Chapter 8:

SOLIDS

Focus Points

- ⋆ Cone can be made by rolling a sectorol sheet.While doing so the arc length of the sector becomes the base perimetre of the cons.Area of the sector becomes lateral surface area of the cone. Lateral surface area is also known as curved surface area .
- ⋆ The radius of the sector becomes slant height of the cone . It can be denoted by l for between convenience.
- ⋆ Since arc length of the sector is equal to base perimetre of the cone we can make the relation given below

$$\frac{2\pi l}{360}x = 2\pi r$$

l radius of the sector , x central angle of the sector and r is the radius of the cone , $lx = 360r$.

- ⋆ Area of the sector becomes the lateral surface area of the cone.We can make a formula to calculate lateral surface area of the cone

Curved surface area= Area of the sector

Curved surface area $= \frac{\pi r^2}{360}x = \frac{\pi \times l \times l \times x}{360}$

$lx = 360r$

Curved surface area $= \frac{\pi \times l \times 360r}{360} = \pi rl$

1) A sectoral sheet of central angle $120°$is cut off from a circular sheet of radius 12cm . It is rolled in such a way as to get a cone.
   a) What is the slant height of the cone?
   b) What is the radius of the cone ?
   c) Find the curved surface area of the cone.

Answers

a) Slant height$l = 12$cm

b) $lx = 360r$
$12 \times 120 = 360 \times r, r = 4$ cm

c) Lateral surface area $= \pi rl = \pi \times 4 \times 12 = 48\pi$sq.cm

2) A cone is made by rolling a semicircular metal sheet of radius 10cm
   a) What is the slant height of the cone.
   b) What is the radius of the cone.
   c) Find the curved surface area of the cone.
   d) Base is made by a suitable circular sheet. What is its total surface area ?

Answers

a) slant height$l = 10$cm

b) $lx = 360r$
$10 \times 180 = 360 \times r, r = 5$cm

c) Lateral surface area $= \pi rl = \pi \times 5 \times 10 = 50\pi$sq.cm

d) Total surface area = Lateral surface area +base face $= 50\pi + 25\pi = 75\pi$sq.cm

3) A circular sheet of card board of radius 12cm .It is cut off into two sectors of central angle $120°$ and $240°$.Both of them are rolled into cones.

a) Name the measure coomon to both comes

b) What is the radius of small cone ?

c) What is the radius of the big cone.

d) How radii of the cones are related to the radius of the circular sheet.

Answers

a) slant height$= 12$ cm

b) $lx = 360r_1 \rightarrow 12 \times 120 = 360 \times r_1$
$r_1 = \frac{12 \times 120}{360} = 4$cm

c) $lx = 360r_2 \rightarrow 12 \times 240 = 360 \times r_2$
$r_2 = \frac{12 \times 240}{360} = 8$cm

d) $r_1 + r_2 = 12$.Sum of the base radii of cones is equal to the radius of the circular sheet

4) A sector of central angle $90°$ is cut off from a circular sheet of radius 16cm .It is rolled in such a way as to get a cone.

a) What is the lateral surface are of the cone?

b) What is the radius of the cone?

c) The remaining part of the circular sheet is also rolled to get a cone . What is its base radius?

d) Which cone has more height ? Explain

Answers

a) Area of sector is $\frac{1}{4}$ of the area of circular sheet.
Area of the sector$= \frac{1}{4} \times \pi \times 16^2 = 64\pi$sq.cm

b) $lx = 360r \rightarrow 16 \times 90 = 360 \times r$
$r = \frac{16 \times 90}{360} = 4$cm

c) $lx = 360r \rightarrow 16 \times 270 = 360 \times r$
$r = \frac{16 \times 270}{360} = 12$cm

d) Radius , height and slant height form a right triangle.Slant height of both pyramids are equal.It is equal to the hypotenuse of the triangle.Whenever the hypotenuse remains same , length of one perpendicular side increases according to the decrease of other side. Cone made from the sector of central angle has less height.

5) A cone is made by a sectoral sheet taken from a circular sheet.The slant height of the cone is two times its radius.

a) What is the relation between lateral surface area and base area?

b) If the base perimetre is 20πcm then what will be its lateral surface area ?

c) What is the central angle of this sector?

d) The remaining part is also rolled to get a cone. What is the ratio of the heights of cones so formed

Answers

a) $l = 2r \rightarrow$ lateral surface area $= \pi rl = \pi \times r \times 2r = 2\pi r^2 = 2\times$base area

b) $2\pi r = 20\pi \rightarrow r = 10$cm
$l = 20$cm .Lareral surface area $\pi \times 10 \times 20 = 200\pi$sq.cm

c) $lx = 360r \rightarrow 2r \times x = 360 \times r$
$x = 180°$

d) This is a semicircle. Remaining part is also a semicircle. Ratio of the height is $1 : 1$

6) A cone is made by taking a sector from a circular sheet.The slant height of the cone is 25cm and its radius 110cm

a) What is the radius of the circular sheet?

b) What is the central angle of the sector?

c) What is the central angle of the remaining part?

d) What is the radius of the cone made by rolling the remaining part?

