32. Statistics

Exercise 32.1

1 A. Question

Calculate the mean deviation about the median of the following observation :

3011, 2780, 3020, 2354, 3541, 4150, 5000

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 3011, 2780, 3020, 2354, 3541, 4150, 5000 are Given.

Since, Median is the middle number of all the observation,

So, To Find the Median, Arrange the numbers in Ascending order, we get

2354, 2780, 3011, 3020, 3541, 4150, 5000

Therefore, The Median = 3020

Deviation |d| = |x-Median|

Now, The Mean Deviation is

X_i	$\lvert d_i \rvert = \lvert x_i - 3020 \rvert$
3011	9
2780	240
3020	0
2354	666
3541	521
4150	1130
5000	1980
Total	4546

Mean Deviation = $\frac{4546}{7}$

Hence, The Mean Deviation is 649.42

1 B. Question

Calculate the mean deviation about the median of the following observation :

38, 70, 48, 34, 42, 55, 63, 46, 54, 44

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 38, 70, 48, 34, 42, 55, 63, 46, 54, 44 are Given.

Since, Median is the middle number of all the observation,

So, To Find the Median, Arrange the numbers in Ascending order, we get

34, 38,42,44,46,48,54,55,63,70

Here the Number of observations are Even then Median = $\frac{46+48}{2}$

Therefore, The Median = 47

Deviation |d| = |x-Median|

And, The number of observations is 10.

Now, The Mean Deviation is

X_i	$\lvert d_i \rvert = \lvert x_i - 47 \rvert$
38	9
70	23
48	1
34	13
42	5
55	8
63	16
46	1
54	7
44	3
Total $\sum di$	86

Mean Deviation = $\frac{86}{10}$

Hence, The Mean Deviation is 8.6

1 C. Question

Calculate the mean deviation about the median of the following observation :

34, 66, 30, 38, 44, 50, 40, 60, 42, 51

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 34, 66, 30, 38, 44, 50, 40, 60, 42, 51 are Given.

Since, Median is the middle number of all the observation,

So, To Find the Median, Arrange the numbers in Ascending order, we get

30, 34, 38, 40, 42, 44, 50,51, 60, 66

Here the Number of observations are Even then the middle terms are 42 and 44

Therefore, The Median = $\frac{42+44}{2}$ = 43

Deviation |d| = |x-Median|

And, The number of observations is 10.

Now, The Mean Deviation is

X_i	$\lvert d_i \rvert = \lvert x_i - 43 \rvert$
30	13
34	9
38	5
40	3
42	1
44	1
50	7
51	8
60	17
66	23
Total $\sum di$	87

Mean Deviation = $\frac{87}{10} = 8.7$

Hence, The Mean Deviation is 8.7

1 D. Question

Calculate the mean deviation about the median of the following observation :

22, 24, 30, 27, 29, 31, 25, 28, 41, 42

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 22, 24, 30, 27, 29, 31, 25, 28, 41, 42are Given.

Since, Median is the middle number of all the observation,

So, To Find the Median, Arrange the numbers in Ascending order, we get

22, 24, 25, 27, 28, 29, 30, 31, 41, 42

Here the Number of observations are Even then the middle terms are 28 and 29

Therefore, The Median = $\frac{28+29}{2}$ = 28.5

Deviation |d| = |x-Median|

And, The number of observations is 10.

Now, The Mean Deviation is

X_i	$\lvert d_i \rvert = \lvert x_i - 28.5 \rvert$
22	6.5
24	4.5
30	1.5
27	1.5
29	0.5
31	2.5
25	3.5
28	0.5
41	12.5
42	13.5
Total $\sum di$	47

Mean Deviation = $\frac{47}{10} = 4.7$

Hence, The Mean Deviation is 8.7

1 E. Question

Calculate the mean deviation about the median of the following observation :

38, 70, 48, 34, 63, 42, 55, 44, 53, 47

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 38, 70, 48, 34, 63, 42, 55, 44, 53, 47 are Given.

Since, Median is the middle number of all the observation,

So, To Find the Median, Arrange the numbers in Ascending order, we get

34, 38, 43, 44, 47, 48, 53, 55, 63, 70

Here the Number of observations are Even then the middle terms are 47 and 48.

Therefore, The Median = $\frac{47+48}{2}$ = 47.5

Deviation |d| = |x-Median|

And, The number of observations are 10.

Now, The Mean Deviation is

X_i	$|d_i|=|x_i-47.5|$
38	9.5
70	22.5
48	0.5
34	13.5
63	15.5
42	5.5
55	7.5
44	3.5
53	5.5
47	0.5
Total $\sum di$	84

Mean Deviation = $\frac{84}{10} = 8.4$

Hence, The Mean Deviation is 8.4

2 A. Question

Calculate the mean deviation from the mean for the following data :

4, 7, 8, 9, 10, 12, 13, 17

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from mean.

Formula Used:

Explanation: Here, Observations 4, 7, 8, 9, 10, 12, 13, 17 are Given.

Deviation |d| = |x-Mean|

Mean = $\sum \frac{|x_1|}{n}$

Mean of the Given Observations = $\frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8}$

And, The number of observations is 8.

Now, The Mean Deviation is

X_i	$|d_i| = |x_i - 10|$
4	6
7	3
8	2
9	1
10	0
12	2
13	3
17	7
Total $\sum xi$ =80	24

Mean Deviation = $\frac{\sum d_i}{n}$

Mean Deviation of the given Observations = $\frac{24}{8} = 3$

Hence, The Mean Deviation is 3

2 B. Question

Calculate the mean deviation from the mean for the following data :

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from Mean.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17 are Given.

Deviation |d| = |x-Mean|

Mean = $\sum \frac{|x_1|}{n}$

Mean of the Given Observations = $\frac{13+17+16+14+11+13+10+16+11+18+12+17}{12} = \frac{168}{12}$

And, The number of observations is 12.

Now, The Mean Deviation is

X_i	$\lvert d_i\rvert = \lvert x_i - 14\rvert$
13	1
17	3
16	2
14	0
11	3
13	1
10	4
16	2
11	3
18	4
12	2
17	3
Total x_i = 168	28

Mean Deviation = $\frac{\sum d_i}{n}$

Mean Deviation of the given Observations = $\frac{28}{12} = 2.33$

Hence, The Mean Deviation is 2.33

2 C. Question

Calculate the mean deviation from the mean for the following data :

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 are Given.

Deviation |d| = |x-Mean|

Mean = $\sum \frac{|x_i|}{n}$

Mean of the Given Observations = $\frac{38+70+48+40+42+55+63+46+54+44}{10} = \frac{500}{10}$

And, The number of observations is 10.

Now, The Mean Deviation is

X_i	$\lvert d_i\rvert = \lvert x_i - 50\rvert$
38	12
70	20
48	2
40	10
42	8
55	5
63	13
46	4
54	4
44	6
Total $\sum x_i = 500$	84

Mean Deviation = $\frac{\sum d_i}{n}$

Mean Deviation of the given Observations = $\frac{84}{10} = 8.4$

Hence, The Mean Deviation is 8.4

2 D. Question

Calculate the mean deviation from the mean for the following data :

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 36, 72, 46, 42, 60, 45, 53, 46, 51, 49 are Given.

Deviation |d| = |x-Mean|

Mean = $\sum \frac{|x_i|}{n}$

Mean of the Given Observations = $\frac{36+72+46+42+60+45+53+46+51+49}{10} = \frac{500}{10}$

And, The number of observations is 10

Now, The Mean Deviation is

X_i	$\lvert d_i \rvert = \lvert x_i - 50 \rvert$
38	12
70	20
48	2
40	10
42	8
55	5
63	13
46	4
54	4
44	6
Total $\sum x_i = 500$	84

Mean Deviation = $\frac{\sum d_i}{n}$

Mean Deviation of the given Observations = $\frac{84}{10} = 8.4$

Hence, The Mean Deviation is 7.4

2 E. Question

Calculate the mean deviation from the mean for the following data :

57, 64, 43, 67, 49, 59, 44, 47, 61, 59

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 57, 64, 43, 67, 49, 59, 44, 47, 61, 59 are Given.

Deviation |d| = |x-Mean|

Mean = $\sum \frac{|x_i|}{n}$

Mean of the Given Observations = $\frac{57+64+43+67+49+59+44+47+61+59}{10} = \frac{550}{10}$

And, The number of observations is 10

Now, The Mean Deviation is

x_i	$\lvert d_i \rvert = \lvert x_i - 55 \rvert$
43	12
44	11
47	8
49	6
57	2
59	4
59	4
61	6
64	9
67	12
Total $\sum x_i = 500$	74

Mean Deviation = $\frac{\sum d_i}{n}$

Mean Deviation of the given Observations = $\frac{74}{10} = 7.4$

Hence, The Mean Deviation is 7.4

3. Question

Calculate the mean deviation of the following income groups of five and seven members from their medians:

I Income in ₹	II Income in ₹
4000	3800
4200	4000
4400	4200
4600	4400
4800	4600
	4800
	5800

Answer

Given, Numbers of observations are given in two groups.

To Find: Calculate the Mean Deviation from their Median.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

For Group 1: Since, Median is the middle number of all the observation,

So, To Find the Median, Arrange the Income of Group 1 in Ascending order, we get

4000, 4200, 4400, 4600, 4800

Therefore, The Median = 4400

Deviation |d| = |x-Median|

Now, The Mean Deviation is

X_i	$|d_i| = |x_i - 4400|$
4000	400
4200	200
4400	0
4600	200
4800	400
Total	1200

Mean Deviation = $\frac{\sum d_i}{n}$

Mean Deviation Of Group 1 = $\frac{1200}{5} = 240$

For Group 2: Since, Median is the middle number of all the observation,

So, To Find the Median, Arrange the Income of Group 2 in Ascending order, we get

3800,4000,4200,4400,4600,4800,5800

Therefore, The Median = 4400

Deviation |d| = |x-Median|

Now, The Mean Deviation is

X_i	$|d_i| = |x_i - 4400|$
3800	600
4000	400
4200	200
4400	0
4600	200
4800	400
5800	1400
Total	3200

Mean Deviation = $\frac{\sum d_i}{n}$

Mean Deviation Of Group 2 = $\frac{3200}{7} = 457.14$

Hence, The Mean Deviation of Group 1 is 240 and Group 2 is 457.14

4. Question

The lengths (in cm) of 10 rods in a shop are given below :

40.0, 52.3, 55.2, 72.9, 52.8, 79.0, 32.5, 15.2, 27.9, 30.2

(i) Find the mean deviation from the median

(ii) Find the mean deviation from the mean also.

Answer

(i) **Given,** Numbers of observations are given .

To Find: Calculate the Mean Deviation from their Median.

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Since, Median is the middle number of all the observation,

So, To Find the Median, Arrange the given length of shops in Ascending order, we get

15.2, 27.9, 30.2, 32.5, 40.0, 52.3, 52.8, 55.2, 72.9, 79.0

Here the Number of observations are Even then Median $= \frac{40.0+52.3}{2}$

Therefore, The Median = 46.15

Deviation |d| = |x-Median|

Now, The Mean Deviation is

X_i	$\lvert d_i \rvert = \lvert x_i - 46.15 \rvert$
40.0	6.15
52.3	6.15
55.2	9.05
72.9	26.75
52.8	6.65
79.0	32.85
32.5	13.65
15.2	30.95
27.9	19.25
30.2	15.95
Total	$\sum d_i$167.4

Mean Deviation $= \frac{\sum d_i}{n}$

Mean Deviation From Median $= \frac{167.4}{10} = 16.74$

Hence, The Mean Deviation is 16.74

(ii) Here, Observations 15.2, 27.9, 30.2, 32.5, 40.0, 52.3, 52.8, 55.2, 72.9, 79.0

are Given.

Deviation |d| = |x-Mean|

Mean $= \sum \frac{|x_1|}{n}$

Mean of the Given Observations $= \frac{15.2+27.9+30.2+32.5+40.0+52.3+52.8+55.2+72.9+79.0}{10} = \frac{458}{10}$

And, The number of observations is 10

Now, The Mean Deviation is

X_i	$\lvert d_i \rvert = \lvert x_i - 45.8 \rvert$
40.0	5.8
52.3	6.5
55.2	9.4
72.9	27.1
52.8	7
79.0	33.2
32.5	13.3
15.2	30.6
27.9	17.9
30.2	15.6
Total	166.4

Mean Deviation $= \frac{\sum d_i}{n}$

Mean Deviation of the given Observations $= \frac{166.4}{10} = 16.64$

Hence, The Mean Deviation is 16.64

5. Question

In question 1(iii), (iv), (v) find the number of observations lying between $\overline{X} - M.D.$ and $\overline{X} + M.D.$, where

M.D. is the mean deviation from the mean.

Answer

(iii) 34, 66, 30, 38, 44, 50, 40, 60, 42, 51

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from Mean

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Here, Observations 34, 66, 30, 38, 44, 50, 40, 60, 42, 51 are Given.

Mean = $\frac{\text{Sum of all observation}}{\text{Total No.of observation}}$

Mean for given data $\overline{X} = \frac{34+66+30+38+44+50+40+60+42+51}{10} = \frac{455}{10}$

Deviation |d| = |x-Mean|

And, The number of observations is 10.

Now, The Mean Deviation is

X_i	$\lvert d_i \rvert = \lvert x_i - 45.5 \rvert$
34	11.5
66	20.5
30	15.5
38	7.5
44	1.5
50	4.5
40	5.5
60	14.5
42	3.5
51	5.5
Total	$\sum di = 90$

Mean Deviation M.D= $\frac{90}{10} = 9$

Now,

$\overline{X}$- M.D = 45.5-9=36.5

$\overline{X}$+ M.D = 45.5+9=54.5

So, There are total 6 observation between $\overline{X}$- M.D and $\overline{X}$+ M.D

(iv) Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from Mean

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Numbers of observations are 22,24,30,27,29,31,25,28,41,42.

Mean = $\frac{\text{Sum of all observation}}{\text{Total No.of observation}}$

Mean for given data $\overline{X} = \frac{(22+24+30+27+29+31+25+28+41+42)}{10} = \frac{299}{10}$

Deviation |d| = |x-Mean|

And, The number of observations is 10.

Now, The Mean Deviation is

X_i	$\|d_i\|=\|x_i-29.9\|$
22	7.9
24	5.9
30	0.1
27	2.9
29	0.9
31	1.1
25	4.9
28	1.9
41	11.1
42	12.1
Total	$\sum di$ =48.8

Mean Deviation M.D= $\frac{48.8}{10} = 4.88$

Now,

$\overline{X}$- M.D = 29.9-4.88= 25.02

$\overline{X}$+ M.D = 29.9+4.88=34.78

So, There are total 6 observation between $\overline{X}$- M.D and $\overline{X}$+ M.D

(V) Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from Mean

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation: Numbers of observations are 38,70,48,34,63,42,55,44,53,47.

