PART # 04

CALCULUS

EXERCISE # 01

SECTION-1 : (ONE OPTION CORRECT TYPE)

651. $\int \frac{1-7\cos^2 x}{\sin^7 x \cos^2 x} dx = \frac{f(x)}{(\sin x)^7} + C,$ then f(x) is equal to

(A) sin x (B) cos x (C) tan x (D) cot x

652. Let f(x) = (x + 1) (x + 2) (x + 3) . . . (x + 100) and g(x) = f(x). $f''(x) - (f'(x))^2$, then g(x) = 0, has

(A) no solution (B) exactly one solution
(C) exactly two solutions (D) minimum three solutions

653. Let f(x) = min $(\ell n(\tan x), \ell n(\cot x))$, which of the following statement are incorrect

(A) f(x) is continuous for x ∈ $\left(0, \frac{\pi}{2}\right)$

(B) Lagrange's mean value theorem is applicable on f(x) for x ∈ $\left[\frac{\pi}{8}, \frac{\pi}{4}\right]$

(C) Rolle's theorem is not applicable on f(x) for x ∈ $\left[\frac{\pi}{4}, \frac{3\pi}{8}\right]$

(D) Rolle's theorem is applicable on f(x) for x ∈ $\left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$

654. Let 'n' be the number of elements in the Domain set of the function f(x) = $\left|\ln\sqrt{{}^{x^2+4x}C_{2x^2+3}}\right|$ and 'Y' be the global maximum value of f(x), then $[n + [Y]]$ is (where [·] = Greatest Integer function)

(A) 4 (B) 5 (C) 6 (D) 7

655. The value of the integral $\int_0^{\infty} e^{-2\theta}(\sin 2\theta + \cos 2\theta)\, d\theta$ is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) does not exist (D) none of these

656. The value of $\int_0^{\pi/3} [\sqrt{3}\tan x]\, dx$ (where [.] denotes the greatest integer function) is

(A) $\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (B) $\frac{5\pi}{6} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (C) $\frac{\pi}{6} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (D) none of these

657. The value of $\lim_{x\to\infty}\left(\frac{a_1^{1/x} + a_2^{1/x} + ... + a_n^{1/x}}{n}\right)^{nx}$; $a_i > 0$, i = 1, 2, …, n is

(A) $a_1 + a_2 + \ldots + a_n$ (B) $e^{a_1+a_2+...a_n}$ (C) $\frac{a_1 + a_2 + ...a_n}{n}$ (D) $a_1a_2a_3\ldots a_n$

658. The value of integral $\int \cot^{-1}\left(\frac{\sqrt{4x-x^2}}{x-2}\right)dx$ is equal to

(A) $\frac{x-2}{2}\sin^{-1}\frac{x-2}{2}+\sqrt{4x-x^2}+c$ (B) $(x-2)\sin^{-1}\left(\frac{x-2}{2}\right)+\sqrt{4x-x^2}+c$

(C) $\frac{x-2}{2}\sin^{-1}\left(\frac{x-2}{2}\right)-\sqrt{\frac{4x-x^2}{2}}+c$ (D) none of these

659. Let f: [– 2, 2] → R, where f(x) = $x^3+\sin x+\left[\frac{x^2+1}{a}\right]$ be an odd function, then

(A) a < 3 (B) a > 5 (C) a < 1 (D) a < – 2

660. $\lim_{n\to\infty}\left(\frac{3n+8}{3n+5}\right)^{5n+9}$ is equal to

(A) $3e^5$ (B) e^5 (C) e^3 (D) None of these

661. The differential equation of the orthogonal trajectories of the system of curves $x + \tan^{-1}(y/x) = c$, is

(A) $xdx + ydy = x^2 + y^2$ (B) $xdy + ydx = x^2 + y^2$

(C) $xdx + ydy = (x^2 + y^2)\, dy$ (D) $xdx + ydy = (x^2 - y^2)\, dy$

662. The area bounded by the curves $f(x) = x^2 - 2x + 2$ and its inverse i.e. f^{-1} is given by

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{5}{3}$ (D) $\frac{4}{3}$

663. The value of the series $\frac{C_0}{5}-\frac{C_1}{6}+\frac{C_2}{7}-\frac{C_3}{8}+...+(-1)\frac{{}^nC_n}{n+4}$

(A) $\int_0^1 x^2(1-x)^n\,dx$ (B) $\int_0^1 x^3(1-x)^n\,dx$ (C) $\int_0^1 x^4(1-x)^n$ dx (D) none of these

664. If $f(n) = \frac{1}{n}[(n + 1)(n + 2) \dots (n + n)]^{1/n}$ then $\lim_{n\to\infty} f(n)$ equals

(A) e (B) 1/e (C) 2/e (D) 4/e

665. If $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ where $y \neq 0$ for all x, y ∈ R and $f'(1) = 2$. then the function f(x) is symmetric about

(A) x-axis (B) y-axis (C) origin (D) y = x

666. Let f : R → R be such that f(1) = 3, f′(1) = 0 and f′′(1) = 6, then $\lim_{x\to 0}\left[\frac{f(1+x)}{f(1)}\right]^{1/x^2}$ equal to

(A) e (B) $e^{1/2}$ (C) e^2 (D) e^3

667. Let f(x) and g(x) be differentiable for $0 \le x \le 1$ such that f(0) = 2, g(0) = 0 f(1) = 6. Let there exists a real numbers x in [0, 1] such that f′(c) = 2g′(c) then the value of g(1) must be

(A) 1 (B) 2 (C) –1 (D) none of these

668. The area of the greatest circle inscribed in 2 |x| + 2 |y| = 4 is given by

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) π^2 (D) π

669. $\int_0^{\pi/2} \sin x \ \log(\sin x) dx$ is equal to

(A) $\log_e(e/2)$ (B) log2 – e (C) $\log_e((2/e)$ (D) loge – 2

670. f $I_n = \int_0^{\pi/4} \tan^n x dx$, then $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}, \frac{1}{I_5+I_7}$ are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

671. $\int \frac{\ln(1+x^{2/3}+2x^{1/3})}{x+x^{2/3}} dx$ is equal to

(A) $3 \ln(1 + x^{1/3})^2 + c$ (B) $\ln(1 + x^{1/3}) + c$
(C) $\ln(x^{1/3} - 1) + c$ (D) none of these

672. $\lim_{x\to 0}\left[\frac{e^x-1}{x}\right]$ is equal to [.] represents G.I. function

(A) 1 (B) 0
(C) does not exist (D) none of these

673. The differential equation of all ellipses centred at origin is :

(A) $y_2 + xy_1^2 - yy_1 = 0$ (B) $xyy_2 + xy_1^2 - yy_1 = 0$
(C) $yy_2 + xy_1^2 - xy_1 = 0$ (D) none of these

674. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represent

(A) straight lines (B) circles (C) parabolas (D) ellipses

675. If $y = e^{4x} + 2e^{-x}$ satisfies the relation $\frac{d^3y}{dx^3} + A\frac{dy}{dx} + By = 0$ then value of A and B respectively are:

(A) –13, 14 (B) –13, –12 (C) –13, 12 (D) 12, –13

676. A particle moves in a straight line with velocity given by $\frac{dx}{dt} = x + 1$ (x being the distance described). The time taken by the particle to describe 99 metres is :

(A) $\log_{10}e$ (B) $2 \log_e 10$ (C) $2 \log_{10} e$ (D) $\frac{1}{2}\log_{10} e$

677. The acute angle between the curve $y = |x^2 - 1|$ and $y = |x^2 - 3|$ at their point of intersection is

(A) $\frac{\pi}{4}$ (B) $\tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$ (C) $\tan^{-1}(4\sqrt{7})$ (D) None of these

678. The range of the function $y = \sqrt{2\{x\} - \{x\}^2 - \frac{3}{4}}$ is (where {.} denotes fractional part)

(A) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (B) $\left[0, \frac{1}{2}\right]$ (C) $\left[0, \frac{1}{4}\right]$ (D) $\left[\frac{1}{4}, \frac{1}{2}\right]$

679. If $f(x + y) = f(x) - f(y) + 2xy - 1 \ \forall x, y \in R$. Also if f(x) is differentiable and $f'(0) = b$ also $f(x) > 0 \ \forall x$, then the set of values of b

(A) ϕ (B) {1} (C) {1, 2} (D) none of these

680. $\lim_{k\to\infty}\int_0^{k[x]}(kx-[kx])^k\,dx$; k $\in$ N is equal to (where [.] denotes the greatest integer function)

(A) [kx] (B) [x] (C) $\left[\frac{x}{k}\right]$ (D) $[x^k]$

681. The equation of curve passing through (1, 1) in which the subtangent is always bisected at the origin is

(A) $y^2 = x$ (B) $2x^2 - y^2 = 1$ (C) $x^2 + y^2 = 2$ (D) $x + y = 2$

682. If f'(3) = 5 then $\lim_{h\to 0}\frac{f(3+h^2)-f(3-h^2)}{2h^2}$ is :

(A) 5 (B) 1/5

(C) 2 (D) None of these

683. If f is twice differentiable function then $\lim_{h\to 0}\frac{f(a+h)-2f(a)+f(a-h)}{h^2}$ is :

(A) 2f'(a) (B) f''(a)

(C) f'(a) (D) f'(a) + f''(a)

684. If f(x) = sinx, g(x) = x^2, h(x) = logx and F(x) = (hogof)(x) then F''(x) is :

(A) $2\text{cosec}^3 x$ (B) $2\cot x^2 - 4x^2\text{cosec}^2 x^2$

(C) $2x\cot x^2$ (D) $-2\text{cosec}^2 x$

685. If $x = \sec\theta - \cos\theta,\ y = \sec^n\theta - \cos^n\theta$ then $\left(\frac{dy}{dx}\right)^2$ is equal to :

(A) $\frac{n^2(y^2+4)}{x^2+4}$ (B) $\frac{n^2(y^2-4)}{x^2}$ (C) $n\frac{y^2-4}{x^2-4}$ (D) $\left(\frac{ny}{x}\right)^2-4$

686. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and f'(x) = sin x^2 then $\frac{dy}{dx}$ is:

(A) $\cos x^2.f'(x)$ (B) $-\cos x^2.f'(x)$

(C) $\frac{2(1+x-x^2)}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$ (D) None of these

687. If y^2 = p(x), a polynomial of degree 3 then $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$ is equal to:

(A) p'''(x) + p'(x) (B) p'''(x) + p''(x)

(C) p(x) p'''(x) (D) a constant

688. $I = \int\frac{(x+x^{\frac{2}{3}}+x^{\frac{1}{6}})}{x(1+x^{\frac{1}{3}})}dx$ is equal to

(A) $\frac{3}{2}x^{\frac{2}{3}}+6\tan^{-1}\left(x^{\frac{1}{6}}\right)+c$ (B) $\frac{3}{2}x^{\frac{2}{3}}-6\tan^{-1}\left(x^{\frac{1}{6}}\right)+c$

(C) $=\frac{3}{2}x^{\frac{2}{3}}+\tan^{-1}\left(x^{\frac{1}{6}}\right)+c$ (D) none of these

689. $\int \frac{\left(\sqrt{x^2+1}\right)\left\{\ln\left(x^2+1\right)-2\ln x\right\}}{x^4}$ dx is equal to :

(A) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]$ + c (B) $\frac{1}{9}\;\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]$ + c

(C) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]$ + c (D) $\frac{1}{9}\;\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]$ + c

690. If the positive number x and y are connected by the relation $x^2 - xy + y^2 = 12$, then maximum value of 2x + 3y, is

(A) $\frac{20}{\sqrt{19}}$ (B) $\frac{74}{\sqrt{19}}$ (C) $\frac{67}{\sqrt{19}}$ (D) $\frac{76}{\sqrt{19}}$

691. f(x) = Minimum {tanx, cot x} $\forall$ x $\in \left(0, \frac{\pi}{2}\right)$. Then $\int_0^{\pi/3} f(x)$ dx is equal to :

(A) $\ell n\left(\frac{\sqrt{3}}{2}\right)$ (B) $\ell n\left(\sqrt{\frac{3}{2}}\right)$ (C) $\ell n\left(\sqrt{2}\right)$ (D) $\ell n\left(\sqrt{3}\right)$

692. If f(x) is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non-zero x, then $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)\,dx$ equals

(A) sinθ + cosecθ (B) $\sin^2\theta$ (C) $\operatorname{cosec}^2\theta$ (D) none of these

693. If $\int_0^{100} f(x)$ dx = a, then $\sum_{r=1}^{100}\left(\int_0^1 f\left(r-1+x\right)dx\right) =$

(A) 100 a (B) a (C) 0 (D) 10 a

694. The area bounded by the curves y = x (1 – ln x); x = e^{-1} and a positive X–axis between x = e^{-1} and x = e is :

(A) $\left(\frac{e^2-4e^{-2}}{5}\right)$ (B) $\left(\frac{e^2-5e^{-2}}{4}\right)$ (C) $\left(\frac{4e^2-e^{-2}}{5}\right)$ (D) $\left(\frac{5e^2-e^{-2}}{4}\right)$

695. The area bounded by $y = x^2$, y = [x + 1], x ≤ 1 and the y-axis is :

(A) 1/3 (B) 2/3 (C) 1 (D) 7/3

696. The area bounded by the curve y = f(x), x-axis and the ordinates x = 1 and x = b is (b – 1) sin (3b + 4), $\forall$ b $\in$ R, then f(x) =

(A) (x – 1) cos (3x + 4) (B) sin (3x + 4)

(C) sin (3x + 4) + 3(x – 1) cos (3x + 4) (D) None of these

697. The areas of the figure into which curve $y^2 = 6x$ divides the circle $x^2 + y^2 = 16$ are in the ratio

(A) $\frac{2}{3}$ (B) $\frac{4\pi-\sqrt{3}}{8\pi+\sqrt{3}}$ (C) $\frac{4\pi+\sqrt{3}}{8\pi-\sqrt{3}}$ (D) none of these

698. If $\sqrt{(x^2+y^2)} = ae^{\tan^{-1}(y/x)}$, a > 0. Then y″(0), equals

(A) $\frac{a}{2}e^{\pi/2}$ (B) $ae^{\pi/2}$

(C) $-\frac{2}{a}e^{-\pi/2}$ (D) $\frac{a}{2}e^{-\pi/2}$

699. The function $f(\theta) = \frac{d}{d\theta}\int_0^{\theta}\frac{dx}{1-\cos\theta\cos x}$ satisfies the differential equation

(A) $\frac{df}{d\theta} + 2f(\theta)\cot\theta = 0$ (B) $\frac{df}{d\theta} - 2f(\theta)\cot\theta = 0$

(C) $\frac{df}{d\theta} + 2f(\theta) = 0$ (D) $\frac{df}{d\theta} - 2f(\theta) = 0$

700. The solution of the differential equation

$$(x^2 \sin^3 y - y^2 \cos x)\, dx + (x^3 \cos y \sin^2 y - 2y \sin x)\, dy = 0 \text{ is :}$$

(A) $x^3 \sin^3 y = 3y^2 \sin x + C$ (B) $x^3 \sin^3 y + 3y^2 \sin x = C$

(C) $x^2 \sin^3 y + y^3 \sin x = C$ (D) $2x^2 \sin y + y^2 \sin x = C$

SECTION-2 (MORE THAN ONE OPTION CORRECT TYPE)

701. A curve that passes through (2, 4) and having subnormal of constant length of 8 units can be;

(A) $y^2 = 16x - 8$ (B) $y^2 = -16x + 24$

(C) $x^2 = 16y - 60$ (D) $x^2 = -16y + 68$

702. Let f : R → R, such that $f''(x) - 2f'(x) + f(x) = 2e^x$ and $f'(x) > 0$ $\forall$ x ∈ R, then which of the following can be correct

(A) $|f(x)| = -f(x), \forall x \in R$ (B) $|f(x)| = f(x), \forall x \in R$

(C) $f(3) = -5$ (D) f(3) = 7

703. Let $|f(x)| \le \sin^2 x, \forall\ x \in R$, then

(A) f(x) is continuous at x = 0

(B) f(x) is differentiable at x = 0

(C) f(x) is continuous but not differentiable at x = 0

(D) f(0) = 0

704. $f(x) = x^3 + x^2 f'(1) + xf''(2) = f'''(3)$ $\forall$ x ∈ R, then

(A) f (0) + f (2) = f (1) (B) f (0) + f (3) = 0

(C) f (1) + f (3) = f (2) (D) none of these

705. The function f (x) = 9 + |sin x| is

(A) continuous every where (B) continuous nowhere

(C) differentiable at infinite number of points (D) not differentiable at infinite number of points

706. If $\lim_{x\to 0}\frac{x(1+a\cos x) - b\sin x}{x^3} = 1$, then

(A) $a = -\frac{5}{2}$ (B) $a = -\frac{3}{2}$

(C) $a = -\frac{7}{2}$ (D) $b = -\frac{3}{2}$

707. Let h (x) = min $\{x^2, x^4\}$ for every real number of a, then

(A) h is not differentiable at two points (B) h is differentiable $\forall$ x

(C) h is continuous $\forall$ x (D) none of these

708. The value of the integral $\int_0^{\pi} xf(\sin x)dx$ is

(A) $\pi\int_0^{\pi/2} f(\sin x)dx$ (B) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ (C) 0 (D) none of these

709. If f(x) = $|\log_{10}x|$, then at x = 1

(A) f(x) is continuous and $f'(1^-) = -\log_e 10$ (B) f(x) is continuous and $f'(1^-) = -1$

(C) f(x) is differentiable on R – {1} (D) f(x) is differentiable on R.

