9 CHAPTER

Deflection of Beams

Q.1 A simply supported beam carrying a concentrated load W at mid-span deflects by δ_1 under the load. If the same beam carries the load W such that it is distributed uniformly over entire length and undergoes a deflection δ_2 at its mid-span then $\delta_1 : \delta_2$ is

(a) 5 : 8 (b) 8 : 5
(c) 1 : 1 (d) 3 : 2

Q.2 Consider the following statements:
In a cantilever subjected to a concentrated load at the free end,
1. the bending stress is maximum at the free end.
2. the maximum shear stress is constant along the length of the beam.
3. the slope of the elastic curve is zero at the fixed end.

Which of these statement(s) is/are correct?

(a) 1, 2 and 3 (b) 2 and 3
(c) 1 and 3 (d) 1 and 2

Q.3 Two identical cantilevers are loaded as shown in the figure. If slope at the free end of the cantilever in figure (i) is θ, then the slope at free end of cantilever in figure (ii) will be

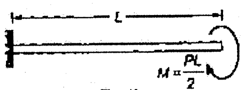


Fig. (i)

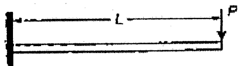


Fig (ii)

(a) $\frac{1}{3}\theta$ (b) $\frac{1}{2}\theta$
(c) $\frac{2}{3}\theta$ (d) θ

Q.4 Assertion (A): In a simply supported beam subjected to a concentrated load P at mid-span, the elastic curve slope becomes zero under the load.
Reason (R): The deflection of the beam is maximum at mid-span.

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.5 A simply supported beam having rectangular section 4 cm wide and 6 cm deep carries a mid-span concentrated load. The deflection under the load is δ. If the beam is now supported with 4 cm side parallel to line of action of loading, then the deflection under the load will be

(a) 0.44 δ (b) 0.67 δ
(c) 1.5 δ (d) 2.25 δ

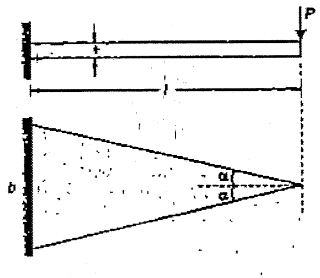
Q.6 In a cantilever beam, if the length is doubled while keeping the cross-section and the concentrated load acting at the free end the same, the deflection at the free end will increase by

(a) 2.66 times (b) 3 times
(c) 6 times (d) 8 times

Q.7 Which one of the following is represented by area of the shear force diagram from one end upto a given location on the beam?

(a) BM at the location
(b) Load at the location
(c) Slope at the location
(d) Deflection at the location

Q.8 A triangular shaped cantilever beam of uniform thickness is as shown in figure. The Young's modulus of the beam is E. A concentrated load P is applied at the free end of the beam. The maximum deflection of the beam is

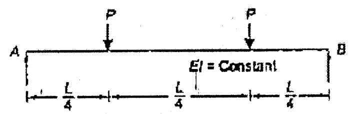


(a) $\frac{24Pl^3}{Ebt^3}$ (b) $\frac{12Pl^3}{Ebt^3}$
(c) $\frac{8Pl^3}{Ebt^3}$ (d) $\frac{6Pl^3}{Ebt^3}$

Q.9 A beam of overall length L rests on two simple supports with equal overhangs on both sides. Two equal loads act at the free ends. If the deflection at the centre of the beam is the same as that at the either end, then the length of either overhang is

(a) 0.152 L (b) 0.207 L
(c) 0.252 L (d) 0.277 L

Q.10 The slope at the support A of a simply supported beam loaded as shown in figure will be



(a) $\frac{3PL^2}{32EI}$ (b) $\frac{PL^2}{16EI}$
(c) $\frac{PL^2}{8EI}$ (d) $\frac{PL^2}{6EI}$

Q.11 Match List-I (Nature of beam) with List-II (Maximum deflection) and select the correct answer using the codes given below the lists:

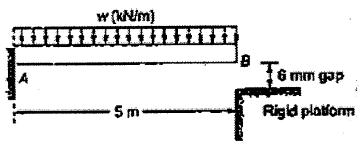
List-I
A. Cantilever beam subjected to concentrated load W at free end
B. Simply supported beam subjected to a point load W at the centre
C. Cantilever beam subjected to a hydrostatic load with zero intensity at the free end and W at the fixed end
D. Simply supported beam subjected to a triangularly distributed load with its apex of magnitude W at the mid span

List-II
1. $\frac{WL^3}{15EI}$ 2. $\frac{WL^3}{3EI}$
3. $\frac{WL^3}{60EI}$ 4. $\frac{WL^3}{48EI}$

