Chapter 5

Deflection of Beams

CHAPTER HIGHLIGHTS

- *Introduction*
- *Deformation of beam under transverse loading*
- *Equation of the elastic curve*
- *Double integration method*
- *Macaulay's method*
- *Moment area methods*

Introduction

Deflection of Beams

When a beam is loaded with concentrated or distributed loads, the axis of beam deflects.

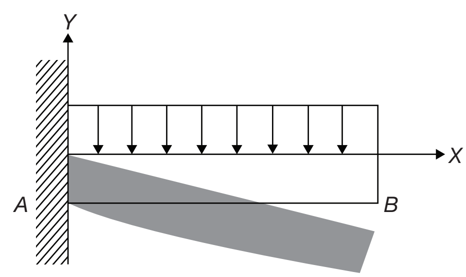
To prevent misalignment, maintain dimensional accuracy, etc., deflection should be within permissible limits.

Therefore, while designing, not only the strength, but deflection also as an important factor to be taken into consideration.

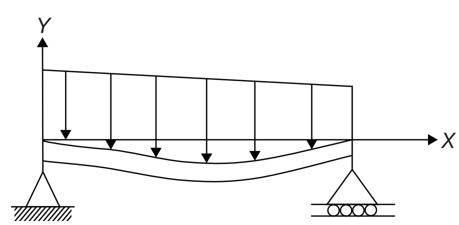
Of particular interest is the determination of the maximum deflection of a beam under a given loading, since the design specifications of a beam generally includes a maximum allowable value for its deflection.

A prismatic beam subjected to pure bending is bent into an arc of circle, and that within the elastic range, the curvature of the neutral surface can be expressed as $\frac{1}{R} = \frac{M}{EI}$.

Where, 'M' is the bending moment, 'E' the modulus of elasticity, and 'I' the moment of inertia of the cross-section about its neutral axis.



Cantilever beam



Simply supported beam

To determine the slope and deflection of the beam at any given point, at first we derive the following second order linear differential equation which governs the elastic curve characterizing the shape of the deformed beam.

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

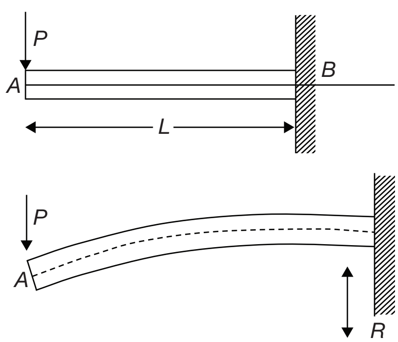
If the bending moment can be represented for all values of 'x' by a single function $M(x)$ as in the case of the beams and loadings, shown in the given figures, the slope $\theta = \frac{dy}{dx}$ and the deflection 'y' at any point of the beam may be obtained through two successive integrations. The two constants of integration introduced in the process will be determined from the boundary conditions indicated in the figure.

However, if different analytical functions are required to represent the bending moment in various portions of the beam, different differential equations will also be required, leading to different functions defining the elastic curve in various portions of the beam.

Deformation of Beam under Transverse Loading

$$\frac{1}{R} = \frac{M(x)}{EI}$$

Consider for example a cantilever beam 'AB' of length L, subjected to a concentrated load 'P' at its free end 'A' as shown in the following figure.



We have, $M(x) = -P \times x$,

and $\frac{1}{R} = \frac{-Px}{EI}$,

which shows that the curvature of the neutral surface varies linearly with x, form zero at 'A', where R_A itself is infinite to $\frac{-PL}{EI}$ at B, where $R_B = \frac{EI}{PL}$, R_A and R_B being the radius of curvature at A and B.

Equation of the Elastic Curve

From elementary calculus, we first recall that the curvature of a plane curve at a point $Q(x, y)$ of the curve can be expressed as:

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

where $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are the first and second derivatives of the function $y(x)$ represented by that curve. But, in the case of the elastic curve of a beam, the slope $\frac{dy}{dx}$ is very small, and its square is negligible compared to unity. Therefore, we write:

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

The equation obtained is a second-order linear differential equation. It is the governing differential equation for the elastic curve.

This is the differential equation for deflection.

Here, y = deflection

$$\theta = \frac{dy}{dx} = \text{Slope}$$

$$M = EI\frac{d^2y}{dx^2} = \text{Moment}$$

It is also to be noted that shear force:

$$F = -\frac{dM}{dx} = -EI\frac{d^3y}{dx^3}$$

Load intensity: $q = \frac{dF}{dx} = -E\frac{d^4y}{dx^4}$

The product EI is known as flexural rigidity. In the case of a prismatic beam, it is taken as constant.

Double Integration Method

Taking x from one end (usually from the left end) and with sagging moment as positive:

$$EI\frac{d^2y}{dx^2} = M$$

$$EI\frac{dy}{dx} = \int_0^x M\,dx + C_1$$

$$EI\frac{d^2y}{dx} = \int_0^x\int_0^x M\,dx + C_1 + C_2$$

The constants C_1 and C_2 are derived by applying boundary conditions.

Some Boundary Conditions

1. At simply supported/roller ends, $y = 0$.
2. At fixed ends, $y = 0\dfrac{dy}{dx} = 0$.
3. At point of symmetry, $\dfrac{dy}{dx} = 0$.

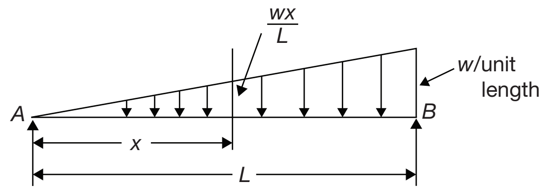
Some General Cases

1. Cantilever subjected to moment at free end.

   At $x = 0$,
   $$\frac{dy}{dx} = \frac{mL}{EI}$$
   $$y = -\frac{mL^2}{2EI}$$
2. Cantilever subjected to concentrated load at free end.
   At $x = 0$,
   $$\frac{dy}{dx} = \frac{wL^2}{2EI}$$
   $$y = -\frac{wL^3}{3EI}$$
3. Cantilever subjected uniform load w/unit length.
   At $x = 0$,
   $$\frac{dy}{dx} = \frac{wL^3}{6EI}$$
   $$y = \frac{-wL^4}{8EI}$$
4. Cantilever subjected uniformly varying load, zero at free end to w/unit length at fixed end.
   At $x = 0$,
   $$\frac{dy}{dx} = \frac{wL^3}{24EI}$$
   $$y = \frac{-wL^4}{30EI}$$
5. Simply supported beam with central concentrated load.

