8 Permutations and Combinations

QUICK LOOK

Fundamental Principles

- **Factorial Notation:** Let n be a positive integer. Then, the continued product of first n natural numbers is called factorial n, to be denoted by $n!$ or n. Also, we define $0! = 1$. when n is negative or a fraction, n ! is not defined.
  Thus, $n! = n(n-1)(n-2)......3.2.1.$
  Deduction: $n! = n(n-1)(n-2)(n-3)......3.2.1$
  $= n[(n-1)(n-2)(n-3)......3.2.1] = n[(n-1)!]$
  Thus, $5! = 5\times(4!), 3! = 3\times(2!)$ and $2! = 2\times(1!)$
  Also, $1! = 1\times(0!) \Rightarrow 0! = 1$.
- **Exponent of Prime p in $n!$:** Let p be a prime number and n be a positive integer. Then the last integer amongst 1, 2, 3, ...$(n-1)$, n which is divisible by p is $\left[\frac{n}{p}\right]p$, where $\left[\frac{n}{p}\right]$ denote the greatest integer less than or equal to $\frac{n}{p}$.

Counting of Number of Ways to do Some Work: If a work W consists of two parts W_1, W_2 of which one part can be done in m ways and the other part in n ways then

- The work W can be done in $m+n$ ways, if by doing any of the parts the work W is done. (Addition law of counting)
- The work W can be done in mn ways, if both the parts are to be done one after the other to do the work W. (Multiplication law of counting). Similar is the law for works that have 3 or more parts. If a work is to be done under some restriction then
- The number of ways to do the work under the restriction = (the number of ways to do the work without restriction) – (the number of ways to do the work under opposite restriction).

Counting Formulae for Permutation

- The number of permutations (arrangements) of n different things taking r at a time $= {}^nP_r = \frac{n!}{(n-r)!}$ where $n! = 1.2.3...n$
- The number of permutations of n things taking at a time of which p things are identical, q things are identical of another type and the rest are different $= \frac{n!}{p!\,q!}$.
- The number of arrangements of n different things round a closed curve = $(n-1)!$ if clockwise and anticlockwise arrangements are considered different, $\frac{1}{2}(n-1)!$ if clockwise and anticlockwise arrangements are considered identical. (Circular permutation)

Counting Formulae for Combination

- The number of combinations (selections) of n different things taking r at a time $= {}^nC_r = \frac{n!}{r!(n-r)!}$.
- The total number of selections of one ore more objects from n different objects $= 2^n - 1 (= {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n)$.
- Total number of selections of any number of things from n identical things = $n+1$ (when selection of 0 things is allowed) n(when at least one thing is to be selected).
- The total number of selections from p like things, q like things of another type and r distinct things $= (p+1)(q+1)2^r - 1$ (if at least one thing is to be selected)
  $(p+1)(q+1)2^r - 2$ (if none or all cannot be selected)
- The total number of selections of r things from n different things when each thing can be repeated unlimited number of times $= {}^{n+r-1}C_r$.

Number of Distributions

- The number of ways to distribute n different things between two persons, one receiving p things and the other q things, where $p+q=n$, $n = {}^nC_p \times {}^{n-p}C_p$
  $= \frac{n!}{p!(n-p)!} \times \frac{(n-p)!}{q!(n-p-q)!} = \frac{n!}{p!q!} \{\because n = p+q\}$
  Similarly for 3 persons, the number of ways
  $= \frac{n!}{p!q!r!}$, where $p+q+r=n$.
- The number of ways to distribute $m\times n$ different things among n persons equally $\frac{(mn)!}{(m!)^n}$.
- The number of ways to divide n different things into three bundles of p, q and r things $= \frac{n!}{p!q!r!}.\frac{1}{3!}$.
- The number of ways to divide $m\times n$ different things into n

equal bundles $\dfrac{(mn)!}{(m!)^n}\cdot\dfrac{1}{n!}$.

- The total number of ways to divide n identical things among r persons $= {}^{n+r-1}C_{r-1}$.
- The total number of ways to divide n identical things among r persons so that each gets at least one $= {}^{n-1}C_{r-1}$.

Note

Gap Method: Suppose 5 males A, B, C, D, E are arranged in a row as × A × B × C × D × E ×. There will be six gaps between these five. Four in between and two at either end. Now if three females P, Q,R are to be arranged so that no two are together we shall use gap method i.e., arrange them in between these 6 gaps. Hence the answer will be ${}^6P_3$.

Together: Suppose we have to arrange 5 persons in a row which can be done in 5! = 120 ways. But if two particular persons are to be together always, then we tie these two particular persons with a string. Thus we have 5 – 2 + 1 (1 corresponding to these two together) = 3 +1 = 4 units, which can be arranged in 4! ways. Now we loosen the string and these two particular can be arranged in 2! ways. Thus total arrangements = 24 × 2 = 48.

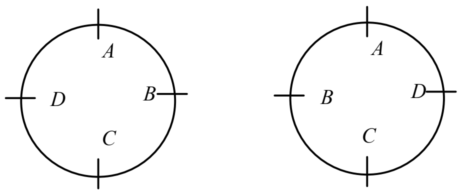
Never together = Total –Together = 120 – 48 = 72.

Circular Permutations

In circular permutations, what really matters is the position of an object relative to the others. Thus, in circular permutations, we fix the position of the one of the objects and then arrange the other objects in all possible ways. There are two types of circular permutations:

- The circular permutations in which clockwise and the anticlockwise arrangements give rise to different permutations, *e.g.* Seating arrangements of persons round a table.
- The circular permutations in which clockwise and the anticlockwise arrangements give rise to same permutations, e.g. arranging some beads to form a necklace.

  Look at the circular permutations, given below:



  Suppose *A, B, C, D* are the four beads forming a necklace. They have been arranged in clockwise and anticlockwise directions in the first and second arrangements respectively.
- **Difference between Clockwise and Anticlockwise Arrangement:** If anticlockwise and clockwise order of arrangement are not distinct *e.g.*, arrangement of beads in a necklace, arrangement of flowers in garland etc. then the number of circular permutations of n distinct items is $\dfrac{(n-1)!}{2}$
- **Theorem on Circular Permutations**

  Theorem 1: The number of circular permutations of n different objects is $(n – 1)!$

  Theorem 2: The number of ways in which n persons can be seated round a table is $(n – 1)!$

  Theorem 3: The number of ways in which n different beads can be arranged to form a necklace, is $\dfrac{1}{2}(n-1)!$.

Note

- When the positions are numbered, circular arrangement is treated as a linear arrangement.
- In a linear arrangement, it does not make difference whether the positions are numbered or not.

Combination and its Operations

The number of all combinations of n things, taken r at a time is denoted by $C(n,r)$ or ${}^nC_r$ or $\binom{n}{r}$.

- **Difference between a Permutation and Combination:** In a combination only selection is made whereas in a permutation not only a selection is made but also an arrangement in a definite order is considered.

  In a combination, the ordering of the selected objects is immaterial whereas in a permutation, the ordering is essential. For example *A,B* and *B,A* are same as combination but different as permutations.

  Practically to find the permutation of n different items, taken r at a time, we first select r items from n items and then arrange them. So usually the number of permutations exceeds the number of combinations.
- **Number of Combinations without Repetition:** The number of combinations (selections or groups) that can be formed from n different objects taken $r (0 \le r \le n)$ at a time is ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

  Let the total number of selections (or groups) = x. Each group contains r objects, which can be arranged in r ! ways. Hence the number of arrangements of r objects = $x \times (r!)$. But the number of arrangements = ${}^nP_r$.

$\Rightarrow x \times (r!) = {}^nP_r$.

$$\Rightarrow \quad x = \frac{{}^nP_r}{r!}$$

$\Rightarrow \quad x = \dfrac{n!}{r!(n-r)!} = {}^nC_r$

- Number of combinations of n dissimilar things taken all at a time ${}^nC_n = \dfrac{n!}{n!(n-n)!} = \dfrac{1}{0!} = 1, \ (\because 0! = 1)$.

Note

- ${}^nC_r$ is a natural number.
- ${}^nC_0 = {}^nC_n = 1, {}^nC_1 = n$
- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_x = {}^nC_y \Leftrightarrow x = y$ or $x + y = n$
- $n.{}^{n-1}C_{r-1} = (n-r+1){}^nC_{r-1}$
- If n is even then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.
- If n is odd then the greatest value of ${}^nC_r$ is $\dfrac{{}^nC_{n+1}}{2}$ or $\dfrac{{}^nC_{n-1}}{2}$.
- ${}^nC_r = \dfrac{n}{r}.{}^{n-1}C_{r-1}$
- $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$
- ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n$
- ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\ldots = 2^{n-1}$
- ${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots + {}^{2n+1}C_n = 2^{2n}$
- ${}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + {}^{n+3}C_n + \ldots. + {}^{2n-1}C_n = {}^{2n}C_{n+1}$

Number of Combinations with Repetition and All Possible Selections

- The number of combinations of n distinct objects taken r at a time when any object may be repeated any number of times.

  = coefficient of x^r in $(1 + x + x^2 + \ldots\ldots. + x^r)^n$

  = coefficient of x^r in $(1-x)^{-n} = {}^{n+r-1}C_r$
- The total number of ways in which it is possible to form groups by taking some or all of n things at a time is $2^n - 1$.
- The total number of ways in which it is possible to make groups by taking some or all out of $n = (n_1 + n_2 + \ldots.)$ things, when n_1 are alike of one kind, n_2 are alike of second kind, and so on is $\{(n_1+1)(n_2+1)\ldots\ldots\} - 1$.
- The number of selections of r objects out of n identical objects is 1.
- Total number of selections of zero or more objects from n identical objects is $n+1$.
- The number of selections taking at least one out of $a_1 + a_2 + a_3 + \ldots\ldots + a_n + k$ objects, where a_1 are alike (of one kind), a_2 are alike (of second kind) and so on...... $.a_n$ are alike (of n[th] kind) and k are distinct

  $= [(a_1+1)(a_2+1)(a_3+1)\ldots\ldots.(a_n+1)]2^k - 1$.