Codes:

	A	B	C	D
(a)	2	1	4	3
(b)	3	1	2	4
(c)	2	4	1	3
(d)	1	4	2	3

Q.12 For the linear elastic beam as shown in the figure, the flexural rigidity, EI, is 781250 kN-m². When w = 10 kN/m, the vertical reaction R_A at A is 50 kN. The value of R_A for w = 100 kN/m is



(a) 500 kN (b) 425 kN
(c) 250 kN (d) 75 kN

Q.13 A cantilever of length l carries a concentrated load W at the mid span, and is also propped at the free end. Following conclusions are drawn:

1. The reaction at the prop is $\frac{5W}{16}$.
2. The bending moment is zero at a distance $\frac{3L}{11}$ from the fixed end.
3. Maximum bending moment is $\frac{5}{32}WL$.

Which of the above conclusion/s is/are correct?

(a) only 1 (b) both 1 and 3
(c) both 2 and 3 (d) 1, 2 and 3

Q.14 A cantilever beam of length l carries a uniformly distributed load 'w' per unit length, and is propped at free end. Following points are made:

1. The maximum deflection occurs at 0.422 l from the propped end.
2. Reaction at the propped end is $\frac{5}{8}wl$.
3. Deflection at mid span of the beam is $\frac{wl^4}{192EI}$.

Which of these statements is/are correct?

(a) only 2 (b) both 1 and 2
(c) both 1 and 3 (d) both 2 and 3

Q.15 Consider the following statements:

1. Deflection in fixed beam is less than that of simply supported beam.
2. Fixed beam is subjected to lesser bending moment in comparison to simply supported beam.
3. Temperature stresses produce large stresses in a fixed beam.

Which of these statements are correct?

(a) both 1 and 2 (b) both 2 and 3
(c) both 1 and 3 (d) 1, 2 and 3

Q.16 The deflection caused by the transverse shear force is not negligible in

(a) slender beams
(b) circular cross-sections only
(c) deep beams
(d) I-beams

Q.17 An increase in temperature on the top fibre of a simply supported beam will cause

(a) upward deflection
(b) downward deflection
(c) no deflection
(d) upward or downward deflection

Q.18 Consider the following statements regarding the beams and their loading as shown in figures (a) and (b).

Deflection at B of beam in figure (a) would equal

1. $(1/EI) \times BM$ at B of beam in figure (b).
2. $(1/EI) \times$ moment of the area about B of BMD of beam in figure (a).
3. $(1/EI) \times$ moment of the area of BMD about B of beam in figure (b).

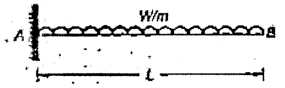


Figure (a)

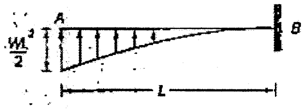


Figure (b)

Which of these statements is/are correct?

(a) 1 and 3 (b) 1 and 2
(c) 2 only (d) 3 only

Q.19 A horizontal beam of length l of uniform cross-section is pinned at ends A and B. At end A, there is a anticlockwise moment M and at end B there is clockwise moment $2M$. The slope at end A and end B are respectively

(a) $\frac{2ML}{3EI}$ and $\frac{5ML}{7EI}$ (b) $\frac{ML}{3EI}$ and $\frac{5ML}{6EI}$
(c) $\frac{ML}{EI}$ and $\frac{5ML}{7EI}$ (d) $\frac{2ML}{3EI}$ and $\frac{5ML}{6EI}$

Q.20 A 4-metre long beam, simply supported at its ends; carries a point load 'W' at its centre. If the slope at the ends of beam is 1°, then deflection at the centre of beam is

(a) 10.56 mm (b) 18.32 mm
(c) 23.27 mm (d) 39.37 mm

Q.21 Which of the following statements are correct? Macaulay's method for calculation of slope and deflection in a beam is suitable for

1. prismatic beams only.
2. several concentrated loads and can be extended to uniformly distributed loads.
3. both prismatic and non-prismatic beams.

(a) Only 1 (b) 1 and 2
(c) Only 3 (d) 2 and 3

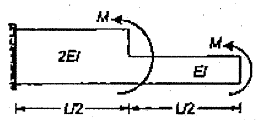
Q.22 Consider the following statements.

1. Conjugate beam can be used to determine slope and deflection in a non-prismatic beam.
2. Conjugate beam may be statically indeterminate.
3. Conjugate beam method gives absolute slope and deflection.