   Deflection at centre, $y_c = \dfrac{wL^3}{48EI}$

   Slope at end, $\theta = \dfrac{-wL^3}{16EI}$
6. Simply supported beam with uniform load w/unit length.
   $$y_c = \frac{5}{384}\frac{wL^4}{EI}$$
   $$\theta_A = \frac{-wL^2}{24EI}$$
7. Simply supported beam with uniformly varying load, zero at end A and w/unit length at end B.



$$EI_y = -\frac{7wL^3x}{360} + \frac{wLx^3}{36} - \frac{wx^5}{120L}$$

$$EI\frac{dy}{dx} = -\frac{7wL^3}{360} + \frac{wLx^2}{12} - \frac{wx^4}{24L}$$

$\dfrac{dy}{dx} = 0$. At the point of maximum deflection, $y_{\max}$.

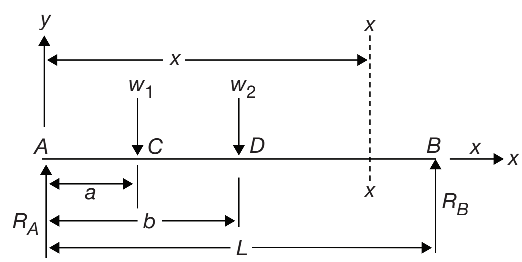
This occur at $x = 0.5193L$

$$y_{\max} = -0.006523\frac{wL^4}{EI}.$$

Macaulay's Method

Macaulay's method is a simplified version of double integration method. It gives a continuous expression for bending moment applicable for all portions of the beam. The constants of integration determined by using boundary conditions are also applicable for all portions of the beam.

For example, consider the case of a beam with concentrated loads as shown in the following figure.



Here, the expression for moment at a distance x from A is $M_x =$

$$EI\frac{d^2y}{dx} = R_Ax - w_1(x-a) - w_2(x-b)$$

The same expression can be used for other portions also, if we ignore the quantities $(x - a)$, $(x - b)$, becoming negative.

Integrating the expression, we get: $EI\dfrac{dy}{dx}$

$$= C_1 + R_A\frac{x^2}{2} - w_1\frac{(x-a)^2}{2} - w_2\frac{(x-b)^2}{2}$$

$$EIy = C_2 + C_1x + R_A\frac{x^3}{6} - w_1\frac{(x-a)^3}{6} - w_2\frac{(x-b)^3}{6}$$

Applying boundary condition, values of C_1 and C_2 are obtained.

Now the given expression can be used for finding out slope and deflection of any portion of the beam. If term $(x - a)$, $(x - b)$, etc., become negative, they are ignored.

In the case of uniformly distributed loads, it is extended upto the section x–x, and an equal and opposite uniformly distributed load is applied to nullify it.

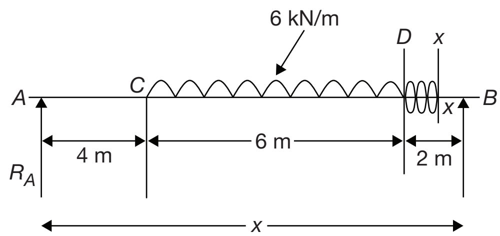
SOLVED EXAMPLES

Example 1

A 12 m long beam simply supported at ends is loaded as shown in the diagram. (A uniformly distributed load of 6 kN/m acts over a length of $CD = 6$ m.)

Determine the slopes at A and B

($E = 2000$ N/cm^2 and $I = 5000$ cm^4)



Solution

Taking moments,

i.e., $R_A \times 12 - 6 \times 6 \times (3 + 2) = 0$

$R_A \times 12 = 180$

$R_A = 15$ kN

i.e., $R_B \times 12 - 6 \times 6 \times (3 + 4) = 0$

$R_B \times 12 = 252$

$R_B = 21$ kN

Now, consider a section at x–x in the portion DB at a distance x from A.

Extend the uniformly distributed load to x–x and an equal and opposite uniformly distributed load from D to section x–x.

Bending moment at any section x–x,

$$M_x = R_A(x) - W\frac{(x-4)^2}{2} + \frac{W(x-10)^2}{2}$$

That is, $$EI\frac{d^2y}{dx^2} = 15x - \frac{6(x-4)^2}{2} + \frac{6(x-10)^2}{2}$$

Integrating

$$EI\frac{dy}{dx} = C_1 + \frac{15x^2}{2} - \frac{3(x-4)^3}{3} + \frac{3(x-10)^3}{3}$$

$$EI\,y = C_2 + C_1 x + \frac{15x^3}{6} - \frac{(x-4)^4}{4} + \frac{(x-10)^4}{4}$$

Applying boundary conditions and omitting negative terms:

$C_2 = 0$ and $C_1 = -275$

Equation for slope:

$$EI\frac{dy}{dx} = -275 + 7.5x^2 - (x-4)^3 + (x-10)^3$$

At A, $x = 0$

Slope, $\dfrac{dy}{dx} = \theta_A$

$EI = 1000$ kNm^2

$\therefore\ EI\,\theta_A = -275 + 7.5 \times 0 = -275$

$\theta_A = -15.76°$.

At B, $x = 12$ m

$EI\,\theta_B = -275 + 7.5 \times 12^2 - 8^3 + 2^3$

$\theta_B = 17.25°$

Example 2

In the above problem, find the maximum deflection.

Solution

Maximum deflection occurs at $\dfrac{dy}{dx} = 0$.

Assume that it occur in the CD portion. The term '$(x - 10)$' will be negative.

$\therefore$ From the slope equation,

$0 = -275 + 7.5x^2 - (x - 4)^3$ for zero slope.

Let, $f(x) = -275 + 7.5x^2 - (x - 4)^3$

At $x = 5$, $f(x) = -275 + 7.5 \times 25 - (1)^3 = -88.5$

By trial $x \sim 6.2$ for $f(x) = 0$.