Conditional Combinations

- The number of ways in which r objects can be selected from n different objects if k particular objects are: Always included = ${}^{n-k}C_{r-k}$ and Never included = ${}^{n-k}C_r$
- The number of combinations of n objects, of which p are identical, taken r at a time is

  $= {}^{n-p}C_r + {}^{n-p}C_{r-1} + {}^{n-p}C_{r-2} + \ldots\ldots. + {}^{n-p}C_0$ if $r \le p$ and

  $= {}^{n-p}C_r + {}^{n-p}C_{r-1} + {}^{n-p}C_{r-2} + \ldots\ldots. + {}^{n-p}C_{r-p}$ if $r > p$

Derangement and Geometrical Problems

- **Derangement:** Any change in the given order of the things is called a derangement. If n things form an arrangement in a row, the number of ways in which they can be deranged so that no one of them occupies its original place is

  $n!\left(1 - \dfrac{1}{1!} + \dfrac{1}{2!} - \dfrac{1}{3!} + \ldots\ldots + (-1)^n.\dfrac{1}{n!}\right)$.

Some Important Results for Geometrical Problems

- Number of total different straight lines formed by joining the n points on a plane of which $m(<n)$ are collinear is ${}^nC_2 - {}^mC_2 + 1$.
- Number of total triangles formed by joining the n points on a plane of which m ($< n$) are collinear is ${}^nC_3 - {}^mC_3$.
- Number of diagonals in a polygon of n sides is ${}^nC_2 - n$.
- If m parallel lines in a plane are intersected by a family of other n parallel lines. Then total number of parallelograms so formed is ${}^mC_2 \times {}^nC_2$ i.e $\dfrac{mn(m-1)(n-1)}{4}$
- Given n points on the circumference of a circle, then

  Number of straight lines = ${}^nC_2$

  Number of triangles = ${}^nC_3$

  Number of quadrilaterals = ${}^nC_4$. .
- If n straight lines are drawn in the plane such that no two lines are parallel and no three lines are concurrent. Then the number of part into which these lines divide the plane is = $1 + \Sigma n$.
- Number of rectangles of any size in a square of $n \times n$ is $\sum_{r=1}^{n} r^3$ and number of squares of any size is $\sum_{r=1}^{n} r^2$.

- In a rectangle of $n \times p\ (n < p)$ number of rectangles of any size is $\frac{np}{4}(n+1)(p+1)$ and number of squares of any size is $\sum_{r=1}^{n}(n+1-r)(p+1-r)$.

Use of Solution of Linear Equations and Coefficient of a Power in Expansion to Find the Number of Ways of Distribution

- The number of integral solutions of $x_1 + x_2 + x_3 + \ldots + x_r = n$ where $x_1 \geq 0, x_2 \geq 0, \ldots x_r \geq 0$ is the same as the number of ways to distribute n identical things among r persons. This is also equal to the coefficient of x^n in the expansion of $(x^0 + x^1 + x^2 + x^3 + ..)^r$

$= $ coefficient of x^n in $\left(\frac{1}{1-x}\right)^r$ = coefficient of x^n in $(1-x)^{-r}$

= coefficient of x^n in

$$\left\{1 + rx + \frac{r(r+1)}{2!}x^2 + \ldots + \frac{r(r+1)(r+2)\ldots(r+n-1)}{n!}x^n + \ldots\right\}$$

$$= \frac{r(r+1)(r+2)\ldots(r+n-1)}{n!} = \frac{(r+n-1)!}{n!(r-1)!} = {}^{n+r-1}C_{r-1}.$$

- The number of integral solutions of $x_1 + x_2 + x_3 + \ldots + x_r = n$ where $x_1 \geq 1, x_2 \geq 1, \ldots, x_r \geq 1$ is the same as the number of ways to distribute n identical things among r persons each getting at least 1. This is also equal to the coefficient of x^n in the expansion of $(x_1 + x_2 + x_3 + \ldots)^r$ = coefficient of x^n in $\left(\frac{x}{1-x}\right)^r$

= coefficient of x^n in $x^r(1-x)^{-r}$ = coefficient of x^n in

$$x^r\left\{1 + rx + \frac{r(r+1)}{2!}x^2 + \ldots + \frac{r(r+1)(r+2)\ldots(r+n-1)}{n!}x^n + \ldots\right\}$$

= coefficient of x^{n-r} in

$$\left\{1 + rx + \frac{r(r+1)}{2!}x^2 + \ldots + \frac{r(r+1)(r+2)\ldots(r+n-1)}{n!}x^n + \ldots\right\}$$

$$= \frac{r(r+1)(r+2)\ldots(r+n-r-1)}{(n-r)!} = \frac{r(r+1)(r+2)\ldots(n-1)}{(n-r)!}$$

$$= \frac{(n-1)!}{(n-r)!(r-1)!} = {}^{n-1}C_{r-1}$$

Note

The number of solutions of $x_1 + x_2 + x_3 + x_4 = 20$ where $x_1 \geq 0, x_2 \geq 1, x_3 \geq 3, x_4 \geq 5$ is equal to the coefficient of x^{20} in $(x^0 + x^1 + x^2 + \ldots)\ (x^1 + x^2 + x^3 + \ldots) \times (x^3 + x^4 + x^5 + \ldots)\ (x^5 + x^6 + x^7 + \ldots)$.

Number of Divisors

Let $N = p_1^{\alpha_1}.p_2^{\alpha_2}.p_3^{\alpha_3}\ldots\ldots p_k^{\alpha_k}$, where $p_1, p_2, p_3, \ldots\ldots p_k$ are different primes and $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_k$ are natural numbers then:

- The total number of divisors of N including 1 and N is $= (\alpha_1+1)(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1)$
- The total number of divisors of N excluding 1 and N is $= (\alpha_1+1)(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1) - 2$
- The total number of divisors of N excluding 1 or N is $= (\alpha_1+1)(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1) - 1$
- The sum of these divisors is $= (p_1^0 + p_1^1 + p_1^2 + \ldots + p_1^{\alpha_1})(p_2^0 + p_2^1 + p_2^2 + \ldots + p_2^{\alpha_2})\ldots (p_k^0 + p_k^1 + p_k^2 + \ldots + p_k^{\alpha_k})$
- The number of ways in which N can be resolved as a product of two factors is

$$\begin{cases} \frac{1}{2}(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1), \text{If } N \text{ is not a perfect square} \\ \frac{1}{2}[(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1)+1], \text{If } N \text{ is a perfect square} \end{cases}$$

- The number of ways in which a composite number N can be resolved into two factors which are relatively prime (or co-prime) to each other is equal to 2^{n-1} where n is the number of different factors in N.

Note

- All the numbers whose last digit is an even number 0, 2, 4, 6 or 8 are divisible by 2.
- All the numbers sum of whose digits are divisible by 3,is divisible by 3 e.g. 534. Sum of the digits is 12, which are divisible by 3, and hence 534 is also divisible by 3.
- All those numbers whose last two-digit number is divisible by 4 are divisible by 4 e.g. 7312, 8936, are such that 12, 36 are divisible by 4 and hence the given numbers are also divisible by 4.
- All those numbers, which have either 0 or 5 as the last digit, are divisible by 5.
- All those numbers, which are divisible by 2 and 3 simultaneously, are divisible by 6. e.g., 108, 756 etc.
- All those numbers whose last three-digit number is divisible by 8 are divisible by 8.
- All those numbers sum of whose digit is divisible by 9 are divisible by 9.
- All those numbers whose last two digits are divisible by 25 are divisible by 25 e.g., 73125, 2400 etc.

MULTIPLE CHOICE QUESTIONS

Fundamental Principles

1. A college offers 7 courses in the morning and 5 in the evening. The number of ways a student can select exactly one course, either in the morning or in the evening:

a. 27 **b.** 15
c. 12 **d.** 35

2. In a monthly test, the teacher decides that there will be three questions, one from each of exercise 7, 8 and 9 of the text book. If there are 12 questions in exercise 7, 18 in exercise 8 and 9 in exercise 9, in how many ways can three questions be selected?

a. 1944 **b.** 1499
c. 4991 **d.** None of these

Operations of Permutation

3. How many numbers can be made with the help of the digits 0, 1, 2, 3, 4, 5 which are greater than 3000: (repetition is not allowed)

a. 180 **b.** 360
c. 1380 **d.** 1500

4. The number of arrangement of the letters of the word "CALCUTTA"?

a. 2520 **b.** 5040
c. 10080 **d.** 40320

5. How many words can be made from the letters of the word 'COMMITTEE?'

a. $\frac{9!}{(2!)^2}$ **b.** $\frac{9!}{(2!)^3}$
c. $\frac{9!}{2!}$ **d.** 9 !

