of an angle at the bottom of the beam which is shop-riveted to the beam and an angle at the top of which is field riveted.
d.	Seated connection	**4.**	When a beam is connected to a beam or stanchion by means of two angles riveted to them.

Codes:

	a	b	c	d
(A)	2	4	3	1
(B)	4	2	1	3
(C)	2	4	1	3
(D)	4	2	3	1

7. Which one of the following pairs is correctly matched?
(A) Truss : Bending
(B) Beam : Twisting
(C) Column : Buckling
(D) Shaft : Shortening

8. What is the failure of a section shown in the following figure called?



I. Web buckling
II. Web crippling
III. Web crimpling
IV. Column buckling
Select the correct answer using the codes given below:
(A) I and II (B) II and III
(C) I and IV (D) II and IV

9. Consider the following statements:
Web crippling due to excessive bearing stress can be avoided by
I. increasing the web thickness.
II. providing suitable stiffeners.
III. increasing the length of the bearing plates.
Which of these statements are correct?
(A) I and II only
(B) II and III only
(C) I and III only
(D) I, II and III

10. The problems of lateral buckling can arise only in those steel beams which have
(A) moment of inertia about the bending axis larger than the other.
(B) moment of inertia about the bending axis smaller than the other.
(C) fully supported compression flange.
(D) None of these

11. If bolts in any bolt group is subjected to shear and tension; the interaction equation need to be satisfy as per IS:800–2007 is [V_b, V_{db}: Factored shear force and design shear strength and T_b, T_{db}: Factored tensile force and design tensile strength]
(A) $\left[\frac{V_b}{V_{db}}\right]+\left[\frac{T_b}{T_{db}}\right]\leq 2.0$
(B) $\left[\frac{V_b}{V_{db}}\right]^2+\left[\frac{T_b}{T_{db}}\right]^2\leq 2.0$
(C) $\left[\frac{V_b}{V_{db}}\right]^2+\left[\frac{T_b}{T_{db}}\right]^2\leq 1.0$
(D) None of these

12. A beam section is selected and provided on the basis of
(A) shear
(B) deflection
(C) section modulus
(D) All of these

13. A steel beam of circular c/s is clamped at both ends. Deformation is just observed when the UDL on the beam is 20 kN-m. At the instant of collapse, the load on the beam will be
(A) 15 kN-m (B) 30 kN-m
(C) 20 kN-m (D) 45 kN-m

14. For an rectangular beam the shape factor is 1.5. The factor of safety in bending is 1.5. If the allowable stress is increased by 15% for wind and earth quake loads, then the load factor is
(A) 1.95 (B) 1.40
(C) 1.65 (D) 1.80

15. A beam section is classed as high shear case when the factored shear force is greater than (where V_d = design shear strength)
(A) $0.6V_d$ (B) $0.5V_d$
(C) V_d (D) $0.4V_d$

16. The design bending strength of laterally supported-beam with plastic section and having plastic section modulus to be 850 × 10^3 mm^3 is _____. (Assume simply supported beam with low shear case and f_y as 250 MPa and Z_e to be 750 × 10^3 mm^3).
(A) 195 kN-m (B) 255 kN-m
(C) 105 kN-m (D) 55 kN-m

17. A 8 m simply supported floor beam of an industrial building supported by a concrete floor load. As per IS:800–2007, the maximum deflection in mm is _____. (assume brittle clading supporting).
(A) 26.67 (B) 32
(C) 13.33 (D) 15

18. A simply supported beam of span 'L' supports a concentrated load 'P' at its mid span. If the cross-section of the beam is an rectangular section, then the length of elastic-plastic zone of the plastic hinge will be
(A) $\frac{L}{3}$ (B) $\frac{2L}{3}$
(C) $\frac{L}{2}$ (D) $\frac{L}{4}$

Previous Years' Questions

1. A steel section is subjected to a combination of shear and bending actions. The applied shear force is V and the shear capacity of the section is V_s. For such a section, high shear force (as per IS:800–2007) is defined as **[GATE, 2014]**
(A) $V > 0.6V_s$
(B) $V > 0.7V_s$
(C) $V > 0.8V_s$
(D) $V > 0.9V_s$

Answer Keys

Exercises

1. A	**2.** D	**3.** B	**4.** C	**5.** C	**6.** C	**7.** C	**8.** B	**9.** A	**10.** A
11. C	**12.** D	**13.** D	**14.** A	**15.** A	**16.** A	**17.** A	**18.** A		

Previous Years' Questions

1. A