710. $\int_0^{\pi/2}\left(\frac{\pi}{2}-x\right)\sec x dx$ is equal to

(A) $-2\int_0^1 \frac{\cot^{-1}x}{x}dx$ (B) $\int_0^1 \tan^{-1}x dx$ (C) $2\int_0^1 \frac{\tan^{-1}x}{x}dx$ (D) $\int_0^1 \cot^{-1}x dx$

711. For f(x) = $\int_0^x 2|t|\,dt$, then tangent parallel to bisector of positive co-ordinate axes are

(A) $y = x - \frac{1}{4}$ (B) $y = x + \frac{1}{4}$ (C) $y = x - \frac{3}{2}$ (D) $y = x + \frac{3}{2}$

712. Let f(x) = $\begin{cases} -2, & -3 \le x \le 0 \\ |x-2|, & 0 \le x \le 3 \end{cases}$ and g(x) = $\int_{-3}^{x} f(t)dt$, then

(A) g(1) = – 3 (B) g(2) = – 4 (C) g′(1) = 1 (D) g′(2) does not exist

713. The domain of the definition of the function f(x) = $\left([x]-|x-1|\right)^{-1/2} + \sec^{-1}[\cos x]$, in the region [–π, 2π] where [.] denotes greater integer function lies in the internal

(A) $\left(\frac{\pi}{2}, \pi\right)$ (B) $\left[-\pi, -\frac{\pi}{2}\right) \cup \{2\pi\}$ (C) $\left[\pi, \frac{3\pi}{2}\right) \cup \{2\pi\}$ (D) [1, 2π]

714. If f(x) = sinπ(x – [x]) (where [.] denotes the greatest integer function), then

(A) f(x) has period 1 (B) f(x) is non-differentiable at x = 1, 2, 3

(C) $\int_0^{100} f(x)dx = \frac{200}{\pi}$ (D) $\int_0^{100} f(x)dx = 200\pi$

715. Let f : R → R be defined by f(x) = $3^{[x]} + 3^{-x}$, (where [.] denotes the greatest integer function) then which of the following statements are current ?

(A) f(x) is many-one (B) f(x) is into

(C) f(x) is bijective (D) neither even nor odd

716. Let f(x) = (x + |x|)|x| then for all x

(A) f is continuous (B) f′ is differentiable for all x

(C) f′ is continuous (D) f″ is continuous

717. If $\int \log\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$ = x f(x) + Ax + B $\sin^{-1}$x + c

(A) f(x) = $\log\left(\sqrt{1-x}+\sqrt{1+x}\right)$ (B) A = $-\frac{1}{2}$

(C) B = $\frac{2}{3}$ (D) B = $-\frac{1}{2}$

718. If f(x) = [x(x – 1)] + |2x – 1|, then f(x) is (where [.] denotes the greatest integer function)

(A) continuous at x = 10 (B) differentiable at x = 10

(C) discontinuous at x = 10 (D) nondifferentiable at x = 10

719. If $\lim_{x\to\infty}\left(1+\frac{\lambda}{x}+\frac{\mu}{x^2}\right)^{2x}$ = e^2, then

(A) $\lambda = 1, \mu = 2$ (B) $\lambda = 2, \mu = 1$ (C) $\lambda = 1, \mu$ = any R (D) $\lambda = \mu = 1$

720. In which of the following intervals $2x^3 - 24x + 5$ increases

(A) (– 2, 2) (B) (2, ∞) (C) (– ∞, – 2) (D) None of these

721. If f(x) = $\int_1^x \frac{\ln t}{1+t}dt$, then

(A) $f\left(\frac{1}{x}\right) = -\int_1^x \frac{\ln t}{t(1+t)}dt$ (B) $f\left(\frac{1}{x}\right) = \int_1^x \frac{\ln t}{t(1+t)}dt$

(C) $f(x)+f\left(\frac{1}{x}\right)=0$ (D) $f(x)+f\left(\frac{1}{x}\right)=\frac{1}{2}(\ln x)^2$

722. Let f and g be functions from the interval [0, ∞) to the interval [0, ∞) f being an increasing function and g being a decreasing function. If f {g (0)} = 0, then

(A) f {g (x)} ≥ f {g (0)} (B) g {f (x)} ≤ g {f (0)} (C) f {g (1)} = 0 (D) none of these

723. If the line ax + by + c = 0 is a normal to the curve xy = 1, then

(A) a > 0, b > 0 (B) a > 0, b < 0 (C) a < 0, b > 0 (D) a < 0, b < 0

724. If y = x $\log\left(\frac{x}{a+bx}\right)$, $x^3\frac{d^2y}{dx^2}$ =

(A) $\left(x\frac{dy}{dx}-y\right)^2$ (B) $\frac{a^2x^2}{(a+bx)^2}$ (C) $\left(\frac{dy}{dx}-y\right)^2$ (D) $\left(x\frac{dy}{dx}+y\right)^2$

725. On the interval I = [–2, 2] the function $f(x)=\begin{cases}(x+1)e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x\neq 0\\ 0 \quad , & x=0\end{cases}$

(A) is continuous for all x ∈ I – {0} (B) is continuous for all x ∈ I

(C) assumes all intermediate values for f(–2) to f(2)

(D) has a maximum values equal to 3/e

726. If I = $\int_{-\pi/3}^{\pi/3}\frac{e^{\sec x}|\sin x|\sec^2 x}{(1+e^{\operatorname{cosec} x})}dx$, then

(A) I can be evaluated using the substitution secx = t

(B) I is irrational number

(C) I = e^2 – e (D) I = e – 1

727. Let f : R → R be a function defined by $f(x+1)=\frac{f(x)-5}{f(x)-3}$ ∀ x ∈ R. Then which of the following statement(s) is/are true

(A) f(2008) = f(2004) (B) f(2006) = f(2010)

(C) f(2006) = f(2002) (D) f(2006) = f(2018)

728. If $\lim_{x\to 0}\frac{ae^{2x}-b\cos 2x+ce^{-2x}-x\sin x}{x\sin x}=1$ and f (t) = (a + b)t^2 + (a – b)t + c, then

(A) a + b + c = 1 (B) a + b + c = 2 (C) f(1) = $\frac{3}{4}$ (D) f (1) = 1

729. If f(x) = $\sec^{-1}\left(\frac{x+2}{2x-3}\right)+\sin^{-1}\left(\frac{2x-3}{x+2}\right)$ then in their domain of definition

(A) f is non decreasing (B) f is non increasing
(C) f′(1) = 10 (D) f′(0) = 0

730. If f : R → R is decreasing and g : R → R is increasing then which of the following function is increasing

(A) f o f (B) g o g (C) f o g (D) g o f

731. If f : R → R^- (set of all negative reals) is decreasing and g : R → R^- is increasing then which of the following is decreasing

(A) fof (B) gof (C) f^2 (D) g^2

732. If f′(sin x) = $\cos^2 x$ for all x and f(1) = 1 then

(A) f is increasing (B) f is injective (C) f(0) = 1/3 (D) f(–1) = –1/3

733. If f(x + 1/x) = x^3 + 1/x^3 (x ≠ 0) then

(A) f(x) is increasing function (B) f(x) has a local maximum at x = –1
(C) f(x) is injective in its domain of definition
(D) The equation f(x) = 3 has a unique real root

734. The function f(x) = 4x^2 – 1/x increases over the interval

(A) (0, ∞) (B) (–∞, –1/2)
(C) (–1/2, 0) (D) (1, ∞)

735. The function f(x) = 2 ln |x| – x |x| decreases over the interval

(A) (1, ∞) (B) (–∞, –1) (C) (0, 1) (D) (–1, 0)

736. The function f(x) = 2|x| + 1/x^2 is increasing in the interval

(A) (–∞, –1) (B) (–1, 0) (C) (0, 1) (D) (1, ∞)

737. Which of the following is/are true

(A) e^π/π^e (B) $(1 + \sin \pi/3)^{1+\cos \pi/3} > (1 + \cos \pi/3)^{1+\sin\pi/3}$
(C) $101^{202} > 202^{101}$ (D) $(4/3)^{9/4} > (9/4)^{4/3}$

738. If f is differentiable at x = a ; then which of the following is FALSE

(A) If f(a) is an extreme value of f(x), then f′(a) = 0
(B) If f′(a) = 0, then f(a) is an extreme value of f(x)
(C) If f(a) = is not an extreme value of f(x) then f′(a) ≠ 0
(D) only one of these statement is false

739. If f(x) = $\frac{x}{\sin x}$ *and* $g(x)=\frac{x}{\tan x}$ where 0≤ x ≤ 1, then in this interval

(A) both f(x) and g(x) are increasing function (B) both f(x) and f(x) are decreasing function
(C) f(x) is an increasing function (D) g(x) is a decreasing function

740. Which of the following DOESNOT hold Rolle’s theorem in [–1, 1]

(A) f(x) = |x| (B) f(x) = x^2 – 1 (C) f(x) = x^2 + x + 1 (D) f(x) = ln|x|

741. Let f(x) = f(x) + f(1 – x) and $f''(x) < 0$, $0 \le x \le 1$. Then

(A) $g(x)$ *increases on* $\left[\frac{1}{2}, 1\right]$ (B) $g(x)$ *decreases on* $\left[\frac{1}{2}, 1\right]$

(C) $g(x)$ *decreases on* $\left[0, \frac{1}{2}\right]$ (D) $g(x)$ *increases on* $\left[0, \frac{1}{2}\right]$

742. If $f'(x) = g(x)(x - a)^2$, where $g(a) \ne 0$ and g is continuous at x = a, then f is

(A) increasing in the nbd. of a if g(a) > 0 (B) increasing in the nbd. of a if g(a) < 0

(C) decreasing in the nbd. of a if g(a) > 0 (D) decreasing in the nbd. of a if g(a) < 0

743. Let $f(x) = 1 + 2^2x^2 + 3^2x^4 + 4^2x^6 + \ldots\ldots + n^2 x^{2n-2}$ then f(x) has

(A) exactly one critical point (B) at least one maximum

(C) exactly one minimum (D) None of these

744. Let f(x) = $\left|x^2 - 3x - 4\right|, -1 \le x \le 4$. Then

(A) f(x) is m.i. in $\left[-1, \frac{3}{2}\right]$ (B) f(x) is m.d. in $\left(\frac{3}{2}, 4\right)$

(C) the maximum value of f(x) is 25/4 (D) the minimum value of f(x) is 0

745. A particle is moving in a straight line such that its distance at any time t is $S = \frac{t^4}{4} - 2t^3 + 4t^2 + 7$, then

(A) velocity is max at t = $\frac{\left(6 - 2\sqrt{3}\right)}{3}$ (B) acceleration is min at t = 2

(C) the distance is min at t = 0, 4 (D) None of these

746. Let f : R → (–1, 1) defined by f(x) = $\frac{e^{x^3} + e^{-x^3}}{e^{x^3} - e^{-x^3}}$, then f is

(A) a one – one function (B) an increasing function

(C) a decreasing function (D) onto function

747. Let f(x) = $\frac{x^2 + 2}{[x]}$, $1 \le x \le 4.9$, where [x] denotes the integral part of x. Then

(A) f(x) is m.i. in [1, 4.9] (B) least value of f(x) = 3

(C) greatest value of f(x) = 6.0075 (D) f(x) is m.d. in [1, 4.9]

748. Let $f(x) = ax^3 + bx^2 + cx + 1$ have extrema at x = α, β such that $\alpha\beta < 0$ and $f(\alpha).\ f(\beta) < 0$ then the equation f(x) = 0 has

(A) three equal real roots (B) three distinct real roots

(C) one positive root if $f(\alpha) < 0$ & $f(\beta) > 0$ (D) one negative root if $f(\alpha) > 0$ & $f(\beta) < 0$

749. Let $h(x) = \{f(x)\}^3 + \{f(x)\}^2 + 10f(x)$. Then

(A) h increases as f increases (B) h decreases as f decreases

(C) h increases as f decreases (D) None of these

750. Let $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$ for all real values of x. Then

(A) h is increasing if f(x) is increasing (B) h is increasing if $f'(x) < 0$

(C) h is decreasing if f is decreasing

(D) nothing can be said in general

751. Let f(x) = cos x sin 2x then

(A) $\min_{x\in(-\pi,\pi)} f(x) > -\frac{7}{9}$

(B) $\min_{x\in(-\pi,\pi)} f(x) > -\frac{9}{7}$

(C) $\min_{x\in[-\pi,\pi]} f(x) > -\frac{1}{9}$

(D) $\min_{x\in[-\pi,\pi]} f(x) > -\frac{2}{9}$

752. If OT and ON are perpendiculars dropped from the origin to the tangent and normal to the curve $x = a\sin^3 t$, $y = a\cos^3 t$ at an arbitrary point, then

(A) $4OT^2 + ON^2 = a^2$

(B) the length of the tangent = $\left|\frac{y}{\cos t}\right|$

(C) the length of the normal = $\left|\frac{y}{\sin t}\right|$

(D) None of these

753. If F(x) = f(x) g(x) and f′(x) g′(x) = c, then

(A) $F' = c\left[\frac{f}{f'} + \frac{g}{g'}\right]$

(B) $\frac{F''}{F} = \frac{f''}{f} + \frac{g''}{g} + \frac{2c}{fg}$

(C) $\frac{F'''}{F} = \frac{f'''}{f} + \frac{g'''}{g}$

(D) $\frac{F'''}{F''} = \frac{f'''}{f''} + \frac{g'''}{g''}$

754. Let $x^{\cos y} + y^{\cos x} = 5$. Then

(A) at x = 0, y = 0, y′ = 0

(B) at x = 0, y = 1, y′ = 0

(C) at x = y, y = 1, y′ = 1

(D) at x = 1, y = 0, y′ = 1

755. Let f(x) = (ax + b) cos x + (cx + d) sin x and f′(x) = x cos x be an identify in x, then

(A) a = 0

(B) b = 1

(C) c = 1

(D) d = 0

756. The function f(x) = max{(1 – x), (1 + x), 2}, $x \in (-\infty,\infty)$ is

(A) continuous for all x

(B) differentiable for all x

(C) except x = 1 and x = –1 differentiable for all x

(D) None of these

757. If $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$ then $\frac{d}{dx}\{f_n(x)\}$ is equal to

(A) $f_n(x).\frac{d}{dx}\{f_{n-1}(x)\}$

(B) $f_n(x).f_{n-1}(x)$

(C) $f_n(x).f_{n-1}(x).\ldots.f_2(x).f_1(x)$

(D) None of these

758. Let $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1).x^2 + xf'(x) + f''(x)$ then

(A) f′(1) + f′(2) = 0

(B) g′(2) = g′(1)

(C) g″(2) + f″(3) = 6

(D) None of these

759. Let f(t) = ln t. Then $\frac{d}{dx}\left\{\int_{x^2}^{x^3} f(t)\,dt\right\}$

(A) has a value 0 when x = 0

(B) has a value 0 when x = 1, x = $\frac{4}{9}$

(C) has a value $9e^2 - 4e$ when x = e

(D) has a d.c. 27e – 8 when x = e

760. If –1, f(x) = $\begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta\sec x & \tan x & x \\ 1 & \tan x - \tan\theta & 0 \end{vmatrix}$ and 6^n $\sin\left(2x + \frac{n\pi}{2}\right)\cos\left(\frac{3x + n\pi}{2}\right)$ are in AP for all x, y, and y_n, then

(A) 0

(B) $y = \int_0^x f(t)\sin\{k(x-t)\}\,dt$

(C) $y = \sin^2\alpha + \cos^2(\alpha + \beta) + 2\sin\alpha\sin\beta\cos(\alpha + \beta)$

(D) $\frac{1}{\sqrt{1+9y^2}}$

761. If $f(x) = \cos^{-1}\cos(x - \pi/4)$ then

(A) $f'\left(\frac{\pi}{2}\right) = 1$

(B) $f'(0) = -1$

(C) $f'(\pi) = 0$

(D) $f'\left(\frac{\pi}{4}\right) = 0$

762. If f(x) = $\left|\sin x - \cos x\right|$ then

(A) $f'\left(\frac{\pi}{2}\right) = 1$

(B) $f(\pi) = -1$

(C) $f'\left(\frac{3\pi}{4}\right) = -1$

(D) $f'(0) = -1$

763. If f(x) = (ax + b) sin x – (cx + d) cosx and $f'(x)$ = x sin x then

(A) a = d = 0

(B) b = 1, c = –1

(C) b = c = 1

(D) a = 0, d = –1

764. If f(x + y) = f(x) + f(y) for all x, y $\in$ R and $f'(0)$ exists then

(A) $f'(x) = f'(0)$

(B) f(x) = kx

(C) f(x) = xf(1)

(D) f(x) is an even as well as periodic function

765. If f(x – y), f(x) f(y) and f(x + y) are in A.P. for all x, y $\in$ R and f(0) $\neq$ 0, then

(A) f(x) is an even function

(B) $f'(1) + f'(-1) = 0$

(C) $f'(2) - f'(-2) = 0$

(D) f(3) + f(–3) = 0

766. For the function $f(x) = \ell n \quad (\sin^{-1} \ell og_2 x)$,

(A) Domain is $\left[\frac{1}{2}, 2\right]$ (B) Range is $\left(-\infty, \ell n\frac{\pi}{2}\right]$

(C) Domain is (1, 2] (D) Range is R

767. A function 'f' from the set of natural numbers to integers defined by,

$$f(n) = \begin{cases} \frac{n-1}{2} & , \text{ when n is odd} \\ -\frac{n}{2} & , \text{ when n is even} \end{cases} \text{ is:}$$

(A) one-one (B) many-one (C) onto (D) into

768. If $F(x) = \frac{\sin \pi [x]}{\{x\}}$, then F (x) is:

(A) periodic with fundamental period 1

(B) even

(C) range is singleton

(D) identical to $\text{sgn}\left(\text{sgn}\frac{\{x\}}{\sqrt{\{x\}}}\right) - 1$, where {x} denotes fractional part function and [.] denotes greatest integer function and sgn (x) is a signum function.