This is the point at which maximum deflection occur.

The required eflection equation is:

$$EI_y = 0 - 275x + 2.5x^3 - \frac{(x-4)^4}{4}$$

$\therefore$ when $x = 6.2$

$$EI_{y\max} = -1115.04$$

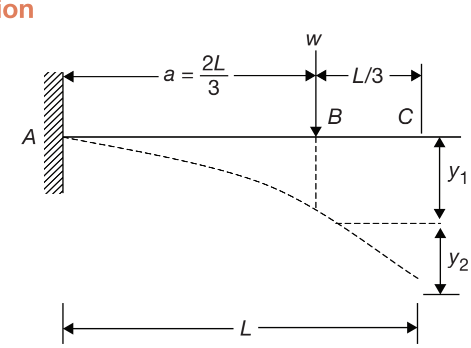
$$Y_{\max} = \frac{-1115.04}{1000} = -1.115 \text{ m}$$

$= 1115$ mm downward.

Example 3

A cantilever beam of length L is subjected to a concentrated load w at distance $\dfrac{L}{3}$ from the free end. Find the deflection of the free end.

Solution



Deflection at $B = \dfrac{wa^3}{3EI} = y_1$.

Since, there is no load in the BC portion, slope $\left(\dfrac{dy}{dx}\right)_B$ is maintained throughout the portion at $B = \dfrac{wa^2}{2EI}$.

Deflection at $C = y_1 + y_2$

$$= \frac{w_a{}^3}{3EI}, \frac{w_a{}^2}{2EI} \times \frac{L}{3}$$

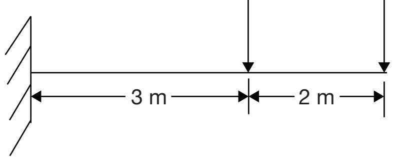
$$= \frac{wa^2}{3EI}\left(a + \frac{L}{2}\right),$$

putting $a = \dfrac{2L}{3}$, $C = \dfrac{14}{81}\dfrac{wL^3}{EI}$.

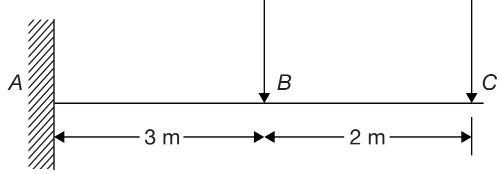
Example 4

Find the deflection at the end of the cantilever beam shown in the figure.

Take $EI = 4 \times 10^4$ kNm^2



Solution



Equation for deflection at end $= \dfrac{-wl^3}{3EI}$.

Equation for slope $= \dfrac{wl^2}{2EI}$.

Total deflection = Deflection due to 20 kN + Deflection due to 15 kN

$$= \frac{20 \times 5^3}{3EI} + \frac{w_B l^3}{3EI} + BC\,\frac{w_B l^2}{2EI}$$

$$= \frac{20 \times 5^3}{3EI} + \frac{15 \times 3^3}{3EI} + \frac{2 \times 15 \times 3^2}{2EI}$$

$$= \frac{1}{EI}(833.33 + 135 + 135)$$

$$= \frac{1103.33}{4 \times 10^4} = 0.0276 \text{ m}$$

$= 27.6$ mm.

Moment Area Methods

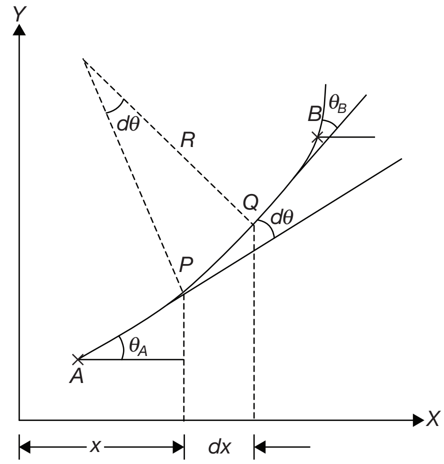
Change in slope and deflection between two points on a beam can be obtained using the moment area theorems (or Mohr's theorems).

We have seen that:

$$\frac{1}{R} = \frac{d^2y}{dx^2} = \frac{M}{EI},$$

where M_x is the bending moment and EI is the flexural rigidity.

First Moment Area Theorem



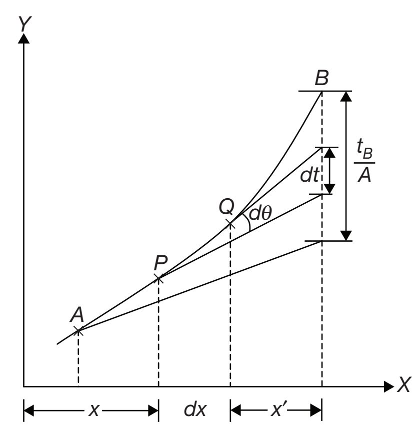
The given figure shows elastic curve AB of an initially straight beam (exaggerated)

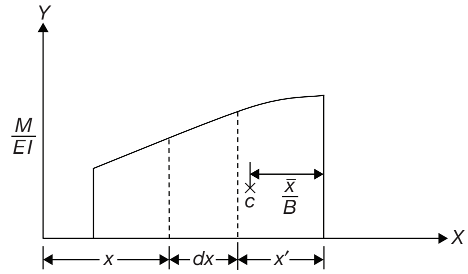
For the infinitesimally small distance dx, from the geometry of the figure, we can see that $dx = Rd\theta$ or $d\theta = \dfrac{dx}{R} = \dfrac{M}{EI}dx$.

On integration over the segment AB, $\int_A^B d\theta = \int_A^B \dfrac{M}{EI}\,dx$ or change in slope,

$\theta_B - \theta_A =$ Area of $\dfrac{M}{EI}$ diagram between A and B. This is the first moment area theorem.

Second Moment Area Theorem





Let, $t_{B/A}$ be the vertical distance of point B from the tangent to the elastic curve at point A. This distance is termed as the tangential deviation of B with respect to A.

