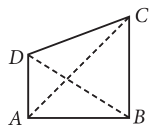
CHAPTER 2

Quadrilaterals

Recap Notes

QUADRILATERAL

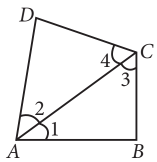
- A closed figure in a plane formed by joining four points in an order is called a quadrilateral. In figure, *ABCD* is a quadrilateral.



- Each diagonal *AC* and *BD* divides the quadrilateral into two triangles.

ANGLE SUM PROPERTY OF A QUADRILATERAL

- The sum of the angles of a quadrilateral is 360°.



Given : We have a quadrilateral *ABCD*. *AC* is one of its diagonals.

Proof : In ΔABC, we have

$\angle 1 + \angle 3 + \angle B = 180°$ …(i)

[By angle sum property of a triangle]

In ΔADC, we have

$\angle 2 + \angle 4 + \angle D = 180°$ …(ii)

[By angle sum property of a triangle]

Adding (i) and (ii), we have

$(\angle 1 + \angle 2) + (\angle 3 + \angle 4) + \angle B + \angle D = 180° + 180°$

$\Rightarrow \angle BAD + \angle BCD + \angle B + \angle D = 360°$

So, $\angle A + \angle C + \angle B + \angle D = 360°$

i.e., Sum of the angles of a quadrilateral is 360°.

TYPES OF QUADRILATERAL

Name of quadrilateral	Definition
Trapezium	Only one pair of opposite sides is parallel.
Parallelogram	Both the pairs of opposite sides are parallel and equal.
Rectangle	It is a parallelogram with each angle of measure 90°.
Rhombus	It is a parallelogram having all sides equal.
Square	It is a rhombus whose each angle is of measure 90°.
Kite	It is a quadrilateral with two pairs of equal adjacent sides.

Note : (i) Square, rectangle and rhombus are all parallelograms.

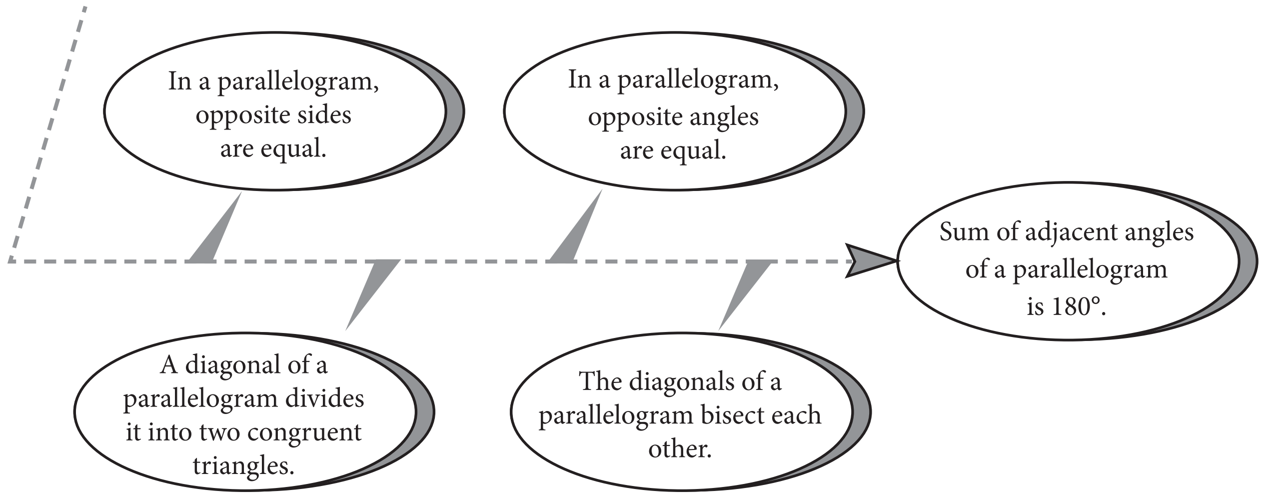
(ii) A kite is not a parallelogram.

(iii) A rectangle or a rhombus is not a square.

(iv) A parallelogram is a trapezium but a trapezium is not a parallelogram.

(v) A square is a rectangle and also a rhombus.

PROPERTIES OF A PARALLELOGRAM



SOME CONDITIONS FOR A QUADRILATERAL TO BE A PARALLELOGRAM

- If each pair of opposite sides of a quadrilateral are equal, then it is a parallelogram.
- If each pair of opposite angles of a quadrilateral are equal, then it is a parallelogram.
- If diagonals of a quadrilateral bisect each other, then it is a parallelogram.
- If a pair of opposite sides of a quadrilateral is equal and parallel, then it is a parallelogram.

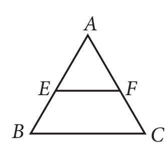
Properties of Diagonals of Special Parallelograms

Properties	Rectangle	Square	Rhombus
Diagonals bisects each other.	✓	✓	✓
Diagonals are equal.	✓	✓	×
Diagonals bisects each vertex angle.	×	✓	✓
Diagonals are perpendicular to each other.	×	✓	✓
Diagonals divides into 4 congruent triangles.	×	✓	✓

MID-POINT THEOREM

- The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

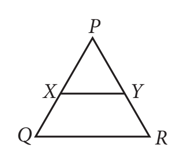
  In the figure, ABC is a triangle in which E and F are mid-points of side AB and AC respectively. Then, by mid-point theorem, $EF \parallel BC$ and $EF = \frac{1}{2} BC$.



Converse of Mid-Point Theorem

- The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

  In the figure, PQR is a triangle in which X is the mid-point of side PQ and $XY \parallel QR$. Then, by mid-point theorem, Y is the mid-point of PR *i.e.*, $PY = YR$.



Practice Time

Multiple Choice Questions (MCQs)

1. How many angles are there in a quadrilateral?

(a) 4 (b) 2 (c) 1 (d) 3

2. The three consecutive angles of a quadrilateral are 70°, 120° and 50°. The fourth angle of the quadrilateral is

(a) 45° (b) 60° (c) 120° (d) 30°

3. If the sum of angles of a triangle is X and the sum of the angles of a quadrilateral is Y, then

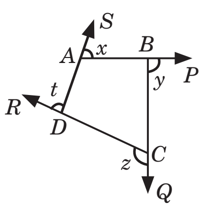
(a) $X = 2Y$ (b) $2X = Y$

(c) $X = Y$ (d) $X + Y = 360°$

4. One of the angles of a quadrilateral is 90° and the remaining three angles are in the ratio 2 : 3 : 4. Find the largest angle of the quadrilateral.

(a) 120° (b) 90° (c) 140° (d) 100°

5. In the figure, $ABCD$ is a quadrilateral whose sides AB, BC, CD and DA are produced in order to P, Q, R and S. Then $x + y + z + t$ is equal to



(a) 180° (b) 360° (c) 380° (d) 270°

6. If only one pair of opposite sides of a quadrilateral are parallel, then the quadrilateral is a

(a) Parallelogram (b) Trapezium

(c) Rhombus (d) Rectangle

7. A blackboard is in the shape of a

(a) Parallelogram (b) Rhombus

(c) Rectangle (d) Kite

8. The angle between the diagonals of a rhombus is

(a) 45° (b) 90° (c) 30° (d) 60°

9. A quadrilateral whose all the four sides and all the four angles are equal is called a

(a) Rectangle (b) Rhombus

(c) Square (d) Parallelogram

10. Which of the following is not true?

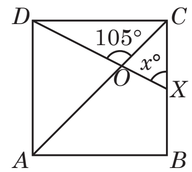
(a) The diagonals of a rectangle are equal.

(b) Diagonals of a square are equal.

(c) Diagonals of a parallelogram are not always equal.

(d) Diagonals of a kite are equal.

11. In the adjoining figure, $ABCD$ is a square. A line segment DX cuts the side BC at X and the diagonal AC at O such that $\angle COD = 105°$ and $\angle OXC = x$. Find the value of x.



(a) 75° (b) 80° (c) 60° (d) 45°

12. If angles A, B, C and D of the quadrilateral $ABCD$, taken in order, are in the ratio 3 : 7 : 6 : 4, then $ABCD$ is a

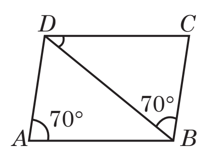
(a) rhombus (b) parallelogram

(c) trapezium (d) kite

13. In a parallelogram $ABCD$, if $\angle A = 75°$, then the measure of $\angle B$ is

(a) 10° (b) 20° (c) 105° (d) 90°

14. In parallelogram $ABCD$, $\angle DAB = 70°$, $\angle DBC = 70°$, then $\angle CDB$ is equal to



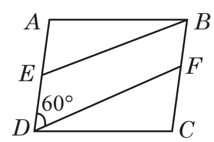
(a) 40°

(b) 60°

(c) 70°

(d) 30°

15. In the given figure, $ABCD$ is a parallelogram. E and F are points on opposite sides AD and BC respectively, such that $ED = \frac{1}{2}AD$ and $BF = \frac{1}{3}BC$. If $\angle ADF = 60°$, then find $\angle BFD$.



(a) 120° (b) 130° (c) 125° (d) 115°

16. Two angles of a quadrilateral are 55° and 65°. The other two angles are in the ratio 3 : 5. The two angles are

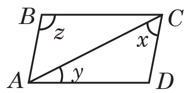
(a) 100°, 110° (b) 85°, 125°

(c) 100°, 120° (d) 90°, 150°

17. In a quadrilateral $ABCD$, diagonals bisect each other at right angle. Also, $AB = BC = AD = 5$ cm, then find the length of CD.