Conditional Permutations

6. m men and n women are to be seated in a row, so that no two women sit together. If $m > n$, then the number of ways in which they can be seated is:

a. $\frac{m!(m+1)!}{(m-n+1)!}$ **b.** $\frac{m!(m-1)!}{(m-n+1)!}$
c. $\frac{(m-1)!(m+1)!}{(m-n+1)!}$ **d.** None of these

7. If the letters of the word 'KRISNA' are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word 'KRISNA' is:

a. 324 **b.** 341
c. 359 **d.** None of these

8. We are to form different words with the letters of the word 'INTEGER'. Let m_1 be the number of words in which I and N are never together, and m_2 be the number of words which begin with I and end with R. Then m_1 / m_2 is equal to:

a. 30 **b.** 60 **c.** 90 **d.** 180

9. An n digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is:

a. 6 **b.** 7 **c.** 8 **d.** 9

10. The number of numbers that can be formed with the help of the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy odd places, is:

a. 24 **b.** 18
c. 12 **d.** 30

Circular Permutations

11. The number of ways in which 5 male and 2 female members of a committee can be seated around a round table so that the two female are not seated together is:

a. 480 **b.** 600
c. 720 **d.** 840

12. If eleven members of a committee sit at a round table so that the President and Secretary always sit together, then the number of arrangements is:

a. $10!\times 2$ **b.** 10!
c. $9!\times 2$ **d.** None of these

13. In how many ways can 5 boys and 5 girls sit in a circle so that no two boys sit together?

a. $5!\times 5!$ **b.** $4!\times 5!$
c. $\frac{5!\times 5!}{2}$ **d.** None of these

14. In how many ways can 15 members of a council sit along a circular table, when the Secretary is to sit on one side of the Chairman and the Deputy Secretary on the other side?

a. $2\times 12!$ **b.** 24
c. $2\times 15!$ **d.** None of these

15. 20 persons are invited for a party. In how many different ways can they and the host be seated at a circular table, if the two particular persons are to be seated on either side of the host?

a. 20! **b.** 2.18!
c. 18 ! **d.** None of these

16. The number of ways in which 5 beads of different colours form a necklace is:
a. 12 **b.** 24 **c.** 120 **d.** 60

17. In how many ways 7 men and 7 women can be seated around a round table such that no two women can sit together:
a. $(7!)^2$ **b.** $7!\times 6!$ **c.** $(6!)^2$ **d.** $7!$

18. The number of ways that 8 beads of different colours be string as a necklace is:
a. 2520 **b.** 2880 **c.** 5040 **d.** 4320

Combination and its Operations

19. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, then the value of r is:
a. 3 **b.** 4 **c.** 5 **d.** 8

20. $\frac{{}^nC_r}{{}^nC_{r-1}} = ?$
a. $\frac{n-r}{r}$ **b.** $\frac{n+r-1}{r}$ **c.** $\frac{n-r+1}{r}$ **d.** $\frac{n-r-1}{r}$

21. If ${}^{n+1}C_3 = 2\,{}^nC_2$, then $n = ?$
a. 3 **b.** 4
c. 5 **d.** 6

22. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is?
a. 10^2 **b.** 1023
c. 2^{10} **d.** 10!

23. Numbers greater than 1000 but not greater than 4000 which can be formed with the digits 0, 1, 2, 3, 4 (repetition of digits is allowed), are:
a. 350 **b.** 375
c. 450 **d.** 576

Conditional Combinations Division into Groups, Derangements

24. In the 13 cricket players 4 are bowlers, then how many ways can form a cricket team of 11 players in which at least 2 bowlers included:
a. 55 **b.** 72
c. 78 **d.** None of these

25. In how many ways a team of 10 players out of 22 players can be made if 6 particular players are always to be included and 4 particular players are always excluded:
a. ${}^{22}C_{10}$ **b.** ${}^{18}C_3$
c. ${}^{12}C_4$ **d.** ${}^{18}C_4$

26. In how many ways can 5 prizes be distributed among four students when every student can take one or more prizes?
a. 1024 **b.** 625 **c.** 120 **d.** 60

27. The number of ways in which 9 persons can be divided into three equal groups is:
a. 1680 **b.** 840
c. 560 **d.** 280

28. A man has 7 friends. In how many ways he can invite one or more of them for a tea party:
a. 128 **b.** 256 **c.** 127 **d.** 130

29. In how many ways can a girl and a boy be selected from a group of 15 boys and 8 girls?
a. 15×8 **b.** $15+8$ **c.** ${}^{23}P_2$ **d.** ${}^{23}C_2$

30. If ${}^{2n}C_3 : {}^nC_2 = 44 : 3$, then for which of the following values of r, the value of ${}^nC_r$ will be 15?
a. $r = 3$ **b.** $r = 4$
c. $r = 6$ **d.** $r = 5$

31. If ${}^{n^2-n}C_2 = {}^{n^2-n}C_{10}$, then $n = ?$
a. 12 **b.** 4 only
c. –3 only **d.** 4 or –3

32. In a conference of 8 persons, if each person shake hand with the other one only, then the total number of shake hands shall be:
a. 64 **b.** 56
c. 49 **d.** 28

33. If ${}^8C_r = {}^8C_{r+2}$, then the value of ${}^rC_2$ is:
a. 8 **b.** 3
c. 5 **d.** 2

34. Everybody in a room shakes hand with everybody else. The total number of hand shakes is 66. The total number of persons in the room is:
a. 11 **b.** 12 **c.** 13 **d.** 14

35. $\sum_{r=0}^{m} {}^{n+r}C_n = ?$
a. ${}^{n+m+1}C_{n+1}$ **b.** ${}^{n+m+2}C_n$
c. ${}^{n+m+3}C_{n-1}$ **d.** None of these

36. In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct, is:
a. 11 **b.** 12
c. 27 **d.** 63

37. In a city no two persons have identical set of teeth and there is no person without a tooth. Also no person has more than 32 teeth. If we disregard the shape and size of tooth and consider only the positioning of the teeth, then the maximum population of the city is:

a. 2^{32} **b.** $(32)^2 - 1$
c. $2^{32} - 1$ **d.** 2^{32-1}

38. If ${}^{10}C_r = {}^{10}C_{r+2}$, then ${}^5C_r$ equals ?

a. 120 **b.** 10 **c.** 360 **d.** 5

39. If ${}^nC_3 + {}^nC_4 > {}^{n+1}C_3$, then:

a. $n > 6$ **b.** $n > 7$
c. $n < 6$ **d .**None of these

40. The least value of natural number n satisfying $C(n,5) + C(n,6) > C(n+1,5)$ is:

a. 11 **b.** 10 **c.** 12 **d.** 13

41. If n and r are two positive integers such that $n \ge r$, then ${}^nC_{r-1} + {}^nC_r = ?$

a. ${}^nC_{n-r}$ **b.** ${}^nC_r$
c. ${}^{n-1}C_r$ **d.** ${}^{n+1}C_r$

42. In an election there are 8 candidates, out of which 5 are to be choosen. If a voter may vote for any number of candidates but not greater than the number to be choosen, then in how many ways can a voter vote:

a. 216 **b.** 114
c. 218 **d.** None of these

43. In how many ways can 21 English and 19 Hindi books be placed in a row so that no two Hindi books are together?

a. 1540 **b.** 1450
c. 1504 **d.** 1405

44. In how many ways a team of 11 players can be formed out of 25 players, if 6 out of them are always to be included and 5 are always to be excluded:

a. 2020 **b.** 2002 **c.** 2008 **d.** 8002

45. Out of 10 white, 9 black and 7 red balls, the number of ways in which selection of one or more balls can be made, is:

a. 881 **b.** 891 **c.** 879 **d.** 892

46. In a touring cricket team there are 16 players in all including 5 bowlers and 2 wicket-keepers. How many teams of 11 players from these, can be chosen, so as to include three bowlers and one wicket-keeper:

a. 650 **b.** 720 **c.** 750 **d.** 800

47. To fill 12 vacancies there are 25 candidates of which five are from scheduled caste. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, then the number of ways in which the selection can be made:

a. ${}^5C_3 \times {}^{22}C_9$ **b.** ${}^{22}C_9 - {}^5C_3$
c. ${}^{22}C_3 + {}^5C_3$ **d.** None of these

48. There are 9 chairs in a room on which 6 persons are to be seated, out of which one is guest with one specific chair. In how many ways they can sit:

a. 6720 **b.** 60480
c. 30 **d.** 346

49. The number of ways in which 10 persons can go in two boats so that there may be 5 on each boat, supposing that two particular persons will not go in the same boat is

a. $\frac{1}{2}({}^{10}C_5)$ **b.** $2({}^8C_4)$
c. $\frac{1}{2}({}^8C_5)$ **d.** None of these

50. A father with 8 children takes them 3 at a time to the Zoological gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is:

a. 336 **b.** 112
c. 56 **d.** None of these

Geometrical Problems

51. There are four balls of different colours and four boxes of colurs same as those of the balls. The number of ways in which the balls, one in each box, could be placed such that a ball doesn't go to box of its own colour is:

a. 8 **b.** 7
c. 9 **d.** None of these

52. Out of 18 points in a plane, no three are in the same straight line except five points which are collinear. The number of:

(i) Straight lines

a. 140 **b.** 142 **c.** 144 **d.** 146

(ii) Triangles which can be formed by joining them

a. 816 **b.** 806 **c.** 800 **d.** 750

53. The number of diagonals in a octagon will be:

a. 28 **b.** 20
c. 10 **d.** 16

54. How many triangles can be formed by joining four points on a circle?

a. 4 **b.** 6 **c.** 8 **d.** 10

55. How many triangles can be drawn by means of 9 non-collinear points?
a. 84 **b.** 72 **c.** 144 **d.** 126

56. The number of straight lines joining 8 points on a circle is:
a. 8 **b.** 16
c. 24 **d.** 28

57. In a plane there are 10 points out of which 4 are collinear, then the number of triangles that can be formed by joining these points are:
a. 60 **b.** 116
c. 120 **d.** None of these

58. The straight lines I_1, I_2, I_3 are parallel and lie in the same plane. A total number of m points are taken on I_1, n points on I_2, k points on I_3. The maximum number of triangles formed with vertices at these points are:
a. ${}^{m+n+k}C_3$ **b.** ${}^{m+n+k}C_3 - {}^{m}C_3 - {}^{n}C_3 - {}^{k}C_3$
c. ${}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3$ **d.** None of these

59. Six points in a plane be joined in all possible ways by indefinite straight lines, and if no two of them be coincident or parallel, and no three pass through the same point (with the exception of the original 6 points). The number of distinct points of intersection is equal to:
a. 105 **b.** 45
c. 51 **d.** None of these