Mean = $\frac{\text{Sum of all observation}}{\text{Total No.of observation}}$

Mean for given data $\overline{X} = \frac{(38+70+48+34+63+42+55+44+53+47)}{10} = \frac{494}{10}$

Deviation |d| = |x-Mean|

And, The number of observations is 10.

Now, The Mean Deviation is

X_i	$\|d_i\|=\|x_i-49.4\|$
38	11.4
70	20.6
48	1.4
34	15.4
63	13.6
42	7.4
55	5.6
44	5.4
53	3.6
47	2.4
Total	$\sum di$ =86.8

Mean Deviation M.D= $\frac{86.8}{10} = 8.68$

Now,

$\overline{X}$- M.D = 49.4-8.68=40.72

$\overline{X}$+ M.D = 49.4+8.68=58.08

So, There are total 6 observation between $\overline{X}$- M.D and $\overline{X}$+ M.D

Exercise 32.2

1. Question

Calculate the mean deviation from the median of the following frequency distribution :

Heights in inches	58	59	60	61	62	63	64	65	66
No. of students	15	20	32	35	35	22	10	8	

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum d_i}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

We know, Median is the Middle term,

Therefore, Median = 61

Let x_i =Heights in inches

And, f_i = Number of students

X_i	f_i	Cumulative Frequency	$\|d_i\|=\|x_i-61\|$	$F_i\|d_i\|$
58	15	15	3	45
59	20	35	2	40
60	32	67	1	32
61	35	102	0	0
62	35	137	1	35
63	22	159	2	44
64	20	179	3	60
65	10	189	4	40
66	8	197	5	40
	N=197			Total=336

N=197

Mean deviation=$\frac{336}{197} = 1.70$

Hence, The mean deviation is 1.70.

2. Question

The number of telephone calls received at an exchange in 245 successive on2-minute intervals is shown in the following frequency distribution :

Number of calls	0	1	2	3	4	5	6	7
Frequency	14	21	25	43	51	40	39	12

Compute the mean deviation about the median.

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

We know, Median in the even terms $\frac{3+5}{2}$,

Therefore, Median = 4

Let x_i =Number of calls

And, f_i = Frequency

X_i	f_i	Cumulative Frequency	$\lvert d_i\rvert = \lvert x_i - 4\rvert$	$F_i\lvert d_i\rvert$
0	14	14	4	56
1	21	35	3	63
2	25	60	2	50
3	43	103	1	43
4	51	154	0	0
5	40	194	1	40
6	39	233	2	78
7	12	245	3	36
				Total=366
	Total=245			

N = 245

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data =$\frac{336}{245} = 1.49$

Hence, The mean deviation is 1.49

3. Question

Calculate the mean deviation about the median of the following frequency distribution :

x_i	5	7	9	11	13	15	17
f_i	2	4	6	8	10	12	8

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

Here, N =50

Then, N/2 = $\frac{50}{2} = 25$

SO, The median Corresponding to 25 is 13

X_i	f_i	Cumulative Frequency	$\|d_i\|=\|x_i-13\|$	$F_i\|d_i\|$
5	2	2	8	16
7	4	6	6	24
9	6	12	4	24
11	8	20	2	16
13	10	30	0	0
15	12	42	2	24
17	8	50	4	32
	Total=50			Total= 136

N = 50

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $=\frac{136}{50} = 2.72$

Hence, The mean Deviation is 2.72.

4 A. Question

Find the mean deviation from the mean for the following data :

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from the mean.

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from Mean So,

Mean = $\frac{\sum f_i x_i}{f_i}$

X_i	f_i	Cumulative Frequency ($x_i f_i$)	$\|d_i\|=\|x_i$-Mean$\|$	$F_i\|d_i\|$
5	8	40	4	32
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	6	90	6	36
	Total =26	Total=234		Total = 88

Mean = $\frac{234}{26}$=9

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $=\frac{88}{26} = 3.3$

Hence, The mean Deviation is 3.3.

4 B. Question

Find the mean deviation from the mean for the following data :

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from the mean.

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from Mean So,

Mean = $\frac{\sum f_i x_i}{f_i}$

X_i	f_i	Cumulative Frequency ($x_i f_i$)	$\lvert d_i \rvert = \lvert x_i$- Mean$\rvert$	$F_i \lvert d_i \rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	Total=25	Total=350		Total=158

Mean = $\frac{350}{25}$=14

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data =$\frac{158}{25} = 6.32$

Hence, The mean Deviation is 6.32.

4 C. Question

Find the mean deviation from the mean for the following data :

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from the mean.

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from Mean So,

Mean = $\frac{\sum f_i x_i}{f_i}$

X_i	f_i	Cumulative Frequency ($x_i f_i$)	$\lvert d_i \rvert = \lvert x_i\text{-Mean} \rvert$	$F_i \lvert d_i \rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	Total=25	Total=350		Total=158

Mean = $\frac{350}{25}$=14

Mean Deviation = $\frac{\sum f|d_i|}{N}$

Mean deviation for given data =$\frac{158}{25} = 6.32$

Hence, The mean Deviation is 6.32.

4 D. Question

Find the mean deviation from the mean for the following data :

Size:	20	21	22	23	24
Frequency:	6	4	5	1	4

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from the mean.

Formula Used: Mean Deviation = $\frac{\sum f|d_i|}{n}$

Explanation.

Here we have to calculate the mean deviation from Mean So,

Mean = $\frac{\sum f_i x_i}{f_i}$

Mean of given Data = $\frac{4000}{80}$=50

X_i	f_i	Cumulative Frequency ($x_i f_i$)	$\lvert d_i \rvert = \lvert x_i\text{-Mean} \rvert$	$F_i \lvert d_i \rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	Total=80	Total=4000		Total=1280

$$\text{Mean} = \frac{4000}{80} = 50$$

Mean Deviation = $\frac{\sum f|d_i|}{N}$

Mean deviation for given data =$\frac{1280}{80} = 16$

Hence, The mean Deviation is 16

4 E. Question

Find the mean deviation from the mean for the following data :

Size:	1	3	5	7	9	11	13	15
Frequency:	3	3	4	14	7	4	3	4

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation from the mean.

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from Mean So,

Mean = $\frac{\sum f_i x_i}{f_i}$

Mean of the given data is $\frac{433}{20}$ =21.65

X_i	f_i	Cumulative Frequency ($x_i f_i$)	$\|d_i\|=\|x_i$-Mean$\|$	$F_i\|d_i\|$
20	6	120	1.65	9.9
21	4	84	0.65	2.6
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
	Total =20	Total = 433		Total=25

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data is $\frac{25}{20} = 1.25$

Hence, The mean Deviation is 1.25.

5 A. Question

Find the mean deviation from the median for the following data :

x_i	15	21	27	30
f_i	3	5	6	7

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

N = 21

$\frac{N}{2} = 10.5$

SO, The median Corresponding to 10.5 is 27

X_i	f_i	Cumulative Frequency	$\lvert d_i \rvert = \lvert x_i\text{-Med} \rvert$	$F_i \lvert d_i \rvert$
15	3	3	15	45
21	5	8	9	45
27	6	14	0	0
30	7	21	3	21
	N=21	Total=46		Total =101

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data =$\frac{101}{21} = 4.80$

Hence, The Mean Deviation is 4.80.

5 B. Question

Find the mean deviation from the median for the following data :

x_i	74	89	42	54	91	94	35
f_i	20	12	2	4	5	3	4

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

N = 50

$\frac{N}{2} = 25$

SO, The median Corresponding to 12.5 is 74

X_i	f_i	Cumulative Frequency	$\lvert d_i \rvert = \lvert x_i\text{-Med} \rvert$	$F_i \lvert d_i \rvert$
35	4	4	39	156
42	2	6	32	64
54	4	10	20	80
74	20	30	0	0
89	12	42	15	180
91	5	47	17	85
94	3	50	20	60
	Total=50			Total = 625

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data =$\frac{625}{50} = 12.5$

Hence, The Mean Deviation is 12.5.

5 C. Question

Find the mean deviation from the median for the following data :

Mark obtained	10	11	12	14	15
No. of students	2	3	8	3	4

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

N = 20

$\frac{N}{2} = 10$

SO, The median Corresponding to 10 is 12

X_i	f_i	Cumulative Frequency	$\|d_i\|=\|x_i\text{-Med}\|$	$F_i\|d_i\|$
10	2	2	2	4
11	3	5	1	3
12	8	13	0	0
14	3	16	2	6
15	4	20	3	12
	Total=20			Total =25

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $\frac{25}{20} = 1.25$

Hence, The Mean Deviation is 1.25.

Exercise 32.3

1. Question

Compute the mean deviation from the median of the following distribution :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	10	20	5	10

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

Median is the middle term of the X_i,

Here, The middle term is 25

Therefore, Median = 25

Class Interval	X_i	F_i	Cumulative Frequency	d_i=(x-median)	F_id_i
0-10	5	5	5	20	100
10-20	15	10	15	10	100
20-30	25	20	35	0	0
30-40	35	5	91	10	50
40-50	45	10	101	20	200
		Total=50			Total=450

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $\frac{450}{50} = 9$

Hence, The Mean Deviation is 9

2 A. Question

Find the mean deviation from the mean for the following data :

Classes	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Frequencies	4	8	9	10	7	5	4	3

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the mean. So,

Mean = $\sum \frac{f_i x_i}{n}$

Here, Mean = $\frac{17900}{50}$

Therefore, Mean = 358

Class Interval	X_i	F_i	F_iX_i	d_i=(x-mean)	F_id_i
0-100	50	4	200	308	1232
100-200	150	8	1200	208	1664
200-300	250	9	2250	108	972
300-400	350	10	3500	8	80
400-500	450	7	3150	92	644
500-600	550	5	2750	192	960
600-700	650	4	2600	292	1168
700-800	750	3	2250	392	1176
		Total=50	Total=17900		Total=7896

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $\frac{7896}{50} = 157.92$

Hence, The Mean Deviation is 157.92

2 B. Question

Find the mean deviation from the mean for the following data :

Classes	95-105	105-115	115-125	125-135	135-145	145-155
Frequencies	9	13	16	26	30	12

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the mean. So,

Mean = $\sum \frac{f_i x_i}{n}$

Here, Mean = $\frac{13630}{106} = 128.6$

Therefore, Mean = 49

Class Interval	X_i	F_i	F_iX_i	d_i=(x-mean)	Fi |di|
95 – 105	100	9	900	-28.6	257.4
105 – 115	110	13	1430	-18.6	241.8
115 – 125	120	16	1920	-8.6	137.6
125 – 135	130	26	3380	1.4	36.4
135 – 145	140	30	4200	12.4	372
145 - 155	150	12	1800	22.4	268.8
		N=106	Total = 13630		Total=1314

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data = $\frac{1314}{106} = 12.39$

2 C. Question

Find the mean deviation from the mean for the following data :

Classes	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	8	14	16	4	2

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the mean. So,

Mean = $\sum \frac{f_i x_i}{n}$

Here, Mean = $\frac{1350}{50}$

Therefore, Mean = 27

Class Interval	X_i	F_i	F_iX_i	d_i=(x-mean)	F_id_i
0-10	5	6	30	22	132
10-20	15	8	120	12	96
20-30	25	14	350	2	28
30-40	35	16	560	8	128
40-50	45	4	180	18	72
50-60	55	2	110	28	56
		Total=50	Total=1350		Total=512

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $\frac{512}{50} = 10.24$

Hence, The Mean Deviation is 10.24

3. Question

Compute mean deviation from mean of the following distribution:

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students	8	10	15	25	20	18	9	5

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the mean. So,

Mean = $\sum \frac{f_i x_i}{n}$

Here, Mean = $\frac{5390}{110}$

Therefore, Mean = 49

Class Interval	X_i	F_i	F_iX_i	d_i=(x-mean)	F_id_i
10-20	15	8	120	34	272
20-30	25	10	250	24	240
30-40	35	15	525	14	210
40-50	45	25	1125	4	100
50-60	55	20	1100	6	120
60-70	65	18	1170	16	288
70-80	75	9	675	26	234
80-90	85	5	425	36	180
		N=110	Total=5390		Total=1644

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $\frac{1644}{110} = 14.95$

Hence, The Mean Deviation is 14.95

4. Question

The age distribution of 100 life-insurance policy holders is as follows :

Age (on nearest birth day)	17-19.5	20-25.5	26-35.5	36-40.5	41-50.5	51-55.5	56-60.5	61-70.5
No. of persons	5	16	12	26	14	12	6	5

Calculate the mean deviation from the median age.

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

N = 96

$\frac{N}{2} = 48$

SO, The cumulative frequency just greater than 48 is 59, and the corresponding value of x is 38.25

Median = 38.25

Class Interval	X_i	F_i	Cumulative Frequency	d_i=(x-median)	F_id_i
17-19.5	18.25	5	5	20	100
20-25.5	22.75	16	21	15.5	248
26-35.5	30.75	12	33	7.5	90
36-40.5	38.25	26	59	0	0
41-50.5	45.75	14	73	7.5	105
51-55.5	53.25	12	85	15	180
56-60.5	58.25	6	91	20	120
61-70.5	65.75	5	96	27.5	137.5
		Total=96			Total=980.5

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $\frac{980.5}{96} = 10.21$

Hence, The Mean Deviation is 10.21.

5. Question

Find the mean deviation from the mean and from a median of the following distribution :

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	5	8	15	16	6

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

N = 50

$$\frac{N}{2} = 25$$

SO, The cumulative frequency just greater than 25 is 58, and the corresponding value of x is 28

Median = 28

Now, Mean = $\sum \frac{f_i x_i}{N}$

$$\frac{1350}{50} = 27$$

Mean = 27

Class Interval	X_i	F_i	Cumulative Frequency	d_i=(x-median)	$F_i d_i$	$F_i X_i$	|x_i-mean|	F_i|x_i-mean|
0-10	5	5	5	23	115	25	22	110
10-20	15	8	13	13	104	120	12	96
20-30	25	15	28	3	45	375	2	30
30-40	35	16	44	7	112	560	8	128
40-50	45	6	50	17	102	270	18	108
		N=50			Total=478	Total=1350		Total=472

Mean deviation from Median $\frac{478}{50} = 9.56$

And, Mean deviation from Median $\frac{472}{50} = 9.44$

Hence, The Mean Deviation from the median is 9.56 and from mean is 9.44.

6. Question

Calculate mean deviation about median age for the age distribution of 100 persons given below :

Age :	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Number of persons	5	6	12	14	26	12	16	9

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation. Here, The Given class interval does not continue, So, we subtract 0.5 from a lower limit of the class and add 0.5 to the upper limit of the class,By this Class interval remain same while the function becomes continues.

