

CHAPTER 11

Limits, Continuity and Differentiability

Section-A

JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Let $f(x) = \begin{cases} (x-1)^2 \sin \frac{1}{(x-1)} |x| & \text{if } x \neq 1 \\ -1, & \text{if } x = 1 \end{cases}$
be a real-valued function. Then the set of points where $f(x)$ is not differentiable is (1981 - 2 Marks)

2. Let $f(x) = \begin{cases} \frac{(x^3 + x^2 - 16x + 20)}{(x-2)^2}, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases}$
If $f(x)$ is continuous for all x , then $k = \dots\dots\dots$ (1981 - 2 Marks)

3. A discontinuous function $y = f(x)$ satisfying $x^2 + y^2 = 4$ is given by $f(x) = \dots\dots\dots$ (1982 - 2 Marks)

4. $\lim_{x \rightarrow 1} (1-x) \tan \frac{\pi x}{2} = \dots\dots\dots$ (1984 - 2 Marks)

5. If $f(x) = \sin x$, $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \dots\dots\dots$
 $= 2$, otherwise
and $g(x) = x^2 + 1$, $x \neq 0, 2$
 $= 4, x = 0$
 $= 5, x = 2$,

then $\lim_{x \rightarrow 0} g[f(x)]$ is (1986 - 2 Marks)

6. $\lim_{x \rightarrow -\infty} \left[\frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{(1 + |x|^3)} \right] = \dots\dots\dots$ (1987 - 2 Marks)

7. If $f(9) = 9$, $f'(9) = 4$, then $\lim_{x \rightarrow 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ equals (1988 - 2 Marks)

8. ABC is an isosceles triangle inscribed in a circle of radius r . If $AB = AC$ and h is the altitude from A to BC then the triangle ABC has perimeter $P = 2(\sqrt{2hr} - h^2) + \sqrt{2hr}$ and area $A = \dots\dots\dots$ also $\lim_{h \rightarrow 0} \frac{A}{P^3} = \dots\dots\dots$ (1989 - 2 Marks)

9. $\lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4} = \dots\dots\dots$ (1990 - 2 Marks)

10. Let $f(x) = x |x|$. The set of points where $f(x)$ is twice differentiable is (1992 - 2 Marks)

11. Let $f(x) = [x] \sin \left(\frac{\pi}{[x+1]} \right)$, where $[\bullet]$ denotes the greatest integer function. The domain of f is... and the points of discontinuity of f in the domain are..... (1996 - 2 Marks)

12. $\lim_{x \rightarrow 0} \left(\frac{1+5x^2}{1+3x^2} \right)^{1/x^2} = \dots\dots\dots$ (1996 - 1 Mark)

13. Let $f(x)$ be a continuous function defined for $1 \leq x \leq 3$. If $f(x)$ takes rational values for all x and $f(2) = 10$, then $f(1.5) = \dots\dots\dots$ (1997 - 2 Marks)

B True / False

1. If $\lim_{x \rightarrow a} [f(x)g(x)]$ exists then both $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist. (1981 - 2 Marks)

C MCQs with One Correct Answer

1. If $f(x) = \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$, then $\lim_{x \rightarrow \infty} f(x)$ is (1979)
(a) 0 (b) ∞
(c) 1 (d) none of these

2. For a real number y , let $[y]$ denotes the greatest integer less than or equal to y . Then the function $f(x) = \frac{\tan(\pi[x - \pi])}{1 + [x]^2}$ is (1981 - 2 Marks)
(a) discontinuous at some x
(b) continuous at all x , but the derivative $f'(x)$ does not exist for some x
(c) $f'(x)$ exists for all x , but the second derivative $f''(x)$ does not exist for some x
(d) $f'(x)$ exists for all x