60. There are n straight lines in a plane, no two of which are parallel and no three pass through the same point. Their points of intersection are joined. Then the number of fresh lines thus obtained is:
a. $\frac{n(n-1)(n-2)}{8}$ **b.** $\frac{n(n-1)(n-2)(n-3)}{6}$
c. $\frac{n(n-1)(n-2)(n-3)}{8}$ **d.** None of these

61. In a plane there are 37 straight lines of which 13 pass through the point A and 11 pass through the point B. Besides no three lines pass through one point, no line passes through both points A and B and no two are parallel. Then the number of intersection points the lines have is equal to:
a. 535 **b.** 601
c. 728 **d.** None of these

62. There are 16 points in a plane, no three of which are in a straight line except 8 which are all in a straight line. The number of triangles that can be formed by joining them equals:
a. 504 **b.** 552 **c.** 560 **d.** 1120

63. The number of straight lines that can be formed by joining 20 points no three of which are in the same straight line except 4 of them which are in the same line:
a. 183 **b.** 186 **c.** 197 **d.** 185

Multinomial Theorem

64. A student is allowed to select utmost n books from a collection of $(2n+1)$ books. If the total number of ways in which he can select one book is 63, then the value of n is:
a. 2 **b.** 3
c. 4 **d.** None of these

65. If x, y and r are positive integers, then ${}^{x}C_r + {}^{x}C_{r-1}\,{}^{y}C_1 + {}^{x}C_{r-2}\,{}^{y}C_2 + \ldots.. + {}^{y}C_r = ?$
a. $\frac{x!y!}{r!}$ **b.** $\frac{(x+y)!}{r!}$
c. ${}^{x+y}C_r$ **d.** ${}^{xy}C_r$

Number of Divisors

66. The number of divisors of 9600 including 1 and 9600 are:
a. 60 **b.** 58
c. 48 **d.** 46

NCERT EXEMPLAR PROBLEMS

More than One Answer

67. The number of ways of painting the faces of a cube with six different colours is:
a. 1 **b.** 6
c. 6! **d.** ${}^{9}C_2$

68. Sanjay has 10 friends among whom two are married to each other. She wishes to invite 5 of the them for a party. If the married couple refuse to attend separately, the number of different ways in which she can invite five friends is:
a. ${}^{8}C_5$ **b.** $2 \times {}^{8}C_3$
c. ${}^{10}C_5 - 2 \times {}^{8}C_4$ **d.** none of these

69. There are n seats round a table marked 1,2,3,…n. The number of ways in which $m(\leq n)$ persons can takes seats is:
a. ${}^{n}P_m$ **b.** ${}^{n}C_m \times (m-1)!$
c. ${}^{n}C_m \times m!$ **d.** ${}^{n-1}P_{m-1}$

70. The number of ways in which 10 candidates $A_1, A_2, \ldots, A_{10}$ can be ranked, so that A_1 is always above A_2 is:
a. $\frac{10!}{2}$ **b.** $8! \times {}^{10}C_2$
c. ${}^{10}P_2$ **d.** ${}^{10}P_2$

71. In a class tournament when the participants were to play one game with another, two class players fell ill, having played 3 games each. If the total number of games played is 84, the number of participants at the beginning was:

a. 15 **b.** 30

c. $^6C_2$ **d.** 48

72. The number of ways of distributing 10 different books among 4 students $(S_1 - S_4)$ such that S_1 and S_2 get 2 books each and S_3 and S_4 get books each is:

a. 12600 **b.** 25200 **c.** $^{10}C_4$ **d.** $\frac{10!}{2!2!3!3!}$

73 The number of ways to select 2 numbers from {0,1,2,3,4} such that the sum of the squares of the selected numbers is divisible by 5 are: (repetition of digits is allowed)

a. $^9C_1$ **b.** $^9C_8$ **c.** 9 **d.** 7

74. The number of ways of arranging seven persons (having *A*, *B*, *C* and *D* among them) in a row so that *A*, *B*, *C* and *D* are always in order *A–B–C–D* (not necessarily together) is:

a. 210 **b.** 5040

c. $6 \times {}^7C_4$ **d.** $^7C_3$

75. Total number of ways of giving at least one coin out of three 25 paise and two 50 pasise coins to a beggar is:

a. 32 **b.** 12 **c.** 11 **d.** $^{12}P_1 - 1$

76. If $\alpha = x_1x_2x_3$ and $\beta = y_1y_2y_3$ be two three digits numbers, the number of pairs of α and β can be formed so that α can be subtracted from β without borrowing is:

a. 2!10!10! **b.** $(45)(55)^2$

c. $3^2.5^3.11^2$ **d.** 136125

Assertion and Reason

Note: Read the Assertion (A) and Reason (R) carefully to mark the correct option out of the options given below:

a. If both assertion and reason are true and the reason is the correct explanation of the assertion.

b. If both assertion and reason are true but reason is not the correct explanation of the assertion.

c. If assertion is true but reason is false.

d. If the assertion and reason both are false.

e. If assertion is false but reason is true.

77. **Assertion:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^9C_3$

Reason: The number of ways of choosing 3 places from 9 different place is $^9C_3$.

78. **Assertion:** If n is a natural number then $\frac{(n^2)!}{(n!)^{n+1}}$ is a natural number.

Reason: The number of ways of dividing mn students into m groups each containing n students is $\frac{(mn)!}{m!(n!)^m}$

79. Let $n \in N$, and $f(n) = \begin{vmatrix} n & n+1 & n+2 \\ ^nP_n & ^{n+1}P_{n+1} & ^{n+2}P_{n+2} \\ ^nC_n & ^{n+1}C_{n+1} & ^{n+2}C_{n+2} \end{vmatrix}$

Assertion: $f(n)$ is an integer for all $n \in N$.

Reason: If elements of a determinant are integers, then determinant itself is an integer.

80. **Assertion:** $\sum_{j=1}^{n}\sum_{k=j}^{n}\binom{n}{k}\binom{k}{j} = 3^n - 2^n$

Reason: $\sum_{k=1}^{n}(^nC_k)^2 = {}^{2n}C_n$

81. **Assertion:** The number of ways of distributing at most 12 toys to three children A_1, A_2 and A_3 so that A_1 gets at least one, A_2 at least three and A_3 at most five, is 145.

Reason: the number of non-negative integral solutions of $x_1 + x_2 + x_3 \le m\, b$ is $m^{-1}P_2$.

82. **Assertion:** The expression $\binom{40}{4}\binom{60}{0} + \binom{40}{r-1}\binom{60}{1} + \ldots$ attains maximum value when $r = 50$.

Reason: $\binom{2n}{r}$ is maximum when $r = n$.

83. **Assertion:** The number of non-negative integral solution of $x_1 + x_2 + \ldots + x_{20} = 100$ is $\binom{120}{20}$.

Reason: The number of ways of distributing n identical objects among r persons giving zero or more objects to a person is $\binom{n+r-1}{r-1}$.

84. **Assertion:** The sum of divisors of $n = 2^{10}3^25^37^211^2$ is $\frac{1}{48}(2^{11}-1)(3^3-1)\ (5^4-1)\ (7^3-1)\ (11^3-1)$

Reason: The number of divisor of $m = p_1^{\alpha_1}p_2^{\alpha_2}...p_r^{\alpha_r}$ where $p_1, p_2, ...p_r$ are distinct primes and $\alpha_1, \alpha_2, ...\alpha_r$ are natural numbers is $(\alpha_1+1)(\alpha_2+1)...(\alpha_r+1)$.

85. **Assertion:** If p is a prime, the exponent of p in $n!$ is $\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots$

Reason: where $[x]$ denotes the greatest integer $\le x$.

86. **Assertion:** A student is allowed to select at most n books from a collection of $(2n+1)$ books. If the total number of ways in which he can select at least one book is 255, then $n = 3$.

Reason: $\binom{2n+1}{0}+\binom{2n+1}{1}+...+\binom{2n+1}{n}=4^n$

Comprehension Based

Different words are being formed by arranging the letters of the word "SUCCESS". All the words obtained by written in the form of a dictionary.

87. The number of words in which the two C are together but no two S are together is:

a. 120 **b.** 96 **c.** 24 **d.** 420

88. The number of words in which no two C and no two S are together is:

a. 120 **b.** 96 **c.** 24 **d.** 420

89. The number of words in which the consonants appear in alphabetic order is:

a. 42 **b.** 40 **c.** 420 **d.** 280

90. The rank of the word 'SUCCESS' in the dictionary is:

a. 328 **b.** 329 **c.** 330 **d.** 331

91. The number of words in which the relative positions of vowels and consonants unaltered is:

a. 20 **b.** 60 **c.** 180 **d.** 540

Match the Column

92. Consider all possible permutations of the letters of the word ENDEANOEL:

Column I	Column II
(A) The number of permutations containing the word ENDEA, is	**1.** 5!
(B) The number of permutations in which the letter E occurs in the first and the last position, is	**2.** 2 × 5!
(C) The number of permutations in which none of the letters D, L, N occurs in the last five positions, is	**3.** 7 × 5!
(D) The number of permutations in which the letters A, E, O occur only in odd positions, is	**4.** 21 × 5!

a. A→1; B→4; C→2; D→2 **b.** A→2; B→3; C→4; D→1
c. A→4; B→2; C→1; D→3 **d.** A→3; B→3; C→2; D→1

93. Observe the following columns:

Column I	Column II
(A) If λ be the number of ways in which 6 boys and 5 girls can be arranged in a line so that they are alternate, the λ is divisible by.	**1.** 5!
(B) If λ be number of ways in which 6 boys and 5 girl can be seated in a row such that two girls are never together, then λis divisible by.	**2.** 6!
(C) If λbe the number of ways in which 6 boys and 5 girls can be seated around a round table if all the five girls do not sit together, thenλis divisible by	**3.** 7!
	4. 5!6!
	5. 5!7!

