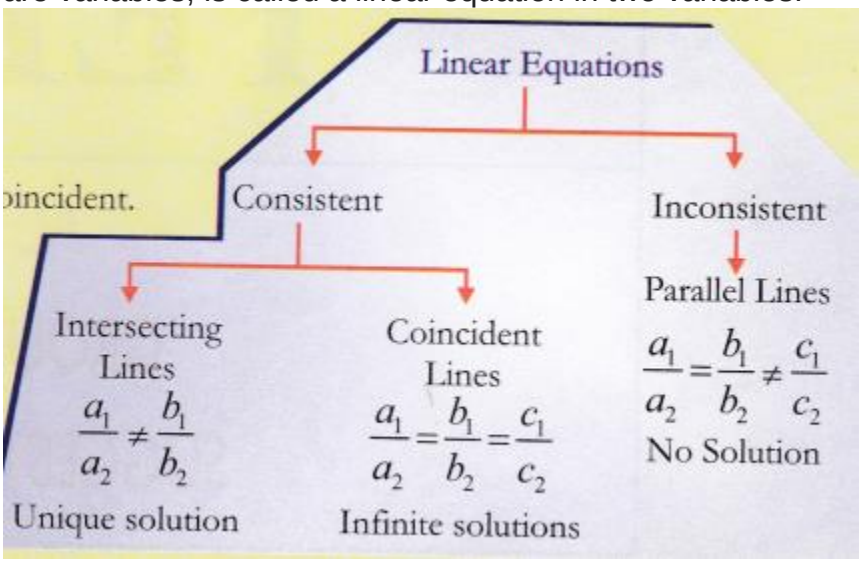
Linear Equations

Objective

To verify the conditions for consistency of a system of linear equations in two variables by graphical representation.

Linear Equation

An equation of the form ax+by+c = 0, where a, b, c are real numbers, a≠0, b≠0 and x, y are variables; is called a linear equation in two variables.



Prerequisite Knowledge

1. Plotting of points on a graph paper.
2. Condition of consistency of lines parallel, intersecting, coincident,

Materials Required

Graph papers, fevicol, geometry box, cardboard.

Procedure

Consider the three pairs of linear equations
1stpair: 2x-5y+4=0, 2x+y-8 = 0
2nd pair: 4x + 6y = 24, 2x + 3y =6
3rd pair: x-2y=5, 3x-6y=15

1. Take the 1st pair of linear equations in two variables, e.g., 2x – 5y +4=0, 2x +y-8 = 0.

2. Obtain a table of at least three such pairs (x, y) which satisfy the given equations.

For $2x-5y+4=0$

x	-2	0.5	3
y	0	1	2

For $2x+y-8=0$

x	2	3	4
y	4	2	0

3. Plot the points of two equations on the graph paper as shown in fig. (i).

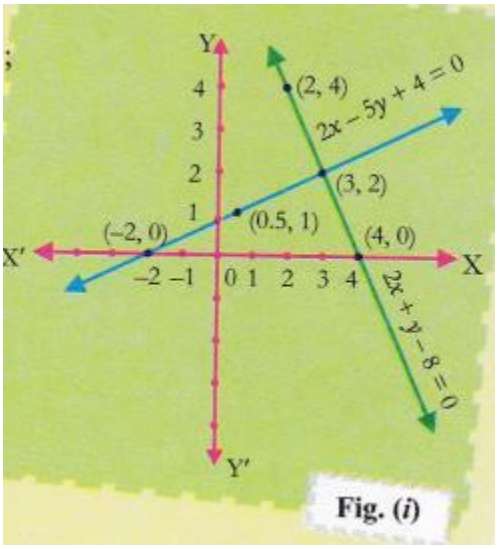


Fig. (*i*)

4. Observe whether the lines are intersecting, parallel or coincident. Write the values in observation table.
   Also, check ; $\frac{a_1}{a_2}$; $\frac{b_1}{b_2}$; $\frac{c_1}{c_2}$

5. Take the second pair of linear equations in two variables

For $4x+6y=24$

x	0	6	3
y	4	0	2

For $2x+3y=6$

x	0	3	1.5
y	2	0	1

6. Repeat the steps 3 and 4.

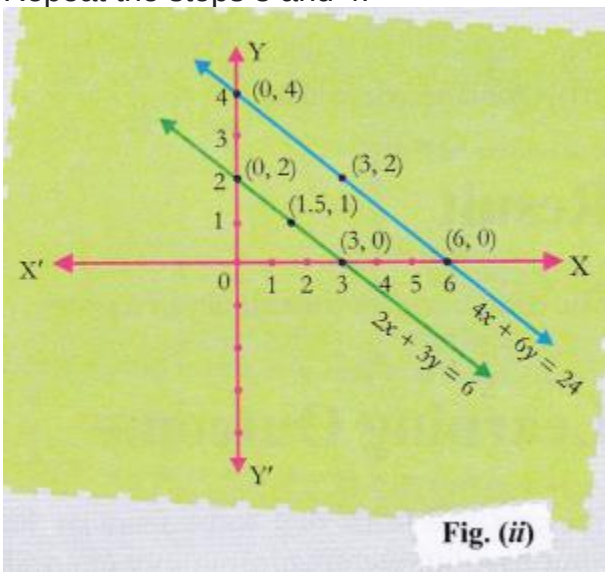


Fig. (*ii*)