Class Interval	X_i	F_i	Cumulative Frequency	d_i=(x-median)	$F_i d_i$
15.5-20.5	18	5	5	20	100
20.5-25.5	23	6	11	15	90
25.5-30.5	28	12	23	10	120
30.5-35.5	33	14	37	5	70
35.5-40.5	38	26	63	0	0
40.5-45.5	43	12	75	5	60
45.5-50.5	48	16	91	10	160
50.5-55.5	53	9	100	15	135
		N = 100			$\sum fd = 735$

Here we have to calculate the mean deviation from the median. So,

N = 100

$$\frac{N}{2} = 50$$

Thus, the cumulative frequency slightly greater than 50 is 63 and lie under class 35.5-40.5.

Median = $l + \frac{\frac{N}{2} - F}{f} \times h$

$35.5 + \frac{(50 - 37)}{26} \times 5$

Median = 38

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $\frac{735}{100} = 73.5$

Hence, The Mean Deviation is 73.5

7. Question

Calculate the mean deviation about mean for the following frequency distribution :

Class interval :	0-4	4-8	8-12	12-16	16-20
Frequency :	4	6	8	5	2

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i|}}{n}$

Explanation.

Here we have to calculate the mean deviation from the mean. So,

Mean = $\sum \frac{f_i x_i}{n}$

Here, Mean = $\frac{230}{25}$

Therefore, Mean = 9.2

Class Interval	X_i	F_i	F_iX_i	d_i=(x-mean)	F_id_i
0-4	2	4	8	7.2	28.8
4-8	6	6	36	3.2	19.2
8-12	10	8	80	0.8	6.4
12-16	14	5	70	4.8	24
16-20	18	2	36	8.8	17.6
		Total=25	Total=230		Total=96

Mean Deviation = $\frac{\sum f|d_{i|}}{N}$

Mean deviation for given data $\frac{96}{25} = 3.84$

Hence, The Mean Deviation is 3.84

8. Question

Calculate mean deviation from the median of the following data :

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency :	4	5	3	6	2

Answer

Given, Numbers of observations are given.

To Find: Calculate the Mean Deviation

Formula Used: Mean Deviation = $\frac{\sum f|d_{i}|}{n}$

Explanation.

Here we have to calculate the mean deviation from the median. So,

N = 20

$\frac{N}{2} = 10$

So, the cumulative frequency just greater than 10 is 12, and the corresponding value of x is 15

Median = 15

Now, Mean = $\sum \frac{f_i x_i}{N}$

$\frac{282}{20} = 14.1$

Mean = 14.1

Class Interval	X_i	F_i	Cumulative Frequency	d_i=(x-median)	$F_i d_i$	$F_i X_i$	$\|x_i$-mean$\|$	$F_i\|x_i$-mean$\|$
0 - 6	3	4	4	12	48	12	11.1	44.4
6 – 12	9	5	9	6	30	45	5.1	25.5
12 – 18	15	3	12	0	0	45	0.9	2.7
18 – 24	21	6	18	6	36	126	6.9	41.4
24 – 30	27	2	20	12	24	54	12.9	25.8
		N=20			Total=138	Total=282		Total=139.8

Mean deviation from Median $= \frac{138}{20} = 6.9$

And, Mean deviation from Median $\frac{139.8}{20} = 96.99$

Hence, The Mean Deviation from the median is 6.9 and from mean is 6.99.

Exercise 32.4

1 A. Question

Find the mean, variance and standard deviation for the following data :

2, 4, 5, 6, 8, 17

Answer

Explanation: Here, Mean $\overline{X} = \frac{2+4+5+6+8+17}{6}$

$\Rightarrow \overline{X} = \frac{42}{6} = 7$

X_i	$(x_i$-X) =$(x_i$-7)	$(x_i-7)^2$
2	-3	25
4	-3	9
5	-2	4
6	-1	1
8	1	1
17	10	100
		$\sum_1^6 (x_i - \bar{X})^2 = 140$

And, n=6

Variance (X) $= \frac{1}{n}\sum_{i=1}^{6}(x_i - \overline{X})$

$\frac{140}{6} = 23.33$

Variance = 23.33

Standard deviation = $\sqrt{Var(X)}$

$\sigma = \sqrt{23.33}$

Standard deviation = 4.83

1 B. Question

Find the mean, variance and standard deviation for the following data :

6, 7, 10, 12, 13, 4, 8, 12

Answer

Explanation: Here, Mean $\overline{X} = \frac{6+7+10+12+13+4+8+12}{8}$

$\Rightarrow \overline{X} = \frac{72}{8} = 9$

X_i	$(x_i\text{-}X)=(x_i\text{-}7)$	$(x_i\text{-}7)^2$
6	-3	9
7	-2	4
10	1	1
12	3	9
13	4	16
4	-5	25
12	3	9
		$\sum_1^8 (x_i - \bar{X})^2 = 74$

And, n=8

Variance (X) $= \frac{1}{n}\sum_{i=1}^{8}(x_i - \overline{X})$

$\frac{74}{8}$

Variance = 9.25

Standard deviation = $\sqrt{Var(X)}$

$\sigma = \sqrt{9.25}$

Standard deviation = 3.04

1 C. Question

Find the mean, variance and standard deviation for the following data :

227, 235, 255, 269, 292, 312, 321, 333, 348

Answer

Explanation: Here, Mean $\overline{X} = \frac{227+235+255+269+292+312+321+333+348}{10}$

$\Rightarrow \overline{X} = \frac{2891}{10} = 289.1$

X_i	$(x_i-X)=(x_i-7)$	$(x_i-7)^2$
227	-62.1	3856.41
235	-54.1	2926.81
255	-34.1	1162.81
269	-20.1	404.01
292	2.9	8.41
299	9.9	98.01
312	22.9	524.41
321	31.9	1017
333	43.9	1927.21
348	58.9	3469.21
		$\sum_1^{10}(x_i-\bar{X})^2 = 15394.9$

And, n=10

Variance (X) $= \frac{1}{n}\sum_{i=1}^{10}(x_i - \overline{X})$

$$\frac{15394.9}{10}$$

Variance = 15394.9

Standard deviation = $\sqrt{\text{Var}(X)}$

$$\sigma = \sqrt{1539.49}$$

Standard deviation = 39.24

1 D. Question

Find the mean, variance and standard deviation for the following data :

15, 22, 27, 11, 9, 21, 14, 9

Answer

Explanation: Here, Mean $\overline{X} = \frac{15+22+27+11+9+21+14+9}{8}$

$$\Rightarrow \overline{X} = \frac{128}{8} = 16$$

X_i	$(x_i-X)=(x_i-7)$	$(x_i-7)^2$
15	-1	1
22	6	36
27	11	121
11	5	25
9	-7	49
21	5	25
14	-2	4
9	-7	49
		$\sum_1^{8}(x_i-\bar{X})^2 = 310$

And, n=8

Variance (X) $= \frac{1}{n}\sum_{i=1}^{8}(x_i - \overline{X})$

$$\frac{310}{8}$$

Variance = 38.75

Standard deviation = $\sqrt{\mathrm{Var(X)}}$

$\sigma = \sqrt{38.75}$

Standard deviation = 6.22

2. Question

The variance of 20 observations is 4. If each observation is multiplied by 2, find the variance of the resulting observations.

Answer

Given, The variance of 20 observations is 4.

To Find: Find the variance of resulting observations.

Explanation: Let Assume, $x_1, x_2, x_3, \ldots, x_{20}$ be the given observations.

So, Variance (X) = 5 (Given)

X = $\frac{1}{n} \times \sum (x_i - \overline{X})^2$

Now, Let $u_1, u_2, \ldots u_{20}$ be the new observation,

When we multiply the new observation by 2, then

$U_i = 2x_i$ (for i=1,2,3…,20) ---(i)

Now,

Mean = $\overline{U} = \frac{\sum_1^{20} U_i}{n}$

$\frac{\sum_{i=1}^{20} 2x_i}{20}$

Mean = $2\overline{X}$

Since, $u_i - \overline{U} = 2x_i - 2\overline{X}$

$2(x_i - \overline{X})$

Now, $(u_i - \overline{U})^2 = (2(x_i - \overline{X}))^2$

$4(x_i - \overline{X})^2$

Comparing Both the observations,

$$\frac{\sum_{20}^{i=1} (u_i - \overline{U})^2}{20} = \frac{\sum_{20}^{i=1} 4(x_i - \overline{X})^2}{20}$$

$$4 \times \frac{\sum_{20}^{i=1} (x_i - \overline{X})^2}{20}$$

Variance (U) =4×Variance (X)

4×5

20

Hence, The variance of new observations is 20.

3. Question

The variance of 15 observations is 4. If each observation is increased by 9, find the variance of the resulting observations.

Answer

Given, The variance of 20 observations is 4.

To Find: Find the variance of resulting observations.

Explanation: Let Assume, $x_1,x_2,x_3,\ldots,x_{15}$ be the given observations.

So, Variance (X) = 4 (Given)

$X = \frac{1}{n} \times \sum(x_i - \overline{X})^2$

Now, Let $u_1,u_2,\ldots u_{20}$ be the new observation,

When new observation increase by 9, then

$U_i=x_i+9$(for i=1,2,3…,20) ---(i)

Now,

$$\overline{U} = \frac{1}{n}\sum_{i=1}^{15} u_i$$

$$\frac{1}{15}\sum_{i=1}^{15}(x_i + 9)$$

$$\frac{1}{15}\sum_{i=1}^{15}x_i + \frac{9 \times 15}{15}$$

$$\overline{U} = 9 + \overline{X}$$

$$u_i - \overline{U} = (x_i + 9) - (9 + \overline{X})$$

$$u_i - \overline{U} = x_i - \overline{X}$$

$$\frac{\sum_{i=1}^{15}(u_i - \overline{U})^2}{15} = \frac{\sum_{i=1}^{15}4(x_i - \overline{X})^2}{15}$$

$$\frac{\sum_{i=1}^{15}(u_i - \overline{U})^2}{15} = 4$$

Variance (U) =60

Hence, The variance of new observations is 60.

4. Question

The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6, find the other two observations.

Answer

Given, Mean of 5 observation is 4.4 and variance is 8.24.

To Find: Find the other two observation

Assumption: Let x and y be the other two observation. And Mean is 4.4

Here, Mean = $\frac{1+2+6+x+y}{5} = 4.4$

9+x+y=22

X+y=13 ……(1)

Now, Let Variance (X) be the variance of this observation which is to be 8.24

If $\overline{X}$ is the mean than we get,

$8.24 = \frac{1}{5}(1^2 + 2^2 + 6^2 + x^2 + y^2) - (\overline{x})^2$

$8.24 = \frac{1}{5}(1^2 + 2^2 + 6^2 + x^2 + y^2) - (4.4)^2$

$8.24 = \frac{1}{5}(41 + x^2 + y^2) - 19.36$

$X^2+y^2 = 97$ ......(2)

$(x+y)^2+(x-y)^2=2(x^2+y^2)$

By Subsititute the value we get,

$13^2+(x-y)^2=2\times 97$

$(x-y)^2=194-169$

$(x-y)^2=25$

$x-y = \pm 5$ ......(3)

On solving equations (1) and (3) we get,

2x = 18

X = 9

And, y =4

Hence, The other two observations are 9 and 4.

5. Question

The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Answer

Given, The mean is 8 and SD is 4 for 6 observation

To Find: Find a new mean and new standard Deviation.

Formula Used: Standard Deviation ($\sigma^2 = \text{Variance}$)

Explanation:

Let Assume, $x_1, x_2, x_3, \ldots, x_6$ be the given observations.

So, Variance (X) = 8 (Given)

n=6

and $\sigma = 4$ (SD)

$X = \frac{1}{n} \times \sum x_i$

$$8 = \frac{1}{6} \times \sum_{i=1}^{6} x_i$$

Now, Let $u_1, u_2, \ldots u_{20}$ be the new observation,

When we multiply the new observation by 3, then

$U_i = 3x_i$ (for i=1,2,3...,6) ......(i)

Now,

$$\overline{U} = \frac{1}{n}\sum_{i=1}^{15} u_i$$

$$\frac{1}{6}\sum_{i=1}^{6}(3x_i)$$

$$3 \times \frac{1}{6}\sum_{i=1}^{6}(x_i)$$

$$\overline{U} = 3\overline{X}$$

3×8 = 25

U = 24

Therefore, The Mean of new observation is 24

Now,

Standard Deviation $\sigma_x = 4$

$\sigma_x^2 = \text{Variance X}$

Since, Variance (X) =16

Variance (U) = $\frac{1}{6}\sum_{i=1}^{6}(3x_i - 3X)$

$$3^2 \times \frac{1}{6} \times \sum (x_i - X)^2$$

9×16

$\sigma_u^2 = \text{Variance (U)}$

$\sigma_u^2 = 144$

$\sigma = 12$

Hence, The mean of new observation is 24 and Standard deviation of new data is 12.

6. Question

The mean and variance of 8 observations are 9 and 9.25 respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Answer

Given, Mean of 8 observation is 9 and variance is 9.25.

To Find: Find the other two observation

Assumption: Let x and y be the other two observation. And Mean is 9

Here, Mean = $\frac{6+7+10+12+12+13+x+y}{8} = 9$

60+x+y=72

X+y=12 ......(1)

Now, Let Variance (X) be the variance of this observation which is to be 9.25

If $\overline{X}$ is the mean than we get,

9.25 = $\frac{1}{8}(6^2 + 7^2 + 10^2 + 12^2 + 12^2 + 13^2 + x^2 + y^2) - (\overline{x})^2$

$9.25 = \frac{1}{8}(6^2 + 7^2 + 10^2 + 12^2 + 12^2 + 13^2 + x^2 + y^2) - (9)^2$

$642 + x^2 + y^2 = 722$

$X^2 + y^2 = 80$ ---(2)

$(x+y)^2 + (x-y)^2 = 2(x^2 + y^2)$

By Subsititute the value we get,

$12^2 + (x-y)^2 = 2 \times 80$

$(x-y)^2 = 160 - 144$

$(x-y)^2 = 14$

x-y =±4 ……(3)

On solving equations (1) and (3) we get,

X= 8, 4

And y = 4,8

Hence, The other two observations are 8 and 4.

7. Question

For a group of 200 candidates, the mean and the standard deviations of scores were found to be 40 and 15 respectively. Later on, it was discovered that the scores of 43 and 35 were misread as 34 and 53 respectively. Find the correct mean and standard deviation.

Answer

To Find: Find the correct mean and standard deviation.