The correct answer is

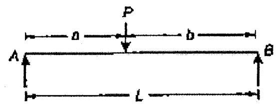
(a) 1 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1, 2 and 3

Q.23 The stepped cantilever is subjected to moments, M as shown in the figure below. The vertical deflection at the free end (neglecting self weight) is



(a) $\frac{ML^2}{8EI}$ (b) $\frac{ML^2}{4EI}$
(c) $\frac{ML^2}{2EI}$ (d) $\frac{7ML^2}{8EI}$

Q.24 A simply supported beam of length L carries a point load P as shown. The maximum deflection occurs at



(a) $\sqrt{\frac{L^2-b^2}{3}}$ from A
(b) $\sqrt{\frac{L^2-b^2}{3}}$ from B
(c) $\frac{L^2-b^2}{3}$ from A
(d) $\frac{L^2-b^2}{3}$ from B

Q.25 For the application of moment area method for finding deflection at a section in a beam

(a) the position of atleast one tangent to the elastic curve, at any section should be known
(b) the M/EI diagram must be triangle
(c) the beam must be of uniform moment of inertia
(d) the BM diagram if known is sufficient

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Answers **Deflection of Beams**

1. (b)	2. (b)	3. (d)	4. (a)	5. (d)	6. (d)	7. (a)	8. (d)	9. (a)	10. (a)
11. (c)	12. (b)	13. (d)	14. (c)	15. (d)	16. (c)	17. (a)	18. (d)	19. (d)	20. (c)
21. (b)	22. (a)	23. (d)	24. (a)	25. (d)					

Explanations Deflection of Beams

1. (b)

$$\delta_1 = \frac{Wl^3}{48EI}$$

$$\delta_2 = \frac{5wl^4}{384EI} = \frac{5Wl^3}{384EI} \qquad (\because W = wl)$$

$$\frac{\delta_1}{\delta_2} = \frac{\left(\frac{1}{48}\right)}{\left(\frac{5}{384}\right)} = \frac{8}{5}$$

3. (d)

$$\theta_1 = \theta = \frac{ML}{EI} = \frac{(PL/2)L}{EI} = \frac{PL^2}{2EI}$$

$$\theta_2 = \frac{PL^2}{2EI} = \theta$$

4. (a)

$$\text{Slope} = \frac{dy}{dx}$$

y is maximuma at mid-span.

5. (d)

$$\delta = \frac{WL^3}{48EI}$$

$$\Rightarrow \quad \delta \propto \frac{1}{I}$$

$$I = \frac{bd^3}{12}$$

$$\therefore \frac{\delta_{4\times6}}{\delta_{6\times4}} = \frac{4\times6^3}{6\times4^3} = \frac{6^2}{4^2} = 2.25$$

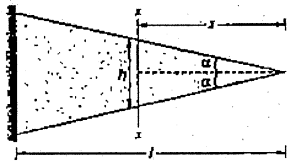
6. (d)

$$\delta = \frac{WL^3}{3EI}$$

$$\Rightarrow \quad \delta \propto L^3$$

$$\therefore \frac{\delta_2}{\delta_1} = \frac{(2L)^3}{L^3} = 8$$

8. (d)



$$I_x = \frac{\frac{b}{l}\times x\times t^3}{12} = \frac{bxt^3}{12l}$$

Strain energy,

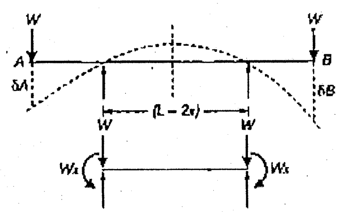
$$U = \int_0^l \frac{M^2dx}{2EI} \qquad (M = Px)$$

$$= \int_0^l \frac{P^2x^2}{2E\frac{bxt^3}{12l}}dx$$

$$= \frac{6lP^2}{Ebt^3}\int_0^l x\,dx = \frac{3l^3P^2}{Ebt^3}$$

$$\delta = \frac{\partial U}{\partial P} = \frac{6Pl^3}{Ebt^3}$$

9. (a)



$$\delta_A = \delta_B = \frac{Wx^3}{3EI}$$

$$\frac{Wx^3}{3EI} = \frac{Wx(L-2x)^2}{8EI}$$

$$\Rightarrow \quad x = 0.152\,L$$

12. (b)

$$\Delta = \frac{wL^4}{8EI}$$

when $w = 10$ kN/m

$$\Delta = \frac{10\times5^4}{8\times781250} = 1 \text{ mm}$$

when $w = 100$ kN/m

$\Delta = 10$ mm (if there is no rigid platform at B but Δ allowable = 6 mm)

