MATHEMATICAL REASONING

13

MCQs with One Correct Answer

1. For the statement "17 is a real number or a positive integer", the "or" is
(a) Inclusive (b) Exclusive
(c) Only (a) (d) None of these

2. Let p and q be any two logical statements and $r : p \rightarrow (\sim p \vee q)$. If r has a truth value F, then the truth values of p and q are respectively :
(a) F, F (b) T, T (c) T, F (d) F, T

3. If p : Ashok works hard
q : Ashok gets good grade
The verbal form for $(\sim p \rightarrow q)$ is
(a) If Ashok works hard then gets good grade
(b) If Ashok does not work hard then he gets good grade
(c) If Ashok does not work hard then he does not get good grade
(d) Ashok works hard if and only if he gets grade

4. If p is false and q is true, then
(a) $p \wedge q$ is true (b) $p \vee \sim q$ is true
(c) $q \wedge p$ is true (d) $p \Rightarrow q$ is true

5. $\sim p \wedge q$ is logically equivalent to
(a) $p \rightarrow q$ (b) $q \rightarrow p$
(c) $\sim (p \rightarrow q)$ (d) $\sim (q \rightarrow p)$

6. Which of the following is a contradiction?
(a) $(p \wedge q) \wedge \sim (p \vee q)$ (b) $p \vee (\sim p \wedge q)$
(c) $(p \Rightarrow q) \Rightarrow p$ (d) None of these

7. $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is
(a) A tautology
(b) A contradiction
(c) Both a tautology and a contradiction
(d) Neither a tautology nor a contradiction

8. The false statement in the following is
(a) $p \wedge (\sim p)$ is contradiction
(b) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a contradiction
(c) $\sim (\sim p) \Leftrightarrow p$ is a tautology
(d) $p \vee (\sim p) \Leftrightarrow p$ is a tautology

9. The conditional $(p \wedge q) \Rightarrow p$ is
(a) A tautology
(b) A fallacy *i.e.*, contradiction
(c) Neither tautology nor fallacy
(d) None of these

10. If p and q are two statements, then $(p \Rightarrow q) \Leftrightarrow (-q \Rightarrow \sim p)$ is a
(a) contradiction (b) tautology
(c) neither (a) nor (b) (d) None of these

11. Which of the following is false?
(a) $p \vee \sim p$ is a tautology
(b) $\sim (\sim p) \leftrightarrow p$ is a tautology
(c) $p \wedge \sim p$ is a contradiction
(d) $((p \wedge q) \rightarrow q) \rightarrow p$ is a tautology

12. In the truth table for the statement $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$, the last column has the truth value in the following order is
(a) TTFF (b) FFFF
(c) TTTT (d) FTFT

13. If $p \Rightarrow (\sim p \vee q)$ is false, then truth values of p and q are respectively
(a) F. T (b) F, F
(c) T, T (d) T, F

14. Negation of "2 + 3 = 5 and 8 < 10" is
(a) $2+3 \neq 5$ and < 10 (b) $2+3=5$ and $8 \nless 10$
(c) $2+3 \neq 5$ or $8 \nless 10$ (d) None of these

15. If the compound statement $p \to (\sim p \vee q)$ is false then the truth value of p and q are respectively
(a) T, T (b) T, F (c) F, T (d) F, F

16. The contrapositive of $p \to (\sim q \to \sim r)$ is
(a) $(\sim q \wedge r) \to \sim p$ (b) $(q \to r) \to \sim p$
(c) $(q \vee \sim r) \to \sim p$ (d) None of these

17. The negation of the compound proposition $p \vee (\sim p \vee q)$ is
(a) $(p \wedge \sim q) \wedge \sim p$ (b) $(p \wedge \sim q) \vee \sim p$
(c) $(p \vee \sim q) \vee \sim p$ (d) None of these

18. The inverse of the statement $(p \wedge \sim q) \to r$ is
(a) $\sim(p \vee \sim q) \to \sim r$ (b) $(\sim p \wedge q) \to \sim r$
(c) $(\sim p \vee q) \to \sim r$ (d) None of these

19. $\sim((\sim p) \wedge q)$ is equal to
(a) $p \vee (\sim q)$ (b) $p \vee q$
(c) $p \wedge (\sim q)$ (d) $\sim p \wedge \sim q$