Explanation: Here, n=200, $\overline{X} = 40, \sigma = 15$

$X = \frac{1}{n} \times \sum x_i$

$$40 = \frac{1}{200} \times \sum_{i=1}^{200} x_i$$

$$\sum x_i = 40 \times 200$$

Since, the score was incorrect,

Now, The sum is incorrect

Corrected $\sum x_i = 8000 - 34 - 53 + 43 + 35$

8000-7

The correct score is 7993

So, The mean of correct score $= \frac{\sum x}{n}$

$$\frac{7993}{200}$$

Mean = 39.95

Now, Standard variance σ = 15

Since, Variance = σ^2

Variance = 255

X = $\frac{1}{n} \times \sum(x_i - \overline{X})^2$

255 = $\frac{1}{200} \times \sum(x_i)^2 - (40)^2$

255 = $\frac{1}{200} \times \sum(x_i)^2 - 1600$

$$\sum(x_i)^2 = 200 \times 1825$$

$$\sum(x_i)^2 = 365000$$

Now, the correct$\sum(x_i)^2 = 365000$-34^2-53^2+43^2+35^2

365000-1156-2809+1849+1225

$$\sum(x_i)^2 = 364109$$

Corrected Variance = $\left(\frac{1}{n} \times \text{corrected } \sum x_i\right) - (\text{Corrected mean})^2$

$$\left(\frac{1}{200} \times 364109\right) - (39.95)^2$$

1820.54-1596.40

Corrected variance =224.14

Now, Corrected Standard Deviation = $\sqrt{\text{Corrected variance}}$

$$\sigma = \sqrt{224.14}$$

Correct Deviation is 14.97

8. Question

The mean and standard deviation of 100 observations were calculated as 40 and 5.1 respectively by a student who took by mistake 50 instead of 40 for one observation. What are the correct mean and standard deviation?

Answer

To Find: Find the correct mean and standard deviation.

Explanation: Here, n=100,$\overline{X} = 40, \sigma = 5.1$

X = $\frac{1}{n} \times \sum x_i$

$$40 = \frac{1}{100} \times \sum_{i=1}^{100} x_i$$

$$\sum x_i = 40 \times 100$$

Now,

Corrected $\sum x_i = 4000 - 50 + 40$

3990

So, The mean of correct score = $\frac{\text{Corrected Sum}}{n}$

$\frac{3990}{100}$

Mean = 39.9

Now, Standard variance $\sigma = 5.1$

Since, Variance = σ^2

Variance = 26.01

X = $\frac{1}{n} \times \sum(x_i - \overline{X})^2$

26.01 = $\frac{1}{100} \times \sum(x_i)^2 - (40)^2$

26.01 = $\frac{1}{100} \times \sum(x_i)^2 - 1600$

$\sum(x_i)^2 = 162601$

Corrected $\sum(x_i)^2 = 162601$-50+40

Corrected $\sum(x_i)^2 = 162591$

Corrected Variance = $\left(\frac{1}{n} \times \text{corrected } \sum x_i\right) - (\text{Corrected mean})^2$

$\left(\frac{1}{100} \times 162591\right) - (39.9)^2$

1625.91-1592.01

Corrected variance =34 (Approx)

Now, Corrected Standard Deviation = $\sqrt{\text{Corrected variance}}$

$\sigma = \sqrt{34}$

Correct Deviation is 5.83

Hence, The correct Mean is 39.9 and Correct SD is 5.83

9. Question

The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases :

(i) If wrong item is omitted

(ii) if it is replaced by 12.

Answer

Given: Mean = 10

And, standard deviation = 2

We know that,

$\frac{\sum x_i}{n} = \text{Mean}$

$\sum x_i = 10 \times 20 = 200$

When wrong item is omitted, $\sum x_i = 200 - 8 = 192$

Corrected mean $= \frac{192}{19} = 10.10$

When it is replaced by 12, $\sum x_i = 192 + 12 = 204$

Corrected mean$= \frac{204}{20} = 10.73$

Now, for standard deviation,

Variance = $(2)^2 = 4$

And we know that,

$$\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = 4$$

$$\frac{\sum x_i^2}{n} - 100 = 4$$

$$\sum x_i^2 = 104 \times 20 = 2080$$

Now, **omitting the wrong observation**, we get,

$$\sum x_i^2 = 2080 - (8)^2 = 2016$$

Corrected Deviation $= \sqrt{\left(\frac{2016}{19}\right) - \left(\frac{192}{19}\right)} = \frac{1}{19}\sqrt{38304 - 36864} = 1.99$

Now, **replacing the observation**

$$\sum x_i^2 = 2016 + (12)^2 = 2160$$

Corrected Deviation $= \sqrt{\frac{2160}{20} - \left(\frac{204}{20}\right)} = \sqrt{108 - 10.2} = 9.88$

10. Question

The mean and standard deviation of a group of 100 observations were found to be 20 and 3 respectively. Later on, it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations were omitted.

Answer

To Find: Find the correct mean and standard deviation.

Explanation: Here, n=100,$\overline{X} = 20, \sigma = 3$

X $= \frac{1}{n} \times \sum x_i$

$$20 = \frac{1}{100} \times \sum_{i=1}^{100} x_i$$

$$\sum x_i = 20 \times 100$$

So, Corrected $\sum x_i = 2000 - 21 - 21 - 18$

Corrected $\sum x_i = 1940$

Now, Standard variance $\sigma = 3$

Since, Variance $= \sigma^2$

Variance = 9

X = $\frac{1}{n} \times \sum(x_i - \overline{X})^2$

9 = $\frac{1}{100} \times \sum(x_i)^2 - (20)^2$

9 = $\frac{1}{100} \times \sum(x_i)^2 - 400$

$\sum(x_i)^2 = 40900$

Corrected $\sum(x_i)^2 = 40900 - 21^2 - 21^2 - 18^2$

Corrected $\sum(x_i)^2 = 39694$

Correct mean = $\frac{1940}{97} = 20$

Corrected Variance = $\left(\frac{1}{n} \times \text{corrected } \sum x_i\right) - (\text{Corrected mean})^2$

$\left(\frac{1}{97} \times 39694\right) - (20)^2$

409.22-400

Corrected variance =9.22

Now, Corrected Standard Deviation = $\sqrt{\text{Corrected variance}}$

$\sigma = \sqrt{9.22}$

Correct Deviation is 3.04

Hence, The correct Mean is 20 and Correct SD is 3.04

11. Question

Show that the two formula for the standard deviation of ungrouped data

$\sigma = \sqrt{\frac{1}{n}\sum(x_i - \overline{X})^2}$ and $\sigma' = \sqrt{\frac{1}{n}\sum x_i^2 - \overline{X}^2}$ are equivalent, where $\overline{X} = \frac{1}{n}\sum x_i$.

Answer

We know, $\sigma = \sqrt{\text{Variance}(X)}$

$$\sum(x_i - \overline{x})^2 = \sum(x_i^2 - 2x_i\overline{X} + \overline{X}^2)$$

$$\sum(x_i - \overline{x})^2 = \sum(x_i^2) - 2\overline{X}\sum x_i + \overline{X}^2\sum 1$$

$$\sum(x_i - \overline{x})^2 = \sum(x_i^2) - 2\overline{X}(\overline{X}) + \overline{X}^2$$

$$\sum(x_i - \overline{x})^2 = \sum(x_i^2) - (\overline{X}^2)$$

On Diving both sides by $\frac{1}{n}$, we get

$$\frac{1}{n}\sum(x_i - \overline{x})^2 = \frac{1}{n}\sum(x_i^2) - (\overline{X}^2)$$

Taking square root both side

$$\sqrt{\frac{1}{n}\sum(x_i-\bar{x})^2} = \sqrt{\frac{1}{n}\sum(x_i^2)-(\bar{X}^2)}$$

Hence, Proved

Exercise 32.5

1. Question

Find the standard deviation for the following distribution :

x:	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f:	1	5	12	22	17	9	4

Answer

Given, The data is given in table/

To Find: Find the standard deviation

The formula used: SD = $\sqrt{Var(X)}$

Explanation: Here, Mean = $\sum\frac{f_i x_i}{f_i}$

Mean = $\frac{4.5+14.5+24+34.5+44.4+54.5+64.5}{7} = 34.4$

X_i	F_i	$d_i=(x_i-\text{mean})$	$u_i=\frac{x_i-mean}{10}$	f_iu_i	U_i^2	$f_iu_i^2$
4.5	1	-30	-3	-3	9	9
14.5	5	-20	-2	-10	4	20
24	12	-10	-1	-12	1	12
34.5	22	0	0	0	0	0
44.5	17	10	1	17	1	17
54.5	9	20	2	18	4	36
64.5	4	30	3	12	9	36
	$\sum f_i = 70$			$\sum u_i f_i = 22$		$\sum u_i^2 f_i = 130$

Now, N=70, $\sum u_i f_i = 22$, $\sum u_i^2 f_i = 130$

$$Var(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$Var(X) = 10^2\left[\frac{1}{70}\times 130 - \left(\frac{1}{70}\times 22\right)^2\right]$$

$$100\left[\frac{130}{70} - \left(\frac{22}{70}\right)^2\right]$$

$$100\left[\frac{13}{7} - \frac{121}{1225}\right]$$

100[1.857-0.0987]

100[1.7583]

$Var(X) = 175.83$

Standard Deviation $\sigma = \sqrt{Var(X)}$

$\sqrt{175.83}$

SD = 13.26

Hence, The standard deviation is 13.26

2. Question

Table below shows the frequency f with which ‘x’ alpha particles were radiated from a diskette

x:	0	1	2	3	4	5	6	7	8	9	10	11	12
f:	51	203	383	525	532	408	273	139	43	27	10	4	2

Calculate the mean and variance.

Answer

Given, The data is given in table

To Find: Find the mean and variance

Explanation: Mean = $\sum \frac{f_i x_i}{x_i}$

Mean = $\frac{10078}{2600} = 3.88$

X_i	F_i	F_iX_i	$(X_i - X)$	$(X_i - X)^2$	$F_i(X_i - X)^2$
0	51	0	-3.88	15.05	767.55
1	203	203	-2.88	8.29	1682.87
2	383	766	-1.88	3.53	1351.99
3	525	1575	-0.88	0.77	404.25
4	532	2128	0.12	0.014	7.448
5	408	2040	1.12	1.25	510
6	273	1638	2.12	4.49	1225.77
7	139	973	3.12	9.73	1352.47
8	42	344	4.12	16.97	729.71
9	27	243	5.12	26.21	707.67
10	10	100	6.12	37.45	374.5
11	4	44	7.12	50.69	202.76
12	2	24	8.12	65.93	131.86
	N=2600	$\sum f_i x_i = 10078$			$\sum f_i(x_i - \bar{X})^2$=9448.848

Now, N=70

$$\text{Variance}(X) = \frac{\sum f_i (x_i - \bar{X})^2}{N}$$

$$\sigma^2 = \frac{9448.848}{2600} = 3.63$$

Hence, The mean is 3.88 and variance is 3.63

3 A. Question

Find the mean, and standard deviation for the following data :

Year render:	10	20	30	40	50	60
No. of persons (cumulative):	15	32	51	78	97	109

Answer

Given, The data is given in table/

To Find: Find the standard deviation

The formula used: SD = $\sqrt{\text{Var}(X)}$

Explanation:

X_i	F_i	f_i	$u_i=\frac{x_i-mean}{10}$	f_iu_i	U_i^2	$f_iu_i^2$
10	15	15	-2.5	-37.5	6.25	93.75
20	32	17	-1.5	-25.5	2.25	38.25
30	51	19	-0.5	-9.5	0.25	4.75
40	78	27	0.5	13.5	0.25	6.75
50	97	19	1.5	28.5	2.25	42.75
60	109	12	2.5	30	6.25	75
		$\sum f_i = 109$		$\sum u_if_i = -0.5$		$\sum u_i^2f_i = 261.2$

Now, N=109, $\sum u_if_i = -0.5, \sum u_i^2f_i = 261.2$

Mean $\overline{X} = A + h\left(\frac{\sum u_if_i}{N}\right)$

$\overline{X} = 35 + 10\left(\frac{-0.5}{109}\right)$

$\overline{X} = 34.96$

$$Var(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_iu_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_if_i\right)^2\right]$$

$$Var(X) = 100\left[\frac{261.25}{109} - \frac{0.25}{11881}\right]$$

100×2.396

Variance = 239.6

Standard Deviation $\sigma = \sqrt{239.6}$

SD = 15.47 years

Hence, The standard deviation is 15.47

3 B. Question

Find the mean, and standard deviation for the following data :

Marks:	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency:	1	6	6	8	8	2	2	3	0	2	1	0	0	0	1

Answer

Given, The data is given in table/

To Find: Find the standard deviation

The formula used: SD = $\sqrt{Var(X)}$

Explanation:

x_i	f_i	f_ix_i	$f_ix_i^2$
2	1	2	4
3	6	18	54
4	6	24	96
5	8	40	200
6	8	48	288
7	2	14	98
8	2	16	128
9	3	27	243
10	0	0	0
11	2	22	242
12	1	12	144
13	0	0	0
14	0	0	0
15	0	0	0
16	1	16	256
	N=40	Total=239	Total=1753

Now, N=40, $\sum x_i f_i = 239$, $\sum x_i^2 f_i = 1753$

Mean $\overline{X} = \left(\frac{\sum x_i f_i}{N}\right)$

$$\overline{X} = \frac{239}{40}$$

$$\overline{X} = 5.975$$

$$\mathrm{Var}(X) = \frac{1753}{40} - (5.97)^2$$

Variance = 8.12

Standard Deviation $\sigma = \sqrt{8.12}$

SD = 2.85 years

Hence, The standard deviation is 2.85

4 A. Question

Find the standard deviation for the following data :

x:	3	8	13	18	23
f:	7	10	15	10	6

Answer

Given, The data is given in table/

To Find: Find the standard deviation

The formula used: SD = $\sqrt{\mathrm{Var}(X)}$

Explanation:

X_i	F_i	F_iX_i	$(x_i - \bar{X})$	$(x_i - \bar{X})^2$	$(x_i - \bar{X})^2 f$
3	7	21	-9.79	95.84	670.88
8	10	80	-4.79	22.94	229.4
13	15	195	0.21	0.04	0.6
18	10	180	5.21	27.14	271.4
23	6	138	10.21	104.24	625.44
	$\sum f_i = 48$	$\sum f_i x_i = 614$			$\sum (x_i - \bar{X})^2 f = 1797.32$

$$\mathrm{Var}(X) = \frac{\sum (x_i - \overline{X})^2 f}{\sum f_i}$$

$\text{Var}(X) = \frac{1797.32}{48}$

Variance = 37.44

Standard Deviation $\sigma = \sqrt{37.44}$

SD = 6.12

Hence, The standard deviation is 6.12

4 B. Question

Find the standard deviation for the following data :

x:	2	3	4	5	6	7
f:	4	9	16	14	11	6

Answer

Given, The data is given in table/

To Find: Find the standard deviation

The formula used: SD = $\sqrt{\text{Var}(X)}$

Explanation:

x_i	f_i	f_ix_i	$f_ix_i^2$
2	4	8	16
3	9	27	81
4	16	64	256
5	14	70	350
6	11	66	396
7	6	42	294
	N=60	Total = 277	Total=1393