3. There exist a function $f(x)$, satisfying $f(0) = 1, f'(0) = -1, f(x) > 0$ for all x , and (1982 - 2 Marks)
- (a) $f''(x) > 0$ for all x (b) $-1 < f''(x) < 0$ for all x
 (c) $-2 \leq f''(x) \leq -1$ for all x (d) $f''(x) < -2$ for all x
4. If $G(x) = -\sqrt{25 - x^2}$ then $\lim_{x \rightarrow 1} \frac{G(x) - G(1)}{x - 1}$ has the value (1983 - 1 Mark)
- (a) $\frac{1}{24}$ (b) $\frac{1}{5}$
 (c) $-\sqrt{24}$ (d) none of these
5. If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2$, then the value of $\lim_{x \rightarrow a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is (1983 - 1 Mark)
- (a) -5 (b) $\frac{1}{5}$
 (c) 5 (d) none of these
6. The function $f(x) = \frac{\ln(1+ax) - \ln(1-bx)}{x}$ is not defined at $x = 0$. The value which should be assigned to f at $x = 0$ so that it is continuous at $x = 0$, is (1983 - 1 Mark)
- (a) $a - b$ (b) $a + b$
 (c) $\ln a - \ln b$ (d) none of these
7. $\lim_{n \rightarrow \infty} \left\{ \frac{1}{1-n^2} + \frac{2}{1-n^2} + \dots + \frac{n}{1-n^2} \right\}$ is equal to (1984 - 2 Marks)
- (a) 0 (b) $-\frac{1}{2}$
 (c) $\frac{1}{2}$ (d) none of these
8. If $f(x) = \frac{\sin[x]}{[x]}$, $[x] \neq 0$ (1985 - 2 Marks)
 $= 0, [x] = 0$
- Where $[x]$ denotes the greatest integer less than or equal to x . then $\lim_{x \rightarrow 0} f(x)$ equals -
- (a) 1 (b) 0
 (c) -1 (d) none of these
9. Let $f: R \rightarrow R$ be a differentiable function and $f(1) = 4$. Then the value of $\lim_{x \rightarrow 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is (1990 - 2 Marks)
- (a) $8f'(1)$ (b) $4f'(1)$ (c) $2f'(1)$ (d) $f'(1)$
10. Let $[.]$ denote the greatest integer function and $f(x) = [\tan^2 x]$, then: (1993 - 1 Mark)
- (a) $\lim_{x \rightarrow 0} f(x)$ does not exist
 (b) $f(x)$ is continuous at $x = 0$
 (c) $f(x)$ is not differentiable at $x = 0$
 (d) $f'(0) = 1$
11. The function $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, $[.]$ denotes the greatest integer function, is discontinuous at (1995S)
- (a) All x (b) All integer points
 (c) No x (d) x which is not an integer
12. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ equals (1997 - 2 Marks)
- (a) $1 + \sqrt{5}$ (b) $-1 + \sqrt{5}$ (c) $-1 + \sqrt{2}$ (d) $1 + \sqrt{2}$
13. The function $f(x) = [x]^2 - [x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at (1999 - 2 Marks)
- (a) all integers
 (b) all integers except 0 and 1
 (c) all integers except 0
 (d) all integers except 1
14. The function $f(x) = (x^2 - 1) |x^2 - 3x + 2| + \cos(|x|)$ is NOT differentiable at (1999 - 2 Marks)
- (a) -1 (b) 0 (c) 1 (d) 2
15. $\lim_{x \rightarrow 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ is (1999 - 2 Marks)
- (a) 2 (b) -2 (c) $1/2$ (d) $-1/2$
16. For $x \in R, \lim_{x \rightarrow \infty} \left(\frac{x-3}{x+2} \right)^x =$ (2000S)
- (a) e (b) e^{-1} (c) e^{-5} (d) e^5
17. $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals (2001S)
- (a) $-\pi$ (b) π (c) $\pi/2$ (d) 1
18. The left-hand derivative of $f(x) = [x] \sin(\pi x)$ at $x = k, k$ an integer, is (2001S)
- (a) $(-1)^k(k-1)\pi$ (b) $(-1)^{k-1}(k-1)\pi$
 (c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$
19. Let $f: R \rightarrow R$ be a function defined by $f(x) = \max\{x, x^3\}$. The set of all points where $f(x)$ is NOT differentiable is (2001S)
- (a) $\{-1, 1\}$ (b) $\{-1, 0\}$ (c) $\{0, 1\}$ (d) $\{-1, 0, 1\}$
20. Which of the following functions is differentiable at $x = 0$? (2001S)
- (a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$
 (c) $\sin(|x|) + |x|$ (d) $\sin(|x|) - |x|$
21. The domain of the derivative of the function $f(x) = \begin{cases} \tan^{-1} x & \text{if } |x| \leq 1 \\ \frac{1}{2}(|x| - 1) & \text{if } |x| > 1 \end{cases}$ is (2002S)
- (a) $R - \{0\}$ (b) $R - \{1\}$
 (c) $R - \{-1\}$ (d) $R - \{-1, 1\}$