Answers

a) 25cm

b) $lx = 360r \rightarrow 25 \times x = 360 \times 10, x = \frac{360\times 10}{25} = 144°$

c) Central angle of sector $= 360 - 144 = 216°$

d) Radius of the cone $25 - 10 = 15$cm

7) The base perimetre of a cone is 20π cm, slant height 18cm . It is rolled to get a cone.

a) What is the radius of the sector?

b) What is the radius of the cone?

c) What is the central angle of the sector?

d) Find the lateral surface area of the cone?

Answers

a) 18cm

b) $2\pi r = 20\pi, r = 10$cm

c) $lx = 360r \rightarrow 18 \times x = 360 \times 10, x = \frac{360\times 10}{18} = 200°$

d) Lateral surface area $\pi rl = 180\pi$sq.cm

8) A sector of central angle $288°$ and radius 25cm is taken from a circulat sheet .

a) What is the radius of the cone?

b) What is the height of the cone ?

c) Find the lateral surface area of the cone?

d) What is the radius of the cone made by rolling the remaining part?

Answers

a) $lx = 360r \rightarrow 288 \times 25 = 360 \times r, r = \frac{288 \times 25}{360} = 20$ cm

b) $l^2 = h^2 + r^2$, $25^2 = h^2 + 20^2 \rightarrow h^2 = 625 - 400 = 225, h = \sqrt{225} = 15$cm

c) Lateral surface area $\pi rl = \pi \times 20 \times 25 = 500\pi$ sq.cm

d) Radius of the remaining part$25 - 20 = 5$cm

9) Radius of a cone is 5cm, height 12cm

a) what is the slant height of the cone?

b) What is the total surface area of the cone?

c) What is the volume of the cone ?

d) In a cone , radius and height are equal. If the volume and curved surface area are equal then what is its radius ? What is its slant height?

Answers

a) $l^2 = r^2 + h^2 \rightarrow l = \sqrt{r^2 + h^2} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$cm

b) Total surface area $\pi r^2 + \pi rl = \pi \times 5^2 + \pi \times 5 \times 13 = 25\pi + 65\pi = 90\pi$

c) Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 5^2 \times 12 = 100\pi$cubic cm

d) If $r = h$then slant height $l = \sqrt{2}r$ .Volume $= \frac{1}{3} \times \pi \times r^2 \times r = \frac{1}{3}\pi \times r^3$
Total surface area of the cone $= \pi \times r \times \sqrt{2}r$.
$\frac{1}{3}\pi r^3 = \pi r \times \sqrt{2}r \rightarrow r = 3\sqrt{2}$cm
slant height$= \sqrt{2} \times 3\sqrt{2} = 6$cm

10) The base perimetre of a cone is 30π cm, height 20 cm

a) What is the radius and slant height of the cone ?

b) What is the total surface area?

c) Find the volume of the cone?

d) What is the volume of a cyclidrical vessel of radius and height equal to that of the cone.

Answers

a) $2\pi r = 30\pi, r = 15$cm
slant height$l = \sqrt{r^2 + h^2} = \sqrt{15^2 + 20^2} = \sqrt{625} = 25$cm

b) Total surface area $\pi r^2 + \pi rl = 225\pi + 375\pi = 600\pi$sq.cm

c) Volume $= \frac{1}{3}\pi r^2 \times h = \frac{1}{3} \times \pi \times 15^2 \times 20 = 1500\pi$cubic cm

d) Volume becomes 3 times .Volume $= 4500\pi$cubic cm

11) Diametre and height of a cone are equal.

a) What is the relation between radius and slant height?

b) What is the ratio of radius , height and slant height?

c) If the radius is 6 cm then what is its volume?

d) If the radius is 6 cm then what is the total surface area ?

Answers

a) $h = 2r, l = \sqrt{r^2 + (2r)^2} = \sqrt{5r^2} = \sqrt{5}r$

b) $r : h : l = r : 2r : \sqrt{5}r$
$r : h : l = 1 : 2 : \sqrt{5}$

c) If radius is 6cm then height 10cm .Volume $= \frac{1}{3}\pi \times 6^2 \times 10 = 360\pi$cubic cm

d) If radius is 6cm then $l = \sqrt{5} \times 6 = 6\sqrt{5}$cm.
Total surface area $= \pi \times 6^2 + \pi \times 6 \times 6\sqrt{5} = 36 + 36\sqrt{6}$
$= 36(1 + \sqrt{6})$sq.cm

12) Radius of a cone is r and height h.

a) What will be the change in volume if radius and height are doubled?

b) What will be the change in volume if radius is doubled and height is halved?

c) How many solid cones can be made by melting a solid cone of radius 10cm and height 6cm with half the radius and height of the melted cone?