Now, N=60, $\sum x_i f_i = 277$, $\sum x_i^2 f_i = 1393$

Mean $\overline{X} = \left(\frac{\sum x_i f_i}{N}\right)$

$\overline{X} = \frac{277}{60}$

$\overline{X} = 4.62$

$\text{Var}(X) = \frac{1393}{60} - (4.62)^2$

Variance = 1.88

Standard Deviation $\sigma = \sqrt{1.88}$

SD = 1.37

Hence, The standard deviation is 1.37

Exercise 32.6

1. Question

Calculate the mean and S.D. for the following data:

Expenditure (in ₹):	0-10	10-20	20-30	30-40	40-50
Frequency:	14	13	27	21	15

Answer

Given, The data is given in table/

To Find: Find the standard deviation

The formula used: SD = $\sqrt{\text{Var}(X)}$

Explanation:

Expenditure	Mid Point(X_i)	F_i	F_iX_i	$(x_i - \bar{X})$	$(x_i - \bar{X})^2$	$(x_i - \bar{X})^2 f$
0-10	5	14	70	-21.1	445.21	6233.94
10-20	15	13	195	-11.1	123.21	1601.1
20-30	25	27	675	-1.1	1.21	34.67
30-40	35	21	735	8.9	79.21	1663.41
40-50	45	15	675	18.9	357.21	53.58
		$\sum f_i = 90$	$\sum f_i x_i = 2350$			$\sum (x_i - \bar{X})^2 f = 1797.32$

Mean $\bar{X} = \sum \frac{f_i x_i}{f_i}$

$$\bar{X} = \frac{2350}{90}$$

Mean = 26.11

$$\text{Var}(X) = \frac{14891.9}{90}$$

Variance = 165.47

Standard Deviation $\sigma = \sqrt{165.47}$

SD = 12.86

Hence, The standard deviation is 12.86

2. Question

Calculate the standard deviation for the following data:

Class:	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequency:	9	17	43	82	81	44	24

Answer

Given, The data is given in table/

To Find: Find the standard deviation

The formula used: SD = $\sqrt{\text{Var}(X)}$

Explanation:

Class	F_i	x_i	$u_i=\frac{x_i-mean}{30}$	f_iu_i	U_i^2	$f_iu_i^2$
0-30	9	15	-3	-27	9	81
30-60	17	45	-2	-34	4	68
60-90	43	75	-1	-43	1	43
90-120	82	105	0	0	0	0
120-150	81	135	1	81	1	81
150-180	44	165	2	88	4	176
180-210	24	195	3	72	9	216
		$\sum f_i = 300$		$\sum u_if_i = 137$		$\sum u_i^2f_i = 665$

Now, N=300, $\sum u_if_i = 137, \sum u_i^2f_i = 665$

Mean $\overline{X} = A + h\left(\frac{\sum u_if_i}{N}\right)$

$\overline{X} = 105 + 30\left(\frac{137}{300}\right)$

$\overline{X} = 118.7$

$$\mathrm{Var}(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_iu_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_if_i\right)^2\right]$$

$$\mathrm{Var}(X) = \frac{900}{90000}[300 \times 665 - 18769]$$

$$\frac{1}{100}[199500 - 18769]$$

Variance = 1807.31

Standard Deviation $\sigma = \sqrt{1807.31}$

SD = 42.51

Hence, The standard deviation is 42.51

3. Question

Calculate the A.M. and S.D. for the following distribution:

Class:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency:	18	16	15	12	10	5	2	1

Answer

Given, The data is given in table/

To Find: Find the standard deviation

The formula used: SD = $\sqrt{\mathrm{Var(X)}}$

Explanation:

Class	F_i	x_i	$u_i=\frac{x_i-mean}{10}$	f_iu_i	$f_iu_i^2$
0-10	18	5	-3	-54	162
10-20	16	15	-2	-32	64
20-30	15	25	-1	-15	15
30-40	12	35	0	0	0
40-50	10	45	1	10	10
50-60	5	55	2	10	20
60-70	2	65	3	6	18
70-80	1	75	4	4	16
	$\sum f_i = 79$			$\sum u_i f_i = -71$	$\sum u_i^2 f_i = 305$

Now, N=79, $\sum u_i f_i = -71, \sum u_i^2 f_i = 305$

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$$\overline{X} = 35 + 10\left(\frac{-71}{79}\right)$$

$$\overline{X} = 26.01$$

$$Var(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$Var(X) = 100\left[\frac{305}{79} - \frac{5041}{6241}\right]$$

Variance = 305.20

Standard Deviation $\sigma = \sqrt{305.20}$

SD = 17.47

Hence, The standard deviation is 17.47

4. Question

A student obtained the mean and standard deviation of 100 observations as 40 and 5.1 respectively. It was later found that one observation was wrongly copied as 50, the correct figure is 40. Find the correct mean and S.D.

Answer

Given, Uncorrected mean is 40 and corrected SD is 5.1 and N = 100

To Find: Find the correct mean and SD

Explanation: Here, $\overline{x} = 40, \sigma = 5.1 \text{ and } n = 100$

Then, $\sum x_o = 4000$

The corrected sum of observation $\sum x_n = 4000 - 50 + 40$

$$\sum x_n = 3990$$

So, $\overline{x_n} = \frac{\sum x_n}{n}$

$$\overline{x_n} = \frac{3990}{100}$$

$$\overline{x_n} = 39.90$$

Now, Given Incorrect SD = 5.1

$\sigma = 5.1$

$$\sum (x_i - \overline{x_o})^2 = 2601$$

$$\sum (x_i - \overline{x_o})^2 = 2601 - 100 + 0.01 = 2501.1$$

Corrected $\sigma_n = \sqrt{\frac{\sum(x_i - \overline{x_o})^2}{n}}$

$$\sigma_n = \sqrt{\frac{2501.01}{100}}$$

Correct SD is 5

Hence, Correct mean is 39.90 and correct SD is 5

5. Question

Calculate the mean, median and standard deviation of the following distribution

Class-interval:	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66 -70
Frequency:	2	3	8	12	16	5	2	3

Answer

Given, The data is given in table/

To Find: Find the standard deviation

The formula used: SD = $\sqrt{Var(X)}$

Explanation:

Class	F_i	x_i	$u_i=\frac{x_i - mean}{4}$	f_iu_i	$f_iu_i^2$
31-35	2	33	-4	-8	32
36-40	3	38	-3	-9	27
41-45	8	43	-2	-16	32
46-50	12	48	-1	-12	12
51-55	16	53	0	0	0
56-60	5	58	1	5	5
61-65	2	63	2	4	8
66-70	2	68	3	6	18
	$\sum f_i = 50$			$\sum u_i f_i = -30$	$\sum u_i^2 f_i = 134$

Now, N=50, $\sum u_i f_i = -30$, $\sum u_i^2 f_i = 134$

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$$\overline{X} = 53 + 5\left(-\frac{30}{50}\right)$$

$$\overline{X} = 50$$

$$Var(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$Var(X) = 25\left[\frac{134}{50} - \frac{9}{25}\right]$$

Variance = 58

Standard Deviation $\sigma = \sqrt{58}$

SD = 7.62

Hence, The standard deviation is 7.62

6. Question

Find the mean and variance of frequency distribution given below:

x_i:	$1 \le x < 3$	$3 \le x < 5$	$5 \le x < 7$	$7 \le x < 10$
f_1:	6	4	5	1

Answer

Given, The data is given in table

To Find: Find the mean and, the variance of the frequency

Explanation: Here, the class interval is not continues frequency distribution, So we have to convert into continues frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit then we get,

Class	F_i	x_i	$u_i=\frac{x_i - mean}{1}$	f_iu_i	$f_iu_i^2$
1-2	6	1.5	-4	-24	96
3-4	4	3.5	-2	-8	16
5-6	5	5.5	0	0	0
7-8	1	7.5	2	2	4
	$\sum f_i = 16$			$\sum u_if_i = -30$	$\sum u_i^2f_i = 116$

Now, N=16, $\sum u_if_i = 30$, $\sum u_i^2 f_i = 116$

Mean $\overline{X} = A + h\left(\frac{\sum u_if_i}{N}\right)$

$$\overline{X} = 5.5 + 1\left(\frac{1}{16} \times (-30)\right)$$

$$\overline{X} = 3.625$$

$$\mathrm{Var}(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_iu_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_if_i\right)^2\right]$$

$$\mathrm{Var}(X) = 1\left[\left(\frac{1}{16} \times 116\right) - \left(\frac{1}{16} \times (-30)\right)^2\right]$$

Variance = 3.74

Hence, The variance is 3.74

7. Question

The weight of coffee in 70 jars is shown in the following table:

Weight (in grams):	200-201	201-202	202-203	203-204	204-205	205-206
Frequency:	13	27	18	10	1	1

Determine the variance and standard deviation of the above distribution.

Answer

Given, The data is given in table

To Find: Find the mean and, the variance of the frequency

Explanation: Here, the class interval in not continues frequency distribution, So we have to convert into continues frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit then we get,

Class	F_i	x_i	$u_i = \frac{x_i - mean}{1}$	$f_i u_i$	$f_i u_i^2$
200-201	13	200.5	-1.5	-19.5	29.25
201-202	27	201.5	-1	-27	27
202-203	18	202.5	-0.5	-9	4.5
203-204	10	203.5	0	0	0
204-205	1	204.5	0.5	0.5	0.25
205-206	1	205.5	1	1	1
	$\sum f_i = 70$			$\sum u_i f_i = -54$	$\sum u_i^2 f_i = 62$

Now, N=70, $-54, \sum u_i^2 f_i = 62$

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$$\overline{X} = 203.5 + 2\left(-\frac{54}{70}\right)$$

$$\overline{X} = 201.9$$

$$Var(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$Var(X) = 4\left[\left(\frac{62}{70}\right) - \left(-\frac{54}{70}\right)^2\right]$$

Variance = 0.98

Standard Deviation $\sigma = \sqrt{0.98}$

SD = 0.99

Hence, The standard deviation is 0.99

8. Question

Mean, and standard deviation of 100 observations was found to be 40 and 10 respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

Answer

Given, Uncorrected mean is 40 and corrected SD is 10 and N = 100

To Find: Find the correct mean and SD

Explanation: Here, $\overline{x} = 40, \sigma = 10 \text{ and } n = 100$

Then, $\sum x_o = 4000$

The corrected sum of observation $\sum x_n = 4000 - 30 - 70 + 3 + 27$

$$\sum x_n = 3930$$

So, $\overline{x_n} = \frac{\sum x_n}{n}$

$$\overline{x_n} = \frac{3930}{100}$$

$\overline{x_n} = 39.30$

Variance =100

$100 = \frac{\sum x_i^2}{100} - (40)^2$

10000+160000

Incorrect $\sum x_i^2 = 170000$

Correct $\sum x_i^2 = 170000 - (900 + 4900) + (9 + 729)$

Correct $\sum x_i^2 = 164938$

Correct SD, $\sigma = \sqrt{\frac{\text{Correct} \sum x^2}{n} - (\text{Correct Mean})^2}$

Correct SD, $\sigma = \sqrt{\frac{164938}{100} - (39.3)^2}$

Hence, SD = 10.24

9. Question

While calculating the mean and variance of 10 readings, a student wrongly used the reading of 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance.

Answer

Given, Uncorrected mean is 40 and corrected SD is 10 and N = 100

To Find: Find the correct mean and SD

Explanation: Here, $\bar{x} = 45, \text{Var} = 16 \text{ and } n = 10$

Then, $\sum x_o = 450$

The corrected sum of observation $\sum x_n = 450 - 52 + 25$

$$\sum x_n = 423$$

So, $\overline{x_n} = \frac{\sum x_n}{n}$

$\overline{x_n} = \frac{423}{10}$

$\overline{x_n} = 42.3$

Variance =16

$16 = \frac{\sum x_i^2}{10} - (45)^2$

Incorrect $\sum x_i^2 = 20410$

Correct $\sum x_i^2 = 20410 - 2704 + 625$

Correct $\sum x_i^2 = 18331$

Correct SD, $\sigma = \sqrt{\frac{\text{Correct} \sum x^2}{n} - (\text{Correct Mean})^2}$

Correct SD, $\sigma = \sqrt{\frac{18331}{10} - (42.3)^2}$

SD = 6.62

Variance = 6.62×6.62

Hence, Variance = 43.82

10. Question

Calculate the mean, variance and standard deviation of the following frequency distribution:

Class:	1-10	10-20	20-30	30-40	40-50	50-60
Frequency:	11	29	18	4	5	3

Answer

Given, The data is given in table

To Find: Find the standard deviation of the frequency

Explanation:

Class	F_i	x_i	$u_i = \frac{x_i - mean}{10}$	f_iu_i	$f_iu_i^2$
0-10	11	5	-3	-33	99
10-20	29	15	-2	-58	116
20-30	18	25	-1	-18	18
30-40	4	35	0	0	0
40-50	5	45	1	5	5
50-60	3	55	2	6	12
	$\sum f_i = 70$			$\sum u_i f_i = -98$	$\sum u_i^2 f_i = 250$

Now, N=70, -98, $\sum u_i^2 f_i = 250$

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$\overline{X} = 35 + 10\left(-\frac{98}{70}\right)$

$\overline{X} = -21$

$$Var(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$Var(X) = 100\left[\left(\frac{1}{70} \times 250\right) - \left(\frac{1}{70} \times (-98)\right)^2\right]$$

Variance = 100(3.57-1.96)

Variance = 161

Standard Deviation $\sigma = \sqrt{161}$

SD = 12.7

Hence, The standard deviation is 12.7

Exercise 32.7

1. Question

Two plants A and B of a factory show the following results about the number of workers and the wages paid to them

	Plant A	Plant B
No. of workers	5000	6000
Average monthly wages	₹2500	₹2500
The variance of distribution of wages	81	100

In which plant A or B is there greater variability in individual wages?

Answer

Variation of the distribution of wages in plant A(σ^2=18)

So, Standard deviation of the distribution A($\sigma - 9$)

Similarly, the Variation of the distribution of wages in plant B(σ^2=100)

So, Standard deviation of the distribution B($\sigma - 10$)

And, Average monthly wages in both the plants is 2500,

Since The plant with a greater value of SD will have more variability in salary.

Hence, Plant B has more variability in individual wages than plant A

2. Question

The means and standard deviations of heights and weights of 50 students in a class are as follows:

	Weights	Heights
Mean	63.2 kg	63.2 inch
Standard deviation	5.6 kg	11.5 inch

Which shows more variability, heights or weights?