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22. The integer n for which $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is (2002S)
 (a) 1 (b) 2 (c) 3 (d) 4
23. Let $f: R \rightarrow R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x \rightarrow 0} \left(\frac{f(1+x)}{f(1)} \right)^{1/x}$ equals (2002S)
 (a) 1 (b) $e^{1/2}$ (c) e^2 (d) e^3
24. If $\lim_{x \rightarrow 0} \frac{((a-n)x - \tan x) \sin nx}{x^2} = 0$, where n is nonzero real number, then a is equal to (2003S)
 (a) 0 (b) $\frac{n+1}{n}$ (c) n (d) $n + \frac{1}{n}$
25. $\lim_{h \rightarrow 0} \frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)}$, given that $f'(2) = 6$ and $f'(1) = 4$
 (a) does not exist (b) is equal to $-3/2$
 (c) is equal to $3/2$ (d) is equal to 3 (2003S)
26. If $f(x)$ is differentiable and strictly increasing function, then the value of $\lim_{x \rightarrow 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is (2004S)
 (a) 1 (b) 0 (c) -1 (d) 2
27. The function given by $y = ||x| - 1|$ is differentiable for all real numbers except the points (2005S)
 (a) $\{0, 1, -1\}$ (b) ± 1 (c) 1 (d) -1
28. If $f(x)$ is continuous and differentiable function and $f(1/n) = 0 \forall n \geq 1$ and $n \in I$, then (2005S)
 (a) $f(x) = 0, x \in (0, 1]$
 (b) $f(0) = 0, f'(0) = 0$
 (c) $f(0) = 0 = f'(0), x \in (0, 1]$
 (d) $f(0) = 0$ and $f'(0)$ need not to be zero
29. The value of $\lim_{x \rightarrow 0} \left((\sin x)^{1/x} + (1+x)^{\sin x} \right)$, where $x > 0$ is (2006 - 3M, -1)
 (a) 0 (b) -1 (c) 1 (d) 2
30. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and $\lim_{t \rightarrow x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $f(x)$ is (2007 - 3 marks)
 (a) $\frac{1}{3x} + \frac{2x^2}{3}$ (b) $\frac{-1}{3x} + \frac{4x^2}{3}$ (c) $\frac{-1}{x} + \frac{2}{x^2}$ (d) $\frac{1}{x}$
31. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\int_{\frac{\pi}{4}}^x \sec^2 t \, dt}{x^2 - \frac{\pi^2}{16}}$ equals (2007 - 3 marks)
 (a) $\frac{8}{\pi} f(2)$ (b) $\frac{2}{\pi} f(2)$ (c) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$ (d) $4f(2)$
32. Let $g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0$, $n > 0$, and let p be the left hand derivative of $|x-1|$ at $x = 1$. If $\lim_{x \rightarrow 1^+} g(x) = p$, then (2008)
 (a) $n = 1, m = 1$ (b) $n = 1, m = -1$
 (c) $n = 2, m = 2$ (d) $n > 2, m = n$
33. If $\lim_{x \rightarrow 0} \left[1 + x \ln(1+b^2) \right]^{1/x} = 2b \sin^2 \theta$, $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is (2011)
 (a) $\pm \frac{\pi}{4}$ (b) $\pm \frac{\pi}{3}$ (c) $\pm \frac{\pi}{6}$ (d) $\pm \frac{\pi}{2}$
34. If $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then (2012)
 (a) $a = 1, b = 4$ (b) $a = 1, b = -4$
 (c) $a = 2, b = -3$ (d) $a = 2, b = 3$
35. Let $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{x} \right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in R$ then f is (2012)
 (a) differentiable both at $x = 0$ and at $x = 2$
 (b) differentiable at $x = 0$ but not differentiable at $x = 2$
 (c) not differentiable at $x = 0$ but differentiable at $x = 2$
 (d) differentiable neither at $x = 0$ nor at $x = 2$
36. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $(\sqrt[3]{1+a} - 1)x^2 + (\sqrt{1+a} - 1)x + (\sqrt[6]{1+a} - 1) = 0$ where $a > -1$. Then $\lim_{a \rightarrow 0^+} \alpha(a)$ and $\lim_{a \rightarrow 0^+} \beta(a)$ are (2012)
 (a) $-\frac{5}{2}$ and 1 (b) $-\frac{1}{2}$ and -1
 (c) $-\frac{7}{2}$ and 2 (d) $-\frac{9}{2}$ and 3

D MCQs with One or More than One Correct

1. If $x + |y| = 2y$, then y as a function of x is (1984 - 3 Marks)
 (a) defined for all real x
 (b) continuous at $x = 0$
 (c) differentiable for all x
 (d) such that $\frac{dy}{dx} = \frac{1}{3}$ for $x < 0$
2. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then— (1985 - 2 Marks)
 (a) $f(x)$ is continuous but not differentiable at $x = 0$
 (b) $f(x)$ is differentiable at $x = 0$
 (c) $f(x)$ is not differentiable at $x = 0$
 (d) none of these