20. Which of the following is true?
(a) $p \Rightarrow q \equiv \sim p \Rightarrow \sim q$
(b) $\sim(p \Rightarrow \sim q) \equiv \sim p \wedge q$
(c) $\sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$
(d) $\sim(\sim p \Leftrightarrow q) \equiv [\sim(p \Rightarrow q) \wedge \sim(q \Rightarrow p)]$

21. The negation of $(p \vee q) \wedge (p \vee \sim r)$ is
(a) $(\sim p \wedge \sim q) \vee (q \wedge \sim r)$
(b) $(\sim p \wedge \sim q) \vee (\sim q \wedge r)$
(c) $(\sim p \wedge \sim q) \vee (\sim q \wedge r)$
(d) $(p \wedge q) \vee (\sim q \wedge \sim r)$

22. Let p, q and r be any three logical statements. Which of the following is true?
(a) $\sim[p \wedge (\sim q)] \equiv (\sim p) \wedge q$
(b) $\sim[(p \vee q) \wedge (\sim r) \equiv (\sim p) \vee (\sim q) \vee (\sim r)$
(c) $\sim[p \vee (\sim q)] \equiv (\sim p) \wedge q$
(d) $\sim[p \vee (\sim q)] \equiv (\sim p) \wedge \sim q$

23. Identify the false statements
(a) $\sim[p \vee (\sim q)] \equiv (\sim p) \vee q$
(b) $[p \vee q] \vee (\sim p)$ is a tautology
(c) $[p \wedge q) \wedge (\sim p)$ is a contradiction
(d) $\sim[p \vee q] \equiv (\sim p) \vee (\sim q)$

24. Negation of the statement $(p \wedge r) \to (r \vee q)$ is
(a) $\sim(p \wedge r) \to \sim(r \vee q)$
(b) $(\sim p \vee \sim r) \vee (r \vee q)$
(c) $(p \wedge r) \wedge (r \wedge q)$
(d) $(p \wedge r) \wedge (\sim r \wedge \sim q)$

25. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is:
(a) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$
(b) If $A \subseteq C$, then $B \subset A$ or $D \subset B$
(c) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$
(d) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

ANSWER KEY

1	(a)	**4**	(d)	**7**	(b)	**10**	(b)	**13**	(d)	**16**	(a)	**19**	(a)	**22**	(c)	**25**	(d)
2	(c)	**5**	(d)	**8**	(b)	**11**	(b)	**14**	(c)	**17**	(a)	**20**	(c)	**23**	(d)		
3	(b)	**6**	(a)	**9**	(a)	**12**	(c)	**15**	(b)	**18**	(c)	**21**	(c)	**24**	(d)		

Hints & Solutions

CHAPTER 13 Mathematical Reasoning

1. **(a)** Inclusive "or". 17 is a real number or a positive integer or both.

2. **(c)** $p \to (\sim p \vee q)$ has truth value F.
It means $p \to (\sim p \vee q)$ is false.
It means p is true and $\sim p \vee q$ is false.
$\Rightarrow$ p is true and both $\sim p$ and q are false.
$\Rightarrow$ p is true and q is false.

3. **(b)** $\sim p$: Ashok does not work hard
Use '$\to$' symbol for then
($\sim p \to q$) mean = If Ashok does not work hard then he gets good grade.

4. **(d)** When p is false and q is true, then $p \wedge q$ is false, $p \vee \sim q$ is false.
($\because$ both p and $\sim q$ are false)
and $q \Rightarrow p$ is also false,
only $p \Rightarrow q$ is true.

5. **(d)** $\sim p \wedge q = \sim (q \to p)$

6. **(a)**

p	q	p ∧ q	p ∨ q	~(p ∨ q)	(p ∧ q) ∧ ~ (p ∨ q)
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$\therefore (p \wedge q) \wedge (\sim (p \vee q))$ is a contradiction.

7. **(b)** $(p \wedge \sim q) \wedge (\sim p \wedge q) = (p \wedge \sim q) \wedge (\sim q \wedge q)$
$= f \wedge f = f$
(By using associative laws and commutative laws)
$\therefore (p \wedge \sim q) \wedge (\sim p \wedge q)$ is a contradiction.

8. **(b)** $p \Rightarrow q$ is logically equivalent to $\sim q \Rightarrow \sim p$
$\therefore$ $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tautology but not a contradiction.

9. **(a)**

p	q	$p \wedge q$	$(p \wedge q) \Rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$\therefore (p \wedge q) \Rightarrow p$ is a tautology.