$t_{B/A} = \int_A^B dt$ where dt is the vertical distance at B corresponding to the tangents at P and Q subtending angle $d\theta$. From the geometry, we can see that $dt = x'd\theta$ for infinitesimal values of dt and $d\theta$.

$$\therefore t_{B/A} = \int_A^B dt = \int_A^B x'd\theta = \int_A^B \frac{M}{EI}x'dx$$

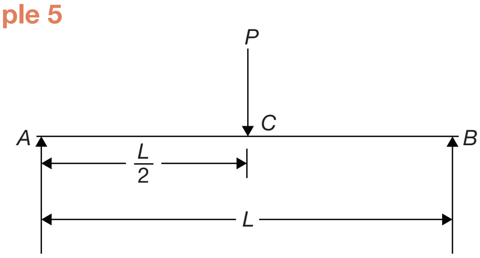
The right-hand side of the given equation represents the first moment of area of the $\frac{M}{EI}$ diagram and is equal to (the area of $\frac{M}{EI}$ diagram between A and B) × (horizontal distance of centroid of the area from B)

Or

$$t_{B/A} = \text{area of } \frac{M}{EI} \text{ diagram}\Big]_B^A \cdot \frac{\bar{x}}{B}$$

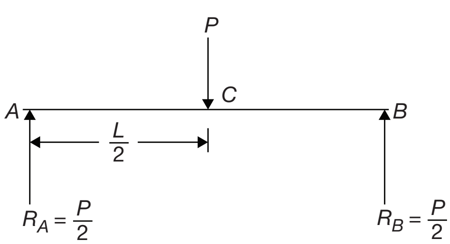
where $\bar{x}/B$ is the distance of centroid of area from B. It is to be noted here that $t_{B/A}$ need not be equal to $t_{A/B}$.

Example 5

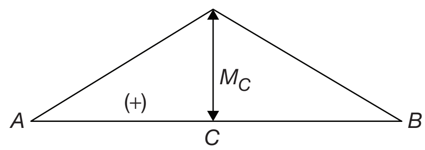


For the simply supported beam loaded as shown in the given figure, determine slope at B and deflection at C by area moment method (flexural rigidity = EI).

Solution



From symmetry, reaction $R_A = R_B = \frac{P}{2}$



Bending Moment Diagram

Bending moment at centre (C)

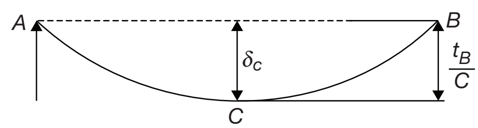
$$= \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$$

Since, tangent at C of the deflection curve is horizontal, slope at B,

$$\theta_B = \text{Area of } \frac{M}{EI} \text{ diagram between } B \text{ and } C$$

$$= \frac{PL}{4} \times \frac{L}{2} \times \frac{1}{2} \times \frac{1}{EI} = \frac{PL^2}{16EI}$$

Deflection at C:



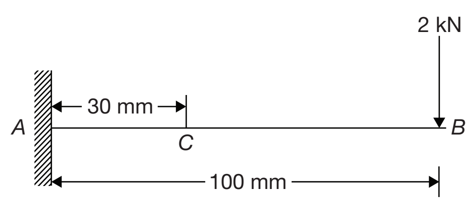
From geometry of elastic curve, deflection at C, δ_c = Tangential deviation $t_{B/C}$

= Area of $\frac{M}{EI}$ diagram between B and C × Distance of centroid of the area

$$= \frac{PL^2}{16EI} \times \frac{2}{3} \times \frac{L}{2}$$

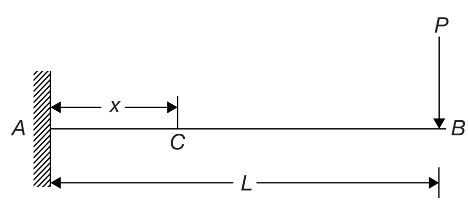
$$= \frac{PL^3}{48EI}.$$

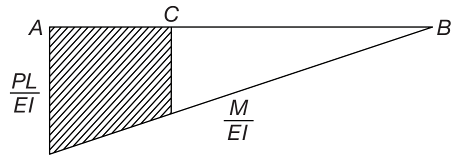
Example 6



A cantilever beam ACB is loaded as shown in the figure. If the flexural rigidity is 22 Nm^2, determine the slope at point C using moment area method.

Solution





$EI = 22$ Nm^2

Slope at C,

$$\theta_c = \text{Area of } \frac{M}{EI} \text{ diagram between } A \text{ and } C$$

$$= -\left[\frac{PL}{EI}\times\frac{L}{2} - \frac{PL}{EI}\times\frac{(L-x)}{L}\times\frac{(L-x)}{2}\right]$$

$$= -\frac{P}{2EI}[L^2-(L-x)^2]$$

$$= -\frac{Px}{2EI}[2L-x]$$

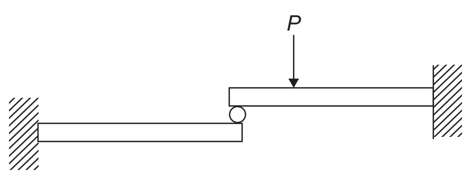
$$= -\frac{-2000\times 0.03}{2\times 22}[2\times 0.1-0.03]$$

$$= -0.2318 \text{ radian}$$

Magnitude of slope is 0.2318 radian.

Exercises

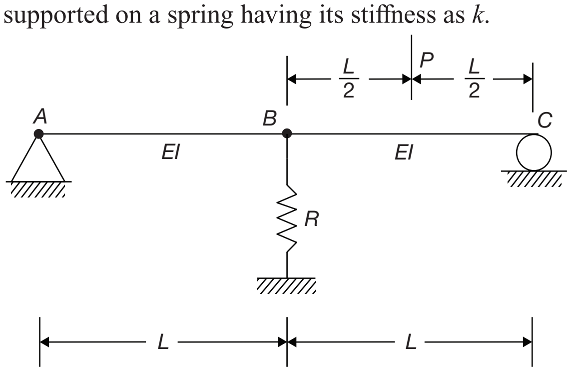
1. Two identical cantilever beams are supported as shown in the figure, with their free ends in contact through a rigid roller. After the load P is applied, the free ends will have



(A) equal deflections but not equal slopes.
(B) equal slopes but not equal deflections.
(C) equal slopes as well as equal deflections.
(D) neither equal slopes nor equal deflections.