3. The function $f(x) = 1 + |\sin x|$ is (1986 - 2 Marks)
 (a) continuous nowhere
 (b) continuous everywhere
 (c) differentiable nowhere
 (d) not differentiable at $x = 0$
 (e) not differentiable at infinite number of points.
4. Let $[x]$ denote the greatest integer less than or equal to x . If $f(x) = [x \sin \pi x]$, then $f(x)$ is (1986 - 2 Marks)
 (a) continuous at $x = 0$ (b) continuous in $(-1, 0)$
 (c) differentiable at $x = 1$ (d) differentiable in $(-1, 1)$
 (e) none of these
5. The set of all points where the function $f(x) = \frac{x}{(1 + |x|)}$ is differentiable, is (1987 - 2 Marks)
 (a) $(-\infty, \infty)$ (b) $[0, \infty)$
 (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(0, \infty)$
 (e) None
6. The function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ is (1988 - 2 Marks)
 (a) continuous at $x = 1$ (b) differentiable at $x = 1$
 (c) continuous at $x = 3$ (d) differentiable at $x = 3$.
7. If $f(x) = \frac{1}{2}x - 1$, then on the interval $[0, \pi]$ (1989 - 2 Marks)
 (a) $\tan [f(x)]$ and $1/f(x)$ are both continuous
 (b) $\tan [f(x)]$ and $1/f(x)$ are both discontinuous
 (c) $\tan [f(x)]$ and $f^{-1}(x)$ are both continuous
 (d) $\tan [f(x)]$ is continuous but $1/f(x)$ is not.
8. The value of $\lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ (1991 - 2 Marks)
 (a) 1 (b) -1
 (c) 0 (d) none of these
9. The following functions are continuous on $(0, \pi)$. (1991 - 2 Marks)
 (a) $\tan x$
 (b) $\int_0^x t \sin \frac{1}{t} dt$
 (c) $\begin{cases} 1, & 0 < x \leq \frac{3\pi}{4} \\ 2 \sin \frac{2}{9}x, & \frac{3\pi}{4} < x < \pi \end{cases}$
 (d) $\begin{cases} x \sin x, & 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x), & \frac{\pi}{2} < x < \pi \end{cases}$
10. Let $f(x) = \begin{cases} 0, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ then for all x (1994)
 (a) f' is differentiable (b) f is differentiable
 (c) f' is continuous (d) f is continuous
11. Let $g(x) = xf(x)$, where $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. At $x = 0$
 (a) g is differentiable but g' is not continuous (1994)
 (b) g is differentiable while f is not
 (c) both f and g are differentiable
 (d) g is differentiable and g' is continuous
12. The function $f(x) = \max \{(1-x), (1+x), 2\}$, $x \in (-\infty, \infty)$ is (1995)
 (a) continuous at all points
 (b) differentiable at all points
 (c) differentiable at all points except at $x = 1$ and $x = -1$
 (d) continuous at all points except at $x = 1$ and $x = -1$, where it is discontinuous
13. Let $h(x) = \min \{x, x^2\}$, for every real number of x , Then (1998 - 2 Marks)
 (a) h is continuous for all x
 (b) h is differentiable for all x
 (c) $h'(x) = 1$, for all $x > 1$
 (d) h is not differentiable at two values of x .
14. $\lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$ (1998 - 2 Marks)
 (a) exists and it equals $\sqrt{2}$
 (b) exists and it equals $-\sqrt{2}$
 (c) does not exist because $x-1 \rightarrow 0$
 (d) does not exist because the left hand limit is not equal to the right hand limit.
15. If $f(x) = \min \{1, x^2, x^3\}$, then (2006 - 5M, -1)
 (a) $f(x)$ is continuous $\forall x \in \mathbb{R}$
 (b) $f(x)$ is continuous and differentiable everywhere.
 (c) $f(x)$ is not differentiable at two points
 (d) $f(x)$ is not differentiable at one point
16. Let $L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$.
 If L is finite, then (2009)
 (a) $a = 2$ (b) $a = 1$ (c) $L = \frac{1}{64}$ (d) $L = \frac{1}{32}$
17. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$, $\forall x, y \in \mathbb{R}$. If $f(x)$ is differentiable at $x = 0$, then (2011)
 (a) $f(x)$ is differentiable only in a finite interval containing zero
 (b) $f(x)$ is continuous $\forall x \in \mathbb{R}$
 (c) $f'(x)$ is constant $\forall x \in \mathbb{R}$
 (d) $f(x)$ is differentiable except at finitely many points.

18. If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \leq 0, \\ x-1, & 0 < x \leq 1 \\ \ln x, & x > 1 \end{cases}$ then (2011)

- (a) $f(x)$ is continuous at $x = -\frac{\pi}{2}$
 (b) $f(x)$ is not differentiable at $x = 0$
 (c) $f(x)$ is differentiable at $x = 1$
 (d) $f(x)$ is differentiable at $x = -\frac{3}{2}$

19. For every integer n , let a_n and b_n be real numbers. Let function $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by (2012)

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}$$

for all integers n . If f is continuous, then which of the following hold(s) for all n ?

- (a) $a_{n-1} - b_{n-1} = 0$ (b) $a_n - b_n = 1$
 (c) $a_n - b_{n+1} = 1$ (d) $a_{n-1} - b_n = -1$

20. For $a \in \mathbb{R}$ (the set of all real numbers), $a \neq -1$, (JEE Adv. 2013)

$$\lim_{n \rightarrow \infty} \frac{(1^a + 2^a + \dots + n^a)}{(n+1)^{a-1}[(na+1) + (na+2) + \dots + (na+n)]} = \frac{1}{60}.$$

Then $a =$

- (a) 5 (b) 7 (c) $\frac{-15}{2}$ (d) $\frac{-17}{2}$

21. Let $f: [a, b] \rightarrow [1, \infty)$ be a continuous function and let $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined as (JEE Adv. 2014)

$$g(x) = \begin{cases} 0, & \text{if } x < a, \\ \int_a^x f(t) dt, & \text{if } a \leq x \leq b; \\ \int_a^b f(t) dt, & \text{if } x > b. \end{cases}$$

- (a) $g(x)$ is continuous but not differentiable at a
 (b) $g(x)$ is differentiable on \mathbb{R}
 (c) $g(x)$ is continuous but not differentiable at b
 (d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

22. For every pair of continuous functions $f, g: [0, 1] \rightarrow \mathbb{R}$ such that $\max \{f(x) : x \in [0, 1]\} = \max \{g(x) : x \in [0, 1]\}$, the correct statement(s) is (are): (JEE Adv. 2014)

- (a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
 (b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
 (c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$
 (d) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

23. Let $g: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function with $g(0) = 0$,

$$g'(0) = 0 \text{ and } g'(1) \neq 0. \text{ Let } f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and $h(x) = e^{|x|}$ for all $x \in \mathbb{R}$. Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is (are) true?