(a) 5 cm (b) 4 cm (c) 2 cm (d) 6 cm

18. In the given figure, $ABCD$ is a parallelogram, what is the sum of the angles x, y and z?

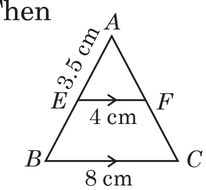


(a) 180° (b) 45° (c) 60° (d) 90°

19. If a pair of opposite sides of a quadrilateral is equal and parallel, then the quadrilateral is a

(a) parallelogram (b) rectangle
(c) rhombus (d) square

20. In ΔABC, $EF \parallel BC$, F is the mid-point of AC and AE = 3.5 cm. Then AB is equal to



(a) 7 cm
(b) 5 cm
(c) 5.5 cm
(d) 4.5 cm

21. The triangle formed by joining the mid-points of the sides of an equilateral triangle is

(a) scalene (b) right angled
(c) equilateral (d) isosceles

22. The four triangles formed by joining the mid-points of the sides of a triangle are

(a) congruent to each other
(b) non- congruent to each other
(c) always right angled triangle
(d) can't be determined

23. If M and N are the mid-points of non parallel sides of a trapezium $PQRS$, then which of the following conditions is/are true?

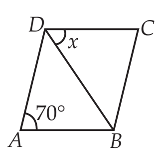
(a) $MN \parallel PQ$

(b) $MN = \frac{1}{2}(PQ + RS)$

(c) $MN = \frac{1}{2}(PQ - RS)$

(d) Both (a) and(b)

24. In the given figure, $ABCD$ is a rhombus. If $\angle A = 70°$, then $\angle CDB$ is equal to



(a) 65°
(b) 55°
(c) 75°
(d) 80°

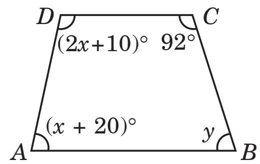
25. Two adjacent angles of a parallelogram are $(2x + 25)°$ and $(3x - 5)°$. The value of x is

(a) 28 (b) 32 (c) 36 (d) 42

26. In a quadrilateral $STAR$, if $\angle S = 120°$, and $\angle T : \angle A : \angle R = 5 : 3 : 7$, then measure of $\angle R$ =

(a) 112° (b) 120°
(c) 110° (d) None of these

27. In figure, $ABCD$ is a trapezium. Find the values of x and y.



(a) $x = 50°, y = 80°$
(b) $x = 50°, y = 88°$
(c) $x = 80°, y = 50°$
(d) None of these

28. In a quadrilateral $ABCD$, $\angle A + \angle C$ is 2 times $\angle B + \angle D$. If $\angle A = 140°$ and $\angle D = 60°$, then $\angle B$ =

(a) 60° (b) 80°
(c) 120° (d) None of these

29. The measure of all the angles of a parallelogram, if an angle is 24 less than twice the smallest angle, is

(a) 37°, 143°, 37°, 143°
(b) 108°, 72°, 108°, 72°
(c) 68°, 112°, 68°, 112°
(d) None of these

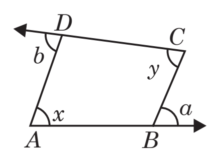
30. Which type of quadrilateral is formed when the angles A, B, C and D are in the ratio 2 : 4 : 5 : 7 ?

(a) Rhombus (b) Square
(c) Trapezium (d) Rectangle

31. In ΔPQR, A and B are respectively the mid-points of sides PQ and PR. If $\angle PAB = 60°$, then $\angle PQR$ =

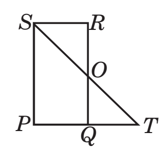
(a) 40° (b) 80° (c) 60° (d) 70°

32. Sides AB and CD of a quadrilateral $ABCD$ are extended as in figure. Then $a + b$ is equal to



(a) $x + 2y$
(b) $x - y$
(c) $x + y$
(d) $2x + y$

33. In the adjoining figure, $PQRS$ is a parallelogram in which PQ is produced to T such that $QT = PQ$. Then, OQ is equal to



(a) OS (b) OR
(c) OT (d) None of these

34. If consecutive sides of a parallelogram are equal, then it is necessarily a

(a) Rectangle (b) Rhombus
(c) Trapezium (d) None of these

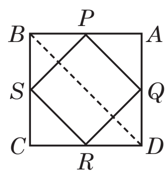
35. The triangle formed by joining the mid-points of the sides of a right angled triangle is

(a) scalene (b) isosceles
(c) equilateral (d) right angled

Case Based MCQs

Case I. Read the following passage and answer the questions from 36 to 40.

Laveena's class teacher gave students some colourful papers in the shape of quadrilaterals. She asked students to make a parallelogram from it using paper folding. Laveena made the following parallelogram.



36. How can a parallelogram be formed by using paper folding?
(a) Joining the sides of quadrilateral
(b) Joining the mid-points of sides of quadrilateral
(c) Joining the various quadrilaterals
(d) None of these

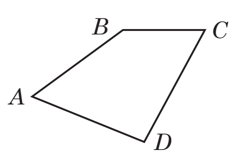
37. Which of the following is true?
(a) $PQ = BD$ (b) $PQ = \frac{1}{2}BD$
(c) $3PQ = BD$ (d) $PQ = 2BD$

38. Which of the following is correct combination?
(a) $2RS = BD$ (b) $RS = \frac{1}{3}BD$
(c) $RS = BD$ (d) $RS = 2BD$

39. Which of the following is correct?
(a) $SR = 2PQ$ (b) $PQ = SR$
(c) $SR = 3PQ$ (d) $SR = 4PQ$

40. Write the formula used to find the perimeter of quadrilateral *PQRS*.
(a) $PQ + QR + RS + SP$
(b) $PQ - QR + RS - SP$
(c) $\frac{PQ + QR + RS + SP}{2}$
(d) $\frac{PQ + QR + RS + SP}{3}$

Case II. Read the following passage and answer the questions from 41 to 45.

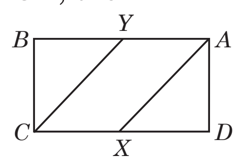


After summervacation, Manit's class teacher organised a small MCQ quiz, based on the properties of quadrilaterals. During quiz, she asks different questions to students.
Some of the questions are listed below.

41. Which of the following is/are the condition(s) for *ABCD* to be a quadrilateral?
(a) The four points *A*, *B*, *C* and *D* must be distinct and co-planar.
(b) No three of points *A*, *B*, *C* and *D* are collinear.
(c) Line segments *i.e.*, *AB*, *BC*, *CD*, *DA* intersect at their end points only.
(d) All of these

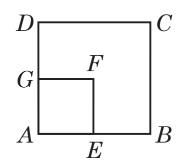
42. Which of the following is wrong condition for a quadrilateral said to be a parallelogram?
(a) Opposite sides are equal
(b) Opposite angles are equal
(c) Diagonal can't bisect each other
(d) None of these

43. If *AX* and *CY* are the bisectors of the angles *A* and *C* of a parallelogram *ABCD*, then



(a) $AX \parallel CY$
(b) $AX \parallel CD$
(c) $AX \parallel AB$
(d) None of these

44. *ABCD* and *AEFG* are two parallelograms. If $\angle C = 63°$, then determine $\angle G$.



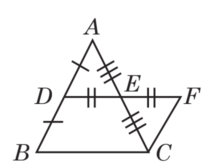
(a) 63°
(b) 117°
(c) 90°
(d) 120°

45. If angles of a quadrilateral are in ratio 3 : 5 : 5 : 7, then find all the angles.
(a) 54°, 80°, 80°, 146° (b) 34°, 100°, 100°, 126°
(c) 54°, 90°, 90°, 126° (d) None of these

Case III. Read the following passage and answer the questions from 46 to 50.

Anjali and Meena were trying to prove mid point theorem.

They draw a triangle *ABC*, where *D* and *E* are found to be the midpoints of *AB* and *AC* respectively. *DE* was joined and extended to *F* such that $DE = EF$ and *FC* is also joined.



46. ΔADE and ΔCFE are congruent by which criterion?
(a) SSS (b) SAS (c) RHS (d) ASA

47. $\angle EFC$ is equal to which angle?
(a) $\angle DAE$ (b) $\angle EDA$ (c) $\angle AED$ (d) $\angle DBC$

48. $\angle ECF$ is equal to which angle?
(a) $\angle EAD$ (b) $\angle ADE$ (c) $\angle AED$ (d) $\angle B$

49. CF is equal to
(a) EC (b) BE (c) BC (d) AD

50. CF is parallel to
(a) AE (b) CE (c) BD (d) AC

Assertion & Reasoning Based MCQs

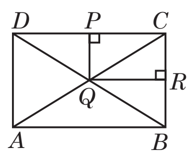
Directions (Q.51 to 55) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.
(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.
(c) Assertion is correct statement but Reason is wrong statement.
(d) Assertion is wrong statement but Reason is correct statement.

51. Assertion : In ΔABC, median AD is produced to X such that $AD = DX$. Then $ABXC$ is a parallelogram.

Reason : Diagonals of a parallelogram are perpendicular to each other.

52. Assertion : $ABCD$ and $PQRC$ are rectangles and Q is the mid-point of AC. Then $DP = PC$.



Reason : The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

53. Assertion : Two opposite angles of a parallelogram are $(3x - 2)°$ and $(50 - x)°$. The measure of one of the angle is 37°.
Reason : Opposite angles of a parallelogram are equal.

54. Assertion : $ABCD$ is a square. AC and BD intersect at O. The measure of $\angle AOB = 90°$.
Reason : Diagonals of a square bisect each other at right angles.