2. In a real beam, at an end, the boundary condition of zero slope and zero vertical displacement exists. In the corresponding conjugate beam, the boundary conditions at this and will be:
(A) Shear forces = 0 and bending moment = 0
(B) Slope = 0 and vertical displacement = 0
(C) Slope = 0 and bending moment = 0
(D) Shear force = 0 and vertical displacement = 0

3. Two elastic rod AB and BC are hinged at B. The joint A is a hinged one, joint C is over a roller and the joint B is supported on a spring having its stiffness as k.



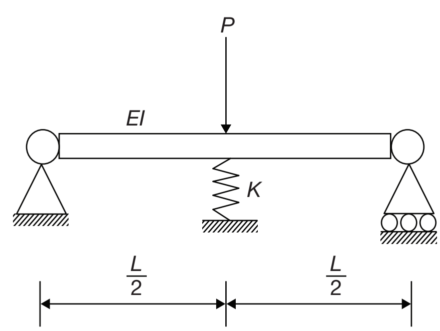
A load P acts at mid-point of the rod BC. The downward deflection of joint B is:

(A) $\frac{P}{k}$ (B) $\frac{2P}{k}$

(C) $\frac{P}{2k}$ (D) 0

4. A cantilever beam of span, 'L' is subjected to a downward load of 800 kN uniformly distributed over its length and a concentrated upward load P at its free end. For vertical displacement to be zero at the free end, the value of P is
(A) 300 kN
(B) 500 kN
(C) 800 kN
(D) 1000 kN

5. A simply supported beam of span length L and flexural stiffness EI has another spring support at the center span of stiffness K as shown in the figure. The central deflection of the beam due to a central concentrated load of P would be



(A) $\left[\frac{PL^3}{48EI}\right]+\left(\frac{P}{K}\right)$ (B) $\frac{P}{\left(\frac{48EI}{L^3}\right)}-k$

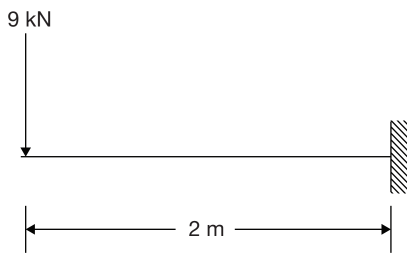
(C) $\left[\frac{PL^3}{48EI}\right]-\left(\frac{P}{K}\right)$ (D) $\frac{P}{\left(\frac{48EI}{L^3}\right)}+k$

6. A cantilever beam of span 'L' is loaded with a concentrated load 'P' at the free end. Deflection of the beam at the free end is

(A) $\frac{PL^3}{48EI}$ (B) $\frac{5PL^3}{384EI}$

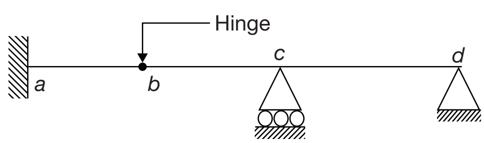
(C) $\frac{PL^3}{3EI}$ (D) $\frac{PL^3}{6EI}$

7. A cantilever beam is shown in the figure. The moment to be applied at free end for zero vertical deflection at that point is:

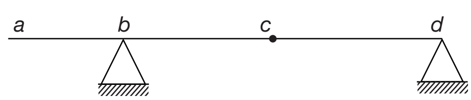


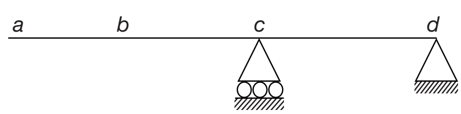
(A) 9 kN-m clockwise
(B) 9 kN-m anti-clockwise
(C) 12 kN-m clockwise
(D) 12 kN-m anti-clockwise

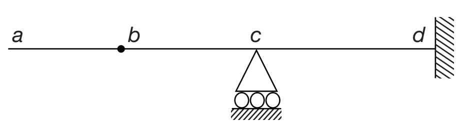
8. A two span beam with an internal hinge is shown below.

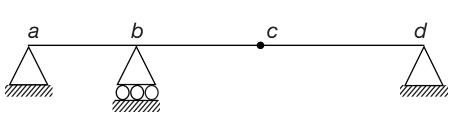


Conjugate beam corresponding to this beam is

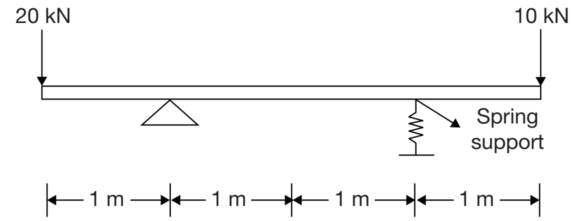
(A)


(B)


(C)


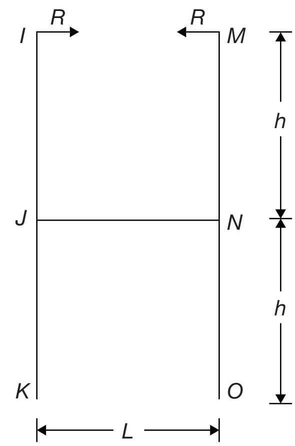
(D)


9. The bending moment (in kN-m units) at the mid-span location X in the beam with overhangs shown in the following figure is equal



(A) 0 (B) −10
(C) −15 (D) −20

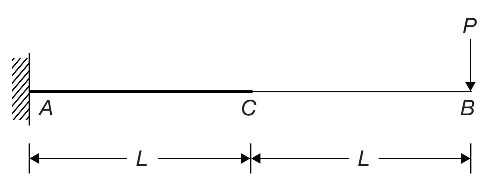
10. A 'H' Shaped frame of uniform flexural rigidity EI is loaded as shown in the figure. The relative outward displacement between points K and O is



(A) $\frac{RLh^2}{EI}$ (B) $\frac{RL^2h}{EI}$

(C) $\frac{RLh^2}{3EI}$ (D) $\frac{RL^2h}{4EI}$

11. Consider the beam AB shown in the following figure. Part AC of the beam is rigid while Part CB has the flexural rigidity EI. Identify the correct combination of deflection at end B and bending moment at end A respectively.