a. A→1,2,4; B→2,4; C→3,5 **b.**A→2,1,4; B→4,2, C→3,5
c. A→1,2,5; B→2,5; C→3,4 **d.**A→4,2,1; B→2,4; C→5,3

Integer

94. If the number of ways of selecting n coupons out of an unlimited number of coupons bearing the letters A, T, C so that they cannot be used to spell to the used CAT is 189, then Σn^2 must be:

95. If ${}^{12}P_r = 11880$, then $\sum_{i=1}^{\lambda} {}^{\lambda}C_i$ must be (where $\lambda = r+3$)

96. The letters of the word PATNA are arrange in all possible ways as in a dictionary, then rank of the word PATNA from last is:

97. The sum of all values of r in

$\binom{18}{r-2}+2\binom{18}{r-2}+\binom{18}{r}\ge\binom{20}{13}$ must be:

98. The number of integral solutions of $a+b+c=0$, $a\ge -5, b\ge -5, c\ge -5$ must be:

99. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1+n_2+n_3+n_4+n_5 = 20$. The number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is:

100. Let $n \ge 2$ be an integer. Taken n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is:

ANSWER

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	a	c	b	b	a	a	a	b	b
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
a	c	b	a	b	a	b	a	a	c
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
c	b	b	c	c	a	d	c	a	b
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
d	d	b	b	a	d	c	d	a	a
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
d	c	a	b	c	b	a	a	b	c
51.	**52.**	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**
c	c,d	b	a	a	d	b	b	c	c
61.	**62.**	**63.**	**64.**	**65.**	**66.**	**67.**	**68.**	**69.**	**70.**
a	a	d	b	c	c	a,d	b,c	a,c	a,b
71.	**72.**	**73.**	**74.**	**75.**	**76.**	**77.**	**78.**	**79.**	**80.**
a,c	b,d	a,b,c	a,c,d	c,d	b,c,d	b	a	a	c
81.	**82.**	**83.**	**84.**	**85.**	**86.**	**87.**	**88.**	**89.**	**90.**
d	a	b	b	a	d	c	b	a	d
91.	**92.**	**93.**	**94.**	**95.**	**96.**	**97.**	**98.**	**99.**	**100.**
a	a	a	91	127	19	70	136	7	5

SOLUTION

Multiple Choice Questions

1. **(c)** The student has seven choices from the morning courses out of which he can select one course in 7 ways.
For the evening course, he has 5 choices out of which he can select one course in 5 ways.
Hence he has total number of 7 + 5 = 12 choices.

2. **(a)** There are 12 questions in exercise 7. So, one question from exercise 7 can be selected in 12 ways. Exercise 8 contains 18 questions. So, second question can be selected in 18 ways. There are 9 questions in exercise 9. So, third question can be selected in 9 ways. Hence, three questions can be selected in 12 × 18 × 9 = 1944 ways.

3. **(c)** All the 5 digit numbers and 6 digit numbers are greater than 3000. Therefore number of 5 digit numbers $= {}^6P_5 - {}^5P_5 = 600$.
{Since the case that 0 will be at ten thousand place should be omit}.
Similarly number of 6 digit numbers 6! – 5! = 600.
Now the numbers of 4 digit numbers which are greater than 3000, having 3, 4 or 5 at first place, this can be done in 3 ways and remaining 3 digit may be filled from remaining 5 digits *i.e.,* required number of 4 digit numbers are ${}^5P_3 \times 3 = 180$.
Hence total required number of numbers
= 600 + 600 + 180 = 1380.

4. **(b)** Required number of ways $= \dfrac{8!}{2!2!2!} = 5040$.
[since here 2C's, 2T's and 2A's]

5. **(b)** Number of words $= \dfrac{9!}{2!2!2!} = \dfrac{9!}{(2!)^3}$ [Since here total number of letters is 9 and 2M's, 2T's and 2E's]

6. **(a)** First arrange m men, in a row in $m!$ ways. Since $n < m$ and no two women can sit together, in any one of the $m!$ arrangement, there are $(m + 1)$ places in which n women can be arranged in ${}^{m+1}P_n$ ways.
∴ By the fundamental theorem, the required number of arrangement $= m!\ {}^{m+1}P_n = \dfrac{m!(m+1)!}{(m-n+1)!}$.

7. **(a)** Words starting from A are 5 ! = 120;
Words starting from I are 5 ! = 120
Words starting from KA are 4! = 24;
Words starting from KI are 4 ! = 24
Words starting from KN are 4 ! = 24; Words starting from KRA are 3 ! = 6
Words starting from $KRIA$ are 2 ! = 2; Words starting from $KRIN$ are 2 ! = 2
Words starting from $KRIS$ are 1 ! = 1 Words starting from $KRISNA$ are 1 ! = 1
Hence rank of the word KRISNA is 324

8. **(a)** We have 5 letters other than 'I' and 'N' of which two are identical (E's). We can arrange these letters in a line in $\dfrac{5!}{2!}$ ways. In any such arrangement 'I' and 'N' can be placed in 6 available gaps in ${}^6P_2$ ways, so required number $= \dfrac{5!}{2!}{}^6P_2 = m_1$.
Now, if word start with I and end with R then the remaining letters are 5. So, total number of ways $= \dfrac{5!}{2!} = m_2$.
∴ $\dfrac{m_1}{m_2} = \dfrac{5!}{2!}.\dfrac{6!}{4!}.\dfrac{2!}{5!} = 30$.

9. **(b)** Since at any place, any of the digits 2, 5 and 7 can be used total number of such positive n-digit numbers are 3^n. Since we have to form 900 distinct numbers,
Hence $3^n \ge 900 \Rightarrow n = 7$.

10. **(b)** The 4 odd digits 1, 3, 3, 1 can be arranged in the 4 odd places, in $\frac{4!}{2!2!}=6$ ways and 3 even digits 2, 4, 2 can be arranged in the three even places $\frac{3!}{2!}=3$ ways.

Hence the required number of ways = 6 × 3 = 18.

11. **(a)** Fix up a male and the remaining 4 male can be seated in 4! ways. Now no two female are to sit together and as such the 2 female are to be arranged in five empty seats between two consecutive male and number of arrangement will be $^5P_2$. Hence by fundamental theorem the total number of ways is $= 4!\times {}^5P_2 = 24\times 20 = 480$ ways.

12. **(c)** Required number of ways $9!\times 2$.

{By fundamental property of circular permutation}.

13. **(b)** Since total number of ways in which boys can occupy any place is $(5-1)!=4!$ and the 5 girls can be sit accordingly in $5!$ ways.

Hence required number of ways are $4!\times 5!$.

14. **(a)** Since total members are 15, but one is to left, because of circular condition, therefore remaining members are 14 but three special member constitute a member. Therefore required number of arrangements are $12!\times 2$, because, chairman remains between the two specified persons and the person can sit in two ways.

15. **(b)** There are 20 + 1 = 21 persons in all. The two particular persons and the host be taken as one unit so that these remain 21 – 3 + 1 = 19 persons to be arranged in 18! ways. But the two person on either side of the host can themselves be arranged in 2! ways. Hence there are 2! 18! ways or 2.18! ways.

16. **(a)** The number of ways in which 5 beads of different colours can be arranged in a circle to form a necklace are $(5-1)!=4!$.

But the clockwise and anticlockwise arrangement are not different (because when the necklace is turned over one gives rise to another)

Hence the total number of ways of arranging the beads $=\frac{1}{2}(4!)=12$.

17. **(b)** Fix up 1 man and the remaining 6 men can be seated in 6! ways. Now no two women are to sit together and as such the 7 women are to be arranged in seven empty seats between two consecutive men and number of arrangement will be 7!. Hence by fundamental theorem the total number of ways = 7! × 6!

18. **(a)** 8 different beads can be arranged in circular form in (8 – 1)! = 7! ways. Since there is no distinction between the clockwise and anticlockwise arrangement. So the required number of arrangements = $\frac{7!}{2}$ = 2520.

19. **(a)** $^{15}C_{3r} = {}^{15}C_{r+3}$

$\Rightarrow$ $^{15}C_{15-3r} = {}^{15}C_{r+3}$

$\Rightarrow$ $15-3r = r+3$

$\Rightarrow$ $r=3$.

20. **(c)** $\frac{^nC_r}{^nC_{r-1}} = \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}}$

$\Rightarrow$ $\frac{n!}{r!(n-r)!}\times\frac{(r-1)!(n-r+1)!}{n!}$

$=\frac{(n-r+1)(r-1)!(n-r)!}{r(r-1)!(n-r)!} = \frac{(n-r+1)}{r}$.

21. **(c)** $^{n+1}C_3 = 2.{}^nC_2$

$\Rightarrow$ $\frac{(n+1)!}{3!(n-2)!} = 2.\frac{n!}{2!(n-2)!}$

$\Rightarrow$ $\frac{n+1}{3.2!} = \frac{2}{2!}$

$\Rightarrow$ $n+1=6$

$\Rightarrow$ $n=5$.

$^5C_3\times {}^{22}C_9$.

22. **(b)** Number of ways are $=2^{10}-1=1023$

[– 1 corresponds to none of the lamps is being switched on.]

23. **(b)** Numbers greater than 1000 and less than or equal to 4000 will be of 4 digits and will have either 1 (except 1000) or 2 or 3 in the first place with 0 in each of remaining places.

After fixing 1st place, the second place can be filled by any of the 5 numbers. Similarly third place can be filled up in 5 ways and 4th place can be filled up in 5 ways. Thus there will be 5 × 5 × 5 = 125 ways in which 1 will be in first place but this include 1000 also hence there will be

124 numbers having 1 in the first place. Similarly 125 for each 2 or 3. One number will be in which 4 in the first place and *i.e.,* 4000. Hence the required numbers are 124 + 125 + 125 +1 = 375 ways.