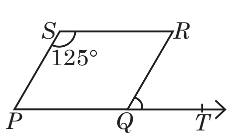
55. Assertion : In ΔABC, E and F are the midpoints of AC and AB respectively. The altitude AP at BC intersects FE at Q. Then, $AQ = QP$.
Reason : If Q is the midpoint of AP, then $AQ = QP$.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

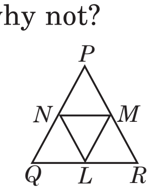
1. Two consecutive angles of a parallelogram are $(x + 60°)$ and $(2x + 30°)$. What special name can you give to this parallelogram?

2. In the given figure, $PQRS$ is a parallelogram in which $\angle PSR = 125°$. Find the measure of $\angle RQT$.



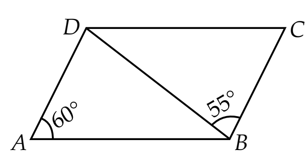
3. Can the angles 110°, 80°, 70° and 95° be the angles of a quadrilateral ? Why or why not?

4. In the figure, it is given that $QLMN$ and $NLRM$ are parallelograms. Can you say that $QL = LR$? Why or why not?

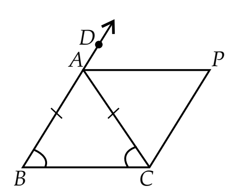


5. $ABCD$ is a parallelogram in which $\angle A = 78°$. Compute $\angle B$, $\angle C$ and $\angle D$.

6. In the given figure, $ABCD$ is a parallelogram in which $\angle DAB = 60°$ and $\angle DBC = 55°$. Compute $\angle CDB$ and $\angle ADB$.



7. In the given figure, $AB = AC$ and $CP \parallel BA$ and AP is the bisector of exterior $\angle CAD$ of ΔABC. Prove that $\angle PAC = \angle BCA$ and $ABCP$ is a parallelogram.



8. If one angle of a rhombus is a right angle, then it is necessarily a _______ .

9. In a rhombus $ABCD$, if $\angle A = 60°$, then find the sum of $\angle A$ and $\angle C$.

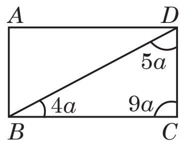
10. $ABCD$ is a trapezium in which $AB \parallel DC$ and $\angle A = \angle B = 45°$. Find angles C and D of the trapezium.

Short Answer Type Questions (SA-I)

11. In a quadrilateral $ABCD$, CO and DO are the bisectors of $\angle C$ and $\angle D$ respectively.

Prove that $\angle COD = \frac{1}{2}(\angle A + \angle B)$.

12. In the given parallelogram $ABCD$, the sum of any two consecutive angles is 180° and opposite angles are equal. Find the value of $\angle A$.

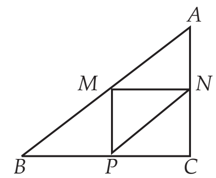


13. Diagonals of a quadrilateral $ABCD$ bisect each other. $\angle A = 45°$ and $\angle B = 135°$. Is it true? Justify your answer.

14. D and E are the mid-points of sides AB and AC respectively of triangle ABC. If the perimeter of ΔABC = 35 cm, then find the perimeter of ΔADE.

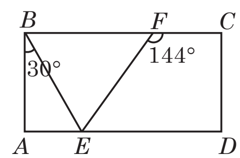
15. In ΔABC, AD is the median and $DE \parallel AB$, such that E is a point on AC. Prove that BE is another median.

16. In the given figure, M, N and P are the midpoints of AB, AC and BC respectively. If MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm, then find $(BC + AC) - AB$.



17. Let ΔABC be an isosceles triangle with $AB = AC$ and let D, E and F be the mid-points of BC, CA and AB respectively. Show that $AD \perp FE$ and AD is bisected by FE.

18. In the given rectangle $ABCD$, $\angle ABE = 30°$ and $\angle CFE = 144°$. Find the measure of $\angle BEF$.



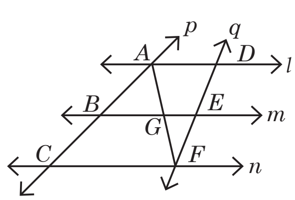
19. The perimeter of a parallelogram is 30 cm. If longer side is 9.5 cm, then find the length of shorter side.

20. In a parallelogram $ABCD$, if $\angle A = (3x - 20)°$, $\angle B = (y + 15)°$ and $\angle C = (x + 40)°$, then find $x + y$ (in degrees).

Short Answer Type Questions (SA-II)

21. In a parallelogram $PQRS$, if $\angle QRS = 2x$, $\angle PQS = 4x$ and $\angle PSQ = 4x$, then find the angles of the parallelogram.

22. l, m and n are three parallel lines intersected by transversals p and q such that l, m and n cut off equal intercepts AB and BC on p (see figure).

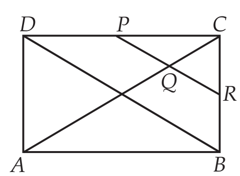


Show that l, m and n cut off equal intercepts DE and EF on q also.

23. The side of a rhombus is 10 cm. The smaller diagonal is $\frac{1}{3}$ of the greater diagonal. Find the length of the greater diagonal.

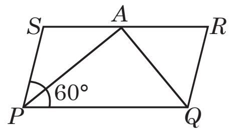
24. In ΔABC, $\angle A = 50°$, $\angle B = 60°$ and $\angle C = 70°$. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

25. In given figure, $ABCD$ is a parallelogram in which P is the midpoint of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$. If PQ produced meet BC at R, then prove that R is a midpoint of BC.

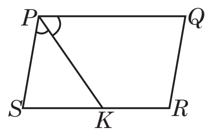


26. $ABCD$ is parallelogram. P is a point on AD such that $AP = \frac{1}{3}AD$ and Q is a point on BC such that $CQ = \frac{1}{3}BC$. Prove that $AQCP$ is a parallelogram.

27. *PQRS* is a parallelogram and $\angle SPQ = 60°$. If the bisectors of $\angle P$ and $\angle Q$ meet at point *A* on *RS*, prove that *A* is the mid-point of *RS*.

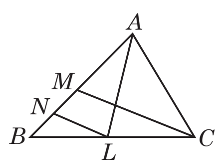


28. In the given figure, *K* is the mid-point of side *SR* of a parallelogram *PQRS* such that $\angle SPK = \angle QPK$. Prove that $PQ = 2QR$.

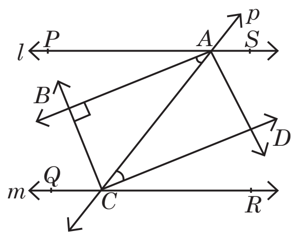


29. Rima has a photo-frame without a photo in the shape of a triangle with sides *a*, *b*, *c* in length. She wants to find the perimeter of a triangle formed by joining the mid-points of the sides of the photo-frame. Find the perimeter of the triangle formed by joining the mid-points of the frame.

30. In the following figure, *AL* and *CM* are medians of ΔABC and $LN \parallel CM$. Prove that $BN = \frac{1}{4} AB$.

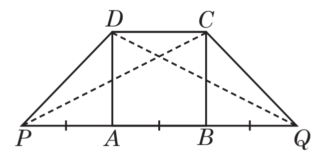


31. Two parallel lines *l* and *m* are intersected by a transversal *p* (see figure). Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

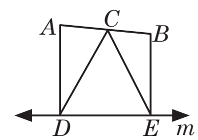


32. *PQRS* is a rhombus with $\angle QPS = 50$. Find $\angle RQS$.

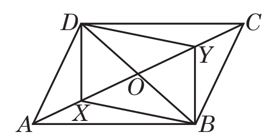
33. In the given figure, *ABCD* is a square, side *AB* is produced to points *P* and *Q* in such a way that $PA = AB = BQ$. Prove that $DQ = CP$.



34. In the adjoining figure, points *A* and *B* are on the same side of a line *m*, $AD \perp m$ and $BE \perp m$ and meet *m* at *D* and *E*, respectively. If *C* is the mid-point of *AB*, then prove that $CD = CE$.

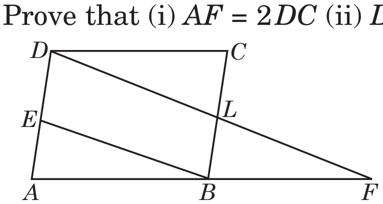


35. In the given quadrilateral *ABCD*, *X* and *Y* are points on diagonal *AC* such that $AX = CY$ and *BXDY* is a parallelogram. Show that *ABCD* is a parallelogram.

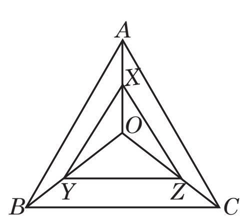


Long Answer Type Questions (LA)

36. In the given figure, *ABCD* is a parallelogram and *E* is the mid-point of *AD*. A line through *D*, drawn parallel to *EB*, meets *AB* produced at *F* and *BC* at *L*. Prove that (i) $AF = 2DC$ (ii) $DF = 2DL$



37. In ΔABC, $AB = 18$ cm, $BC = 19$ cm and $AC = 16$ cm. *X*, *Y* and *Z* are mid-points of *AO*, *BO* and *CO* respectively as shown in the figure. Find the perimeter of ΔXYZ.



38. Prove that the line segment joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and is equal to half the difference of these sides.

39. *ABCD* is a parallelogram. *AB* and *AD* are produced to *P* and *Q* respectively such that $BP = AB$ and $DQ = AD$. Prove that *P*, *C*, *Q* lie on a straight line.

40. *P*, *Q*, *R* and *S* are respectively the mid-points of sides *AB*, *BC*, *CD* and *DA* of quadrilateral *ABCD* in which $AC = BD$ and $AC \perp BD$. Prove that *PQRS* is a square.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(a) :** Number of angles in a quadrilateral = 4.