769. $D \equiv [-1, 1]$ is the domain of the following functions, state which of them are injective.

(A) $f(x) = x^2$ (B) $g(x) = x^3$ (C) $h(x) = \sin 2x$ (D) $k(x) = \sin(\pi x/2)$

770. If $f(x) = \sqrt{x}$ and $g(x) = x - 1$, then

(A) fog is continuous on $[0, \infty)$ (B) gof is continuous on $[0, \infty)$

(C) fog is continuous on $[1, \infty)$ (D) none of these

771. The function $f(x) = \begin{cases} x^m \sin\frac{1}{x} & , \quad x > 0 \\ 0 & , \quad x = 0 \end{cases}$ is continuous at x = 0 if

(A) $m \geq 0$ (B) $m > 0$ (C) $m < 1$ (D) $m \geq 1$

772. Let $f(x) = \frac{1}{[\sin x]}$ ([.] denotes the greatest integer function) then

(A) domain of f(x) is $(2n\pi + \pi, 2n\pi + 2\pi) \cup \{2n\pi + \pi/2\}$

(B) f(x) is continuous when $x \in (2n\pi + \pi, 2n\pi + 2\pi)$

(C) f(x) is continuous at $x = 2n\pi + \pi/2$ (D) f(x) has the period 2π

773. Let $f(x) = [x] + \sqrt{x - [x]}$, where [x] denotes the greatest integer function. Then

(A) f(x) is continuous on R^+ (B) f(x) is continuous on R

(C) f(x) is continuous on $R - I$ (D) discontinuous at x = 1

774. Let f(x) and g(x) be defined by f(x) = [x] and $g(x) = \begin{cases} 0 & , \quad x \in I \\ x^2 & , \quad x \in R - I \end{cases}$ (where [.] denotes the greatest integer function) then

(A) $\lim_{x \to 1} g(x)$ exists, but g is not continuous at x = 1

(B) $\lim_{x \to 1} f(x)$ does not exist and f is not continuous at x = 1

(C) gof is continuous for all x

(D) fog is continuous for all x

775. Which of the following function(s) defined below has/have single point continuity.

(A) $f(x) = \begin{bmatrix} 1 & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{bmatrix}$ (B) $g(x) = \begin{bmatrix} x & \text{if } x \in Q \\ 1-x & \text{if } x \notin Q \end{bmatrix}$

(C) $h(x) = \begin{bmatrix} x & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{bmatrix}$ (D) $k(x) = \begin{bmatrix} x & \text{if } x \in Q \\ -x & \text{if } x \notin Q \end{bmatrix}$

776. Two functions f & g have first & second derivatives at x = 0 & satisfy the relations,

$f(0) = \frac{2}{g(0)}$, $f'(0) = 2g'(0) = 4g(0)$, $g''(0) = 5f''(0) = 6f(0) = 3$ then:

(A) if $h(x) = \frac{f(x)}{g(x)}$ then $h'(0) = \frac{15}{4}$ (B) if $k(x) = f(x).\, g(x) \sin x$ then $k'(0) = 2$

(C) $\underset{x\to 0}{\text{Limit}} \frac{g'(x)}{f'(x)} = \frac{1}{2}$ (D) none

777. If $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_o(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is equal to:

(A) $f_n(x).\frac{d}{dx}\{f_{n-1}(x)\}$ (B) $f_n(x).\, f_{n-1}(x)$

(C) $f_n(x).\, f_{n-1}(x)........ f_2(x).\, f_1(x)$

(D) none of these

778. If f is twice differentiable such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If h(x) is a twice differentiable function such that $h'(x) = [f(x)]^2 + [g(x)]^2$. If h(0) = 2, h(1) = 4, then the equation y = h(x) represents:

(A) a curve of degree 2

(B) a curve passing through the origin

(C) a straight line with slope 2

(D) a straight line with y intercept equal to 2.

779. Given $f(x) = -\frac{x^3}{3} + x^2 \sin 1.5a - x \sin a.\, \sin 2a - 5\sin^{-1}(a^2 - 8a + 17)$ then:

(A) $f'(x) = -x^2 + 2x \sin 6 - \sin 4 \sin 8$ (B) $f'(\sin 8) > 0$

(C) $f'(x)$ is not defined at $x = \sin 8$ (D) $f'(\sin 8) < 0$

780. $P(x)$ is a fourth degree polynomial such that

(a) $P(-x) = P(x)$ (b) $P(-x) > 0 \;\; \forall x \in R$ (c) $P(0) = 1$

(d) $P(x)$ has exactly two local minima at x_1 and x_2 such that $|x_1 - x_2| = 2$

The line $y = 1$ touches the curve at a certain point Q and the enclosed area between the line and the curve is $\frac{8\sqrt{2}}{15}$. Let $g(x) = Ax^2 + Bx + C \;\; (A \neq 0)$ such that $\lim_{x\to 0} \frac{P(x) - g(x) - g(-x)}{x^2}$ is finite and is equal to the slope of the tangent of $g(x)$ at $x = -1$. Also $P(x)$ and $g(x)$ have common tangent at Q parallel to x-axis, Then

(A) the value of A is $\frac{-1}{2}$ (B) the value of $B+C$ is $\frac{-1}{2}$

(C) the value of $A+C$ is 1 (D) the value of $A+B+C$ is

781. If f(x) = (ax + b) sin x + (cx + d) cos x, then the values of a, b, c and d such that f′(x) = x cos x for all x are
(A) a = d = 1 (B) b = 0 (C) c = 0 (D) b = c

782. If $f(x) = \sum_{k=0}^{n} a_k |x|^k$, where a_i's are real constants, then f(x) is
(A) continuous at x = 0 for all a_i (B) differentiable at x = 0 for all $a_i \in R$
(C) differentiable at x = 0 for all $a_{2k+1} = 0$ (D) None of these

783. Consider the curve $f(x) = x^{1/3}$, then
(A) the equation of tangent at (0, 0) is x = 0
(B) the equation of normal at (0, 0) is y = 0
(C) normal to the curve does not exist at (0, 0)
(D) f(x) and its inverse meet at exactly 3 points.

784. The equation of normal to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ ($n \in N$) at the point with abscissa equal to 'a' can be:
(A) $ax + by = a^2 - b^2$ (B) $ax + by = a^2 + b^2$
(C) $ax - by = a^2 - b^2$ (D) $bx - ay = a^2 - b^2$

785. If the line, ax + by + c = 0 is a normal to the curve xy = 2, then:
(A) a < 0, b > 0 (B) a > 0, b < 0
(C) a > 0, b > 0 (D) a < 0, b < 0

786. In the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$, at point (2, – 1)
(A) length of subtangent is 7/6. (B) slope of tangent = 6/7
(C) length of tangent = $\sqrt{(85)}/6$ (D) None of these

787. If y = f(x) be the equation of a parabola which is touched by the line y = x at the point where x = 1. Then
(A) f′(1) = 1 (B) f′(0) = f′(1)
(C) 2f(0) = 1 – f′(0) (D) f(0) + f′(0) + f″(0) = 1

788. If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a, a) cuts off intercepts α, β on co-ordinate axes, where $\alpha^2 + \beta^2 = 61$, then the value of 'a' is equal to:
(A) 20 (B) 25
(C) 30 (D) – 30

789. The curves $ax^2 + by^2 = 1$ and $Ax^2 + By^2 = 1$ intersect orthogonally, then

(A) $\frac{1}{a}+\frac{1}{A}=\frac{1}{b}+\frac{1}{B}$ (B) $\frac{1}{a}-\frac{1}{A}=\frac{1}{b}-\frac{1}{B}$

(C) $\frac{1}{a}+\frac{1}{b}=\frac{1}{B}-\frac{1}{A}$ (D) $\frac{1}{a}-\frac{1}{b}=\frac{1}{A}-\frac{1}{B}$

790. The set of values of a for which the function $f(x) = x^2 + ax + 1$ is an increasing function on [1, 2] is I_1 and decreasing in [1, 2] is I_2, then :

(A) $I_1 : a \in (2, \infty)$ (B) $I_2 : a \in (-\infty, -4)$

(C) $I_2 : a \in (-\infty, -4]$ (D) $I_1 ; a \in [-2, \infty)$

791. If f is an even function then

(A) f^2 increases on (a, b) (B) f cannot be monotonic

(C) f^2 need not increases on (a, b) (D) f has inverse

792. Let $g(x) = 2f(x/2) + f(1 - x)$ and $f''(x) < 0$ in $0 \le x \le 1$ then g(x) :

(A) decreases in $\left[0, \frac{2}{3}\right]$ (B) decreases $\left[\frac{2}{3}, 1\right]$

(C) increases in $\left[0, \frac{2}{3}\right]$ (D) increases in $\left[\frac{2}{3}, 1\right]$

793. On which of the following intervals, the function $x^{100} + \sin x - 1$ is strictly increasing

(A) $(-1, 1)$ (B) $[0, 1]$ (C) $[\pi/2, \pi]$ (D) $[0, \pi/2]$

794. The function $y = \dfrac{2x-1}{x-2}$ $(x \ne 2)$:

(A) is its own inverse (B) decreases for all values of x

(C) has a graph entirely above x-axis (D) is bound for all x.

795. Let f and g be two functions defined on an interval I such that $f(x) \ge 0$ and $g(x) \le 0$ for all $x \in I$ and f is strictly decreasing on I while g is strictly increasing on I then

(A) the product function fg is strictly increasing on I

(B) the product function fg is strictly decreasing on I

(C) fog(x) is monotonically increasing on I

(D) fog (x) is monotonically decreasing on I

796. Let $f(x) = 40/(3x^4 + 8x^3 - 18x^2 + 60)$, consider the following statement about f(x).

(A) f(x) has local minima at x = 0

(B) f(x) has local maxima at x = 0

(C) absolute maximum value of f(x) is not defined

(D) f(x) is local maxima at x = – 3, x = 1

797. Maximum and minimum values of the function,

$$f(x) = \frac{2-x}{\pi}\cos \pi (x + 3) + \frac{1}{\pi^2}\sin \pi (x + 3) \quad 0 < x < 4 \text{ occur at :}$$

(A) x = 1 (B) x = 2 (C) x = 3 (D) $x = \pi$

798. If $\lim_{x \to a} f(x) = \lim_{x \to a} [f(x)]$ ([.] denotes the greater integer function) and f(x) is non-constant continuous function, then

(A) $\lim_{x \to a} f(x)$ is integer (B) $\lim_{x \to a} f(x)$ is non-integer

(C) f (x) has local maximum at x = a (D) f (x) has local minima at x = a

799. If the derivative of an odd cubic polynomial vanishes at two different values of 'x' then

(A) coefficient of x^3 & x in the polynomial must be same in sign

(B) coefficient of x^3 & x in the polynomial must be different in sign

(C) the values of 'x' where derivative vanishes are closer to origin as compared to the respective roots on either side of origin.

(D) the values of 'x' where derivative vanishes are far from origin as compared to the respective roots on either side of origin.

800. Let f(x) = ln $(2x - x^2)$ + sin $\frac{\pi x}{2}$. Then

(A) graph of f is symmetrical about the line x = 1

(B) graph of f is symmetrical about the line x = 2

(C) maximum value of f is 1

(D) minimum value of f does not exist

801. The curve $y = \frac{x+1}{x^2+1}$ has:

(A) x = 1, the point of inflection

(B) $x = -2 + \sqrt{3}$, the point of inflection

(C) x = – 1, the point of minimum

(D) $x = -2 - \sqrt{3}$, the point of inflection

802. If the function y = f (x) is represented as, $x = \phi(t) = t^3 - 5t^2 - 20t + 7$

$y = \psi(t) = 4t^3 - 3t^2 - 18t + 3$ $(-2 < t < 2)$, then:

(A) $y_{max} = 12$ (B) $y_{max} = 14$ (C) $y_{min} = -67/4$ (D) $y_{min} = -69/4$

803. The maximum and minimum values of $y = \frac{ax^2 + 2bx + c}{Ax^2 + 2Bx + C}$ are those for which

(A) $ax^2 + 2bx + c - y(Ax^2 + 2Bx + C)$ is equal to zero

(B) $ax^2 + 2bx + c - y(Ax^2 + 2Bx + C)$ is a perfect square

(C) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} \neq 0$

(D) $ax^2 + 2bx + c - y(Ax^2 + 2Bx + C)$ is not a perfect square

804. If $\int \frac{(x-1)\,dx}{x^2\sqrt{2x^2 - 2x + 1}}$ is equal to $\frac{\sqrt{f(x)}}{g(x)}$ + c then

(A) $f(x) = 2x^2 - 2x + 1$

(B) g(x) = x + 1

(C) g(x) = x

(D) $f(x) = \sqrt{2x^2 - 2x}$

805. $\int \frac{dx}{5 + 4\cos x} = I \tan^{-1}\left(m \tan\frac{x}{2}\right)$ + C then:

(A) I = 2/3 (B) m = 1/3 (C) I = 1/3 (D) m = 2/3

806. If $\int \frac{3\cot 3x - \cot x}{\tan x - 3\tan 3x}\,dx$ = p f(x) + q g(x) + c where 'c' is a constant of integration, then

(A) p = 1; $q = \frac{1}{\sqrt{3}}$; f(x) = x; $g(x) = \ell n \left|\frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x}\right|$

(B) p = 1; $q = -\frac{1}{\sqrt{3}}$; f(x) = x; $g(x) = \ell n \left|\frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x}\right|$

(C) p = 1; $q = -\frac{2}{\sqrt{3}}$; f(x) = x; $g(x) = \ell n \left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right|$

(D) p = 1; $q = -\frac{1}{\sqrt{3}}$; f(x) = x; $g(x) = \ell n \left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right|$

807. $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x}$ dx is equal to:

(A) $\cot^{-1}(\cot^2 x) + c$ (B) $-\cot^{-1}(\tan^2 x) + c$ (C) $\tan^{-1}(\tan^2 x) + c$ (D) $-\tan^{-1}(\cos 2x) + c$

808. If f(x) is integrable over [1, 2], then $\int_1^2 f(x)\, dx$ is equal to

(A) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (B) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

(C) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ (D) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

809. If $f(x) = \int_0^x (\cos^4 t + \sin^4 t)\, dt$, $f(x + \pi)$ will be equal to

(A) $f(x) + f(\pi)$ (B) $f(x) + 2f(\pi)$ (C) $f(x) + f\left(\frac{\pi}{2}\right)$ (D) $f(x) + 2f\left(\frac{\pi}{2}\right)$

810. The value of $\int_0^1 \frac{2x^2+3x+3}{(x+1)\left(x^2+2x+2\right)}$ dx is:

(A) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1} 2$ (B) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1}\frac{1}{3}$

(C) $2\ln 2 - \cot^{-1} 3$ (D) $-\frac{\pi}{4} + \ln 4 + \cot^{-1} 2$

811. The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding subnormal

(A) is linear (B) is homogeneous

(C) has separable variables (D) is none of these

812. The solution of $x^2 y_1^2 + xy\, y_1 - 6y^2 = 0$ are

(A) $y = Cx^2$ (B) $x^2 y = C$ (C) $\frac{1}{2}\log y = C + \log x$ (D) $x^3 y = C$

813. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2 = a/x$ are

(A) $9a(y + c) = 4x^3$ (B) $y + C = \frac{-2}{3\sqrt{a}} x^{3/2}$ (C) $y + C = \frac{2}{3\sqrt{a}} x^{3/2}$ (D) None

814. The solution of $\left(\frac{dy}{dx}\right)(x^2 y^3 + xy) = 1$ is

(A) $1/x = 2 - y^2 + C e^{-y^2}/2$

(B) the solution of an equation which is reducible to linear equation.