24. **(c)** The number of ways can be given as follows:
2 bowlers and 9 other players
$= {}^4C_2 \times {}^9C_9$; 3 bowlers and 8 other players = ${}^4C_3 \times {}^9C_8$
4 bowlers and 7 other players = ${}^4C_4 \times {}^9C_7$
Hence required number of ways
= 6 × 1 + 4 × 9 + 1 × 36 = 78.

25. **(c)** 6 particular players are always to be included and 4 are always excluded, so total number of selection, now 4 players out of 12. Hence number of ways = ${}^{12}C_4$.

26. **(a)** The required number of ways = $4^5 = 1024$
[since each prize can be distributed by 4 ways]

27. **(d)** Total ways = $\frac{9!}{(3!)^3} = \frac{9\times8\times7\times6\times5\times4}{3\times2\times3\times2\times3\times2} = 280.$

28. **(c)** Required number of ways = $2^7 - 1 = 127$.
{Since the case that no friend be invited
i.e., ${}^7C_0$ is excluded}.

29. **(a)** Required number of ways $= {}^{15}C_1 \times {}^8C_1 = 15 \times 8$.

30. **(b)** $\frac{(2n)!}{(2n-3)!.3!} \times \frac{2!\times(n-2)!}{n!} = \frac{44}{3}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{3n(n-1)} = \frac{44}{3}$

$\Rightarrow 4(2n-1) = 44$

$\Rightarrow 2n = 12$

$\Rightarrow n = 6$

Now ${}^6C_r = 15$

$\Rightarrow {}^6C_r = {}^6C_2$

or ${}^6C_4$

$\Rightarrow r = 2, 4$.

31. **(d)** ${}^{n^2-n}C_2 = {}^{n^2-n}C_{10}$

$\Rightarrow {}^{n^2-n}C_{n^2-n-2} = {}^{n^2-n}C_{10}$

$\Rightarrow n^2 - n - 2 = 10$

or $n = 4, -3$.

32. **(d)** Total number of shake hands when each person shake hands with the other once only = ${}^8C_2 = 28$ ways.

33. **(b)** ${}^8C_r = {}^8C_{r+2}$

$\Rightarrow 8 - r = r + 2$

$\Rightarrow r = 3$

Hence ${}^3C_2 = 3$.

34. **(b)** ${}^nC_2 = 66$

$\Rightarrow n(n-1) = 132$

$\Rightarrow n = 12$.

35. **(a)** Since ${}^nC_r = {}^nC_{n-r}$ and ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$ we have

$\sum_{r=0}^{m} {}^{n+r}C_n = \sum_{r=0}^{m} {}^{n+r}C_r = {}^nC_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + ...+ {}^{n+m}C_m$

$= [1 + (n+1)] + {}^{n+2}C_2 + {}^{n+3}C_3 + ...+ {}^{n+m}C_m$

$= {}^{n+m+1}C_{n+1}$

$[\because {}^nC_r = {}^nC_{n-r}]$.

36. **(d)** Each question can be answered in 4 ways and all questions can be answered correctly in only one way, so required number of ways = $4^3 - 1 = 63$.

37. **(c)** We have 32 places for teeth. For each place we have two choices either there is a tooth or there is no tooth. Therefore the number of ways to fill up these places is 2^{32}. As there is no person without a tooth, the maximum population is $2^{32} - 1$.

38. **(d)** ${}^{10}C_r = {}^{10}C_{r+2}$

$\Rightarrow r + r + 2 = 10$

$\Rightarrow r = 4$

$\therefore {}^5C_r = {}^5C_4 = \frac{5!}{1!4!} = 5$.

39. **(a)** ${}^nC_3 + {}^nC_4 > {}^{n+1}C_3$

$\Rightarrow {}^{n+1}C_4 > {}^{n+1}C_3$ $(\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$

$\Rightarrow \frac{{}^{n+1}C_4}{{}^{n+1}C_3} > 1$

$\Rightarrow \frac{n-2}{4} > 1$

$\Rightarrow n > 6$.

40. **(a)** ${}^nC_5 + {}^nC_6 > {}^{n+1}C_5$

$\Rightarrow {}^{n+1}C_6 > {}^{n+1}C_5$

$\Rightarrow \frac{(n+1)!}{6!.(n-5)!}.\frac{5!.(n-4)!}{(n+1)!} > 1 \Rightarrow \frac{(n-4)}{6} > 1$

$\Rightarrow n - 4 > 6 \Rightarrow n > 10$

Hence according to options $n = 11$.

41. **(d)** ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$ is a standard formula.

42. **(c)** Required number of ways

$= {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5$

$= 8 + 28 + 56 + 70 + 56 = 218$

{Since voter may vote to one, two, three, four or all candidates}

43. **(a)** $\bullet E \bullet E \bullet E \bullet ......\bullet E \bullet$

According to condition there are 22 vacant places for Hindi books hence total number of ways are

$= {}^{22}C_{19} = 1540$, {Since books are same}.

44. **(b)** Since 5 are always to be excluded and 6 always to be included, therefore 5 players to be chosen from 14. Hence required number of ways are ${}^{14}C_5 = 2002$.

45. **(c)** The required number of ways are

$(10+1)(9+1)(7+1)-1$

$= 11 \times 10 \times 8 - 1 = 879$.

46. **(b)** Required number of ways

$= {}^5C_3 \times {}^2C_1 \times {}^9C_7$

$= 10 \times 2 \times 36 = 720$.

47. **(a)** The selection can be made in ${}^5C_3 \times {}^{22}C_9$.

{Since 3 vacancies filled from 5 candidates in ${}^5C_3$ ways and now remaining candidates are 22 and remaining seats are 9}.

48. **(a)** 5 persons are to be seated on 8 chairs *i.e.* ${}^8C_3 \times 5!$ or 6720.

{Since 5 chairs can be select in ${}^8C_5$ and then 5 persons can be arranged in $5!$ ways}.

49. **(b)** First omit two particular persons, remaining 8 persons may be 4 in each boat. This can be done in ${}^8C_4$ ways. The two particular persons may be placed in two ways one in each boat. Therefore total number of ways are $= 2 \times {}^8C_4$.

50. **(c)** The number of times he will go to the garden is same as the number of selecting 3 children from 8.

Therefore the required number $= {}^8C_3 = 56$.

51. **(c)** Number of derangement are = $4!\left\{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\}$

$= 12 - 4 + 1 = 9$.

(Since number of derangements in such a problem is given by $n!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} ...... + (-1)^n \frac{1}{n!}\right\}$.

52. **(c, b)** Out of 18 points, 5 are collinear

(i) Number of straight lines

$= {}^{18}C_2 - {}^5C_2 + 1 = 153 - 10 + 1 = 144$

(ii) Number of triangles $= {}^{18}C_3 - {}^5C_3 = 816 - 10 = 806$.

53. **(b)** Required number of ways are ${}^8C_2 - 8 = 20$.

54. **(a)** Required number of ways = ${}^4C_3 = 4$.

55. **(a)** The number of triangles are ${}^9C_3 = 84$.

56. **(d)** Required number of ways ${}^8C_2 = 28$.

57. **(b)** Required number of triangles

$= {}^{10}C_3 - {}^4C_3 = 120 - 4 = 116$.

58. **(b)** Total number of points are $m+n+k$, the $\Delta's$ formed by these points $= {}^{m+n+k}C_3$

Joining 3 points on the same line gives no triangle, such $\Delta's$ are ${}^mC_3 + {}^nC_3 + {}^kC_3$

Required number $= {}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$.

59. **(c)** Number of lines from 6 points $= {}^6C_2 = 15$. Points of intersection obtained from these lines $= {}^{15}C_2 = 105$. Now we find the number of times, the original 6 points come.

Consider one point say A_1. Joining A_1 to remaining 5 points, we get 5 lines, and any two lines from these 5 lines give A_1 as the point of intersection.

$\therefore$ A_1 come ${}^5C_2 = 10$ times in 105 points of intersections.

Similar is the case with other five points.

$\therefore$ 6 original points come $6 \times 10 = 60$ times in points of intersection.

Hence the number of distinct points of intersection $= 105 - 60 + 6 = 51$.

60. **(c)** Since no two lines are parallel and no three are concurrent, therefore n straight lines intersect at ${}^nC_2 = N$ (say) points. Since two points are required to determine a straight line, therefore the total number of lines obtained by joining N points ${}^NC_2$.But in this each old line has been counted ${}^{n-1}C_2$ times, since on each old line there will be $n-1$ points of intersection made by the remaining $(n-1)$ lines.

Hence the required number of fresh lines is

$${}^NC_2 - n \,.\, {}^{n-1}C_2 = \frac{N(N-1)}{2} - \frac{n(n-1)(n-2)}{2}$$

$$= \frac{{}^nC_2({}^nC_2 - 1)}{2} - \frac{n(n-1)(n-2)}{2} = \frac{n(n-1)(n-2)(n-3)}{8}.$$

61. **(a)** The number of points of intersection of 37 straight lines is ${}^{37}C_2$. But 13 of them pass through the point A. Therefore instead of getting ${}^{13}C_2$ points we get merely one point.

Similarly 11 straight lines out of the given 37 straight lines intersect at B. Therefore instead of getting ${}^{11}C_2$ points, we get only one point. Hence the number of intersection points of the lines is

${}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2 = 535$.

62. **(a)** ${}^{16}C_3 - {}^{8}C_3 = 504$.

63. **(d)** Required number $= {}^{20}C_2 - {}^{4}C_2 + 1$

$= \dfrac{20\times19}{2} - \dfrac{4\times3}{2} + 1$

$= 190 - 6 + 1 = 185.$

64. **(b)** Since the student is allowed to select utmost n books out of $(2n+1)$ books. Therefore in order to select one book he has the choice to select one, two, three,......., n books.