Answer

Given, The mean and SD is given of 50 students.

To Find: which shows more variability, height and weight.

The formula used: Coefficient of variations = $\frac{\text{SD}}{\text{Mean}} \times 100$

Explanation:

The coefficient of variations in weights = $\frac{\text{SD}}{\text{Mean}} \times 100$

$$\Rightarrow \frac{5.6}{63.2} \times 100 = 8.86$$

The coefficient of variations in weights = $\frac{\text{SD}}{\text{Mean}} \times 100$

$$\Rightarrow \frac{11.5}{63.2} \times 100 = 18.19$$

As results clearly show that Cv heights is greater than Cv in weights.

Hence, Heights will show more variability than weights

3. Question

The coefficient of variation of two distribution are 60% and 70%, and their standard deviations are 21 and 16 respectively. What is their arithmetic means?

Answer

Here, the Coefficient of variation for the first distribution is 60

And, Coefficient of variation for the first distribution is 70

$SD(\sigma_1)$ =21 and $SD(\sigma_2)$ = 16

We know that, Coefficients variation $= \frac{SD}{Mean} \times 100$

So, Mean $\overline{X} = \frac{SD}{CV} \times 100$

For first distribution

$$\overline{X} = \frac{21}{60} \times 100$$

Mean = 35

For the second distribution

$$\overline{X} = \frac{16}{70} \times 100$$

Mean = 22.86

Hence, Means are 35 and 22.86 .

4. Question

Calculate coefficient of variation from the following data :

Income (in ₹):	1000-1700	1700-2400	2400-3100	3100-3800	3800-4500	4500-5200
No. of families:	12	18	20	25	35	10

Answer

Given, The data is given in table

To Find: Find the standard deviation of the frequency

Explanation:

Class	F_i	x_i	$u_i = \frac{x_i - mean}{700}$	f_iu_i	$f_iu_i^2$
1000-1700	12	1350	-2	-24	48
1700-2400	18	2050	-1	-18	18
2400-3100	20	2750	0	0	0
3100-3800	25	3450	1	25	25
3800-4500	35	4150	2	70	140
4500-5200	10	4850	3	30	90
	$\sum f_i = 120$			$\sum u_if_i = 83$	$\sum u_i^2f_i = 321$

Now, N=120, $\sum u_i^2 f_i = 321$

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$$\overline{X} = 2750 + 700\left(\frac{83}{120}\right)$$

$$\overline{X} = 3234.17$$

$$Var(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$\text{Var}(X) = 490000\left[\left(\frac{321}{120}\right) - \left(\frac{83}{120}\right)^2\right]$$

Variance = 1076332.64

Standard Deviation $\sigma = \sqrt{1076332.64}$

SD = 1037.47

Coefficients variation $= \frac{1037.46}{3234.17} \times 100$

Cv=32.08

Hence, The coefficient variation is 332.08

5. Question

An analysis of the weekly wages paid to workers in two firms A and B, belonging to the same industry gives the following results:

	Firm A	Firm B
No. of wage earners	586	648
Average weekly wages	₹52.5	₹47.5
The variance of the distribution of wages	100	121

(i) Which firm A or B pays out the larger amount as weekly wages?

(ii) Which firm A or B has greater variability in individual wages?

Answer

(i) Average weekly wages = $\frac{\text{Total weekly wages}}{\text{No.of workers}}$

Total weekly wages = (Avg weekly wages)×(No. of workers)

Total weekly wages of Firm A = 52.5×586 = Rs 30765

Total weekly wages of Firm B = 47.5×648 = Rs 30780

Firm B pays a larger amount as Firm A

(ii) Here SD(firm A) 10 and SD (Firm B) = 11

Coefficient variance (Firm A) $= \frac{10}{52.5} \times 100$

Cv (Firm A) = 19.04

Coefficient variance (Firm B) $= \frac{11}{47.5} \times 100$

Cv (Firm B) = 23.15

Hence, Cv of firm B is greater that that of firm A, Firm B has greater variability in individual wages.

6. Question

The following are some particulars of the distribution of weights of boys and girls in a class:

	Boys	Girls
Number	100	50
Mean weight	60 kg	45 kg
Variance	9	4

Which of the distributions is more variable?

Answer

Here SD(Boys) is 3 and SD (girls) = 2

Coefficient variability = $\frac{\text{SD}}{\text{Mean}} \times 100$

Coefficient variance (Boys) =$\frac{3}{60} \times 100$

Cv (Boys) = 5

Coefficient variance (Girls) =$\frac{2}{45} \times 100$

Cv (Girls) = 4.4

Hence, Cv Boys is greater than Cv girls,then the distribution of weights of boys is more variable than that of girls

7. Question

The mean and standard deviation of marks obtained by 50 students of a class in three subjects, mathematics, physics and chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Answer

$\sigma_m = 12, \sigma_p = 15, \sigma_m = 20 \text{ and } \overline{X_m} = 42, \overline{X_p} = 32, \overline{X_c} = 40.9$

Coefficient variability = $\frac{\sigma}{\overline{X}} \times 100$

Cv (Maths) =$\frac{12}{42} \times 100$

Cv (Maths) = 28.57

Cv (physics) =$\frac{15}{32} \times 100$

So, Cv (physics) = 46.87

Cv (chemistry) =$\frac{20}{40.9} \times 100$

So, Cv (chemistry) = 48.89

Hence, Cv of chemistry is greatest, the variability of marks in chemistry is highest and that of Mathematics is lowest.

8. Question

From the data given below state which group is more variable G_1 or G_2?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group G_1	9	17	32	33	40	10	9
Group G_2	10	20	30	25	43	15	7

Answer

Given, The data is given in the table.

To find: Check which group is more variable G1 and G2

Explanation: Let's find the coefficient of the variable for group G1

Class Interval	F	X	U=(X-A)/h	Fu	U²	Fu²
10-20	9	15	-3	-27	9	81
20-30	17	25	-2	-34	4	68
30-40	32	35	-1	-32	1	32
40-50	33	45	0	0	0	0
50-60	40	55	1	40	1	40
60-70	10	65	2	20	4	40
70-80	9	75	3	27	9	81
	Total= 150			Total=-6		Total=342

Here, n=150 a= 45

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$\overline{X} = 45 + 10\left(\frac{-6}{150}\right)$

$\overline{X} = 44.6$

$\mathrm{Var}(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$

$\mathrm{Var}(X) = 100\left[\left(\frac{342}{150}\right) - \left(\frac{-6}{150}\right)^2\right]$

Variance = 227.84

Standard Deviation $\sigma = \sqrt{227.84}$

Standard Deviation $\sigma = 15.09$

The coefficient of variation $= \frac{SD}{\overline{X}} \times 100$

So, Coefficient of variation $= \frac{15.09}{44.6} \times 100$

Cv = 33.83

Now find the coefficient of variable for group G2

Class Interval	F	X	U=(X-A)/h	Fu	U²	Fu²
10-20	10	15	-3	-30	9	90
20-30	20	25	-2	-40	4	80
30-40	30	35	-1	-30	1	30
40-50	25	45	0	0	0	0
50-60	43	55	1	43	1	43
60-70	15	65	2	30	4	60
70-80	7	75	3	21	9	62
	Total= 150			Total=-6		Total=366

Here, n=150 a= 45

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$\overline{X} = 45 + 10\left(\frac{-6}{150}\right)$

$\overline{X} = 44.6$

$$Var(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$Var(X) = 100\left[\left(\frac{366}{150}\right) - \left(\frac{-6}{150}\right)^2\right]$$

Variance = 243.84

Standard Deviation $\sigma = \sqrt{243.84}$

Standard Deviation $\sigma = 15.62$

The coefficient of variation $= \frac{SD}{\overline{X}} \times 100$

So, Coefficient of variation $= \frac{15.62}{44.6} \times 100$

Cv = 35.02

Since, G2 has a high coefficient of variance,

Hence, Group G2 is more variable.

9. Question

Find the coefficient of variation for the following data :

Size (in cms):	10-15	15-20	20-25	25-30	30-35	35-40
No. of items:	2	8	20	35	20	15

Answer

Given, The data is given in table

To Find: Find the coefficient variation.

Explanation:

Class	F_i	x_i	$u_i=\frac{x_i - mean}{5}$	$f_i u_i$	$f_i u_i^2$
10-15	2	12.5	-2	-4	8
15-20	8	17.5	-1	-8	8
20-25	20	22.5	0	0	0
25-30	35	27.5	1	35	35
30-35	20	32.5	2	40	80
35-40	15	37.5	3	45	135
	$\sum f_i = 100$			$\sum u_i f_i = 108$	$\sum u_i^2 f_i = 266$

Now, N=100, $\sum u_i^2 f_i = 266$

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$$\overline{X} = 22.5 + 5\left(\frac{108}{100}\right)$$

$\overline{X} = 27.90$

$$Var(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$\text{Var}(X) = 25\left[\left(\frac{266}{100}\right) - \left(\frac{108}{100}\right)^2\right]$$

Variance = 37.34

Standard Deviation $\sigma = \sqrt{37.34}$

SD = 6.11

Coefficients variation $= \frac{6.11}{27.90} \times 100$

Cv=21.9

Hence, The coefficient variation is 21.9

10. Question

From the prices of shares X and Y given below: Find out which is more stable in value:

X:	35	54	52	53	56	58	52	50	51	49
Y:	108	107	105	105	106	107	104	103	104	101

Answer

Given, Data is given in the form of two table

To Find: Find out which one is more stable in value

Explanation: Let's Find for Value X

X	d=(x-Mean)	d²
35	-13	169
24	-24	576
52	4	16
53	5	25
56	8	64
58	10	100
52	4	16
50	2	4
51	3	9
49	1	1
Total=480		Total=980

Now, N=70

$$\overline{X} = \frac{1}{n}\sum x_i$$

$\overline{X} = \frac{1}{10}[480]$=48

$$\text{Variance}(X) = \frac{\sum(x_i - \overline{X})^2}{N}$$

$$\text{Variance}(X) = \frac{980}{10} = 98$$

SD (X)=$\sqrt{\text{Var}(X)}$

SD (X)=$\sqrt{98}$=9.9

Coefficient of variation = $\frac{SD}{\overline{X}} \times 100$

Coefficient of variation = $\frac{9.9}{48} \times 100 = 20.6$

Let's Find for Value Y

X	d=(x-Mean)	d^2
108	3	9
107	2	4
105	0	0
105	0	0
106	1	1
107	2	4
104	-1	1
103	-2	4
104	-1	1
101	-4	16
Total=1050		Total=40

Now, N=10

$$\overline{Y}=\frac{1}{n}\sum x_i$$

$\overline{Y}=\frac{1}{10}[1050]$=105

$$\text{Variance}(Y)=\frac{\sum(x_i-\overline{X})^2}{N}$$

$$\text{Variance}(Y)=\frac{40}{10}=4$$

SD (Y)=$\sqrt{\text{Var}(Y)}$

SD (Y)=$\sqrt{4}$=2

Coefficient of variation $=\frac{SD}{\overline{Y}}\times 100$

The coefficient of variation $=\frac{2}{105}\times 100=1.90$

Hence, Cv for Y is smaller that Cv of X, So X is more stable that y.

11. Question

Life of bulbs produced by two factories A and B are given below:

Length of life (in hours):	550-650	650-750	750-850	850-950	950-1050
Factory A (No. of bulbs):	10	22	52	20	16
Factory B (No. of bulbs):	8	60	24	16	12

The bulbs of which factory are more consistent from the point of view of the length of life?

Answer

For Factory A

Length of line	Mid Value x_i	F_i	$u_i=\frac{x_i-800}{100}$	F_iu_i	$F_iu_i^2$
550-650	600	10	-2	-20	40
650-750	700	22	-1	-22	22
750-850	800	52	0	0	0
850-950	900	20	1	20	20
950-1050	1000	16	2	32	64
		$\sum f_i=120$		$\sum u_if_i=10$	$\sum u_i^2f_i=146$

Now, N=120, $\sum u_i^2 f_i = 146$

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$\overline{X} = 800 + 100\left(\frac{10}{120}\right)$

$\overline{X} = 808.33$

$$\mathrm{Var}(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$\mathrm{Var}(X) = 10000\left[\left(\frac{1}{120}\times 146\right) - \left(\frac{1}{120}\times 10\right)^2\right]$$

Variance = 12097

Standard Deviation $\sigma = \sqrt{12097}$

SD = 109.98

Coefficients variation $= \frac{109.98}{808.33} \times 100$

Cv=13.61

For Factory B

Length of line	Mid Value x_i	F_i	$u_i = \frac{x_i - 800}{100}$	$F_i u_i$	$F_i u_i^2$
550-650	600	8	-2	-16	32
650-750	700	60	-1	-60	60
750-850	800	24	0	0	0
850-950	900	16	1	16	16
950-1050	1000	12	2	12	48
		$\sum f_i = 120$		$\sum u_i f_i = -48$	$\sum u_i^2 f_i = 156$

Now, N=120, $\sum u_i^2 f_i = 156$

Mean $\overline{X} = A + h\left(\frac{\sum u_i f_i}{N}\right)$

$\overline{X} = 800 + 100\left(-\frac{48}{120}\right)$

$\overline{X} = 760$

$$\mathrm{Var}(X) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} u_i f_i\right)^2\right]$$

$$\mathrm{Var}(X) = 10000\left[\left(\frac{1}{120}\times 156\right) - \left(\frac{1}{120}\times (-48)\right)^2\right]$$

Variance = 11400

Standard Deviation $\sigma = \sqrt{11400}$

SD = 106.77

Coefficients variation $= \frac{110}{770} \times 100$

Cv=14.29

Since, the coefficient of variation of factory B is greater than the coefficient of variation of factory A,

Hence, This means bulbs of factory A are more consistent from the point of view of the length of life.

12. Question

Following are the marks obtained, out of 100, by two students Ravi and Hashina in 10 tests:

Ravi:	25	50	45	30	70	42	36	48	35	60
Hashina:	10	70	50	20	95	55	42	60	48	80

Who is more intelligent and who is more consistent?

Answer

Given, Marks obtained by two students in 10 tests are given in Table.

To Find: Who is more intelligent and who is more consistent?