(JEE Adv. 2015)

- (a) f is differentiable at $x = 0$
 (b) h is differentiable at $x = 0$
 (c) $f \circ h$ is differentiable at $x = 0$
 (d) $h \circ f$ is differentiable at $x = 0$

24. Let $a, b \in \mathbb{R}$ and $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = a \cos(|x^3 - x|) + b|x| \sin(|x^3 + x|)$.

Then f is (JEE Adv. 2016)

- (a) differentiable at $x=0$ if $a=0$ and $b=1$
 (b) differentiable at $x=1$ if $a=1$ and $b=0$
 (c) NOT differentiable at $x=0$ if $a=1$ $b=0$
 (d) NOT differentiable at $x=1$ if $a=1$ and $b=1$

25. Let $f: \left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$ and $g: \left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then (JEE Adv. 2016)

- (a) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
 (b) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
 (c) g is NOT differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$
 (d) g is NOT differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

E Subjective Problems

1. Evaluate $\lim_{x \rightarrow a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$, ($a \neq 0$) (1978)
 2. $f(x)$ is the integral of $\frac{2 \sin x - \sin 2x}{x^3}$, $x \neq 0$, find $\lim_{x \rightarrow 0} f'(x)$ (1979)

3. Evaluate: $\lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$ (1980)

4. Let $f(x+y) = f(x) + f(y)$ for all x and y . If the function $f(x)$ is continuous at $x = 0$, then show that $f(x)$ is continuous at all x . (1981 - 2 Marks)

5. Use the formula $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$ to find

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{(1+x)^{1/2} - 1} \quad (1982 - 2 \text{ Marks})$$

6. Let $f(x) = \begin{cases} 1+x, & 0 \leq x \leq 2 \\ 3-x, & 2 \leq x \leq 3 \end{cases}$ (1983 - 2 Marks)

Determine the form of $g(x) = f[f(x)]$ and hence find the points of discontinuity of g , if any

7. Let $f(x) = \begin{cases} \frac{x^2}{2}, & 0 \leq x < 1 \\ 2x^2 - 3x + \frac{3}{2}, & 1 \leq x \leq 2 \end{cases}$ (1983 - 2 Marks)

Discuss the continuity of f, f' and f'' on $[0, 2]$.

8. Let $f(x) = x^3 - x^2 + x + 1$ and $g(x) = \max\{f(t); 0 \leq t \leq x\}, 0 \leq x \leq 1$ (1985 - 5 Marks)
 $= 3 - x \quad 0 \leq x \leq 2$

Discuss the continuity and differentiability of the function $g(x)$ in the interval $(0, 2)$.

9. Let $f(x)$ be defined in the interval $[-2, 2]$ such that

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x-1, & 0 < x \leq 2 \end{cases}$$

and $g(x) = f(|x|) + |f(x)|$

Test the differentiability of $g(x)$ in $(-2, 2)$. (1986 - 5 Marks)

10. Let $f(x)$ be a continuous and $g(x)$ be a discontinuous function. prove that $f(x) + g(x)$ is a discontinuous function. (1987 - 2 Marks)

11. Let $f(x)$ be a function satisfying the condition $f(-x) = f(x)$ for all real x . If $f'(0)$ exists, find its value. (1987 - 2 Marks)

12. Find the values of a and b so that the function

$$f(x) = \begin{cases} x + a\sqrt{2} \sin x, & 0 \leq x < \pi/4 \\ 2x \cot x + b, & \pi/4 \leq x \leq \pi/2 \\ a \cos 2x - b \sin x, & \pi/2 < x \leq \pi \end{cases}$$

is continuous for $0 \leq x \leq \pi$. (1989 - 2 Marks)

13. Draw a graph of the function $y = [x] + |1-x|, -1 \leq x \leq 3$. Determine the points, if any, where this function is not differentiable. (1989 - 4 Marks)

14. Let $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & x > 0 \end{cases}$ (1990 - 4 Marks)

Determine the value of a , if possible, so that the function is continuous at $x = 0$

15. A function $f: R \rightarrow R$ satisfies the equation $f(x+y) = f(x)f(y)$ for all x, y in R and $f(x) \neq 0$ for any x in R . Let the function be differentiable at $x = 0$ and $f'(0) = 2$. Show that $f'(x) = 2f(x)$ for all x in R . Hence, determine $f(x)$. (1990 - 4 Marks)

16. Find $\lim_{x \rightarrow 0} \{\tan(\pi/4 + x)\}^{1/x}$ (1993 - 2 Marks)

17. Let $f(x) = \begin{cases} \{1 + |\sin x|\}^{a/|\sin x|}; & \frac{\pi}{6} < x < 0 \\ b & x = 0 \\ e^{\tan 2x / \tan 3x} & 0 < x < \frac{\pi}{6} \end{cases}$ (1994 - 4 Marks)

Determine a and b such that $f(x)$ is continuous at $x = 0$

18. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x) + f(y)}{2}$ for all real x and y . If $f'(0)$ exists and equals -1 and $f(0) = 1$, find $f(2)$. (1995 - 5 Marks)

19. Determine the values of x for which the following function fails to be continuous or differentiable: (1997 - 5 Marks)

$$f(x) = \begin{cases} 1-x, & x < 1 \\ (1-x)(2-x), & 1 \leq x \leq 2 \\ 3-x, & x > 2 \end{cases} \text{ Justify your answer.}$$