(A) $\frac{PL^3}{3EI}, 2PL$ (B) $\frac{PL^3}{3EI}, PL$

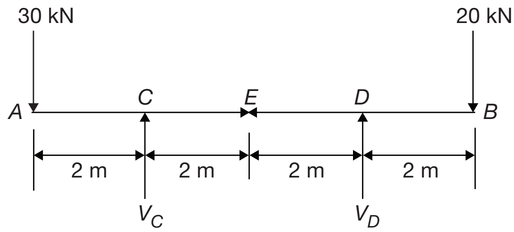
(C) $\frac{8PL^3}{3EI}, 2PL$ (D) $\frac{8PL^3}{3EI}, PL$

12. A cantilever of 4 m span is loaded with a point load of 20 kN/m at a distance of 1 m from the free end. The downward deflection of the cantilever at the free end is [Take $E = 2 \times 10^5$ N/mm^2 and $I = 2 \times 10^8$ mm^4]

(A) 5.25 mm (B) 6.23 mm
(C) 6.75 mm (D) 5.78 mm

Direction for questions 13 and 14:

For the double overhanging beam loaded as shown in the figure, Young's modulus = 200 GPa and moment of inertia = 5×10^6 mm^4.



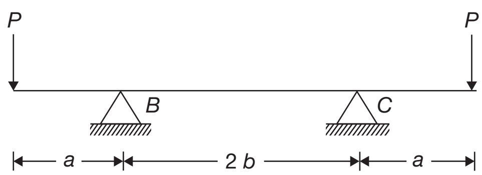
13. Deflection at point A relative to top of support is

(A) 29.33 mm up (B) 32.41 mm up
(C) 29.33 mm down (D) 32.41 mm down

14. Deflection at point E relative to top of support is

(A) 12 mm up (B) 10 mm up
(C) 12 mm down (D) 10 mm down

15. For the beam shown in the following figure, the elastic curve between the supports B and C will be



(A) circular (B) parabolic
(C) elliptical (D) a straight line

16. At a certain section at a distance 'x' from one of the supports of a simply supported beam, the intensity of loading, bending moment and shear force are w_x, M_x and V_x respectively. If the intensity of loading is varying continuously along the length of the beam, then the invalid relation is

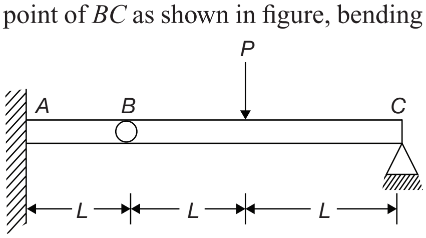
(A) $\theta_x = \dfrac{M_x}{V_x}$ (B) $V_x = \dfrac{dM_x}{dx}$

(C) $w_x = \dfrac{d^2 M_x}{dx^2}$ (D) $w_x = \dfrac{dV_x}{dx}$

17. Slope of a beam under load is

(A) rate of change of deflection.
(B) rate of change of bending moment.
(C) rate of change of bending moment x flexural rigidity.
(D) rate of change deflection x flexural rigidity.

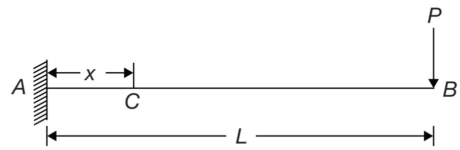
18. A beam is made of 2 bars AB and BC hinged at B, fixed at A and simply supported at C. If it is loaded at mid-point of BC as shown in figure, bending moment at A is



(A) PL (B) $\dfrac{PL}{2}$

(C) $2PL$ (D) $\dfrac{2}{3}PL$

19. A cantilever beam is loaded as shown in the figure.

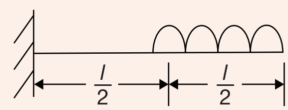
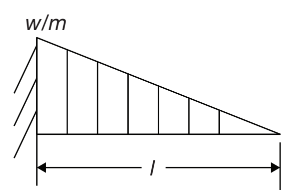
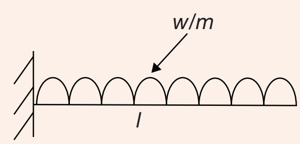
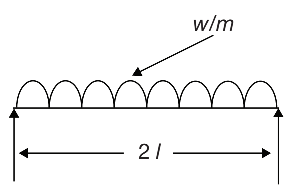


If L is the length of the beam and EI, the flexural rigidity, slope at point C at a distance x from fixed end is

(A) $\dfrac{Px}{EI}(2L - x)$ (B) $-\dfrac{Px}{2EI}(2L - x)$

(C) $\dfrac{Px}{2EI}(L - x)$ (D) $-\dfrac{Px}{EI}(L - x)$

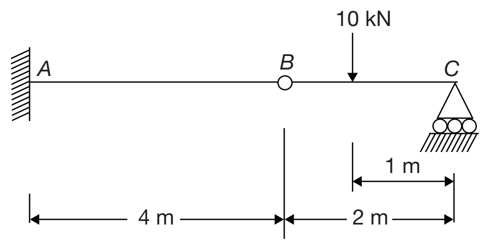
20. Match the following List I (Loaded beam) and List II (Maximum bending moment).

List I		List II	
a.		1.	$\dfrac{wl^2}{2}$
b.		2.	$\dfrac{wl^2}{6}$
c.		3.	$\dfrac{3}{8}wl^2$
d.		4.	$\dfrac{wl^2}{4}$

Codes:

	a	b	c	d
(A)	3	2	4	1
(B)	3	4	2	1
(C)	1	2	3	4
(D)	4	3	2	1

21. A Cantilever beam AB is connected to another beam BC with a pin joint at B as shown in the figure. For the loading as shown in the figure, the magnitude of bending moment at A (in kN-m) is



(A) 5 (B) 10
(C) 20 (D) 50

22. A simply supported beam of length L has a cross-section of depth d and width $\frac{d}{2}$. If it is loaded with a uniformly distributed load of w/unit length, maximum deflection is (Young's modulus = E)