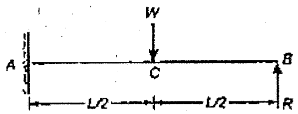
$$\therefore \quad 10 = \frac{R_BL^3}{3EI} + 6$$

$$\Rightarrow \quad R_B = \frac{4\times10^{-3}\times3\times78125}{5^3}$$

$$R_B = 75 \text{ kN}$$

$$R_A = 500 - 75 = 425 \text{ kN}$$

13. (d)



- Deflection at B due to W downwards

$$\delta_B = \frac{5}{48}\frac{WL^3}{EI} \qquad \text{...(i)}$$

Deflection at B due to R upwards

$$\delta_B = \frac{RL^3}{3EI} \qquad \text{...(ii)}$$

But eq. (i) = (ii)

$$R = \frac{5}{16}W$$

where R is the reaction at the prop support.

Therefore

$$V_a = \frac{11}{16}W$$

- Let us find bending moment at x from fixed end.

$$M_x = +R(L-x) - W\left(L - x - \frac{L}{2}\right)$$

$$= R(L-x) - W\left(\frac{L}{2} - x\right)$$

$$= \frac{5W}{16}(L-x) - W\left(\frac{L}{2} - x\right)$$

If $M_x = 0$

$$\Rightarrow \frac{5}{16}(L-x) - \left(\frac{L}{2} - x\right) = 0$$

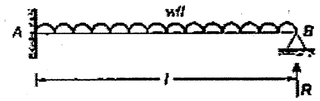
$$\Rightarrow \frac{5}{16}L - \frac{L}{2} = \frac{5}{16}x - x$$

$$\Rightarrow \quad x = \frac{3L}{11}$$

- S.F. changes sign at the centre and hence maximum bending moment occurs at C

$$M_c = M_{max} = R\frac{L}{2} = \frac{5}{32}WL$$

14. (c)



Deflection at B due to UDL $= \dfrac{wl^4}{8EI}$

Deflection at B due to reaction R at propped end

$$= \frac{Rl^3}{3EI}$$

$$\therefore \quad \frac{wl^4}{8EI} = \frac{Rl^3}{3EI}$$

$$\Rightarrow \quad R = \frac{3wl}{8}$$

18. (d)

In moment area method:

Slope $= \theta_B - \theta_A = \dfrac{1}{EI}$ [Area of BMD between A and B]

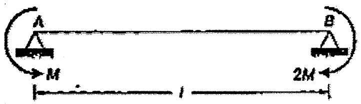
Deflection $= \delta_B - \delta_A = \dfrac{1}{EI}$ [moment of area of BMD between B and A about B]

In conjugate beam method:

Slope at any section of given beam = SF at corresponding section of conjugate beam.

Deflection at any section for the given beam = BM at corresponding section of conjugate beam.

19. (d)



$$\theta_A = \frac{ML}{3EI} + \frac{2ML}{6EI} = \frac{ML}{3EI} + \frac{ML}{3EI} = \frac{2ML}{3EI}$$

$$\theta_B = \frac{2ML}{3EI} + \frac{ML}{6EI} = \frac{5ML}{6EI}$$

20. (c)

$$\text{Deflection at centre of beam} = \frac{w \times (4)^3}{48EI}$$

$$\text{Slope at ends of beam} = \frac{w \times (4)^2}{16EI} = 1° = \frac{\pi}{180}$$

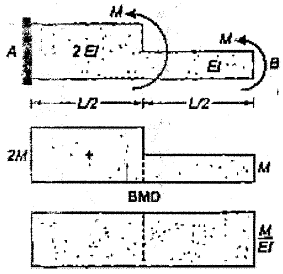
$$\therefore \text{ Deflection} = \frac{4}{3} \times \frac{\pi}{180} \text{m} = 23.27 \text{ mm}$$

21. (b)

Macaulay's method is based on singularity function. It is applicable for prismatic beams only. While Mohr's moment area method can be used for prismatic and non-prismatic beams.

23. (d)

Using moment area method.



$\frac{M}{EI}$ Diagram

Deflection at B w.r.t. A

$= \text{Moment of area of } \frac{M}{EI} \text{ diagram between } A \text{ and } B \text{ about } B$

$$= \frac{2M}{EI} \times \frac{L}{2} \times \frac{3L}{4} + \frac{M}{EI} \times \frac{L}{2} \times \frac{L}{4}$$

$$= \frac{3ML^2}{4EI} + \frac{ML^2}{8EI} = \frac{7ML^2}{8EI}$$

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