20. Let $f(x), x \geq 0$, be a non-negative continuous function, and

$$\text{let } F(x) = \int_0^x f(t) dt, x \geq 0. \text{ If for some } c > 0, f(x) \leq cF(x) \text{ for all}$$

$x \geq 0$, then show that $f(x) = 0$ for all $x \geq 0$. (2001 - 5 Marks)

21. Let $\alpha \in R$. Prove that a function $f: R \rightarrow R$ is differentiable at α if and only if there is a function $g: R \rightarrow R$ which is continuous at α and satisfies $f(x) - f(\alpha) = g(x)(x - \alpha)$ for all $x \in R$. (2001 - 5 Marks)

22. Let $f(x) = \begin{cases} x+a & \text{if } x < 0 \\ |x-1| & \text{if } x \geq 0, \end{cases}$ and (2002 - 5 Marks)

$$g(x) = \begin{cases} x+1 & \text{if } x < 0 \\ (x-1)^2 + b & \text{if } x \geq 0, \end{cases} \text{ where } a \text{ and } b \text{ are}$$

non-negative real numbers. Determine the composite function $g \circ f$. If $(g \circ f)(x)$ is continuous for all real x , determine the values of a and b . Further, for these values of a and b , is $g \circ f$ differentiable at $x = 0$? Justify your answer.

Limits, Continuity and Differentiability

23. If a function $f : [-2a, 2a] \rightarrow R$ is an odd function such that $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left hand derivative at $x = a$ is 0 then find the left hand derivative at $x = -a$.
(2003 - 2 Marks)
24. $f'(0) = \lim_{n \rightarrow \infty} n f\left(\frac{1}{n}\right)$ and $f(0) = 0$. Using this find

$$\lim_{n \rightarrow \infty} \left((n+1) \frac{2}{\pi} \cos^{-1}\left(\frac{1}{n}\right) - n \right) \cdot \left| \cos^{-1} \frac{1}{n} \right| < \frac{\pi}{2}$$

(2004 - 2 Marks)
25. If $|c| \leq \frac{1}{2}$ and $f(x)$ is a differentiable function at $x = 0$ given

$$\text{by } f(x) = \begin{cases} b \sin^{-1}\left(\frac{c+x}{2}\right) & , -\frac{1}{2} < x < 0 \\ \frac{1}{2} & , x = 0 \\ \frac{e^{ax/2} - 1}{x} & , 0 < x < \frac{1}{2} \end{cases}$$

Find the value of 'a' and prove that $64b^2 = 4 - c^2$
(2004 - 4 Marks)
26. If $f(x-y) = f(x) \cdot g(y) - f(y) \cdot g(x)$ and
 $g(x-y) = g(x) \cdot g(y) - f(x) \cdot f(y)$ for all $x, y \in R$.
If right hand derivative at $x = 0$ exists for $f(x)$. Find derivative of $g(x)$ at $x = 0$
(2005 - 4 Marks)

F Integer Value Correct Type

DIRECTIONS (Q. 1 and 2) : Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

| | p | q | r | s | t |
|---|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| A | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input checked="" type="radio"/> |
| B | <input type="radio"/> | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| C | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| D | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> |

1. In this questions there are entries in columns I and II. Each entry in column I is related to exactly one entry in column II. Write the correct letter from column II against the entry number in column I in your answer book.
(1992 - 2 Marks)

Column I

- (A) $\sin(\pi[x])$
(B) $\sin(\pi(x-[x]))$

Column II

- (p) differentiable everywhere
(q) nowhere differentiable
(r) not differentiable at 1 and -1

2. In the following $[x]$ denotes the greatest integer less than or equal to x .

Match the functions in **Column I** with the properties in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.
(2007 - 6 marks)

Column I

- (A) $x|x|$
(B) $\sqrt{|x|}$
(C) $x + [x]$
(D) $|x-1| + |x+1|$

Column II

- (p) continuous in $(-1, 1)$
(q) differentiable in $(-1, 1)$
(r) strictly increasing in $(-1, 1)$
(s) not differentiable at least at one point in $(-1, 1)$

DIRECTIONS (Q. 3) : Following question has matching lists. The codes for the list have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

3. Let $f_1 : R \rightarrow R$, $f_2 : [0, \infty) \rightarrow R$, $f_3 : R \rightarrow R$ and $f_4 : R \rightarrow [0, \infty)$ be defined by $f_1(x) = \begin{cases} |x| & \text{if } x < 0, \\ e^x & \text{if } x \geq 0; \end{cases}$

$$f_2(x) = x^2; f_3(x) = \begin{cases} \sin x & \text{if } x < 0, \\ x & \text{if } x \geq 0; \end{cases} \text{ and } f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0, \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0. \end{cases}$$

(JEE Adv. 2014)

List-I

- P. f_4 is
 Q. f_3 is
 R. $f_2 \circ f_1$ is
 S. f_2 is

P Q R S

(a) 3 1 4 2

(c) 3 1 2 4

I Integer Value Correct Type

1. Let $f: [1, \infty) \rightarrow [2, \infty)$ be a differentiable function such that

$f(1) = 2$. If $6 \int_1^x f(t) dt = 3xf(x) - x^3$ for all $x \geq 1$, then the

value of $f(2)$ is (2011)