(C) $2/x = 1 - y^2 + e^{-y}/2$ (D) $\frac{1-2x}{x} = -y^2 + Ce^{-y^2}/2$

SECTION-3 (COMPREHENSION TYPE)

COMPREHENSION-1

Paragraph for Questions Nos. 815 to 817

Let f : R → R be a continuous function such that $f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2$

815. f(3) is equal to

(A) f(0)
(B) 4 + f(0)
(C) 9 + f(0)
(D) 16 + f(0)

816. The equation f(x) – x – f(0) = 0 have exactly

(A) No solution
(B) One solution
(C) Two solution
(D) infinite solution

817. $f'(0)$ is equal to

(A) 0
(B) 1
(C) f(0)
(D) - f(0)

COMPREHENSION-2

Paragraph for Questions Nos. 818 to 820

If f(x) = max. ($|x^2 - 1|$, $|x - 1|$) and g(x) = $\int_a^x f(t)\,dt$, x ∈ R.

818. The value of f(x) is

(A) $f(x) = \begin{cases} x^2 - 1, & x \le -2 \\ 1 - x, & -2 < x \le 0 \\ 1 - x^2, & 0 < x < 1 \\ x^2 - 1, & x > 1 \end{cases}$

(B) $f(x) = \begin{cases} x^2 - 1, & x \le -2 \\ 1 - x^2, & -2 < x \le 0 \\ 1 - x, & 0 < x \le 1 \\ x - 1, & x > 1 \end{cases}$

(C) $f(x) = \begin{cases} x^2 - 1, & x \le 1 \\ x - 1, & x > 1 \end{cases}$

(D) $f(x) = \begin{cases} x - 1, & x \le 1 \\ x^2 - 1, & x > 1 \end{cases}$

819. The function f(x) is continuous for x belongs to

(A) R – {0, 1}
(B) R – {–2, 0, 1}
(C) R
(D) none of these

820. The function g(x) is differentiable for

(A) R – {0, 1}
(B) R – {–2, 0, 1}
(C) R
(D) None of these

COMPREHENSION-3

Paragraph for Questions Nos. 821 to 823

Let $f(x) = \log_{\{x\}}[x]$
$g(x) = \log_{[x]}\{x\}$
$h(x) = \log_{\{x\}}\{x\}$
where [.], {.} denotes the greatest integer function and fractional part.

821. For $x \in (1, 5)$ the f(x) is not defined at how many points

(A) 5 (B) 4
(C) 3 (D) 2

822. If A = {x: x $\in$ domain of f(x)} and B = {x: x $\in$ domain of g(x)} then A – B will be

(A) (2, 3) (B) (1, 3)
(C) (1, 2) (D) none of these

823. Domain of h(x) is

(A) R (B) I
(C) R – I (D) $R^+ - I$

COMPREHENSION-4

Paragraph for Questions Nos. 824 to 826

Let a function f(x) satisfies the condition $f(x+y) = \frac{f(x)+f(y)}{f(x)}$ such that f′(0) = 2 and f(x) ≥ 0. Using the above information answer the following:

824. The curve y = f(x) is

(A) $y = \sqrt{2(x+1)}$ (B) $y = 2\sqrt{(x+1)}$
(C) y = ln(x + 1) (D) y = ln(x – 1)

825. Area bounded between $y = f(|x|)$ *and* $y = 7 - |x|$ is

(A) $\frac{23}{6}$ sq. unit (B) $\frac{11}{6}$ sq. unit
(C) $\frac{86}{6}$ sq. unit (D) 7 sq. unit

826. The number of points where g(x) = max. {f(x), 6, 7 – |x|} is non differentiable $\forall$ x $\in$[–10, 10] are

(A) 5 (B) 6
(C) 7 (D) 8

COMPREHENSION-5

Paragraph for Questions Nos. 827 to 829

Let f : [2, ∞) → [1, ∞) defined by f(x) = $2^{x^4-4x^2}$ and $g:\left[\frac{\pi}{2},\pi\right]\to A$ defined by $g(x)=\frac{\sin x+4}{\sin x-2}$ be two invertible functions, then

827. $f^{-1}(x)$ is equal to

(A) $-\sqrt{2+\sqrt{4+\log_2 x}}$

(B) $\sqrt{2+\sqrt{4+\log_2 x}}$

(C) $\sqrt{2-\sqrt{4+\log_2 x}}$

(D) None of these

828. The set A is equal to

(A) [–5, –2] (B) [2, 5] (C) [–5, 2] (D) [–3, –2]

829. The domain of $f^{-1}g^{-1}(x)$ is

(A) [–5, sin 1] (B) $\left[-5, \frac{\sin 1}{2-\sin 1}\right]$ (C) $\left[-5, -\frac{(4+\sin 1)}{2-\sin 1}\right]$ (D) $\left[-\frac{(4+\sin 1)}{2-\sin 1}, -2\right]$

COMPREHENSION-6

Paragraph for Questions Nos. 830 to 832

A function is defined as the approaching value of the expression $\frac{1+2(x)^{2n}}{1+x^{2n}}$ as x approaches to infinity.

830. The domain and range of the function is

(A) $(-\infty, 1)\cup(1,\infty), \left\{1, -1, \frac{3}{2}, -\frac{3}{2}, 2, -2\right\}$

(B) $(-\infty,\infty), \left\{1, \frac{3}{2}, 2\right\}$

(C) (1, ∞), {1, – 1}

(D) None of these

831. The points of discontinuity of the function are

(A) 1, – 1 (B) 1, 0, – 1 (C) 1, 3 (D) None of these

832. The composition of the function with y = |x| is

(A) $\begin{cases} 1, & |x|<1 \\ \frac{3}{2}, & |x|=1 \\ 2, & |x|>1 \end{cases}$

(B) $\begin{cases} -2, & x\le -1 \\ -\frac{3}{2}, & -1<x<0 \\ 1, & 0\le x<1 \\ 2, & x\ge 1 \end{cases}$

(C) $\begin{cases} -2, & x\le -1 \\ -\frac{3}{2}, & x=-1 \\ -1, & -1<x<0 \\ 1, & 0\le x<1 \\ \frac{3}{2}, & x=1 \\ 2, & x>1 \end{cases}$

(D) None of these

COMPREHENSION-7

Paragraph for Questions Nos. 833 to 835

Let f(x) be a real valued function not identically zero, such that
$f(x + y^n) = f(x) + (f(y))^n \quad \forall x, y \in R$ where $n \in N$ ($n \neq 1$) and $f'(0) \geq 0$.

833. The value of f′(0) is

(A) 1 (B) 1 + n
(C) n (D) 2

834. The value of f(5) is

(A) 2 (B) 3
(C) 5n (D) 5

835. $\int_0^1 f(x)\,dx$ is equal to

(A) $\frac{1}{2n}$ (B) 2n
(C) $\frac{1}{2}$ (D) 2

COMPREHENSION-8

Paragraph for Questions Nos. 836 to 838

Let f: R → R be function defined as

$$f(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

and g(x) = f(x – 1) + f(x + 1) ∀ x ∈ R.

836. For x ∈ [– 1, 1], g(|x|) is equal to

(A) x (B) – |x|
(C) |x| (D) 2 + |x|

837. Value of $g\left(-\frac{3}{2}\right)$ is equal to

(A) 0 (B) $\frac{1}{2}$
(C) $\frac{7}{2}$ (D) $\frac{3}{2}$

838. The number of points at which y = |g(x)| is non differentiable is

(A) 3 (B) 4
(C) 5 (D) 6

COMPREHENSION-9

Paragraph for Questions Nos. 839 to 841

Let y = f(x) be a function continuous and differentiable every where also,
$g_1(x) = \min\{|f(x)|, |f(x-1)|\}$
$g_2(x) = f(|x|)$
$g_3(x) = -f(|x|)$
If f(x) = x – 1, then

839. The area bounded by $y = g_1(x)$, x-axis and lines x = 0 and x = 3 is equal to

(A) 1 (B) 2
(C) $\frac{5}{4}$ (D) None of these

840. The area bounded by $y = g_2(x)$ and $y = g_3(x)$ is equal to

(A) 2 (B) 4
(C) 1 (D) none of these

841. The area bounded by $y = g_3(x)$ and $y = \ln(|x|)$ is equal to

(A) 2 (B) 3
(C) 4 (D) None of these

COMPREHENSION-10

Paragraph for Questions Nos. 842 to 844

Let f be a polynomial function such that $f(x) f(y) + 2 = f(x) + f(y) + f(xy)\ \forall\ x, y \in R^+ \cup \{0\}$ and f(x) is one-one $\forall\ x \in R^+$ with f(0) = 1 and f′(1) = 2.

842. The function y = f(x) is given by

(A) $x^{1/3} - 1$ (B) $1 + \frac{2x^3}{3}$
(C) $1 + x^2$ (D) $1 - x^2$

843. Area bounded between the curve $y = x^2$ and y = g(x) where $g(x) = \frac{2}{f(x)}$ and x-axis is

(A) $\frac{\pi}{2} - \frac{1}{3}$ (B) $\pi - \frac{1}{3}$
(C) $\frac{\pi}{2} - \frac{1}{6}$ (D) $\pi - \frac{2}{3}$

844. If $h(x) = \min\left\{\frac{2}{f(x)},\ x^2,\ |1-|x||\right\}$, then the number of points of non-differentiability of h(x) is/are

(A) 3 (B) 4
(C) 5 (D) 6

COMPREHENSION-11

Paragraph for Questions Nos. 845 to 847

Let f(x) be a function such that its derivative f ′(x) is continuous in [a, b] and derivable in (a, b). Consider a function $\phi(x) = f(b) - f(x) - (b - x) f'(x) - (b - x)^2 A$. If Rolle's theorem is applicable to $\phi(x)$ on [a, b], answer following questions

845. If there exist some number c (a < c < b) such that $\phi'(c) = 0$ and $f(b) = f(a) + (b - a) f'(a) + \lambda (b - a)^2 f''(c)$, then λ is

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

846. Let $f(x) = x^3 - 3x + 3$, a = 1 and b = 1 + h. If there exists $c \in (1, 1 + h)$ such that $\phi'(c) = 0$ and $\frac{f(1+h) - f(1)}{h^2} = \lambda c$, then λ =

(A) $\frac{1}{2}$ (B) 2 (C) 3 (D) does not exists

847. Let f(x) = sin x , a = α and b = α + h. If there exists a real number t such that 0 < t < 1, $\phi'(\alpha + th)$ = 0 and $\frac{\sin(\alpha + h) - \sin\alpha - h\cos\alpha}{h^2} = \lambda \sin(\alpha + th)$, then λ =

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$

COMPREHENSION-12

Paragraph for Questions Nos. 848 to 850

Sometimes we can find the sum of series by use of differentiation. If we know the sum of a series

e.g. if $f(x) = f_1(x) + f_2(x) + \ldots\ldots\ldots\ldots\ldots$

$f'(x) = f_1'(x) + f_2'(x) + \ldots\ldots\ldots\ldots$

e.g. $(1 - x)^{-1} = 1 + x + x^2 + x^3 \ldots\ldots\ldots\ldots$ $|x| < 1$

Hence the sum of the AGP

$1 + 2x + 3x^2 + \ldots\ldots = (1 - x)^{-2}$ (By differentiation both the sides)

Now answer the question that follows

848. The sum of the series $\frac{2^2}{1!} + \frac{3^2}{2!} + \frac{4^2}{3!} + \ldots\ldots\ldots$ upto ∞ is

(A) 4e – 1 (B) 5e (C) 5e – 1 (D) 4e

849. Sum of the series $1 - \frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \ldots\ldots$ upto ∞ is

(A) $\frac{1}{2} - \ell n2$ (B) $1 - \ell n2$ (C) ∞ (D) $\frac{3}{2} - \ln 2$

850. Sum of the series $1 + 1 + \frac{3}{4} + \frac{4}{8} + \frac{5}{16} + \ldots\ldots$ upto infinite terms, is

(A) 4 (B) 2 (C) 1 (D) $\frac{1}{4}$

COMPREHENSION-13

Paragraph for Questions Nos. 851 to 853

If $y = \int_{u(x)}^{v(x)} f(t)\, dt$, let us define $\frac{dy}{dx}$ in a different manner as $\frac{dy}{dx} = v'(x) f^2(v(x)) - u'(x) f^2(u(x))$ and the equation of the tangent at (a, b) as $y - b = \left(\frac{dy}{dx}\right)_{(a,b)} (x - a)$

851. If $y = \int_{x}^{x^2} t^2\, dt$, then equation of tangent at x = 1 is

(A) $y = x + 1$ (B) $x + y = 1$ (C) $y = x - 1$ (D) $y = x$

852. If $F(x) = \int_{1}^{x} e^{t^2/2} (1 - t^2)\, dt$, then $\frac{d}{dx} F(x)$ at x = 1 is

(A) 0 (B) 1 (C) 2 (D) – 1

853. If $y = \int_{x^3}^{x^4} \ell nt\, dt$, then $\lim_{x \to 0^+} \frac{dy}{dx}$ is

(A) 0 (B) 1 (C) 2 (D) – 1

COMPREHENSION-14

Paragraph for Questions Nos. 854 to 856

Let f be a function defined so that every element of the codomain has at most two pre-images and there is at least one element in the co-domain which has exactly two pre-images we shall call this function as "two-one" function. A two-one function is definitely a many one function but vice-versa is not true. For example, $y = |e^x - 1|$ is a "two-one" function. $y = x^3 - x$ is a many one function but not a "two-one" function. In the light of above definition answer the following questions:

854. In the following functions which one is a "two-one" function :-

(A) $y = |\ell n|x||$ (B) $y = x^2 \sin x$

(C) $y = x^3 + 3x + 1$ (D) $y = x^4 - x + 1$

855. Let f(x) = {x} be the fractional part function. For what domain is the function "two-one"?

(A) $\left[\frac{1}{2}, \frac{5}{2}\right]$ (B) $\left[-\frac{1}{2}, \frac{3}{2}\right)$

(C) [1, 2) (D) None of these

856. A continuous "two-one" function defined for $x \in (a, b)$ has

(A) atmost one point of extremum

(B) atleast two points of extrema

(C) exactly one point of extremum

(D) none of these

COMPREHENSION-15

Paragraph for Questions Nos. 857 to 859

Continuous Probability Distributions. A continuous distribution is one in which the variate may take any value between certain limits a and b, a < b. Suppose that the probability of the variate X falling in the infinitesimal interval $x-\frac{1}{2}dx$ to $x+\frac{1}{2}dx$ is expressible as f(x) dx, where f(x) is a continuous function of x.

Symbolically, $P(x-\frac{1}{2}dx \le X \le x+\frac{1}{2}dx) = f(x)\,dx$

where f(x) is called the probability density function (abbreviated as p.d.f.) or simply density function. The continuous curve y = f(x) is called probability curve ; and when this is symmetrical, the distribution is said to be symmetrical. Clearly, the probability density function possesses the following properties:

(i) $f(x) \ge 0$ for every x in the interval [a, b], a < b

(ii) $\int_a^b f(x)\,dx = 1$, a, b > 0

since the total area under the curve is unity.