Thus, if T is the total number of ways of selecting one book then $T = {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n = 63$.

Again the sum of binomial coefficients

${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} +$

${}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n+1} = (1+1)^{2n+1} = 2^{2n+1}$

or, ${}^{2n+1}C_0 + 2({}^{2n-1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n) + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow$ $1 + 2(T) + 1 = 2^{2n+1}$

$\Rightarrow$ $1 + T = \dfrac{2^{2n+1}}{2} = 2^{2n}$

$\Rightarrow$ $1 + 63 = 2^{2n}$

$\Rightarrow$ $2^6 = 2^{2n}$

$\Rightarrow$ $n = 3$.

65. **(c)** The result ${}^{x+y}C_r$ is trivially true for $r = 1, 2$ it can be easily proved by the principle of mathematical induction that the result is true for r also.

66. **(c)** Since 9600 = $2^7 \times 3^1 \times 5^2$

Hence number of divisors

= $(7+1)(1+1)(2+1) = 48$.

NCERT Exemplar Problems

More than One Answer

67. **(a,d)** $\dfrac{6!}{6!} = 1 = {}^{6}C_6$ ($\because$ All faces are alike)

68. **(b,c)** The number of ways of inviting, with the couple not included = ${}^{8}C_5$ and the number of ways of inviting, with the couple included = ${}^{8}C_3$

$\therefore$ Required number of ways = = ${}^{8}C_5$ + = ${}^{8}C_3$

$= {}^{8}C_3 + {}^{8}C_3$ ($\because {}^{8}C_5 = {}^{8}C_3$) $= 2\times{}^{8}C_3$

Also, ${}^{10}C_5 - 2\times{}^{8}C_4 = \dfrac{10}{5}.{}^{9}C_4 - 2\times{}^{8}C_4$

$= 2({}^{9}C_4 - {}^{8}C_4) = 2({}^{8}C_3 + {}^{8}C_4 - {}^{8}C_4) = 2\times{}^{8}C_3$

69. **(a,c)** The number of ways $= {}^{n}C_m \times m!$

{Here numbers 1, 2, 3, …, n; $\therefore$ $(m-1)!$ fail}

70. **(a,b)** The number of ways of placing A_1 and A_2 in ten places so that A_1 is always above A_2 is ${}^{10}C_2$. There are 8! Ways of arranging the eight other candidates.

Hence, total number of arrangements

$= {}^{10}C_2 \times 8! = \dfrac{10!}{2!8!}8! = \dfrac{10!}{2}$

71. **(a,c)** Suppose the two players did not play at all so that the remaining $(n-2)$ players played ${}^{n-2}C_2$ matches. Since, these two players played 3 matches each, hence the total number of matches is ${}^{n-2}C_2 + 3 + 3 = 84$ (given)

$\Rightarrow$ ${}^{n-2}C_2 = 78 = {}^{13}C_2$

$\Rightarrow$ $n - 2 = 13$

$\therefore$ $n = 15 = {}^{6}C_2$

72. **(b,d)** $\dfrac{10!}{2!2!3!3!}$ ($\because$ $S_1$and S_2 get 2 books each and S_3 and S_4 get 3 books each)

73. **(a,b,c)** *i.e*, $0^2 + 0^2 + 0, 1^2 + 2^2 = 5, 2^2 + 4^2$

$= 20, 3^2 + 4^2 = 25,$

$1^2 + 3^2 = 10, 2^2 + 1^2 = 5, 4^2 + 2^2 = 20, 4^2 + 3^2 = 25,$

$3^2 + 1^2 = 10$

$\therefore$ Required number of ways $= {}^{9}C_1 = 9$

Also, ${}^{9}C_8 = {}^{9}C_{9-8} = {}^{9}C_1 = 9$

74. **(a,c,d)** Total number of arrangements =7!

Number of arrangements of A,B,C,D among themselves =4!

$\therefore$ Number of arrangements when A,B,C,D occur in a particular order $= \dfrac{7!}{4!} = 210 = {}^{7}P_3 = 3! \times {}^{7}C_3$.

75. **(c,d)** Required number = number of selection of one or more out of three 25 paise coins and two 50 paise coins

$= 4\times3 - 1 = 11 = {}^{12}P_1 - 1$

76. **(b,c,d)** α can be subtracted from β without borrowing if $y_i \geq x_i$; for $i = 1,2,3$ Let $x_i = \lambda$

If $i = 1$, then $\lambda = 1,2,3,...9$ and for $i = 0$ and $3, \lambda = 0,2,3,...9$

Hence, total number of ways of choosing the pair α, β is

$$\left(\sum_{\lambda=1}^{9}(10-\lambda)\right)\left(\sum_{\lambda=0}^{9}(10-\lambda)\right)^2 = (45)(55)^2$$

Assertion and Reason

77. **(b)** Let x_i = number of balls put in ith box,

Then $x_1 + x_2 + x_3 + x_4 = 10$ where $x_i \geq 1$. Put $x_i = y_i + 1$, so that equation becomes $y_1 + y_2 + y_3 + y_4 = 6$. where $y_i \geq 0$

Number of non-negative integral solution of the above equation = Number of ways of arranging 6 identical balls and 3 identical separators $= \dfrac{(6+3)!}{6!3!} = {}^9C_3$ number of ways of choosing 3 places out of 9 different places.

78. **(a)** The number of ways of selecting students for the first group is ${}^{mn}C_n$; for the second group is ${}^{mn}C_n$ and so on.

$\therefore$ The number of ways of dividing (mn) students into m numbered groups is $({}^{mn}C_n)\,({}^{mn-n}C_n)\ldots({}^{n}C_n)$

$$= \frac{(mn)!}{n!(mn-n)!}\frac{(mn-n)!}{n!(mn-2n)!}\cdots\frac{n!}{n!0!} = \frac{(mn)!}{(n!)^m}$$

As groups are not to be numbered, the desired number of ways is $\dfrac{(mn)!}{m!(n!)^m}$

$\therefore$ Reason is true. For Assertion, put $m = n$.

79. **(a)** If each element of a determinant is an integer, then its each cofactor is an integer, and hence determinant itself is an integer.

80. **(c)** $\dbinom{n}{k}\dbinom{k}{j} = \dfrac{n!}{k!(n-k)!}\dfrac{k!}{j!(k-j)!}$

$$= \frac{n!}{j!(n-j)!}\frac{(n-j)!}{(k-j)!(n-k)!} = \binom{n}{j}\binom{n-j}{k-j}$$

Thus, $\displaystyle\sum_{j=1}^{n}\sum_{k=j}^{n}\binom{n}{k}\binom{k}{j} = \sum_{j=1}^{n}\binom{n}{j}\sum_{k=j}^{n}\binom{n-j}{k-j}$

But $\displaystyle\sum_{k=j}^{n}\binom{n-j}{k-j} = \sum_{l=0}^{n-j}\binom{n-j}{l} = 2^{n-j}$

$\therefore$ $\displaystyle\sum_{j=1}^{n}\sum_{k=r}^{n}\binom{n}{j}2^{n-j}(2+1)^n - 2^n = 3^n - 2^n$

Reason is false as $\displaystyle\sum_{k=1}^{n}({}^nC_k)^2 = {}^{2n}C_n - 1$

81. **(d)** Suppose A_i gets x_i toys then $x_1 + x_2 + x_3 \leq 12$.

Let $x_4 = 12-(x_1+x_2+x_3)$, then $x_1 + x_2 + x_3 + x_4 = 12$ . . .(i)

The number of non-negative integral solutions of (i)

= coefficient of t^{12} in $(t + t^2 + \ldots)\,(t^3 + t^4 + t^5 + \ldots)$ $(1 + t + \ldots + t^5) \times (1 + t + t^2 + \ldots)$

= coefficient of t^{12} in $t^4\,(1 - t^6)\,(1 - t)^{-4}$

= coefficient of t^8 in $(1 - t^6)\,(1 + {}^4C_1t + {}^5C_2t^2 + \ldots)$

$= {}^{11}C_8 - {}^5C_2 = 165 - 10 = 155$

Reason is false as the number of non-negative integral solution of $x_1 + x_2 + x_3 \leq m$

Equals the number of non-negative integral solution of $x_1 + x_2 + x_3 + x_4 = m$, which equals ${}^{m+3}C_m$.

82. **(a)** We have $\dfrac{\binom{2n}{r}}{\binom{2n}{r+1}} = \dfrac{(2n)!}{r!(2n-r)!}\dfrac{(r+1)!(2n-r-1)!}{(2n)!}$

$$= \frac{r+1}{2n-r}$$

Since for $0 \leq r \leq n-1, \dfrac{r+1}{2n-r} < 1$, we get

$$\binom{2n}{0} < \binom{2n}{1} < \ldots < \binom{2n}{n-1} < \binom{2n}{2}$$

Also, as $\dbinom{2n}{0} = \dbinom{2n}{2n-r}$

$$\binom{2n}{2n} < \binom{2n}{2n-1} \ldots < \binom{2n}{2n+1} < \binom{2n}{n}$$

Thus, $\dbinom{2n}{r}$ is maximum when $r = n$.

Next, $\dbinom{40}{r}\dbinom{60}{0} + \dbinom{40}{r-1}\dbinom{60}{1} + \ldots$

= then number ways of selecting r persons out of 40 men and 60 women $= \dbinom{100}{r}$. Which is maximum when $r = 50$.

83. **(b)** The number of ways of distributing n identical objects among r persons giving zero of more objects to a person is equivalent to arrangement n identical objects of one kind and $(r-1)$ identical objects of second kind in a row, which is equal to $\dfrac{(n+r-1)!}{n!(r-1)!} = \dbinom{n+r-1}{r-1}$. Next, the number of non-negative integral solutions of $x_1+x_2+\ldots x_{20}$ equals the number of ways of distributing 100 identical objects among 20 persons giving zero or more objects to a person, which equal $\dbinom{100+20-2}{20-1} = \dbinom{109}{19}$.

