CHAPTER 15

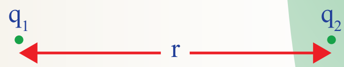
Electrostatics

ELECTRIC CHARGE

Charge of a meterial body is that property due to which it interacts with other charges. There are two kinds of charges- positive and negative. SI unit is coulomb. Charge is quantized and additive.

Coulomb's law:

Force between two charges $\vec{F} = \dfrac{1}{4\pi \in_0 \in_r} \dfrac{q_1 q_2}{r^2} \hat{r}$ $\in_r$ = dielectric constant



NOTES

The Law is applicable only for static point charges.

Principle of Superposition

Force on a point charge due to many charges is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

NOTES

The force due to one charge is not affected by the presence of other charges.

Electric Field or Electric Field Intensity (Vector Quantity)

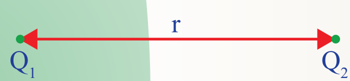
In the surrounding region of a charge there exist a physical property due to which other charges experience a force. The direction of electric field is direction of force experienced by a positively charged particle and magnitude of the field (electric field intenity) is the force experienced by a unit charge.

$$\vec{E} = \frac{\vec{F}}{q} \text{ unit is N/C or V/m.}$$

Electric field due to charge Q

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2} \hat{r}$$

Null point for two charges :



If $|Q_1| > |Q_2|$

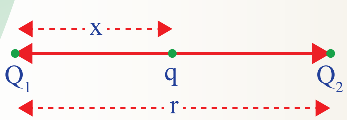
$\Rightarrow$ Null point near Q_2

$$x = \frac{\sqrt{Q_1}\, r}{\sqrt{Q_1} \pm \sqrt{Q_2}}; \; x \to \text{distance of null point from } Q_1 \text{ charge}$$

(+) for like charges

(–) for unlike charges

Equilibrium of three point charges



(i) Two charges must be of like nature.

(ii) Third charge should be of unlike nature.

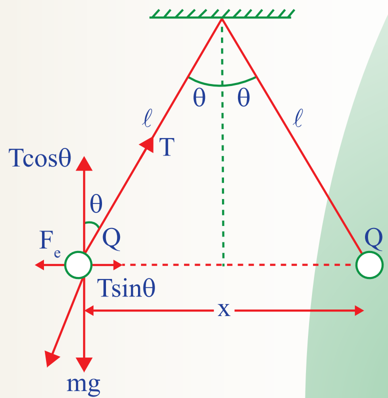
$$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r \quad \text{and} \quad q = \frac{-Q_1 Q_2}{\left(\sqrt{Q_1} + \sqrt{Q_2}\right)^2}$$

Equilibrium of suspended point charge system

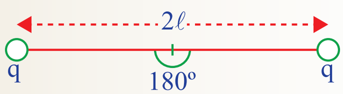
For equilibrium position

$$T\cos\theta = mg \ \& \ T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

$$T = \sqrt{(F_e)^2 + (mg)^2}$$



If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$)



$$T = F_e = \frac{kq^2}{4\ell^2}$$

Electric potential difference $\Delta V = \frac{\text{work}}{\text{charge}} = W/q$

Electric potential $V_p = -\int_{\infty}^{P} \vec{E}.d\vec{r}$

It is the work done against the field to take a unit positive charge from infinity (reference point) to the given point

- For point charge : $V = K\frac{q}{r}$
- For several point charges : $V = K\sum \frac{q_1}{r_1}$

Relation between $\vec{E}$ & V

$$\vec{E} = -\text{grad}\, V = -\nabla V, E = \frac{-\partial V}{\partial r}; \quad \vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}, V = \int -\vec{E}.d\vec{r}$$

Electric potential energy of two charges : $U = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r}$

Electric dipole

- Electric dipole moment $p = qd$
- Torque on dipole placed in uniform electric field $\vec{\tau} = \vec{p} \times \vec{E}$
- Work done in rotating dipole placed in uniform electric field

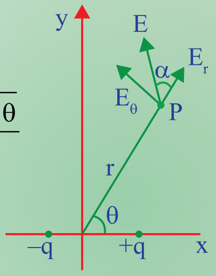
$$W = \int \tau d\theta = \int_{\theta_0}^{\theta} PE \sin\theta d\theta = pE(\cos\theta_0 - \cos\theta)$$

- Potential energy of dipole placed in an uniform field $U = -\vec{p}.\vec{E}$
- At a point which is at a distance r from dipole midpoint and making angle θ with dipole axis.

Potential $V = \frac{1}{4\pi \epsilon_0} \frac{p\cos\theta}{r^2}$

Electric field $E = \frac{1}{4\pi \epsilon_0} \frac{p\sqrt{1+3\cos^2\theta}}{r^3}$

Direction $\tan\alpha = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$



- Electric field at axial point (or End-on) $\vec{E} = \frac{1}{4\pi \epsilon_0} \frac{2\vec{p}}{r^3}$ of dipole
- Electric field at equatorial position (Broad-on) of dipole $\vec{E} = \frac{1}{4\pi \epsilon_0} \frac{(-\vec{p})}{r^3}$

Equipotential Surface and Equipotential Region

In an electric field the locus of points of equal potential is called an equipotential surface. An equipotential surface and the electric field line meet at right angles. The region where E = 0, Potential of the whole region must remain constant as no work is done in displacement of charge in it. It is called as equipotential region like conducting bodies.