(iii) Furthermore, we define for any [c, d], where c, d ∈ [a, b], c < d;

$$P(c \le X \le d) = \int_c^d f(x)\,dx \qquad \text{...............(i)}$$

We define F(x), the cumulative distribution function (abbreviated as c.d.f.) of the random variate X where $F(x) = P(X \le x)$

or $$F(x) = \int_a^x f(x)\,dx \,. \qquad \text{...............(ii)}$$

857. If $f(x) = \begin{cases} 2x & ; \quad 0 \le x \le 1 \\ 0 & ; \quad x > 1 \end{cases}$ then the probability that $x \le \frac{1}{2}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{1}{8}$

858. In Q. No. 857, probability that $x \ge \frac{3}{4}$ given $x \ge \frac{1}{2}$ is

(A) $\frac{7}{16}$ (B) $\frac{3}{4}$ (C) $\frac{3}{7}$ (D) $\frac{7}{12}$

859. Suppose the life in hours (x) of a certain kind of radio tube has the probability density function $f(x) = \frac{100}{x^2}$ when x > 100 and zero when x < 100. Then the probability that none of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation, is

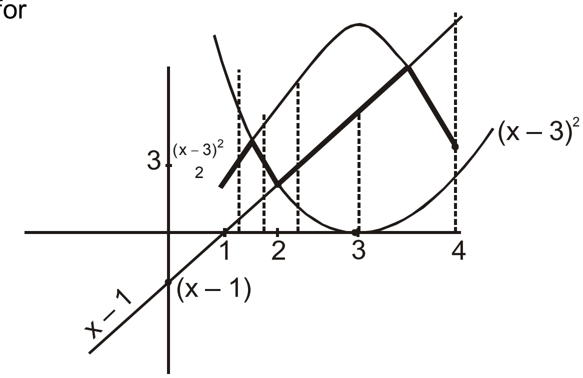
(A) $\frac{1}{27}$ (B) $\frac{8}{27}$ (C) $\frac{1}{225}$ (D) $\frac{26}{27}$

COMPREHENSION-16

Paragraph for Questions Nos. 860 to 862

If f(x) = Mid {g(x), h(x), p(x)} means the function which will be second in order when values of the three function at a particular value of x are arranged, then for

$$f(x) = \text{Mid}\left\{x-1, (x-3)^2, 3-\frac{(x-3)^2}{2}\right\}, x \in [1, 4]$$



860. Numerical value of difference between the LHD and RHD at the point x = 2 for f(x) in $x \in [1, 4]$ will be

(A) 0 (B) 2 (C) 3 (D) 1

861. The greatest value of f(x) in [1, 4] will be

(A) $1 + \sqrt{3}$ (B) $2 + \sqrt{3}$ (C) $3 + \sqrt{3}$ (D) N.O.T.

862. Rate of change of x w.r.t. f(x) at x = 3 will be

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $-\frac{3}{2}$

COMPREHENSION-17

Paragraph for Questions Nos. 863 to 865

A function f(x) having the following properties;

(i) f(x) is continuous except at x = 3

(ii) f(x) is differentiable except at x = – 2 and x = 3

(iii) $f(0) = 0$, $\lim_{x\to 3} f(x) \to -\infty$, $\lim_{x\to -\infty} f(x) = 3$, $\lim_{x\to \infty} f(x) = 0$

(iv) $f'(x) > 0 \ \forall x \in (-\infty, -2) \cup (3, \infty)$ and $f'(x) \le 0 \ \forall x \in (-2, 3)$

(v) $f''(x) > 0 \ \forall x \in (-\infty, -2) \cup (-2, 0)$ and $f''(x) < 0 \ \forall x \in (0, 3) \cup (3, \infty)$

then answer the following questions

863. Range of f(x) is

(A) $(-\infty, \infty)$ (B) $(-\infty, 3]$
(C) $(-\infty, 3)$ (D) $(-\infty, f(-2)]$

864. Graph of function y = f (– | x |) is

(A) differentiable for all x, if $f'(0) = 0$
(B) continuous but not differentiable at two points, if $f'(0) = 0$
(C) continuous but not differentiable at one points, if $f'(0) = 0$
(D) discontinuous at two points, if $f'(0) = 0$

865. f(x) + 3x = 0 has five solutions if

(A) $f(-2) > 6$ (B) $f'(0) < -3$ and $f(-2) > 6$
(C) $f'(0) > -3$ (D) $f'(0) > -3$ and $f(-2) > 6$

COMPREHENSION-18

Paragraph for Questions Nos. 866 to 868

l, m, n are real numbers and x_0 be an arbitrary real number in [p, q] and f is a real valued function such that

l^2 [f(a – x) – f(a + x)] + 4l [f(x) + f(–x)] + { $\underset{x \to x_0}{\text{Lt}}$ f(x) – f(x_0)} = 0,

m^2 [f(a – x) – f(a + x)] + 4m [f(x) + f(–x)] + { $\underset{x \to x_0}{\text{Lt}}$ f(x) – f(x_0)} = 0,

& n^2 [f(a – x) – f(a + x)] + 4n[f(x) + f(–x)] + { $\underset{x \to x_0}{\text{Lt}}$ f(x) – f(x_0)} = 0,

866. The function f is

(A) periodic with period 'a' (B) periodic with period '4a'
(C) periodic with a period 2a (D) non periodic

867. x_1, x_2, ….. x_n $\in$ (p, q) and for some $\xi \in$(p, q), (f(ξ) $\neq$ 0) =

(A) n (B) n + 1 (C) n – 1 (D) 2n

868. If f(x) > 0 $\forall$ x [0, 2a] then $\dfrac{\int_0^{2a} f(x)\,dx}{\int_0^{a} f(x)\,dx} =$

(A) 1 (B) 2 (C) 3 (D) 4

COMPREHENSION-19

Paragraph for Questions Nos. 869 to 872

Let f(x) = x^3+ax^2+bx+c be a cubic polynomial where a, b, c $\in$ R. Now f'(x) = $3x^2+2ax+b$ and let D = $4a^2-12b$ be the discriminant of the equation f'(x) = 0. If d > 0, f'(x) = 0 has two real roots. $\alpha, \beta(\alpha < \beta)$, then x = α will be point of local maxima and x = β will be a point of local minima of f(x), also

If $f(\alpha).f(\beta)$>0, then f(x) = 0 would have just one real root.

$f(\alpha).f(\beta)$<0, then f(x) = 0 would have three real and distinct roots.

$f(\alpha).f(\beta)$=0, then f(x) = 0 would have three real roots.

869. If the function f(x) = $x^3-9x^2+24x+k$ has three real and distinct roots x_1,x_2,x_3 where $x_1<x_2<x_3$. Then the possible value of k will be

(A) k < -20 (B) k > 20 (C) 16 < k < 20 (D) -20<k<-16

870. In the question No. 869, $[x_1]+[x_3]$ is equal to {where [x] is greatest integer function}

(A) 2 (B) 3 (C) 4 (D) 5

871. In the question No. 869, x_2 lies in the interval

(A) (-2, 0) (B) (0, 2) (C) (2, 4) (D) none of these

872. If f(x) = ax^3+bx^2+cx+d has it non-zero local minimum and maximum values at x = 2 and x = 1 respectively. If a be the root of the equation $x^2-2x-15$ = 0, then a is equal to

(A) -3 (B) 5 (C) both (a) and (b) (D) none of these

COMPREHENSION-20

Paragraph for Questions Nos. 873 to 875

One of the most famous functions in calculus is the Dirichlet's function, viz.

D(x) = $\begin{cases} 1, & x \in Q \\ 0, & x \notin Q \end{cases}$. This function is one of the rare functions whose graph cannot be drawn. A number of functions were later defined by imitating Dirichlet's function.

Let f(x) = $\begin{cases} x^3 + 2x^2 & , \ x \in Q \\ -x^3 + 2x^2 + ax, & x \notin Q \end{cases}$

873. The value of a so that this function is differentiable at x = 0 is

(A) 1 (B) –1 (C) 0 (D) none of these

874. For the value of a obtained in above question. f(x) is

(A) one-one and onto (B) many-one and onto
(C) one-one and into (D) many one and into

875. $\lim_{x \to 0} |f'(x)|$

(A) equals 1 (B) equals 2 (C) equals 3 (D) does not exist

COMPREHENSION-21

Paragraph for Questions Nos. 876 to 878

$$\text{Let } f(x) = \begin{cases} e^{\{x^2\}} - 1 & , \ x > 0 \\ \dfrac{\sin x - \tan x + \cos x - 1}{2x^2 + \ell n(2+x) + \tan x} & , \ x < 0, \\ 0 & , \ x = 0 \end{cases}$$

where { } represents fractional part function. Lines L_1 and L_2 represent tangent and normal to curve y = f(x) at x = 0. Consider the family of circles touching both the lines L_1 and L_2

876. Ratio of radii of two circles belonging to this family cutting each other orthogonally is

(A) $2 + \sqrt{3}$ (B) $\sqrt{3}$
(C) $2 + \sqrt{2}$ (D) $2 - \sqrt{2}$

877. A circle having radius unity is inscribed in the triangle formed by L_1 and L_2 and a tangent to it. Then the minimum area of the triangle possible is

(A) $3 + \sqrt{2}$ (B) $3 - \sqrt{2}$
(C) $3 + 2\sqrt{2}$ (D) $3 - 2\sqrt{2}$

878. If centers of circles belonging to family having equal radii 'r' are joined, the area of figure formed is

(A) $2r^2$ (B) $4r^2$
(C) $8r^2$ (D) r^2

COMPREHENSION-22

Paragraph for Questions Nos. 879 to 881

While finding the Sine of a certain angle *x*, an absent minded professor failed to notice that his calculator was not in the correct angular mode. Howevery he was lucky to get the right answer. The two least positive values of *x* for which the Sine of *x* degrees is the same as the Sine of x radians were found by him as $\frac{m\pi}{n-\pi}$ and $\frac{p\pi}{q+\pi}$ where *m, n, p* and *q* are positive integers. Suppose $\frac{mn}{pq}$ be denoted by the quantity 'L'. Now answer the following questions.

879. The value of (*m* + *n* + *p* + *q*) is equal to

(A) 720 (B) 900
(C) 1080 (D) 1260

880. If x is measured in radians and $\lim_{x\to\infty}\left(\sqrt{Ax^2+Bx}-Cx\right)$ = L, the value of $\frac{BC}{A}$ equals (A,B,C $\in$R)

(A) 4 (B) 2 (C) $\frac{1}{2}$ (D) none

881. Assume that f is differentiable for all x. The sign of f' is as follows:

f ' (x) > 0 on (– ∞, – 4)
f ' (x) < 0 on (– 4, 6)
f ' (x) > 0 on (6, ∞)

Let g (x) = f (10 – 2x). The value of g ' (L) is

(A) Positive
(B) negative
(C) zero
(D) the function g is not differentiable at x = 5

COMPREHENSION-23

Paragraph for Questions Nos. 882 to 884

Consider a family of curves, hwere the ordinate is proportional to the cube of the abscissa and let A be a fixed point in the plane which has coordinates (a, b).

882. If tangents be drawn through A to the members of family of curves then the locus of the point of contact is

(A) xy + bx – 3ay = 0 (B) xy – 4bx + 3ay = 0
(C) 2xy + bx – 3ay = 0 (D) 2xy – 4bx + 3ayu + 2 = 0

883. If the tangent through A to a curve cuts the curve again at a point B then the locus of B is

(A) xy + bx – 3ay = 0 (B) xy – 4bx + 3ay = 0
(C) $x^2 - 3y^2 = ax - 3by$ (D) $x^2 + 3y^2 = ax + 3by$

884. If the tangent through A to a curve cuts the curve again at a point B then the locus of B is

(A) xy – 4bx + 3ay = 0 (B) 2xy + bx – 3ay = 0
(C) $x^2 - 3y^2 = ax - 3by$ (D) $a^2x^2 + b^2y^2 = 1$

COMPREHENSION-24

Paragraph for Questions Nos. 885 to 887

A chemical manufacturing company has 1000 k*l* holding tank which it uses to control the release of pollutants into a sewage system. Initially the tank has 360 kl of water containing 2 kg of pollutant per kl. Water containing 3 kg of pollutant per kl enters the tank at the rate 80 kl per hour and is uniformly mixed with the water already in the tank. Simultaneously, water is released from the tank at the rate of 40 kl per hours.

885. If P(t) denotes the amount of pollutant at any given time 't' inside the tank, then the rate at which pollutant is leaving the tank is

(A) $\frac{P(t)}{9-t}$ (B) $\frac{P(t)}{9+t}$ (C) $\frac{P(t)}{10+t}$ (D) $\frac{P(t)}{10-t}$

886. The differential equation giving pollutant at any instant 't' is given by

(A) $\frac{dP}{dt}+\frac{P}{9+t}=240$ (B) $\frac{dP}{dt}-\frac{P}{9+t}=240$

(C) $\frac{dP}{dt}+\frac{P}{10+t}=240$ (D) $\frac{dP}{dt}-\frac{P}{10+t}=240$

887. The amount of pollutant at any time 't' is given by

(A) $P(t) = 120(9-t)-\frac{3240}{9+t}$ (B) $P(t) = 120(9+t)+\frac{3240}{9+t}$

(C) $P(t) = 120(10-t)+\frac{3240}{10-t}$ (D) $P(t) = 120(9+t)-\frac{3240}{9+t}$

COMPREHENSION-25

Paragraph for Questions Nos. 888 to 890

Let the derivative of f(x) be defined as $D^* f(x) = \lim_{h\to 0}\frac{f^2(x+h)-f^2(x)}{h}$, where $f^2(x) = \{f(x)\}^2$.

888. If u = f(x), v = g(x), then the value of D* (u . v) is

(A) (D* u) v + (D* v) u (B) $u^2 D^* v + v^2 D^* u$

(C) D*u + D* v (D) uvD* (u + v)

889. If u = f(x), v = g(x) then the value of $D^*\left\{\frac{u}{v}\right\}$ is

(A) $\frac{u^2 D^* v - v^2 D^* u}{v^4}$ (B) $\frac{uD^* v - v D^* u}{v^2}$ (C) $\frac{v^2 D^* u - u^2 D^* v}{v^4}$ (D) $\frac{vD^* u - u D^* v}{v^2}$

890. The value of D* c, where c is constant, is

(A) non-zero constant (B) 2

(C) does not exist (D) zero

COMPREHENSION-26

Paragraph for Questions Nos. 891 to 893

Consider the implicit equation $x^2 + 5xy + y^2 - 2x + y - 6 = 0$ ....(i)

891. The value of $\frac{dy}{dx}$ at (1, 1) is

(A) $\frac{5}{8}$ (B) $-\frac{5}{8}$

(C) $\frac{8}{5}$ (D) $-\frac{8}{5}$

892. The value of $\frac{d^2y}{dx^2}$ at (1, 1) is

(A) $\frac{111}{256}$ (B) $-\frac{111}{256}$

(C) $\frac{256}{111}$ (D) $-\frac{256}{111}$

893. The equation of normal to the conic (i) at (1, 1) is

(A) $5x - 8y - 3 = 0$ (B) $8y - 5x - 3 = 0$

(C) $8x - 5y - 3 = 0$ (D) $8x - 5y + 3 = 0$

COMPREHENSION-27

Paragraph for Questions Nos. 894 to 896

If $f : [0, 2] \to [0, 2]$ is a bijective function defined by $f(x) = ax^2 + bx + c$, where a, b, c are non zero real numbers, then

894. f(2) is equal to

(A) 2 (B) α where $a \in (0, 2)$

(C) 0 (D) cannot be determined

895. Which of the following is one of the roots f(x) = 0 ?

(A) $\frac{1}{a}$ (B) $\frac{1}{b}$

(C) $\frac{1}{c}$ (D) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

896. Which of the following is not a value of 'a'

(A) $a = -\frac{1}{4}$ (B) $a = \frac{1}{2}$

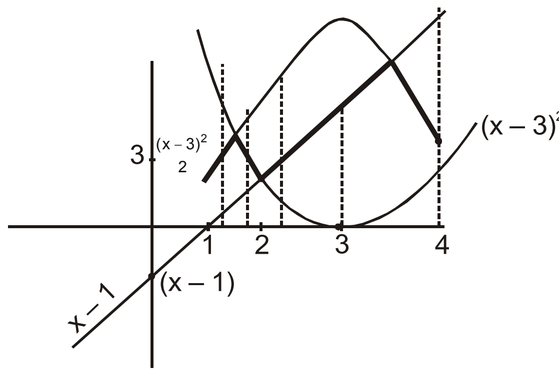
(C) $a = -\frac{1}{2}$ (D) $a = 1$

COMPREHENSION-28

Paragraph for Questions Nos. 897 to 899

If f(x) = Mid {g(x), h(x), p(x)} means the function which will be second in order when values of the three function at a particular x are arranged?

$$f(x) = \text{Mid}\left\{x-1, (x-3)^2, 3-\frac{(x-3)^2}{2}\right\}, x \in [1, 4]$$



897. Numerical value of difference between the LHD and RHD at the point x = 2 for f(x) in x ∈ [1, 4] will be
(A) 0 (B) 2 (C) 3 (D) 1

898. The greatest value of f(x) in [1, 4] will be
(A) $1 + \sqrt{3}$ (B) $2 + \sqrt{3}$
(C) $3 + \sqrt{3}$ (D) N.O.T.

899. Rate of change of x w.r.t. f(x) at x = 3 will be
(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $-\frac{3}{2}$

COMPREHENSION-29

Paragraph for Questions Nos. 900 to 902

Let $f(x) = \begin{cases} 2x+a & : x \ge -1 \\ bx^2+3 & : x < -1 \end{cases}$

and $g(x) = \begin{cases} x+4 & : 0 \le x \le 4 \\ -3x-2 & : -2 < x < 0 \end{cases}$

functions

900. g(f(x)) is not defined if
(A) a ∈ (10, ∞), b ∈ (5, ∞) (B) a ∈ (4, 10), b ∈ (5, ∞)
(C) a ∈ (10, ∞), b ∈ (1, 5) (D) a ∈ (4, 10), b ∈ (1, 5)

901. If domain of g(f(x)) is [–1, 4], then
(A) a = 0, b > 5 (B) a = 2, b > 7
(C) a = 2, b > 10 (D) a = 0, b ∈ R

902. If a = 2 and b = 3 then range of g(f(x)) is
(A) (–2, 8] (B) (0, 8]
(C) [4, 8] (D) [–1, 8]

Comprehension-30

Paragraph for Questions Nos. 903 to 905

Let $f : R \to R$ is a function satisfying $f(2-x) = f(2+x)$ and $f(20-x) = f(x)$, $\forall x \in R$. For this function f answer the following.