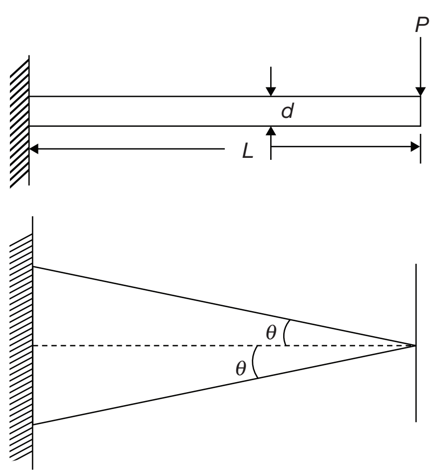
(A) $\frac{5}{8}\frac{wL^4}{Ed^4}$

(B) $\frac{5}{16}\frac{wL^4}{Ed^4}$

(C) $\frac{5}{8}\frac{wL^3}{Ed^4}$

(D) $\frac{5}{16}\frac{wL^3}{Ed^4}$

23. A cantilever beam of varying width and constant depth is loaded as shown in the figure. Maximum bending stress at the fixed end of the beam is



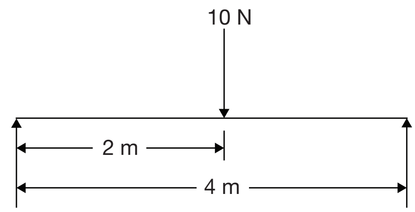
(A) $\frac{PL}{d^2 \tan\theta}$

(B) $\frac{2P}{d^2 \tan\theta}$

(C) $\frac{3P}{d^2 \tan\theta}$

(D) $\frac{3PL}{d^2 \tan\theta}$

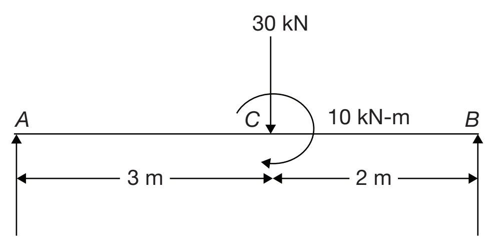
24. A beam with cross-section 10 cm width and 20 cm depth is loaded as shown in the figure. Maximum shear stress at a section 1 m away from end is



(A) 0
(B) 0.375 MPa
(C) 3.75 MPa
(D) 37.5 MPa

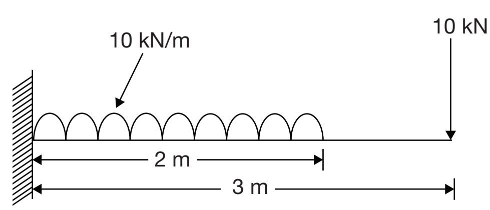
25. A cantilever beam of span length 6 m is loaded by a weight 'W' at the free end. The deflection at the free end is observed to be 1.8 cm. The slope of the beam at free end in radians will be
(A) 0.045
(B) 0.0045
(C) 0.45
(D) 45×10^{-5}

26. For the simply supported beam ABC of 5 m span a concentrated load of 30 kN and a clockwise moment of 10 kN-m acts at the point C, 3 m from end A. Value of flexural rigidity for the beam is 10000 kN-m^2. The deflection at point C is



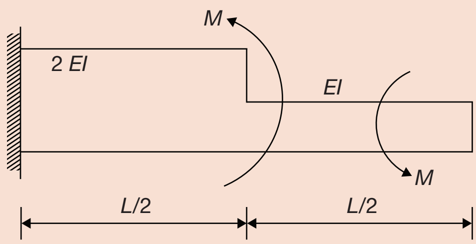
(A) 9.58 mm
(B) 8.22 mm
(C) 10.33 mm
(D) 7.32 mm

27. A cantilever beam is loaded as shown in the figure. Deflection of the beam (in mm) at the free end is (Take flexural rigidity = 36000 kN-m^2)



Previous Years' Questions

1. The stepped cantilever is subjected to moments M as shown in the figure. The vertical deflection at the free end (neglecting the self weight) is **[GATE, 2008]**



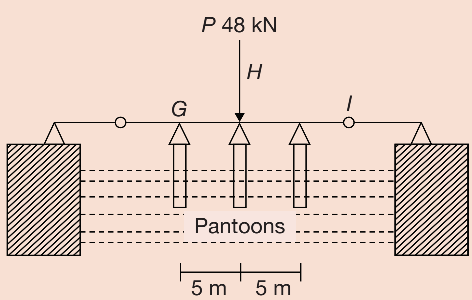
(A) $\dfrac{ML^2}{8EI}$

(B) $\dfrac{ML^2}{4EI}$

(C) $\dfrac{ML^2}{2EI}$

(D) zero

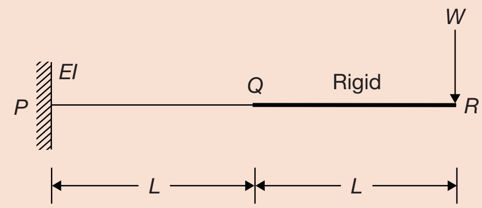
Direction for questions 2 and 3:
Beam GHI is supported by three pantoons as shown in the figure, the horizontal cross-sectional area of each pantoon is 8 m^2, the flexural rigidity of the beam is 10000 kN-m^2 and the unit weight of water is 10 kN-m^3. **[GATE, 2008]**



2. When the middle pantoon is removed, the deflection at H will be
(A) 0.2 m (B) 0.4 m
(C) 0.6 m (D) 0.8 m

3. When the middle pantoon is brought back to its position as shown in the figure above, the reaction at H will be
(A) 8.6 kN (B) 15.7 kN
(C) 19.2 kN (D) 24.2 kN

Direction for questions 4 and 5:
In the cantilever beam PQR shown in the following figure, the segment PQ has flexural EI and the segment QR has infinite flexural rigidity **[GATE, 2010]**



4. The deflection and slope of the beam at 'Q' are respectively

(A) $\dfrac{5WL^3}{6EI}$ and $\dfrac{3WL^2}{2EI}$ (B) $\dfrac{WL^3}{6EI}$ and $\dfrac{WL^2}{2EI}$

(C) $\dfrac{WL^3}{2EI}$ and $\dfrac{WL^2}{EI}$ (D) $\dfrac{WL^3}{3EI}$ and $\dfrac{3WL^2}{2EI}$