2. **(c) :** Let the measure of fourth angle be x.
Now, sum of angles of a quadrilateral = 360°
$\Rightarrow$ $70° + 120° + 50° + x = 360°$
$\Rightarrow$ $240° + x = 360° \Rightarrow x = 120°$

3. **(b) :** Here, X = Sum of angles of a triangle = 180°, Y = Sum of angles of a quadrilateral = 360°
Now, $2X = 2 \times 180° = 360° = Y$
$\therefore$ $2X = Y$

4. **(a) :** Let the quadrilateral be $ABCD$ in which $\angle A = 90°$, $\angle B = 2x$, $\angle C = 3x$ and $\angle D = 4x$.
Then, $\angle A + \angle B + \angle C + \angle D = 360°$
$\Rightarrow$ $90° + 2x + 3x + 4x = 360°$
$\Rightarrow$ $9x = 270° \Rightarrow x = 30°$
$\therefore$ $\angle B = 60°, \angle C = 90°, \angle D = 120°$
Hence, the largest angle is 120°.

5. **(b) :** We have, $x + \angle A = 180°$ (Linear pair)
$\Rightarrow$ $x = 180° - \angle A$ similarly, $y = 180° - \angle B$,
$z = 180° - \angle C$, $t = 180° - \angle D$
$\Rightarrow$ $x + y + z + t = 720° - (\angle A + \angle B + \angle C + \angle D)$
$= 720° - 360° = 360°$

6. **(b) :** In a trapezium, only one pair of opposite sides are parallel.

7. **(c) :** A blackboard is in the shape of a rectangle.

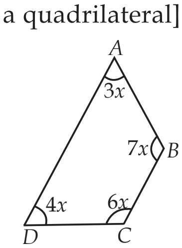
8. **(b) :** Diagonals of a rhombus are perpendicular to each other. So, the angle between them is 90°.

9. **(c) :** In a square, all the four sides are equal and all the angles are of equal measure, *i.e.*, 90°.

10. **(d) :** Diagonals of a kite are not equal.

11. **(c) :** We know, the angles of a square are bisected by the diagonals.
$\therefore$ $\angle OCX = 45°$
Also, $\angle COD + \angle COX = 180°$ (Linear pair)
$\Rightarrow$ $105° + \angle COX = 180° \Rightarrow \angle COX = 180° - 105° = 75°$
Now, in ΔCOX, we have
$\angle OCX + \angle COX + \angle OXC = 180°$
$\Rightarrow$ $45° + 75° + x = 180°$
$\Rightarrow$ $x = 180° - 120° = 60°$.

12. **(c) :** Let the angles of quadrilateral $ABCD$ be $3x$, $7x$, $6x$ and $4x$ respectively.
$\therefore$ $3x + 7x + 6x + 4x = 360°$
[Angle sum property of a quadrilateral]
$\Rightarrow$ $20x = 360°$
$\Rightarrow$ $x = 18°$
$\therefore$ Angles of the quadrilateral are
$\angle A = 3 \times 18° = 54°$
$\angle B = 7 \times 18° = 126°$
$\angle C = 6 \times 18° = 108°$
and $\angle D = 4 \times 18° = 72°$



Now, for the line segments AD and BC, with AB as transversal $\angle A$ and $\angle B$ are co-interior angles.
Also, $\angle A + \angle B = 54° + 126° = 180°$
$\therefore$ $AD \parallel BC$
Thus, $ABCD$ is a trapezium.

13. **(c) :** Sum of adjacent angles of a parallelogram is 180°.
$\therefore$ $\angle A + \angle B = 180° \Rightarrow 75° + \angle B = 180° \Rightarrow \angle B = 105°$

14. **(a) :** $\angle ABC + \angle BAD = 180°$
($\because$ Sum of adjacent angles of a parallelogram is 180°)
$\Rightarrow$ $\angle ABC = 180° - 70° = 110°$
$\Rightarrow$ $\angle ABD = \angle ABC - \angle DBC = 110° - 70° = 40°$
Now, $CD \parallel AB$ and BD is transversal.
$\therefore$ $\angle CDB = \angle ABD = 40°$ (Alternate angles)

15. **(a) :** Given, $ABCD$ is a parallelogram.
$\therefore$ $AD \parallel BC$ and DF is a transversal.
$\therefore$ $\angle ADF = \angle DFC = 60°$ (Alternate angles)
Also, $\angle BFD + \angle DFC = 180°$ (Linear pair)
$\Rightarrow$ $\angle BFD + 60° = 180° \Rightarrow \angle BFD = 180° - 60° = 120°$

16. **(d) :** Let the other two angles be $3x$ and $5x$.
Now, sum of angles of a quadrilateral = 360°.
$\therefore$ $55° + 65° + 3x + 5x = 360°$
$\Rightarrow$ $120° + 8x = 360° \Rightarrow 8x = 240° \Rightarrow x = 30°$
$\therefore$ Two angles are 90° and 150°.

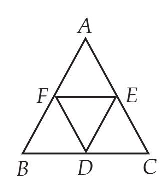
17. **(a) :** Diagonals of quadrilateral bisect each other at right angle.
$\therefore$ It is a square or a rhombus.
Also, all the sides of square or rhombus are equal.
$\therefore$ $CD = 5$ cm.

18. **(a) :** In ΔADC,
$x + y + \angle ADC = 180°$ (By angle sum property of a triangle)
$\Rightarrow$ $\angle ADC = 180° - (x + y)$ ...(i)
$\because$ $\angle ABC = \angle ADC$
($\because$ Opposite angles of parallelogram are equal)
$\therefore$ $z = 180° - (x + y)$ [Using (i)]
$\Rightarrow$ $z + x + y = 180°$

19. **(a) :** If a pair of opposite sides of a quadrilateral is equal and parallel, then it is a parallelogram.

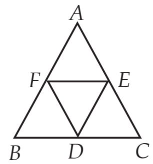
20. **(a) :** Here, $EF \parallel BC$ and F is mid-point of AC.
$\therefore$ By converse of mid-point theorem, E is the mid-point of AB.
$\Rightarrow$ $AB = 2(AE) = 2 \times 3.5$ cm $= 7$ cm

21. **(c) :** Let ABC be an equilateral triangle.
$\therefore$ $AB = BC = AC$ ...(i)
Let D, E, F are mid-points of sides BC, AC, AB respectively.
$\therefore$ By mid-point theorem,
$DE = \frac{1}{2}AB$, $EF = \frac{1}{2}BC$, $DF = \frac{1}{2}AC$



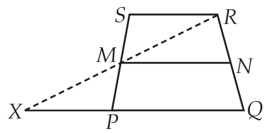
$\therefore$ $DE = EF = DF$ (From (i))
Hence, DEF is an equilateral triangle.

22. (a) : Let ABC be the triangle and D, E, F are mid-points of sides BC, AC, AB respectively.



$\therefore$ By mid-point theorem,
$DE \parallel AB$, $EF \parallel BC$, $DF \parallel AC$
$\therefore$ $DEAF$, $BDEF$, $FDCE$ are all parallelograms.
Now, DE is the diagonal of parallelogram $FDCE$
$\therefore$ $\Delta DEC \cong \Delta EDF$
Similarly, $\Delta FAE \cong \Delta EDF$
and $\Delta BFD \cong \Delta EDF$
Hence, all four triangles are congruent.

23. (d) : Given, M and N are respectively mid-points of non-parallel sides PS and QR of trapezium $PQRS$.



Join RM and produce it to meet QP produced at X.
In ΔSMR and ΔPMX,
$\angle SMR = \angle PMX$ (Vertically opposite angles)
$\angle SRM = \angle PXM$
($\because$ Alternate angles as, $SR \parallel QX$ and XR is transversal)
$SM = PM$ ($\because$ M is mid-point of PS)
$\therefore$ $\Delta SMR \cong \Delta PMX$ (By AAS congruence rule)
$\Rightarrow$ $MR = MX$ and $SR = PX$ (By C.P.C.T.)
Now, in ΔRXQ, M is the mid-point of XR, as $XM = MR$ and N is the mid-point of RQ.
$\therefore$ By mid-point theorem, $MN \parallel XQ$ and $MN = \frac{1}{2}XQ$
$\Rightarrow$ $MN \parallel PQ$ and $MN = \frac{1}{2}(XP + PQ) = \frac{1}{2}(SR + PQ)$
($\because$ $SR = XP$)
Hence, $MN \parallel PQ$ and $MN = \frac{1}{2}(SR + PQ)$

24. (b) : In ΔCDB, we have $CD = CB$
[$\because$ adjacent sides of rhombus are equal]
$\Rightarrow$ $\angle CBD = \angle CDB = x$
In ΔBCD, $\angle BCD = 70°$
and $\angle CDB + \angle CBD + \angle DCB = 180°$
$\Rightarrow$ $x + x + 70° = 180° \Rightarrow x = 55°$
$\Rightarrow$ $\angle CDB = 55°$

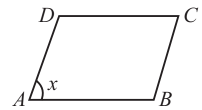
25. (b) : Since, adjacent angles of a parallelogram are supplementary.
So, $2x + 25° + 3x - 5° = 180°$
$\Rightarrow$ $5x = 160° \Rightarrow x = 32$

26. (a) : Let the three angles $\angle T$, $\angle A$ and $\angle R$ be $5x$, $3x$ and $7x$ respectively.
$\because$ $\angle S + \angle T + \angle A + \angle R = 360°$
$\Rightarrow$ $120° + 5x + 3x + 7x = 360°$
$\Rightarrow$ $15x = 240° \Rightarrow x = 16°$
$\therefore$ $\angle R = 7 \times 16 = 112°$