903. If $f(0) = 5$, then minimum possible number of values of x satisfying $f(x) = 5$, for $x \in [0, 170]$, is
(A) 21 (B) 12 (C) 11 (D) 22

904. Graph of $y = f(x)$ is
(A) symmetrical about $x = 18$
(B) symmetrical about $x = 5$
(C) symmetrical about $x = 8$
(D) symmetrical about $x = 20$

905. If $f(2) \neq f(6)$, then
(A) fundamental period of f(x) is 1
(B) fundamental period of f(x) may be 1
(C) period of f(x) can't be 1
(D) fundamental period of f(x) is 8

Comprehension-31

Paragraph for Questions Nos. 906 to 908

If $f : (0, \infty) \to (0, \infty)$ satisfy $f(xf(y)) = x^2y^a$ $(a \in R)$, then

906. Value of a is
(A) 4 (B) 2 (C) $\sqrt{2}$ (D) 1

907. $\sum_{r=1}^{n} f(r)\,{}^nC_r$ is
(A) $n.2^{n-1}$
(B) $n(n-1)\,2^{n-2}$
(C) $n.2^{n-1} + n(n-1)\,2^{n-2}$
(D) 0

908. Number of solutions of $2f(x) = e^x$ is
(A) 1 (B) 2 (C) 3 (D) 4

Comprehension-32

Paragraph for Questions Nos. 909 to 911

Consider two functions $f(x) = \lim_{n\to\infty}\left(\cos\frac{x}{\sqrt{n}}\right)^n$ and $g(x) = -x^{4b}$ where $b = \lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$.

Then

909. f(x)h is
(A) e^{-x^2} (B) $e^{\frac{-x^2}{2}}$ (C) e^{x^2} (D) $e^{\frac{x^2}{2}}$

910. g(x) is
(A) $-x^2$ (B) x^2 (C) x^4 (D) $-x^4$

911. Number of solutions of $f(x) + g(x) = 0$ is
(A) 2 (B) 4 (C) 0 (D) 1

COMPREHENSION-33

Paragraph for Questions Nos. 912 to 914

Let $f(x) = \lim_{n \to \infty} \left(\cos\sqrt{\frac{x}{n}}\right)^n$, $g(x) = \lim_{n\to\infty} \left(1 - x + x\sqrt[n]{e}\right)^n$. Now, consider the function y = h(x), where $h(x) = \tan^{-1}(g^{-1} f^{-1}(x))$.

912. $\lim_{x\to 0} \frac{\ell n\,(f(x))}{\ell n\,(g(x))}$ is equal to

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 0 (D) 1

913. Domain of the function y = h(x) is

(A) $(0, \infty)$ (B) R
(C) (0, 1) (D) [0, 1]

914. Range of the function y = h(x) is

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2}, 0\right)$ (C) R (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

COMPREHENSION-34

Paragraph for Questions Nos. 915 to 917

If f(x) approaches to zero as x approaches to 'a' then

$\lim_{x\to a} \frac{\sin(f(x))}{f(x)} = 1$, $\lim_{x\to a} \frac{\tan(f(x))}{f(x)} = 1$, $\lim_{x\to a} \frac{\ell n(1+f(x))}{f(x)} = 1$

$\lim_{x\to a} \frac{K^{f(x)}-1}{f(x)} = \ell n(K)$, K > 0 (K is independent of x)

915. $\lim_{x\to 0} \frac{\tan(\sin x)}{\sin x}$ is

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) – 1

916. $\lim_{x\to\infty} x \sin\left(\frac{2}{x}\right)$ is

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 0

917 $\lim_{x\to 0} \frac{\sin([x^2])}{x^2}$, where [.] denote the greatest integer function

(A) is 1 (B) is 0
(C) does not exist (D) none of these

SECTION - 4 (MATRIX MATCH Type)

918. Match the following:

List – I	List – II
(A) $\int_0^{\pi/2} \frac{dx}{1+\tan x}$	(i) $\frac{1}{117}$
(B) If $\int_0^{x^2(1+x^5+7x^{12})} f(t)dt$ = x, then f (3) is equal to	(ii) $\frac{\pi}{2} - \log 2$
(C) $\int_0^{\infty} e^{-2x}(\sin 2x + \cos 2x)dx$ is equal to	(iii) $\frac{1}{2\sqrt{2}}$
(D) $\int_0^1 \cot^{-1}(1+x^2-x)dx$ is equal to	(iv) $\frac{\pi}{4}$

919. Match the following:

List – I	List – II
(A) $\lim_{x\to-\infty} \frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{1+\|x\|^3}$ is equal to	(i) – 24
(B) $\lim_{x\to\infty} \left(\frac{x+8}{x+3}\right)^{x+6}$ is equal to	(ii) $\frac{1}{e}$
(C) $\lim_{x\to\pi/3} \frac{\tan^3 x - 3\tan x}{\cos\left(x+\frac{\pi}{6}\right)}$ is equal to	(iii) – 1
(D) $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$ is equal to	(iv) e^5

920. Match the following:

List – I	List – II
(A) $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} dx$ =	(i) $\frac{3\pi}{2}$
(B) The value of α which satisfy $\int_{\pi/2}^{\alpha} \sin x dx = \sin 2\alpha$ is	(ii) $\frac{\pi}{2}$
(C) I = $\int_0^{\pi/2} \frac{f(x)}{f(x)+f\left(\frac{\pi}{2}-x\right)} dx$, then value of 2I is	(iii) $-\frac{\pi}{2}$
(D) $\int_{-1}^{1} \left\{\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)\right\} dx$ =	(iv) $\frac{\pi}{12}$

921. Match the following:

List – I	List – II
(A) If $x \cdot e^{xy} = y + \sin^2 x$, then $\left[\frac{dy}{dx}\right]_{x=0}$ is equal to	(i) 4
(B) Derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at x = $\frac{1}{2}$ is	(ii) 0
(C) Let $f(x) = \begin{cases} -x-2, & for\ -3 \le x \le 0 \\ x-2, & for\ 0 < x \le 3 \end{cases}$, g(x) = |f(x)| + f(|x|), then the number of points of non-differentiability of g(x) is	(iii) 1
(D) Let F (x) = f (x) g (x) h (x) ∀ real x, where f, g and h are differentiable functions. At some point x_0, F′ (0) = 21 F (x_0), f′ (x_0) = 4f (x_0) g′ (x_0) = –7g (x_0) and h′ (x_0) = kh (x_0), then $\frac{k}{12}$ is equal to	(iv) 2

922. Match the following

List – I	List – II
(A) Domain of ${}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$	(i) $\left\{-1, 2, \frac{1}{2}\right\}$
(B) Range of f(x) = $\lim_{n\to\infty} \frac{1+x^{2n}}{2x^{2n}-1}$	(ii) {–1, 1}
(C) Domain of $\sin^{-1}\left(\frac{1+x^2}{2x}\right)$	(iii) ϕ
(D) Domain of f(x) = $\frac{1}{\sqrt{x-\|x\|}}$	(iv) {2, 3}

923. If $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$, then match the following :

List – I	List – II
(A) $\int_0^\infty \frac{\sin 5x}{x} dx$	(i) 0
(B) $\int_0^\infty \frac{\sin ax \cos bx}{x} dx$ (a> b > 0)	(ii) $\frac{\pi}{2}$
(C) $\int_0^\infty \frac{\sin^2 x}{x^2} dx$	(iii) $\frac{\pi}{4}$
(D) $\int_0^\infty \frac{\sin^3 x}{x} dx$	(iv) π

924. Match the following :

List – I	List – II
(A) $x^{100} + \sin x - 1$ is decreasing in	(i) $(-\infty, \infty)$
(B) Domain of $\log_4\log_5\log_3(18x - x^2 - 80)$	(ii) $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup (1, \infty)$
(C) Range of $x^3 + 3x^2 + 10x + 2\sin x$	(iii) (8, 10)
(D) $\left(\frac{\sqrt{a+4}}{1-a} - 1\right)x^5 - 3x + \log 5$ decreases for all R the set of values of a	(iv) none of these
	(v) $\left(\frac{\pi}{2}, \pi\right)$

925. Match the following:

List – I	List – II
(A) Let $f : R \to R$ be a periodic function such that $f(T + x) = 1 + [1 - 3f(x) + 3(f(x))^2 - (f(x)^3]^{1/3}$, where T is fixed positive number, then period of f(x) is AT, where A =	(i) 3
(B) The area between the curve $y = 2x^4 - x^2$, the x-axis and the ordinates of two minima of the curve is $\frac{B}{120}$ where B is	(ii) 2
(C) $\int_0^4 \frac{f(x)}{f(x)+f(4-x)}dx$ =	(iii) 7
(D) f(x) = $\begin{cases} \frac{1-\sin x}{(\pi-2x)^2}\frac{\log\sin x}{\log(1+\pi^2-4\pi x+4x^2)} \\ k, \quad x=\frac{\pi}{2} \end{cases}$ is continuous at x $= \frac{\pi}{2}$ then $-\frac{1}{k}$ equals to	(iv) 64

926. Match the list:

List – I	List – II
(A) $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n}\sqrt{\frac{n+r}{n-r}}$	(i) $\frac{\pi}{2}$
(B) $\lim\limits_{x\to\infty}\left[\frac{1}{\sqrt{n^2-1}}+\frac{1}{n^2-2^2}+\cdots+\frac{1}{\sqrt{n^2-(n-1)^2}}\right]$	(ii) $\frac{\pi}{2}+1$
(C) $\lim\limits_{x\to\infty}\left(\frac{n!}{n^n}\right)^{1/n}$	(iii) 2π
(D) $\int_0^{2\pi} e^{\cos x}\cos(\sin x)dx$	(iv) $\frac{1}{e}$
	(v) e

927. Match the following:

List I (Expression)		List II (Value)	
I.	If f(x + y) = f(x) f(y) (x, y are independent $\forall$ x, y $\in$ R and f(2) = f′(2) = 3 then f′(4) =	(A)	**0**
II.	If f(xy) = f(x) f(y) and f′(4) = 2f′(8), then f(2) =	**(B)**	**10**
III.	If f(x) is a diff. function such that f(xy) = f(x) + f(y) $\forall$ x, y $\in$ R then $f(e) + f\left(\frac{1}{e}\right) =$	**(C)**	**9**
IV.	If f is a twice diff. function Such the f′(x) = –f(x) If $h(x) = (f(x)^2 + (g(x))^2$ And h(5) = 10. Then h(10) =	**(D)**	**1**

928. Match the following:

(A) $\frac{\sin 1}{\sin 2} - \frac{\sin 5}{\sin 6}$ (1) positive

(B) $\tan\frac{3}{2} - \frac{9}{4}$ (2) negative

(C) $\lim_{x\to 0}\left[\frac{e^x - 1}{x}\right]$ (where [.] denotes the greatest integer function) (3) 1

(D) If $f'(\alpha) = 0$ and $f'(x) > 0\ \forall\ x \in R - \{\alpha\}$, then $f''(\alpha)$ is (4) does not exist

(5) 0

929. If $y = \cos^{-1}\left(\frac{a\cos x + b}{a + b\cos x}\right) - 2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$, then match the following :

Column– I		Column – II	
(A)	If a = 4, b = 3, then $\frac{dy}{dx}$ at x = 0	(P)	0
(B)	Number of points of local minima	(Q)	$\frac{4}{5}$
(C)	For a = 4, b = 3, value of y at x = 0	(R)	$\frac{3}{5}$
(D)	Number of tangents parallel to the y axis	(S)	2

930. Let $f(x) = ax^2 + bx + c$, Given that $f'(1) = 8, f(2) + f''(2) = 33$ and 2a + 3b + 6c = 14, then match the following

Column– I		Column – II	
(A)	Global maximum value of f(x)	(P)	Not defined
(B)	If global minimum value of f(x) = k then 28k is equal to	(Q)	48
(C)	Number of real roots of f(x) = 0	(R)	0

(D)	Number of real roots of f(x) = 3	(S)	2

931.

	Column I		Column II
(A)	The number of non-differentiability points on the curve $y = \|e^{\|x\|} - 3\|$ is/are	(p)	1
(B)	Length of the latus-rectum of the parabola defined by x = cos t – sint and y = sin 2t	(q)	0
(C)	The number of real solution of the equation $x^{2\log_x(x+3)} = 16$ is	(r)	3
(D)	If in a triangle 2R + r = r_1, then $\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right)$ is equal to	(s)	2

932.

(A)	Area of the rectangle formed by asymptotes of the hyperbola xy – 3y – 2x = 0 and co-ordinate axes is	(p)	32
(B)	Area bounded by min (|x|, |y|) = 1 and max (|x|, |y|) = 3 is,	(q)	2
(C)	The number of common tangents of the two circles $x^2 + y^2 - 10x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 2y - 11 = 0$ are	(r)	5
(D)	The greatest value of $f(x) = 2\cos(2xe^x + 7x^4 - \log(1 + x^2))$	(s)	6

933. List I with List II and select the correct answers using the codes given below the lists :

List I LIMIT		**List II** VALUE
1.	$\lim\limits_{x\to 0} \frac{(x)}{\tan(x)}$	(A) $-\log_{16}e$
2.	$\lim\limits_{x\to 0} \frac{\sqrt{1-x+x^2}-\sqrt{1+x^2}}{4^x-1}$	(B) e^{-1}
3.	$\lim\limits_{x\to 0} \frac{2e^{\sin x}-(1+\sin x)^2}{2(\tan^{-1}(\sin x))^2}$	(C) 1
4.	$\lim\limits_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$	(D) 0

934.

List I Limits		**List II** Value
I.	$\lim\limits_{x\to 2} \frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$	(A) $-\frac{4}{3}$
II.	$\lim\limits_{x\to 2} \frac{3^x+3^{3-x}-12}{3^{3-x}-3^{x/2}}$	(B) $\frac{1}{2}$
III.	$\lim\limits_{x\to 2} \left(\frac{x^3-4x}{x^3-8}\right)^{-1} - \left(\frac{x+\sqrt{2}x}{x-2} - \frac{\sqrt{2}}{\sqrt{x}-\sqrt{2}}\right)^{-1}$	(C) 8
IV.	$\lim\limits_{x\to 2} \frac{2^x+2^{3/2}-6}{\sqrt{2^{-x}}+2^{1-x}}$	(D) $\frac{1}{4(3+\sqrt{3})}$

935. **List I** **List II**

Function f(x) **The No.of points of discontinuity**

I. f(x) = $\frac{1+\cos 5x}{1-\cos 4x}$ in $0 \le x \le \pi$ (A) 0

II. f(x) = $[\cos x + \sin x]$ $0 < x < 2\pi$ (B) Infinite

III. f(x) = 4x + 7[x] + 2log (1 + x) (C) 5

IV. f(x) = $\int_0^x t \sin\frac{1}{t} dt$ where $0 < x < \pi$ (D) 3

936. **List I** **List II**

(Function) **(Limit at x = 0)**

I. f(x) = $\left[\frac{x}{\sin(x)}\right]$ (A) $\underset{x\to 0}{Lt} f(x) = 0$

II. f(x) = $\frac{a^{[x]+x}-1}{[x]+x}$ (B) $\underset{x\to 0}{Lt} f(x) = e^{\sin^2 y}$

III. f(x) = $\frac{\sin[\cos x]}{1+[\cos x]}$ (C) $\underset{x\to 0-}{Lt} f(x) = 1-\frac{1}{a}$

IV. f(x) = $\frac{1}{x}\left(\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt\right)$ (D) $\underset{x\to 0}{Lt} f(x) = 1$

Where [.] denotes G.I.F..