7. Take the third pair of linear equations in two variables,i.e. x-2y=5, 3x-6y=15

For $x-2y=5$

x	1	5	0
y	–2	0	–2.5

For $3x-6y=15$

x	0	1	5
y	–2.5	–2	0

8. Repeat steps 3 and 4

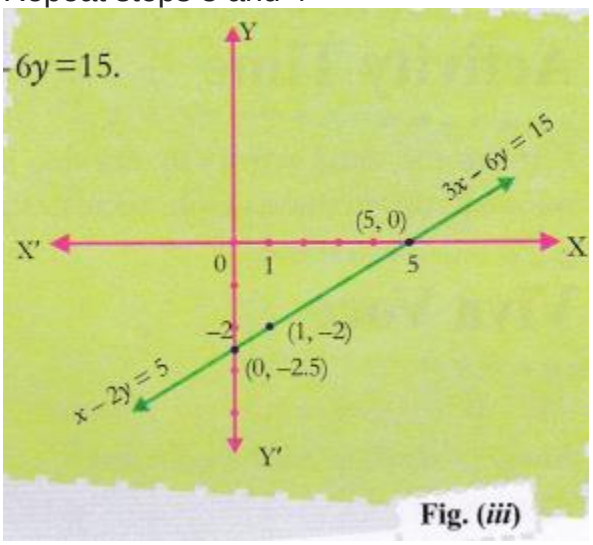


Fig. (*iii*)

Obtain the condition for two lines to be intersecting, parallel or coincident from the observation table by

comparing the values of $\frac{a_1}{a_2}, \frac{b_1}{b_2} and \frac{c_1}{c_2}$

Observation Table					
Pairs of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios and write conditions	Types of lines
1st pair					
2nd pair					
3rd pair					

Observation

Students will observe that

1. for intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
2. for parallel lines,
3. for coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Result

The conditions for consistency of a system of linear equations in two variables is verified.

Learning Outcome

Students will learn that some pairs of linear equations in two variables have a unique solution (intersecting lines), some have infinitely many solutions (coincident lines) and some have no solutions (parallel lines).

Activity Time

Perform the same activity by drawing graphs of x-y+1=0 and 3x + 2y – 12 =0. Show that there is a unique solution. Also from the graph, calculate the area bounded by these linear equations and x-axis.

Viva Voce

Question 1.

What is the equation of a line parallel to x-axis ?

Answer:

y = a, where a is any constant.

Question 2.
What is the equation of a line parallel to y-axis ?
Answer:
x = b, where b is any constant.

Question 3.
If x = 0 and y = 0, where would the point lie on graph ?
Answer:
At origin (0,0)

Question 4.
What is the condition for inconsistent and consistent solution for the system of linear equations ?
Answer:
Linear equations are
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
Inconsistent solution,
Consistent solution,
(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Question 5.
Is the pair of linear equations 2x + 3y – 9 = 0 and 4x + 6y -18 = 0, consistent ?
Answer:
Here
⇒ Given system of equations is consistent and has infinitely many solutions.

Question 6.
For what value of p does the pair of linear equations given below has unique solution ?
4x + 8 = 0, 2x + 2y + 2 = 0
Answer:
For unique solution,

Question 7.
What does the graph of a linear equation represent ?
Answer:
A straight line

Question 8.
If the graphical solutions of two linear equations of two lines are parallel to each other in plane, then what type of solution do they have ?

Answer:
No solution

Question 9.
If the graphical solutions of two linear equations of two lines intersect in a plane, then what type of the solution do they have ?
Answer:
Unique solution

Multiple Choice Questions

Question 1.
Is x= -1, y=5 a solution of the equation 4x + 3y = 11 ?
(a) yes
(b) no
(c) can't say
(d) none of these

Question 2.
Equations 5x + 2y=16 and 7x-4y = 2 have:
(a) no solution
(b) a unique solution
(c) infinitely many solutions
(d) none of these

Question 3.
Equations —3x + 4y = 5 and $\frac{9}{2}x - 6y = \frac{15}{2}$
(a) a unique solution
(b) infinitely many solutions
(c) no solution
(d) none of these

Question 4.
Equations -3x + 4y = 5 and $\frac{9}{2}x - 6y + \frac{15}{2} = 0$ have:
(a) many solutions
(b) a unique solution
(c) no solution
(d) none of these

Question 5.
Condition for the system of linear equations ax + by = c; lx + my = n to have a unique solution is:
(a) am ≠ bl

(b) am = bl
(c) $\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$
(d) none of these

Question 6.
When l_1 and l_2 are parallel lines, then the graphical solution of system of linear equations has
(a) many solutions
(b) no solution
(c) a unique solution
(d) none of these

Question 7.
When lines l_1 and l_2 are coincident, then the graphical solution of system of linear equations has
(a) infinitely many solutions
(b) a unique solution
(c) no solution
(d) parallel lines

Question 8.
Values of x and y for the pair of linear equations x+y=14 and x-y = 4 are respectively
(a) 9 and 5
(b) 5 and 9
(c) 5 and 5
(d) 9 and 9

Question 9.
The difference between two numbers is 26 and one number is three times the other. The numbers are
(a) 39 and 12
(b) 39 and 13
(c) 38 and 13
(d) 13 and 13

Question 10.
In a cyclic quadrilateral BACD, ∠A = (2x – 4)°, ∠B = (y + 5)°, ∠C = (2y + 10)° and ∠D = (4x – 2)°. Find four angles.
(a) ∠A = 58°, ∠B = 60°, ∠C = 120°, ∠D = 122°
(b) ∠A = 65°, ∠B = 55°, ∠C = 115°, ∠D = 125°
(c) ∠A = 70°, ∠B = 110°, ∠C = 55°, ∠D = 125°
(d) ∠A = 65°, ∠B = 55°, ∠C = 110°, ∠D = 127°

Answers

1. (a)
2. (b)
3. (c)
4. (a)
5. (a)
6. (b)
7. (a)
8. (a)
9. (b)
10. (a)