27. (b) : Since $ABCD$ is a trapezium.
$\therefore$ $x + 20° + 2x + 10° = 180°$
(Sum of measure of interior angles is 180°)
$\Rightarrow$ $3x + 30 = 180° \Rightarrow x = 50°$
and $y + 92° = 180° \Rightarrow y = 88°$

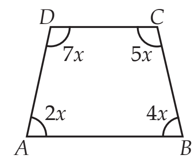
28. (a) : Given $\angle A + \angle C = 2(\angle B + \angle D)$
$\Rightarrow$ $140° + \angle C = 2\angle B + 2 \times 60°$
$\Rightarrow$ $2\angle B - \angle C = 20°$ ...(i)
Also, $\angle A + \angle B + \angle C + \angle D = 360°$
$\Rightarrow$ $140° + \angle B + \angle C + 60° = 360°$
$\Rightarrow$ $\angle B + \angle C = 160°$ ...(ii)
Using (i) and (ii), we get $\angle B = 60°$

29. (c) : Let the smallest angle be $\angle A = x°$, and other angle be $\angle B = (2x - 24)°$



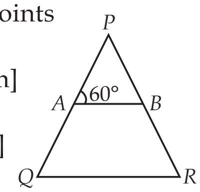
$\therefore$ $\angle A + \angle B = 180°$
$\Rightarrow$ $x + 2x - 24 = 180$
$\Rightarrow$ $3x = 204 \Rightarrow x = 68$
$\therefore$ $\angle A = 68°$
and $\angle B = (2x - 24)° = (2 \times 68 - 24)° = 112°$
Since, opposite angles of a parallelogram are equal.
So, $\angle A = \angle C = 68°$, $\angle B = \angle D = 112°$

30. (c) : Let the measures of the angles be $2x$, $4x$, $5x$ and $7x$.



$\Rightarrow$ $2x + 4x + 5x + 7x = 360°$
(Angle sum property)
$\Rightarrow$ $18x = 360° \Rightarrow x = 20°$
$\therefore$ $\angle A = 40°$, $\angle B = 80°$, $\angle C = 100°$, $\angle D = 140°$
As $\angle A + \angle D = 180°$ and $\angle B + \angle C = 180°$
$\Rightarrow$ $CD \parallel AB$
$\therefore$ $ABCD$ is a trapezium.

31. (c) : In ΔPQR, A and B are mid-points of PQ and PR respectively.



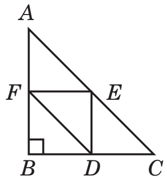
$\therefore$ $AB \parallel QR$ [By mid-point theorem]
$\therefore$ $\angle AQR = \angle PAB$
[Corresponding angles]
$\therefore$ $\angle PQR = \angle PAB = 60°$

32. (c) : We have, $\angle ADC + b = 180°$ [Linear pair]
$\Rightarrow$ $\angle ADC = 180° - b$ ...(i)
Also, $\angle ABC + a = 180°$ [Linear pair]
$\Rightarrow$ $\angle ABC = 180° - a$ ...(ii)
In quadrilateral $ABCD$, we have
$\angle ABC + \angle BCD + \angle ADC + \angle BAD = 360°$
[By angle sum property of a quadrilateral]
$\Rightarrow$ $180° - a + y + 180° - b + x = 360°$ [Using (i) and (ii)]
$\Rightarrow$ $360° - a - b + x + y = 360°$
$\Rightarrow$ $x + y = a + b$

33. (b) : Given, $PQRS$ is a parallelogram.
$\therefore$ $SR \parallel PQ$ and $SR = PQ$ ...(i)
But, $QT = PQ$ (Given) ...(ii)
From (i) and (ii), we have $SR = PQ = QT$
In ΔSRO and ΔTQO
$\angle RSO = \angle QTO$ (Alternate angles)
$SR = QT$ (Proved above)
$\angle SRO = \angle TQO$ (Alternate angles)
$\therefore$ $\Delta SRO \cong \Delta TQO$ (By ASA congruency criteria)
$\Rightarrow$ $RO = OQ$ (By C.P.C.T.)

34. (b) : If consecutive sides of a parallelogram are equal, then it is necessarily a rhombus.

35. (d) : Let ABC be right angled triangle and $\angle ABC = 90°$.



Let D, E, F are mid-points of sides BC, AC and AB respectively.

$\therefore$ $EF \parallel BD$ and $BF \parallel DE$ (By mid-point theorem)

$\Rightarrow$ $BDEF$ is a parallelogram.

$\therefore$ $\angle FED = \angle FBD = 90°$ ($\because$ Opposite angles of a parallelogram are equal)

$\therefore$ DEF is right angled triangle.

36. (b) : A parallelogram can be formed by joining the mid points of sides of quadrilateral.

37. (b) : As P and Q are mid points of AB and AD respectively.

$\therefore$ $PQ = \frac{1}{2}BD$ ...(1)

and $PQ \parallel BD$ [By midpoint theorem]

38. (a) : As, R and S are mid points of CD and BC respectively.

$\therefore$ $RS \parallel BD$ and $RS = \frac{1}{2}BD$ *i.e.*, $BD = 2RS$ ...(2)

39. (b) : From (1) and (2), $RS = PQ = \frac{1}{2}BD$

40. (a) : Perimeter of quadrilateral $PQRS$
$= PQ + QR + RS + SP$

41. (d) : All the conditions given in options (a), (b) and (c) are necessary for $ABCD$ to be a quadrilateral.

42. (c) : In a parallelogram, diagonal can't bisect each other.

43. (a) : $\angle A = \angle C \Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow$ $\angle YAX = \angle YCX$

Also, $\angle AYC + \angle YCX = 180°$ [$\because$ $CX \parallel AY$]

$\therefore$ $\angle AYC + \angle YAX = 180°$

So, $AX \parallel CY$ ($\because$ Interior angles on the same side of the transversal are supplementary)

44. (b) : As $ABCD$ is a parallelogram.

$\therefore$ $\angle A = \angle C = 63°$ (Opposite angles of a parallelogram are equal)

Also, $AEFG$ is a parallelogram.

$\therefore \angle A + \angle G = 180°$ (Adjacent angles are supplementary)

$\therefore$ $\angle G = 180° - 63° = 117°$

45. (c) : Let the angles be $3x$, $5x$, $5x$ and $7x$.

Now, $3x + 5x + 5x + 7x = 360°$

$\Rightarrow$ $20x = 360° \Rightarrow x = 18°$

$\therefore$ All angles are 54°, 90°, 90°, 126°

46. (b) : In ΔADE and ΔCFE, we have

$AE = CE$ (Given)

$DE = FE$ (Given)

$\angle AED = \angle CEF$ (Vertically opposite angles)

$\therefore$ $\Delta ADE \cong \Delta CFE$ (By SAS congruency criterion)

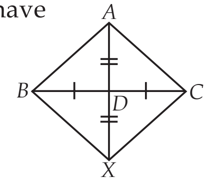
47. (b) : $\angle EFC = \angle EDA$ (By CPCT)

48. (a) : $\angle ECF = \angle EAD$ (By CPCT)

49. (d) : $CF = AD$ (By CPCT)

50. (c) : $CF \parallel BD$ ($\because$ $\angle ECF = \angle EAD$)

51. (c) : In quadrilateral $ABXC$, we have



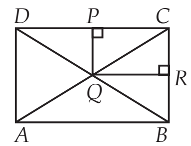
$AD = DX$ [Given]

$BD = DC$ [Since AD is median]

So, diagonals AX and BC bisect each other but not at right angles.

Therefore, $ABXC$ is a parallelogram.

52. (a) : Clearly, statement-II is true.



Now, in ΔADC, Q is the mid-point of AC such that $PQ \parallel AD$.

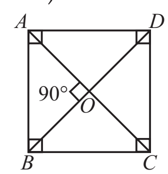
$\therefore$ P is the mid-point of DC.
[By converse of mid-point theorem]

$\Rightarrow$ $DP = PC$

53. (a) : Since, opposite angles of a parallelogram are equal. Therefore, $3x - 2 = 50 - x \Rightarrow x = 13$.

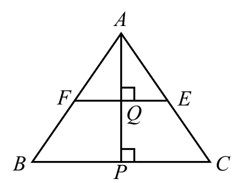
So, angles are $(3 \times 13 - 2)° = 37°$ and $(50 - 13)° = 37°$.

54. (a) : Since, diagonals of a square bisect each other at right angles.



$\therefore$ $\angle AOB = 90°$

55. (b) : In ΔABC, E and F are midpoint of the sides AC and AB respectively.



$\therefore$ $FE \parallel BC$ [By mid-point theorem

Now, in ΔABP, F is mid-point of AB and $FQ \parallel BP$

$\Rightarrow$ Q is mid-point of AP

$\Rightarrow$ $AQ = QP$.

SUBJECTIVE TYPE QUESTIONS

1. We know that consecutive interior angles of a parallelogram are supplementary.

$\therefore$ $(x + 60°) + (2x + 30°) = 180°$

$\therefore$ $3x + 90° = 180° \Rightarrow 3x = 90° \Rightarrow x = 30°$

Thus, two consecutive angles are $(30° + 60°)$, $(2 \times 30° + 30°)$ *i.e.*, 90° and 90°.

Hence, the special name of the given parallelogram is rectangle.

2. $\angle PQR = \angle PSR = 125°$
($\because$ Opposite angles of a parallelogram are equal)

Now, $\angle PQR + \angle RQT = 180°$ (Linear pair)

$\Rightarrow$ $125° + \angle RQT = 180° \Rightarrow \angle RQT = 55°$

3. No.

$\because$ Sum of the angles $= 110° + 80° + 70° + 95°$
$= 355° \neq 360°$

Thus, the given angles cannot be the angles of a quadrilateral.

4. Yes, $QL = LR$

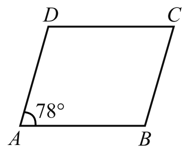
As, opposite sides of a parallelogram are equal.