937. **List I** **List II**

(Function) **(Derivative)**

I. f(x) = $\begin{cases} 1, & x \le 0 \\ 1+\sin x, & 0 \le x < \frac{\pi}{2} \end{cases}$ (A) L→' (0) = –1, R*f*'(0) = 0

II. = x($\sqrt{x} - \sqrt{x+1}$) (B) f′(0) = 0

III. = x^3 sgn x (C) f′(0) = 1

IV. g(x) = $\begin{cases} -1, & -2 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases}$

f(x) = g(|x|) + |g(x)| (D) f'(0) = –1 does not exist

938. Match the columns -

Column – I		Column - II	
(a)	The number of values of c for which $\int_0^1 \lvert (c-x) \rvert \, dx = \frac{1}{2}$ is	(P)	2
(b)	If $\int \frac{\operatorname{cosec}\left(2x-\frac{5\pi}{6}\right)}{\sin\left(2x-\frac{\pi}{6}\right)} dx = \frac{k}{\sqrt{3}} \ell n \left(\frac{\sin\left(2x-\frac{5\pi}{6}\right)}{\sin\left(2x-\frac{\pi}{6}\right)}\right)$ + c, then k =	(Q)	1
(c)	Area of the region bounded by the $x^2 + y^2 - 2x \le 0$, $x + y \le 1$, $y \ge 0$ is	(R)	$\frac{\pi}{8}$
(d)	$\frac{1}{\pi}\int_{-\pi}^{\pi} \frac{x\sin x}{e^x+1}$ dx is equal to	(S)	8

939. Match the following

Column – I		Column – II	
(a)	The number of solutions of the equation $x.2^x = x+1$ is	(P)	4
(b)	$\lim_{n\to\infty}\left(\frac{1+\sqrt[n]{4}}{2}\right)^n$ is equal to	(Q)	8
(c)	The number of points at which $g(x) = \frac{1}{1+\frac{2}{f(x)}}$ is not differentiable where $f(x) = \frac{1}{1+\frac{1}{x}}$, is :	(R)	2
(d)	$f\left(x+\frac{1}{2}\right)+f\left(x-\frac{1}{2}\right) = f(x)$ for all $x \in R$, then period of $f(x)$ is	(S)	3

940. Match the following

Column – I		Column – II	
(a)	The least positive integral solution of $x^2 - 4x > \cot^{-1} x$	(P)	1
(b)	The least positive integral value of x for which $f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$ is increasing	(Q)	2
(c)	Let $f : R \to R$ be such that $f(a) = 1, f'(a) = 2,$ then $\lim_{x\to 0}\left(\frac{f^2(a+x)}{f(a)}\right)^{\frac{1}{x}} = e^k,$ then $k =$	(R)	5
(d)	Number of integral values of x which satisfy equation $\sin^{-1}\left((3x-x)(x-1)\right) + \sin^{-1}\left(2-\lvert x\rvert\right) = \frac{\pi}{2}$ is/are	(S)	4

941. Consider f(x) = $t^{|x^2-4x+3|}$, where t is a real number greater than 1. Then

Column I		Column II
(A) f(x) increases in the interval	(P)	$(-\infty, 1)$
(B) f(x) decreases in the interval	(Q)	(0, 2)
(C) Local maxima of f(x) occurs in the interval	(R)	$(1, 2) \cup (3, \infty)$
(D) f(x) has a local minima in the interval	(S)	(1,3)

942. Match the column

	Column I		Column II
(A)	Let a, b, c be positive and a + b + c = abc the maximum value of square of least among a, b, c is	(P)	1
(B)	The fundamental period of the function y = $\sin^2\left(\frac{\sqrt{2}t+3}{6\pi}\right)$ is $\lambda\pi^2$ then the value of $\frac{\lambda}{\sqrt{2}}$ is	(Q)	2
(C)	If $(x + y)^m$ has three consecutive cofficients in A.P. ($m \in N$) for which the sum of first 'n' values of m is $an^3 + bn^2 + cn + d$. The value of greatest integer of $\left(\frac{a+b+c+d}{2}\right)$ is	(R)	3
(D)	If equation of tangent to the curve y = $\int_{x^2}^{x^3} \frac{dt}{\sqrt{1+t^2}}$ at x = 1 is $\sqrt{2}$ x = by + $\sqrt{2}$ then value of $\frac{b}{2}$ is	(S)	Number of solution of $\cos x + \cos\sqrt{2}\, x = 2$
		(T)	Number of values of x for which f (x) = $\frac{1}{\ell n\,|x|}$ is not defined

943. Match the column

	Column I		Column II
(A)	The area of the quadrilateral formed by the tangents from the point (4, 0) to the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ and the pair of radii through the points of contanct of the tangent is	(P)	1
(B)	The number of points at which the function f(x) = $\frac{1}{\ell n|x|}$ is discontinuous is	(Q)	3
(C)	Let f(x + y) = f(x). f(y) for all x, y $\in$ R if f (5) = 2 and $f'(0)$ = 3, then $f'(5)$ =	(R)	6
(D)	If the normal to the curve y = f (x) at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3)$ =	(S)	4

944. **Match the following**

Column-I		Column-II
(a)	If the point (6, k) is closest to the curve $x^2 = 2y$ at (2, 2), then k =	(P) 0
(b)	If the curve $y = px^2 + qx + r$ passes through the point (1, 2) and touches the line y = x at the origin, then the value of p – q + r =	(Q) –2
(c)	Let $f(x) = kx^3 + 9kx^2 + 9x + 3$ be a strictly increasing function and has non stationary point. The greatest value of k is	(R) 1
(d)	Let $0 < a < b < \frac{\pi}{2}$. If $f(x) = \begin{bmatrix} \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \\ \tan x & \tan a & \tan b \end{bmatrix}$, then minimum	(S) does not exist

possiable number of roots of $f'(x) = 0$ lying in (a, b) is

945. **Column I** **Column II**

(A) if $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$ where $g(x) = \int_0^{\cos x} (1 + \sin t^2)\, dt$ (P) 3

then the value of $f'(\pi/2)$

(B) If $f(x)$ is a non zero differentiable function such that (Q) 2

$\int_0^x f(t)dt = (f(x))^2$ for all x, then $f(2)$ equals

(C) If $\int_a^b (2 + x - x^2)\, dx$ is maximum then (a + b) is eual to (R) 1

(D) If $\lim\limits_{x\to 0} \left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2}\right) = 0$ then (3a + b) has the (S) –1

value equals to

946. **Column I** **Column II**

(A) Number of integers which do not lie inthe range (P) 0

of the function $f(x) = \sec\left(2\sin^{-1}\frac{1}{x}\right)$

(B) Let f : (0, ∞) onto (0, ∞) be a derivable function for which there exists its primitive F such that $2(F(x) - f(x)) = f^2(x)$ for any real (Q) 1

positive x. Then $\lim\limits_{x\to\infty} \frac{f(x)}{x}$ equals

(C) How many of the following derivatives are correct (on their domains)? (R) 2

I. $\frac{d}{dx} \ln|\sec x| = \tan x$ **II.** $\frac{d}{dx} \ln(x + e^x) = 1 + \frac{1}{x}$

III. $\frac{d}{dx} x^{\ln x} = (\ln x)\, x^{\ln(x)-1}$

(D) A differentiable function satisfies $f'(x) = f(x)\, 2e^x$ with initial conditions f (0) = 0. The area enclosed f (x) and the x-axis is (S) 3

947. **Column I** **Column II**

(A) Suppose, $f(n) = \log_2(3) . \log_3(4).\log_4(5)......\log_{n-1}(n)$ then the sum $\sum_{k=2}^{100} f(2^k)$ equals (P) 5010

(B) Let $f(x) = \sqrt{1+x\sqrt{1+(x+1)\sqrt{1+(x+2)(x+4)}}}$ then $\int_0^{100} f(x)\,dx$ is (Q) 5050

(C) In an A.P. the series containing 99 terms, the sum of all the odd numbered terms is 2550. The sum of all the 99 terms of the A.P. is (R) 5100

(D) $\lim_{x\to 0} \frac{\prod_{r=1}^{100}(1+rx)-1}{x}$ equals (S) 5049

948. **Column - I** **Column - II**

(A) Let f be continuous and the function F is defined as (P) 0

$F(x) = \int_0^x \left(t^2 . \int_1^t f(u)du \right) dt$ where $f(1) = 3$, then $F'(1) + F''(1)$ has the value equal to (Q) 1

(B) For each value of x a function f(x) is defined as (R) 2

min $\{2x + 3, \frac{(x+4)}{3}, 3(6 - x)\}$ Maximum value of f(x) is .

(C) $\lim_{x\to 1^+} (\ell n\, x)^{\frac{1}{(x-1)\tan x}}$ (S) 3

(D) Exponent of 2 in the binomial coefficient ${}^{500}C_{212}$ is

949. **Column I** **Column II**

(A) If three normals can be drawn to the curve $y^2 = x$ from the point (c, 0), then c can be equal to

(B) Subnormal length to $xy = c^2$ at any point varies directly as

(C) If the sides and angles of a plane triangle vary in such a way that its circum radius remains constant, then

$$\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} =$$

where da, db, dc are small increments in the sides a, b, c, respectively

(P) 1

(Q) 0

(R) $\frac{5}{4}$

(S) Cube of ordinate

(T) 2

950. **Column-I** **Column-II**

	Column-I		Column-II
(A)	$\int_{-\pi/2}^{\pi/2} \frac{\ell n(\cos x)}{1+e^{x}.e^{\sin x}} dx =$	(P)	$-2\pi \ \ell n\ 2$
(B)	$\int_{0}^{2\pi} \ell n\,(1+\sin x)\,dx =$	(Q)	$-\frac{\pi}{4}\ell n2$
(C)	$\int_{-\pi/4}^{\pi/4} \ell n\sqrt{1+\sin 2x}\,dx =$	(R)	$-\pi\ \ell n2$
(D)	$\int_{\infty}^{0} \frac{xe^{-x}}{\sqrt{1-e^{-2x}}}dx.$	(S)	$-\frac{\pi}{2}\ell n2$

951. **Column I** **Column II**

	Column I		Column II
(A)	The function $f(x)-(x-[x])^2$, (where [x] is greatest integer function $\le x$) is	(P)	periodic
		(Q)	non - periodic
(B)	The function $f(x)=\log_a\left(x+\sqrt{(x^2+1)}\right)$; $a>0,\ a\neq 1$ is (assume it to be an onto)	(R)	one - one
		(S)	many one
(C)	The function f(x) = cos(5x + 2) is	(T)	invertible

952. **Column I** **Column II**

	Column I		Column II
(A)	The area bounded by the curve max $\{\lvert x\rvert, \lvert y\rvert\}=1$ is	(P)	0
(B)	If the point (a, a) lies between the lines |x + y| = 6, then [|a|] is (where [.] denotes the greatest integer function)	(Q)	1
		(R)	2
(C)	Number of integral values of b for which the origin and the point (1, 1) lie on the same side of the st. line $a^2x+aby+1=0$ for all $a\in R\sim\{0\}$	(S)	3
		(T)	4

953. **Column I** **Column II**

	Column I		Column II
(A)	The slope of the curve $2y^2=ax^2+b$ at $(1,-1)$ is -1, then	(P)	a - b = 2
(B)	If (a, b) be the point on the curve $9y^2=x^3$ where normal to the curve makes equal intercepts with the axes, then	(Q)	$a-b=\frac{7}{2}$
		(R)	$a-b=\frac{4}{3}$
(C)	If the tangent at any point (1, 2) on the curve $y=ax^2+bx+\frac{7}{2}$ be parallel to the normal at (-2, 2) on the curve $y = x^2 + 6x + 10$, then	(S)	$\alpha+\lvert b\rvert=\frac{20}{3}$
		(T)	5a + 2b = 0

954. Match the column

Column – I		Column – II	
(A)	The number of possible values of k if fundamental period of $\sin^{-1}(\sin kx)$ is $\frac{\pi}{2}$	(p)	1
(B)	Numbers of points in the domain of $f(x) = \tan^{-1}x + \sin^{-1}x + \sec^{-1}x$	(q)	2
(C)	$f(x) = \sin\left(\frac{\pi x}{2}\right) . \operatorname{cosec}\left(\frac{\pi x}{2}\right)$ is periodic with period	(r)	3
(D)	If range of the function $f(x) = \cos^{-1}[5x]$ where [.] denotes greatest integer, is {a, b, c}, then a + b + c is	(s)	4

955. Match the column

Column – I		Column – II	
[.] and {.} represent the greatest integer and fractional part functions respectively.			
(A)	Number of solutions of $[x] = \cos^{-1}x$	(p)	3
(B)	Number of solutions of $\sin^{-1}x = \text{sgn}(x)$	(q)	2
(C)	Number of solutions of $\{x\} = e^{x^2}$	(r)	1
(D)	Number of solutions of $\frac{\sin^{-1}x + \cos^{-1}x}{2} = \{x\}$	(s)	0

956. Match the column

Column – I		Column – II	
(A)	Smallest positive integral value of x for which $x^2 - x + \sin^{-1}(\sin 2) < 0$ is	(p)	$\frac{3\pi}{2}$
(B)	Number of solution of $2[x] = x + 2\{x\}$ is where [x], {x} are greatest integer and least integer functions respectively.	(q)	3
(C)	If $x^2 + y^2 = 1$, then maximum value of x + y is	(r)	1
(D)	$f\left(x+\frac{1}{2}\right) + f\left(x-\frac{1}{2}\right) = f(x)$ for all $x \in R$, then period of f(x) is	(s)	2

957. Match the column

Column – I		**Column – II**	
(A)	If function f(x) is defined in [–2, 2], then domain of f(|x| + 1) is	(p)	$\left[\frac{1}{4}, \frac{3}{4}\right]$
(B)	If range of the function f(x) = $\frac{\sin^{-1}x + \cos^{-1}x + \tan^{-1}x}{\pi}$ is	(q)	[–1, 1]
(C)	Range of the function f(x) = 3 |sin x| – 4 |cos x| is	(r)	[– 4, 3]
(D)	Range of f(x) = $(\sin^{-1}x) \sin x$ is	(s)	$\left[0, \frac{\pi}{2}\sin 1\right]$

958.

Column - I		**Column - II**	
(A)	Range of sgn {x} is (where {.} represents fractional part function)	(p)	{1}
(B)	Domain of $\sin^{-1} x + \sin^{-1}(1 - x)$ is	(q)	[0, 1)
(C)	Range of $\sqrt{\frac{2\tan^{-1}x}{\pi}}$ is	(r)	{0, 1}
(D)	Range of $\frac{2}{\pi}\sin^{-1}[x^2 + x + 1]$ is (where [.] represent greatest integer function)	(s)	[0, 1]

959.