10. **(b)**

$p \Rightarrow q$	$\sim q \Rightarrow \sim p$	$p \Rightarrow q \Leftrightarrow \sim q \Rightarrow \sim p$
T	T	T
F	F	T
T	T	T
T	T	T

11. **(b)** The truth value of $\sim(\sim p) \leftrightarrow p$ as follow

p	$\sim p$	$\sim(\sim p)$	$\sim(\sim p) \to p$	$p \to \sim(\sim p)$	$\sim(\sim p) \leftrightarrow p$
T	F	T	T	T	T
F	T	F	T	T	T

Since last column of above truth table contains only T.
Hence $\sim(\sim p) \to p$ is a tautology.

12. **(c)**

p	q	p→q	~p	~p∨q	(p→q)↔~(p∨q)
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

13. **(d)** $p \Rightarrow (\sim p \vee q)$ is false means p is true and $\sim p \vee q$ is false.
$\Rightarrow p$ is true and both $\sim p$ and q are false. $\Rightarrow p$ is true and q is false

14. **(c)** Let $p : 2 + 3 = 5, q : 8 < 10$

Given proposition is : $p \wedge q$.

Its negation is $\sim (p \wedge q) = \sim p \vee \sim q$

$\therefore$ we have $2 + 3 \neq 5$ or $8 \not< 10$.

15. **(b)** We know that $p \to q$ is false only when p is true and q is false.

So $p \to (\sim p \vee q)$ is false only when p is true and $(\sim p \vee q)$ is false.
But $(\sim p \vee q)$ is false if q is false because $\sim p$ is false.
Hence $p \to (\sim p \vee q)$ is false when truth value of p and q are T and F respectively.

16. **(a)** We know that the contrapositive of $p \to q$ is $\sim q \to \sim p$.

So contrapositive of $p \to (\sim q \to \sim r)$ is
$\sim (\sim q \to \sim r) \to \sim p \equiv \sim q \wedge [\sim (\sim r)]\ \sim p$
$\because \sim (p \to q) \equiv p \wedge \sim q \equiv \sim q \wedge r \to \sim p$

17. **(a)** $\sim [p \vee (\sim p \vee q)] \equiv \sim p \wedge \sim (\sim p \vee q)$

$\equiv \sim p \wedge (\sim (\sim p) \wedge \sim q) \ \equiv \sim p \wedge (p \wedge \sim q)$.

18. **(c)** The inverse of the proposition
$(p \wedge \sim q) \to r$ is $\sim (p \wedge \sim q) \to \sim r$
$\equiv \sim p \vee \sim (\sim q) \to \sim r \equiv \sim p \vee q \to \sim r$

19. **(a)** $\sim ((\sim p) \wedge q) \equiv \sim (\sim p) \vee \sim q \equiv p \vee (\sim q)$

20. **(c)** $\sim (p \Rightarrow q) \equiv p \wedge \sim q$

$\therefore \sim (\sim p \Rightarrow \sim q) \equiv \sim p \wedge \sim (\sim q) \equiv \sim p \wedge q$.

Thus $\sim (\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$

21. **(c)** $\sim [(p \vee q) \wedge (q \vee \sim r)]$
$\equiv \sim (p \vee q) \vee \sim (q \vee \sim r)$
$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge r)$

22. **(c)** Statement given in option (c) is correct.
$\sim [p \vee (\sim q)] = (\sim p) \wedge \sim (\sim q) = (\sim p) \wedge q$.

23. **(d)** Since $\sim (p \vee q) \equiv \sim p \wedge \sim q$
(By De-Morgans' law)

$\therefore \sim (p \vee q) \neq \sim p \vee \sim q$

$\therefore$ (d) is the false statement.

24. **(d)** We know that $\sim (p \to q) \equiv p \wedge \sim q$
$\therefore \sim ((p \wedge r) \to (r \vee q)) \equiv (p \wedge r) \wedge [\sim (r \vee q)]$
$\equiv (p \wedge r) \wedge (\sim r \wedge \sim q)$

25. **(d)** Let $P = A \subseteq B, Q = B \subseteq D, R = A \subseteq C$

Contrapositive of $(P \wedge Q) \to R$ is $(\to \sim R \to \sim (P \wedge Q))$.
$\sim R \to \sim P \vee \sim Q$