84. **(b)** Sum of the divisors of n

$= (1 + 2 + \ldots – 2^{10})(1+3+3^2)(1+5+5^2+5^3)$
$(1+7+7^2)\ (1+11+11^2)$

$$=(2^{11}-1)\left(\frac{3^3-1}{3-1}\right)\left(\frac{5^4-1}{5-1}\right)\left(\frac{7^3-1}{7-}\right)\left(\frac{11^3-1}{11-1}\right)$$

$$=\frac{1}{480}(2^{11}-1)(3^3-1)(5^4-1)(7^3-1)(11^3-1)$$

A divisors of m is of the form $p_1^{\beta_1}p_2^{\beta_1}\ldots p_r^{\beta_r}$ where $0\le\beta_i\le\alpha_i$ for $i=1,2,\ldots,r$. That is, β_i can take α_i+1 values. Thus, the number of divisors of m is $(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_r+1)$

85. **(a)** We have $\binom{1000}{500}=\frac{1000!}{500!500!}$

Exponent of 11 in 1000! Is

$$\left[\frac{1000}{11}\right]+\left[\frac{1000}{11^2}\right]=90+8=98$$

Exponent of 11 in 500! is $\left[\frac{500}{11}\right]+\left[\frac{500}{11^2}\right]=45+4=49.$

Thus, exponent of 11 in $\binom{1000}{500}$ is 0.

$\Rightarrow$ $\binom{1000}{500}$ is not divisible by 11.

86. **(d)** Let $S=\binom{2n+1}{0}+\binom{2n+1}{1}+\ldots+\binom{2n+1}{n}$ $\quad\ldots(i)$

Using $\binom{n}{r}=\binom{n}{n-r}$, we can write (i) as

$$S=\binom{2n+1}{2n+1}+\binom{2n+1}{2n}+\ldots+\binom{2n+1}{n+1}\quad\ldots(ii)$$

Adding (i) and (ii), we get

$$2S=\binom{2n+1}{0}+\ldots+\binom{2n+1}{2n+1}=2^{2n+1}$$

$\Rightarrow$ $S=2^{2n}=4^n$

The number of ways of choosing r books out of $2n+1\binom{2n+1}{r}$. We are given

$$\Rightarrow\quad\binom{2n+1}{1}+\binom{2n+1}{2}+\ldots+\binom{2n+1}{n}=255$$

$\Rightarrow$ $4^n - 1 = 255 \Rightarrow 4^n = 256 = 4^4 \Rightarrow n = 4.$

Comprehension Based

87. **(c)** Considering CC as single object, U,CC,E can be arranged in 3!Ways $\times\ U \times CC \times E \times$

Now the three S are to be place in four available places.

Hence, required number of ways $= {}^4C_3 \times 3! = 24$

88. **(b)** Let us first find the words in no two S are together.

(i) Arrange the remaining letters $=\frac{4!}{2!}=12$ ways

(ii) $\times\ U \times C \times C \times E \times$

Hence, total number of ways no two S together

$= 12 \times {}^5C_3 = 120$

$\therefore$ Hence, number of words having CC separated and SSS separated= 120 – 24 = 96.

89. **(a)** Total number of ways $=\frac{7!}{2!3!}=420$

Consonants in SUCCESS are S,C,C,S,S

Number of ways arranging consonants $=\frac{5!}{2!3!}=10$

Hence, number of words in which the consonants appear

In alphabetic order $=\frac{420}{10}=42$

90. **(d)** The alphabetic order is C,E,S,U. The number of words beginning with C is $\frac{6!}{3!}=120$ ways and those beginning

With E is $\frac{6!}{2!3!}$ = 60 ways.

Then come wards beginning SC, numbering $\frac{5!}{2!}$ = 60,

SE, numbering $\frac{5!}{2!2!}$ = 30 and SS, numbering $\frac{5!}{2!}$ = 60.

After which come the word SUCCESS.

Thus the rank of SUCCESS is 120+60+60+30+60+1= 331

91. **(a)** Vowels are EU

These vowels can be arranged themselves in 2! = 2 ways. The consonants are $CCSSS$ these consonants can be arranged themselves in $\frac{5!}{2!3!}=10$ ways.

$\therefore$ Required number of words = 2 × 10 = 20 ways.

Match the Column

92. **(a)** **(A)** If ENDEA is fixed word, then assume this as a single letter. Total number of letters = 5

Total number of arrangements = 5!

(B) If E is at first and last place, then total number of permutations = 7!/2! = 21 × 5!

(C) If D, L, N are not in last five positions

$\leftarrow D,L,N,\rightarrow\leftarrow E,E,E,A,O\rightarrow$

Total number of permutations $=\frac{4!}{2!}\times\frac{5!}{3!}=2\times5!$

(D) Total number of odd positions = 5

Permutations of AEEEO are $\frac{5!}{3!}$

Total number of even positions = 4

Number of permutations of $N, N, D, L = \frac{4!}{2!}$

Hence, total number of permutations $= \frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$

93. **(a) (A)** $B\ G\ B\ G\ B\ G\ B\ G\ B\ G\ B$

Required number of ways = 6!5!

(B) Since here restriction on girls $\times B \times B \times B \times B \times B \times B \times$

Let us seat the boys first, which can be done in 6! ways.

For five girls there are 7 places shown by '×' which can be done in ${}^7P_5$

Required number of ways

Required number of ways = ${}^7P_5 \times 6!$

$= \frac{7!}{2!} \times 6! = \frac{7 \times 6 \times 5! \times 6!}{2} = 21 \times 5!6!$ Or $3 \times 5!\ 7!$ (R, T)

(C) Total ways without restriction = (11 – 1)! = 10!

Number of ways in which all the girls can be seated together = (7 – 1)! × 5! = 6! 5!

∴ Required number of ways = 10! –6!5! = 41×5!6!

Integer

94. **(91)** Number of ways of selecting n coupons consisting of C or $A = 2^n$ (∵ The work CAT cannot be written if at least one letters is not selected)

Now number of ways of selecting n coupons bearing only $A = 1^n$

∴ Total number of ways

$= 2^n + 2^n + 2^n - 1^n - 1^n - 1^n = 3(2^n - 1)$

Given $3(2^n - 1) = 189 \Rightarrow 2^n - 1 = 63$

⇒ $2^n = 64 = 2^6$ ∴ $n = 6$

Then $\sum n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{6 \cdot 7 \cdot 13}{6} = 91$

95. **(127)** ${}^{12}P_r = 12 \times 11 \times 10 \times 9 = {}^{12}P_4$

∴ $r = 4$

Then, $\lambda = r + 3 = 7$

∴ $\sum_{i=1}^{7} {}^7C_i = 2^7 - 1 = 127$

96. **(19)** The letters of PATNA can be arranged $= \frac{5!}{2!} = 60$ ways

The alphabetic order is A,A, N, P, T. The number of words beginning with A is 4! = 24, Number of words beginning with N is $\frac{4!}{2!} = 12$, Number of words beginning with PAA is 2! = 2,

Number of words beginning with PAN is 2! = 2,

The total $(24 + 12 + 2 + 2 = 40)$

∴ Next 41th words are PATNA, then 42th word is PATNA,

∴ Rank of the word PATNA from end = 60 – 42 = 19

97. **(70)** We have $\binom{18}{r-2} + 2\binom{18}{r-2} + \binom{18}{r} \geq \binom{20}{13}$ it means that ${}^{18}C_{r-2} + 2 \cdot {}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$

⇒ $({}^{18}C_{r-2} + {}^{18}C_{r-1}) + ({}^{18}C_{r-1} + {}^{18}C_r) \geq {}^{20}C_{13}$

⇒ ${}^{19}C_{r-2} + {}^{19}C_r \geq {}^{20}C_{13}$

⇒ ${}^{20}C_r \geq {}^{20}C_{13}$ or ${}^{20}C_r \geq {}^{20}C_7$

Hence $7 \leq r \leq 13$

∴ r = 7, 8, 9, 10, 11, 12, 13

∴ Required sum = 7 + 8 + 9 + 10 + 11 + 12 + 13 = 70

98. **(136)** $a \geq -5 \Rightarrow a + 6 \geq 1$

Similarly $b + 6 \geq 1$ and $c + 6 \geq 1$

Let $a + 6 = \alpha, b + 6 = \beta$ and $c + 6 = \gamma$

Then $\alpha + \beta + \gamma = 18, \alpha \geq 1, \beta \geq 1, \gamma \geq 1$

∴ Required number of solution

$= {}^{18-1}C_{3-1} = {}^{17}C_2 = \frac{17 \cdot 16}{2} = 136$

99. **(7)** As, $n_1 \geq 1, n_2 \geq 2, n_3 \geq 3, n_4 \geq 4, n_5 \geq 5$

Let $n_1 - 1 = x_1 \geq 0, n_2 - 2 = x_2 \geq 0, \ldots, n_5 - 5 = x_5 \geq 0$

⇒ New equation will be $x_1 + 1 + x_2 + 2 + \ldots + x_5 + 5 = 20$

⇒ $x_1 + x_2 + x_3 + x_4 + x_5 = 20 - 15 = 5$

Now, $x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5$

x_1	x_2	x_3	x_4	x_5
0	0	0	0	5
0	0	0	1	4
0	0	0	2	3
0	0	1	1	3
0	0	1	2	2
0	1	1	1	2
1	1	1	1	1

So, 7 possible cases will be there.

100. **(5)** Number of red line segments = ${}^nC_2 - n$

Number of blue line segments = n

∴ ${}^nC_2 - n = n$

⇒ $\frac{n(n-1)}{2} = 2n$

⇒ $n = 5$

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