Explanation: Marks obtained by Ravi

Mean, $\overline{X} = A + \frac{\sum d_i}{n}$

$\overline{X} = 45 + \frac{-9}{10} = 44.1$

SD (σ)= $\sqrt{\left(\frac{\sum d_i^2}{n} - \left(\frac{\sum d}{n}\right)^2\right)}$

SD (σ)= $\sqrt{\left(\frac{1699}{10} - \left(\frac{-9}{10}\right)^2\right)}$

SD (σ)= $\sqrt{169.09}$

SD (σ)=13.003

Coefficient of variation = $\frac{SD}{Mean}$

Cv = $\frac{13.003}{44.1} \times 100 = 29.49$

For Hashima

Mean, $\overline{X} = A + \frac{\sum d_i}{n}$

$\overline{X} = 55 + \frac{-20}{10} = 53$

SD (σ)= $\sqrt{\left(\frac{\sum d_i^2}{n} - \left(\frac{\sum d}{n}\right)^2\right)}$

SD (σ)= $\sqrt{\left(\frac{6368}{10} - \left(\frac{-20}{10}\right)^2\right)}$

SD (σ)= $\sqrt{632.8}$

SD (σ)=25.16

Coefficient of variation = $\frac{SD}{Mean}$

Cv = $\frac{25.16}{53} \times 100 = 47.47$

Since, The coefficient of variation in mark obtained by Hashima is greater than the coefficient of Variation in mark obtained by Ravi,

Hence, Hasima is more consistent and intelligent

Very Short Answer

1. Question

Write the variance of first n natural numbers.

Answer

Let the numbers be 1,2,3,...,n

Sum of First n natural numbers is $\frac{n(n+1)}{2}$

Mean $\overline{X} = \frac{\text{Sum of all observation}}{\text{Total number of observation}}$

$$\Rightarrow \overline{X} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\Rightarrow \sigma^2 = \frac{\sum(x_i - \overline{X})}{n}$$

$$\Rightarrow \sigma^2 = \frac{\sum\left(x_i - \frac{n+1}{2}\right)^2}{n}$$

$$\Rightarrow \sigma^2 = \frac{1}{n}\left[\sum x_i^2 - x_1(n+1) + \left(\frac{n+1}{2}\right)^2\right]$$

$$\Rightarrow \sigma^2 = \frac{n(n+1)(2n+1)}{6n} - \left[\frac{n(n+1)}{2}\right]\left(\frac{n+1}{n}\right) + \frac{(n+1)^2}{4n} \times n$$

$$\Rightarrow \sigma^2 = \frac{(n+1)(2n+1)}{6} - \left[\frac{n(n+1)}{2}\right]\left(\frac{n+1}{n}\right) + \frac{(n+1)^2}{4}$$

$$\Rightarrow \sigma^2 = \frac{(n+1)(2n+1)}{6} - \left[\frac{(n+1)^2}{2}\right] + \frac{(n+1)^2}{4}$$

$$\Rightarrow \sigma^2 = \frac{(n+1)(n-1)}{12}$$

Hence, $\sigma^2 = \frac{(n+1)(n-1)}{12}$

2. Question

If the sum of the squares of deviations for 10 observations taken from their mean is 2.5, then write the value of standard deviation.

Answer

Given, The sum of the squares of derivations for 10 observation, and mean is 2.5.

To Find: Find the standard derivation

Explanation: Here, N=10 and mean = 2.5

The square of each division = $\frac{2.5}{10} = 0.25$

Standard deviation $\sigma = \sqrt{0.25}$

$\sigma = 0.5$

Hence, standard deviation is 0.5

3. Question

If $x_1, x_2, \ldots\ldots, x_n$ are n values of a variable X and $y_1, y_2, \ldots\ldots., y_n$ are n values of variable Y such that $y_i = ax_i + b$, i = 1,2, ..., n, then write Var(Y) in terms of Var(X).

Answer

$\mathrm{Var}(X) = \frac{\sum(x_i - \overline{X})^2}{n}$

$$\mathrm{Var}(Y) = \frac{\sum(y_i - \overline{Y})^2}{n}$$

And, $y_i = ax_1 + b$

$$\bar{y} = \frac{\sum y_i}{n}$$

$$\bar{y} = \frac{a\sum ax_i + nb}{n}$$

$$\bar{y} = a\overline{X} + b$$

$$\mathrm{Var}(Y) = \frac{\sum(ax_i + b - a\overline{X} - b)^2}{n}$$

$$\mathrm{Var}(Y) = \frac{\sum(ax_i - a\overline{X})^2}{n}$$

$$\mathrm{Var}(Y) = a^2\frac{\sum(x_i - \overline{X})^2}{n}$$

$$\mathrm{Var}(Y) = a^2\mathrm{Var}(X)$$

Hence, proved

4. Question

If X and Y are two variates connected by the relation $Y = \frac{aX + b}{c}$ and $\mathrm{Var}(X) = \sigma^2$, then write the expression for the standard deviation of Y.

Answer

Given, $Y = \frac{aX+b}{c}, \mathrm{Var}(X) = \sigma^2$

To Find: Write the expression for the standard deviation of Y.

Explanation: We have $Y = \frac{aX+b}{c}$

Mean (y) = $\frac{\sum y_i}{n}$

We can write as Mean (y) = $\frac{\left(\frac{a\sum X + nb}{c}\right)}{n}$

Mean (y) = $\frac{a\sum \overline{X}}{nc} + \frac{nb}{nc}$

Var(X) = $\sum\frac{(x_i - \overline{X})^2}{n}$

But, Var(X)= σ^2

Then, Var(Y) = $\sum \frac{(y_i-\overline{Y})^2}{n}$

Now, Substitute the value of $y_i$and Y, then we get

$$Var(Y) = \frac{\sum\left(\frac{aX}{c}+\frac{b}{c}-\frac{a}{c}\overline{X}-\frac{b}{c}\right)^2}{n}$$

$$Var(Y) = \frac{\sum\left(\frac{aX}{c}-\frac{a}{c}\overline{X}\right)^2}{n}$$

$$Var(Y) = \left(\frac{a}{c}\right)^2\frac{\sum(x_i-\overline{X})^2}{n}$$

$$Var(Y) = \left(\frac{a}{c}\right)^2\sigma^2$$

$$SD\ (\sigma) = \sqrt{\left(\frac{a}{c}\right)^2\sigma^2}$$

$$(Y\sigma) = \left|\frac{a}{c}\right|\sigma$$

Hence, Proved

5. Question

In a series of 20 observations, 10 observations are each equal t k, and each of the remaining halves is equal to –k. If the standard deviation of the observation is 2, then write the value of k.

Answer

Given, n=20, d_i=x_i-a

$$d_i = x_i - a$$

Where $a = \frac{\sum x_i}{n}$

$$d_i = x_i - \frac{\sum x_i}{20}$$

$$d_i = x_i - 0$$

$$d_i = x_i$$

$$\sum d_i = \sum x_i = 0$$

$$\sum d_i^2 = \sum 20k^2$$

$$\sigma^2 = \frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2$$

$$\sigma^2 = \frac{20k^2}{20} - 0$$

$$\sigma^2 = k^2$$

$$\sigma = 2 = \sqrt{k^2}$$

K=±2

6. Question

If each observation of a raw data whose standard deviation is σ is multiplied by a, then write the S.D. of the new set of observations.

Answer

We know,

Standard deviation $\sigma = \sqrt{\frac{\sum(x_i - \overline{X})^2}{n}}$

And, mean $\overline{X} = \frac{1}{n}\sum x_i$

Now, multiply by a in x_i

$$\overline{x_{new}} = \frac{1}{n}\sum ax_i$$

$$\overline{x_{new}} = a \times \frac{1}{n}\sum x_i$$

$$\overline{x_{new}} = a\overline{x_{old}}$$

New standard deviation, $\sigma_{new} = \sqrt{\frac{\sum(ax_i - \overline{X})^2}{n}}$

$$\sigma_{new} = \sqrt{\frac{\sum(a^2x_i - \overline{X})^2}{n}}$$

$$\sigma_{new} = |a|\sqrt{\frac{\sum(x_i - \overline{X})^2}{n}}$$

$$\sigma_{new} = |a|\sigma_{old}$$

Hence, Proved

7. Question

If a variable X takes values 0, 1, 2, ..., n with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ...${}^nC_n$, then write variance X.

Answer

we know, mean x = $\frac{\sum x_i f_i}{\sum f_i}$

$$\overline{x} = \frac{\sum x_i f_i}{\sum f_i}$$

$$\overline{x} = \frac{0 \times {}_0^nC + 1 \times {}_1^nC + \cdots + n \times {}_n^nC}{{}_0^nC + {}_1^nC + \ldots {}_n^nC}$$

$$\overline{x} = \frac{n \times 2^{n-1}}{\frac{2^n}{n+1}}$$

Hence, $\overline{x} = \frac{n(n+1)}{2}$

MCQ

1. Question

For a frequency distribution mean deviation from mean is computed by

A. $M.D. = \frac{\sum f}{\sum f|d|}$

B. $M.D. = \frac{\sum d}{\sum f}$

C. $M.D. = \frac{\sum fd}{\sum f}$

D. $M.D. = \frac{\sum f|d|}{\sum f}$

Answer

The general formula of Mean is $\bar{x} = \frac{\sum x_i f_i}{\sum f_i}$

For mean deviation, d = (x-mean)

M.**D =** $\frac{\sum fd}{\sum f}$

2. Question

For a frequency distribution standard deviation is computed by applying the formula

A. $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$

B. $\sigma = \sqrt{\left(\frac{\sum fd}{\sum f}\right)^2 - \frac{\sum fd^2}{\sum f}}$

C. $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \frac{\sum fd}{\sum f}}$

D. $\sigma = \sqrt{\left(\frac{\sum fd}{\sum f}\right)^2 - \frac{\sum fd^2}{\sum f}}$

Answer

We know,

M.**D =** $\frac{\sum fd}{\sum f}$

Variance = $\left(\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2\right)$

SD, $\sigma = \sqrt{\text{Variance}}$

Hence, $\sigma = \sqrt{\left(\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2\right)}$

3. Question

If v is the variance and σ is the standard deviation, then

A. $v = \frac{1}{\sigma^2}$

B. $v = \frac{1}{\sigma}$

C. $v = \sigma^2$

D. $v^2 = \sigma$

Answer

If v is the variance and σ is the standard deviation, then

We know that the formula of standard variance is

$\sigma = \sqrt{\text{Variance}}$

So, $\text{variacne} = \sigma^2$

4. Question

The mean deviation from the median is

A. equal to that measured from another value

B. Maximum if all observation are positive

C. greater than that measured from any other value.

D. less than that measured from any other value.

Answer

equal to that measured from the another value

5. Question

If n = 10, $\overline{X} = 12$ and $\sum x_i^2 = 1530$, then the coefficient of variation is

A. 36%

B. 41%

C. 25%

D. none of these

Answer

Given, n=10, $\overline{X} = 12$ and $\sum x_i^2 = 1530$

We know, Variance = $\left(\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2\right)$

Variance = $\left(\frac{1530}{10} - (12)^2\right)$

Variance = (153-144)

$\sigma = \sqrt{\text{Variance}}$

$\sigma = \sqrt{9}$

SD = 3

Coefficient of variance = $\frac{\text{SD}}{\text{Mean}} \times 100$

Coefficient of variance = $\frac{3}{12} \times 100$

Cv = 25

Hence, Cv = 15

6. Question

The standard deviation of the data:

x:	1	a	a^2	.....	a^n
f:	$^nC_0$	$^nC_1$	$^nC_2$	......	^nC_n

Is

A. $\left(\frac{1+a^2}{2}\right)^n - \left(\frac{1+a}{2}\right)^n$

B. $\left(\frac{1+a^2}{2}\right)^{2n} - \left(\frac{1+a}{2}\right)^n$

C. $\left(\frac{1+a}{2}\right)^{2n} - \left(\frac{1+a^2}{2}\right)^n$

D. none of these

Answer

Let us calculate the mean first,

Mean $= \frac{\sum f_i x_i}{\sum f_i}$

$$\bar{x} = \frac{1 \times {}^n_0C + a \times {}^n_1C + \cdots + a^n \times {}^n_nC}{{}^n_0C + {}^n_1C + \ldots {}^n_nC}$$

$$\bar{x} = n\left(\frac{1+a^2}{2}\right)^{2n}$$

Standard deviation $= \left(\frac{1+a^2}{2}\right)^{2n} - \left(\frac{1+a}{2}\right)^n$

7. Question

The mean deviation of the series a, a+d, a+2d, ……, a+2n from its mean is

A. $\frac{(n+1)d}{2n+1}$

B. $\frac{nd}{2n+1}$

C. $\frac{n(n+1)d}{2n+1}$

D. $\frac{(2n+1)d}{n(n+1)}$

Answer

Given, Series is a, a+d, a+2d,...,+a+2n

Mean (X) = $\frac{a+a+d+a+2d+\cdots+a+2nd}{2n+1}$

$$\overline{X} = \frac{a+a+d+a+2d+\cdots+a+2nd}{2n+1}$$

$$\overline{X} = \frac{a(2n+1)+(d+2d+\cdots+2nd)}{2n+1}$$

$$\overline{X} = \frac{a(2n+1)+d(1+2+3\ldots+2n)}{2n+1}$$

$$\overline{X} = \frac{a(2n+1)+d\frac{2n(2n+1)}{2}}{2n+1}$$

$$\overline{X} = \frac{a(2n+1)+dn(2n+1)}{2n+1}$$

$$\overline{X} = \frac{(2n+1)(a+nd)}{2n+1}$$

$$\overline{X} = a+nd$$

Now, deviation from mean is x_i-X

M.D = $\frac{nd+(n-1)d+(n-2)d+\cdots+0+d+2d+\cdots+(n-1)d+(n-2)d+nd}{2n+1}$

M.D = $\frac{2d(1+2+3+\cdots(n-1)+(n-2)+n)}{2n+1}$

M.D = $\frac{2\left(\frac{n(n+1)}{2}\right)d}{2n+1}$

Hence, M.D = $\frac{n(n+1)d}{2n+1}$

8. Question

A batsman scores runs in 10 innings as 38, 70, 48, 34, 42, 55, 63, 46, 54 and 44. The mean deviation about mean is

A. 8.6

B. 6.4

C. 10.6

D. 7.6

Answer

Mean (X)=$\frac{38+70+48+34+42+55+63+46+54+44}{10}$

$$(\overline{X}) = \frac{494}{10}$$

x_i	d=(x_i-Mean)
38	-11.4
70	20.6
48	-1.4
34	-15.4
42	-7.4
55	5.6
63	13.6
46	-3.4
54	4.6
44	-5.4
	$\sum d_i = 0$

Here, N= 10 $\sum \mathrm{d} = 0$

Mean deviation = $\left(\frac{\sum \mathrm{d_i}}{\mathrm{N}}\right)$

$\mathrm{M.D} = \left(\frac{0}{10}\right)$

Hence, MD is 0

9. Question

The mean deviation of the numbers 3, 4, 5, 6, 7 from the mean is

A. 25

B. 5

C. 1.2

D. 0

Answer

The mean deviation of the numbers 3, 4, 5, 6, 7

Mean $\overline{\mathrm{X}} = \frac{3+4+5+6+7}{5}$

$\overline{\mathrm{X}} = \frac{25}{5}$

$\overline{\mathrm{X}}$=5

x_i	(x_i-mean)
3	-2
4	-1
5	0
6	1
7	2
$\sum x_i = 25$	$\sum d_i = 0$

Mean deviation = $\frac{\sum \mathrm{d_i}}{\sum \mathrm{x_i}}$

Mean deviation = $\frac{0}{25}$

Hence, Mean deviation is 0

10. Question

The sum of the squares deviations for 10 observations for 10 observations taken from their mean 50 is 250. The coefficient of variation is