$\therefore$ In parallelogram $QLMN$, $QL = NM$ ...(i)

In parallelogram $NLRM$, $NM = LR$ ...(ii)

From (i) and (ii), $QL = LR$

5. Since, $\angle A + \angle B = 180°$ [Co-interior angles]



$\Rightarrow$ $\angle B = 180° - 78° = 102°$

Now, $\angle B = \angle D = 102°$

and, $\angle A = \angle C = 78°$

[$\because$ opposite angles of a parallelogram are equal]

6. We have, $\angle A + \angle B = 180°$ [Co-interior angles]

$\Rightarrow$ $60° + \angle ABD + 55° = 180° \Rightarrow \angle ABD = 65°$

Also, $\angle ABD = \angle CDB$

[Alternate interior angles are equal]

$\therefore$ $\angle CDB = \angle ABD = 65°$

We have, $\angle ADB = \angle DBC$

[Alternate interior angles are equal]

$\Rightarrow$ $\angle ADB = 55°$

7. We have, $AB = AC \Rightarrow \angle BCA = \angle B$

Now, $\angle CAD = \angle B + \angle BCA$ [Exterior angle property]

$\Rightarrow$ $2\angle CAP = 2\angle BCA$ [$\because$ AP is the bisector of $\angle CAD$]

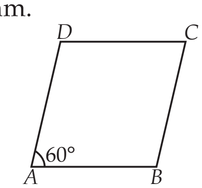
$\Rightarrow$ $\angle CAP = \angle BCA \Rightarrow AP \parallel BC$

Also, $AB \parallel CP$ [Given]

Hence, $ABCP$ is a parallelogram.

8. If one angle of a rhombus is a right angle, then it is necessarily a square.

9. Since a rhombus is a parallelogram.



$\therefore$ Its opposite angles are equal.

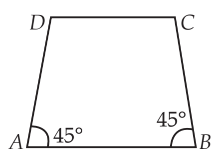
$\Rightarrow$ $\angle A = \angle C$

$\therefore$ $\angle C = 60°$ [$\because \angle A = 60°$ (Given)]

Now, required sum $= \angle A + \angle C$

$= 60° + 60° = 120°$

10. We have given, a trapezium $ABCD$, whose parallel sides are AB and DC.



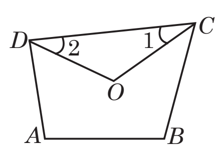
Since, $AB \parallel CD$ and AD is a transversal.

$\therefore$ $\angle A + \angle D = 180°$ [Angles on same side of transversal]

$\Rightarrow$ $\angle D = 180° - \angle A = 180° - 45° = 135°$

Similarly, $\angle C = 135°$

11. In ΔCOD, we have



$\angle COD + \angle 1 + \angle 2 = 180°$

$\Rightarrow$ $\angle COD = 180° - (\angle 1 + \angle 2)$

$\Rightarrow$ $\angle COD = 180° - \left(\frac{1}{2}\angle C + \frac{1}{2}\angle D\right)$

$\Rightarrow$ $\angle COD = 180° - \frac{1}{2}(\angle C + \angle D)$

$\Rightarrow$ $\angle COD = 180° - \frac{1}{2}\{360° - (\angle A + \angle B)\}$

[$\because \angle A + \angle B + \angle C + \angle D = 360°$]

$\Rightarrow$ $\angle COD = \frac{1}{2}(\angle A + \angle B)$

12. In ΔBCD, we have

$\angle BDC + \angle DCB + \angle CBD = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $5a + 9a + 4a = 180°$

$\Rightarrow$ $18a = 180° \Rightarrow a = 10°$

$\therefore$ $\angle C = 9 \times 10° = 90°$

Since, opposite angles of a parallelogram are equal

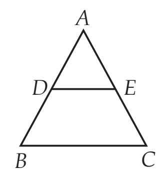
Therefore, $\angle A = \angle C \Rightarrow \angle A = 90°$

13. True. Given, $ABCD$ is a quadrilateral whose diagonals bisect each other. Then, it should be a parallelogram.

Also, $\angle A$ and $\angle B$ are adjacent angles of parallelogram $ABCD$. So, their sum should be 180°.

Now, $\angle A + \angle B = 45° + 135° = 180°$

14. Since, D and E are the mid-point of sides AB and AC respectively.



$\therefore$ $AD = \frac{1}{2}AB$ and $AE = \frac{1}{2}AC$

By mid-point theorem, $DE = \frac{1}{2}BC$

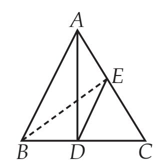
$\therefore$ $AD + AE + DE = \frac{1}{2}(AB + AC + BC)$

Perimeter of $\Delta ADE = \frac{1}{2} \times$ perimeter of ΔABC

$= \frac{1}{2} \times 35$ cm $= 17.5$ cm

Hence, the perimeter of ΔADE is 17.5 cm.

15. In ΔABC, $DE \parallel AB$ and AD is the median.



So, D is the mid-point of BC.

By converse of mid-point theorem, E is the mid-point of AC.

Hence, BE is median.

16. We have,

$MN = \frac{1}{2}BC$, $MP = \frac{1}{2}AC$ and $NP = \frac{1}{2}AB$

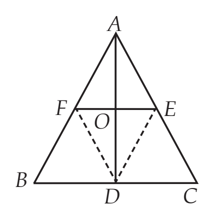
[By midpoint theorem]

$\Rightarrow$ $BC = 6$ cm, $AC = 5$ cm and $AB = 7$ cm.

The value of $(BC + AC) - AB$

$= (6 + 5) - 7 = 4$ cm.

17. ABC is an isosceles triangle with $AB = AC$ and D, E and F as the mid-points of sides BC, CA and AB respectively. AD intersects FE at O. Join DE and DF.



Since, D, E and F are mid-points of sides BC, AC and AB respectively.

$\therefore$ $DE \parallel AB$ and $DE = \frac{1}{2}AB$ [By mid-point theorem]

Also, $DF \parallel AC$ and $DF = \frac{1}{2}AC$

But, $AB = AC$ [Given]

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow \quad DE = DF$ ...(i)

Now, $DE = \frac{1}{2}AB \Rightarrow DE = AF$ ...(ii)

and, $DF = \frac{1}{2}AC \Rightarrow DF = AE$ ...(iii)

From (i), (ii) and (iii), we have

$DE = AE = AF = DF \Rightarrow$ $DEAF$ is a rhombus.

Since, diagonals of a rhombus bisect each other at right angles.

$\therefore$ $AD \perp FE$ and AD is bisected by FE.

18. Here, $\angle ABE + \angle EBF = 90°$

$\Rightarrow \quad 30° + \angle EBF = 90°$

$\Rightarrow \quad \angle EBF = 60°$ ...(i)

and $\angle BFE + \angle CFE = 180°$ [Linear pair]

$\Rightarrow \quad \angle BFE + 144° = 180°$

$\Rightarrow \quad \angle BFE = 180° - 144° = 36°$ ...(ii)

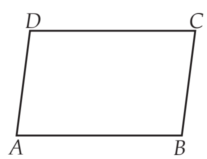
Now, in ΔBEF,

$\angle EBF + \angle BFE + \angle BEF = 180°$ (Angle sum property)

$\Rightarrow \quad 60° + 36° + \angle BEF = 180°$ [Using (i) and (ii)]

$\Rightarrow \quad \angle BEF = 180° - 96° = 84°$

19. Let $ABCD$ be a parallelogram with AB and DC as longer sides and AD and BC as shorter sides.



Now, $AB = DC = 9.5$ cm [Opposite sides of a parallelogram are equal and longer side = 9.5 cm (Given)]

Let $AD = BC = x$

Now, $AB + BC + CD + DA = 30$

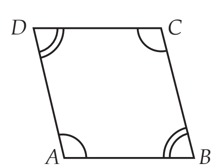
[Perimeter = 30 cm (Given)]

$\Rightarrow \quad 9.5 + x + 9.5 + x = 30$

$\Rightarrow \quad 2x = 30 - 19 = 11 \Rightarrow x = 5.5$ cm

$\therefore$ Length of shorter side = 5.5 cm

20. Since, $ABCD$ is a parallelogram.



$\therefore \quad \angle A = \angle C$

$\Rightarrow \quad (3x - 20)° = (x + 40)°$

$\Rightarrow \quad 3x - x = 40 + 20$

$\Rightarrow \quad 2x = 60 \Rightarrow x = 30$

Also, $\angle A + \angle B = 180°$

$\Rightarrow \quad (3x - 20)° + (y + 15)° = 180°$

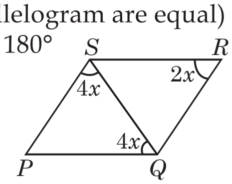
$\Rightarrow \quad 3x + y = 185 \Rightarrow y = 185 - 90 = 95$

$\therefore \quad x + y = 30 + 95 = 125$

21. $\angle SPQ = \angle QRS = 2x$

($\because$ Opposite angles of a parallelogram are equal)

In ΔPSQ, $\angle PSQ + \angle PQS + \angle SPQ = 180°$



$\Rightarrow \quad 4x + 4x + 2x = 180°$

$\Rightarrow \quad 10x = 180°$

$\Rightarrow \quad x = 18°$

Now, $\angle PSR = \angle PQR$

($\because$ Opposite angles of a parallelogram are equal)

$\Rightarrow \quad 4x + \angle QSR = 4x + \angle SQR$

$\Rightarrow \quad \angle QSR = \angle SQR$ ...(i)

In ΔSRQ, $\angle SRQ + \angle RSQ + \angle SQR = 180°$

$\Rightarrow \quad 2 \times 18° + 2\angle RSQ = 180°$ [From (i)]

$\Rightarrow \quad 2\angle RSQ = 180° - 36° = 144° \Rightarrow \angle RSQ = 72°$

Hence, $\angle P = \angle R = 2 \times 18° = 36°$,

$\angle Q = \angle S = 4x + 72° = 4 \times 18° + 72° = 144°$

22. We have, $AB = BC$ and have to prove that $DE = EF$.

Now, trapezium $ACFD$ is divided into two triangles namely ΔACF and ΔAFD.