2. The largest value of non-negative integer a for which

$\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$ is (JEE Adv. 2014)

3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be respectively given by

$f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h: \mathbb{R} \rightarrow \mathbb{R}$ by

List-II

1. Onto but not one-one
 2. Neither continuous nor one-one
 3. Differentiable but not one-one
 4. Continuous and one-one

P Q R S

(b) 1 3 4 2

(d) 1 3 2 4

$$h(x) = \begin{cases} \max \{f(x), g(x)\} & \text{if } x \leq 0, \\ \min \{f(x), g(x)\} & \text{if } x > 0. \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

(JEE Adv. 2014)

4. Let m and n be two positive integers greater than 1. If

$$\lim_{\alpha \rightarrow 0} \left(\frac{e^{\cos(\alpha^n)} - e}{\alpha^m} \right) = -\left(\frac{e}{2}\right) \text{ then the value of } \frac{m}{n} \text{ is}$$

(JEE Adv. 2015)

5. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x \rightarrow 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then

$6(\alpha + \beta)$ equals.

(JEE Adv. 2016)

Section-B

JEE Main / AIEEE

1. $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2x}}$ is

[2002]

- (a) 1 (b) -1
 (c) zero (d) does not exist

2. $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x$

[2002]

- (a) e^4 (b) e^2 (c) e^3 (d) 1

3. Let $f(x) = 4$ and $f'(x) = 4$. Then $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x-2}$ is

given by [2002]

- (a) 2 (b) -2 (c) -4 (d) 3

4. $\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}}$ is

[2002]

- (a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$ (c) $\frac{1}{p} - \frac{1}{p-1}$ (d) $\frac{1}{p+2}$

5. $\lim_{x \rightarrow 0} \frac{\log x^n - [x]}{[x]}$, $n \in \mathbb{N}$, ($[x]$ denotes greatest integer less

than or equal to x)

[2002]

- (a) has value -1 (b) has value 0
 (c) has value 1 (d) does not exist

6. If $f(1) = 1, f'(1) = 2$, then $\lim_{x \rightarrow 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$ is [2002]

- (a) 2 (b) 4 (c) 1 (d) 1/2

7. f is defined in $[-5, 5]$ as [2002]

$f(x) = x$ if x is rational

$= -x$ if x is irrational. Then

- (a) $f(x)$ is continuous at every x , except $x = 0$
 (b) $f(x)$ is discontinuous at every x , except $x = 0$
 (c) $f(x)$ is continuous everywhere
 (d) $f(x)$ is discontinuous everywhere

8. $f(x)$ and $g(x)$ are two differentiable functions on $[0, 2]$ such that $f''(x) - g''(x) = 0$, $f'(1) = 2g'(1) = 4f(2) = 3g(2) = 9$ then $f(x) - g(x)$ at $x = 3/2$ is [2002]

- (a) 0 (b) 2 (c) 10 (d) 5

Limits, Continuity and Differentiability

9. If $f(x+y) = f(x) \cdot f(y) \forall x, y$ and $f(5) = 2, f'(0) = 3$, then $f'(5)$ is [2002]
 (a) 0 (b) 1 (c) 6 (d) 2
10. $\lim_{n \rightarrow \infty} \frac{1+2^4+3^4+\dots+n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1+2^3+3^3+\dots+n^3}{n^5}$ [2003]
 (a) $\frac{1}{5}$ (b) $\frac{1}{30}$ (c) Zero (d) $\frac{1}{4}$
11. If $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is [2003]
 (a) $-\frac{2}{3}$ (b) 0 (c) $-\frac{1}{3}$ (d) $\frac{2}{3}$
12. The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$ is [2003]
 (a) 0 (b) 3 (c) 2 (d) 1
13. Let $f(a) = g(a) = k$ and their n th derivatives $f^n(a), g^n(a)$ exist and are not equal for some n . Further if $\lim_{x \rightarrow a} \frac{f(a)g(x) - f(a) - g(a)f(x) + f(a)}{g(x) - f(x)} = 4$ then the value of k is [2003]
 (a) 0 (b) 4 (c) 2 (d) 1
14. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\left[1 - \tan\left(\frac{x}{2}\right)\right][1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right)\right][\pi - 2x]^3}$ is [2003]
 (a) ∞ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$
15. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then $f(x)$ is [2003]
 (a) discontinuous every where
 (b) continuous as well as differentiable for all x
 (c) continuous for all x but not differentiable at $x = 0$
 (d) neither differentiable nor continuous at $x = 0$
16. If $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b , are [2004]
 (a) $a = 1$ and $b = 2$ (b) $a = 1, b \in \mathbb{R}$
 (c) $a \in \mathbb{R}, b = 2$ (d) $a \in \mathbb{R}, b \in \mathbb{R}$
17. Let $f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is [2004]
 (a) -1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1
18. $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right]$ equals [2005]
 (a) $\frac{1}{2} \sec 1$ (b) $\frac{1}{2} \operatorname{cosec} 1$
 (c) $\tan 1$ (d) $\frac{1}{2} \tan 1$
19. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to [2005]
 (a) $\frac{a^2}{2}(\alpha - \beta)^2$ (b) 0
 (c) $\frac{-a^2}{2}(\alpha - \beta)^2$ (d) $\frac{1}{2}(\alpha - \beta)^2$
20. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h \rightarrow 0} \frac{1}{h} f(1+h) = 5$, then $f'(1)$ equals [2005]
 (a) 3 (b) 4 (c) 5 (d) 6
21. Let f be differentiable for all x . If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then [2005]
 (a) $f(6) \geq 8$ (b) $f(6) < 8$ (c) $f(6) < 5$ (d) $f(6) = 5$
22. If f is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2, x, y \in \mathbb{R}$ and $f(0) = 0$, then $f(1)$ equals [2005]
 (a) -1 (b) 0 (c) 2 (d) 1
23. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \min \{x+1, |x|+1\}$, Then which of the following is true?
 (a) $f(x)$ is differentiable everywhere [2007]
 (b) $f(x)$ is not differentiable at $x = 0$
 (c) $f(x) \geq 1$ for all $x \in \mathbb{R}$
 (d) $f(x)$ is not differentiable at $x = 1$
24. The function $f: \mathbb{R}/\{0\} \rightarrow \mathbb{R}$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ [2007]
 can be made continuous at $x = 0$ by defining $f(0)$ as
 (a) 0 (b) 1 (c) 2 (d) -1