Answers

a) $V = \frac{1}{3}\pi r^2 h.$
radius $2r$, height $2h$ Volume $= \frac{1}{3}\pi(2r)^2 \times (2h) = 8 \times \frac{1}{3}\pi r^2 h = 8V$
Volume becomes 8times

b) If radius is $2r$ and height $\frac{h}{2}$ then
volume $\frac{1}{3}\pi \times (2r)^2 \times \frac{h}{2} = 2 \times \frac{1}{3}\pi r^2 h = 2V$
Volume becomes 2 times .

c) When radius and height become halved then volume become $\frac{1}{8}$part . 8 cones can be made.

13) A conical measuring vessel is made by rolling a sectoral sheet of central angle $288°$ and radius 10cm.

a) What is the radius of the vessel?

b) What is the height of the vessel?

c) What is the capacity of the vessel in litres ?

Answers

a) $lx = 360r \rightarrow 10 \times 288 = 360 \times r$
$r = \frac{10 \times 288}{360} = 8$ cm

b) $h = \sqrt{l^2 - r^2} = \sqrt{10^2 - 8^2} = 6$cm

c) Volume $= \frac{1}{3}\pi \times r^2 \times h = \frac{1}{3} \times \pi \times 8^2 \times 6 = 128\pi$cubic cm
1000cubic cm $= 1$litre
Volume $= \frac{128 \times 3.14}{1000} = 0.4$litre

14) Radius of a cone is 21cm , height 28cm.

a) Calculate slant height.

b) Find the leteral surface area .

c) Calculate the total surface area

d) Calcualte the volume of the cone..

Answers

a) $l^2 = r^2 + h^2, l^2 = 21^2 + 28^2 = 1225, l = \sqrt{1225} = 35$ cm

b) Lateral surface area $= \pi rl = \pi \times 21 \times 35 = 735\pi$ sq.cm

c) Total surface area $= \pi r^2 + \pi rl = \pi \times 21^2 + \pi \times 21 \times 35 = 441\pi + 735\pi = 1176\pi$ sq.cm

d) Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 21^2 \times 28 = 4116\pi$ cubic cm

15) Ratio of radius and height of a cone are $3 : 4$.Volume of the cone is 301.44 cubic cm

a) Find the radius of the cone.

b) Find the height of the cone.

c) Calculate the slant height of the cone.

d) Calculate the lateral surface area of the cone.

Answers

a) $r : h = 3 : 4, r = 3x, h = 4x$
$\frac{1}{3} \times \pi r^2 h = 301.44, \frac{1}{3} \times 3.14 \times (3x)^2 \times (4x) = 301.44$
$x^3 = \frac{301.44 \times 3 \times 7}{22 \times 9 \times 4} = 8$
$x = 2, r = 3x = 6$cm

b) Height $h = 4x = 8$cm

c) $l = \sqrt{r^2 + h^2} = \sqrt{100} = 10$cm

d) Lateral surface area $= \pi \times r \times l = 60\pi$ sq.cm

16) Lateral surface area of a cone is 4070 sq.cm , diametre 70 cm

a) Find the slant height of the cone .

b) Find the height of the cone?

c) Calcualte the volume of the cone.

Answers

a) $\pi rl = 4070, \frac{22}{7} \times 35 \times l = 4070, l = \frac{4070 \times 35}{22 \times 5} = 37$ cm

b) $h = \sqrt{l^2 - r^2} = \sqrt{37^2 - 35^2} = \sqrt{144}, h = 12$cm

c) Volume$= \frac{1}{3} \times \pi \times r^2 \times h = \frac{1}{3} \times \pi \times 35^2 \times 12 = 4900\pi$ cubic cm

17) The height of a cone is 24cm , its lateral surface area 550 sq.cm

a) What is the radius of the cone?

b) Find the height of the cone?

c) Calcualte the volume of the cone?

Answers

a) $\pi rl = 550, \frac{22}{7} \times rl = 550, rl = 175.$
$r^2 l^2 = 175^2, r^2(r^2 + h^2) = 175^2$
If $r^2 = x$ then, $x(x + 24^2) = 175^2$
$x^2 + 576x = 30625, x^2 + 576 - 30625 = 0$
$x = 49 \rightarrow r^2 = 49, r = 7$

b) $rl = 175, l = \frac{175}{7} = 25$cm

c) Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 7^2 \times 24 = 49 \times 8 \times \pi = 392\pi$ cubic cm

18) A semicircular sheet of radius 28cm is rolled in such a way as to get a cone.

a) What is the slant height of the cone?

b) Find the radius of the cone?

c) Find the height of the cone?

d) Calcualte the volume of the cone

Answers

a) Slant height $= 14$cm

b) $lx = 360r \rightarrow 14 \times 180 = 360 \times r, r = \frac{14 \times 180}{360} = 7$cm

c) $h = \sqrt{l^2 - r^2} = \sqrt{14^2 - 7^2 = 147}$. $h = \sqrt{147} = 12.12$ cm

d) Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 7^2 \times 12.12 = 621.6$cubic cm