In ΔACF, $AB = BC \Rightarrow B$ is mid-point of AC

and $BG \parallel CF$ [$\because m \parallel n$]

So, G is the mid-point of AF.

[By converse of mid-point theorem]

Now, in ΔAFD, G is the mid-point of AF.

and $GE \parallel AD$ [$\because m \parallel l$]

$\therefore$ E is the mid-point of FD.

[By converse of mid-point theorem]

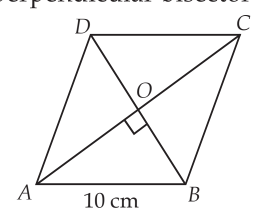
$\Rightarrow \quad DE = EF$

$\therefore$ l, m and n cut off equal intercepts on q also.

23. Let $ABCD$ be the rhombus and greater diagonal AC be x cm.

$\therefore$ Smaller diagonal, $BD = \frac{1}{3}AC = \frac{x}{3}$cm

Since diagonals of rhombus are perpendicular bisector of each other.



$\therefore \; OA = \frac{x}{2}$cm and $OB = \frac{x}{6}$cm

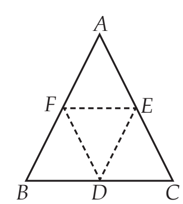
In ΔAOB, we have

$AB^2 = OA^2 + OB^2$

$\Rightarrow \quad 10^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{x}{6}\right)^2$

$\Rightarrow \quad 100 = \frac{x^2}{4} + \frac{x^2}{36} \quad \Rightarrow \quad 100 = \frac{10}{36}x^2 \Rightarrow x = 6\sqrt{10}$ cm

24. Let D, E and F be the mid-points of sides BC, CA and AB respectively.



In ΔABC, F and E are mid-points of AB and AC.

$\therefore \quad FE \parallel BC$ and $FE = \frac{1}{2}BC$

$\therefore \quad FE \parallel BD$ and $FE = BD$

$\therefore \quad FEDB$ is a parallelogram.

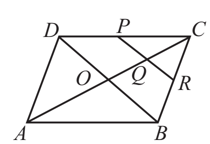
Similarly, $CDFE$ and $AFDE$ are also parallelograms.

$\therefore \quad \angle B = \angle DEF$, $\angle C = \angle DFE$ and $\angle FDE = \angle A$

$\Rightarrow \quad \angle DEF = 60°$, $\angle DFE = 70°$ and $\angle FDE = 50°$

25. Suppose AC and BD intersect at O.

Then, $OC = \frac{1}{2}AC$



Now, $CQ = \frac{1}{4}AC$ [Given]

$\Rightarrow \quad CQ = \frac{1}{2}OC$

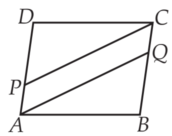
In ΔCOD, P and Q are the midpoints of DC and OC respectively.

$\therefore \quad PQ \parallel DO$ [By mid-point theorem]

Also, in ΔCOB, Q is the midpoint of OC and $QR \parallel OB$ $\therefore$ R is the midpoint of BC.

[By converse of mid-point theorem]

26. $\because$ $ABCD$ is parallelogram.



$\Rightarrow$ $AD = BC$ and $AD \parallel BC$

$\Rightarrow$ $\frac{1}{3}AD = \frac{1}{3}BC$ and $AD \parallel BC$

$\Rightarrow$ $AP = CQ$ and $AP \parallel CQ$

Thus, $APCQ$ is a quadrilateral such that one pair of opposite sides AP and CQ are parallel and equal.

Hence, $APCQ$ is a parallelogram.

27. $\angle P + \angle Q = 180°$ (Adjacent angles of parallelogram)

$\Rightarrow$ $60° + \angle Q = 180° \Rightarrow \angle Q = 120°$

Since, PA and QA are bisectors of angles P and Q

$\therefore$ $\angle SPA = \angle APQ = \frac{1}{2}\angle P = \frac{1}{2} \times 60° = 30°$

And $\angle RQA = \angle AQP = \frac{1}{2}\angle Q = \frac{1}{2} \times 120° = 60°$

Now, $SR \parallel PQ$ and AP is transversal.

$\therefore$ $\angle SAP = \angle APQ = 30°$ [Alternate interior angles]

In ΔASP, we have

$\angle SAP = \angle APS = 30°$

$\Rightarrow$ $SP = AS$ ...(i)

(Sides opposite to equal angles are equal)

Similarly, $QR = AR$ ...(ii)

But, $QR = SP$ [Opposite sides of parallelogram] ...(iii)

From (i), (ii) and (iii), we have $AS = AR$

$\Rightarrow$ A is the mid-point of SR.

28. We have, $\angle SPK = \angle QPK$ ...(i)

Now, $PQ \parallel RS$ and PK is a transversal

$\therefore$ $\angle SKP = \angle QPK$ [Alternate angles] ...(ii)

From (i) and (ii), $\angle SPK = \angle SKP$

$\Rightarrow$ $PS = SK$ ...(iii)

($\because$ Sides opposite to equal angles are equal)

But K is the mid-point of SR.

$\therefore$ $SK = KR$ ...(iv)

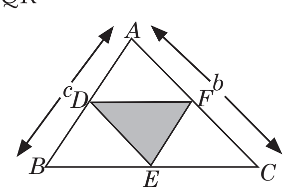
$PS = QR$ (Opposite sides of parallelogram are equal) ...(v)

From (iii) and (v), $SK = PS = QR$

Also, $PQ = SR = SK + KR = 2SK$ [From (i)]

$= 2QR$

29. Let the photo-frame be ABC such that $BC = a$, $CA = b$ and $AB = c$ and the mid-points of AB, BC and CA are D, E and F respectively. We have to determine the perimeter of ΔDEF.



In ΔABC, DF is the line-segment joining the mid-points of sides AB and AC.

By mid-point theorem, $DF \parallel BC$ and $DF = \frac{BC}{2} = \frac{a}{2}$

Similarly, $DE = \frac{AC}{2} = \frac{b}{2}$ and $EF = \frac{AB}{2} = \frac{c}{2}$

$\therefore$ Perimeter of $\Delta DEF = DF + DE + EF$

$= \frac{a}{2} + \frac{b}{2} + \frac{c}{2} = \frac{a+b+c}{2}$

30. We have, AL and CM are medians of ΔABC, *i.e.*, L and M are the mid-points of BC and AB respectively.

$\therefore$ $LC = BL = \frac{1}{2}BC$ and $BM = AM = \frac{1}{2}AB$ ...(i)

In ΔBMC, L is the mid-point of BC and $LN \parallel CM$. So, by converse of mid-point theorem, N is mid-point of BM.

i.e., $BN = NM = \frac{1}{2}BM$ ...(ii)

From (i) and (ii), we get

$BN = \frac{1}{2}\left(\frac{1}{2}AB\right) \Rightarrow BN = \frac{1}{4}AB$

31. It is given that $l \parallel m$ and transversal p intersects them at points A and C respectively.

The bisectors of $\angle PAC$ and $\angle ACQ$ intersect at B and bisectors of $\angle ACR$ and $\angle SAC$ intersect at D.

Now, $\angle PAC = \angle ACR$

[Alternate angles as $l \parallel m$ and p is a transversal]

So, $\frac{1}{2}\angle PAC = \frac{1}{2}\angle ACR$

$\Rightarrow$ $\angle BAC = \angle ACD$

These form a pair of alternate angles for lines AB and DC with AC as transversal and they are equal also.

So, $AB \parallel DC$

Similarly, $BC \parallel AD$

Therefore, quadrilateral $ABCD$ is a parallelogram.

Also, $\angle PAC + \angle CAS = 180°$ [Linear pair]

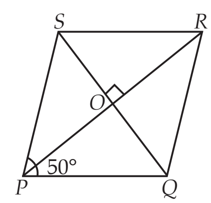
So, $\frac{1}{2}\angle PAC + \frac{1}{2}\angle CAS = \frac{1}{2} \times 180° = 90°$

$\Rightarrow$ $\angle BAC + \angle CAD = 90° \Rightarrow \angle BAD = 90°$

So, $ABCD$ is a parallelogram in which one angle is 90°.

Therefore, $ABCD$ is a rectangle.

32. Since a rhombus satisfies all the properties of a parallelogram.



$\therefore$ $\angle QPS = \angle QRS$

[Opposite angles of a parallelogram]

$\Rightarrow$ $\angle QRS = 50°$

[$\because$ $\angle QPS = 50°$ (Given)]

$\because$ Diagonals of a rhombus bisect the opposite angles.