5. The deflection of the beam at 'R' is

(A) $\dfrac{8WL^3}{EI}$ (B) $\dfrac{5WL^3}{6EI}$

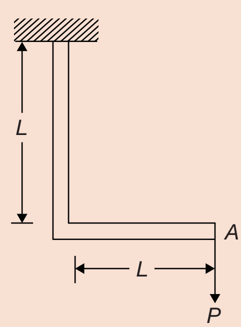
(C) $\dfrac{7WL^3}{3EI}$ (D) $\dfrac{8WL^3}{6EI}$

6. A simply supported beam is subjected to a uniformly distributed load of intensity w per unit length, on half of the span from one end. The length of the span and the flexural stiffness are denoted as l and EI respectively. The deflection at mid-span of the beam is **[GATE, 2012]**

(A) $\dfrac{5}{6144}\dfrac{wl^4}{EI}$ (B) $\dfrac{5}{768}\dfrac{wl^4}{EI}$

(C) $\dfrac{5}{384}\dfrac{wl^4}{EI}$ (D) $\dfrac{5}{192}\dfrac{wl^4}{EI}$

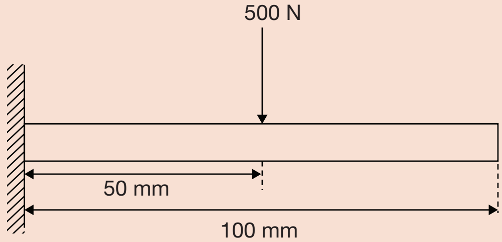
7. A frame is subjected to a load P as shown in the figure. The frame has a constant flexural rigidity EI. The effect of axial load is neglected. The deflection at point A due to the applied load P is **[GATE, 2014]**



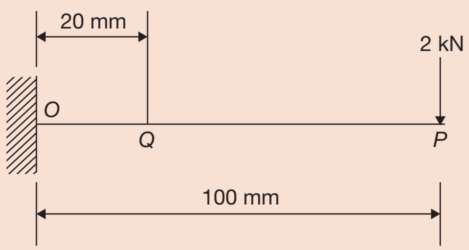
(A) $\dfrac{1}{3}\dfrac{PL^3}{EI}$ (B) $\dfrac{2}{3}\dfrac{PL^3}{EI}$

(C) $\dfrac{PL^3}{EI}$ (D) $\dfrac{4}{3}\dfrac{PL^3}{EI}$

8. A cantilever beam with flexural rigidity of 200 Nm^2 is loaded as shown in the figure. The deflection (in mm) at the tip of the beam is ________. **[GATE, 2015]**

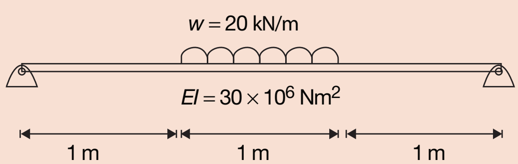


9. A cantilever beam with square cross-section of 6 mm side is subjected to a load of 2 kN normal to the top surface as shown in the figure. The Young's modulus of elasticity of the material of the beam is 210 GPa. The magnitude of slope (in radian) at Q (20 mm from the fixed end) is ________. **[GATE, 2015]**



10. A simply supported reinforced concrete beam of length 10 m sags while undergoing shrinkage. Assuming a uniform curvature of 0.004 m^{-1} along the span, the maximum deflection (in m) of the beam at mid-span is __________. **[GATE, 2015]**

11. A 3 m long simply supported beam of uniform cross section is subjected to a uniformly distributed load of $w = 20$ kN/m in the central 1 m as shown in the figure. **[GATE, 2016]**



If the flexural rigidity (EI) of the beam is 30 × 106 Nm^2, the maximum slope (expressed in radians) of the deformed beam is

(A) 0.681×10^{-7} (B) 0.943×10^{-7}
(C) 4.310×10^{-7} (D) 5.910×10^{-7}

12. Two beams PQ (fixed at P and with a roller support at Q, as shown in Figure I, which allows vertical movement) and XZ (with a hinge at Y) are shown in the Figures I and II respectively. The spans of PQ and XZ are L and $2L$ respectively. Both the beams are under the action of uniformly distributed load (W) and have the same flexural stiffness, EI (where, E and I respectively denote modulus of elasticity and moment of inertia about axis of bending). Let the maximum deflection and maximum rotation be δ_{max1} and θ_{max1}, respectively, in the case of beam PQ and the corresponding quantities for the beam XZ be δ_{max2} and θ_{max2}, respectively. **[GATE, 2016]**

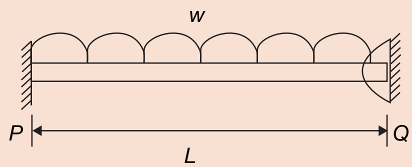


Figure 1

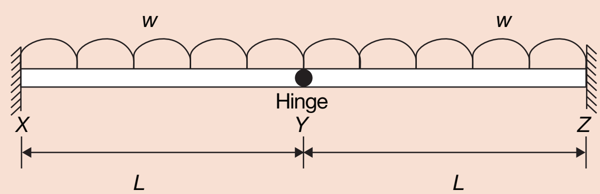


Figure 2

Which one of the following relationships is true?

(A) $\delta_{max1} \neq \delta_{max2}$ and $\theta_{max1} \neq \theta_{max2}$
(B) $\delta_{max1} = \delta_{max2}$ and $\theta_{max1} \neq \theta_{max2}$
(C) $\delta_{max1} \neq \delta_{max2}$ and $\theta_{max1} = \theta_{max2}$
(D) $\delta_{max1} = \delta_{max2}$ and $\theta_{max1} = \theta_{max2}$

Answer Keys

Exercises

1. A	**2.** A	**3.** C	**4.** A	**5.** A	**6.** C	**7.** C	**8.** D	**9.** C	**10.** A
11. A	**12.** C	**13.** C	**14.** B	**15.** D	**16.** A	**17.** A	**18.** B	**19.** B	**20.** A
21. C	**22.** B	**23.** C	**24.** B	**25.** B	**26.** A	**27.** 3.4 to 3.5			

Previous Years' Questions

1. C	**2.** B	**3.** C	**4.** A	**5.** C	**6.** B	**7.** D	**8.** 0.24 to 0.28
9. 0.15 to 0.17		**10.** 0.05	**11.** C	**12.** D			