SAMPLE QUESTION PAPER - 04

Time: 3 Hrs 15 Min **Subject : Mathematics (35)** **Max Marks: 100**

Instructions: (1) The question paper has five Parts namely A, B, C, D and E. Answer all the parts.
(2) Use the graph sheet for the question on linear programming in PART-E.

PART-A

Answer any Ten of the following. (One Mark each) $\mathbf{10 \times 1 = 10}$

1. Define binary operation on a set.
2. A relation R on A={1,2,3}defined by R={(1,1),(2,2),(3,3),(1,2),((2,3)} is not transitive. Why?
3. Write the set of principal values of y, if $sin^{-1}x = y$.
4. Prove that $sin(tan^{-1}x) = \frac{x}{\sqrt{1+x^2}}$, $|x| < 1$
5. If a matrix has 13 elements, what are the possible order it can have?.
6. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, find the value of $|A^{-1}|$.
7. Differentiate $\log(cose^x)$ with respect to x.
8. Find the derivative of $2\sqrt{cot(x^2)}$ with respect to x
9. Find $\int tan^2 x dx$.
10. Evaluate $\int_a^b x\, dx$
11. Define Zero vector or Null vector
12. If the vectors $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ and $4\hat{\imath} - m\hat{\jmath} - 12\hat{k}$ are parallel, find the value of m.
13. Write the direction cosines of negative z-axis.
14. Define optimal solution in the linear programming problem.
15. What is conditional probability?

PART-B

Answer any Ten of the following.(Two Marks each) $\mathbf{10 \times 2 = 20}$

16. Let $*$ be a binary operation on the set of natural numbers N given by $a * b = H.C.F\ of\ a\ and\ b$. Is $*$ associative?
17. Evaluate $sin\left[\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right]$
18. Write $tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$, $|x| < 1$ in simplest form
19. Find the value of x and y in $\begin{bmatrix} x+2y & 2 \\ 4 & x+y \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} = O$ where O is null matrix.
20. Find the equation of line joining $(3,1)\ and\ (9,3)$ using determinants.
21. If $sin^2 x + \cos(xy) = k$, find $\frac{dy}{dx}$.
22. Differentiate x^x with respect to x
23. If $y = \sin(logx)$, then prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{x}$
24. Find point on the curve $y^2 = 4x$ at which tangents are parallel to the line $y = x + 1$.
25. Find $\int sin2x.cos3x dx$.
26. Find $\int \frac{1}{x-\sqrt{x}} dx$.
27. Evaluate $\int_0^{\pi/2} cos^2 x\, dx$.
28. Find the order and degree of differential equation

$\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y^5 = 0$ OR $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

29. Find the projection of vector $\vec{a} = 3\hat{\imath} - 5\hat{\jmath} + 7\hat{k}$ on vector $\vec{b} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k}$.
30. Find the vector of magnitude 8 units in the direction of the vector $\vec{a} = 5\hat{\imath} - \hat{\jmath} + 2\hat{k}$
31. Find the vector equation of plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}$.
32. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other
33. If the probability distribution of X is

X	0	1	2	3	4
P(X)	0.1	k	2k	2k	k

Find the value of k and $P(X \geq 2)$.