Electric flux : $\phi = \int \vec{E}.\vec{ds}$

Gauss's Law : $\oint \vec{E}.\vec{ds} = \frac{\sum q}{\epsilon_0}$ (Applicable only on closed surface)

Net flux emerging out of a closed surface is $\frac{q_{en}}{\varepsilon_0}$

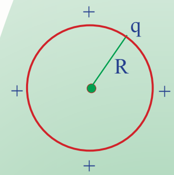
$$\phi = \oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$ where q_{en} = net charge enclosed by the closed surface.

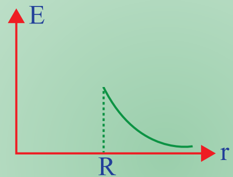
ϕ does not depend on the

(i) Shape and size of the closed surface

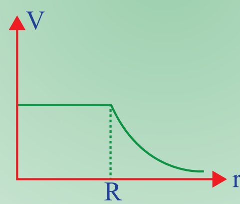
(ii) The charges located outside the closed surface.

For a conducting sphere



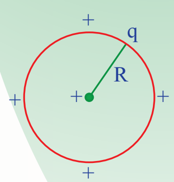


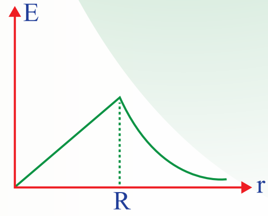
For $r \geq R : E = \frac{1}{4\pi \in_0} \frac{q}{r^2}, V = \frac{1}{4\pi \in_0} \frac{q}{r}$



For $r < R : E = 0, \; V = \frac{1}{4\pi \in_0} \frac{q}{R}$

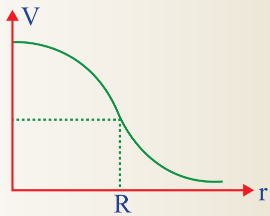
For a non-conducting sphere





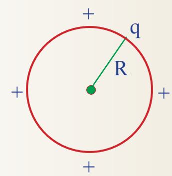
For $r \geq R : E = \frac{1}{4\pi \in_0} \frac{q}{r^2}, V = \frac{1}{4\pi \in_0} \frac{q}{r}$

$$\text{For } r < R : E = \frac{1}{4\pi \epsilon_0}\frac{qr}{R^3},\ V = \frac{1}{4\pi \epsilon_0}\frac{q(3R^2 - r^2)}{2R^3}$$



$$V_c = V_{max} = \frac{3}{2}\frac{Kq}{R} = 1.5V_{surface}$$

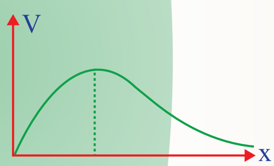
For a conducting/non conducting spherical shell

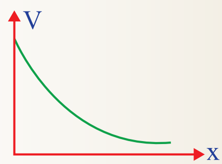


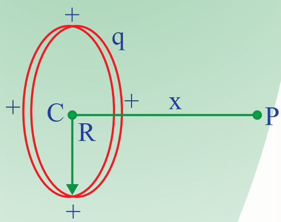
$$\text{For } r \geq R : E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2},\ V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}$$

$$\text{For } r < R : E = 0,\ V = \frac{1}{4\pi \epsilon_0}\frac{q}{R}$$

For a charged circular ring



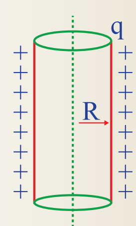




$$E_P = \frac{1}{4\pi \epsilon_0}\frac{qx}{(x^2 + R^2)^{3/2}},\ V_p = \frac{q}{4\pi \epsilon_0 (x^2 + R^2)^{1/2}}$$

Electric field will be maximum at $x = \pm\frac{R}{\sqrt{2}}$

For a charged long conducting cylinder



- For $r \geq R : E = \frac{q}{2\pi \epsilon_0 r}$
- For $r < R : E = 0$

Electric field intensity at a point near a charged conductor $E = \dfrac{\sigma}{\epsilon_0}$

Mechanical pressure on a charged conductor $P = \dfrac{\sigma^2}{2\epsilon_0}$

For non-conducting infinite sheet of surface charged density $E = \dfrac{\sigma}{2\epsilon_0}$

For conducting infinite sheet of surface charge density $E = \dfrac{\sigma}{\epsilon_0}$

Energy density in electric field $U = \dfrac{\epsilon_0}{2}E^2$

Electric lines of force

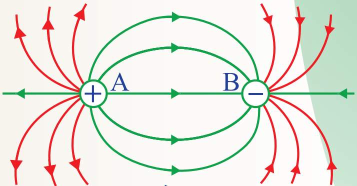
Electric lines of electrostatic field have following properties.

(i) Imaginary

(ii) Can never cross each other

(iii) Can never be closed loops

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge.



$q_A > q_B$

(v) Lines of force ends or starts normally at the surface of a conductor.

(vi) If there is no electric field there will be no lines of force.

(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field.

(viii) Tangent to the line of force at a point in an electric field gives the direction of Electric Field.

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