Column – I		**Column – II**	
(A)	Domain of f(x) = $\sin^{-1}\left(\frac{2-x}{2x}\right)$ is	(p)	$[-2, \infty)$
(B)	Range of f(x) = $\frac{2x^2 - 2}{3x^2 + 1}$ is	(q)	$(-\infty, -1] \cup [1, \infty)$
(C)	Set of all values of p for which the function f(x) = px + sin x is bijective is	(r)	$(-\infty, -2] \cup [2/3, \infty)$
(D)	If f : $(-\infty, 1] \to A$ is defined by f(x) = $x^2 - 3x$, then set A for which f(x) becomes invertiable, is	(s)	[–2, 2/3)

960. Match the column

Column – I		Column – II	
(A)	$\lim\limits_{x\to\frac{\pi}{2}^+} \dfrac{\cos(\tan^{-1}(\tan x))}{x-\frac{\pi}{2}}$	(p)	$\dfrac{3\pi}{2}$
(B)	The number of solutions of the equation $2\cos x = \lvert \sin x \rvert$ $0 \le x \le 4\pi$ is	(q)	$-\dfrac{1}{\sqrt{2}}$
(C)	If $y = \tan^{-1}\sqrt{\dfrac{1-\sin x}{1+\sin x}}$, then the value of $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{6}$ is	(r)	1
(D)	If $f(x) = e^{\{2x\}} + \sin 2\pi x$, the period of f(x) is { } represents fractional part function	(s)	Does not exist

961. Match the column

Column – I		Column – II	
(A)	Let $f : R \to R$ be a differentiable function and $f(1) = 1$, $f'(1) = 3$. Then the value of $\lim\limits_{x\to 1} \displaystyle\int_1^{x^2} \dfrac{(f(t)-t)}{(x-1)^2}\,dt$ is	(p)	0
(B)	$\lim\limits_{n\to\infty} \left(\dfrac{1+\sqrt[n]{4}}{2}\right)^n$ is equal to	(q)	– 1
(C)	If $f(x) = \lim\limits_{n\to\infty} \dfrac{2x}{\pi} \cdot \tan^{-1}(nx)$, $x > 0$ then $\lim\limits_{x\to 0^+} [f(x) - 1]$ is, {where [ ] represents greatest integer function}	(r)	2
(D)	$\lim\limits_{n\to\infty} \left[\displaystyle\sum_{r=1}^{n} \dfrac{1}{2^r}\right]$, where [ ] denotes the greatest integer function	(s)	4
		(t)	1

962. **Column - I** **Column - II**

	Column - I	Column - II
(A)	Number of points of discontinuity of $f(x) = \tan^2 x - \sec^2 x$ in $(0, 2\pi)$ is	(p) 1
(B)	Number of points at which $f(x) = \sin^{-1}x + \tan^{-1}x + \cot^{-1}x$ is non-differentiable in $(-1, 1)$ is	(q) 2
(C)	Number of points of discontinuity of $y = [\sin x]$, $x \in [0, 2\pi)$ where [.] represents greatest integer function	(r) 0
(D)	Number of points where $y = \|(x-1)^3\| + \|(x-2)^5\| + \|x-3\|$ is non-differentiable	(s) 3

963. **Column – I** **Column – II**

For $x \in R$,

	Column – I		Column – II
(A)	$f(x) = \{\sin(\pi x)\}$ is discontinuous for $x \in$	(p)	$[0, 1)$
(B)	$g(x) = \left\{\dfrac{\sin x}{x}\right\}$ is discontinuous for $x \in$	(q)	$\{1, 2\}$
(C)	$h(x) = \dfrac{\{\sin x\}}{\{x\}}$ is non-differentiable for $x \in$	(r)	$\{0\}$
(D)	$u(x) = \dfrac{(\sin x)}{[x]}$ is discontinuous function for $x \in$	(s)	$\left\{\dfrac{1}{2}\right\}$

964. **Column – I** **Column – II**

	Column – I		Column – II
(A)	Point of discontinuity of $y = \dfrac{1}{t^2 - t - 2}$ where $t = \dfrac{1}{x+1}$	(p)	$-\dfrac{1}{2}$
(B)	Points of continuity of $y = [x] + [-x]$	(q)	-2
(C)	$y = [\sin(\pi x)]$ is non differentiable at	(r)	-1
(D)	$f(x) = \|2x + 1\| + \|x + 2\| - \|x + 1\| - \|x - 4\|$ is non differentiable at	(s)	4

965. Match the column

Column – I		Column – II	
(A)	$\lim_{x\to\frac{1}{\sqrt{2}}} \dfrac{x-\cos(\sin^{-1}x)}{1-\tan(\sin^{-1}x)}$ is equal to	(p)	does not exist
(B)	If f(x) = $\log_{x^2}(\log x)$, then $f'\left(\frac{1}{2}\right)$ is equal to	(q)	0
(C)	For the function f(x) = $\ell n \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ if $\frac{dy}{dx}$ = sec x + p, then p is equal to	(r)	28
(D)	$\lim_{x\to 0} \frac{1}{x}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ is equal to	(s)	4

966. Match the column

Column – I		Column – II	
(A)	If y = $\cos^{-1}(\cos x)$, then y′ at x = 5 is equal to	(p)	–1
(B)	The value of $\frac{1}{2^{11}} \sum\sum_{0\le i\le j\le 8} i\,.\,{}^{8}C_j$ is	(q)	$-\frac{1}{2}$
(C)	The derivative of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ at x = 1 is	(r)	$\frac{1}{2}$
(D)	The derivative of $\frac{\log\lvert x\rvert}{x}$ at x = – 1 is	(s)	1

SECTION-5 (INTEGER TYPE)

967. Let f (x) = $\max\{x^2, (1-x)^2, 2x(1-x)\}$, where $0 \le x \le 1$. Then the integral part of area of the region bounded by the curves y = f (x), x-axis x = 0 and x = 1 is ___________

968. If g (x) = 2 + cos x cos $\left(x+\frac{\pi}{3}\right)-\left(\cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)\right)$ and $f\left(\frac{5}{4}\right)$ = 9, then the value of fog (x) is _____________

969. Let f (x) be a polynomial of degree 3 if the curve y = f (x) has relative extremities at x = $\pm\frac{2}{\sqrt{3}}$ and passes through (0, 0) and (1, – 2) dividing the circle $x^2 + y^2 = 4$ in two parts. Then the integral part of areas of these two parts is ______________

970. I = $\int_0^{1.5}\left[x^2\right]dx$ where [.] is greatest integer function then the value of [I] is ________

971. From a point A on the curve $x = 3y^2 - 2y + 7$, subnormal and subtangent are drawn. If they measure 1 unit each, distance of A from (4, 1) is ___________

972. The value of $\frac{8\sqrt{2}}{\pi}\int_0^1\left(\frac{1-x^2}{1+x^2}\right)\frac{dx}{\sqrt{1+x^4}}$ is _________

973. Let A be the area of the region bounded by the curve $a^4y^2 = (2a - x)x^5$ and B be the area of the circle whose radius is $\frac{a}{2}$, then $\frac{A}{B}$ is ___________

974. The area bounded by curves y = $\left[6+x\left[\frac{1}{x}\right]\right]$, $y^2 - 18x + 18 = 0$ and $6x - 5y - 6 = 0$, (where [.] denotes the greatest integer function) is _________.

975. $\left[\int_{-1}^{\sqrt{3}}\frac{\sin^{-1}\frac{2x}{1+x^2}}{1+x^2}dx\right]$ = ____________; where [.] denotes G.I.F.

976. The value of $\lim_{x\to 0}\frac{16-16\cos(1-\cos x)}{x^4}$ is ______________

977. The altitude of a right circular cone of minimum volume circumscribed about a sphere of radius 2 is _________ .

978. $\int 7\left\{\frac{x^2(x^4-4x-3)}{(x^3-1)^2}\cos x-\frac{(x^4+1)}{(x^3-1)}\sin x\right\}dx$ = ___________.

979. The value of $\lim_{x\to\pi/2} 12\tan^2 x\left[\sqrt{6+3\sin x-2\cos^2 x}-\sqrt{3+6\sin x-\cos^2 x}\right]$ is _______________

980. If $\int_0^{\pi/2}\frac{dx}{\left(\sqrt{\cos x}+\sqrt{\sin x}\right)^4}=A$. Then the values of 6A is ___________

981. A closed right circular cylinder has volume 2156 cubic units. The radius of its base so that its total surface area may be minimum is ___________.

982. If g(x) is a polynomial satisfying g(x) g(y) = g(x) + g(y) + g(xy) – 2 $\forall$ x, y $\in$ R and g(2) = 5, then the value of g(3) is ____________.

983. Let A be the area of the region in the first quadrant bounded by the x-axis the line 2y = x and the ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1$. Let A′ be the area of the region in the first quadrant bounded by the y-axis, the line y = kx and the ellipse. The value of 9k such that A and A′ are equal is ____________

984. If A be the area bounded by y = f (x), y = f^{-1} (x) and line 4x + 4y – 5 = 0 where f (x) is a polynomial of 2nd degree passing through the origin and having maximum value of 1/4 at x = 1, then 96 A is equal to ______________

985. Let y = g(x) be the image of f(x) = x + sinx about the line x + y = 0. If the area bounded by y = g(x), x-axis, x = 0 and x = 2π is A, then $\frac{A}{\pi^2}$ is ___________

986. If $I_n = \int_0^\infty e^{-x}(\sin x)^n dx$ (n > 1), then the value of $\frac{101 I_{10}}{I_8}$ is equal to __________

987. The number of solutions of the equation $\left[\sin^{-1}x\right] = x-[x]$, where [.] denotes the greatest integer function is

988. Find the value of a + c so that:

$$\lim_{x\to\infty}\left(\sqrt{x^4+ax^3+3x^2+bx+2}-\sqrt{x^4+2x^3-cx^2+3x-d}\right)=4$$

989. Find the value of limits using expansion : $2\lim_{x\to 0}\left[\frac{\ell n(1+x)^{(1+x)}}{x^2}-\frac{1}{x}\right]$

990. Evaluate $3\lim_{x\to 0^+}\left\{\lim_{n\to\infty}\left(\frac{[1^2(\sin x)^x]+[2^2(\sin x)^x]+\ldots\ldots+[n^2(\sin x)^x]}{n^3}\right)\right\}$,

where [.] denotes the greatest integer function.

991. Evaluate the following limit $\lim_{x\to\infty}\log_{x-1}(x).\log_x(x+1).\log_{x+1}(x+2).\log_{x+2}(x+3).....\log_k(x^5)$;

where $k = x^5 - 1$.

992. Let $P_n = \frac{2^3-1}{2^3+1}\cdot\frac{3^3-1}{3^3+1}\cdot\frac{4^3-1}{4^3+1}\ldots\ldots\ldots\frac{n^3-1}{n^3+1}$. find the value of $\lim_{n\to\infty}6P_n$.

993. $\lim_{n\to\infty}\frac{(n+2)!+(n+1)!}{(n+3)!}$, $n\in N=$

994. If $f(x)=\begin{cases}x-1, & x\ge 1\\ 2x^2-2, & x<1\end{cases}$, $g(x)=\begin{cases}x+1, & x>0\\ -x^2+1, & x\le 0\end{cases}$, and $h(x)=|x|$ then find $\lim_{x\to 0}f(g(h(x)))$

995. Let [x] denote the greatest integer function & $f(x)$ be defined in a neighbourhood of 2 by

$$f(x) = \begin{cases} \dfrac{\exp\left((x+2)\dfrac{1}{4}[x+1]\ \ln 4\right) - 16}{4^x - 16}, & x < 2 \\ A\dfrac{1-\cos(x-2)}{(x-2)\tan(x-2)}, & x > 2 \end{cases}.$$

Find the value of $A+2f(2)$ in order that $f(x)$ may be continuous at $x = 2$.

996. Let the greatest and the least values of the function $f(x)$ be respectively a and b

$f(x) =$ minimum of $\{3t^4 - 8t^3 - 6t^2 + 24t; 1 \le t \le x\}, 1 \le x < 2.$

maximum of $\left\{3t + \frac{1}{4}\sin^2 \pi t + 2\,;\, 2 \le t \le x\right\}, 2 \le x \le 4$. Then find the value of $a+b$.

997. Find the area of the largest rectangle with lower base on the x-axis & upper vertices on the curve $y = 12 - x^2$.

998. The cosine of the angle at the vertex of an isosceles triangle having the greatest area for the given constant length ℓ of the median drawn to its lateral side is p. Find 100p.

999. The fuel charges for running a train are proportional to the square of the speed generated in m.p.h. & costs Rs. 48/- per hour at 16 mph. What is the most economical speed if the fixed charges i.e. salaries etc. amount to Rs. 300/- per hour.

1000. A figure is bounded by the curves, $y = x^2 + 1, y = 0, x = 0$ & $x = 1$. At a point (a,b), a tangent should be drawn to the curve, $y = x^2 + 1$ for it to cut off a trapezium of the greatest area from the figure. Find $2a + 12b$

Answer Key

Qs.	Ans.	Qs.	Ans.	Qs.	Ans.
651	C	701	AB	751	AB
652	A	702	BD	752	ABC
653	D	703	ABD	753	ABC
654	B	704	ABC	754	BC
655	A	705	AD	755	ABC
656	A	706	AD	756	AC
657	D	707	AC	757	AC
658	B	708	AB	758	ABC
659	B	709	AC	759	BC
660	B	710	AC	760	AC
661	C	711	AB	761	AB
662	A	712	BC	762	ABD
663	C	713	AC	763	AC
664	D	714	ABC	764	ABC
665	B	715	ABC	765	AB
666	A	716	AC	766	BC
667	B	717	AB	767	AC
668	A	718	CD	768	ABCD
669	C	719	ACD	769	BD
670	A	720	BC	770	BC
671	A	721	BD	771	BD
672	C	722	AB	772	ABD
673	B	723	AD	773	ABC
674	C	724	AB	774	ABC
675	B	725	ACD	775	BCD
676	B	726	ABC	776	ABC
677	B	727		777	AC
678	B	728	AC	778	CD
679		729	AB	779	AD
680	B	730	AB	780	ABD
681	A	731	ABD	781	ABC
682	A	732	CD	782	AC
683	B	733	ACD	783	ABD
684	D	734	ACD	784	AC
685	A	735	ABD	785	AB
686	C	736	AD	786	ABC
687	C	737	ABC	787	AC
688	A	738	BC	788	CD
689	D	739	CD	789	BD
690	D	740	ACD	790	CD
691	D	741	BD	791	BC
692	D	742	AD	792	BC
693	B	743	AC	793	ABD
694	B	744	ABCD	794	AB
695	B	745	ABC	795	AD
696	C	746	ACD	796	ACD
697	C	747	BC	797	AC
698	C	748	BCD	798	AD
699	A	749	AB	799	BC
700	A	750	AC	800	ACD

Qs.	Ans.	Qs.	Ans.	Qs.	Ans.	Qs.	Ans.
801	ABD	851	C	901	A	951	A-(PS), B-(QRT), C-(PS)
802	BD	852	A	902	C	952	A-(T), B-(PQR), C-(S)
803	BD	853	A	903	A	953	A-(P), B-(R), C-(QT)
804	AC	854	D	904	A	954	A-(s), B-(q), C-(q), D-(s)
805	ABD	855	B	905	C	955	A-(s), B-(p), C-(s), D-(q)
806	AD	856	D	906	A	956	A-(q), B-(q), C-(q), D-(q)
807	ABCD	857	B	907	C	957	A-(q), B-(p), C-(r), D-(s)
808	BC	858	D	908	C	958	A-(r), B-(s), C-(q), D-(r)
809	AD	859	B	909	B	959	A-(r), B-(s), C-(q), D-(p)
810	AD	860	C	910	A	960	A-(r), B-(s), C-(p), D-(q)
811	AB	861	A	911	D	961	A-(s), B-(r), C-(q), D-(p)
812	ACD	862	A	912	B	962	A-(q), B-(r), C-(q), D-(s)
813	ABC	863	D	913	C	963	A-(q), B-(p), C-(s), D-(p)
814	ABD	864	B	914	D	964	A-(p,q,r), B-(p), C-(q,r,s), D-(p,q,r,s)
815	D	865	D	915	B	965	A-(q), B-(q), C-(r), D-(p)
816	C	866	B	916	A	966	A-(q), B-(p), C-(r), D-(s)
817	A	867	A	917	B	967	0
818	A	868	B	918	A-(iv), B-(i), C-(iii), D-(ii)	968	9
819	C	869	D	919	A-(iii), B-(iv), C-(i), D-(ii)	969	6
820	B	870	D	920	A-(iv), B-(iii), C-(ii), D-(iii)	970	0
821	C	871	C	921	A-(iii), B-(i), C-(iv), D-(iv)	971	4
822	C	872	A	922	A-(iv), B-(i), C-(ii), D-(iii)	972	2
823	C	873	C	923	A-(ii), B-(ii), C-(ii), D-(iii, i)	973	5
824	B	874		924	A-(v), B-(iv), C-(i), D-(ii)	974	11
825	C	875		925	A-(ii), B-(iii), C-(ii), D-(iv)	975	0
826	A	876		926	A-(ii), B-(i), C-(iv), D-(iii)	976	2
827	B	877	C	927	A-(C), B-(D), C-(A), D-(B)	977	8
828	A	878	B	928	A-(2), B-(1), C-(4), D-(5)	978	24
829	C	879	B	929	A-(P), B-(P), C-(P), D-(P)	979	1
830	B	880	A	930	A-(P), B-(Q), C-(R), D-(S)	980	2
831	A	881		931	A-(r), B-(p), C-(q), D-(s)	981	7
832	A	882	C	932	A-(s), B-(p), C-(q), D-(q)	982	10
833	A	883	D	933	A-(C), B-(A), C-(D), D-(B)	983	2
834	D	884	A	934	A-(D), B-(A), C-(B), D-(C)	984	34
835	C	885	B	935	A-(D), B-(C), C-(B), D-(A)	985	2
836	C	886	A	936	A-(D), B-(C), C-(A), D-(B)	986	90
837	B	887	D	937	A-(D), B-(C), C-(B), D-(A)	987	1
838	B	888	B	938	A-(P), B-(Q), C-(R), D-(Q)	988	7
839	C	889	C	939	A-(R), B-(R), C-(S), D-(S)	989	1
840	A	890	D	940	A-(R), B-(P), C-(S), D-(Q)	990	1
841	B	891	B	941	A-(R), B-(P), C-(S), D-(Q)	991	5
842	C	892	A	942	A-(RT), B-(RT), C-(RT), D-(PS)	992	4
843	D	893	C	943	A-(R), B-(Q), C-(R), D-(P)	993	0
844	D	894	C	944	A-(P), B-(P), C-(S), D-(R)	994	0
845	C	895	A	945	A-(S), B-(R), C-(R), D-(Q)	995	2
846	C	896	D	946	A-(R), B-(Q), C-(Q), D-(R)	996	22
847	B	897	C	947	A-(S), B-(R), C-(S), D-(Q)	997	32 sq.
848	C	898	A	948	A-(S), B-(S), C-(P), D-(P)	998	80
849		899	A	949	A-(P,R,T), B-(S), C-(Q)	999	40 mph
850	A	900		950	A-(S), B-(P), C-(Q), D-(S)	1000	16