25. Let $f(x) = \begin{cases} (x-1)\sin \frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$ [2008]

Then which one of the following is true?

- (a) f is neither differentiable at $x=0$ nor at $x=1$
 (b) f is differentiable at $x=0$ and at $x=1$
 (c) f is differentiable at $x=0$ but not at $x=1$
 (d) f is differentiable at $x=1$ but not at $x=0$

26. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be a positive increasing function with

$\lim_{x \rightarrow \infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} =$ [2010]

- (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 1

27. $\lim_{x \rightarrow 2} \left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x-2} \right)$ [2011]

- (a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$
 (c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

28. The values of p and q for which the function [2011]

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

$x=0$ is continuous for all x in \mathbf{R} , are

- (a) $p = \frac{5}{2}, q = \frac{1}{2}$ (b) $p = -\frac{3}{2}, q = \frac{1}{2}$
 (c) $p = \frac{1}{2}, q = \frac{3}{2}$ (d) $p = \frac{1}{2}, q = -\frac{3}{2}$

29. Let $f: \mathbf{R} \rightarrow [0, \infty)$ be such that $\lim_{x \rightarrow 5} f(x)$ exists and

$\lim_{x \rightarrow 5} \frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$. Then $\lim_{x \rightarrow 5} f(x)$ equals:

- (a) 0 (b) 1 (c) 2 (d) 3

30. If $f: \mathbf{R} \rightarrow \mathbf{R}$ is a function defined by $f(x) = [x]$

$\cos \left(\frac{2x-1}{2} \right) \pi$, where $[x]$ denotes the greatest integer function, then f is. [2012]

- (a) continuous for every real x .
 (b) discontinuous only at $x=0$
 (c) discontinuous only at non-zero integral values of x .
 (d) continuous only at $x=0$.

31. Consider the function, $f(x) = |x-2| + |x-5|, x \in \mathbf{R}$.

Statement-1: $f'(4) = 0$

Statement-2: f is continuous in $[2, 5]$, differentiable in $(2, 5)$ and $f(2) = f(5)$. [2012]

- (a) Statement-1 is false, Statement-2 is true.
 (b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
 (c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
 (d) Statement-1 is true, statement-2 is false.

32. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to [JEE M 2013]

- (a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

33. $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to: [JEE M 2014]

- (a) $-\pi$ (b) π (c) $\frac{\pi}{2}$ (d) 1

34. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to: [JEE M 2015]

- (a) 2 (b) $\frac{1}{2}$ (c) 4 (d) 3

35. If the function.

$g(x) = \begin{cases} k\sqrt{x+1}, & 0 \leq x \leq 3 \\ mx+2, & 3 < x \leq 5 \end{cases}$ is differentiable, then the value of $k+m$ is: [JEE M 2015]

- (a) $\frac{10}{3}$ (b) 4 (c) 2 (d) $\frac{16}{5}$

36. For $x \in \mathbf{R}$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then:

[JEE M 2016]

- (a) $g'(0) = -\cos(\log 2)$
 (b) g is differentiable at $x=0$ and $g'(0) = -\sin(\log 2)$
 (c) g is not differentiable at $x=0$
 (d) $g'(0) = \cos(\log 2)$

37. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)(n+2)\dots 3n}{n^{2n}} \right)^{\frac{1}{n}}$ is equal to: [JEE M 2016]

- (a) $\frac{9}{e^2}$ (b) $3 \log 3 - 2$
 (c) $\frac{18}{e^4}$ (d) $\frac{27}{e^2}$

38. Let $p = \lim_{x \rightarrow 0^+} \left(1 + \tan^2 \sqrt{x} \right)^{\frac{1}{2x}}$ then $\log p$ is equal to:

[JEE M 2016]

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$
 (c) 2 (d) 1