PART-C

Answer any Ten of the following. (Three Marks each) **$10 \times 3 = 30$**

34. Show that the relation R in the set $A = \{x/x \in Z,\ \ and\ 0 \leq x \leq 12\}$ given by $R = \{(a, b): a = b\}$ is an equivalence relation.
35. Find the value of x, if $tan^{-1}(2x) + tan^{-1}(3x) = \frac{\pi}{4}\ ,\ \ x > 0$
36. By using elementary transformation find the Inverse of matrix $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$
37. Consider the determinant $\begin{vmatrix} 1 & -2 \\ 3 & 3 \end{vmatrix}$ then find the minor and cofactors of all the elements
38. If $x = a(cos\theta + \theta sin\theta)\ and\ y = a(sin\theta - \theta cos\theta)$ then find $\frac{dy}{dx}$.
39. If $y = x^3 logx$ then find $\frac{d^2y}{dx^2}$
40. Verify Rolle's theorem for the function $f(x) = x^2 + 2,$ in the interval $[-2,2]$
41. Find the point on the curve $x^2 = 2y$ which is nearest to the point (0,5).
42. Evaluate $\int \frac{2x}{(x^2+1)(x^2+2)} dx$
43. Integrate $xtan^{-1}x$ with respect to x.
44. Evaluate $\int_0^1 \frac{tan^{-1}x}{1+x^2} dx$
45. Find the Area of region bounded by the curve $y^2 = x$ and the lines $x = 4\ and\ x = 9$ and the x-axis in first quadrant.
46. Form the differential equation representing family of circles having centre on y-axis and radius 3 units.
47. Find the general solution of differential equation $ylog_e ydx - xdy = 0$.
48. Show that the position vector of the point P which divides the line joining the points A and B having the position vectors $\vec{a}\ and\ \vec{b}$ internally in the ratio $m{:}\,n$ is $\frac{m\vec{b}+n\vec{a}}{m+n}$
49. Find unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}\ and\ \vec{a} - \vec{b}$ where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.
50. Find the equation of a line which passes through the point (1,2,3) and parallel to the vector $3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ both in vector and Cartesian form
51. Probability that A speaks truth is 4/5. A coin is tossed. A reports that is a head appear. Find the probability that it is actually head.

PART-D

Answer any Six of the following. (Five Marks each) $\qquad 6 \times 5 = 30$

52. Show that $f: N \to N$ given by $f(x) = \begin{cases} x-1 & if\ x\ is\ odd \\ x+1 & if\ x\ is\ even \end{cases}$ is both one-one and onto
53. Prove that the function $f: N \to Y$ is defined as $f(x) = 4x + 3$, where, $Y = \{y \in N : y = 4x + 3\ for\ some\ x \in N\}$ is invertible. write the inverse of $f(x)$
54. If $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}\ and\ B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ then
    Show that i) $(A+B)' = A' + B'$ ii) $(A-B)' = A' - B'$
55. Solve the following system of linear equation by matrix method
    $x + y + z = 6,$
    $y + 3z = 11,$
    $x + z = 2y.$
56. If $y = 3\cos(log_e x) + 4\sin(log_e x)$ show that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$
57. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall.
58. Find the integral $\frac{1}{\sqrt{x^2-a^2}}$ w.r.t x and hence evaluate $\int \frac{1}{\sqrt{x^2+6x-7}} dx.$
59. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by the method of integration
60. Solve the differential equation $(1 + x^2)dy + 2xydx = cotxdx,\ (x \neq 0).$
61. Derive the Equation of a plane perpendicular to a given vector and passing through a given point.
62. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is 1/100. What is the probability that he will win a prize
    (i) At least once (ii) Exactly once (iii) At least twice?
63. Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6.
    Find (i) P(A and B) (ii) P(A and not B) (iii) P(A or B) (iv) P(neither A nor B)

PART-E

Answer any One of the following. (Ten Marks) $\qquad 1 \times 10 = 10$

64. a) Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$ and find $\int_0^{2\pi} cos^5x\, dx.$

    b) If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, satisfies the equation $A^2 - 4A + I = O$, then find the inverse of A using this equation, where I is the identity matrix of order 2
65. a) Solve the following problem graphically:
    Minimise and Maximise $Z = 5x + 10y$
    Subject $to\ x + 2y \leq 120$
    $x + y \geq 60$
    $x - 2y \geq 0$
    $x, y \geq 0.$

    b) Find the value of k if $f(x) = \begin{cases} kx^2 & if\ x \leq 2 \\ 3 & if\ x > 2 \end{cases}$ is continuous at $x = 2.$
66. a) Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $tan^{-1}\sqrt{2}$

    b) Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$