A. 10%

B. 40%

C. 50%

D. none of these

Answer

Given, n=10 mean 250

SD, $\sigma = \sqrt{\left(\frac{250}{10}\right)}$

$\sigma = \sqrt{25}$

SD = 5

Now, Coefficient of variance = $\frac{\text{SD}}{\text{Mean}} \times 100$

Cv = $\frac{5}{50} \times 100$

Cv = 50

Hence, Coefficient of Variation is 10

11. Question

Let x_1, x_2, ...x_n be values taken by a variable X and y_1, y_2, ...y_n be the values taken by a variable Y such that $y_i = ax_i + b$, i = 1,2, ...,, n. Then,

A. Var (Y) = a^2 Var (X)

B. Var (X) = a^2 Var (Y)

C. Var (X) = Var (X) + b

D. none of these

Answer

we have given, $y_i=ax_i+b$

Mean (Y) = $\frac{\sum f_i}{n}$

$$\overline{Y} = \frac{a\sum x_n + bn}{n}$$

Mean (y) = $\frac{a\sum \overline{X}}{n} + \frac{nb}{n}$

Then, Var(Y) = $\sum \frac{(y_i - \overline{Y})^2}{n}$

And, Var(X) = $\sum \frac{(x_i - \overline{X})^2}{n}$

$$\text{Var}(Y) = \frac{\sum(aX + b - a\overline{X} - b)^2}{n}$$

$$\text{Var}(Y) = \frac{\sum(a - a\overline{X})^2}{n}$$

$$\text{Var}(Y) = a^2 \frac{\sum(x_i - \overline{X})^2}{n}$$

$$\text{Var}(Y) = a^2 \text{Var}(X)$$

Hence, Var(Y) = a^2 Var(X)

12. Question

If the standard deviation of a variable X is σ, then the standard deviation of the variable $\frac{aX+b}{c}$ is

A. a σ

B. $\frac{a}{c}\sigma$

C. $\left|\frac{a}{c}\right|\sigma$

D. $\frac{a\sigma+b}{c}$

Answer

We have $X = \frac{aX+b}{c}$

Mean (X) = $\frac{\sum y_i}{n}$

We can write as: Mean (X) = $\frac{\left(\frac{a\sum x+nb}{c}\right)}{n}$

Mean (X) = $\frac{a\sum \bar{X}}{nc} + \frac{nb}{nc}$

Var(X) = $\sum \frac{(x_i-\bar{X})^2}{n}$

Now, Substitute the value of $y_i$and Y, then we get

$$\mathrm{Var}(X) = \frac{\sum\left(\frac{aX}{c}+\frac{b}{c}-\frac{a}{c}\bar{X}-\frac{b}{c}\right)^2}{n}$$

$$\mathrm{Var}(X) = \frac{\sum\left(\frac{aX}{c}-\frac{a}{c}\bar{X}\right)^2}{n}$$

$$\mathrm{Var}(X) = \left(\frac{a}{c}\right)^2 \frac{\sum(x_i-\bar{X})^2}{n}$$

$$\mathrm{Var}(X) = \left(\frac{a}{c}\right)^2 \sigma^2$$

$$\mathrm{SD}\ (\sigma) = \sqrt{\left(\frac{a}{c}\right)^2 \sigma^2}$$

$$(X\sigma) = \left|\frac{a}{c}\right|\sigma$$

Hence, Proved

13. Question

If the S.D. of a set of observations is 8 and if each observation is divided by -2, the S.D. of the new set of observations will be

A. -4

B. -8

C. 8

D. 4

Answer

Let take two observation 16 and 32

Now,

x_i	x_i^2
16	256
32	1024
$\sum x_i = 48$	$\sum x_i^2 = 1280$

$$\text{Variance} = \left(\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2\right)$$

$$\sigma = \sqrt{\left(\frac{1280}{2} - \left(\frac{48}{2}\right)^2\right)}$$

$$\sigma = \sqrt{640 - 576}$$

$$\sigma = \sqrt{64}$$

SD = 8

Now, If we divide each observation then SD is

x_i	x_i^2
8	64
16	256
$\sum x_i = 24$	$\sum x_i^2 = 320$

$$\text{Variance} = \left(\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2\right)$$

$$\sigma = \sqrt{\left(\frac{320}{2}\right) - \left(\frac{24}{2}\right)^2}$$

$$\sigma = \sqrt{160 - 144}$$

$$\sigma = \sqrt{16}$$

SD = 4

Hence, SD will also be half.

14. Question

If two variates X and Y are connected by the relation $Y = \frac{aX + b}{c}$, where a, b, c are constants such that ac < o, then

A. $\sigma Y = \frac{a}{c} \sigma X$

B. $\sigma Y = -\frac{a}{c}\sigma X$

C. $\sigma Y = \frac{a}{c}\sigma X + b$

D. none of these

Answer

Given, $Y = \frac{aX+b}{c}$

To Find: Write the expression for the standard deviation of Y.

Explanation: We have $Y = \frac{aX+b}{c}$

Mean (y) = $\frac{\sum y_i}{n}$

We can write as: Mean (y) = $\frac{\left(\frac{a\sum X+nb}{c}\right)}{n}$

Mean (y) = $\frac{a\sum \bar{X}}{nc} + \frac{nb}{nc}$

Var(X) = $\sum \frac{(x_i - \bar{X})^2}{n}$

Then, Var(Y) = $\sum \frac{(y_i - \bar{Y})^2}{n}$

Now, Substitute the value of $y_i$and Y, then we get

$$\mathrm{Var}(Y) = \frac{\sum\left(\frac{aX}{c} + \frac{b}{c} - \frac{a}{c}\bar{X} - \frac{b}{c}\right)^2}{n}$$

$$\mathrm{Var}(Y) = \frac{\sum\left(\frac{aX}{c} - \frac{a}{c}\bar{X}\right)^2}{n}$$

$$\mathrm{Var}(Y) = \left(\frac{a}{c}\right)^2 \frac{\sum(x_i - \bar{X})^2}{n}$$

$$\mathrm{Var}(Y) = \left(\frac{a}{c}\right)^2 \sigma^2$$

$$\mathrm{SD}\ (\sigma) = \sqrt{\left(\frac{a}{c}\right)^2 \sigma^2}$$

$$(Y\sigma) = \left|\frac{a}{c}\right| \sigma$$

15. Question

If for a sample of size 60, we have the following information $\sum x_i^2 = 18000$ and $\sum x_i = 960$, then the variance is

A. 6.63

B. 16

C. 22

D. 44

Answer

Given, N=60,$\sum x_i = 55$, $\sum x_i^2 = 385$

$$\text{Variance} = \left(\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2\right)$$

$$\text{Variance} = \left(\frac{18000}{60} - \left(\frac{960}{60}\right)^2\right)$$

$$\text{Variance} = (300 - 256)$$

Variance = 44

Hence, variance = 44

16. Question

Let a, b, c, d, e be the observations with mean m and standard deviation s. The standard deviation of the observations a + k, b + k, c + k, d + k, e + k is

A. s

B. ks

C. s + k

D. $\frac{s}{k}$

Answer

Let a, b, c,d,e be the observation and mean is m

$$m = \frac{a + b + c + d + e}{5}$$

Let suppose new mean be m_1

$$m_1 = \frac{a + k + b + k + c + k + d + k + e + k}{5}$$

$$m_1 = \frac{5k}{5} + \frac{a + b + c + d + e}{5}$$

M_1 =m+k

Now, The standard deviation

$$S = \sqrt{\frac{(a - m)^2 + (b - m)^2 + (c - m)^2 + (e - m)^2}{5}}$$

So, The standard deviation for new observation

$$s_1 = \sqrt{\frac{(a + k - n)^2 + (b + k - n)^2 + (c + k - n)^2 + (e + k - n)^2 + (d + k - n)^2}{5}}$$

Now, we can compare both observation

a+k-n=a+k-(m+k)

a+k–n = a+k–m- k

a+k-n=a-m

Similary

b+k-n=b-m

c+k-n=c-m

d+k-n=d-m

e+k-n=e-m

when we substitute the values, we get,

$S_1 = S$

Hence, The Sd is S

17. Question

The standard deviation of first 10 natural numbers is

A. 5.5

B. 3.87

C. 2.97

D. 2.87

Answer

First 10 natural numbers are 1,2,3,4,5,6,7,8,9,10

So, N =10

x_i	x_i^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
$\sum x_i = 55$	$\sum x_i^2 = 385$

$$\text{Variance} = \left(\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2\right)$$

$$\sigma = \sqrt{\left(\frac{385}{10} - \left(\frac{55}{10}\right)^2\right)}$$

$$\sigma = \sqrt{38.5 - 30.25}$$

$$\sigma = \sqrt{8.25}$$

Hence, SD is 2.87

18. Question

Consider the first 10 positive integers. If we multiply each numbers by -1 and then add 1 to each number, the variance of the numbers so obtained is

A. 8.25

B. 6.5

C. 3.87

D. 2.87

Answer

Consider 10 positive integer

Let Assume, 1,2,3,4,5,6,7,8,9,10

Now, If we multiply by -1 in each number we get,

-1,-2,-3,-4,-5,-6,-7,-8,-9,-10

And then we add 1 in each number

0,-1,-2,-3,-4,-5,-6,-7,-8,-9

Now,

x_i	x_i^2
0	0
-1	1
-2	4
-3	9
-4	16
-5	25
-6	36
-7	49
-8	64
-9	81
$\sum x_i = -45$	$\sum x_i^2 = 285$

Standard deviation $\text{Variance} = \left(\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2\right)$

$$\text{Var} = \left(\frac{285}{10} - \left(\frac{-45}{10}\right)^2\right)$$

$$\text{Var} = (28.5 - 20.25)$$

$$\text{Var} = 8.25$$

Hence, variance is 8.25

19. Question

Consider the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. It is added to each number, the variance of the numbers so obtained is

A. 6.5

B. 2.87

C. 3.87

D. 8.25

Answer

Consider numbers are 1,2,3,4,5,6,7,8,9,10

If one is added to each number then, numbers will be

Let say x_i = 2,3,4,5,6,7,8,9,10,11

So, N= 10

x_i	x_i^2
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
$\sum x_i = 65$	$\sum x_i^2 = 505$

Standard deviation $\text{Variance} = \left(\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2\right)$

$$\text{Var} = \left(\frac{505}{10} - \left(\frac{65}{10}\right)^2\right)$$

$\text{Var} = 8.25$

Hence, the variance is 8.25

20. Question

The mean of 100 observations is 50, and their standard deviations is 5. The sum of all squares of all the observations is

A. 50,000

B. 250,000

C. 252500

D. 255000

Answer

Given, $\bar{x} = 50, n = 100 \text{ and } \sigma = 5$

$$\sigma = \frac{\sum x_i}{N}$$

$\sum x_i = 50 \times 100$

$\sum x_i = 5000$

Now, $\sigma^2 = \frac{\sum x_i^2}{N} - (\bar{x})^2$

$$25 = \frac{\sum x_i^2}{100} - (50)^2$$

$$\sum x_i^2 = 252500$$

21. Question

Let x_1, x_2, ...x_n, be n observations. Let $y_i = ax_i + b$ for I = 1, 2, ..., n, where a and b are constant. If the mean of $x_i^{'s}$ is 48 and their standard deviation is 12, the mean of $y_i^{'s}$ is 55 and standard deviation of $y_i^{'s}$ is 15, the values of a and b are

A. a = 1.25, b = -5

B. a = -1.25, b = 5

C. a = 2.5, b = -5

D. a = 2.5, b = 5

Answer

Mean(y) = a.mean(x) + b

Therefore,

55 = a.48 + b

We can see that only first option satisfies this equation. Therefore, a is the correct answer.

22. Question

The mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean is

A. 2

B. 2.57

C. 3

D. 3.57

Answer

Explanation: Mean $\overline{X} = \frac{3+10+10+4+7+10+5}{7}$

$\overline{X} = \frac{49}{7}$

$\overline{X} = 7$

X_i	$D_i = \lvert x_i - x \rvert$
3	4
10	3
10	3
4	3
7	0
10	3
5	2
Total	18

Mean Deviation $= \frac{\sum d_i}{N}$

Mean Deviation $= \frac{18}{7}$

Hence, The MD is 2.57

23. Question

The mean deviation for n observations x_1, x_2, ..., x_n from their mean $\overline{X}$ is given by

A. $\sum_{i=1}^{n}(x_i - \overline{X})$

B. $\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{X})$

C. $\sum_{i=1}^{n}(x_i - \overline{X})^2$

D. $\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{X})^2$

Answer

Let $x_1, x_2, \ldots x_n$ be n observation

And X is the aithemetic mean then,

We know, Standard deviation $\sigma = \sqrt{\left(\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2\right)}$

So, $\sigma = \frac{1}{n}\sqrt{(\sum x_i^2 - \overline{X})^2}$

24. Question

Let x_1, x_2, ..., x_n be n observations and $\overline{X}$ be their arithmetic mean. The standard deviation is given by

A. $\sum_{i=1}^{n}(x_i - \overline{X})^2$

B. $\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{X})^2$

C. $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{X})^2}$

D. $\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2 - \overline{X}^2}$

Answer

Let $x_1, x_2, \ldots x_n$ be n observation

And X is the arithmetic mean then,

We know, Standard deviation $\sigma = \sqrt{\left(\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2\right)}$

So, $\sigma = \frac{1}{n}\sqrt{(\sum x_i^2 - \overline{X})^2}$

25. Question

The standard deviation of the observations 6, 5, 9, 13, 12, 8, 10 is

A. 6

B. $\sqrt{6}$

C. $\frac{52}{7}$

D. $\sqrt{\frac{52}{7}}$

Answer

The standard deviation of the observations 6, 5, 9, 13, 12, 8, 10 is

x_i	x_i^2
6	36
5	25
9	81
13	169
12	144
8	64
10	100
$\sum x_i = 63$	$\sum x_i = 619$

And, N=7

Standard deviation $\sigma = \sqrt{\left(\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2\right)}$

$$\sigma = \sqrt{\left(\frac{619}{7} - \left(\frac{63}{7}\right)^2\right)}$$

$$\sigma = \sqrt{\left(\frac{7 \times 619 \times 3969}{49}\right)}$$

$$\sigma = \sqrt{\frac{396}{49}}$$

$$\sigma = \sqrt{\frac{52}{7}}$$