$\therefore$ $\angle ORQ = \frac{1}{2}\angle QRS \Rightarrow \angle ORQ = 25°$

Now, in ΔORQ, we have

$\angle OQR + \angle ORQ + \angle ROQ = 180°$

$\Rightarrow$ $\angle OQR + 25° + 90° = 180°$

[$\because$ Diagonals of a rhombus are perpendicular to each other $\Rightarrow \angle ROQ = 90°$]

$\Rightarrow$ $\angle OQR = 180° - 115° = 65°$

$\therefore$ $\angle RQS = 65°$

33. Since, $ABCD$ is a square.

$\therefore$ $AB = BC = CD = DA$

Also, $PA = AB = BQ$

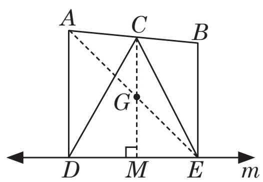
$\therefore$ $AB = BC = CD = DA = PA = BQ$

In ΔPDA and ΔQCB, $PA = BQ$ (Given)

$AD = BC$ (Sides of a square)

$\angle A = \angle B$ (Each 90°)
$\Delta PDA \cong \Delta QCB$ (By SAS congruency rule)
$\Rightarrow$ $PD = QC$ (By C.P.C.T.) …(i)
$\angle PDA = \angle QCB$ (By C.P.C.T.) …(ii)
Now, $\angle PDC = \angle PDA + \angle ADC$
$= \angle PDA + 90°$ …(iii)
$\angle QCD = \angle QCB + \angle BCD$
$= \angle QCB + 90°$ …(iv)
From (ii), (iii) and (iv), we have $\angle PDC = \angle QCD$
Now, in ΔPDC and ΔQCD, $PD = QC$ (Given)
$DC = DC$ (Common)
$\angle PDC = \angle QCD$ (Proved above)
$\Delta PDC \cong \Delta QCD$ (By SAS congruency rule)
$\therefore$ $DQ = CP$ [By C.P.C.T.]

34. We have, C is the mid-point of AB



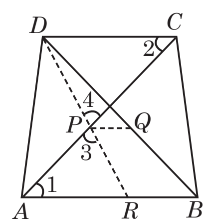
$\therefore$ $AC = BC$.
Draw, $CM \perp m$ and join AE.
We have, $AD \perp m$,
$CM \perp m$ and $BE \perp m$.
$\therefore$ $AD \parallel CM \parallel BE$
In ΔABE, $CG \parallel BE$ [$\because CM \parallel BE$]
and C is the mid-point of AB.
Thus, by converse of mid-point theorem, G is the mid-point of AE.
In ΔADE, G is the mid-point of AE and $GM \parallel AD$.
[$\because CM \parallel AD$]
Thus, by converse of mid-point theorem, M is mid-point of DE.
In ΔCMD and ΔCME,
$DM = EM$ ($\because M$ is the mid-point of DE)
$\angle CMD = \angle CME = 90°$ ($\because CM \perp m$)
$CM = CM$ (Common)
$\therefore$ $\Delta CMD \cong \Delta CME$ (By SAS congruence rule)
So, $CD = CE$ (By C.P.C.T.)

35. Since $BXDY$ is a parallelogram.
$\therefore$ $XO = YO$ …(i)
and $DO = BO$ …(ii)
[$\because$ Diagonals of a parallelogram bisect each other]
Also, $AX = CY$ (Given) …(iii)
Adding (i) and (iii), we have $XO + AX = YO + CY$
$\Rightarrow$ $AO = CO$ …(iv)
From (ii) and (iv), we have
$AO = CO$ and $DO = BO$
Thus, $ABCD$ is a parallelogram, because diagonals AC and BD bisect each other at O.

36. (i) As $EB \parallel DF \Rightarrow EB \parallel DL$ and $ED \parallel BL$.
Therefore, $EBLD$ is a parallelogram.
$\therefore$ $BL = ED = \frac{1}{2} AD = \frac{1}{2} BC = CL$ ...(i)
[$\because ABCD$ is a parallelogram $\therefore AD = BC$]
Now, in ΔDCL and ΔFBL, we have
$CL = BL$ [from (i)]
$\angle DLC = \angle FLB$ (Vertically opposite angles)
$\angle DCL = \angle FBL$ (Alternate angles)
$\therefore$ $\Delta DCL \cong \Delta FBL$ (By ASA congruency criteria)
$\Rightarrow$ $CD = BF$ and $DL = FL$ (By C.P.C.T.)
Now, $BF = DC = AB$ …(ii)
$\Rightarrow$ $2AB = 2DC \Rightarrow AB + AB = 2DC$
$\Rightarrow$ $AB + BF = 2DC$ [Using (ii)]
$\Rightarrow$ $AF = 2DC$
(ii) $\because DL = FL \Rightarrow DF = 2DL$

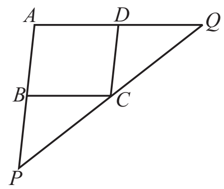
37. Here, in ΔABC, $AB = 18$ cm, $BC = 19$ cm, $AC = 16$ cm.
In ΔAOB, X and Y are the mid-points of AO and BO.
$\therefore$ By mid-point theorem, we have
$XY = \frac{1}{2} AB = \frac{1}{2} \times 18$ cm $= 9$ cm
In ΔBOC, Y and Z are the mid-points of BO and CO.
$\therefore$ By mid-point theorem, we have
$YZ = \frac{1}{2} BC = \frac{1}{2} \times 19$ cm $= 9.5$ cm
And, in ΔCOA, Z and X are the mid-points of CO and AO.
$\therefore$ By mid-point theorem, we have
$\therefore$ $ZX = \frac{1}{2} AC = \frac{1}{2} \times 16$ cm $= 8$ cm
Hence, the perimeter of $\Delta XYZ = 9 + 9.5 + 8 = 26.5$ cm

38. Let, trapezium $ABCD$ in which, $AB \parallel DC$ and P and Q are the mid-points of its diagonals AC and BD respectively.
We have to prove (i) $PQ \parallel AB$ and $PQ \parallel DC$
(ii) $PQ = \frac{1}{2}(AB - DC)$



Join D and P and produce DP to meet AB at R.
(i) Since $AB \parallel DC$ and transversal AC cuts them at A and C respectively.
$\therefore$ $\angle 1 = \angle 2$ (Alternate angles) ...(1)
In ΔAPR and ΔCPD,
$\angle 1 = \angle 2$ (From (1))
$AP = CP$ ($\because$ P is the mid-point of AC)
$\angle 3 = \angle 4$ (Vertically opposite angles)
$\therefore$ $\Delta APR \cong \Delta CPD$ (By ASA congruence rule)
$\Rightarrow$ $AR = DC$ and $PR = DP$ (By C.P.C.T.)
In ΔDRB, P and Q are the mid-points of side DR and DB respectively.
$\therefore$ $PQ \parallel RB$ (By mid-point theorem)
$\Rightarrow$ $PQ \parallel AB$ ($\because$ RB is a part of AB)
$\Rightarrow$ $PQ \parallel AB$ and $PQ \parallel DC$ ($\because$ $AB \parallel CD$)
(ii) In ΔDRB, P and Q are the mid-points of side DR and DB respectively.
$\therefore$ $PQ = \frac{1}{2}RB$ (By mid-point theorem)
$\Rightarrow$ $PQ = \frac{1}{2}(AB - AR)$ $\Rightarrow$ $PQ = \frac{1}{2}(AB - DC)$
[From part (i), $AR = DC$]

39. *CP* and *CQ* are joined.



$\because$ *ABCD* is a parallelogram.

So, $BC = AD$, $AB = DC$

[Opposite sides of parallelogram]

and $\angle ABC = \angle ADC$

[Opposite angles of parallelogram]

$\therefore$ Their supplementary angles are equal

So, $\angle PBC = \angle CDQ$

In ΔPBC and ΔCDQ, we have

$BC = DQ$ [$BC = AD$ and $AD = DQ$ (Given)]

$BP = DC$ [$AB = DC$ and $AB = BP$ (given)]

$\angle PBC = \angle CDQ$ [Proved above]

$\therefore$ $\Delta PBC \cong \Delta CDQ$ [By SAS congruency]

$\Rightarrow$ $\angle BPC = \angle DCQ$ and $\angle BCP = \angle DQC$ [By C.P.C.T.]

Again, $\angle BCD = \angle PBC$ [since, $AP \parallel DC$]

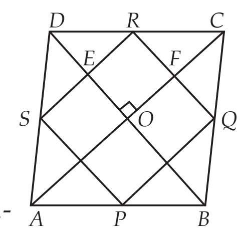
Now, $\angle BCP + \angle BCD + \angle DCQ$

$= \angle BCP + \angle PBC + \angle BPC = 2$ right angles

i.e., $\angle PCQ$ is a straight angle.

i.e., *P*, *C*, *Q* lie on a straight line.

40. In quadrilateral *ABCD*, $AC \perp BD$ and $AC = BD$.



In ΔADC, *S* and *R* are the mid-points of the sides *AD* and *DC* respectively.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(i)

[By mid-point theorem]

In ΔABC, *P* and *Q* are the mid-points of *AB* and *BC* respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(ii)

[By mid-point theorem]

From (i) and (ii), $PQ \parallel SR$

and $PQ = SR = \frac{1}{2}AC$ ...(iii)

Similarly, in ΔABD,

$SP \parallel BD$ and $SP = \frac{1}{2}BD$ [By mid-point theorem]

$\therefore$ $SP = \frac{1}{2}AC$ [$\because$ $AC = BD$] ...(iv)

Now in ΔBCD, $RQ \parallel BD$ and $RQ = \frac{1}{2}BD$

[By mid-point theorem]

$\therefore$ $RQ = \frac{1}{2}AC$ [$\because$ $BD = AC$] ...(v)

From (iv) and (v), $SP = RQ = \frac{1}{2}AC$ ...(vi)

From (iii) and (vi), $PQ = SR = SP = RQ$ ...(vii)

$\therefore$ All four sides are equal.

Now, in quadrilateral *OERF*,

$OE \parallel FR$ and $OF \parallel ER$

$\therefore$ $\angle EOF = \angle ERF = 90°$ [$\because AC \perp DB$]

$\therefore$ $\angle QRS = 90°$ ...(viii)

From (vii) and (viii), we get

PQRS is a square.

☺☺☺